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Abstract

This paper presents a multi-period vehicle routing problem for a large-scale production and distribution network. The vehicles must be routed in such a way as to minimize travel and inventory costs over a multi-period horizon, while also taking retailer demands and the availability of products at a central production facility into account. The network is composed of one distribution center and hundreds of retailers. Each retailer has its demand schedule representing the total number of units of a given product that should have been received on a given day. Many high value products are distributed. Product availability is determined by the production facility, whose production schedule determines how many units of each product must be available on a given day. To distribute these products, the routes of a heterogeneous fleet must be determined for a multiple period horizon. The objective of our research is to minimize the cost of distributing products to the retailers and the cost of maintaining inventory at the facility. In addition to considering product availability, the routing schedule must respect many constraints, such as capacity restrictions on the routes and the possibility of multiple vehicle trips over the time horizon. In the situation studied, no more than 20 product units could be carried by a single vehicle, which generally limited the number of retailers that could be supplied to one or two per route. This article proposes a mathematical formulation, as well as some heuristics, for solving this single-retailer-route vehicle routing problem. Extensions are then proposed to deal with the multiple-retailer-route situation.

Key words: Vehicle routing, inventory, synchronization, heterogeneous fleet.

1. Introduction

Manufacturers of seasonal products have traditionally imposed a policy of one order per retailer on their distribution network. All orders are contracted prior to the selling season, before the manufacturers have launched their production and constituted their supply. Generally, it is not possible to reorder during the selling season. The market is continually putting pressure on seasonal manufacturers to allow the possibility of in-season reordering and to reduce the pre-season commitment requirements. For many of these manufacturers, the scope of transformation required to make the supply chain more flexible, coupled with the increased market risk that these transformations would engender, is such that either they resist any move from the single order mode that retailers dislike or they embark on a multi-year transformation program that has no immediate effect but will enable more flexibility in the future.

In this paper, we present an intermediary collaborative relationship in which the manufacturer sticks with the single pre-season order per retailer, but allows each retailer to specify a preferred demand schedule. Traditionally, the contract between the manufacturer and each retailer indicates only the quantity of each product type (SKU: stock-keeping unit) purchased. The delivery dates are determined entirely by the manufacturer within the minimal guidelines set for such essentials as minimal coverage prior to the season launch. In the collaborative relationship studied in this paper, each retailer is allowed to express when he wants to receive each product unit purchased. Subject to the limits of overall production capacity, the manufacturer signs a contract with each retailer, stating the quantity of each product type purchased and the requested demand schedule for each product type. The manufacturer is thus bound by contract to ship each SKU unit on or before the date specified in the demand schedule.

The contracts signed with each retailer are aggregated to form the production requirements. These requirements are transmitted to the assembly center, which develops an optimized production plan that will allow the on-time delivery of the overall quantities of each SKU to each retailer, while also respecting the constraints of the production system and its supply chain. This production plan is then transmitted to the distribution center, which concurrently generates delivery schedules for each retailer and shipping tours for its vehicle fleet, taking into account the information contained in the production plan and the demand schedules contracted with each retailer, as well as the cost structure and constraints of its own transportation and inventory system. In this paper, we focus on developing solution approaches for this synchronized routing problem involving the production-distribution network of a seasonal products manufacturer. First, the situation in which each vehicle delivers

to only one retailer is analyzed, allowing a tractable mathematical model and efficient heuristics to be formulated. Then, these heuristics are generalized to make it possible for vehicles to deliver to more than one retailer.

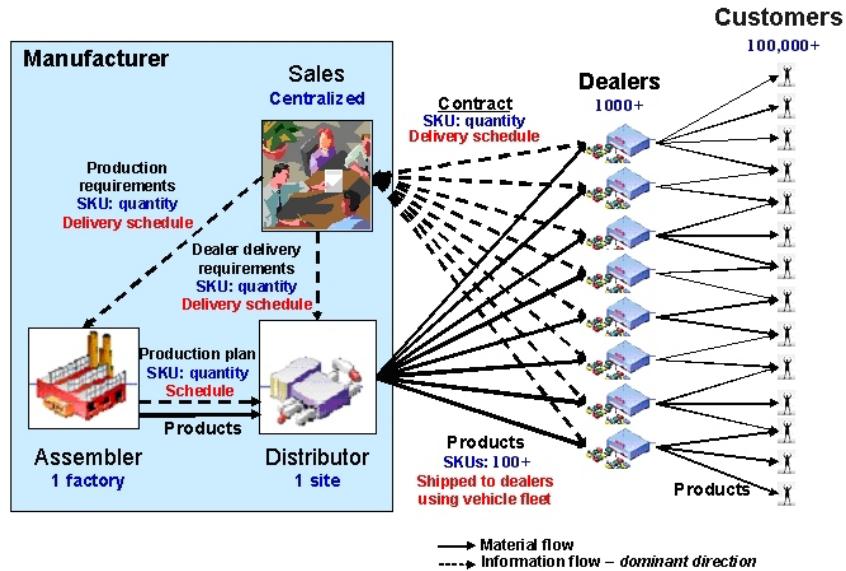


Figure I – The production-distribution network under study

Figure I illustrates the production-distribution network of the manufacturer studied for this article. The manufacturer sells high-value seasonal products to consumers through an independent dealer network that spreads throughout the U.S.A. and Canada. The products, dealers and consumers number respectively in the hundreds, thousands and hundreds of thousands. The individual products include such bulky items as motorcycles, snowmobiles and agricultural equipment costing many thousands of dollars. The products are all assembled in one plant and are distributed from the adjacent distribution center. Due to the limited number of products that can be shipped in any one trailer or container, many shipments are necessary to transport the products to the dealerships, with each delivery route covering only a few dealerships. The manufacturer plans to evolve toward allowing retailers to reorder during the season, but does not think this is feasible for several years given the extent of the transformations necessary in the factory and the supply network, not to mention the marketing, sales and financial operations.

The collaborative relationship described at the beginning of this introductory section is being investigated as a possible transitional alternative. The primary objective of this paper is to examine the

operational implications of such an alternative for the manufacturer, especially in terms of the assignment and routing of product units to dealers. From the central position of the distribution center, caught between the supply conditions imposed by the production schedule and the demand schedule imposed by the dealerships, our job is to optimize the synchronized assignment and routing decisions, working to minimize the inventory and transport costs while respecting both the constraints of the production and demand schedules and those imposed by the vehicle fleet and travel time.

The scientific literature dealing with these kinds of problems can be divided according to the type of problem targeted: vehicle routing problems or supply chain synchronizing problems. The research related to the *Vehicle Routing Problem* (VRP) is extensive, and concerns a variety of problems that are easy to explain but hard to solve, such as the traveling salesman problem (Lawler *et al.* 1985) and the vehicle routing problem itself (Golden & Assad 1988, Toth & Vigo 2002). Many authors – Bodin *et al.* (1983), Laporte (1992a, 1992b, 1993) and Laporte & Osman (1995) – have written articles surveying this topic at length. At the moment, the best exact algorithm is able to solve problems with up to 50 nodes (Toth & Vigo 1998). However, the research into branch-and-cut algorithms is working to find solutions for even bigger problems (Achuthan, Cacetta & Hill 2003). In order to solve large-scale practical instances, many classic heuristics (Laporte *et al.* 2000) and metaheuristics (Cordeau *et al.* 2005 and Gendreau, Laporte & Potvin 2002) have been developed. Reimann, Doerner & Hartl (2004) have proposed what is currently the most efficient metaheuristic, based on ants colony optimization. In addition, many generalizations of the classic VRP have been introduced, which include such practical constraints as time windows (Cordeau *et al.* 2002), pickups and deliveries (Desaulniers *et al.* 2002) and the use of a heterogeneous fleet (Gendreau *et al.* 1999, Renaud & Boctor 2002 and Tarantilis, Kiranoudis & Vassiliadis 2004). An excellent overview of recent research in VRP can be found in Toth & Vigo (2002).

Supply chain management involves making a lot of complex, interconnected decisions (Lakhal *et al.* 2001). The idea of coordination was first presented by Clark & Scarf (1960) in the area of inventory/distribution planning. Erengüç, Simpson & Vakharia (1999) reviewed production/distribution planning by highlighting the relationships between actors. Thomas & Griffin (1996) have presented general problems of coordination between the various levels of the supply chain: replenishment, production and distribution. They emphasized the relationships between operational activities, identifying the pairs supply/purchasing, production/distribution, and inventory/distribution. The profitability of coordination has been demonstrated by Burns *et al.* (1985) in the context of inventory/distribution and by Blumenfeld, Burns & Daganzo (1991) in the context of transportation/production. Chandra &

Fisher (1994) pushed this demonstration farther, showing the advantage of coordinating production and distribution by identifying the relationships between the production/distribution parameters and the related value added. Finally, Gavirneni (2001) illustrated the profitability of cooperation using a network composed of one supplier, one product and many retailers, which is itself a reduced version of the supply chain studied in this paper.

The remainder of this paper is organized as follows. In Section 2, the problem is described and a mathematical model is introduced for a variant of the problem that is solvable for small instances. Heuristics for solving large-scale instances of the problem are presented in Section 3, the problem generator is described in Section 4, and an illustrative example is provided in Section 5. Our computational results are presented in Section 6, and a summary of our findings, as well as our concluding remarks and suggestions for future research, are offered in the last section of the paper.

2. Problem statement and model

Problem definition

The problem addressed in this paper is defined on a directed graph $G(V, A)$, where $V = \{0, \dots, n\}$ is a set of vertices representing the distribution center (0) and n independent retailers (1 to n), and A is the set of arcs linking the vertices of V . A fleet of m_k vehicles of type $k = \{1, \dots, K\}$ is available for transporting products. The values c_{ijk} and t_{ijk} are associated to each arc (i, j) , $i \neq j$, respectively defining the cost and time (including the service time at retailer j) needed to travel from retailer i to retailer j in a type k vehicle. The capacity W_k of type k vehicles is expressed in terms of the maximum number of product units that can be simultaneously carried by the vehicle. It is assumed that one unit of each product occupies one unit of volume in each vehicle.

The distribution center receives an *a priori* production schedule from the factory; this schedule is designed to make delivery to all retailers i feasible. For each product $p = \{1, \dots, P\}$ and time period $t = \{1, \dots, T\}$, the production schedule g_{pt} is defined as the number of units of product p to be made available at the beginning of period t ; g_{pt} is not cumulative and varies from period to period. Finished products p can be stored at

the distribution center, incurring a daily inventory holding cost h_p for each unit of product p kept in stock.

Each retailer i has contracted a demand schedule with the distribution center such that guaranteed quantities of specified products will be delivered to the retailer no later than the negotiated delivery times. Thus, for each product p in period t , the demand schedule d_{pit} specifies the minimum number of units of product p that must arrive at retailer i no later than the beginning of period t ; like g_{pt} , d_{pit} is not cumulative and varies from period to period. There is no bound or cost relative to early delivery and the structure of the production and demand schedules is unrestricted.

The objective is to minimize the total distribution cost, which includes the combined product routing costs and daily inventory holding costs. The delivery schedule must respect the following constraints:

- i) Each vehicle tour starts and ends at the distribution center,
- ii) The number m_k of each type k vehicle used is respected during the horizon,
- iii) The capacity W_k of each type k vehicle is respected on each tour,
- iv) Each product unit can be loaded only once, on a single vehicle during a single tour,
- v) Products are made available for shipment or inventory according to the production schedule,
- vi) Retailers receive product units in the quantities and in the lapse of time specified in their contracted demand schedule.

Mathematical model

Solving the entire problem defined in the previous section optimally is far beyond the current capabilities of mixed integer programming solvers. In addition, formulating the problem mathematically is cumbersome due to a combination of tour feasibility constraints, material balance constraints and vehicles availability constraints. A slightly more restrictive version of the problem, which restricts each tour to a single retailer delivery, can be solved optimally for small instances. Such a restricted version has a simpler mathematical formulation, but retains most of the original problem features. This single-retailer-route version of the problem is formulated below.

Define the following binary variables:

$$Z_{imkt} = \begin{cases} 1, & \text{if some products are shipped from the distribution center to} \\ & \text{the retailer } i \text{ using the } m^{\text{th}} \text{ type } k \text{ vehicle at the beginning of} \\ & \text{the period } t \\ 0, & \text{otherwise} \end{cases}$$

Define the continuous variables:

- s_{pt} : Stock of product p in the distribution center at the end of period t ,
- q_{pimkt} : Quantities of product p leaving the distribution center to be delivered to retailer i using the m^{th} type k vehicle at the beginning of period t .

The studied problem formulation is

$$\text{Minimize} \quad \sum_{i=1}^n \sum_{k=1}^K \sum_{m=1}^{m_k} \sum_{t=1}^T (c_{0ik} + c_{t0k}) Z_{imkt} + \sum_{p=1}^P \sum_{t=1}^T h_p s_{pt} \quad (1)$$

$$\text{subject to} \quad s_{pt} = s_{p(t-1)} + g_{pt} - \sum_{i=1}^n \sum_{k=1}^K \sum_{m=1}^{m_k} q_{pimkt} \quad \forall p, t \quad (2)$$

$$\sum_{k=1}^K \sum_{m=1}^{m_k} \sum_{a=1}^{t-t_{0ik}} q_{pimka} \geq \sum_{a=1}^t d_{pia} \quad \forall p, i, t \quad (3)$$

$$\sum_{p=1}^P q_{pimkt} \leq W_k Z_{imkt} \quad \forall i, m, k, t \quad (4)$$

$$\sum_{i'=1}^n \sum_{t'=t+1}^{t+(t_{0ik}+t_{i'0k})-1} Z_{i'mkt'} \leq 1 - Z_{imkt} \quad \forall i, m, k, t \quad (5)$$

$$\sum_{i=1}^n Z_{imkt} \leq 1 \quad \forall m, k, t \quad (6)$$

$$s_{pt} \geq 0 \quad \forall p, i, t \quad (7)$$

$$q_{pimkt} \geq 0 \quad \forall p, i, m, k, t \quad (8)$$

$$Z_{imkt} \in \{0, 1\} \quad \forall i, m, k, t \quad (9)$$

In this formulation, the objective (1) is to minimize both the transportation cost and the distribution center's inventory holding cost. Constraint set (2) insures material balance by adjusting the distribution center's stock according to incoming production and outgoing shipments. Constraint set (3) assures that the contracted retailer demand schedule is satisfied.

Constraint set (4) limits vehicle loads to a specified vehicle capacity for each vehicle tour. Constraint set (5) insures that vehicles must have finished their previous tour before starting on a new tour. Constraint set (6) imposes a limit of one delivery per retailer per tour. Constraint sets (7) and (8) respectively require inventory and shipped quantities to be non-negative. Finally, constraint set (9) forces binary conditions on the Z_{imkt} variables.

The above formulation has such a high degree of complexity that, currently, optimal solutions are possible for only small instances of the problem. There are four reasons for this complexity. First, the number of binary variables grows rapidly, in proportion to the number of retailers, trucks and periods involved. Second, there is an intricate operational relationship between the routing and inventory decision variables, meaning that minor route modifications have a direct impact on product inventory, which in turn has an impact on other routes, and so on. Third, the arbitrary production schedule imposed as input to the problem interacts with this complex routing/inventory relationship. Fourth, the goals of minimizing inventory costs and minimizing transportation costs are basically contradictory, in that minimizing inventory cost usually entails shipping products as soon as they become available, while minimizing transportation costs usually means postponing shipments so as to transport full loads.

Section 5 illustrates the difficulty of solving small instances of this problem optimally. Note that the situation can be modeled to allow vehicles to deliver to up to two retailers per tour by using many more variables and constraints. However, even this small generalization proved impossible to solve for the small instances in the experiment. Clearly, given that real-life problems involve orders of magnitude much larger than these small instances, solving such problems is currently impossible. The next section introduces some heuristics that can be used to solve the problem more rapidly, with up to near-optimal results.

3. Heuristics

In this section, four heuristics for the single-retailer-route version of the problem are proposed: the Earliest Due Date (EDD) heuristic, the Demand by Demand Increase/Advance (DDIA) heuristic, the Demand by Demand Advance/Increase (DDAI) heuristic and the Demand Advance/Demand Increase (DADI) heuristic. Then, two heuristics allowing deliveries to multiple retailers per tour are introduced; these heuristics are modified versions of EDD and DADI, called EDD2 and DADI2.

3.1. Earliest due date heuristic for the single-retailer-route problem

In the Earliest Due Date (EDD) heuristic, deliveries are made to all retailers as far in advance of their demand schedules as possible. This heuristic prioritizes the retailers with delivery demands in the current period, where the current period is iteratively advanced throughout the horizon. Globally, this heuristic works as follows.

Step 1 Initialization of latest delivery schedule

The latest delivery schedule (LDS) is planned for each retailer i . The LDS corresponds to the latest moment at which a vehicle can leave the distribution center and still arrive at the retailer on time. This is done by moving each retailer demand ahead by $(t_{0i} + 1)$ periods, which is the duration of the tour to retailer i . Let $lds_{pit} = d_{pi(t+t_{0i}+1)}$, $\forall p, t$, be the quantity of product p to be shipped to retailer i at the beginning of period t . Let the period counter $t_1 = 1$.

Step 2 Selection of the shipping period

This step examines each period t_1 and tries to move any demand to a preceding period t_2 ($t_2 \leq t_1$). For period t_1 , all retailers whose LDS requires a delivery (those having a $lds_{pit_1} > 0$) are considered in order, and the following six substeps are repeated.

- a) Set $t_2 = t_1$.
- b) If the distribution center's inventory of all products in period t_2 can respond to the retailer's demand lds_{pit_1} (i.e., if $s_{pt_2} \geq lds_{pit_1}$, $\forall p$) and if a truck is available, then set $t_2 = t_2 - 1$.
- c) Repeat substep b) until t_2 does not change or $t_2 = 0$.
- d) If the demand cannot be delivered in period t_2 , let $t_2 = t_2 + 1$.
- e) Save the demand of product p in the delivery schedule (DS) of retailer i at period t_2 (i.e., $ds_{pit_2} = ds_{pit_2} + lds_{pit_1}$, $\forall p$). This quantity now corresponds to a shipment to retailer i .
- f) Update distribution center stock quantities (i.e., $s_{pt_2} = s_{pt_2} - lds_{pit_1}$, $\forall p$).

Step 3 Period update

The period is updated by setting $t_1 = t_1 + 1$; Step 2 is repeated until $t_1 > T$ or all demands are saved in the DS.

Step 4 Assignment of demands to trucks

Starting with period $t_1 = 1$, all retailers to whom the DS requires the delivery of at least one product are considered in order. The demand is assigned to the smallest truck that can handle the total quantity of the delivery demand; the delivery begins in period t_1 . This truck is now unavailable until $t_1 + t_{0i} + t_{i0}$. Let $t_1 = t_1 + 1$. This step is repeated until $t_1 > T$ or all demands are included in the delivery plan.

This first heuristic is simple and is able to maximize early retailer deliveries; however, it also has the potential to permit vehicles that are not fully loaded to leave on delivery tours. Our second kind of heuristic deals with this limitation.

3.2. Three heuristics that manipulate slack stock to solve the single-retailer-route problem

To optimize the use of space in vehicle tours, Chandra & Fisher (1994) proposed the idea of combining two deliveries going to a retailer i . In heuristics based on this idea, we exploit the idea of *slack stock* to better manage use of truck capacity. For a specific product and period, the slack stock is the minimum value of the residual inventory for all remaining periods. Basically, for each period, the distribution center evaluates the amount of slack stock for each product that can be delivered early without causing backorders for the other retailers, and then draws on this stock to fill the available space in the truck.

Figure II presents sample data for a slack stock calculation involving a network with one product delivered to three retailers. This figure shows the production schedule and the latest delivery schedule of the three retailers. The slack stock for the first period (bottom of Figure II) is calculated as follows. For each period, the residual inventory is evaluated by adding the residual inventory for the preceding period to the production quantity and then subtracting the period demand (the classic stock balance equation). Figure II shows six units of slack stock, which is the minimum residual inventory left over after deliveries in period 1. This value can be used to update the delivery schedule.

If, as shown in Figure III Part A, the six units are sent to retailer A in period 1, this retailer's shipping quantity is increased to ten units for period 1. The available slack stock for period 1 is then re-evaluated, revealing a 0 value, which means that no other delivery can be increased during period 1 without causing an inventory shortage.

Next, the slack stock for period 2 is evaluated. As is shown in Part B of Figure III, the slack stock value for period 2 is two units. Since only retailer B has requested a delivery during this period, this retailer's delivery schedule can be increased from three to five units (Figure III Part C). This process is repeated for all 10 periods.

	Period	1	2	3	4	5	6	7	8	9	10
Production schedule											
		14	5	0	0	30	0	0	0	9	0
Latest delivery schedule											
Retailer A		4	0	0	0	8	0	4	0	8	0
Retailer B		0	3	0	0	4	0	0	0	0	0
Retailer C		4	0	0	0	0	8	4	0	0	0
Total		8	3	0	0	12	8	8	0	8	0
Residual inventory											
		6	8	8	8	26	18	10	10	11	11
		slack stock period 1									
											6

Figure II – Initial schedules for a slack stock calculation

	Delivery schedule update											
A	Retailer A	10	0	0	0	2	0	4	0	8	0	
	Retailer B	0	3	0	0	4	0	0	0	0	0	
	Retailer C	4	0	0	0	0	8	4	0	0	0	
	Total	14	3	0	0	6	8	8	0	8	0	
	Residual inventory											
		0	2	2	2	26	18	10	10	11	11	
		slack stock period 1										0
B	Residual inventory											
		2	2	2	26	18	10	10	11	11		
		slack stock period 2										2
C	Delivery schedule update											
	Retailer A	10	0	0	0	2	0	4	0	8	0	
	Retailer B	0	5	0	0	2	0	0	0	0	0	
	Retailer C	4	0	0	0	0	8	4	0	0	0	
	Total	14	5	0	0	4	8	8	0	8	0	
	Residual inventory											
		0	0	0	26	18	10	10	11	11		
		slack stock period 2										0

Figure III – Possible results for the above slack stock calculation

Based on the slack stock idea, we propose three other heuristics for the single-retailer-route problem. The DDIA (Demand by Demand Increase/Advance) heuristic increases the quantity of each demanded delivery, one by one, and then advances the delivery as much as possible. The DDAI (Demand by Demand Advance/Increase) heuristic works in the opposite way. One by one, the demanded delivery dates are moved forward and then the quantity to be delivered is increased. The DADI (Demand Advance/Demand Increase) heuristic simply increases the quantities of all the demands and then moves them all forward in time. The DDIA heuristic works as follows.

Step 1 Initialization of latest delivery schedule

The latest delivery schedule (LDS) is planned for each retailer i . This is done by advancing each retailer's demand by $(t_{0i} + 1)$ periods, which is the duration of the trip to retailer i . Let $lds_{pit} = d_{pi(t+t_{0i}+1)}$, $\forall p, t$.

In Steps 2 and 3, the slack stock for each product and period is calculated, and then this stock is assigned to retailers as early as possible. Starting with period $t_1 = 1$, Steps 2 and 3 are implemented one after the other for each retailer expecting a delivery according to the LDS (those having a $lds_{pit_1} > 0$).

Step 2 Increasing demands

- a) The slack stock for each product is evaluated:

$$\min_{t \in \{t_1, \dots, T\}} \left(s_{p(t-1)} + g_{pt} - \sum_{i=1}^N lds_{pit} \right), \quad \forall p.$$

- b) The demand of the current retailer is upgraded for all products by the minimum value, either the slack stock or the available space on the truck. A bigger truck is allowed only if all demands for a future period can be moved ahead, thus saving time by avoiding a trip to this retailer.
- c) Steps a) and b) are repeated until there is no slack stock or the truck is full.

This heuristic continues by adopting steps 2, 3 and 4 of EDD heuristic as its steps 3, 4 and 5.

The two other heuristics, DDAI (Demand by Demand Advance/Increase) and DADI (Demand Advance/Demand Increase), are variants of DDIA heuristic. Though the DDAI heuristic is essentially the same as the DDIA heuristic, it reverses the order of DDIA's second and third steps. The DADI heuristic, on the other hand, begins by advancing deliveries to all

retailers and then tries to increase the quantities of the deliveries using the slack stocks. It follows the steps of the DDIA heuristic out of order: 1, 2, 4, 3 and 5.

3.3. Delivering to many retailers on a single tour

For certain instances, delivery trucks are sometimes not used to their full capacity. In order to avoid this situation, we propose the following procedure, which tries to combine deliveries to many retailers in the same tour. This procedure is applied to a feasible complete solution in which each retailer on a single-retailer-route is visited individually.

For each period, every possible pair of tours is examined to determine whether combining them into a single route will save money. The savings associated with delivering to retailers on routes r_1 and r_2 by combining the two tours in a larger route r_3 is defined as

$\delta_{12} = Z(r_1) + Z(r_2) - Z(r_3)$, where $Z(r)$ is the total routing cost of delivering to retailers on route r (i.e., a traveling salesman problem, see Laporte 1992a). This is a simple generalization of Clarke & Wright's classic algorithm (1964). In our procedure, the first feasible savings is implemented. Regrouping two tours can be done if and only if the total demand for all the retailers on the resulting route respects the truck capacity, and if all retailers can still receive their deliveries on time. At the end of this procedure, the resulting routes can be optimized using any traveling salesman problem algorithm, such as Helsgaun's state-of-the-art algorithm (2000), for example. In practice, in the context studied here, the combined routes rarely permit deliveries to more than four retailers, which makes the resulting problem fairly straightforward.

The procedure described above, coupled with the single-retailer-route heuristics, generates the multiple-retailer-route heuristics. Thus, EDD2 and DADI2 are the multiple-retailer-route version of the EDD and DADI heuristics. We chose to apply the procedure to these two heuristics because, of the four described in sections 3.1 and 3.2, EDD and DADI generate the best results for the single-retailer-route.

4. Problem generator

To evaluate the relative performance of each heuristic, two sets of 30 instances were generated randomly. These sets, A and B, correspond respectively to small and large instances. Table 1 presents the characteristics of each set. For both sets, each day is divided into two planning periods. The instances within each set differ in terms of the underlying average preferred delivery frequency for each retailer, which is the average number of days between two consecutive deliveries according to the retailer's contracted demand schedule. The instances

also differ in terms of the imposed production schedule, determined from the average production for each product and the average number of days between two consecutive product batches.

Table 1 – The characteristics of the instances

	Set A (Small instances)	Set B (Large instances)
Number of products (p)	2	3
Number of retailers (N)	3	75
Number of truck types (k)	2	2
Number of each type of truck ($m(k)$)	5	75
Planning horizon (days)	5	30
Period/day	2	2
Planning horizon in period (T)	10	60
<i>Retailer information</i>		
Demand (units/day)	[1; 10]	[1; 10]
Preferred delivery frequency (days)	[1; 5]	[1; 5]
<i>Production information</i>		
Safety factor (%)	10	10
Production cycle (days)	[0.5; 5]	[0.5; 5]

The location of all sites – the factory, the distribution center and the retailers – were intentionally restricted to an area measuring 1,200 km (latitude) by 1,000 km (longitude). Both the factory and the distribution center were situated in the south-east (i.e., between 1,000 km and 1,100 km in latitude and 100 km and 200 km in longitude) on the same site, and the retailers were randomly distributed throughout the entire area. All distances were calculated using Euclidian metrics.

In all instances, vehicles are assumed to be truck-trailer units. The vehicle fleet includes two vehicle types, 1 and 2, whose respective capacities are 16 and 20 units. All vehicles are considered to move at an average speed of 80 km/h. The traveling costs depend on the type of vehicle. Specifically, type 1 vehicles incur both a fixed cost of \$100 and a variable cost, set randomly between \$1 and \$1.10/km. For type 2 vehicles, the fixed cost is \$110 and the variable cost is between \$1.10 and \$1.20/km.

The distribution center's holding costs for each product were randomly set between \$1 and \$1.50/unit-day, based on an average product cost of \$6,000 capitalized at 8%. The retailers' daily demand was randomly generated within the limits presented in Table 1 and then randomly allocated to the different products. The generator then determined a preferred delivery frequency (e.g. every four days) and the retailer demand for these days was combined in a single preferred delivery.

The production schedule for each product was constructed according to a cyclic policy. For example, if a product's production cycle is five days, then the product is produced every five days. The production lot is equal to the accumulated demand between two consecutive production lots, plus a preset safety factor.

5. Illustrative example

To illustrate the heuristics, we selected the most difficult generated instance from set A (instance 12, detailed computational results will be presented in Section 6). Applying model (1) to (9) to this instance generates 922 variables, including 300 binary variables, and 620 constraints. Using Ilog Cplex 9.0 on a Pentium III 1.26 Ghz computer with 1,800 MB of RAM, the problem is optimally solved in 50 minutes. The optimal solution has a global cost of \$12,036.52. All heuristics solve the problem almost instantaneously on a slower Pentium III, 750 MHz, 128 MB of RAM computer. The solution values of EDD, DDIA, DDAI, DADI, EDD2 and DADI2 are respectively \$14,579; \$14,549; \$14,605; \$14,518; \$13,809 and \$14,518. These values correspond to optimality gaps of 17.44%, 17.27%, 17.59%, 17.09%, 12.84% and 17.09% respectively.

Table 2 shows the production schedule for each product. Tables 3, 4 and 5 present the latest delivery schedule (Demand row), the optimal delivery schedule and the delivery schedules produced by the heuristics for each of the different retailers. Table 6 describes the evolution of inventory levels. Finally, Figures IV and V illustrate the use of the vehicle fleet. The grey cells correspond to the periods when vehicles are used and the number in these cells indicates the retailer who receives a delivery. For example, in the optimal solution presented in Figure IV, the first type 1 vehicle is used in periods 1 and 2 to deliver products to the retailer 1 (denoted R1). This same vehicle is also used to supply the same retailer during periods 5 and 6.

Table 2 – Production schedule

Product	Period										Total
	1	2	3	4	5	6	7	8	9	10	
1	56	0	0	0	0	0	0	0	0	0	56
2	18	0	0	0	49	0	0	0	9	0	76

Table 3 – Delivery schedule for retailer 1

Product	Period										Total
	1	2	3	4	5	6	7	8	9	10	
Demand	1	4		4		4		4		0	16
	2	4		4		4		4		8	24
Optimal	1	8			8				2		18
	2	8			8				10		26
EDD	1	8			8				0		16
	2	8			8				8		24
DDIA	1	12			6				0		18
	2	8			14				10		32
DDAI	1	12			5				3		20
	2	8			11				13		32
DADI	1	8			8				0		16
	2	8			8				16		32
EDD2	1	8			8				0		16
	2	8			8				8		24
DADI2	1	8			8				0		16
	2	8			8				16		32

Table 4 – Delivery schedule for retailer 2

Product	Period										Total
	1	2	3	4	5	6	7	8	9	10	
Demand	1				8		8				16
	2				24		0				24
Optimal	1				16						16
	2				24						24
EDD	1	8			8						16
	2	0			24						24
DDIA	1	8			8						16
	2	0			24						24
DDAI	1	11			5						16
	2	0			24						24
DADI	1	14			2						16
	2	2			22						24
EDD2	1	8			8						16
	2	0			24						24
DADI2	1	14			2						16
	2	2			22						24

Table 5 – Delivery schedule for retailer 3

Product	Period										Total
	1	2	3	4	5	6	7	8	9	10	
Demand	1	8		8		4	0				20
	2	4		4		8	4				20
Optimal	1	22			0						22
	2	10			16						26
EDD	1	16			4						20
	2	8			12						20
DDIA	1	22			0						22
	2	10			10						20
DDAI	1	20			0						20
	2	10			10						20
DADI	1	24			0						24
	2	8			12						20
EDD2	1	16			4						20
	2	8			12						20
DADI2	1	24			0						24
	2	8			12						20

Table 6 – Inventory levels

Product	Period										Total
	1	2	3	4	5	6	7	8	9	10	
Optimal	1	26	26	26	26	2	2	2	2	0	112
	2	0	0	0	0	1	1	1	1	0	4
EDD	1	24	24	24	24	4	4	4	4	4	120
	2	2	2	2	2	7	7	7	7	8	52
DDIA	1	14	14	14	14	0	0	0	0	0	56
	2	0	0	0	0	1	1	1	1	0	4
DDAI	1	13	13	13	13	3	3	3	3	0	64
	2	0	0	0	0	4	4	4	4	0	16
DADI	1	10	10	10	10	0	0	0	0	0	40
	2	0	0	0	0	7	7	7	7	0	28
EDD2	1	24	24	24	24	4	4	4	4	4	120
	2	2	2	2	2	7	7	7	7	8	52
DADI2	1	10	10	10	10	0	0	0	0	0	40
	2	0	0	0	0	7	7	7	7	0	28

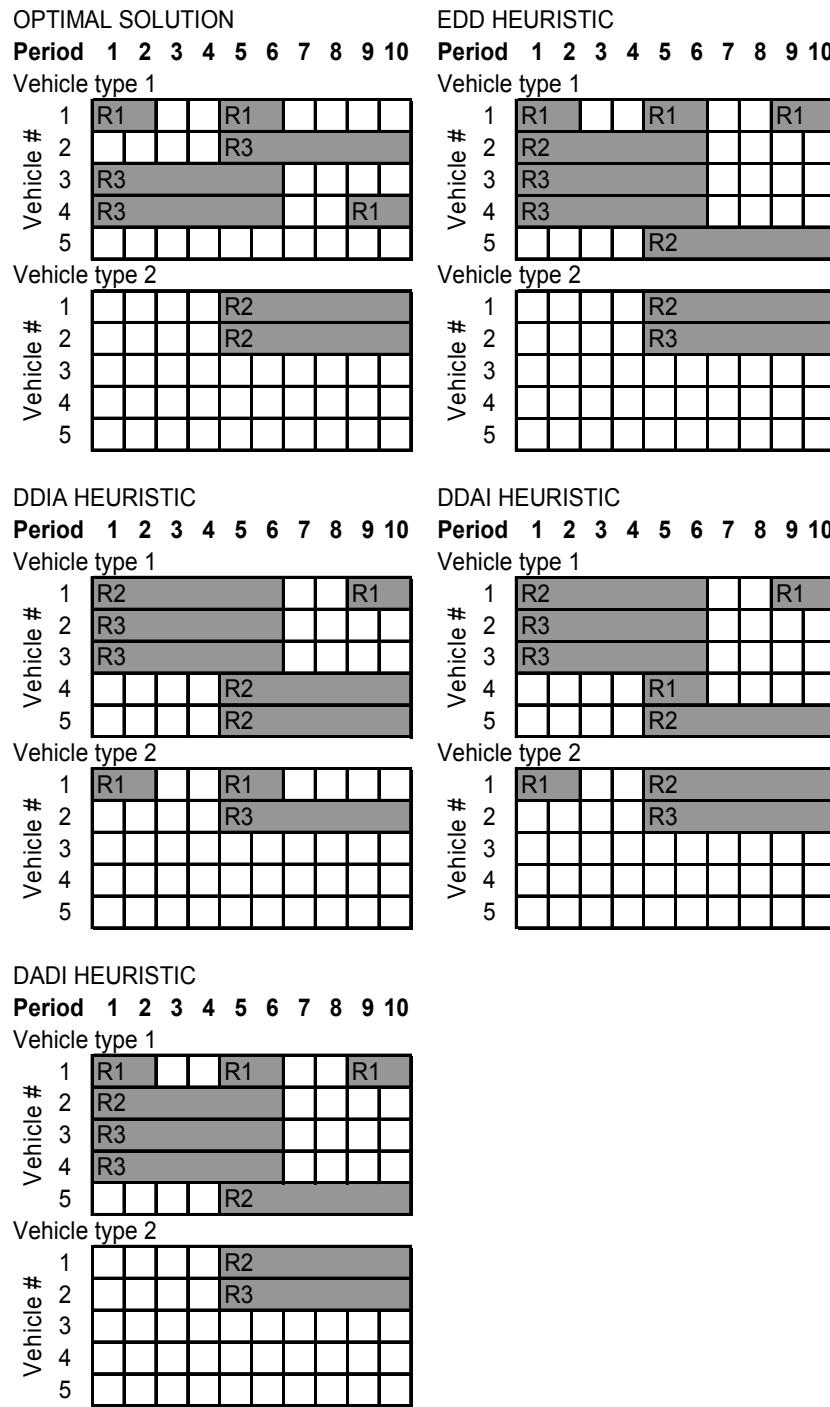


Figure IV – Fleet use given by single-retailer-route heuristics and optimal solution

EDD2 HEURISTIC											DADI2 HEURISTIC										
Period	1	2	3	4	5	6	7	8	9	10	Period	1	2	3	4	5	6	7	8	9	10
Vehicle type 1											Vehicle type 1										
Vehicle #	1	R1			R1				R1		Vehicle #	1	R1			R1				R1	
1	R1										2	R2									
2	R3 + R2										3	R3									
3	R3										4	R3									
4					R2						5				R2						
5					R2						Vehicle type 2										
Vehicle type 2	1				R3						Vehicle #	1				R2					
1					R3						2				R3						
2											3										
3											4										
4											5										
5																					

Figure V – Fleet use for multiple-retailer-route heuristics

The DADI heuristic generates the single-retailer-route solution closest to the optimal solution, with an optimality gap of 17.09%. In the optimal solution, retailer 2 is supplied only in period 5 while all the heuristic solutions make two deliveries, always in periods 1 and 5, as shown in Table 4 and Figure IV. For this instance, the heuristics attempted to reduce inventory cost by adding a delivery to retailer 2 in period 1, which leads to a bad routing decision that ends up increasing the overall cost. Figure V shows the pertinence of the route heuristics. In this example, the solution produced with the EDD2 heuristic uses one vehicle less than the EDD, while the solution produced by the DADI2 heuristic is identical to the DADI solution. As shown in Table 6, the various heuristics all result in different inventory decisions.

6. Computational results

This section first presents the computational results for the instances in set A. These instances are small enough to be solved to optimality using the mathematical model presented in Section 2. These optimal solutions can be used to evaluate the exact empirical performance of each of the four single-retailer-route heuristics: EDD, DDIA, DDAI and DADI. Then, based on the optimal single-retailer-route solutions, the benefits of delivering to many retailers on the same tour can be evaluated, using heuristics EDD2 and DADI2. The same computations were then done for the set B instances; the results are given in Section 6.2. All the heuristics were coded on Visual Basic 6.0 and run under a Windows XP system with a Pentium III, 750MHz, 128 MB of RAM.

6.1. Results for problem set A

Table 7 presents the detailed computational results for the instances in set A. Solution values (Z) and the deviation from the optimal solutions (%) are given for each single-retailer-route heuristic. These figures are also reported for the best heuristic solution found for each instance (BEST). The last column presents the optimal solution values obtained by solving the mathematical models using Ilog Cplex 9.0. The average computing time for these optimal solutions is 12,826 seconds.

Table 7 – Single-retailer-route results for set A instances

Instance	EDD		DDIA		DDAI		DADI		BEST		OPTIMAL
	Z	%	Z	%	Z	%	Z	%	Z	%	Z
1	11,619	0.22	11,613	0.16	11,613	0.16	11,613	0.16	11,613	0.16	11,593
2	14,615	2.61	16,001	11.04	16,001	11.04	16,000	11.04	14,615	2.61	14,234
3	13,882	0.33	13,849	0.09	13,849	0.09	14,030	1.38	13,849	0.09	13,837
4	9,526	0.44	9,487	0.03	9,503	0.20	9,487	0.03	9,487	0.03	9,484
5	10,173	0.45	10,317	1.83	10,317	1.83	11,182	9.43	10,173	0.45	10,128
6	12,834	0.41	12,806	0.19	12,812	0.24	12,806	0.19	12,806	0.19	12,781
7	10,599	0.37	10,682	1.14	10,583	0.22	10,682	1.14	10,583	0.22	10,560
8	11,761	0.22	12,818	8.45	12,943	9.33	13,341	12.04	11,761	0.22	11,735
9	11,342	9.84	12,746	19.78	11,749	12.96	11,288	9.41	11,288	9.41	10,226
10	6,605	0.25	6,588	0.00	6,588	0.00	6,588	0.00	6,588	0.00	6,588
11	12,331	0.26	12,300	0.01	12,300	0.01	12,300	0.01	12,300	0.01	12,299
12	14,580	17.44	14,550	17.27	14,605	17.59	14,518	17.09	14,518	17.09	12,037
13	16,572	0.26	18,226	9.31	18,226	9.31	18,236	9.36	16,572	0.26	16,529
14	13,989	0.25	16,025	12.92	16,025	12.92	13,956	0.02	13,956	0.02	13,954
15	9,335	0.43	9,781	4.97	9,781	4.97	9,781	4.97	9,335	0.43	9,295
16	8,526	12.67	7,499	0.71	7,499	0.71	8,479	12.19	7,499	0.71	7,446
17	10,282	0.45	11,299	9.41	10,342	1.03	10,342	1.03	10,282	0.45	10,235
18	9,673	3.40	9,452	1.14	9,452	1.14	9,519	1.84	9,452	1.14	9,344
19	7,143	0.97	8,904	20.55	8,904	20.55	7,138	0.91	7,138	0.91	7,074
20	4,845	1.13	5,704	16.01	5,712	16.14	4,816	0.53	4,816	0.53	4,790
21	14,645	0.52	15,249	4.46	15,249	4.46	15,260	4.53	14,645	0.52	14,569
22	6,965	0.39	6,944	0.09	6,950	0.18	6,944	0.09	6,944	0.09	6,938
23	10,646	0.45	12,180	12.99	12,204	13.15	10,847	2.29	10,646	0.45	10,599
24	8,253	2.67	9,923	19.05	9,923	19.05	8,228	2.38	8,228	2.38	8,032
25	18,507	8.74	16,967	0.46	16,967	0.46	16,958	0.40	16,958	0.40	16,890
26	12,228	0.34	12,188	0.01	12,188	0.01	12,187	0.00	12,187	0.00	12,187
27	8,625	0.29	8,645	0.52	8,645	0.52	8,645	0.52	8,625	0.29	8,600
28	10,675	0.23	12,930	17.63	12,922	17.57	11,607	8.23	10,675	0.23	10,651
29	6,025	1.71	5,978	0.93	5,978	0.93	5,983	1.01	5,978	0.93	5,922
30	14,173	0.25	16,354	13.55	16,354	13.55	14,146	0.06	14,146	0.06	14,137
Average deviation	2.27		6.82		6.34		3.74		1.34		
Average solution time (sec.)	0.004		0.009		0.008		0.007				12,826

Please note that, respectively, the EDD, DDIA, DDAI and DADI heuristics generate the best heuristic solution 10, 9, 8 and 15 times out of 30. However, although the DADI heuristic generates the largest number of

best solutions, its average deviation above the optimal solution (3.74%) is higher than the deviation produced with EDD (2.27%). By selecting the best solution over all the heuristics for each instance, the average deviation above the optimum can be reduced to 1.34%. Given that all the heuristics can be run within a fraction of a second, this strategy is not time consuming, and the results are promising. For the best obtained solutions, for example, a deviation less than 0.10% is obtained 8 times and optimal solutions are obtained twice.

Table 8 – Multiple-retailer-route results for set A instances

Instance	EDD2			DADI2		
	Z	IMP(EDD)	IMP(OPT)	Z	IMP(DADI)	IMP(OPT)
1	11,619	0.00	0.22	11,613	0.00	0.16
2	14,615	0.00	2.61	14,602	-9.58	2.52
3	13,882	0.00	0.33	14,030	0.00	1.38
4	9,526	0.00	0.44	9,487	0.00	0.03
5	9,437	-7.80	-7.32	10,446	-7.05	3.04
6	11,979	-7.14	-6.70	11,951	-7.16	-6.95
7	10,599	0.00	0.37	10,682	0.00	1.14
8	11,761	0.00	0.22	13,341	0.00	12.04
9	9,095	-24.71	-12.44	11,288	0.00	9.41
10	6,605	0.00	0.25	6,588	0.00	0.00
11	10,936	-12.75	-12.46	10,905	-12.79	-12.78
12	13,810	-5.57	12.84	14,518	0.00	17.09
13	14,678	-12.90	-12.61	18,236	0.00	9.36
14	12,557	-11.40	-11.12	12,524	-11.43	-11.42
15	9,335	0.00	0.43	9,781	0.00	4.97
16	8,526	0.00	12.67	8,479	0.00	12.19
17	10,282	0.00	0.45	10,342	0.00	1.03
18	9,673	0.00	3.40	9,519	0.00	1.84
19	7,143	0.00	0.97	7,138	0.00	0.91
20	4,845	0.00	1.13	4,242	-13.54	-12.94
21	14,645	0.00	0.52	15,260	0.00	4.53
22	6,965	0.00	0.39	6,944	0.00	0.09
23	10,646	0.00	0.45	10,847	0.00	2.29
24	5,929	-39.18	-35.46	6,992	-17.68	-14.88
25	18,507	0.00	8.74	16,958	0.00	0.40
26	11,181	-9.36	-8.99	12,187	0.00	0.00
27	8,625	0.00	0.29	8,645	0.00	0.52
28	10,675	0.00	0.23	11,607	0.00	8.23
29	6,025	0.00	1.71	5,983	0.00	1.01
30	14,173	0.00	0.25	14,146	0.00	0.06
Average improvement	-4.36	-1.94		-2.64	1.18	
Average solution time (sec.)	0.006			0.007		

Table 8 shows the results when multiple-retailer-route are allowed. To evaluate the performance of EDD2 and DADI2, the solution of the optimal

model cannot be used directly as a reference because allowing multiple retailers on each tour is a relaxation of this model. Thus, the results presented in Table 8 should be viewed as the improvement that can be obtained by relaxing this condition. In this table, IMP(EDD) and IMP(DADI) represent, respectively, the improvement obtained by EDD2 and DADI2 over EDD and DADI. A zero value means that the same solution was obtained, while a negative value indicates a cost reduction. To provide better insight into the behaviour of these heuristics, their performance was also compared to the optimal single-retailer-route solutions; the results of this comparison appear in the columns IMP(OPT). In these columns, a positive value means that the solution obtained has a higher cost than the optimal single-retailer-route solution.

Clearly, EDD2 is the best combination, reducing as it does the initial EDD solution value by an average of 4.36%. This improvement is large enough to produce an average reduction of 1.94% compared to the initial optimal solution value to the single-retailer-route problem. This result clearly demonstrates the efficiency of allowing multiple deliveries on a single tour. Again, considering the best solutions produced by the combination of EDDH2 and DADI2, the improvement over the initial optimal solution is 2.85%.

6.2. Results for problem set B

For the larger problems in set B, optimal solutions are not available so the heuristics have been evaluated with respect to their deviation above the best solution found for each instance. The results presented in Table 9 show that, for the single-retailer-route problem, the EDD heuristic generates the best solutions 28 times out of 30 (instances 1 and 25 having their best results with DADI heuristic) in addition to having the lowest average deviation at 0.01%. DADI comes in second with an average deviation of 1.55% above the best known solutions. More importantly, given the context, the solutions are produced in a few minutes. Remember that resolving these problems corresponds to making tactical decisions covering a horizon of several weeks. Such low computational times demonstrate the capacity of our method for generating real-life delivery schedules for 30 working days (a month and a half).

Table 9 – Single-retailer-route results for set B instances

Instance	EDD		DDIA		DDAI		DADI		BEST
	Z	%	Z	%	Z	%	Z	%	Z
1	1,450,272	0.14	1,540,482	5.98	1,529,800	5.33	1,448,291	0.00	1,448,291
2	1,432,623	0.00	1,517,758	5.61	1,515,540	5.47	1,448,959	1.13	1,432,623
3	1,598,425	0.00	1,677,662	4.72	1,662,074	3.83	1,598,455	0.00	1,598,425
4	1,436,668	0.00	1,514,830	5.16	1,504,566	4.51	1,448,667	0.83	1,436,668
5	1,405,698	0.00	1,478,030	4.89	1,465,542	4.08	1,435,895	2.10	1,405,698
6	1,470,349	0.00	1,518,370	3.16	1,495,854	1.71	1,495,222	1.66	1,470,349
7	1,530,128	0.00	1,583,049	3.34	1,570,928	2.60	1,550,333	1.30	1,530,128
8	1,570,256	0.00	1,629,450	3.63	1,614,522	2.74	1,602,429	2.01	1,570,256
9	1,648,863	0.00	1,733,200	4.87	1,702,935	3.18	1,688,617	2.35	1,648,863
10	1,647,434	0.00	1,714,427	3.91	1,701,943	3.20	1,673,174	1.54	1,647,434
11	1,621,870	0.00	1,682,051	3.58	1,673,486	3.08	1,644,989	1.41	1,621,870
12	1,502,114	0.00	1,567,001	4.14	1,576,275	4.70	1,507,969	0.39	1,502,114
13	1,711,846	0.00	1,846,055	7.27	1,812,789	5.57	1,745,215	1.91	1,711,846
14	1,556,825	0.00	1,606,218	3.08	1,592,005	2.21	1,586,908	1.90	1,556,825
15	1,603,208	0.00	1,689,693	5.12	1,687,708	5.01	1,617,524	0.89	1,603,208
16	1,626,030	0.00	1,679,077	3.16	1,661,910	2.16	1,631,963	0.36	1,626,030
17	1,571,281	0.00	1,649,692	4.75	1,642,750	4.35	1,648,494	4.68	1,571,281
18	1,557,075	0.00	1,676,843	7.14	1,669,989	6.76	1,576,747	1.25	1,557,075
19	1,648,990	0.00	1,724,475	4.38	1,723,883	4.34	1,687,097	2.26	1,648,990
20	1,497,191	0.00	1,597,194	6.26	1,572,244	4.77	1,503,730	0.43	1,497,191
21	1,522,923	0.00	1,593,307	4.42	1,571,518	3.09	1,603,061	5.00	1,522,923
22	1,736,838	0.00	1,800,994	3.56	1,802,594	3.65	1,751,877	0.86	1,736,838
23	1,540,927	0.00	1,571,303	1.93	1,568,081	1.73	1,563,194	1.42	1,540,927
24	1,526,645	0.00	1,548,728	1.43	1,546,212	1.27	1,540,229	0.88	1,526,645
25	1,485,990	0.25	1,585,480	6.51	1,560,288	5.00	1,482,245	0.00	1,482,245
26	1,539,287	0.00	1,661,090	7.33	1,650,676	6.75	1,567,364	1.79	1,539,287
27	1,564,828	0.00	1,613,890	3.04	1,606,439	2.59	1,590,266	1.60	1,564,828
28	1,398,928	0.00	1,494,659	6.40	1,493,095	6.31	1,405,697	0.48	1,398,928
29	1,474,219	0.00	1,545,787	4.63	1,528,290	3.54	1,500,875	1.78	1,474,219
30	1,686,908	0.00	1,742,089	3.17	1,730,685	2.53	1,759,881	4.15	1,686,908
Average deviation	0.01		4.55		3.87		1.55		
Average solution time (sec.)	25		219		227		174		

Table 10 presents the results for EDD2 and DADI2 for the larger instances in set B, allowing multiple retailers routes. Again, EDD2 generates the best tours 28 times out of 30. EDD2 reduces the solution values of EDD by an average of 11.39%. These results show that greater improvements may be obtained for larger instances by allowing multiple deliveries during the same tour.

Table 10 – Multiple-retailer-route results for set B instances

Instance	EDD2			DADI2		
	Z	IMP(EDD)	IMP(BEST)	Z	IMP(DADI)	IMP(BEST)
1	1,249,143	-16.10	-15.94	1,247,690	-16.08	-16.08
2	1,270,074	-12.80	-12.80	1,289,290	-12.38	-11.12
3	1,464,775	-9.12	-9.12	1,465,957	-9.04	-9.04
4	1,296,010	-10.85	-10.85	1,312,372	-10.39	-9.47
5	1,282,988	-9.56	-9.56	1,322,952	-8.54	-6.25
6	1,371,059	-7.24	-7.24	1,398,865	-6.89	-5.11
7	1,388,370	-10.21	-10.21	1,417,631	-9.36	-7.94
8	1,403,410	-11.89	-11.89	1,442,318	-11.10	-8.87
9	1,496,445	-10.19	-10.19	1,546,466	-9.19	-6.62
10	1,500,525	-9.79	-9.79	1,536,172	-8.92	-7.24
11	1,412,814	-14.80	-14.80	1,442,416	-14.04	-12.44
12	1,367,399	-9.85	-9.85	1,373,278	-9.81	-9.38
13	1,568,979	-9.11	-9.11	1,603,487	-8.84	-6.76
14	1,434,716	-8.51	-8.51	1,472,987	-7.73	-5.69
15	1,439,375	-11.38	-11.38	1,449,289	-11.61	-10.62
16	1,469,605	-10.64	-10.64	1,485,830	-9.84	-9.44
17	1,411,831	-11.29	-11.29	1,491,556	-10.52	-5.35
18	1,346,958	-15.60	-15.60	1,371,246	-14.99	-13.55
19	1,444,231	-14.18	-14.18	1,481,449	-13.88	-11.31
20	1,336,482	-12.02	-12.02	1,352,078	-11.22	-10.73
21	1,346,751	-13.08	-13.08	1,427,721	-12.28	-6.67
22	1,508,241	-15.16	-15.16	1,533,164	-14.27	-13.28
23	1,383,604	-11.37	-11.37	1,412,259	-10.69	-9.11
24	1,428,349	-6.88	-6.88	1,442,297	-6.79	-5.85
25	1,340,701	-10.84	-10.56	1,340,461	-10.58	-10.58
26	1,352,166	-13.84	-13.84	1,377,066	-13.82	-11.78
27	1,429,622	-9.46	-9.46	1,462,636	-8.73	-6.99
28	1,278,239	-9.44	-9.44	1,292,942	-8.72	-8.20
29	1,282,491	-14.95	-14.95	1,309,405	-14.62	-12.59
30	1,510,767	-11.66	-11.66	1,590,204	-10.67	-6.08
Average improvement	-11.39	-11.38		-10.85	-9.14	
Average solution time (sec.)	73			231		

7. Conclusion

This paper deals with a synchronized production-distribution problem for a large-scale distribution network. This is a complex vehicle routing problem in a multi-period context, which must take into account the production schedule, the distribution center's inventory cost and the demand schedules contracted with retailers. We first presented the mathematical model for the single-retailer-route problem. Then, given the complexity of the solutions and resulting high solution times, which made the optimal resolution of large-scale problems unachievable, we proposed four heuristics for solving the single-retailer-route problem quickly. Our results show that these heuristics generated excellent results

for small problems, both in terms of solution quality and resolution time. For small instances, the EDD heuristic produced the lowest average optimality gap of 2.27%. For large instances, all the heuristics managed to solve the problems quickly. For the studied large instances, the EDD heuristic had the best performance, with an average deviation of 0.01% for the best known heuristic solutions. For dealing with the multiple-retailer-route problem, we introduced two heuristics EDD2 and DADI2, which improved results from 10.85% to 11.39% as compared to the solutions involving one delivery per retailer per tour.

In summary, we have made a formal presentation of a highly complex, large-scale industrial problem, whose resolution requires that conflicting constraints and costs related to routing, inventory and production be taken into account in an integrated manner. We have presented heuristic approaches that permit near-optimal solutions to be obtained rapidly. We have also shown the potential gain associated with a better coordination of the vehicle fleet as well as the better results that are possible with a multiple-retailer-route approach.

The paper suggests a variety of subjects for future research. In terms of the specific problem addressed in this paper, innovative new heuristics could be developed to further exploit the potential of multiple-retailer-route. The problem itself could be relaxed by transforming the demand schedule contracted with retailers into a desired demand schedule that would allow the manufacturer to deliver later than the retailer desired, subject to penalty costs. In addition, variations of the problem could be studied and compared. For example, the problem could be stated in terms of a case in which the manufacturer deals with a third-party transportation company or logistic provider, and thus is subject to specific cost structures.

The problem can also be integrated with the production planning problem, in which production is not considered to be a given input, but rather an integrated set of decision variables, each with its own cost structure and constraints. Finally, retailers could be allowed to dynamically modify their preferred demand schedule and even reorder during the selling season, which again opens a wealth of challenging versions of the problem.

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