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The Effect of Multi-Scenario Policies on Empty Container Repositioning under Uncertainty and Data Shortage

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Abstract. This study addresses a repositioning problem where some ports impose several restrictions on the storage of empty containers, sailing distances are short, information becomes available close to decision times and decisions are made in a highly uncertain environment. Although point forecasts are available and probabilistic distributions can be derived from the historical database, specific changes in the operational environment may give rise to the realization of parameters that were never observed in the past. Since historical statistics are useless for decision-making processes, we propose a time-extended multi-scenario optimization model in which scenarios are generated by shipping company opinions. We then show the importance of adopting multi-scenario policies compared to standard deterministic ones.

Keywords. Empty container repositioning, mathematical modeling, time-extended optimization models, multicommodity flow problems, optimization under uncertainty.

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1. Introduction

Freight transportation systems are often characterized by directional imbalances in the flows of goods, leading to the accumulation of transportation equipment in import-dominant areas and to its shortage in export-dominant ones. To correct these imbalances, transportation equipment must be repositioned from areas with a surplus to areas with a scarcity. This paper addresses the repositioning of empty containers in a scheduled maritime system. This distribution process consists in the planning of empty container distribution so as to minimize inventory, handling and transportation costs, while at the same time meeting demand and supply requirements in every port. The maritime repositioning of empty containers represents a crucial activity for shipping companies. According to their strategies, the demand for empty containers at ports must be met, to take advantage of future transportation opportunities and to reduce the risk of competitors providing containers as requested (Crainic, 2003).

This paper is motivated by the analysis of the operational context of a shipping company operating in the Mediterranean Sea. This case study exhibits some characteristics not described in previous papers. Some ports do not offer long-term storage space for empty containers, whereas others do. Navigation times are significantly short. Typically, while vessels are berthing in a given port, the shipping company must make decisions on loading and unloading operations for the next ports. Moreover, empty container loading operations can be interrupted, to reduce delays accumulated during terminal operations. As a result, precise information on vessel compositions is usually not available. Generally speaking, information becomes available close to decision times and some crucial parameters are still uncertain when the shipping company must decide how to reposition its empty containers.

An important feature of this issue is the lack of adequate statistics based on historical data, to estimate uncertain parameters. Although point forecasts are available and probabilistic distributions can be derived, they are useless for decision-making processes, because the operational environment is expected to change significantly. As a consequence, uncertain parameters may take values that are not correlated to past ones.

In order to solve empty container repositioning problems, the literature has proposed both deterministic and stochastic optimization models. On the one hand, deterministic models assume perfect knowledge of what information will be available. They may yield low-quality repositioning plans for the needs of shipping companies. Since they allocate empty containers according to expected demand and supply values, they can provide an insufficient number of empty containers when larger demand or lower supply values will be eventually observed. On the other hand, stochastic optimization models require a good knowledge of uncertain parameter distributions to be adopted successfully. To our knowledge, there is currently no optimization model aimed at the repositioning of empty containers, when historical data are inappropriate for estimating uncertain parameters.

This paper aims to illustrate a modeling process to take into account the uncertain nature of parameters, which cannot be estimated by historical data. In this approach, shipping company opinions are used to determine the distributions of uncertain parameters as sets of a limited number of values. Then, we generate one scenario for every joint realization of uncertain parameters. All scenarios are collected in a multi-scenario multi-commodity time-extended optimization model and linked by non-anticipativity constraints. Our experimental tests show that multi-scenario policies are by far better for shipping company needs, because they can allocate more empty containers compared to deterministic ones and satisfy a larger number of potential requests.

The remainder of the paper is organized as follows. Section 2 describes the operational context for a shipping company in dealing with empty container repositioning. In Section 3 we present the most important parameters of the maritime repositioning issue and their connections with shipping company decisions. Section 4 presents a brief survey of the literature. Section 5 is dedicated to the description of how we model the problem. For the sake of clarity, we describe system dynamics in Section 5.1, we introduce a deterministic single-scenario formulation in Section 5.2 and we present the multi-scenario one in Section 5.3. In Section 6 we compare multi-scenario and deterministic policies. Finally conclusions are drawn in Section 7.

2. Problem context

Directional imbalances in trade result in accumulations of unnecessary containers in import-dominant ports and in shortages in export-dominant ones. As a consequence, empty containers must be repositioned between ports, to ensure the continuity of shipping companies' activity and meet future transportation opportunities.

Some shortcomings have been observed in the repositioning processes of a shipping company operating in the Mediterranean Sea. At the moment, this company does not adopt any decision support system for this activity. Typically it allocates its empty containers to export-dominant areas (for example the Far East), where transportation demand is relevant, but it does not take into account requests arising in other ports of the Mediterranean Sea. As a consequence, this logistic practice risks failing to provide containers where they may be needed and missing some transportation opportunities.

Inefficiencies also arise because of usual recourse to the so-called "Cut-and-Run" practice. It is the conclusion of terminal operations for vessels, that leave ports without loading part of empty containers which were supposed to be loaded. Typically, the shipping company cuts empties while terminal operations are in progress, in order to reduce delays accumulated in ports and avoid rushing to the next ports. Failure to load empty containers on vessels may result in the risk of not providing containers where they may be required and may also generate avoidable storage costs. Moreover, "Cut-and-Run" practices and the short distances of some maritime segments result in imprecise information on the composition of vessels, when loading and unloading decisions must be made.

In order to avoid the informal decision-making processes observed during cut-and-runs, the shipping company will significantly benefit from a decision support system for repositioning its empty containers. This tool is expected to determine how many empties must be loaded, unloaded, repositioned on vessels and stored in ports. Such decisions must be implemented as planned. It is worth noting that, since this tool targets the elimination of cut-and-runs, the composition of vessels will no longer represent a source of uncertainty, when the shipping company must decide how to reposition its containers.

Nevertheless, significant sources of uncertainty still affect this issue. Information on the number of empty containers requested in each port is imprecise, because unexpected transportation demands may arise. Moreover the number of empty containers available in ports is uncertain, because we do not know precisely when they will be returned by import customers. Since loaded containers have greater priority than empties and unexpected transportation opportunities may arise, the residual transportation capacity for empty containers on vessels also fails to provide certain information (Cheung and Chen, 1998). Moreover, the maximum number of empty containers that can be loaded and unloaded in ports is uncertain as well.

Low-quality information may be captured from the historical database to estimate how many empty containers will be available in ports. It provides point forecasts and some probabilistic distributions can be derived as well. According to this database, the expected number of empty containers available in ports is notably increased by the massive application of cut-and-runs. Moreover the expected elimination of cut-and-runs will enable the shipping company to satisfy a larger number of transportation opportunities than in the past. Therefore, the company expects a raise in the distribution of loaded containers, that will be put on vessels, hauled and delivered to customers. The historical database currently does not expect this growth in the distribution of loaded containers, resulting in an overestimation of loading-unloading capacity and residual transportation capacity for empty containers. Finally, the expected delivery of larger volumes of loaded containers will result in larger numbers of empty containers available in the landside, reducing the demand for them at ports. At the moment, the historical database significantly overestimates the request for empty containers in ports with respect to the subjective expectations of the shipping company.

3. Problem definition

In this section we describe the main decisions and elements characterizing the maritime repositioning of a heterogeneous fleet of empty containers, that may come in different types and two main sizes (*20* ft and *40* ft containers). Fundamentally decisions are concerned with where and when logistic operations start and finish. Such decisions require a large array of information, such as supply and demand levels at ports, the available transportation capacity and the specific relations with the operating environment (e.g. possible restrictions imposed by ports). These decisions are highly important, because they influence the inventory levels at ports and the subsequent distribution of containers in the landside.

The physical network is made up of ports. They are linked by vessels, where containers can be loaded and unloaded. An illustrative network is drawn in Figure 1. It shows two vessels, represented by rhombus shaped nodes and denoted by numbers *1* and *2*. They sail between ports represented by circular nodes and denoted by letters from *A* to *E*. A link from a

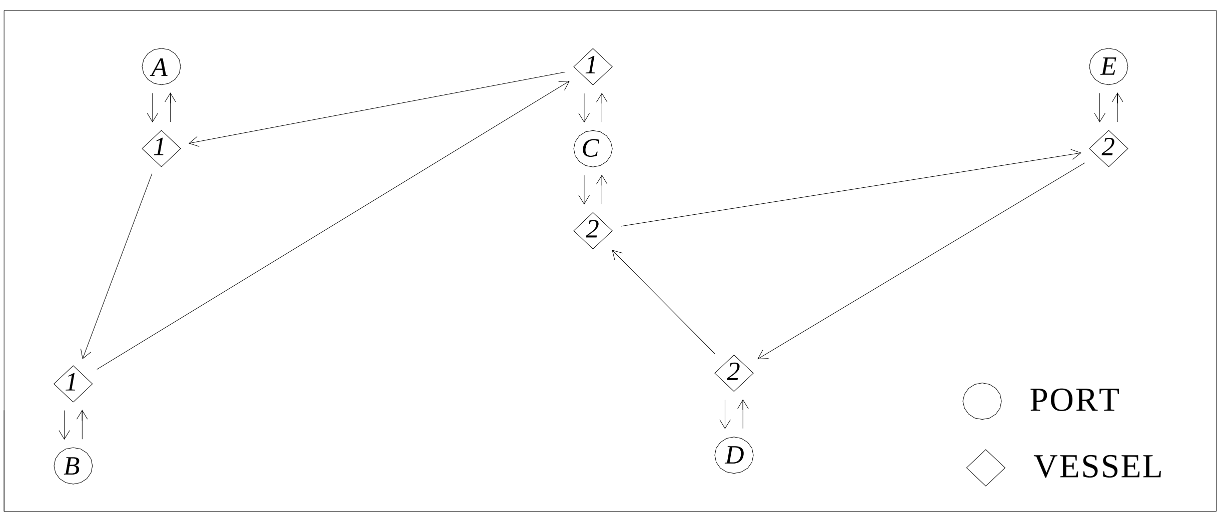

Figure 1. Possible movements of two vessels in a maritime network made up of five ports

port to a vessel represents the option of loading empty containers on that vessel, whereas a link in the opposite direction represents the possibility of unloading containers. In this picture, vessel *1* sails from port *A* to port *B*, then from *B* to *C* and subsequently returns to *A*, from where this transportation line is repeated. Both vessels berth in port *C*, so that containers shipped from one port can reach any other port.

The backbone of the maritime repositioning system is represented by ports. Since door-to-door transportation must be ensured, ports are intermodal transit facilities for empty containers in order to meet potential transportation requests arising in the landside (they also store loaded containers, but this aspect is not addressed in this paper). At this planning level the set of ports to be considered is fixed. Shipping companies must make decisions on the number of empty containers of different types to be stored in ports. Ports impose restrictions on the storage of empty containers. Typically, large ports have specific areas for depositing empties, which may be kept in stock, for a price, for as long as shipping companies need. Small ports do not have dedicated areas for empty containers, which have limited dwelling times. Due to significant storage costs charged by ports, shipping companies sometimes prefer storing empty containers in cheap depots on the landside.

In order to introduce transportation decisions, we need to define the demand and the supply of empty containers at ports. In some ports the number of inbound loaded containers is larger than the number of outbound loaded containers. These ports have a surplus of empty containers of different types, that become available over time. In these cases, we refer to the empty container supply, that represents the number of empty containers of a given type available in a given port at any given time. They can be repositioned in other ports where they may be requested or stored, according to ports' restrictions.

In other ports the number of inbound loaded containers is lower than the number of outbound loaded containers. These ports have a deficit of empty containers, that must be provided on time to take advantage of transportation opportunities arising in the landside. Therefore, empty container demand can be defined as the number of empty containers of a given type requested in a given port at any given time. It is worth noting that a port may provide a supply of containers of a given type in a given period and may require containers of different types in the same period.

The demand for empty containers must be met using the available supply, that is shipping companies need to reposition empty containers between ports. This activity is performed by vessels sailing well-established routes according to tight schedules. They provide timing information about the routes of vessels, that is arrival and departure times at ports. Vessels carry both loaded and empty containers. Loaded containers have greater priority than empties, because they generate profits for shipping companies. Therefore, shipping companies must decide how many containers of each type must be repositioned by vessels, using their residual transportation capacity.

The dynamic characteristics of supply and demand at ports generate a need for decisions as to when empty containers must be moved from one port to another. As a consequence, the time-dependency of transportation decisions characterizes the relationship between shipping companies and ports. Shipping companies must decide how many empty containers will be loaded and unloaded from vessels before their arrival in ports, so that ports can organize their internal activities on time and all containers can be repositioned as requested.

It is worth distinguishing between loading decisions in small and large ports. Due to space limitations, small ports receive empty containers arriving from the landside, only if they are assigned to the first vessels scheduled to berth. In this case, shipping companies must devote their attention to the expected number of empty containers arriving from the landside and decide the most "suitable" vessels for them before their arrival in ports. On the other hand, large ports do not impose restrictions on empty containers, that can be long-term stored or assigned to vessels one day before their arrival. It is worth noting that both assigned and unassigned containers use available storage capacities, while awaiting incoming vessels.

Moreover, shipping companies must decide the number of empty containers to be unloaded one day before the arrival of vessels. These containers can be dispatched in the landside to meet transportation requests or stored, depending on the different restrictions imposed by ports. Small ports allow keeping in stock these containers for few days, whereas they can be long-term stored in large ports.

One of the major difficulties in this problem is that some parameters are uncertain, when decisions must be made. The uncertain parameters we consider in this issue are future demands at ports, future numbers of containers arriving from the landside, residual transportation capacities and maximum number of empty containers that can be loaded and unloaded. Uncertainty on empty container demand at ports depends on unexpected transportation opportunities arising in the landside. The number of empty containers available in ports is uncertain, because customers can hold containers for several days. It is not simple to estimate when they will be returned and how much time is needed to move them to ports. Moreover, unexpected transportation opportunities result in loaded containers modifying both the residual transportation capacity and the loading-unloading capacity, which depends on the time spent by vessels in ports and on the time necessary to place and discharge loaded containers from vessels. While a part of these parameters is known, others do not offer precise information, especially if they are associated with farther periods in the planning horizon.

The current shortage of adequate information on uncertain parameters prevents adopting deterministic optimization models to solve this issue, because they assume knowledge of what information is going to arrive in the future. Since shipping companies must meet the demand for empty containers at ports, deterministic formulations are highly unsuitable for their needs, because they do not ensure high demand fulfillment percentages for the different values that may be taken by uncertain parameters. On the other hand, we cannot adopt stochastic programming techniques based on historical data, because this is not the case. In fact, they cannot be exploited to derive probabilistic distributions and generate an appropriate scenario tree (Powell and Topaloglu, 2003).

In the next section we will briefly review the literature on this issue.

4. Literature review

In recent years some shipping companies have adopted decision support systems based on mathematical programming models and algorithms, to reposition empty containers. This is a consequence of the intensive research developed over past years in this field.

Dejax and Crainic (1987) reviewed past papers in the management of empty flows. In their opinion, although this topic had received much attention since the sixties, little consideration had been dedicated to the development of specific models in the container transportation issue. They mentioned few authors addressing the problem of allocating empty containers in a dynamic environment, taking into account a single joint realization of uncertain parameters. Network optimization and linear programming algorithms were usually adopted as resolution techniques.

Crainic *et al.* (1989) discussed the problem of locating empty containers in an intercity freight transportation system on behalf of a shipping company. They proposed an optimization model in order to minimize the cost of depot opening and container transportation, while satisfying customer demands. This issue belongs to a strategic planning level and aims to identify a suitable set of inland depots supporting the activity of ports.

Crainic *et al.* (1993) presented a general framework to address the specific characteristics of the empty container allocation problem in the context of an inland distribution system for a maritime shipping company. They developed a deterministic time-extended single-commodity model and a multicommodity formulation in order to minimize total inland operating costs. They also provided a restricted recourse formulation to deal with the stochastic characters of demands and supplies.

Lai *et al.* (1995) evaluated several allocation policies in the maritime reposition issue to reduce operational costs and prevent empty container shortage when the demand for them is uncertain. They employed simulation models. They exploited probability distributions based on historical data to estimate uncertain parameters, whereas we cannot rely on useful statistics for our case-study.

Shen and Khoong (1995), focusing on the business perspective of the shipping industry, developed a decision supporting system for repositioning empty containers among a set of undifferentiated ports. They proposed a time-extended formulation based on a network optimization model to minimize repositioning costs. Their formulation was deterministic.

Since in real-world repositioning problems it is necessary to include stochastics, Cheung and Chen (1998) considered a set of undifferentiated ports in a maritime system and applied a two-stage stochastic model to the dynamic single-commodity allocation of empty containers. Randomness arose from demands for and supplies of empty containers in ports and from vessel capacities for empty containers, whereas we also consider the uncertain nature of loading-unloading capacities for empty containers. Moreover, while these authors rely on probabilistic rules to estimate uncertain parameters, in our case useful statistics are not available.

Choong *et al.* (2002) addressed the end-of-horizon issue to manage a homogenous fleet of empty containers in the context of an inland distribution system. They proposed for this problem a time-extended deterministic optimization model. To deal with uncertain parameters, decisions were implemented in a rolling horizon fashion.

Leung and Wu (2004) developed a dynamic optimization model for repositioning empty containers, to generate solutions insensitive to the realization of uncertain parameters. They considered a set of undifferentiated ports and assumed customer demands as a single source of uncertainty. They exploited the reliability of available data to estimate uncertain parameters.

Erera *et al.* (2005) proposed a time-extended optimization model to manage loaded and empty tank containers simultaneously. The incorporation of routing and repositioning decisions in a single multicommodity model resulted in a significant drop of operating costs. However, their formulation is still deterministic and recourse actions are performed implementing decisions in a rolling horizon fashion.

Olivo *et al.* (2005) proposed a time-extended optimization model to support the decisions of shipping companies in the context of multimodal networks. However, their formulation is still deterministic and uncertain parameters are taken into account by implementing decisions in a rolling horizon fashion. Moreover, their model failed to capture some specific characteristics of the maritime repositioning issue, such as restrictions imposed by different ports and the intervals between decision and implementation times.

Erera *et al.* (2007) proposed a robust optimization framework to reposition empty resources over time, such that flow balance constraints and flow bounds are satisfied for the nominal values and feasibility can be reestablished for any outcome by recovery actions. Since the costs of recovery actions are requested to be negligible, in our problem we should not modify nominal decisions involving loading and unloading operations. Moreover, if we allow only recovery actions on inventory decisions, we must determine appropriate flow lower bounds and solutions may become infeasible due to limited storage capacities.

To conclude, the literature in the field does not seem to address the problem described above. Therefore, we propose a multi-scenario optimization model, where scenarios are generated following expert-based opinions.

5. Modeling

To facilitate the understanding of the modeling process, in the next sections we first describe system dynamics, then we model the problem by a deterministic single-scenario formulation, and finally we present the multi-scenario model. These formulations must be used in a rolling horizon fashion. They determine decisions to be implemented immediately and assess their impacts on system evolution. Then, in the next period, when new information becomes available, these models must be run again to make new decisions.

5.1 System dynamics

Some assumptions are made during the modeling. All demands must be met by the same type of containers, that is substitutions between container types are not included. Moreover empty container demand cannot be postponed, due to the high risk of competitors providing containers as requested. Short-term leased containers are not taken into account, whereas long-term leased containers are modeled as company-owned ones. When demand cannot be met, containers cannot be provided by other partners or from external sources.

We refer to a graph, where nodes represent ports replicated in every period of the planning horizon and vessels in the periods when they stay at ports. Therefore, while the configuration of ports is invariant with the time periods, the set of vessels berthing in ports is time-dependent. We consider several types of arcs, representing several types of decisions. Arcs from a port in a period to the same port in the next period represent the flow of empty containers to be stored. Arcs from a vessel in a period to the same vessel berthing in another port in another period represent the flow of empty containers to be repositioned. Arcs from a vessel berthing in a port in a period to the same vessel in the same port in the next period represent the flow of empty containers to be kept on this vessel. Arcs from ports to vessels represent the flow of empty containers to be loaded, whereas arcs from vessels to ports represent the flow of empty containers to be unloaded.

To clarify, we present in Figure 2 a time-extended network made up of five ports and two vessels. In each period we replicate these ports, represented by circular shaped nodes denoted by letters from A to E. Moreover we consider two vessels represented by rhombus shaped nodes above the ports where they berth, denoted by numbers *1* and *2*. Vessel *1* arrives at port *B* in period *1*, at port *C* in period *2*, at port *A* in period *3* and then at *B* in period *5*, from where this transportation line is repeated. It means that this vessel will berth in port *C* in period *6*, in port *A* in period *7* and so on. Vessel *2* arrives at port *D* in period *2*, at port *C* in period *3*, at port *E* in period *4*, it also stays at period *5* in this port, from where it repeats the route.

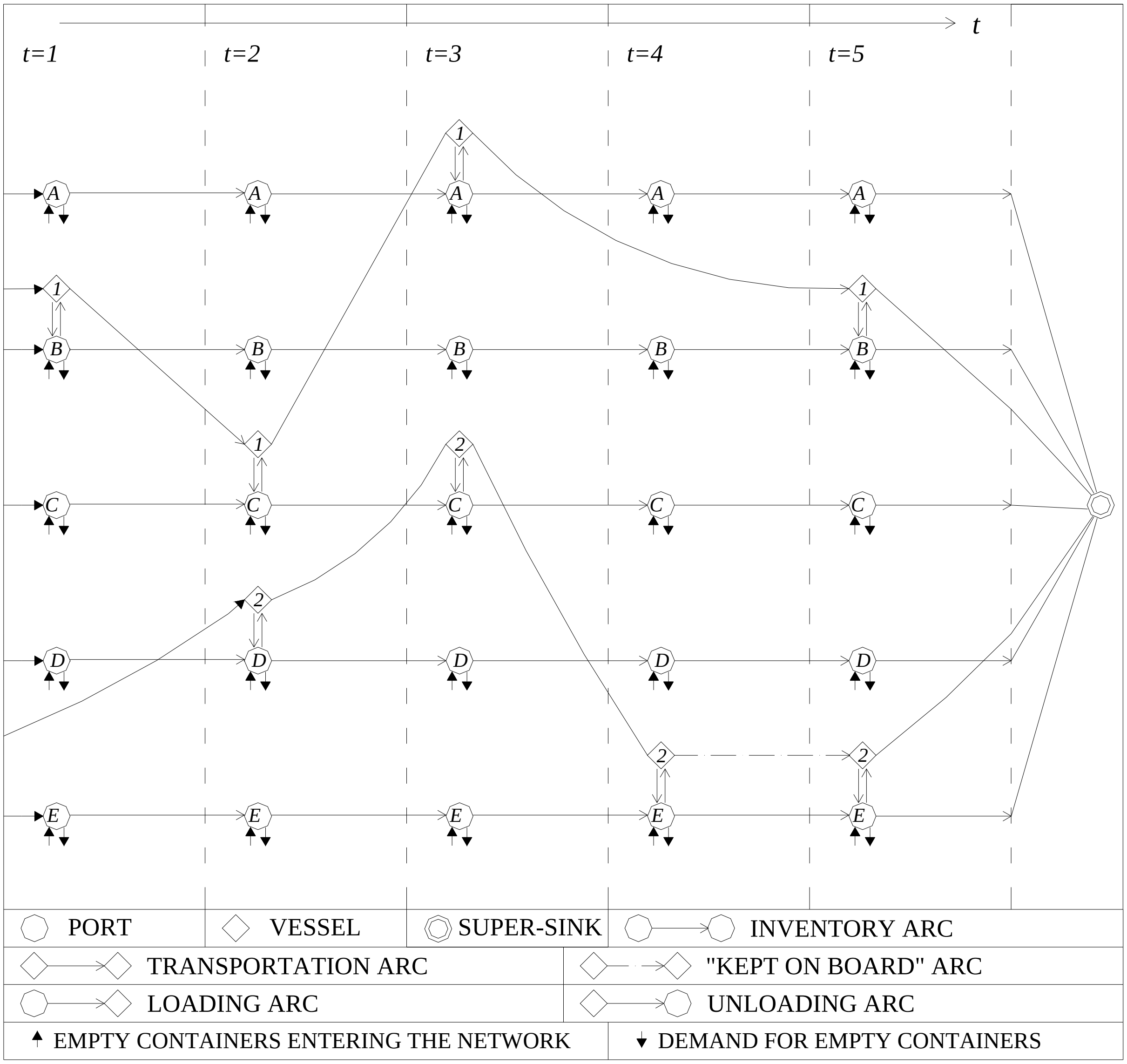

Figure 2. The flows of empty containers in a time-dependent network

According to classical graph notation (Ahuja et al., 1993), arrows entering nodes represent empty containers entering the network, whereas arrows leaving nodes represent the demand. Empty containers enter this maritime network when they arrive from the landside. Moreover each port or vessel may have on-hand empty containers at the beginning of the planning horizon. A super-sink node is introduced at the end of the planning horizon. It is linked to all arcs, whose tail is in the planning horizon and whose head is beyond the fifth period. A proper cost is set for these arcs to reduce end-of-horizon effects (Powell, 1988).

This network does not reflect several characteristics of the problem previously described. In Figure 2, empty container arriving from the landside can be stored in every port or loaded on vessels. However, according to our problem, in small ports these containers cannot be stored, because they must be assigned to vessels. Moreover, Figure 2 indicates how many containers must be loaded and unloaded from every vessel in each period, as Cheung and Chen (1998) did. However, in our problem we aim to determine the total number of empty containers to be loaded and unloaded during the whole interval of time spent in ports, without taking into account the single period when they are loaded and unloaded. Moreover, while in Figure 2 the supply of empty containers in a period can be loaded on vessels berthing in that port in that period, in our problem loading decisions involve empty containers available in ports before the arrival of vessels.

To properly model our problem, we represent vessels only in the period when they arrive at ports. We do not consider arcs representing the flow of empty containers kept on board and unloading arcs have the head in the period when vessels leave ports. As a result, if unloading arcs have tail and head in the same period, vessels stay in port for one period. If they have tail in one period and head in the next one, vessels stay in port for two periods and so on. Moreover, loading arcs must have the tail in a period previous to the arrival of vessels at ports.

Figure 3 illustrates decisions to be made in the network illustrated in Figure 2. Let us assume that *A*, *C* and *E* are small ports, where containers arriving from the seaside can be stored for some periods, whereas those arriving from the landside must be assigned to the first vessel berthing in these ports. We also assume that *B* and *D* are large ports, where empty containers arriving from both the landside and the seaside can be stored or loaded on vessels. To take into account the restrictions imposed by ports *A*, *C* and *E*, they are modeled by a pair of nodes in each period. Therefore, we denote port *A* by A^s and A^d, port *C* by C^s and C^d and port *E* by E^s and E^d. Nodes A^s, C^s and E^s are associated with empty containers arriving from the landside, whereas nodes A^d, C^d and E^d are related to empty containers unloaded from vessels and past inventories. Nodes *B* and *D* are associated with containers arriving from both the seaside and the landside. Vessels *1* and *2* are represented above ports in the periods when they arrive at ports, whereas in Figure 2 they were indicated in every period when they stay at ports (for instance, we represent no longer vessel *2* in period *5*).

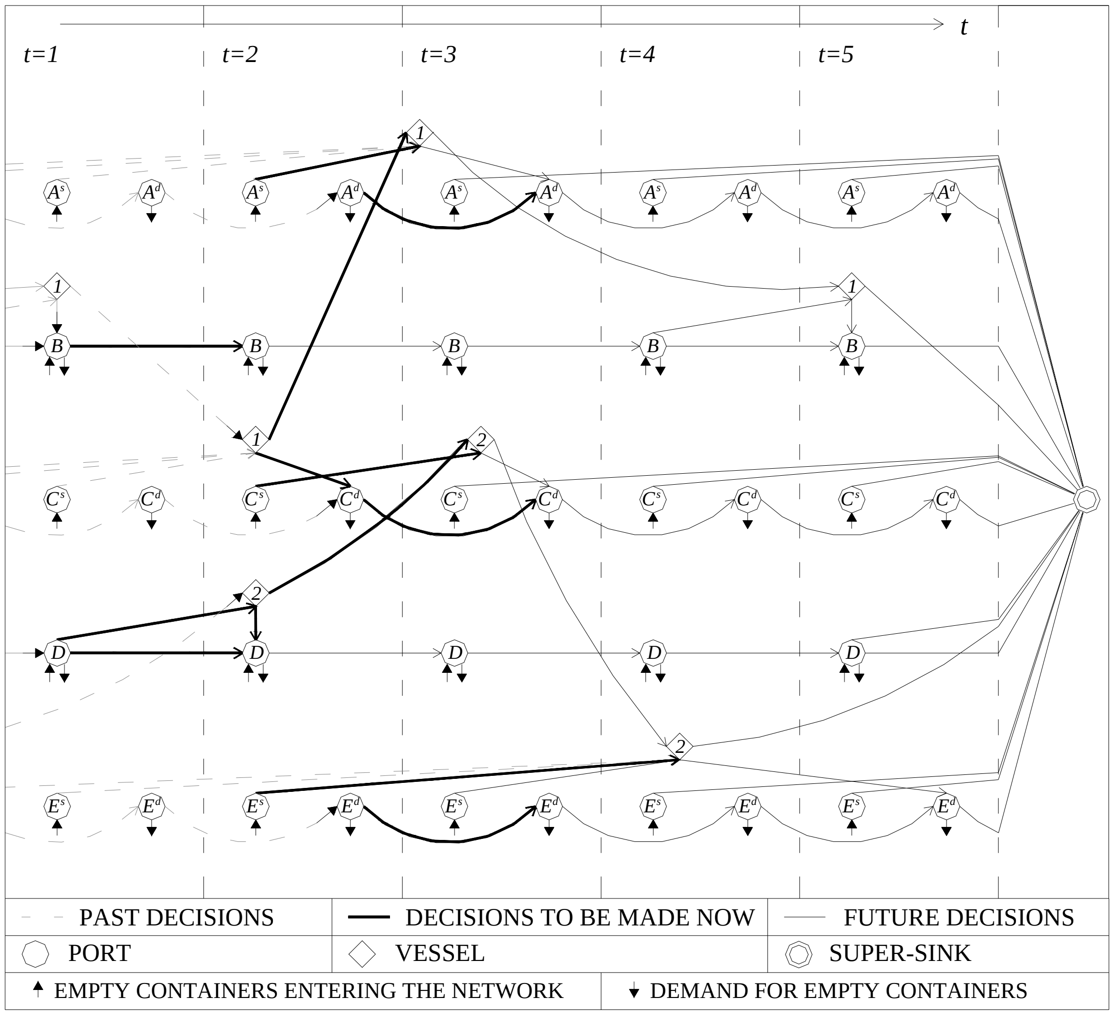

Figure 3. The decisions to be made in the time-dependent network shown in Figure 2

In ports *B* and *D* shipping companies must focus on the number of empty containers available in the first period and decide now how many of them will be stored or loaded on vessels arriving the next period. It is worth noting that in ports *A*, *C* and *E* shipping companies do not decide now the number of empty containers available in the first period to be loaded on vessels, because they were requested to make these decisions before the arrival of containers from the landside. They must focus on the number of containers that will arrive at ports in period *2* and decide how they must be allocated to arriving vessels (in this picture we assume for the sake of simplicity that empty containers arriving from the landside must be assigned to the first vessel berthing after their arrival, whereas in real-world operations they can also be assigned to the first two or three vessels).

Moreover, shipping companies must decide immediately how many empty containers will be unloaded from vessels arriving at every port in period *2*. These containers can be dispatched in the landside, kept in stock for some periods in small ports or stored in large ports as far as shipping companies need. Finally, shipping companies must also determine now the number of empty containers that will stay on vessels arriving in period *2*, once they leave ports (see for instance vessel *2*, leaving port *D* in the second period). Figure 3 also represents all decisions to be made immediately by thick lines, past decisions by discontinuous lines and future decisions by thin lines.

5.2 Deterministic formulation

We refer to a graph $G(N, A)$, where nodes represents ports replicated in every period of the planning horizon and vessels in the periods when they arrive at ports. Let $\overline{H}$ be the set of large ports, where empty containers can be stored with no restriction. Let H be the set of small ports, where empty containers arriving from the landside must be assigned to vessels. Moreover, we consider a set P of container types, a set T of contiguous time-periods and a set V of vessels. We also denote by $V(i,t)$ the set of vessels arriving in port $i \in H \bigcup \overline{H}$ at time $t \in T$.

Every port in $i \in \overline{H}$ is modeled in a given period for a given container type by a single node. Each port $i \in H$ is modeled in a given period for a given container type by the pair of nodes $i^s \in H^s$ and $i^d \in H^d$, such that $H^s \bigcup H^d = H$. Each node $i^s \in H^s$ is associated with the number $s^i_{p,t} \geq 0$ of empty containers of type $p \in P$ arriving from the landside at time $t \in T$. Each node $i^d \in H^d$ is related to the demand $d^i_{p,t} \geq 0$ of empty containers of type $p \in P$ requested in this port at time $t \in T$. The notation $b^i_{p,t}$ represents the number of empty containers of type $p \in P$ arriving from the landside in port $i \in \overline{H}$ at time $t \in T$, if it is positive, whereas it represents the demand for $p-$type containers requested in this port at time $t \in T$, if it is negative.

Inventory decisions are denoted as follows:

- Variable $x^h_{p,t}(i)$ indicates the decision made at time $t \in T$ on the number of empty containers of type $p \in P$ to be stored in port $i \in \overline{H}$ at time $t \in T$; $c^h_{p,t}(i)$ represents the related unitary cost.
- Variable $x^h_{p,t}(i^d)$ indicates the decision made at time $t-1 \in T$ on the number of empty containers of type $p \in P$ to be stored in port $i \in H$ at time $t \in T$; $c^h_{p,t}(i^d)$ represents the related cost.

To define transportation decisions, we need to introduce vessel schedules. The schedule of a given vessel $v \in V$ is denoted by $R_v = \{\ldots,(i,t,t'),(j,t'',t'''),\ldots\}$, which means that vessel $v \in V$ arrives in port $i \in H \bigcup \overline{H}$ at time $t \in T$ and stays in this port up to time $t' \in T$. Then, this vessel moves to port $j \in H \bigcup \overline{H}$, where it arrives at time $t'' \in T$ and leaves at time $t''' \in T$.

Transportation links are denoted as follows:

- Variable $x^m_{p,t}(i,j)$ indicates the decision made at time $t-1 \in T$ on the number of empty containers of type $p \in P$ to be moved by vessel $v \in V(i,t)$ from port $i \in H \bigcup \overline{H}$ to port $j \in H \bigcup \overline{H}$, where the arrival time is indicated in its schedule R_v; $c^m_{p,t}(i,j)$ represents the related unitary cost.

Loading links are denoted as follows:

- Variable $x^l_{p,t}(i,v)$ indicates the decision made at time $t \in T$ on the number of empty containers of type $p \in P$ available in port $i \in \overline{H}$ at time $t \in T$ to be loaded on vessel $v \in V(i,t+1)$; $c^l_{p,t}(i,v)$ represents the related unitary cost.
- Variable $x^l_{p,t}(i^s,v)$ indicates the decision made at time $t-1 \in T$ on the number of empty containers of type $p \in P$ available in port $i \in H$ at time $t \in T$ to be loaded on

vessel $v \in V$, that arrives in this port in a future period indicated in its schedule R_v; $c^l_{p,t}(i^s, v)$ represents the related unitary cost.

Unloading arcs are denoted as follows:

- Variable $x^u_{p,t}(v,i)$ indicates the decision made at time $t-1 \in T$ on the number of empty containers of type $p \in P$ to be unloaded from vessel $v \in V(i,t)$ in port $i \in \overline{H}$; $c^u_{p,t}(v,i)$ represents the related unitary cost.
- Variable $x^u_{p,t}(v,i^d)$ indicates the decision made at time $t-1 \in T$ on the number of empty containers of type $p \in P$ to be unloaded from vessel $v \in V(i,t)$ in port $i \in H$; $c^u_{p,t}(v,i^d)$ represents the related unitary cost.

Capacity constraints are imposed to load, unload, reposition and store a number of empty containers that can be implemented in daily operations. Since we consider a heterogeneous fleet of containers coming in about fifteen types and two sizes (*20* ft and *40* ft containers), we consider the largest container type $r \in P$ and express capacities in terms of space available for r- type containers. The available space for empty containers of type $q \neq r$ can be determined by conversion factors denoted by a_{qr}, which represent the number of r- type containers taking up the space of one q- type container. To clarify, since the available space is be expressed in terms of *40* ft containers, the conversion factor for *20* ft containers is equal to ½. Storage capacities are negotiated with large ports, whereas in small ports they are calculated assuming the maximum dwelling time for empty containers, which can be about *15* days. Since the planning horizon is supposed to be shorter that the maximum dwelling time, storage capacities in small ports are known.

Let us consider a generic port $i \in \overline{H}$. Empty containers available in this port in a given period can be loaded on vessels arriving in the next period or can be stored. The demand for empties it this port in a given period can be met by containers unloaded from vessels leaving in that period and by past stocks. As for notation, let us denote by $W(i,t-\alpha)$ the set of vessels arriving in port $i \in \overline{H}$ at time $t-\alpha \in T$ and leaving at time $t \in T$ (where α takes value *0* if vessel $v \in V$ stays in port for one period, it takes value *1* if it stays for two periods and so on). Then, we have

$$\sum_{v \in V(i,t+1)} x^l_{p,t}(i,v) + x^h_{p,t}(i) - \sum_{v \in W(i,t-\alpha)} x^u_{p,t-\alpha}(v,i) - x^h_{p,t-1}(i) = b^i_{p,t} \qquad \forall i \in \overline{H}, \forall t \in T, \forall p \in P \qquad (1)$$

Let us consider a generic port $i \in H$. Empty container arriving from the landside in a given period $t \in T$ must be loaded on a given number $\beta(i)$ of vessels arriving in the next periods. This number depends on the specific restrictions imposed by ports. Hence, we have

$$\sum_{v=1}^{\beta(i)} x^l_{p,t}(i^s, v) = s^i_{p,t} \qquad \forall i^s \in H, \forall t \in T, \forall p \in P \qquad (2)$$

The demand for empty containers at ports of set H in a given period $t \in T$ can be met by past stocks and by containers unloaded from vessels leaving at time $t \in T$. Other containers can also be stored to satisfy future demands. Let us denote by $W(i,t-\alpha)$ the set of vessels arriving in port $i \in H$ at time $t - \alpha \in T$ and leaving at time $t \in T$, where α takes values as in constraint set (1). Then we have

$$x^h_{p,t}(i^d) - \sum_{v \in W(i,t-\alpha)} x^u_{p,t-\alpha}(v, i^d) - x^h_{p,t-1}(i^d) = -d^i_{p,t} \qquad \forall i^d \in H, \forall t \in T, \forall p \in P \qquad (3)$$

Vessels berthing in ports carry empty containers, that can be unloaded or repositioned to other ports. Moreover, empty containers available in ports can be loaded on vessels. Let us consider a vessel $v \in V(i,t)$ arriving in port $i \in \overline{H}$ at time $t \in T$. It navigates from port $k \in H \cup \overline{H}$, where it arrived at time $t - \tau \in T$, to port $i \in \overline{H}$ and next to port $j \in H \cup \overline{H}$, where it arrives in a period indicated in its schedule. The flow conservation of $p-$ type containers for this vessel can be expressed as follows

$$x^m_{p,t}(i,j) + x^u_{p,t}(v,i) - x^m_{p,t-\tau}(k,i) - x^l_{p,t-1}(i,v) = 0$$
$$\forall v \in V(i,t), \forall i \in \overline{H}, \forall t \in T, \forall p \in P \quad (4)$$

Let us consider a vessel $v \in V(i,t)$ arriving in port $i \in H$ at time $t \in T$. It navigates from port $k \in H \cup \overline{H}$, where it arrived at time $t - \tau \in T$, to port $i \in H$ and next to port $j \in H \cup \overline{H}$, according to its schedule. To capture restrictions imposed by ports in set H, let us denote by $\chi(i)$ the maximum number of periods between the arrival time of empty containers from the landside in port $i \in H$ and the arrival time of this vessel, where these containers can be loaded. Then we have

$$x^m_{p,t}(i,j) + x^u_{p,t}(v,i^d) - x^m_{p,t-\tau}(k,i) - \sum_{z=1}^{\chi(i)} x^l_{p,t-z}(i^s, v) = 0$$
$$\forall v \in V(i,t), \forall i \in H, \forall t \in T, \forall p \in P \quad (5)$$

We denote by $U^{lu}_t(v,i)$ the maximum number of empty containers that can be loaded and unloaded from vessel $v \in V(i,t)$. It is worth noting that this capacity does not depend on container types, because we assume the same loading and unloading times for every container type. Constraint set (6) ensures an upper capacity on the number of empty containers that can be loaded and unloaded from any vessel $v \in V(i,t)$ arriving at port $i \in \overline{H}$. Then, we have

$$\sum_{p \in P} x^l_{p,t-1}(i,v) + \sum_{p \in P} x^u_{p,t}(v,i) \le U^{lu}_t(v,i) \qquad \forall v \in V(i,t), \forall i \in \overline{H}, \forall t \in T \quad (6)$$

Let us consider a vessel $v \in V(i,t)$ arriving at port $i \in H$. Constraint set (7) imposes an upper capacity on the number of empty containers that can be loaded and unloaded on this vessel. Let us recall the definition of $\chi(i)$ used in constraint set (5). Then we have

$$\sum_{p \in P} \sum_{z=1}^{\chi(i)} x^l_{p,t-z}(i^s, v) + \sum_{p \in P} x^u_{p,t}(v, i^d) \le U^{lu}_t(v,i) \qquad \forall v \in V(i,t), \forall i \in H, \forall t \in T \quad (7)$$

We denote by $U^h_{r,t}(i)$ the maximum number of empty containers of type $r \in P$ that can be stored in port $i \in H \bigcup \overline{H}$ at period $t \in T$. According to constraint set (8), the number of empty containers available in port $i \in \overline{H}$ at time $t \in T$, which can be stored or loaded on vessels arriving at time $t+1 \in T$, must not exceed the storage capacity expressed in terms of $r-$ type containers.

$$x^h_{r,t}(i) + \sum_{q\in P|q\neq r} a_{qr} \cdot x^h_{q,t}(i) + \sum_{v\in V(i,t+1)} x^l_{r,t}(i,v) + \sum_{q\in P|q\neq r} \sum_{v\in V(i,t+1)} x^l_{q,t}(i,v) \leq U^h_{r,t}(i) \qquad \forall i \in \overline{H}, \forall t \in T \tag{8}$$

According to constraint set (9), the number of empty containers available in port $i \in H$ at time $t \in T$ must not exceed the storage capacity. It is worth noting that storage spaces are used at time $t \in T$ by empty containers stored and already assigned to vessels arriving in future periods, indicated in their schedules. To model restrictions imposed by ports in set H, let us recall the definition of $\chi(i)$ used in constraint set (5) and (7). Then, we have

$$x^h_{r,t}(i^d) + \sum_{q\in P|q\neq \overline{p}} a_{qr} \cdot x^h_{q,t}(i^d) + \sum_{v\in V(i,t-z)} \sum_{z=0}^{\chi(i)-1} x^l_{r,t-z}(i^s,v) + \sum_{q\in P|q\neq r} \sum_{v\in V(i,t-z)} \sum_{z=0}^{\chi(i)-1} x^l_{q,t-z}(i^s,v) \leq U^h_{r,t}(i) \qquad \forall i \in H, \forall t \in T \tag{9}$$

Let $U^m_{r,t}(i,j)$ represent the residual capacity for empty containers moved by vessel $v \in V(i,t)$ from port $i \in H \bigcup \overline{H}$ to port $j \in H \bigcup \overline{H}$, where the arrival time is specified by its schedule R_v. Constraint set (10) guarantees that the volume of containers repositioned between ports does not exceed the space available on vessels. Then we have

$$x^m_{r,t}(i,j) + \sum_{q\in P|q\neq r} a_{qr} \cdot x^m_{q,t}(i,j) \leq U^m_{r,t}(i,j) \qquad \forall v \in V(i,t), \forall i \in H \bigcup \overline{H}, \forall t \in T \tag{10}$$

We minimize the cost of loading, unloading, repositioning and storing empty containers over the maritime network.

$$\begin{aligned}
\min \sum_{t\in T} \sum_{p\in P} \Bigg[& \sum_{\mathrm{v\in V(i,t)}} \left(c^m_{p,t}(i,j) x^m_{p,t}(i,j) + \sum_{i\in \overline{H}} c^u_{p,t}(v,i) x^u_{p,t}(v,i) + \sum_{i^d\in H} c^u_{p,t}(v,i^d) x^u_{p,t}(v,i^d) \right) + \\
& + \sum_{i\in\overline{H}} \left(c^h_{p,t}(i) x^h_{p,t}(i) + \sum_{v\in V(i,t+1)} c^l_{p,t}(i,v) x^l_{p,t}(i,v) \right) + \sum_{i^d\in H} c^h_{p,t}(i^d) x^h_{p,t}(i^d) + \\
& + \sum_{i^s\in H} \sum_{v=1}^{\beta(\mathrm{i})} c^l_{p,t}(i^s,v) x^l_{p,t}(i^s,v) \Bigg]
\end{aligned} \tag{11}$$

Finally, all decision variables are requested to be integer.

5.3 Handling uncertain parameters

The previous formulation provides effective policies only if problem data are known accurately and future events unfold as expected. However, the future may not be exactly as planned and shipping companies risk providing an inadequate number of empty containers, when uncertain parameters take different values with respect to expected ones. Therefore, we aim to adopt a formulation ensuring an effective repositioning of empty containers for any realization of uncertain parameters. The effectiveness of repositioning policies will be evaluated in the next paragraph in terms of total operating costs and demand fulfillment percentages.

To take specifically into account the uncertain nature of demand at ports, empty containers arriving from the landside, transportation and loading-unloading capacities, we exploit expert-based opinions to generate several possible futures or scenarios. More precise information on uncertain parameters is available in the first part of the planning horizon, whereas they are known with less certainty in the following time periods.

As for notation, all parameters are assumed to be certain up to period θ, whereas time $\theta+1$ is the first period when uncertain parameters may appear. In most real-world applications the time θ corresponds to the second period. In fact, although unexpected transportation opportunities may arise, empty container demand can be assumed to be certain in the first two periods of the planning horizon, because customers book transportation requests several days before the arrival of vessels. Moreover, we know the number of containers available in ports in period *1*, because they can be observed. Empty container supply at period *2* can be straightforwardly estimated, because shipping companies know the number of empty containers arriving from the landside. If no failure or incident occurs, these containers will reach ports. Furthermore, the number of loaded containers to be placed and discharged from vessels arriving at ports in period two is certain, because customers book their requests some days before shipping their goods. As a result, we also know the residual transportation capacity and the loading-unloading capacity for vessels berthing in ports in period *2*. As a result, uncertain parameters are likely to appear in the third period.

All scenarios are collected in an overall mathematical model linked by non-anticipativity constraints. They guarantee that decision variables are identical up to time θ, so that current decisions do not take advantage of information not yet available. Different decisions will be made in the future, depending on the values taken by uncertain parameters. A weight can be assigned to each scenario to characterize its relative importance.

To clarify, Figure 4 shows two different scenarios for the network illustrated in Figure 3. Positive numbers close to nodes indicate empty containers arriving from the landside or already on board. Negative numbers represent empty container demands. The difference between these scenarios consists in the different demands for empty containers at port *C* in period *4*. Figure 4 also indicates through the black arrows getting into and out of nodes the certain number of empty containers entering the network and the certain demand, which appear in the first two periods. For the third and later periods, possible realizations of these parameters are shown and are indicated by grey arrows.

We present the multi-scenario formulation in a compact form. For the sake of clarity, a compact form is provided for the deterministic single-scenario formulation as well. Let us denote by G the set of scenarios and by g a scenario representing one possible realization of the future. Since scenario g represents a deterministic problem instance, the integer programming model presented Section 5.2 can be expressed as follows:

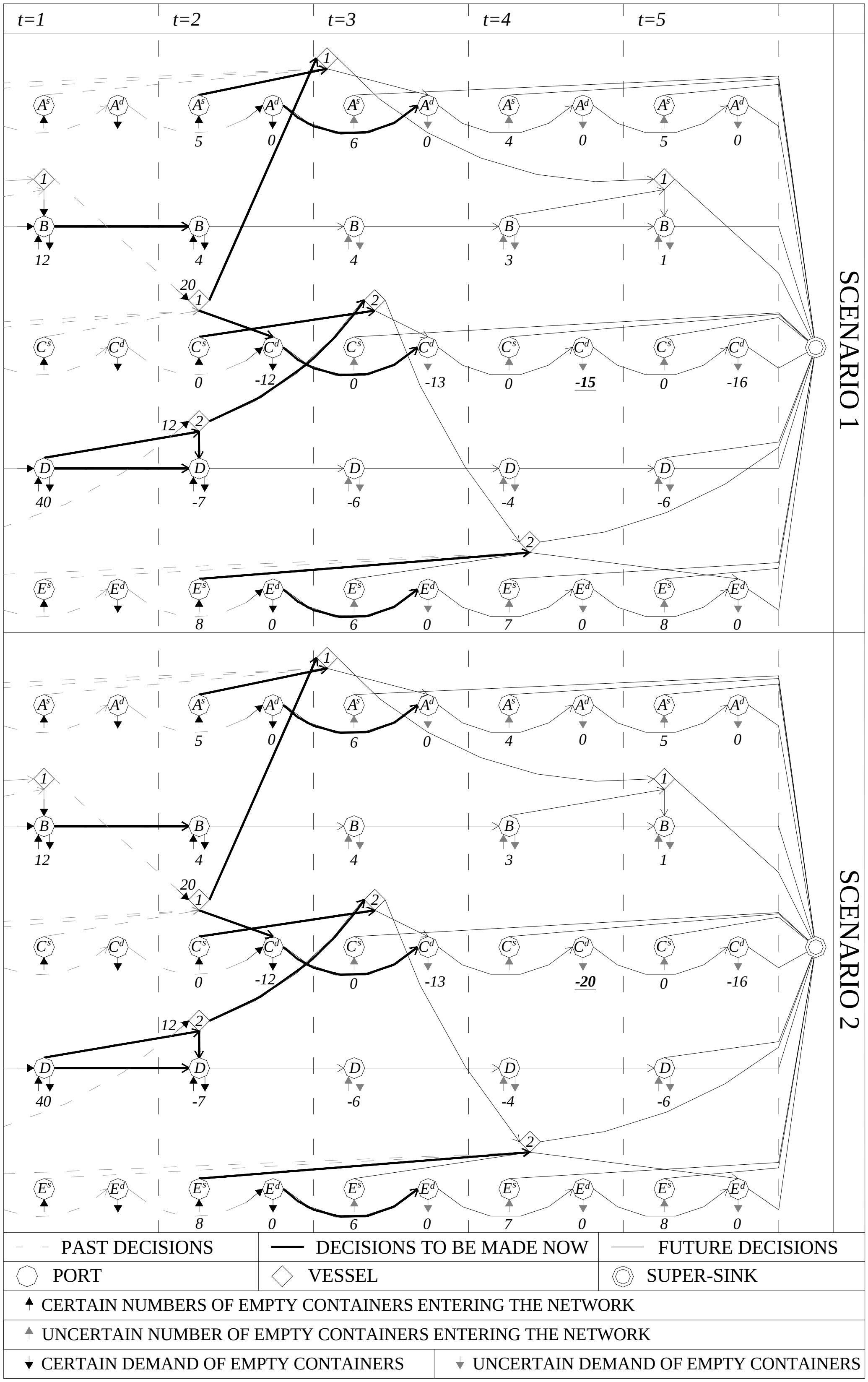

Figure 4. Two scenarios with known and uncertain parameters

$$\min \; cx_g \tag{12}$$

s.t.

$$Ax_g = b_g \tag{13}$$

$$x_g \in Z^+ \cup \{0\} \tag{14}$$

The index g identifies the vector of decision variables x_g and the values of the RHS vector b_g associated with this specific scenario. The cost vector c in the objective function (12) includes loading, unloading, storage and transportation costs, as detailed in (11). The matrix A in constraint set (13) represents the coefficient matrix. We indicate the equality in constraint set (13), because all constraints from (1) to (10) can be expressed in terms of equality constraints by adding slack variables. Constraint set (14) ensures the integrality of all decision variables.

Let w_g denote the weight associated with each scenario $g \in G$. Weights are determined on the basis of subjective knowledge of the problem. We denote by x_{tg} the set of decision variables to be implemented at time $t \in T$ in scenario $g \in G$. The multi-scenario model can be expressed as follows:

$$\min \sum_{g \in G} w_g c x_g \tag{15}$$

s.t.

$$Ax_g = b_g \qquad \forall g \in G \tag{16}$$

$$x_g \in Z^+ \cup \{0\} \qquad \forall g \in G \tag{17}$$

$$x_{tg} = x_{tf} \qquad \forall t \in \{1 \dots \vartheta\}, \forall g, f \in G \tag{18}$$

where (18) represents non-anticipativity constraints, to require that decision variables x_{tg} to be implemented at time $t \in T$ are identical up to time θ for every scenario $g \in G$. According to (15), we minimize the cost of loading, unloading, repositioning and storing empty containers over all scenarios. According to (16) and (17), constraint systems (13) and integrality constraints (14) are replicated for every scenario $g \in G$. Since the uncertain parameters we consider are future demands, the number of empty containers arriving from the landside, transportation and loading/unloading capacities, the index g is used to denote the RHS vector, as well as decision variables.

6. Experimentation

To illustrate the interest of the multi-scenario formulation, we simulate the behavior of the system over a number of periods and compare multi-scenario policies with deterministic ones. We aim to show that multi-scenario solutions are significantly better, yielding larger demand fulfillment percentages, even though they may yield slightly higher repositioning costs.

For the sake of clarity, we refer to the five periods network depicted in Figure 3 and we consider a single container type. The number of container arriving from the landside and the demand in ports *C* and *D* in periods *3* and *4* are supposed to represent the only sources of uncertainty, controlling the values taken by the other uncertain parameters. Point forecasts about the last period of the planning horizon are provided by expert-based opinions. A given number of empty containers is supposed to be available in port *D* in period *1*. As a result, in the first period we must compare how many containers are stored in this port or shipped to port *C* according to multi-scenario policies and deterministic ones.

Expert-based opinions are used to estimate the uncertain parameters we are considering. Shipping company suggestions indicate the minimum, the most probable and the maximum values for each uncertain parameter. Such information is collected in triangular distributions. To generate scenarios, we consider three values of these uncertain parameters for each port in each period: the mode, the minimum and the maximum. Then, we build one scenario for each joint realization and, according to previous assumptions, the multi-scenario formulation includes $3^4 = 81$ scenarios. We link all scenarios by non-anticipativity conditions. We assume that every joint realization of uncertain parameters is independent one from the other, so scenarios' weight can be calculated by multiplying probabilities deriving from triangular distributions.

We solve the multi-scenario formulation and the deterministic one in the first period and we determine the resulting policies. To evaluate these policies day-by-day, we assume which values are taken by uncertain parameters, when they are observed in the next period. Since many joint realizations of uncertain parameters may become true in the next day, we consider three significant cases, to limit our analysis to a reasonable size:

- The expected values of these uncertain parameters will become true;
- The worst-case realization of these uncertain parameters will be observed (i.e. the largest values for the demand and the lowest ones for number of empty containers arriving from the landside);
- A random combination of these uncertain parameters will occur.

Then we add a new period at the end of the planning horizon, we run the models once again, we implement decisions, we evaluate the performances of the system and so on.

Policies are evaluated in terms of demand fulfillment percentages and total operating costs (storage, transportation, loading and unloading costs). They are presented in Table 1 and Table 2. Rows *1*, *2* and *3* concern the deterministic policy, while rows *4, 5* and *6* focus on the multi-scenario one. Rows *1* and *4* show results in the case that the average values of all uncertain parameters are observed in the next period. Rows *2* and *5* indicate results in the case that the worst case values occur in both ports in the next period. Finally rows *3* and *6* represent system performances when a random set of uncertain parameters is observed in the next period. This simulation is performed for five periods.

DEMAND FULFILLMENT (%)	Period *1*	Period *2*	Period *3*	Period *4*	Period *5*
1. Deterministic Solution Expected realization	*100%*	*100%*	*100%*	*100%*	*100%*
2. Deterministic Solution The worst-case realization	*100%*	*92%*	*65%*	*58%*	*86%*
3. Deterministic Solution A random realization	*100%*	*100%*	*79%*	*89%*	*96%*
4. Multi-scenario Solution Expected realization	*100%*	*100%*	*100%*	*100%*	*100%*
5. Multi-scenario Solution The worst-case realization	*100%*	*100%*	*100%*	*100%*	*89%*
6. Multi-scenario Solution A random realization	*100%*	*100%*	*100%*	*100%*	*100%*

Table 1. Demand fulfillment evaluation between multi-scenario and deterministic solutions

TOTAL OPERATING COSTS ($)	Period *1*	Period *2*	Period *3*	Period *4*	Period *5*	Total
1. Deterministic Solution Expected realization	*11 617*	*7 853*	*1 833*	*3 002*	*4 393*	*28 698*
2. Deterministic Solution The worst-case realization	*11 617*	*8 353*	*1 823*	*3 002*	*4 877*	*29 672*
3. Deterministic Solution A random realization	*11 617*	*8 053*	*1 826*	*3 002*	*4 584*	*29 082*
4. Multi-scenario Solution Expected realization	*10 134*	*8 879*	*1 859*	*3 226*	*6 993*	*31 091*
5. Multi-scenario Solution The worst-case realization	*10 134*	*9 379*	*1 846*	*3 200*	*8 539*	*33 098*
6. Multi-scenario Solution A random realization	*10 134*	*9 079*	*1 852*	*3 222*	*7 080*	*31 367*

Table 2. Total operating cost comparison between multi-scenario and deterministic solutions

According to our tests, in the first day the deterministic policy suggests storing a number of empty containers to meet the average demand in port *D* and moving to port *C* the residual part of containers. On the other hand, the multi-scenario one recommends to store a larger number of empty containers in port *D*, to meet the maximum demand that may arise in this port. Since loading costs are higher than storage costs, in Table 2 the multi-scenario plan yields lower operating costs in period *1*. In periods *2*, *3* and *4* the multi-scenario policy exhibit slightly higher operating costs than the deterministic one. In period *5* the multi-scenario policy suggests loading a larger number of empty containers in port *D* to meet the extra demand that may arise in port *C*. Although it yields higher operating costs in this period, it makes for a larger number of empty containers on vessels. As a result, shipping companies are put in the position of promptly satisfying unexpected demands in future periods, whereas this is not possible when the deterministic policy is adopted.

If expected values occur in every period, at the end of the simulation the multi-scenario policy yields larger operating costs than the deterministic one ($ *31 091* > $ *28 69*8). The difference represents what shipping companies can pay to ensure a robust repositioning plan over all scenarios. When the worst combination of uncertain parameters occurs, the deterministic policy is particularly inappropriate, because typically there is no container to meet extra demand in ports *C* and *D*. Table 1 indicates that demand fulfillment percentages significantly decrease in this case, whereas they are significantly higher when the multi-scenario policy is adopted. At the end of the simulation, we are not able to meet *31* requests for empty containers, when uncertain parameters take the worst-case values and the deterministic policy is adopted. If we adopt the multi-scenario one, we cannot satisfy *4* requests only. This different behavior of multi-scenario and deterministic policies also occurs in the case that a random set of uncertain parameters is observed in the next periods.

It is worth noting that in real-world operations shipping companies cannot afford to not satisfy empty container demand and reject transportation opportunities, because customers are not willing to delay the shipment of their goods. To face this problem, during the pick season shipping companies usually rent containers, thus generating additional operating costs. During the slack season, usually containers are not rented, due to internal operating policies adopted by shipping companies or specific clauses in leasing contracts. As a result, some transportation requests may not be satisfied on time and some profits may be lost. Let us assume that we are simulating the system in the slack season and that the revenue for each requests is $ *400*. If the shipping company adopts the deterministic policy, the total possible loss is $ $400 \cdot 31$ = $ *12 400*. On the other hand, if the multi-scenario one had been implemented, the shipping company would have lost $ $400 \cdot 4$ = $ *1 600* only. If we add the possible loss to the total operating cost, one can notice that the multi-scenario policy will by far more profitable than the deterministic one: $ *29 672* + $ *12 400* >> $ *33 098* + $ *1 600*.

7. Conclusion

In this paper we have outlined the case-study of a shipping company needing to properly reposition its empty containers in the Mediterranean region and we have introduced a new variant of the empty container management problem. Two different types of ports have been considered for the first time, to take into account the different restrictions imposed on the storage of empty containers. One of the most relevant characteristics in this issue has been the lack of reliable forecasts to be used during the decision-making process. Therefore, we have exploited the subjective knowledge of this shipping company to generate a set of scenarios, which have been incorporated in a time-extended optimization model. All scenarios have been linked by non-anticipativity constraints, so that current decisions will not “see” which scenario will occur.

We have shown why it is crucial to adopt multi-scenario policies in this highly uncertain environment. They make for insight with respect to deterministic ones, that generate high-quality repositioning plans only if the expected realizations of uncertain parameters are observed. Multi-scenario policies contain the option of allocating more empty containers where larger demands may be observed. Deterministic policies do not include these options, because the future has no allowance for surprises. As a result, repositioning is done as cheaply as possible. However, low demand fulfillment percentages may be obtained. This is an unappealing situation for shipping companies, because in their highly competitive industry several transportation opportunities might be lost. This problem can be faced by paying slightly higher repositioning costs, to exploit repositioning options included in multi-scenario-policies.

A natural extension of this paper is represented by modeling the extra operating costs generated by rented containers. They add flexibility to fleet management, because they can be picked up and returned by shipping companies, according to their needs and specific clauses indicated in rental contracts. Moreover, it is worth investigating pros and cons resulting from the simultaneous repositioning of empty containers in both land and sea. Finally, since the stochastics blow up problem dimensions, specialized resolution techniques should be developed for repositioning problems that cannot be solved by standard software. For instance, parallel computing can be adopted, exploiting the many algebraic structures arising in the proposed formulation.

Appendix

This appendix describes which problem instances of the multi-scenario formulation can be solved through a standard software, depending on which uncertain parameters we consider.

We refer to the *5* ports network depicted in Figure 3, which is extended to *10* periods and *5* container types. We consider three possible values for every uncertain parameter, we set the number of uncertain parameters and we generate one scenario for every joint realization of them. We use the solver Cplex *10.1* running on a PC with a *2.8* GHz processor and *512* Mb of memory.

Three set of instances are generated in our computational tests:

1. The first uncertain parameter we consider is the demand for empty containers of a given type in period *3*, the second one is the demand for empty containers of the same type in the same port in the next period and so on up to the end of the planning horizon. Then a new port is considered and so on.
2. The first uncertain parameter we consider is the demand for empty containers of a given type in period *3*, the second one is the demand for empty containers of the same type in another port in the same period and so on. Once all ports in period 3 are considered, we take into account the demand for empty containers of the same type in period *4* and so on.
3. The first uncertain parameter we consider is the demand for empty containers of a given type in period *3*, the second one is the demand for empty containers of a different type in the same port in the same period and so on. Once all container types are considered for a given port in a given period, we take into account the demand in another port in the same period for every container type and so on.

Table 3 shows computational tests for the first set of instances.

Number of uncertain parameters (*NUP*)	Uncertain parameters	Number of scenarios (3^{NUP})	Number of variables	Time for Cplex (s)
4	Demand for a container type in port *A* from period *3* to period *6*	*81*	*84 240*	*0.86*
5	Demand for a container type in port *A* from period *3* to period *7*	*243*	*252 720*	*3.21*
6	Demand for a container type in port *A* from period *3* to period *8*	*729*	*758 160*	*10.42*
7	Demand for a container type in port *A* from period *3* to period *9*	*2 187*	*2 274 480*	*37.38*

Table 3. Computational results for the first set of instances.

Table 4 illustrates computational tests for the second set of instances.

Number of uncertain parameters (*NUP*)	Uncertain parameters	Number of scenarios (3^{NUP})	Number of variables	Time for Cplex (s)
4	Demand for a container type in ports *A*, B, *C*, *D* in period *3*	*81*	*84 240*	*0.88*
5	Demand for a container type in ports *A*, *B*, *C*, *D* and *E* in period *3*	*243*	*252 720*	*3.63*
6	Demand for a container type in ports *A*, *B*, *C*, *D*, *E* in period *3* and in port *A* in period *4*	*729*	*758 160*	*11.15*
7	Demand for a container type in ports *A*, *B*, *C*, *D*, *E* in period *3* and in ports *A* and *B* in period *4*	*2 187*	*2 274 480*	*37.48*

Table 4. Computational results for the second set of instances.

Table 5 shows computational tests for the third set of instances.

Number of uncertain parameters (*NUP*)	Uncertain parameters	Number of scenarios (3^{NUP})	Number of variables	Time for Cplex (s)
4	Demand for *4* different container types in port *A* in period *3*	*81*	*84 240*	*0.84*
5	Demand for *5* different container types in port *A* in period *3*	*243*	*252 720*	*3.29*
6	Demand for *5* different container types in port *A* in period *3* and demand for *1* container type in port *B* in period *3*	*729*	*758 160*	*10.47*
7	Demand for *5* different container types in port *A* in period *3* and demand for *2* container types in port *B* in period *3*	*2 187*	*2 274 480*	*37.56*

Table 5. Computational results for the third set of instances.

These tables show that at this point in time we can solve instances having up to *7* uncertain parameters within similar times. Larger instances are not available at the moment, because the generation process is killed due to insufficient memory. Therefore, we can only assume that, if we had more memory, we would be able to solve instances with about *10* uncertain parameters within time limits imposed by planning operations (around *20* minutes). If shipping companies can determine repositioning plans in this short interval of time, they will be able to pass these decisions to ports one day before the arrival of vessels, ports will plan their operations on time and empty containers can be repositioned as requested.

It is worth noting that *7* uncertain parameters seem to be inadequate for this small network, in fact we have *5* (number of ports) · *8* (number of periods with uncertain parameters) · *5* (number of container types) = *200* uncertain values for the demand and *200* uncertain values of supply (and a huge number of scenarios). Moreover, we have not still considered the uncertain nature of residual transportation capacities and loading-unloading capacities.

To conclude, the multi-scenario formulation proposed in this paper is difficult to solve due to the huge size corresponding to the large number of scenarios. Therefore, specialized approaches should be developed in the future for solving effectively this model or computing effective bounds on the optimal solution value.

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