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Robust Production Planning in a Manufacturing Environment with Random Yield: A Case in Sawmill Production Planning

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Abstract. This paper addresses a multi-period, multi-product sawmill production planning problem where the yields of processes are random variables due to non-homogeneous quality of raw materials (logs). In order to determine the production plans with robust customer service level, robust optimization approach is applied. Two robust optimization models with different variability measures are proposed, which can be selected based on the tradeoff between the expected backorder/inventory cost and the decision maker risk aversion level about the variability of customer service level. The implementation results of the proposed approach for a realistic scale sawmill example highlights the significance of using robust optimization in generating more robust production plans in the uncertain environments compared with stochastic programming.

Keywords. Production, sawmill, robust optimization, stochastic programming, random yield.

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1. Introduction

Production planning in many manufacturing environments is based on some parameters with uncertain values. Uncertainties might arise in product demand, yield of processes, etc. Thus, the robustness of a production plan, in term of fulfillment of product demand, is dependent on incorporating the uncertain parameters in production planning models.

This study is concentrated on multi-period, multi-product (MPMP) production planning in the sawing units of sawmills where possible combinations of log classes and cutting patterns produce simultaneously different mix of lumbers. As logs are grown under uncertain natural circumstances, non-homogeneous and random characteristics (in terms of diameter, number of knots, internal defects, etc.) can be observed in different logs in each class. Consequently, the processes yields (quantities of lumbers that can be produced by each cutting pattern) are random variables. Lumber demand in each period is assumed as a deterministic parameter which is determined based on the received orders. In the sawmill production planning problem, we are looking for the optimal combination of log classes and cutting patterns that best fit against lumber demand. The part of demand that cannot be fulfilled on time, due to machine capacities and/or uncertain yield, will be postponed to the next period by considering a lost sale (backorder) cost. The objective is to minimize products inventory and backorder cost and raw material consumption cost, regarding fulfillment of product demand, machine capacities, and raw material (log) inventory. The uncertainty in the yields of cutting patterns in sawmills can be represented with uncertain yield coefficients in the coefficients of constraints matrix. Regarding to the potential significance of yield uncertainty on the production plan, and customer orientation which is at center of attention in the sawmills which are dependent on the export markets, obtaining robust plans with minimum backorder size (service level) variability is an important goal of production planning in sawmills.

Sawmill production planning problem can be considered as the combination of several classical production planning problems in the literature which have been modeled by linear programming (LP). This problem was formulated by a deterministic LP model and was solved based on the average values for processes yields in Gaudrault et al. (2004). However, if decisions are made based on the deterministic model, there is a risk that the demand might not be met with the right products. Consequently, it results high inventory levels of products with low quality and price as well as extra levels of backorder of products with high quality and price (decreased customer service level). The other approach in the literature for sawmill production planning is focused on combined optimization type solutions linked to real-time simulation sub-systems (Maness and Norton, 2002; Maness and Adams, 1991; Mendoza et al., 1991). In this approach, the stochastic characteristics of logs are taken into account by assuming that all the input logs are scanned through an X-ray scanner, before planning. Maness and Norton (2002) developed an integrated multi-period production planning model which is the combination of an LP model and a log sawing optimizer (simulator). The LP model acts as a coordinating problem that allocates limited resources. The log sawing optimization models are used to generate columns for the coordinating LP based on the products' shadow prices. Although the stochastic characteristics of logs are considered in this approach, it includes the following limitations: logs, needed for the next planning horizon, are not always available in the sawmill to be scanned before planning. Furthermore, to implement this method, the logs should be processed in production line in the same order they have been simulated, which is not an easy practice. Finally scanning logs before planning is a time consuming process in the high capacity sawmills which delays the planning process.

There are several techniques to incorporate uncertainty in optimization models, including stochastic programming (Birge and Louveux, 1997; Kall and Wallace, 1994; Kall and Mayer, 2005), and robust optimization (Mulvey et al., 1995). Bakir and Byrune (1998) developed a stochastic LP model based on the two-stage deterministic equivalent problem to incorporate demand uncertainty in a multi-period multi-product (MPMP) production planning model. In Escudero et al. (1993) a multi-stage stochastic programming approach was proposed for solving a MPMP production planning model with random demand. Kazemi et al. (2007b; 2008) proposed a two-stage stochastic model for sawmill production; it was shown in Kazemi et al. (2007b; 2008) that the proposed production plans by stochastic programming approach results a considerably lower expected inventory and backorder cost than the plans of the mean-value deterministic model. It is important to note that stochastic programming approach focuses on optimizing the expected performance (e.g. cost) over a range of possible scenarios for the random parameters. We can expect that the system would behave optimally in the *mean* sense. However, the system might perform poorly at a particular realization of scenarios such as the worst-case scenario. More precisely, unacceptable inventory and backorder size for some scenarios might be observed by implementing the solution of two-stage stochastic model. To handle the tradeoff associated with the expected cost and its variability in stochastic programs, Mulvey et al. (1995) proposed the concept of robust optimization. Leung and Wu. (2004) proposed a robust optimization model for stochastic aggregate production planning. In Leung et al. (2006) a robust optimization model was developed to address a multi-site aggregate production planning problem in an uncertain environment. In Kazemi et al. (2007a) robust optimization approach was proposed as one of the potential methodologies to address MPMP production planning in a manufacturing environment with random yield.

In this paper, a robust optimization (RO) approach is proposed for multi-period sawmill production planning while considering random characteristics of raw materials (logs) and consequently random processes yields. The random yields are modeled as scenarios with a stationary discrete probability distribution during the planning horizon. We are studying a service sensitive company that wants to establish a reputation for always meeting customer service level. We also define the customer service level as the proportion of the customer demand that can be fulfilled on time, and we use the expected backorder size as a measure for evaluating the service level. Thus, the need for robustness has been mainly recognized in term of determining a robust customer service level by minimizing the products backorder size variability in the presence of different scenarios for random yields. The robustness in the products inventory size is also considered in this problem. Two alternative variability measures are used in the robust optimization model which can be selected depending on risk aversion level of decision maker about backorder/inventory size variability and the total cost. The proposed robust optimization (RO) approach is applied for a realistic-scale sawmill production planning example. The resulted large-scale quadratic programming models are solved by CPLEX 10 in a reasonable amount of time. A comparison between the backorder/inventory size variability in the two-stage stochastic model and the two robust optimization models is provided. Finally, the tradeoff between the backorder/inventory size variability and the expected total cost in the two RO models is discussed and a decision framework to select among them is proposed.

The main contributions of this paper can be summarized as follows. Applying robust optimization approach as a robust tool for sawmill production planning, regarding to the limitations of the existing approaches for sawmill production planning; comparing the performance of two different robust optimization models in controlling the robustness of production plans through applying them for a prototype sawmill; proposing a framework for selecting the most appropriate robust optimization model depending on the risk preferences of the decision maker about service level robustness and total expected cost of plans.

The rest of this paper is organized as follows. In section 2, sawmill processes and specific characteristics are introduced. In section 3, the robust optimization formulation for two-stage stochastic programs is provided. In section 4, the proposed robust optimization model for multi-period sawmill production planning is presented. In section 5, the scenario generation approach for random yields is described. In section 6, the computational results of implementing the proposed robust optimization models for a prototype sawmill are provided. Our concluding remarks are given in section 7.

2. Sawmill processes and specific characteristics

There are a number of processes that occur at a sawmill: log sorting, sawing, drying, planing and grading (finishing). Raw materials in the sawmills are the logs which are transported from different districts of forest after bucking the felled trees. The finished and graded lumbers (products) are then transported to the domestic and international markets. Figure 1 illustrates the typical processes. In this paper we focus on operational level production planning in the sawing units of sawmills. In the sawing units, logs are classified according to some attributes namely: diameter class, species, length, taper, etc. Logs are broken down into different dimensions of lumbers by means of different cutting patterns. See figure 2 for three different cutting patterns. Each cutting pattern is a combination of activities that are run on a set of machines. From each log, several pieces of sawn lumber (e.g. 2(in)x4(in)x8(ft), 2(in)x4(in)x10(ft), 2(in)x6(in)x16(ft),…) are produced depending on the cutting pattern. The lumber quality (grade) as well as its quantity yielded by each cutting pattern depends on the quality and characteristics of the input logs. Despite the classification of logs in sawmills, a variety of characteristics might be observed in different logs in each class. In fact, natural variable conditions that occur during the growth period of trees make it impossible to anticipate the exact yields of a log. As it is not possible in many sawmills to scan the logs before planning, the exact yields of cutting patterns for different log classes cannot be determined in priori.

Insert figure 1 here

Insert figure 2 here

3. Robust optimization formulation for two-stage stochastic programs

The robust optimization method developed by (Mulvey et al., 1995) extends stochastic programming by replacing traditional expected cost minimization objective by one that explicitly addresses cost variability.

Consider the following LP model that includes random parameters:

$$\text{Minimize } c^T x \quad (1)$$

Subject to

$$Ax = b, \quad (2)$$

$$Bx \le e, \quad (3)$$

$$x \ge 0, \quad (4)$$

where B and e represent random technological coefficient matrix and right-hand side vector, respectively. Assume a finite set of scenarios $\Omega = \{1, 2, ..., S\}$ to model the uncertain parameters; with each scenario $s \in \Omega$ we associate the subset $\{d^s, B^s, C^s, e^s\}$ and the probability of the scenario p^s, ($\sum_{s=1}^{S} p^s = 1$). The standard two-stage stochastic linear program is formulated as follows.

SP:

$$\text{Minimize } c^T x + \sum_{s \in \Omega} p^s d^{sT} y^s \quad (5)$$

Subject to

$$Ax = b, \quad (6)$$

$$B^s x + C^s y^s = e^s, \qquad s \in \Omega, \quad (7)$$

$$x, y^s \ge 0, \qquad s \in \Omega, \quad (8)$$

where x denotes the vector of first-stage decision variables whose optimal value is not conditioned on the realization of uncertain parameters, y^s denotes the vector of second-stage (recourse) decision variables, corresponding to scenario *s*, that are subject to adjustment once the uncertain parameters are observed. C^s and d^s denote the recourse matrix and the penalty recourse cost vector corresponding to scenario *s*, respectively. The optimal solution of model (5)-(8) will be robust with respect to optimality if it remains close to optimal for any of the scenarios $s \in \Omega$. This is termed *solution robustness*. In other words, the solution robustness measures the variability of the recourse cost in model SP for any of the scenarios $s \in \Omega$. The solution is also robust with respect to feasibility if it remains almost feasible for all scenarios. This is termed *model robustness*. The robust optimization (RO) framework introduced by (Mulvey et al., 1995) is a goal programming approach to balance the tradeoffs between solution robustness and model robustness. Hence, the RO approach is to modify the objective in SP as follows.

RO:

$$\text{Minimize } c^T x + \sum_{s\in\Omega} p^s d^{sT} y^s + \lambda\sigma(y^1,...,y^s) + \omega\rho(\delta^1,...,\delta^s) \qquad (9)$$

Subject to

$$Ax = b, \qquad (10)$$

$$B^s x + C^s y^s + \delta^s = e^s, \qquad s \in \Omega, \qquad (11)$$

$$x, y^s \geq 0, \qquad s \in \Omega. \qquad (12)$$

The term ($\sum_{s\in\Omega} p^s d^{sT} y^s + \lambda\sigma(y^1,...,y^s)$) in the objective function denotes the solution robustness measure, where $\lambda \geq 0$ is a goal programming weight and $\sigma(y^1,...,y^s)$ denotes the recourse cost variability measure. By changing λ, the relative importance of the expectation and variability of the recourse cost in the objective can be controlled. The last term in the objective function $\rho(\delta^1,...,\delta^s)$ is a feasibility penalty function, which is used to penalize the violation of constraints (11) (denoted by δ^s) under some of the scenarios. ω is a goal programming weight. In the following, the recourse cost variability measures existing in the literature, as well as the measures that we use in this work, are presented.

3.1. Variability measures in robust optimization models

The classical approach to model the tradeoff between the expectation and the variability in RO models is to use mean-variance model of Markowitz (1959) which has been implemented in many applications, namely capacity expansion of power systems (Malcolm and Zenios, 1994), stochastic logistic problems (Yu and Li , 2000), stochastic aggregate production planning (Leung and Wu., 2004; Leung et al., 2006). However, there are some exceptions against using mean-variance in some applications: variance is a symmetric risk measure, penalizing the cost both above and below the expected recourse cost, equally. As in the case of production planning it is more convenient to use an asymmetric risk measure that would penalize only costs above the expected value. Shabbir and Shahinidis (1998) proposed to use upper partial mean of the recourse cost as the measure of variability in a robust optimization model for process planning under uncertainty. In List et al. (2003) an upper partial moment (UPM) of order 1 was used in a robust optimization model for fleet planning under uncertainty. Takriti and Shabbir (2004) used the upper partial moment of order 2 for robust optimization of two-stage stochastic models.

3.2. Proposed variability measures

As we have already mentioned, in the production planning problem that we are addressing, using the symmetric mean-variance tradeoff for recourse cost can generate solutions that are inefficient and which would not be considered by a rational manager. Regarding the proposed asymmetric variability measures in the literature and the recent developments on optimization solvers, namely CPLEX 10, which have made it possible to solve the large-scale quadratic programs in a reasonable amount of time, we propose two variability measures of recourse costs in this problem, namely the upper partial moment of order 2 (UPM-2), and the upper partial variance (UPV).

3.2.1. Upper partial moment of order 2 (UPM-2)

The upper partial moment of order 2 (used also in Takriti and Shabbir, 2004) is defined as follows.

$$\bar{\Delta}_+^2 = \sum_{s\in\Omega} p^s \Delta_+^{s^2}, \qquad (13)$$

where

$$\Delta_+^s = \max\left\{0, (d^{sT} y^s - R^*)\right\}, \qquad (14)$$

and R^* is the target recourse cost. For scenario *s*, $\Delta_+^{s^2}$ is the squared positive deviation of that scenario's recourse cost from the target recourse cost. In this way, $\bar{\Delta}_+^2$ is defined as the expectation of the squared positive deviations over all scenarios.

3.2.2. Upper partial variance (UPV)

The upper partial variance is the quadratic version of upper partial mean (UPM) of Shabbir and Shahinidis (1998). It is defined as follows.

$$\bar{\Delta}_+^2 = \sum_{s\in\Omega} p^s \Delta_+^{s^2}, \qquad (15)$$

where

$$\Delta_+^s = \max\left\{0, (d^{sT} y^s - \sum_{s\in\Omega} p^s d^{sT} y^s)\right\}. \tag{16}$$

For scenario s, $\Delta_+^{s^2}$ is the squared positive deviation of that scenario's recourse cost from the expected recourse cost. In this way, $\bar{\Delta}_+^2$ is defined as the expectation of the squared positive deviations over all scenarios. It should be mentioned that the advantage of UPV variability measure over the (UPM-2) is that UPV does not require a priori specification of a target recourse cost and therefore is more flexible.

4. Robust optimization model for multi-period sawmill production planning

Consider a sawing unit with a set of products (lumbers) P, a set of classes of raw materials (logs) C, a set of production processes A, a set of machines R, a planning horizon consisting of T periods, and a scenario set $\Omega = \{1, 2, ..., S\}$ for random processes yields. For modeling simplicity, we define a production process in a sawing unit as a possible combination of a log class and a cutting pattern. The (first-stage) decision variable is the number of times each process should be run in each period (production plan X_{at}). This is equivalent to finding log consumption of each log class as well cutting pattern selection for each log class in each period. The production plan X_{at} cannot anticipate the yield scenarios and must be feasible for all of the scenarios. Inventory (I_{pt}^i) and backorder (B_{pt}^i) size of each product in each period are the recourse decision variables that can be determined based on the first-stage production plan and the realized scenarios for processes yields. To state the robust optimization model for this production planning problem, the following notations are used:

4.1. Notations

<u>*Indices*</u>

p product (lumber)

t period

c raw material (log) class

a production process (combination of a log class and a cutting pattern)

r machine

i scenario

<u>*Parameters*</u>

h_{pt} Inventory holding cost per unit of product p in period t

b_{pt} Backorder cost (lost opportunity and goodwill) per unit of product *p* in period *t*

m_{ct} Raw material cost per unit of class *c* in period *t*

I_{c0} The inventory of raw material class *c* at the beginning of planning horizon

I_{p0} The inventory of product *p* at the beginning of planning horizon

s_{ct} The quantity of raw material of class *c* supplied at the beginning of period *t*

d_{pt} Demand of product *p* by the end of period *t*

ϕ_{ac} The units of class *c* raw material consumed by process *a* (consumption factor)

ρ_{ap}^{i} The units of product *p* produced by process *a* (yield of process *a*) for scenario *i*

p^{i} The probability of scenario *i*

δ_{ar} The capacity consumption of resource *r* by process *a*

M_{rt} The capacity of resource *r* in period *t*

N Number of yield scenarios

λ Goal programming parameter ($\lambda \geq 0$)

R^{*} Target inventory/backorder cost

Decision variables

X_{at} The number of times each process *a* should be run in period *t*

I_{ct} Inventory size of raw material of class *c* by the end of period *t*

I_{pt}^{i} Inventory size of product *p* by the end of period *t* for scenario *i* (recourse decision variable)

B_{pt}^{i} Backorder size of product *p* by the end of period *t* for scenario *i* (recourse decision variable)

Δ_{+}^{i} The variability measure of inventory and backorder cost for scenario *i*

4.2. The robust optimization model

$$\text{Minimize } Z = \sum_{c\in \mathrm{C}}\sum_{t=1}^{T}\sum_{a\in A} m_{ct}\phi_{ac}X_{at} + \sum_{i=1}^{N}\sum_{p\in \mathrm{P}}\sum_{t=1}^{T} p^{i}[h_{pt}I_{pt}^{i} + b_{pt}B_{pt}^{i}] + \lambda\sum_{i=1}^{N} p^{i}\Delta_{+}^{i^{2}} \quad (17)$$

Subject to

Material inventory constraint

$$I_{ct} = I_{ct-1} + s_{ct} - \sum_{a\in A}\phi_{ac}X_{at}, \qquad t = 1,\ldots,T,\ c\in \mathrm{C}, \quad (18)$$

Production capacity constraint

$$\sum_{a \in \mathrm{A}} \delta_{ar} X_{at} \le M_{rt}, \qquad t = 1, 2, \ldots, T, \; r \in \mathrm{R}, \tag{19}$$

Product inventory constraint

$$I_{p1}^{i} - B_{p1}^{i} = I_{p0} + \sum_{a \in \mathrm{A}} \rho_{ap}^{i} X_{a1} - d_{p1},$$

$$I_{pt}^{i} - B_{pt}^{i} = I_{pt-1}^{i} - B_{pt-1}^{i} + \sum_{a \in \mathrm{A}} \rho_{ap}^{i} X_{at} - d_{pt}, \qquad t = 2, \ldots, T, \; p \in \mathrm{P}, \; i = 1, \ldots, N, \tag{20}$$

Recourse cost variability

$$\Delta_{+}^{i} \ge \sum_{p \in \mathrm{P}} \sum_{t=1}^{T} (h_{pt} I_{pt}^{i} + b_{pt} B_{pt}^{i}) - \sum_{i'=1}^{N} \sum_{p \in \mathrm{P}} \sum_{t=1}^{T} p^{i'} (h_{pt} I_{pt}^{i'} + b_{pt} B_{pt}^{i'}), \qquad i = 1, \ldots, N, \qquad \text{(RO-UPV)} \tag{21}$$

$$\Delta_{+}^{i} \ge \sum_{p \in \mathrm{P}} \sum_{t=1}^{T} (h_{pt} I_{pt}^{i} + b_{pt} B_{pt}^{i}) - R^{*}, \qquad i = 1, \ldots, N, \qquad \text{(RO-(UPM-2))}$$

Non-negativity of all variables

$$X_{at} \ge 0, \; I_{ct} \ge 0, \; I_{pt}^{i} \ge 0, \; B_{pt}^{i} \ge 0, \Delta_{+}^{i} \ge 0, \qquad c \in C, \; p \in P, \; t = 1, \ldots, T, \; a \in A, \; i = 1, \ldots, N. \tag{22}$$

The objective function (17) is to minimize the raw material consumption cost, the expected inventory and backorder costs, in addition to inventory and backorder cost variability for all scenarios in the planning horizon. The inventory and backorder costs are computed by multiplying the inventory and backorder unit cost by the inventory and backorder size, respectively. As it was mentioned in section 3, λ is the goal programming parameter that models the tradeoff between the expectation and variability of the recourse cost in the objective function. For $\lambda = 0$, model (17)-(22) would be the two-stage stochastic model in Kazemi et al. (2007b; 2008). Constraint (18) ensures that the total inventory of raw material of class c at the end of period t is equal to its inventory in the previous period plus the quantity of material of class c supplied at the beginning of that period (s_{ct}) minus its total consumption in that period. It should be noted that the total consumption of each class of raw material in each period is calculated by multiplying material consumption factor of each process (ϕ_{ac}) by the number of times that process is executed in that period. Constraint (19) requires that the total production do not exceed the available production capacity. In other words, the sum of capacity consumption of a machine r by the corresponding processes in each period should not be greater than the capacity of that machine in that period. Constraints (20) ensure that the sum of inventory (or backorder) of product p at the end of period t is equal to its inventory (or backorder) in the previous period plus the total production of that product in that period, minus the product demand for that period. Total quantity of production for each product in each period is calculated as the sum of the quantities yielded by each of the corresponding processes regarding the yield (ρ_{ap}) of each process. Due to the randomness of process yields (ρ_{ap}), these constraints are defined for each scenario of processes yields. Constraints (21) compute the inventory and backorder cost variability for each scenario. Depending on the type of variability measure that is used in the RO model, the mentioned cost variability is defined as follows. In the RO-UPV model (see section 3) it is defined as the difference between the total inventory and backorder cost of each scenario and the expected inventory and backorder cost, while in the RO-(UPM-2) (see section 3) it denotes the difference between the total inventory and backorder cost of each scenario and a target inventory and backorder cost (R^*). Note that constraints (21) and the non-negativity of Δ_+^s together with the minimization in the objective function satisfy the definition of upper partial moment of order 2 ((13) – (14)) and upper partial variance ((15) - (16)).

5. Scenario generation

In this section, we explain how the scenarios for random processes yields (ρ_{ap}^i) can be generated in the RO model. We define a scenario in this model as the combinations of scenarios for yields of individual processes. We suppose that the yields of different processes are independent. Therefore, as the first step, all possible scenarios for yields of each process should be determined and then these scenarios should be aggregated to generate the scenarios for the RO model. This approach is illustrated in figure 3. A scenario for the yields of process (*a*) (combination of a log class (*c*) and a cutting pattern (*s*)) in a sawing unit is defined as possible quantities of lumbers that can be produced by cutting pattern (*s*) after sawing each log of class (*c*). As an example of the uncertain yields in sawmills, consider the cutting pattern (*s*) that can produce 6 products (P1, P2, P3, P4, P5, P6) after sawing the logs of class (*c*). Table 1 represents four scenarios among all possible scenarios for the uncertain yield of this process.

Insert figure 3 here

Insert table 1 here

In this work, we assume that all the logs that will be processed in the next planning horizon are supplied from the same discrete of forest. Hence, a stationary probability distribution can be considered for the quality of logs and uncertain processes yields during the planning horizon. Regarding the limited volume of logs and dimensions of lumbers, we assume a discrete probability distribution for processes yields. Furthermore, due to the wide variety of characteristics in each log class a huge number of scenarios for processes yields can be expected. The scenarios for processes yields with their probability distribution in the sawmills can be determined as follows.

1) Take a sample of logs in each class (e.g. 300 logs) and let them to be processed by each cutting pattern.
2) Register the yield of the process (the corresponding products with their quantity) for each individual log and consider the result as a scenario.
3) Having observed all the scenarios, calculate their probabilities as their proportion in the population of scenarios.

It should be noted that the implementation of the above approach is very difficult in the sawmills. In fact, the high production speed in the sawing unit makes it difficult to track the logs through the line and to observe the result of sawing individual logs. In this paper we use yield scenarios generated by a log sawing simulator which will be discussed more in sub-section 6.2.

6. Computational results

In this section, we describe the characteristics of the prototype sawmill, scenario generation approach for uncertain processes yields, and some implementation details. We also provide the results of implementing the proposed robust optimization models for the sawmill example. We compare the recourse cost variability in the two RO models and two-stage stochastic one. We also discuss about the performance of the two robust optimization models in controlling backorder/inventory size variability and provide a framework to choose among them depending on the decision maker’s risk preference.

6.1. Example description

A prototype sawmill is used to illustrate the use of the two robust optimization models. The prototype sawmill is a typical softwood sawmill located in Quebec (Canada). The sawmill focuses on sawing high-grade products to the domestic markets as well as export products to the USA. It is assumed that the input bucked logs into the sawing unit are categorized into 3 classes based on their two ends diameters. 5 different cutting patterns are available. The sawing unit produces 27 products of custom sizes (e.g. 2(in) x4(in), 2(in) x6(in) lumbers) in four lengths. In other words, there are 15 processes all can produce 27 products with random yields. We consider two bottleneck machines: Trimmer and Bull. The planning horizon consists of 30 periods (days). Product demand in each period is assumed to be deterministic which is determined based on the received orders. Lumbers that remain from one period to the next are subject to a unit holding cost. The unsatisfied demand is penalized by a unit backorder cost. We assume that the company is very service sensitive and wishes to fulfill customer demands on time as much as possible. Hence, the inventory costs of products are considered much lower than their backorder costs. The inventory holding cost is computed by multiplying the interest rate (per period) by the lumber price; the lumber price is considered as the backorder cost. It would be worth mentioning that the data used in this example are based on the gathered data from different sawmills in Quebec province (Canada). As the list of custom sizes, machine parameters and prices are proprietary, they are not reported in this paper.

6.2. Scenario generation for the uncertain processes yields

In the prototype sawmill that we considered in this work, due to the lack of historical data on the yields of processes, the yield scenarios already generated by a log sawing simulator (Optitek) were used. “Optitek” was developed by a research company for Canada's solid wood products industry (Forintek Canada Corp.). “Optitek” simulates the sawing process in the sawing units of Quebec sawmills. It was developed based on the characteristics of a sample of logs in different log classes, as well as sawing rules available in Quebec sawmills. The inputs to this simulator include log class, cutting pattern, and the number of logs to be processed. The simulator considers the logs in the requested class with random physical and internal characteristics; afterwards it generates different quantity of lumbers for each log based on the sawing rules of the requested cutting pattern. The yielded lumbers of each log can then be considered as a scenario for the yields of the corresponding process.

Recall from section 4 that a yield scenario in the RO model is the combination of yield scenarios of all the processes in the problem. In this example we have 15 processes, each can produce 27 products. Thus, the RO model (17)-(22) includes 405 (27x15) yield coefficients ρ_{ap}. If we assume that each yield coefficient can take 5 different values, the number of scenarios for random yields in the RO model can be estimated as $5^{405} \approx 1.2\times10^{283}$. As solving the robust optimization model (17)-(22) for all scenarios of random yields is far beyond present computational capacities, a random sample of such scenarios is considered. Thus, we generated 250 scenarios by Monte Carlo sampling among the scenarios generated by “Optitek” for the same log classes and cutting patterns that we considered in this example. It should be noted that the same sample size for yield scenarios was used in a two-stage stochastic model for production planning in the same prototype sawmill in Kazemi et al. (2007b; 2008).

Based on the sample average approximation (SAA) scheme which was applied in Kazemi et al. (2007b; 2008), by considering 250 scenarios a good approximate solution with an acceptable optimality gap can be obtained.

6.3. Implementation details

By considering 250 scenarios for process yields in this example, the quadratic programming model (17)-(22) consists of 202900 constraints and 405790 decision variables. Both of the quadratic robust optimization models (RO-UPV and RO-(UPM-2)) were solved by CPLEX 10 barrier solver and all the calculations of the recourse cost variability for different values of goal programming parameter (λ) as well as threshold values (R^*) were performed by scripts in OPL Studio 5.1. All computations were carried out on a Pentium(R) IV 1.8 GHz PC with 512 MB RAM running Windows XP.

6.4. Results of robust optimization approach for the sawmill example

In this section, we report the results of the implementation of the two robust optimization models for the prototype sawmill described in sub-section 6.1.

6.4.1. RO-(UPM-2) model results

Remember from section 3 that RO-(UPM-2) model requires a target recourse cost R^*. It should be noted that the target cost can be determined based on the desired service level. In this sawmill example, we provide the target cost as a percentage of the optimal expected backorder and inventory cost when $\lambda = 0$ (the standard two-stage stochastic program). For example, the expected backorder and inventory cost for this prototype sawmill, by considering 250 yield scenarios, without any penalty on recourse cost variability is 379367. As we mentioned before, a range of robust optimal solutions can be generated in the robust model as we change the robustness parameter λ. This parameter reflects the decision maker level of concern with exceeding the target cost for all scenarios of random yield. Table 2 presents the results of RO-(UPM-2) model for various $R^* - \lambda$ combinations for the sawmill example.

Insert table 2 here

When table 2 provides the value of 80% in column "R^*", the target cost is 379367 x 80% = 303494. In the second column in table 2 the values of λ are provided in multiples of 10^{-5} since R^* =379367 and a quadratic variability measure is used in the RO-(UPM-2) model. The recourse cost variability measure in column 7 is presented as the square root of (13) used in RO-(UPM-2) model. The last column of table 2 includes CPU time (on minutes) for finding the optimal solution of RO-(UPM-2) model by CPLEX 10. As expected, for a given value of R^*, increasing λ reduces the backorder/inventory cost (size) variability. Thus, we can expect more control on the exceeding of each scenario's backorder/inventory cost over the target cost (R^*) as well as decreased expected backorder cost (size), although at the expense of increased raw material cost and the expected inventory cost (size). In other words, by enforcing the backorder/inventory cost variability measure in the objective function of model (17)-(22) (see section 4), the production level and consequently raw material consumption is increased in order to minimize the exceeding of backorder/inventory cost of all scenarios over the target cost. Furthermore, the increased inventory cost (size) is also the result of increasing the production level and raw material consumption. Figure 4 illustrates better the tradeoff between the backorder/inventory cost variability and raw material cost for different values of λ for each R^*. Figure 5 illustrate the tradeoff between the expected backorder and inventory cost by enforcing the robustness parameter in RO-(UPM-2) model for $R^* = 100\%$.

Insert figure 4 here

Insert figure 5 here

As it can be observed from the results presented in table 2, decreasing the target recourse cost R^* in this example does not necessarily decrease the variability measure. This implies that the control on the exceeding of the backorder/inventory cost of scenarios over a target cost might be limited depending on the yield scenarios as well as problem constraints (i.e. raw material inventory and machine capacity constraints). In other words, by imposing a target cost on the variability measure in the RO model it might not be feasible to achieve a plan with small recourse cost variability. In this example, for the target costs R^* below or equal to the two-stage stochastic model expected recourse cost, by enforcing the value of λ the recourse cost variability can be decreased into a limited value. On the other hand, for higher values of R^* (120% and 140%) more robust production plans with less variable backorder/inventory cost (size) can be achieved at the expense of lower service level (higher expected backorder size).

From the above discussions, it can be concluded that, if the decision maker wishes to use RO-(UPM-2) model to obtain a robust production plan, he/she should choose a value of λ which reflects his/her risk aversion about backorder/inventory cost (size) variability as well as increased raw material consumption and expected inventory cost (size). Moreover, it might not be feasible to achieve a completely robust production plan by considering any desirable service level (target cost R^*), depending on the yield scenarios and problem constraints.

6.4.2. RO-(UPV) model results

Table 3 presents the results of RO-(UPV) models as well as the two-stage stochastic LP (2-stage stochastic LP) for the sawmill example. In the second column in table 3 the values of λ are provided in multiples of 10^{-5} since a quadratic variability measure is used in the RO-(UPV) model. The recourse cost variability measure in column 7 is presented as the square root of (15) used in RO-(UPV) model. The last column of table 3 includes CPU time (on minutes) for finding the optimal solution of RO-(UPV) model by CPLEX 10.

Insert table 3 here

In the RO-(UPV) model, as it can be observed from table 3, by increasing the value of parameter λ, the backorder/inventory cost variability decreases significantly, while the expected backorder cost is augmented considerably and the expected inventory cost and the raw material cost is decreased. In other words, by enforcing the backorder/inventory cost variability measure in the objective function of model (17)-(22) (see section 4), a higher expected backorder/inventory cost is determined by the model to minimize the exceeding of backorder/inventory cost of all scenarios over the expected backorder/inventory cost. Thus, the expected backorder size is increased and consequently production level and raw material consumption are decreased. Furthermore, the decreased expected inventory cost is also the result of decreasing the production level and raw material consumption. Figure 6 illustrates the tradeoff between the backorder/inventory cost variability and the expected backorder/inventory cost in RO-(UPV) model.

Insert figure 6 here

From the above discussions, it can be concluded that if the decision maker wishes to have a robust production plan by using RO-UPV model, he/she should choose a value of λ which reflects his/her risk aversion about backorder size variability as well as increased expected backorder cost (size). Since the customer service level is defined in this work as the proportion of customer demand that can be fulfilled, the increased expected backorder cost (size) leads to decreased customer service level.

It should be noted that, setting a value for λ and R^* in the above robust optimization models requires explicit managerial input regarding the degree of risk aversion that is appropriate for a given situation. In practical sense, it is probably most effective to run the model with a substantial range of λ, and R^* values, creating a set of solutions like the set graphed in table 2, and 3 and figures 4 and 6 and let the mangers pick a desired solution from that set, rather than trying to specify the most appropriate value of λ and R^* in priori.

6.4.3. Comparison between RO-(UPM-2) and RO-UPV models performances

As the expected recourse cost is not limited by a target value in RO-UPV model, the backorder/inventory cost (size) variability can be controlled as much as possible by increasing the value of λ. On the other hand, in RO-(UPM-2) model, the control over recourse cost variability depends on the target cost (target service level) as well as yield scenarios and problem constraints. Figure 7 illustrates the difference between the robustness of optimal solutions in RO-(UPM-2) model and RO-UPV model for different values of R^* and λ. In figure 7, as the target cost R^* increases (the service level decreases) in RO-(UPM-2) model, the robustness of plans proposed by this model gets closer to those of RO-UPV model. However, more robust solution of RO-UPV model might own larger expected backorder cost (size) (lower customer service level) compared with those of RO-(UPM-2) model. The comparison between the total costs of both RO models is presented in figure 8. Finally, as it is shown in tables 2 and 3, the execution time of RO-UPV model is also larger than that of RO-(UPM-2) one.

Insert figure 7 here

Insert figure 8 here

In a very service-sensitive company that wants to establish a reputation for always meeting customer service level, the robust optimization formulation allows a decision maker to see explicitly what possible tradeoffs between backorder/inventory cost (size) variability and the expected cost exists, and to choose a solution that is consistent with his/her willingness to accept risk. In the sawmill example, the following decision framework can be proposed. If the decision maker prefers to determine a robust customer service level that remains near optimal as much as possible for all scenarios of random yield, he/she should select the solution of RO-UPV model which results in less backorder/inventory cost (size) variability. However, the solution of this model might result high expected backorder cost (size) which reduces customer service level. Thus, in the case of choosing the RO-UPV model, a value of robustness term λ should be selected that reflects appropriately the tradeoff between risk aversion level of the decision maker about the robustness of customer service level and the expected backorder cost (size). By choosing the smaller values of λ, lower expected backorder size and consequently better service level can be promised to the customer while this service level is not completely robust. On the other hand, larger values of λ result in promising lower service level to the customer which is considerably robust. In a company where the variability of backorder size (customer service level) is less crucial for the decision-maker, the solution of RO-(UPM-2) can be selected which results an expected backorder size (service level) close to a target one. However the desired robustness level of the plans might not be necessarily achieved depending on the problem constraints and yield scenarios. In this case, λ should be selected that reflects appropriately the tradeoff between the risk aversion level of the decision maker about the robustness of customer service level and raw material and expected inventory cost.

7. Conclusions

In this paper, two robust optimization models with different variability measures were proposed to address multi-period sawmill production planning by considering the uncertainty in quality of raw materials (logs). The computational results of addressing a prototype sawmill by this approach provides evidence supporting the advantages of robust optimization approach in generating more robust production plans over the 2-stage stochastic programming approach. Furthermore, the tradeoff between the plan's robustness (backorder/inventory cost (size) variability) and raw material consumption and expected backorder/ inventory cost (size) for different values of robustness term is discussed for both models. The robust optimization models are compared in terms of their performance in controlling backorder size (customer service level) variability for all scenarios in addition to their total cost (raw material and the expected backorder/inventory cost). A decision framework is also proposed to select among two RO models based on risk aversion level of decision maker for the robustness of backorder size (customer service level) and the increased total cost. Although these results are found for sawmill production planning, the proposed approach in this work can be applied for production planning in other manufacturing environments where non-homogeneous and random characteristics of raw materials result in random yield. Future research will consider also the products demands as random variables in order to obtain more realistic production plans.

8. Acknowledgment

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10. Tables

Table 1. Scenarios for yields of a process in a sawing unit

Scenarios	Products					
	P1	P2	P3	P4	P5	P6
1	1	0	1	0	1	1
2	2	1	1	0	1	0
3	1	0	0	1	1	1
4	2	0	0	1	0	1

Table 2. Results of RO-(UPM-2) model for sawmill example - The values of λ are in multiples of 10^{-5}.

R^*	λ	Raw material cost	Expected recourse (backorder/inventory) cost	Expected backorder cost	Expected inventory cost	Recourse (backorder/inventory) cost variability ($\overline{\Delta}_+$)	CPU time (min.)
-	0	155473	379367	378560	807	-	3.3
60%	1	218392	343974	342942	1032	178009	4.5
	2	236984	340742	339662	1080	176136	4.5
	5	259574	339013	337895	1118	175070	4.5
	10	275420	338259	337121	1138	174592	4.5
	20	285092	338041	336900	1141	174474	4.5
80%	1	210971	346041	345031	1009	137648	4.5
	2	228366	342169	341110	1059	135700	4.5
	5	252038	339526	338420	1106	134252	4.5
	10	269560	338756	337640	1116	133738	4.5
	20	280942	338406	337285	1121	133533	4.5
100%	1	201327	349348	348369	979	98539	4.5
	2	219352	343931	342896	1035	96124	4.5
	5	242070	340381	339292	1089	94284	4.5
	10	258540	339528	338435	1093	93611	4.5
	20	272871	338812	337702	1110	93195	4.5
120%	1	191259	354018	353074	944	62844	4.5
	2	205886	348158	347165	993	60523	4.5
	5	227558	342977	341925	1052	58490	4.5
	10	243688	340873	339787	1085	57644	4.5
	20	257247	340047	338962	1085	57145	4.5
140%	1	181544	359445	353074	944	42515	4.5
	2	194034	353158	347165	993	41033	4.5
	5	213167	346798	341925	1052	39364	4.5
	10	227432	344132	339787	1085	38532	4.5
	20	240147	342989	338962	1085	38091	4.5

Table 3. Results of RO-(UPV) model for sawmill example - The values of λ are in multiples of 10^{-5}.

λ	Raw material cost	Expected recourse (backorder/inventory) cost	Expected backorder cost	Expected inventory cost	Recourse (backorder/inventory) cost variability ($\overline{\Delta}_+$)	CPU time (min.)
0	155473	379367	378560	807	111627	3.3
1	134286	537009	536298	711	50000	12
2	137009	628833	628118	715	25000	12
5	129062	751170	750492	678	10000	12
10	118655	831630	831004	626	5000	12
20	102452	923633	923089	544	2500	12

11. Figures

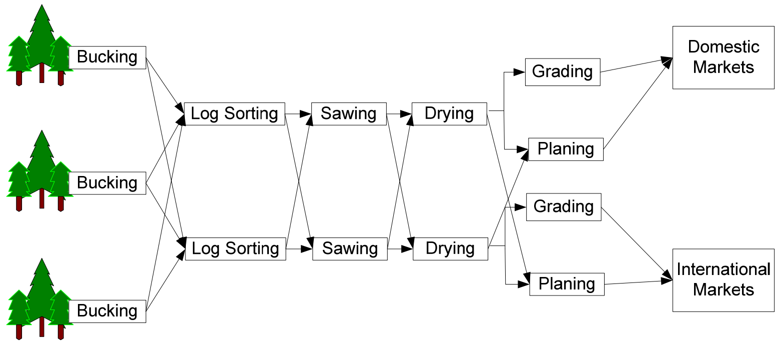

Figure 1.Illustration of sawmills processes

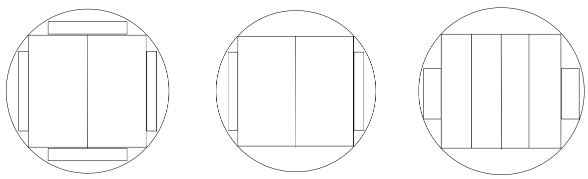

Figure 2. Three possible cutting patterns in a sawmill

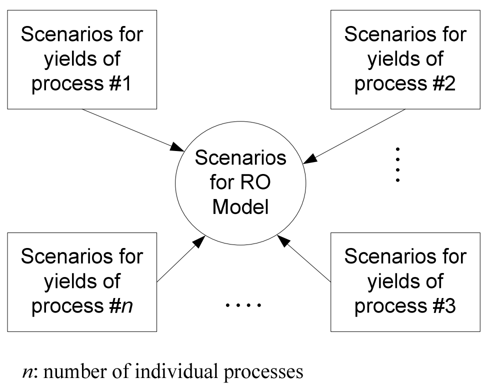

Figure 3. Scenario generation approach

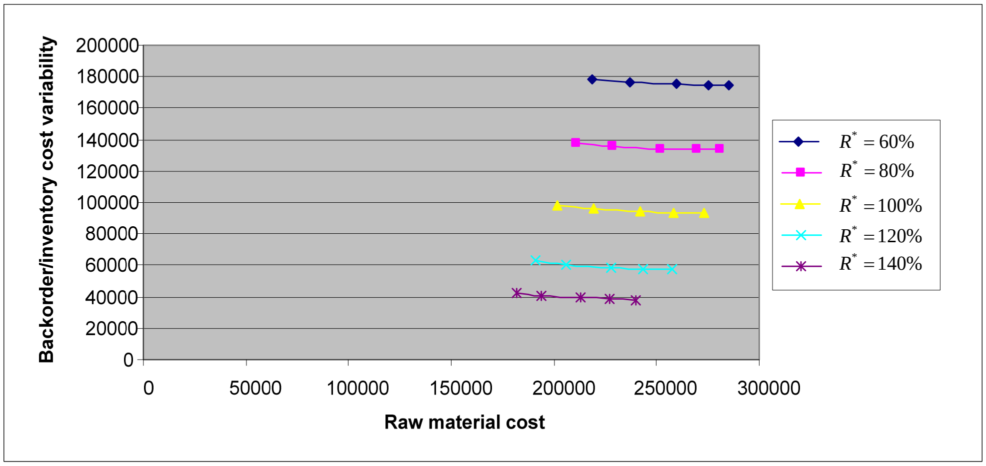

Figure 4. Raw material cost and backorder/inventory cost variability tradeoff in RO-(UPM-2) model for different values of λ and R^*

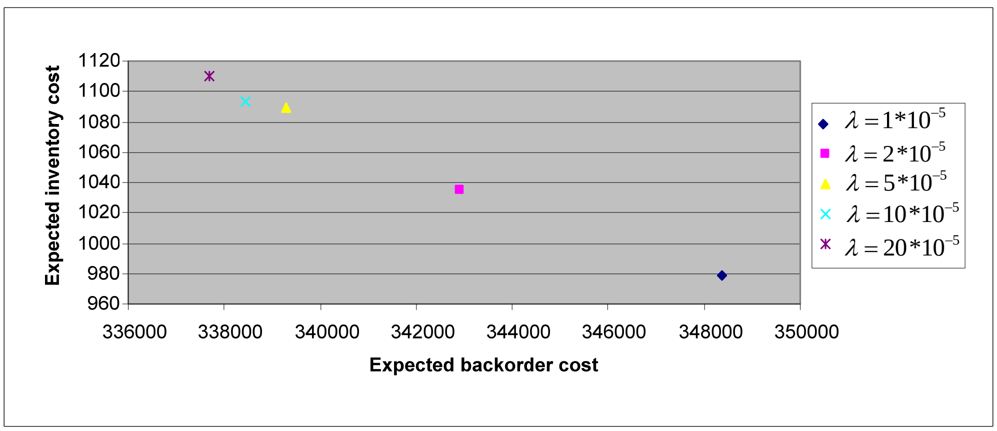

Figure 5. Tradeoff between the expected backorder and inventory costs for different values of λ ($R^* = 100\%$) in RO-(UPM-2) model

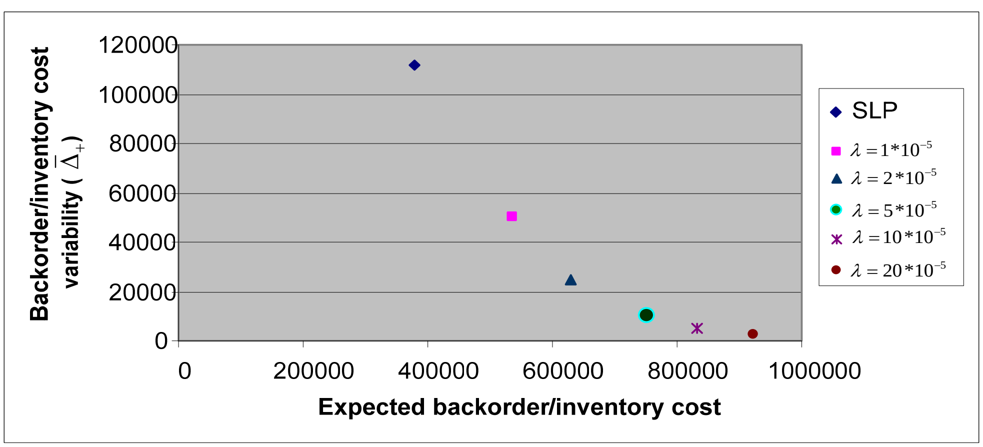

Figure 6. Expected backorder/inventory cost and backorder/inventory cost variability tradeoff in RO-(UPV) model for different values of λ

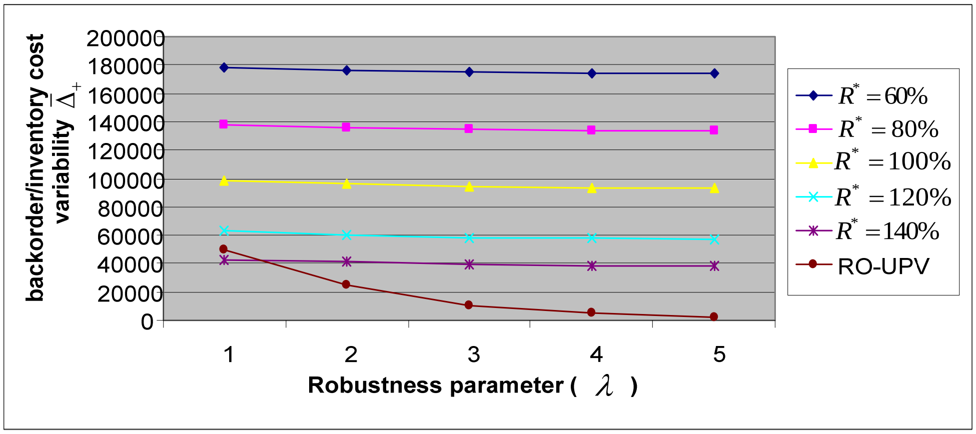

Figure 7. Comparison between the performance of two robust optimization models in controlling recourse cost variability

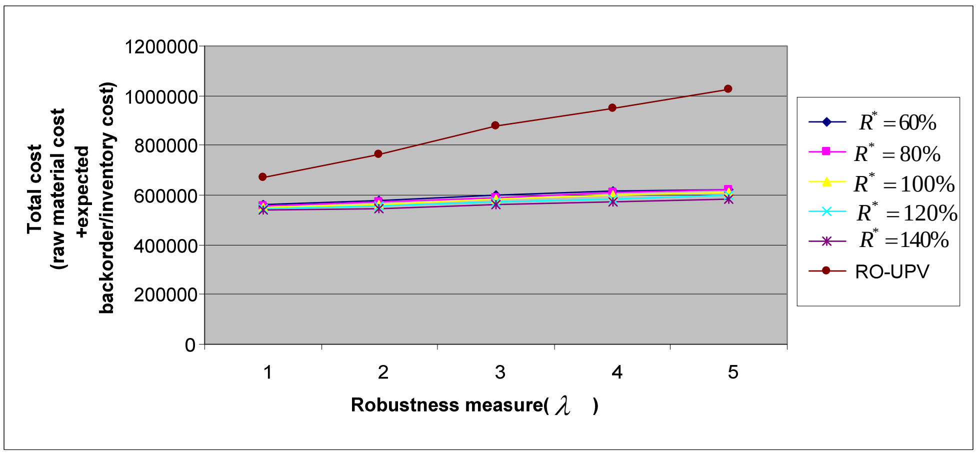

Figure 8. Comparison between the total cost resulted by two robust optimization models