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Abstract. This paper presents a market-driven approach to design production-distribution networks for the lumber industry. The approach is developed to tackle a vast array of issues, from the adaptation of an enterprise supply chain to its evolving environment, such as changing forest policies, to enterprise rationalizations through mergers or acquisitions. The methodology takes into account the specificity of the industry divergent manufacturing processes as well as the lumber market segmentation into contracts, vendor managed inventory (VMI) agreements and spot markets. The approach is based on a comprehensive two-stage stochastic programming with recourse model. A sample average approximation (SAA) method based on Monte Carlo sampling techniques is proposed to solve this stochastic program, and it is shown that this approach outperforms the use of a comparable deterministic model based on averages. Finally, the decision support system developed to implement the approach is used to show how it can contribute to dealing with strategic issues in the Eastern-Canadian lumber industry. Forest policy as well as acquisition and rationalization issues are analyzed through applications of the methodology to a virtual but realistic case called Virtu@l-Lumber.

Keywords. Production-distribution network design, mathematical programming, Monte Carlo sampling methods, strategic analysis, acquisition and rationalization.

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1 Introduction

The sawmill industry is a key economic activity in the Province of Quebec. It generates approximately twenty thousand direct jobs in three hundred sawmills. The average annual harvest is about thirty-six million cubic meters, corresponding to roughly twenty-five percent of the Canadian harvest and it creates one and a half billion Canadian dollars of added value\(^1\). An important specificity of the Quebec lumber industry is that 90 percent of forested area is on public land. Consequently, government is the main fiber supplier and influences the organization and behaviour of companies. For example, the wood allocation is granted to a specific sawmill through governmental contracts (Supply and Forest Management Agreements) which stipulate that logs from a specific area must be processed in a particular sawmill. Historically, the Quebec lumber industry has been strongly influenced by trade relationships with the United-State. Moreover, the exchange rate of the Canadian and U.S. currencies plays a key role. The lumber industry is a commodity industry where buyer concentration and price sensitivity increase the buyer power. Indeed, this concentration phenomenon confers an advantage to large retail companies, for example, in the bargaining process. Moreover, substitutes, such as steel and concrete, represent a real threat to lumber products. Lastly, competition between forest companies is intense. The lumber sector experiences pure and perfect competition in a commodity market where delivery costs are significant. The market share of the top five North American producers is only 22% (Taylor et al., 2002). Moreover, the industry products are in the mature and even declining stage of their life cycle, a position where rivalry is customarily intense, and concentration appears inevitable.

This strategic overview shows that the industry measures up to significant challenges. In this low margins industry, operational excellence and customer intimacy are key success factors. In order to be able to deliver the low prices and high service levels expected by customers, lumber companies must streamline their supply chains. This becomes mandatory when significant forest policy changes or market shifts perturbate a company’s

\(^1\) CIFQ Web site: www.cifq.qc.ca (September 2005).
environment. The aim of this paper is to propose a methodology to capture the dyadic relationship between a lumber company production-distribution network and its market opportunities in order to increase the probability for the enterprise to obtain profitable contracts or VMI agreements and thus increase profits. The approach can also be used by lumber company managers to evaluate enterprise rationalization scenarii or to develop strategies to adapt to dramatic environment evolution such as profound modifications in forest policies. The approach is based on a two-stage stochastic programming model with fixed recourse. A sample average approximation (SAA) method based on Monte Carlo sampling techniques is proposed to solve this stochastic program. In order to demonstrate the scope and strength of the methodology, a realistic lumber industry case, Virtu@l-Lumber, was created with the collaboration of the institutional and industrial partners of the FOR@C research consortium. Virtu@l-Lumber is used to show how the design approach proposed can contribute to dealing with strategic issues in the Eastern-Canadian lumber industry. Forest policy as well as acquisition and rationalization issues are analyzed, and it is shown that the stochastic programming model proposed outperforms a comparable deterministic model based on averages.

The paper is organized as follows. Section 2 presents a review of the relevant production-distribution network design literature. Section 3 develops the concepts underlying the stochastic programming model on which the methodology is based. Section 4 describes the Virtu@l-Lumber case, as well as examples of applications of the modeling approach to the strategic planning of forest products companies. The supply chain network design methodology proposed is applied to the forest product industry but is generic: it could be applied to any divergent process industry where several products are being made from a common raw material.

2 Literature Review

This section presents a concise review of the literature related to the proposed production-distribution network design methodology from the point of view of the lumber industry. Two dimensions must be distinguished to describe the state of the art. The first concerns the supply chain design problem, which is composed of three sub-problems, namely location, capacity acquisition and technology selection problems. The second is the conceptual modeling of markets. Indeed, the proposed approach pleads in favour of
integrating supply chain and market facets in order to capture their dyadic relationship and interaction.

An abundant literature exists on location, capacity acquisition and technology selection problems. Verter and Dincer (1992) review the initial literature in these fields. More recent reviews are found in Goetschalckx et al. (2002), Bhutta (2004) and Martel (2005). Rönqvist (2003) presents a review of optimization models for all planning levels and for all sectors of the forest products industry. At the strategic level, Carlsson and Rönqvist (2005) propose a model combining distribution facility location with ship routing applied to a Swedish pulp company. Martel et al. (2005, 2006) study the international factors in the design of multinational supply chains for Canadian pulp and paper companies. Vila et al. (2006) propose a generic approach to design production-distribution networks for divergent process industries, i.e. for industries producing several products or byproducts from a single raw material such as the lumber industry or the meat industry. One of the objectives of this paper is to extend the modeling framework of Vila et al. (2006) to take probabilistic market opportunities into account.

At the tactical level, Maness and Norton (2002) and Liden and Rönqvist (2000) propose linear programming models combining bucking and production planning for the lumber industry. The production planning problem in the secondary processing sector has been examined by Carino and Lenoir (1988) who propose a wood procurement model for a cabinet manufacturing plant. Also, Carino and Willis (2001a and 2001b) and Farell and Maness (2005) propose production planning models for secondary wood product manufacturing. At the operational level, Rönqvist (1995) proposes a method for the allocation of wood products in order to optimize the cutting process in real time, taking the quality of logs into account, and Rönqvist and Astrand (1998) integrate defect detection to this approach.

In order to be competitive, companies must design their production-distribution network to support their product-market strategies. Shapiro (2001) emphasizes the necessity to integrate strategic marketing and production-distribution decisions in the same model in order to design superior supply chains. In particular, it is important that the production-distribution facilities of a company be located to provide competitive prices and service
levels, given the location of competitors’ facilities. For example, quick response because of proximity may lead to added market shares and/or price premiums. None of the papers cited previously address this type of issues. Vila et al. (2007) present a generic approach, based on a two-stage stochastic programming (Birge and Louveaux, 1997) model and a discrete choice (Louviere et al., 2000) customer preference model, to design production-distribution networks well positioned to capture promising markets. Three types of markets found in several industrial contexts are considered: spot markets, contracts and Vendor Managed Inventory (VMI) agreements. We use this approach to model markets in the current paper. Our objective is to integrate the production-distribution network design approach of Vila et al. (2006) with the product-market modeling framework of Vila et al. (2007), and to use the resulting supply chain design methodology to investigate important issues for the Eastern-Canadian lumber industry.

3 Design Methodology

3.1 The Integrated Approach

The supply chain of timber companies is typically composed of geographically dispersed woodlands, woodyards, sawmills, distribution sites and markets. Generally, the strategic design of forest industry supply chains involves the overall company: the forest operations, manufacturing, logistics and marketing departments must be involved in the strategic planning process. Basically, the problem is to take simultaneously capacity, technology, location and marketing decisions which maximize the profits of the timber company for known woodland locations and capacities, cost structure and international market opportunities.

This paper presents a modeling approach to assist the strategic planner in making these complex high level decisions. It aims at coordinating production-distribution with marketing strategy. On the one hand, as illustrated in Figure 1, the industry manufacturing process is mapped onto potential production-distribution facility locations, layouts and capacity options. A detailed description of the activities \( a \in A \) of the lumber industry supply chain multigraph presented in Figure 1 is found in Vila et al. (2006). The arcs of the multigraph are associated to the product families \( p \in P \) flowing through the supply chain. Manufacturing activity \( a \in A' \subset A \) can be performed using a set of predetermined
production technologies $KM_a$, and warehousing activity $a \in A^w \subset A$ with a set of storage technologies $KS_a$. When activity $a \in A^w$ is performed with technology $k \in KM_a$, a set of possible one-to-many recipes $\varphi \in R_{ak}$ can be used to transform input product $p_{\varphi}$ into the set of output products $P_{\varphi}^{out}$. A set $S^{pd}$ of potential production-distribution (sawmills) sites and a set $S^{d}$ of potential storage sites are considered. A potential sawmill $s \in S^{pd}$ can be implemented using different potential layouts $l \in L_s$ (including the current layout if the sawmill already exists) and different potential capacity options $j \in J_{ls}$ with specified production/storage technologies. Sawmills $s \in S^{pd}$ are supplied from a set of vendors (forest areas) $v \in V_s$ with known minimum and maximum stem/log supply volumes.

Figure 1: Mapping Activities onto Potential Facilities.

On the other hand, the approach integrates market specificities (Figure 2). In our context, each national product-market, i.e. the product-market associated to a country $o \in O$, can be
partitioned into three sub-markets:

- A set of spot markets $m \in SM_o$ characterized each by a product set $P_m$, sets of demand zones $D_{pm}$ and a decreasing price step function defined over a set of levels $\lambda \in \Lambda_{pm}$ based on a reference price. This reference price is determined by the company using price forecasts based on the historical behavior of the firm prices on the spot market.
- A set of customer contracts $C_o$ partitioned into potential contracts $PC_o$ and signed contracts $SC_o$. A contract $c \in C_o$ pertains to a product $p_c$ sold to customer location $d_c$.
- A set of Vendor Managed Inventory (VMI) agreements $VM_o$ that is also partitioned into potential VMI agreements $PVM_o$ and signed VMI agreements $SVM_o$. Under a VMI agreement, the lumber company must keep sufficient inventory at the customer location to guarantee a given fill rate. The products are paid only when the customer takes the lumber out of inventory and, consequently, inventory holding costs are covered by the lumber company. A VMI agreement $c \in VM_o$ applies to a product $p_c$ sold to customer location $d_c$.

Figure 2: Finished Products and Markets.

Figure 2 shows finished product families and their corresponding markets. The products associated to contracts and VMI agreements have specific values and end-users: “Machine Stress Rated (MSR)” for contracts with secondary transformation companies and “Premium” appearance for VMI agreements. The added value of contracts and VMI agreements is materialized by a price premium with respect to the spot market price. The price premium is also conditioned by the characteristics of the logistics policy used. The logistics policy concept describes criteria that qualify the company as a potential supplier (qualifiers) and criteria that win contracts on the marketplace (order winners) as proposed by Hill (1994). For contracts, the basic criteria associated to a logistics policy are price and delivery times, and for VMI agreements they are price and fill rate. In order to provide
good delivery times, distribution centers must be located close to contract ship-to-points, but in order to provide good fill rates, sufficient inventory must be kept by the lumber company at the VMI customer location. A set of logistic policies $i \in I_c$ is considered for each potential customer $c \in PC_o \cup PVM_o$. The spot market can be considered as a recourse which can absorb any amount of product, but for a price decreasing with quantity. Moreover, substitution possibilities allow the manager to downgrade and sell contracts and VMI products on spot markets as discussed in Vila et al.

Signed contracts/agreements yield a deterministic demand to be satisfied but potential contracts/agreements define a stochastic demand process. Customer preferences are captured by econometric discrete choice methods, as presented in Vila et al. (2007), from which one can estimate the probability that a contract or a VMI agreement will be signed when a given logistics policy is implemented. Because of competition, it is assumed that potential customers will sign contracts only if the company can demonstrate that it has the resources required to comply with all the clauses of the contracts/agreements. Consequently, the production-distribution network must be designed to satisfy signed contracts and agreements and to be in a position to satisfy some potential new contracts and agreements, knowing that the uncommitted production can be sold on the spot market.

The goal of the company is to design its production-distribution network anticipating the future by simultaneously selecting adequate logistics policies, and by deploying production capacity and locating distribution centers to support these policies. This is done by solving a stochastic programming model with a Monte-Carlo sample average approximation (SAA) method. This model can then be used to investigate all sorts of strategic options, as will be shown in section 4.

3.2 The Mathematical Model

The mathematical model on which the methodology is based is a two-stage stochastic program with fixed recourse (Birge and Louveaux, 1997). The reader may refer to the Appendix for a presentation of the detailed mathematical model, as well as to Vila et al. (2006 and 2007) for a complete account of the model genesis. The model presented in the Appendix was obtained by applying the network design methodology proposed in Vila et al. (2007) to the lumber industry supply chain design problem examined in Vila et al.
(2006). More specifically, it is the equivalent deterministic program obtained when the stochastic program formulated is solved with the Monte Carlo sample average approximation (SAA) method proposed by Shapiro (2003). The following sub-sections discuss the first and second stage decision variables of our stochastic programming model, as well as the model structure, and they provide an outline of the solution method used.

First stage decisions

The first stage decisions are mostly strategic decisions to be implemented to shape the future of the company. As our approach is cross-functional, the strategic decisions concern several departments of the forest company. Strategic manufacturing decisions involve layout choices for each of the production facilities, the selection of capacity options, and seasonal opening or shutdown during the seasons $t \in T$ of the planning horizon for the capacity options selected. Strategic marketing decisions essentially correspond to logistics policy choices. Strategic distribution decisions concern the selection of the distribution centers to use, among a set of possibilities, in order to satisfy the requirements of logistics policies. This leads to the definition of the following strategic decision variables:

- $Y_{ls} = \text{Binary variable equal to 1 if layout } l \in L_s \text{ is used for production-distribution site } s \in S^{pd} \text{ and to 0 otherwise.}$
- $Y_{gs} = \text{Binary variable equal to 1 if production-distribution site } s \in S^{pd} \text{ is not used and to 0 otherwise. This variable is included to be able to charge fixed shutting down costs when an existing facility is closed.}$
- $Y_s = \text{Binary variable equal to 1 if potential storage center } s \in S^d \text{ is used and to 0 otherwise. Since distribution centers are usually rented, it is assumed that their layout is predetermined.}$
- $Z_j = \text{Binary variable equal to 1 if capacity option } j \text{ is installed and to 0 otherwise.}$
- $\hat{Z}_{jt} = \text{Binary variable equal to 1 if capacity option } j \text{ is used during season } t \text{ and to 0 otherwise.}$
- $\tilde{Z}_i = \text{Binary variable equal to 1 if logistics policy } i \in I_c, c \in PC_o \cup PVM_o \text{ is deployed and to 0 otherwise.}$

Note that only the previous strategic decisions would be implemented in practice. However, additional decision variables must be incorporated in the model in order to anticipate the impact of the network design on sales revenues, and on supply, inventory, production and
transportation costs. In our model, some of these aggregate anticipation variables are considered as first-stage decision variables and others as second-stage recourse variables. The first stage model integrates supply decisions shaping stem and log flows between the forest and the sawmills, inbound transportation decisions depicting the seasonal flow of semi-finished products or bi-products between sawmills, production decisions reflecting the seasonal missions of manufacturing sites, and seasonal inventory level decisions setting end-of-season inventory targets for stems. This leads to the definition of the following anticipation decision variables:

$$F_{p(n,a)(n',a')t} = \text{Flow of product } p \in P \text{ between activity } a \in A \text{ at location } n \in V \cup S \text{ and activity } a' \neq a \text{ at site } n' \in S \text{ during season } t \in T.$$

$$X_{p\varphi st}^\varphi = \text{Quantity of product } p \varphi \text{ processed with recipe } \varphi \in \bigcup_{a \in A, a' \neq a} R_{ak} \text{ in production-distribution site } s \text{ during season } t \in T.$$

$$I_{pks} = \text{Seasonal inventory of raw material } p \text{ stored on site } s \text{ with technology } k \in KS_{s_2} \text{ at the end of season } t \in T.$$

It is reasonable to consider these variables as first as opposed to second stage variables in our stochastic model because production in practice is largely driven by known raw material minimum/maximum supply limits, and because the spot market can absorb any excess production. This also tends to stabilise the mission of production facilities, which is desirable. Finally, at the modeling level, it has the immense advantage that it decreases the number of variables in the model considerably.

**Second stage recourse variables**

In order to explain the second stage recourse variables in our stochastic model adequately, the notion of environment must first be defined. An environment is one of a set of possible future demand outcomes $$\omega \in \Omega$$. In our context, an environment is described by a vector of binary variables indicating whether the customers would sign a contract/agreement ($$\omega_i = 1$$) or not ($$\omega_i = 0$$), for all possible logistics policies $$i \in I$$, as introduced by Vila et al. (2007). The notion of environment describes contractual opportunities. Since spot markets are recourses which can be used to absorb any outstanding production, it is not necessary to include the spot market explicitly in the description of an environment. A set of second stage recourse variables is attached to each environment considered. These variables model seasonal finished product inventories, seasonal flows of finished products to distribution.
centers, and seasonal flows of finished products to markets. More formally, they are defined as follows:

\[ F_{p(n,a)(n',S)}(\omega) = \text{Flow of product } p \in P \text{ between activity } a \text{ at location } n \in S \text{ and storage activity } n' \in S \text{ during season } t \in T \text{ for environment } \omega \in \Omega. \]

\[ I_{pkst}(\omega) = \text{Seasonal inventory of product } p \in FP \text{ stored on site } s \text{ with technology } k \in KS_{s8} \text{ at the end of season } t \in T \text{ for environment } \omega \in \Omega. \]

\[ F_{pp's'd'\lambda}(\omega) = \text{Outbound flow from site } s \text{ of finished product } p' \in FP \text{ used to satisfy the demand for product } p \in FP \text{ in spot market zone } d \in \bigcup_{m \in SM} D_{pm}, \text{ and sold at the level } \lambda \text{ price on the spot market sales price function, during season } t \in T \text{ for environment } \omega \in \Omega. \]

\[ F_{psit}(\omega) = \text{Outbound flow from site } s \text{ of finished product } p \in FP \text{ used to satisfy the demand of product } c_{i(j)} \text{ in demand zone } d_{c_{i(j)}}, \text{ when using logistics policy } i \text{ during season } t \in T \text{ for environment } \omega \in \Omega, \text{ where } c(i) \text{ denotes the contract/agreement associated to policy } i. \]

The model structure

The model formulated maximizes expected corporate profits for a multinational company. First, the operating income for each country \( o \in O \) is calculated: total revenues and costs for each facility and logistics policy costs are taken into account. Facility revenues come from outflows to other sites and to demand zones. Facility costs include inbound flow costs (inflow transfers, raw materials and receptions from other sites), site related costs (facilities and capacity options fixed costs, production and handling costs, holding costs of order cycle stocks, safety stocks and seasonal stocks), and the costs of outflows to other sites and demand zones. The maximization of this objective function is subject to several first-stage and second stage constraints, namely:

- First stage constraints:
  - Supply market constraints (1, 2, 3)
  - Facility layout, space and exclusive options constraints (4, 5, 6)
  - Seasonal capacity option usage constraints (7)
  - Production activities flow equilibrium constraints (8, 9)
  - Storage activities inventory accounting constraints (10, 11)
  - Production and storage capacity constraints (12, 13)
- Second stage constraints:
  - Production activities flow equilibrium constraints (15, 16)
  - Storage activities inventory accounting constraints (17)
  - Storage capacity constraints (18, 19)
  - Sales market constraints (20, 21, 22, 23)

The numbers between brackets in this list refer to the corresponding constraints in the Appendix. The resulting equivalent deterministic model is a very large scale mixed integer program.

**Sample average approximation (SAA) solution method**

The reader is referred to Santoso et al. (2005) for a detailed description of the Sample Average Approximation (SAA) method for solving stochastic programs, and to Vila et al. (2007) for its application to supply chain network design problems. In the model previously defined, the number of environments \( \omega \in \Omega \) to take into account could be huge. The essence of the SAA method is that instead of taking all these environments into account explicitly, it restricts itself to a sample of \( N \) environments, \( \Omega_N \subset \Omega \), generated with Monte-Carlo sampling methods, using the probability that contracts/agreements will be signed. When generating environment \( \omega \), for each customer \( c \in \bigcup_{o \in O} (C_o \cup VM_o) \), this method defines the subset \( I_c(\omega) \subset I_c \) of logistics policies which would lead to a signed contract/agreement if implemented. Each of the \( N \) environments thus generated have an equal probability of occurrence, \( 1/N \), which simplifies the computation of expected profits considerably.

The approach involves the solution of the SAA program for \( M \) different samples of size \( N \). This implies that \( M \) different near-optimal feasible designs could be obtained and the questions to answer are then: which design is the best and how close is it to the true optimum? To answer these questions, a better estimate of the true expected value of the designs found is calculated with a Monte-Carlo evaluation based on a sample of size \( N' \) much bigger than \( N \). This is done by fixing the strategic binary first stage decisions \( (Y_{ib}, Y_{0s}, Z_j, \hat{Z}_{ij}) \) and by solving the resulting linear programs for the sample of size \( N' \). Statistical lower and upper bounds on the true value of the optimal solution of the stochastic program must also be calculated to estimate the optimality gap of the solutions.
obtained. The number of first stage constraints is independent of the size \( N \) of the environment sample \( \Omega_N \) selected for the SAA program. However, the number of second stage constraints (22) grows almost proportionally with the size of the sample.

4 Experimental Evaluation

4.1 Experimental Design

Virtu@l-Lumber, a realistic case, was built based on typical production-distribution network design problems encountered in the forest products industry of the province of Quebec. It was developed in order to test the applicability and feasibility of our approach. The main strategic options considered in the case are presented in Figure 3. Each sawmill has two layouts with their respective capacity options as well as seasonal shutdown or opening options. Note that the supply costs of the River mill are very expensive in comparison to the Mountain and Valley mills. The distribution network is constituted of seven potential distribution centers. The reference market price of softwood lumber is assumed equal to $450/tbf (thousand board feet). The American and Canadian markets are made up of four spot markets, one signed and seven potential contracts, and one signed and six potential VMI agreements. The probabilities of getting the various contracts/agreements considered were randomly generated. In practice, they would be estimated using discrete choice econometric models (Ben-Akiva and Lerman, 1985; Louviere et al., 2000). On the whole, as explicited in Figure 3, 21 potential logistics policies describing pricing and delivery relationship options were defined. For this set of policies, the number of possible environments \( |\Omega| \) is about two millions \( 2^{21} \approx 2\,000\,000 \). Consequently, the need to use the SAA method was deemed justified, even for this moderate size case.

In order to test the mathematical model on the Virtu@l-Lumber case, a plan of experiment was elaborated. Each problem in the experiments was solved with the SAA method using five independent samples of twenty-five environments \((M = 5, \ N = 25)\), which means that at most five different designs could be obtained for a given problem. In order to determine the best design, an estimate of the true expected value of the solutions was calculated through a Monte-Carlo evaluation based on a sample size \( N' = 100 \). In practice, instead of using stochastic programming, logistic network designs are often based on the solution of deterministic models with an average demand. In order to compare these two approaches in
our experiments, a deterministic MIP based on average demand was solved for each case considered. An estimate of the expected value of the solution thus obtained was also calculated with the Monte-Carlo method using a sample of size \( N' = 100 \).

In the following sections, the use of the models to contribute to the solution of various management problems is examined. The impact of price differentials between market segments on the production-distribution network structure and on the number of contract signatures is analyzed first. Subsequently, different issues such as the adaptation of the enterprise to dramatic modifications in forest policy and the evaluation of acquisition and rationalization scenarios are studied using the model.

**4.2 Impact of Market Segments Price Differences**

*Spot market vs Contract/VMI prices differential*

The impact of a price differential between the sport market and contracts (or VMI agreements) on the logistics policies selected are first studied. Figure 4 illustrates the
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evolution of the number of deployed logistics policies as a function of the spot vs contract/VMI prices differential when the average demand model is used. This average demand model is obtained by replacing the random contract demand quantities by their expected value. Figure 5 shows the behaviour of the solution recommended when the SAA method is used to solve the stochastic programming model.

It can be seen that the SAA method deploys less policies than the average demand model. The evolution of total deployed logistics policies with price differentials is similar for the two approaches, but the SAA method prefers to sign VMI agreements rather than contracts. Signing a large number of contracts is not necessarily efficient, as shown in Figure 6. It appears that the difference in the expected value of the designs provided by the two modeling approaches is quite large, which justifies the use of stochastic programming in order to take customer preferences into account adequately.

Figure 4: Average Demand Model.         Figure 5: SAA Method.

Figure 6: Expected Values for the Average Demand Model vs the SAA Method.
Spot-VMI vs Contract prices differential

As explained in the methodology and in the previous section, price differentials are key drivers to design marketing strategies. In order to better understand the specific role of the differential between spot markets and contract prices on the supply chain, four new potential contracts are added. The Valley mill is assumed to be the only site allowed to ship products to these four new locations. Moreover, the price for VMI agreements is assumed equal to the spot market price: then, the impact of the “spot-VMI versus contract prices differential” is studied.

Figure 7 and Figure 8 show the evolution of the number of deployed logistics policies and the layout of the Valley mill for the designs obtained, respectively, with the average demand model and the SAA method. In both cases, as could be expected, the number of VMI agreements remains constant and corresponds to the agreement signed initially (see Figure 3). The layout and the mission of the Valley mill evolve in three phases for the solutions provided by the average demand deterministic model:

1. In the $0-20/tbf range, the Status quo layout is optimal, and the sawmill produces as usual.
2. In the $40-80/tbf range, the Status quo layout remains optimal but some logistic policies are deployed for the four new potential contracts considered. The sawmill produces as usual and becomes the distribution center from which the products to the new contract customers are shipped. Indeed, Valley mill is the only site to ship products for these profitable contracts.
3. For a $100/tbf differential, the Valley mill layout changes and the mill produces and ship the products for the new profitable signed contracts.

Figure 9 shows the results of the evaluation of the expected value of the solutions obtained with the two solution methods. It shows that the average demand model often proposes solutions which are not feasible for some of the environments $\omega \in \Omega^*$. Indeed, the four potential contracts added to the initial configuration require large production quantities. The average environment considers average demand contracts, which yield demand levels lower than to the true demand of several environments. The supply chain designs obtained with the average demand model are thus often infeasible for the true demand of the contracts selected randomly by the Monte Carlo sampling method. On the other hand, the
SAA method proposes efficient feasible designs. These results are a convincing argument for using a stochastic programming model considering the variability in demand patterns explicitly, instead of a static deterministic model.

Figure 7: Average Demand Model Solutions.

Figure 8: SAA Method Solutions.

Figure 9: Expected Values for the Average Demand Model vs the SAA Method.
4.3 Adaptation to Changes in Forest Policy

This section aims at studying the impact of some eventual modifications in Quebec’s forest policy by using the proposed mathematical modeling approach. In the province of Quebec, about 90 percent of commercial forests are on public land. Hence, forest policy is of major importance for taking strategic decisions on the supply side of softwood lumber companies. In our model, the constraints imposed by the actual legislation in Quebec are the following:

\[
\sum_{p \in P_{\text{out}}^{\text{out}}} F_{p(v,v),a}^{(v,v),a} \leq b_{v,a}^{\text{max}}, \quad a \in A_{\text{out}}^{\text{out}}, s \in S^{\text{pd}}, v \in V, t \in T
\]  

\[
\sum_{t \in T} \sum_{a \in A_{\text{out}}} \sum_{p \in P_{\text{out}}^{\text{out}}} F_{p(v,v),a}^{(v,v),a} \geq b_{v,s}^{\text{min}}, \quad s \in S^{\text{nd}}, v \in V
\]

where,

- \(A_{\text{out}}^{\text{out}}\) = Set of immediate successors of activity \(a\) (\(A_{\text{out}}^{\text{out}} \subset A\)).
- \(P_{\text{in}}^{\text{in}}\) = Input product families of activity \(a\) (\(P_{\text{in}}^{\text{in}} \subset P\)).
- \(P_{\text{out}}^{\text{out}}\) = Output product families of activity \(a\) (\(P_{\text{out}}^{\text{out}} \subset P\)).
- \(b_{v,a}^{\text{max}}\) = Upper bound on the seasonal shipments of raw material between source \(v \in V\) and activity \(a \in A_{\text{out}}^{\text{out}}\) on site \(s \in S^{\text{nd}}\) for season \(t\).
- \(b_{v,s}^{\text{min}}\) = Minimum annual level of raw material to be shipped between source \(v \in V\) and site \(s \in S^{\text{nd}}\) in order to comply with governmental supply contracts.

Constraints 1) state that the seasonal supply is restricted to a maximum quantity. Constraints 2) rule that each sawmill has to consume a minimum annual volume of wood from its supply agreement signed with government. Note that when \(b_{v,s}^{\text{min}} > 0\), these constraints implicitly force the company to keep mill \(s\) open, even if it is not profitable. Moreover, inter-sawmill supply flows are forbidden.

All of the following experiments assume that the reference price is of $450/tbf and the Spot vs contract-VMI price differential is equal to $40/tbf. This base case scenario is called scenario #0. Table 1 presents the alternative scenarios examined and the main results obtained by solving the proposed model with the SAA method.

Supply decrease

Recently, the Quebec government, after a thorough review of its forest practices in a public Commision, reduced the Annual allowable cut (AAC) by 20%, hence reducing the log
supply for each sawmill. Taking this change in forest policy into consideration, scenario #1 is devised to represent a corresponding decrease in seasonal supply. The upper bounds $b_{v,s,a}^{\text{max}}$ on the seasonal shipments of raw material between forest and sawmill for scenario #1 are derived from the data of scenario #0 by calculating $b_{v,s,a}^{\text{max}}(\#1) = b_{v,s,a}^{\text{max}}(\#0) \times (1 - 0.2)$. In this context, we also assume that the manager would be allowed to close some sawmills without loosing its supply agreement with government, which is acheived by the relaxation of equations 2), i.e by setting $b_{v,s}^{\text{min}}(\#1) = 0$.

<table>
<thead>
<tr>
<th>ID</th>
<th>Description</th>
<th>Production</th>
<th>DC</th>
<th>Markets</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>#0</td>
<td>Reference price = $450/tbf Spot vs contract/VMI price differential = $40/tbf</td>
<td>Valley</td>
<td>Boston</td>
<td>Sherbrooke</td>
<td>$1,782,499</td>
</tr>
<tr>
<td>#1</td>
<td>Scenario #0 with: $b_{v,s,a}^{\text{max}}(#1) = b_{v,s,a}^{\text{max}}(#0) \times (1 - 0.2)$ in 1) $b_{v,s}^{\text{min}}(#1) = 0$ in 2)</td>
<td>Valley</td>
<td>Boston</td>
<td>Sherbrooke</td>
<td>$3,713,543</td>
</tr>
<tr>
<td>#2</td>
<td>Scenario #1 with supply flows: $V_{\text{Valley}}(#2) = {\text{Valley, Mountain, River}}$ $V_{\text{River}}(#2) = {\text{Valley, Mountain, River}}$ $V_{\text{Mountain}}(#2) = {\text{Valley, Mountain, River}}$</td>
<td>Valley</td>
<td>Boston</td>
<td>Sherbrooke</td>
<td>$3,709,565</td>
</tr>
</tbody>
</table>

**Table 1 : Forest Policy Modification Experiments Results.**

After applying the SAA method, the expected value of the design selected for scenario #1 becomes twice as much as for the design selected for scenario #0, as seen in Table 1. As a result, River mill is shut down because its wood supply is too expensive: this saves seasonal and yearly fixed costs, as well as operations costs. However, the other two sawmills produce for the markets because they are profitable. The VMI logistics policies selected decrease, which leads to a decrease in the volume of production.

**Inter sawmill supply transfers**

Scenario #2 is derived from scenario #1. It evaluates the effect of allowing supply flows between sawmill forests in order to allow for more flexibility. In the current forest policy, the contractual agreement with enterprises specifies that each public forest management unit (PFMU) must be linked to a specific sawmill. This constrainst is very controversial at the moment and Scenario #2 evaluates the impact of relaxing it. In the model, this is specified through the definition of the set $V_s$ of vendors which can supply sawmills. In
scenario #1, for example, the supply set of Valley mill is restricted to \( V_{Valley} (#1) = \{Valley\} \), i.e. to the PFMU of Valley mill. In scenario #2, Valley mill can receive supply from its own forest and also from the River and Mountain PFMU, i.e. \( V_{Valley} (#2) = \{Valley, Mountain, River\} \). The design obtained by the SAA analysis for scenario #2 is identical to the one from scenario #1. Indeed, the River mill shuts down and the two other mills keep producing. There is no supply flow between forest and non-associated sawmills. This result is explained by long distances between the forests and sawmills: in the case considered, it is not profitable to proceed with transfers. However, it should be kept in mind that the permission to transfer wood supply allows a new degree of freedom to the manager to organize its production-distribution network. Other conditions of mill to forest distances and transportation costs would lead to the use of supply transfers, and generate significant differences between Scenario #1 and #2.

4.4 Evaluation of Acquisition and Rationalization Scenarios

This section studies acquisition and rationalization scenarios in order to take advantage of network synergies, considering the fact that, in practice, production equipment can be moved from one mill to another. To do this, scenario #2 is modified by adding the opportunity to acquire a sawmill, called Lone Patch, and to merge it with River mill. Figure 10 exposes the resulting sawmill network: two new potential rationalization layouts are considered at the Lone Patch mill (A merger layout) and at the River mill (B merger layout), respectively. The additional options considered are the following:

- Not acquiring the Lone Patch mill, which would result if the model does not select any of the layouts associated to Lone Patch;
- Acquiring Lone Patch and operating it as is at its current site, which would result if the model selects the Lone Patch Status quo layout;
- Acquiring Lone Patch and merging it with the River mill by moving all the equipment of the later to the Lone Patch mill location, which would result if the A merger layout is selected;
- Acquiring Lone Patch and merging it with the River mill by moving all the equipment of the former to the River mill location, which would result if the B merger layout is selected.

Moreover, the company can take advantage of inter-sawmill supply flows, as for scenario
#2. Table 2 presents two merger scenarios with different capacities for the Lone Patch mill, and different fixed and variable costs for A and B layouts.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>#3</th>
<th>#4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assumption</td>
<td>Lone patch mill capacity = River mill capacity</td>
<td>Lone patch mill capacity = 2 x River mill capacity</td>
</tr>
<tr>
<td>Layout</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>Fixed Cost</td>
<td>75% X (Lone + River)</td>
<td>75% X (Lone + River)</td>
</tr>
<tr>
<td>Variable Cost</td>
<td>90% X (Lone)</td>
<td>90% X (Lone)</td>
</tr>
</tbody>
</table>

Table 2 : Merger and Sawmill Scenarios Data.

Figure 10: Sawmill Network for the Acquisition and Merger Scenarios.

In the model, the following constraints must be added in order to ensure that the merger and rationalization options are considered adequately:

\[
\sum_{s \in S^m} \sum_{l \in L^m_s} Y_{ls} \leq 1 \quad m \in \mathcal{M}
\]

where,

\[
\mathcal{M} = \text{Set of potential merger options considered (} m \in \mathcal{M} \text{).}
\]

\[
L^m_s = \text{Set of potential site } s \in S^{md} \text{ facility layouts considered for merger option } m \in \mathcal{M} \text{ (} L^m_s \subseteq L_s \text{).}
\]
These constraints insure that not more than one layout associated to a merger option is selected. For the case described in Figure 10, they impose that the $A$ and $B$ layouts are not selected simultaneously, even if they are associated to different sites.

The optimal solution of the SAA model proposes the installation of the $A$ layout for the two scenarii as shown in Table 3. It is seen that the acquisition of the Lone Patch mill raises the expected return of the company significantly, to $M12.6, and that, in the optimal design, a new distribution center is installed at Bangor and the number of contract offers (logistics policies) worth pursuing increase significantly. It is interesting to note that, in the optimal design, the River mill wood supply is allocated to the merged Lone Patch mill because the two sawmills are geographically close and logistics supply costs are synergetic. Moreover, the reduction of fixed and variable costs offsets the high price of the River mill supply.

<table>
<thead>
<tr>
<th>Experiments</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>ID</td>
<td>Description</td>
</tr>
<tr>
<td>#3</td>
<td>Scenario #2 with: Lone Patch capacity = River capacity</td>
</tr>
<tr>
<td>#4</td>
<td>Scenario #2 with: Lone Patch capacity = 2*River capacity</td>
</tr>
</tbody>
</table>

Table 3: Acquisition and Rationalization Experiments Results.

These examples of how to use our methodology show amply how it can help managers to cope with significant changes in their business environment. They also show that the use of a stochastic programming model in our context is far superior to the use of a traditional deterministic mixed integer programming model.

5 Conclusion

As was demonstrated in the previous section, the methodology proposed in this paper can effectively capture the relationship between the production-distribution resources deployed by a company and its market opportunities. Instead of optimizing the supply chain network to cope with expected future demands with a low probability of occurrence, as is usually done in the literature, the stochastic programming model proposed considers several future market environments and it deploys the production-distribution network to preposition the
company to capture profitable opportunities. The methodology is also able to guide managers in their reengineering efforts to adapt their supply chain network to changes in governmental forest policies. It can be put to work usefully to analyze business opportunities in areas in line with the global trends presented in introduction, especially the trend towards increasing mergers and rationalization in the very fragmented lumber industry.

The approach is useful not only to industrial managers but also to governmental policy makers. Indeed, the economic consequences of forest policies on the lumber industry, and on the regions where it is implemented, can be analyzed in a comprehensive manner. An application of our model to a representative sample of lumber industry companies could help quantify the overall economic impact of governmental decisions. In particular, the decrease in Annual Allowable Cut or allowing for inter sawmill supply transfers can be analyzed to determine their expected impact on company profitability, and on sawmill shutdowns in the regions where the forest industry is essential. These analyses, combined with studies of indirect economic impacts (indirect jobs and activities, taxation revenues…), can quantify the impact of the policy considered on the prosperity of the regions concerned. By using the model in this way, policy makers can simulate the impact of various policies, and eventually make better decisions.

The methodology proposed is based on a comprehensive two-stage stochastic program with recourse solved with a sample average approximation (SAA) method based on a Monte Carlo sampling technique. Our experiments showed that this approach gives much better supply chain designs than a comparable deterministic model based on averages. When a reasonable environment sample size is used (25 environments in our case), the resulting equivalent deterministic MIP is not much more difficult to solve than the MIP obtained for average demands because the number of binary variables in the model remains the same. It therefore seems that the additional efforts required to model supply chain network design problems as stochastic programs are well worth it.
6 References


Appendix

This appendix presents the mixed integer programming model to solve when the SAA method is used, with a sample $\Omega_N$ of environments, to obtain good solutions for the stochastic programming model proposed to design the supply chain networks of Quebec lumber companies. We start by defining the additional notation required and then we formulate the model.

Additional notation

Notation required to define the manufacturing process and the business environment:

- $A^i_a$ = Set of immediate predecessors of activity $a$ in the process graph ($A^i_a \subset A$).
- $FP$ = Set of product families sold on the market ($p \in FP \subset P$).
- $SP^p$ = Set of substitutes for product family $p \in FP$ ($SP^p \subset FP$).
- $SP_p$ = Set of products which can be substituted by product family $p \in FP$ ($SP_p \subset FP$).
- $T$ = Set of seasons in the planning horizon ($t \in T$).
- $O$ = Set of countries covered by the logistics network ($o \in O$).
- $o(n)$ = Country of geographical location $n$.
- $SM$ = Set of spot markets $m \in SM = \bigcup_{o \in O} SM_o$.
- $C$ = Set of contracts $c \in C = \bigcup_{o \in O} C_o$.
- $VM$ = Set of VMI agreements $c \in VM = \bigcup_{o \in O} VM_o$.
- $x_{ct}$ = Quantity of product $p_c$ demanded in contract/agreement $c \in C \cup VM$ during season $t \in T$.
- $c(i)$ = Contract/agreement $c \in C \cup VM$ for which logistics policy $i$ is considered.

Notation required to define potential facilities and potential moves in the network:

- $S$ = Set of potential network sites ($s \in S = S^{pd} \cup S^d$).
- $S_o$ = Set of sites located in country $o \in O$ ($S_o \subset S$).
- $S^o_{ps}$ = Set of potential sites (output destinations) which can receive product $p$ from site $s \in S$.
- $S^i_{ps}$ = Set of potential sites (input sources) which can ship product $p$ to site $s \in S$. 
The Strategic Design of Forest Industry Supply Chains

\[ S^i_{m} = \text{Set of potential sites (input sources) which can ship product } p \text{ to spot market } m \in SM . \]

\[ S^i = \text{Set of facilities } s \in S \text{ the company could use to comply with the terms of logistics policy } i, \text{i.e. to ship product } p_{c(i)} \text{ to location } d_{c(i)}. \]

\[ SM_{ps} = \text{Set of spot markets which can receive substitute products } p \text{ from node } s, \text{i.e.} \]

\[ SM_{ps} = \left\{ m \mid s \in S^i_{m}, p \in \bigcup_{p \in P_{P_{SM}}} SP \right\}. \]

\[ V_{ps} = \text{Set of forest areas which can supply product } p \text{ to site } s \in S \left( V_{ps} \subseteq V_s \right). \]

Notation required to define technologies, recipes and wood supply flows:

\[ KM_{sa} = \text{Production technologies which can be used to perform activity } a \in A^p \text{ on site } s \left( KM_{sa} \subseteq KM_a \right). \]

\[ KS_{sa} = \text{Storage technologies which can be used to perform activity } a \in A^r \text{ on site } s \left( KS_{sa} \subseteq KS_a \right). \]

\[ b_{p,p}^\varphi = \text{Quantity of product } p \text{ obtained from one unity of product } p_{\varphi} \text{ with recipe } \varphi \in R_{ak}. \]

\[ a^\varphi = \text{Production capacity required to process one unit of product } p_{\varphi} \text{ with recipe } \varphi \in R_{ak}. \]

\[ q_{p,a} \] = Capacity consumption rate per unit of product \( p \in P^a \) for storage activity \( a \in A^r \).

\[ Pr_{p,v,s,a} = \text{Historical proportion of products of family } p \in P^a \text{ in the stems (} a=2 \text{) or logs (} a=4 \text{) supplied by forest area } v \in V \text{ to site } s \in S^{pd}, \text{when bucking in done in the sawmill (} a=2 \text{) or in the forest (} a=4 \text{).} \]

Notation required to define facility layouts and capacity options:

\[ J_s = \text{Potential capacity options which can be installed on site } s \in S^{pd} \left( j \in J = \bigcup_{s \in S^{pd}} J_s \right). \]

\[ J_{ks} = \text{Potential technology } k \text{ capacity options which can be installed on site } s \in S^{pd} \left( J_{ks} \subseteq J_s \right). \]

\[ JR_{ls}^n = \text{Mutually exclusive options subset in } J_{ls} \left( n \in \{1, \ldots, N_{ls}\} \right). \]

\[ N_{ls} = \text{Number of mutually exclusive option subsets (equipment replacement / reconfiguration) in } J_{ls} \subseteq J_s. \]

\[ E_{ls} = \text{Total area of the layout } l \text{ of site } s. \]

\[ e_j = \text{Area required to install capacity option } j. \]
\[ b_j^t \quad = \quad \text{Capacity of the technology associated to option } j \text{ available for season } t. \]
\[ b_{skt} \quad = \quad \text{Technology } k \text{ capacity available in season } t \text{ for distribution site } s \in S^d. \]

Notation required to model costs and revenues:

\[ A_{ls} \quad = \quad \text{Fixed cost of using layout } l \text{ on site } s \in S^{nd} \text{ for the planning horizon.} \]
\[ A_{ps} \quad = \quad \text{Fixed cost of disposing of production-distribution site } s \in S^{nd} \text{ at the beginning of the planning horizon.} \]
\[ A_s \quad = \quad \text{Fixed cost of using distribution site } s \in S^d \text{ for the planning horizon.} \]
\[ a_{j}^d \quad = \quad \text{Fixed cost of disposing of capacity option } j \text{ at the beginning of the planning horizon.} \]
\[ a_{j}^l \quad = \quad \text{Fixed cost of installing or keeping capacity option } j \text{ for the planning horizon.} \]
\[ \hat{a}_{jt} \quad = \quad \text{Fixed cost of using capacity option } j \text{ during season } t. \]
\[ c_{p,\wp}^{\phi} \quad = \quad \text{Cost of producing one unit of product } p_{\wp} \text{ with recipe } \wp \text{ on site } s \text{ during season } t. \]
\[ m_{psnt} \quad = \quad \text{Unit handling cost for the transfer of product } p \text{ to or from its stock in production-distribution site } s \text{ during season } t. \]
\[ f_{p,sn}^{o} \quad = \quad \text{Unit cost of the flow of product } p \text{ between site } s \text{ and node } n \text{ paid by origin } s \text{ during season } t \text{ (this cost includes the customer-order processing cost, the shipping cost, the variable transportation cost and the inventory-in-transit holding cost).} \]
\[ f_{p,sn}^{t} \quad = \quad \text{Unit transportation cost of product } p \text{ from site } s \text{ to node } n \text{ during season } t \text{ (this cost is included in } f_{p,sn}^{o}). \]
\[ f_{p,ds}^{d} \quad = \quad \text{Unit cost of the flow of product } p \text{ between node } n \text{ and site } s \text{ paid by destination } s \text{ during season } t \text{ (this cost includes the supply-order processing costs and the receiving cost).} \]
\[ f_{p,sv,at}^{v} \quad = \quad \text{Unit cost of the flow of product } p \text{ between forest area } v \text{ and activity } a \text{ on site } s \text{ paid by destination } s \text{ during season } t \text{ (this cost includes the product’s price and the variable transportation cost).} \]
\[ h_{psnt} \quad = \quad \text{Unit inventory holding cost of product } p \text{ in facility } s \text{ during season } t. \]
\[ \pi_{p,psnt} \quad = \quad \text{Transfer price of product } p \text{ shipped from site } s \text{ during season } t. \]
\( e_{oo'} = \) Exchange rate, i.e. number of units of country \( o \) currency by units of country \( o' \) currency (the index \( o = 0 \) is given to the base currency, whether it is part of \( O \) or not).

\( \delta_{pns} = \) Import duty rate applied to the CIF price of product \( p \) when transferred from the country of node \( n \) to the country of site \( s \).

\( P_{pms,lt} = \) Price obtained for product \( p \in P_m \) on spot market \( m \in SM \), at the level \( \lambda \in \Lambda_{pnm} \) of the price step function, during season \( t \in T \).

\( X_{pms,lt} = \) Largest quantity of product \( p \in P_m \) which can be sold on spot market \( m \in SM \), at the level \( \lambda \in \Lambda_{pnm} \) of the price step function, during season \( t \in T \) \( (X_{pms,lt} = 0 \) and \( X_{pms,lt} \approx +\infty \).

\( \kappa_d = \) Proportion of the demand of spot market \( m(d) \in SM_o \) in each demand zone \( d \in D_{pm(d)} \) for each product \( p \in P_{m(d)} \).

\( K_i = \) Fixed cost incurred for the implementation of logistics policy \( i \).

\( P_{it} = \) Price of the product associated to logistics policy \( i \) during season \( t \in T \).

Parameter required to compute inventory holding costs:

\( \rho_{pmt} = \) Number of seasons of inventory (order cycle and safety stocks) of product \( p \) kept at site \( s \) for season \( t \) (this is the inverse of the familiar inventory turnover ratio).

**MIP model formulation**

In the model, first stage decision variables are constrained mainly by supply, capacity deployment and flow equilibrium relationships. The supply of wood from forest areas is modeled using constraints 1) and 2), in the text, together with constraints on the historical proportions of stem and log inflows, namely:

\[
F_{p(v,3)(s,a)} = Pr_{p(v,3)(s,a)} \sum_{p' \in P_{p(v,3)(s,a)}^{out}} F_{p(v,3)(s,a)}^{out} a \in A_1^{out}, p \in P_1^{out} \cap P_2^{in}, s \in S^{pvd}, v \in V, t \in T
\]

The following constraints are required to ensure that layouts are selected correctly:

\[
\sum_{l \in L_s} Y_{ls} + Y_{0s} = 1 \quad s \in S^{pad}
\]

\[
\sum_{j \in J_a} e_j Z_j \leq E_{ib} Y_{ib} \quad s \in S^{pad}, l \in L_s
\]
\[ \sum_{j \in P_{\text{pd}}} Z_j \leq 1 \quad s \in S^{pd}, l \in L_s, n = 1, \ldots, N_{ls} \quad 6) \]

Constraints 4) ensure that a single layout is selected for each production-distribution site, constraints 5) that the area required by the selected capacity options does not exceed the area available in the selected layout, and 6) that mutually exclusive options are not selected. The following constraints verify that a capacity option can be used in a season only if it was installed:
\[ \hat{Z}_{jt} \leq Z_j \quad s \in S^d, j \in J_s, t \in T \quad 7) \]

The flow equilibrium constraints of the inventory and production activities are the following:
\[ \sum_{k \in K_{sa}} X^p_{psat} \leq \sum_{a' \in A_a^{in}} F_{p(s,a')(s,a)l} + \sum_{a' \in A_a^{in}} F_{p(s,a)(s,a)l} \quad 8) \]
\[ a \in A^p, p \in P_a^{in}, s \in S^{pd}, t \in T \]

\[ \sum_{a' \in A_a^{out}} \sum_{s \in S^{ps}} F_{p(s,a)(s,a)l} + \sum_{a' \in A_a^{out}} F_{p(s,a)(s,a)l} \leq \sum_{k \in K_{sa}} \varphi_{p,s,a} \sum_{p \in P_a^{out}} g_{p,s,a} X^p_{ps} \quad 9) \]
\[ a \in \{3, 4, 6\}, p \in P_a^{out}, s \in S^{pd}, t \in T \]

\[ \sum_{k \in K_{s2}} I_{pka} = \sum_{k \in K_{s2}} I_{pka-1} + \sum_{a' \in A_a^{is}} F_{p(s,a)(s,a)l} + \sum_{a' \in A_a^{is}} F_{p(s,a)(s,a)l} - \sum_{a' \in A_a^{out}} \sum_{s \in S^{ps}} F_{p(s,2s,a)l} - \sum_{a' \in A_a^{out}} F_{p(s,2s,a)l} \quad 10) \]
\[ p \in P_a^{in}, s \in S, t \in T \]

\[ \sum_{k \in K_{s2}} I_{p0} = \sum_{k \in K_{s2}} I_{p0} + \sum_{k \in K_{s2}} I_{p0} \quad 11) \]

Production capacity restrictions are enforced by these constraints:
\[ \sum_{p \in P_a^{in}} q_{p,s} X^p_{ps} \leq \sum_{j \in J_s} b_j \hat{Z}_{jt} \quad a \in A^p, s \in S^{pd}, k \in K_{sa}, t \in T \quad 12) \]
\[ \sum_{p \in P_a^{in}} q_{p,s} (\sum_{a' \in A_a^{out}} F_{p(s,2s,a)l} + \sum_{a' \in A_a^{out}} F_{p(s,2s,a)l}) \leq \sum_{k \in K_{s2}} \sum_{j \in J_s} b_j \hat{Z}_{jt} \quad s \in S^{pd}, t \in T \quad 13) \]

The second stage decision variables are constrained mainly by capacity, flow equilibrium and demand constraints. In order to simplify the formulation, we first define working variables \( F_{ps}(\omega) \) representing the outflow of product \( p \) from site \( s \) in season \( t \). The definition of these working variables requires the following constraints:
\[ F_{pm}(\omega) = \sum_{m \in SM_p} \sum_{p \in P_m} \sum_{d \in D_p} \sum_{\omega \in \Lambda_{pm}} F'_{pm,d}(\omega) + \sum_{c \in C, \delta(M \cap S_{p_m})} \sum_{i \in I_c} F_{pm,i}(\omega) \quad \omega \in \Omega_N, t \in T, p \in P, s \in S \]

The flow equilibrium constraints of the chipping activity \((a=5)\) are the following:

\[ F_{pm}(\omega) \leq \sum_{k \in K, \omega \in \Omega_N} \sum_{p \in P_{out}^s} g^a_{p,p'} X^a_{p_{out},p'} p \in P^i, s \in S_p, t \in T, \omega \in \Omega_N \]

The flow equilibrium constraints of the planing and grading activity \((a=7)\) are the following:

\[ \sum_{s \in S_{ps}} F_{p(s,7)(s,8)}(\omega) + F_{p(s,7)(s,8)}(\omega) \leq \sum_{k \in K, \omega \in \Omega_N} \sum_{p \in P_{out}^s} g^a_{p,p'} X^a_{p_{out},p'} \quad p \in P^i, s \in S_p, t \in T, \omega \in \Omega_N \]

The flow equilibrium constraints of the storage activity \((a=8)\) are the following:

\[ \sum_{k \in K} I_{pk,0} = \sum_{k \in K} I_{pk,1}(\omega) + \sum_{s \in S_{ps}} F_{p(s,7)(s,8)}(\omega) \]

The capacity constraints for the storage activity \((a=8)\) are the following:

\[ \sum_{p \in P^i} q_{p} F_{pm}(\omega) \leq \sum_{k \in K} \sum_{j \in I_c} b_{k} \tilde{Z}_{j} s \in S_p, t \in T, \omega \in \Omega_N \]

\[ \sum_{p \in P^i} q_{p} F_{pm}(\omega) \leq (\sum_{k \in K} b_{k}) Y_{s} s \in S_p, t \in T, \omega \in \Omega_N \]

Finally, demand constraints and logistics policies are modeled by:

\[ \sum_{p' \in P'} \sum_{s \in S_n} F_{p',s}(\omega) \leq \kappa_{d}(X_{pm,\lambda} - X_{pm,\lambda-\lambda}) \quad \omega \in \Omega_N, t \in T, o \in O, m \in SM_p, p \in P_m, d \in D_p, \lambda \in \Lambda_{pm} \]

\[ \sum_{p' \in P'} \sum_{s \in S_n} F_{p',s}(\omega) = x_{ct} \quad \omega \in \Omega_N, t \in T, o \in O, c \in SC_o \cup SVM_o \]

\[ \sum_{p' \in P'} \sum_{s \in S_n} F_{p',s}(\omega) = X_{ct} \tilde{Z}_{i} \quad \omega \in \Omega_N, t \in T, o \in O, c \in PC_o \cup PVM_o, i \in I_c(\omega) \]

\[ \sum_{i \in I_c} \tilde{Z}_{i} \leq 1 \quad c \in PC_o \cup PVM_o, \forall o \in O \]

To write the objective function, the following additional notation is needed:

\[ \bar{C}_s \quad = \text{Sample average of total site } s \text{ expenses for the planning horizon.} \]
The Strategic Design of Forest Industry Supply Chains

\[ \bar{R}_s = \text{Sample average of total site } s \text{ revenues for the planning horizon.} \]

Then, using the expenditure and revenue elements in Table 4, it is seen that:

\[ \bar{C}_s = \frac{1}{N} \{ (a) + b) + c) + d) + e) + f) + g) + h) + i) + j) \} \quad s \in S^{pd} \tag{24} \]

\[ \bar{C}_s = \frac{1}{N} \{ (a) + b) + c) + e) + f) + g) + i) + j) \} \quad s \in S^d \tag{25} \]

\[ \bar{R}_s = \frac{1}{N} \{ (k) + l) \} \quad s \in S \tag{26} \]

The sample average operating income for each country \( o \in O \), is given by

\[ \bar{M}_o = \sum_{s \in S_o} (\bar{R}_s - \bar{C}_s) - \sum_{c \in C_o} \sum_{i \in I_c} K_i \hat{Z}_i \]

and the sample average corporate net revenues in the reference currency are \( \sum_{o \in O} e_{oo} \bar{M}_o \).

Based on this, it is seen that the Sample Average Approximation program to solve is the following:

\[
\max \sum_{o \in O} e_{oo} \left[ \sum_{s \in S_o} (\bar{R}_s - \bar{C}_s) - \sum_{c \in C_o} \sum_{i \in I_c} K_i \hat{Z}_i \right]
\]

subject to constraints 1) to 26), and to the non-negativity constraints:

\[
Y_{is} \in \{0;1\} \quad s \in S^{pd}, i \in I_s \quad Y_{ox} \in \{0;1\} \quad s \in S^{pd}; \quad \hat{Z}_i \in \{0;1\} \quad i \in I_c, c \in PC_o \cup PVM_o
\]

\[
Z_j \in \{0;1\} \quad s \in S^d, j \in J_s; \quad \hat{Z}_j \in \{0;1\} \quad t \in T, s \in S^{pd}, j \in J_s; \quad Y_s \in \{0;1\} \quad s \in S^d
\]

\[
F_{p(n,a)(n',a')}(\omega) \geq 0 \quad p \in P, (n,a) \in (V \cup S) \times A, (n',a') \in (V \cup D_p) \times A, t \in T
\]

\[
X_{i,t}^{j} \geq 0 \quad s \in S^{pd}, a \in A^r, k \in KM_{sa}, i \in R_{ak}, t \in T \tag{27}
\]

\[
I_{pktr} \geq 0 \quad p \in P, s \in S^{pd}, a \in A^r, k \in KS_{sa}
\]

\[
F_{p(n,a)(n',a')}(\omega) \geq 0 \quad p \in P, (n,a) \in S \times A, (n',a') \in (S) \times A, t \in T, \omega \in \Omega_N
\]

\[
I_{pkr}(\omega) \geq 0 \quad p \in P, s \in S^{pd}, a \in A^r, k \in KS_{sa}, t \in T, \omega \in \Omega_N
\]

\[
F_{psa}(\omega) \geq 0 \quad t \in T, \omega \in \Omega_N, p \in P, s \in S^{pd} \cup S^d
\]

\[
F_{psa}(\omega) \geq 0 \quad \omega \in \Omega_N, o \in O, c \in C_o \cup VM_o, p' \in PS_{pe}, s \in S^{pd} \cup S^d, i \in I_c(\omega), t \in T
\]

\[
F_{ppsd}(\omega) \geq 0 \quad \omega \in \Omega_N, m \in MS_o, p \in P_m, p' \in SP^p, s \in S^{pd} \cup S^d, d \in D_m, \lambda \in \Lambda_{pm}, t \in T
\]

\[
\bar{C}_s \geq 0 \quad s \in S^{pd} \cup S^d \quad \bar{R}_s \geq 0 \quad s \in S^{pd} \cup S^d
\]
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<table>
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<td>l) Outflows to demand zones</td>
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### Table 4: Facilities Expenses and Revenues in Local Currency for a Given Environment.