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Multiple-Round Timber Auction Design and Simulation

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Abstract. This paper presents a multiple-round timber auction simulation, developed in order to study various configurations of auction design. In this study, sequential and simultaneous timber auctions are modelled and analyzed using agent-based simulation technology. As there are many individual items in the auction to be sold, the auction designer defines several rounds that are sequential at pre-defined intervals. Since bidders are offered different items at each round, a mathematical linear programming model for selecting the best set of items to bid for is presented. Different bidding strategies are simulated and compared in various setup configurations. The most advanced of these strategies are adaptive and use agent-learning capability. The comparisons include the success rate of winning the auction and the winning price per m³. This study suggests an efficient strategy for bidders to bid in order to achieve their goal and increase their profit. Similarly, in order to increase profit, the auctioneer (i.e., the government) needs to control several auction parameters including the number of auctions per year, the lot size, the auction periodicity, and the number of bidders. This study also suggests parameters configurations that to maximize revenue for the auctioneer.

Keywords: Timber auction, sequential auction, learning strategy, multi agent system, and allocation.

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1. Introduction

Environmental pressure to reform forest management practices on public land, as well as drastic reduction in industrial activity following forest products markets collapse in the United States, have led to a net decrease in timber sales. At the same times, successful mills or entrepreneurs complained that access to wood supply was impossible under the Québec forest regime, which was based on an exclusive long-term licencing system. Moreover, this licensing system made it difficult to establish a fair price for transactions. In response to these issues, the Québec government decided to make a portion of the annual wood supply (25%) available through an auction system, as soon as 2013. With wood available through auction, buyers can access supplies according to the value of their own forest products market. In such a context, designing an auction system while preserving a certain level of guaranteed supplies is complex. Different goals are pursued such as offering a certain level of stability to traditional user, offering opportunities to new entrepreneurs and assuring a fair financial return for a public asset.

In this paper the sealed first-price Auction protocol is considered as the interaction protocol between the auctioneer agent (i.e., a government agency) and the bidder agents (i.e., forest products companies). In this type of auction, bidders submit their sealed bids, all at the same time, without disclosure of the bid content to competitors. After evaluation, the bidder with the highest bid is announced to pay the proposed price and own the lot. This method of auction is different from the English auction method, in which each bidder can only bid once at each time. Further, bidders cannot adjust their proposed bid, since they do not have information about their competitors' bid. It is most preferable to use sealed first-price technique in tendering, e.g. mining leases and governmental contracts (Milgrom, 1989).

Timber auctions aim at selling timber lots via a bidding process. The multiple-round timber auction is a process, in which the auctioneer announces several different items (i.e., wood lots) periodically to the bidders. The design of a simulation platform of a wood procurement system based on a multiple-round auction requires a framework that captures the basic dynamics of that system. Therefore, agent-based technology is used in this study to design and simulate realistic agents behaviours and bidding strategies in the context of a multiple-round timber auction.

A software agent is a situated autonomous computer system capable of sensing and reacting to change in its environment without the direct intervention of a user. As a consequence, a software agent has a certain level of control over its actions. A software agent can also exhibit goal-directed behaviour by interacting with other agents or humans.(Wooldridge and Jennings, 1994).

In this paper, different combinations of bidding strategies and auction design parameters are simulated and compared in order to better understand the impacts of various factors of the auctions outcomes. The results show the combined influence of several auction design parameters and bidding strategies over both bidders' capacity to achieve procurement target and the seller's total profit.

The remaining of the paper proceeds as follows. Section 2 presents the literature review. The simulation multiple-round auction model is presented in Section 3, followed by the models of the agents' bidding strategies in Section 4. Section 5 presents the results of experiments designed to compare and validate the various bidding strategies. Next, Section 6 presents and discusses the results of experiments designed to specifically study the influence of various auction configurations. Finally, Section 7 concludes and presents the limitations of this research.

2. Literature review

Allocating and pricing limited natural resources, such as oil, mineral rights, spectrum, and timber, are two important questions. In order to solve timber allocation problems, many auction models have been used (Mead (1967), Hansen (1985), Paarsch (1991), Elyakime *et al.* (1994, 1997), Baldwin *et al.* (1997), Athey and Levin (2001), Haile (2001), Athey *et al.* (2011)). In practice, formal and informal processes are used to determine the allocation of natural resources. Auctions are an example of formal process for allocating and pricing natural resources. They have generated research interest in economic, marketing and consumer behaviour fields. Auction maximizes the revenue for the seller, while being transparent and competitive method of allocation. An efficient auction design can achieve both an efficient assignment of rights to bidders, and maximizes revenue for the seller (Cramton, 2007).

The auction process contains three main elements: auction issues, auction protocols, and auction strategies. The bidders apply the auction protocols to express clear rules and procedures. These rules are used to send bids, accept or reject proposals, as well as decide when the auction starts

and ends. The bidders' preference and the need of the bidder at the time of auction are part of the auction strategies (McAfee and McMillan, 1987).

Multiple-round auctions usually consist of a number of auctions that are announced consecutively or concurrently, dealing with multiple goods (Grossklags *et al.*, 2000). One of the important aspects of analysing this type of auction is to attempt to analyze and predict bidders' behaviour. To achieve this goal, different theoretical and empirical studies have been developed (Kagel, 1995). Many of the studies on bidding in multiple-round auctions involve online auction (Anthony and Jennings 2002, Shehory 2002, Airiau and Sen 2003, Greenwald and Boyan 2005, Gerding 2008, Yue *et al.* 2010).

Similarly, several studies have compared the advantages and disadvantages of sequential auction over simultaneous auction (Weber, 1983, Menezes, 1993). In sequential Auctions, Zeithammer (2004) investigated the bidders' forward-looking behaviour. In forward-looking behaviour, bidders intend to underbid if they expect another auction by the seller to happen in the next round of the auction. Along the same line, Ganuza (2004) studies the sale item information to be revealed by auctioneer in a round of the auction. This study shows that to have more competition, the auctioneer should publish less information to the bidders. By using data from Internet auction sites, Pinker *et al.* (2000), and Karuga *et al.* (2005) studies the number of items to be sold in each round of a sequential auction. However, in these studies, they did not consider both sequential and simultaneous auction.

Similarly, bidding strategies have been studied in various kinds of auction systems. One such strategy is Zero-Intelligence-Plus (ZIP) strategy (Gode and Sunder 1993, Cliff and Bruton 1997). However the advantage of ZIP strategy is unknown over other strategies. In sequential and simultaneous auctions, Boutilier *et al.* (1999) and Tesauro and Bredlin (2001) investigated bidding strategies that use history (past auctions). Mathematical functions are widely used by different strategies to calculate optimal bid(s) value, or to calculate the amount of bid at every time step for the bidder.

The application of multi-agent technology to simulate and study auction systems is generating increasing interest (Vidal, 2007). Indeed, such a technology allows researchers to study the interactions among agents and process large amounts of data. Furthermore, multi-agent

simulation enables the modeling of bidders' interactions and bidding strategies in complex environments. Mehlenbacher (2007) explain that multi-agent simulation has some advantages, as it does not require simplifying assumptions of mathematical analysis, nor assumptions about Bayesian Nash equilibrium used by econometric methods.

However, although bidding processes have been used in the field of distributed artificial intelligence, such as in the Contract-Net, the design of a simulation system of auctions requires to address and overcome challenges. One of these challenges is the randomness of bidders' preferences (Shoham and Leyton-Brown, 2009 and Vidal, 2007).

In order to analyze different bidding strategies in different applications, several studies use autonomous agents. These agents are autonomous and intelligent software entities that are designed to conduct different task with minimum human supervision. Andreoni and Miller (1995) implemented experiments with artificial adaptive agents systems and investigate learning in auctions. To study the interaction of agents they used genetic algorithm to implement adaptive learning algorithm. However, their method is not compatible with the context of time pressure, as many factors should be considered. They explained that adaptive learning is very hard to investigate in auction. Bapna *et al.* (2003) applied different types of agents to simulate auctions, aiming to maximize both seller and buyer profit. They introduced agents with virtual behavior that can play with real human bidders. They also proposed hybrid bidding strategies, which consist three different bidding strategies.

In an auction, bidders need to consider the other participants when they offer their bids. In contrast, a seller (i.e., the auctioneer) should consider the protocol of the auction, potential buyers, and other competing sellers in order to sell items with highest profit (Park *et al.* 1999). Agents use different models to find their best moves (i.e. equilibrium strategies); one model in game theory is to use a prediction of other bidder's possible moves and payoffs (Kreps, 1990). Other researchers have reported the design of an agent with ability of predicting opponents move in the bid, as well as opponents' idea about other participants (Gmytrasiewicz and Durfee, 1995, Vidal and Durfee, 1996). However, when the model is complex and dynamic with a large number of bidders, the behaviour modeling of other agents is impossible. Even if some models exist, using them is difficult and implementation is complex (Park *et al.* 1999).

Cramton (2007) studied the design of auctions and highlighted the reason why auction is appropriate for the allocation of natural resources to individual companies. For instance, the author claimed that the structure of bidder preference and the competition level are two examples of settings that determine the best auction format. Simultaneous sealed first-price auction is one of the best options for a weak competition and for bidders with additive values. It is indeed easy to implement. It requires no price discovery. It controls weak competition and bidder collusion (Milgrom 1987). Sealed-bid auction is less disposed to collusion, while in open bidding, bidders use predetermined agreements through their bids. Similarly, sealed-bid auction returns higher revenues when bidders have different preferences (Maskin and Riley 2000, Klemperer 2002).

Athey *et al.* (2011) used some data of timber sales for auction design to compare the results of open and sealed-bid timber auctions. As an observed outcome, small bidders are attracted more to sealed-bid timber auctions, which generate greater revenue for the U.S. in some forests.

In collaboration with the partner of this project, the *Bureau de mise en marché des bois* of the Québec government, a simulation platform was developed and implemented in order to study various configurations of multiple-round sealed-bid timber auctions. This type of auctions consists of a number of auctions announced at predefined time periods and concurrently (i.e., at each time period, a set of timber lots are announced simultaneously). It was selected by the *Bureau de mise en marché des bois* as the Québec timber auction system. The aim of this paper is to present a multi-agent auction simulation model and the results of various experiments, and to analyse these results in order to better understand the impacts of these configurations.

3. Multiple-Round Timber Auction Model

The proposed model contains three main components: the seller (i.e., government), the buyers (i.e., mills, entrepreneurs), and the auctioneer (i.e., a governmental agency). The auctioneer manages the publication and general organisation of the auction. The seller wants to sell several items (i.e., timber lots). The auctioneer announces the items periodically in several rounds of auctions. In other words, at each round, there are many items to be sold. At the start of each round, which is decided by the auctioneer, the items for sale in this period are announced. These items have specific characteristics such as their location, their timber volume, and their species and quality, which make them different from each other. Once the auctions are initiated, bidders

must decide whether or not they wish to bid on these items, and how much. Because bidders can be located anywhere, transform different types of timber, and supply different forest products markets, they have different valuation and interest on each of the items.

The design of such an auction system includes several decision variables and parameters. First, there is a set of items (I) that the auctioneer announces to the bidders. Each item i is unique, with a specific set of features. In other words, the potential value of each item is different from the others. In the simulation, these features are randomly assigned to each item. More specifically, it is assumed that each item consists of two species including hardwood and softwood, two different levels of quality for each species, a predefined volume, a ground slope, and a geographic location. In other words, each item to be sold is represented by a volume of hardwood of quality 1, a volume of hardwood of quality 2, a volume of softwood of quality 1, a volume of softwood of quality 2, a location, and a reserved price.

Next, the simulation model contains different types of agent. Bidder agents, also called bidders (J), participate in the auction and bid for items. Here, three kinds of bidding agents are defined according to their needs for specific types of products. These three types include the paper mill, the lumber mill, and the entrepreneur. Paper mills mostly require softwood; lumber mills mainly need hardwood; and, entrepreneurs are interested in both softwood and hardwood. The parameters of each bidder include the type of bidder as well as its transformation capacity per year (i.e., both paper mills and lumber mills), their supply need per year, their location, their bidding strategy, and their forest products market price. Similarly, in order to study the impacts of various auction configurations, the proposed simulation model includes also several parameters such as the number of bidders, the average lot size, the auction periodicity, and the number of auction per year.

At the start of the auction, the auctioneer calls all of the potential bidders. These bidders may not be interested in all items. Therefore, at each round, several individual items are generated and announced to the bidders. For each individual item auction, each bidder i has an entry cost k_i of gathering information and entering the auction and a private value for the item v_i depending on their valuation of the item. Other factors include distance, supply need, transformation capacity, and the market price that mills can obtain for their products. Concerning entrepreneurs, their

capacity is defined as the forecast of their buyers' aggregated demand. Similarly, their need is defined as their capacity minus the volume of their past wins.

After announcing items and receiving bids, the auctioneer chooses the winner that has offered the highest price for each item. Some bidders may win one or more items, while others may obtain none. If the item is not assigned at a specific round, it remains in the set of items to be sold and is announced again during the next round of the auctions until it is sold. The developed model allows bidder agents to update their needs in order to reflect changes in their environment.

4. Agents' model

The design of such a simulation platform requires the modelling of the behaviour and interactions of two types of agents, namely the auctioneer and the bidders. In sequential simultaneous timber auctions, bidders face two non-trivial decisions: (1) Which sub-set of items (i.e., timber lots) is more profitable for them to bid on; and (2) How much should they bid for each item. The first question depends on the characteristics of the lot and on the supply needs of the mill. The second depends on their valuation of each lot, as well as on the bidder's bidding strategy. The design of our simulation platform proposes several elements to address these questions. The next section explains the auctioneer's and bidders' decision problems and the processes design to solve and simulate them.

4.1 Auctioneer

In a sequential auction, at the start of each round, the auctioneer announces the auction. Once the auction is closed after a specific time period, the auctioneer identifies the winners. The design of a simulated auction system requires the auctioneer to consider the potential issues regarding the auction process and the behaviour of the bidders. One of the key issues in sequential auction is collusion. For a single unit auction, Graham and Marshall (1987) and Mailath and Zemsky (1991) address the collusion problem caused by a group of bidders who cooperatively agree to bid in an auction. Such cooperation usually occurs via meetings outside the auction. By colluding, bidders collaborate to decrease the level of competition to pay less for the auctioned items.

In order to avoid collusion, the *Bureau de mise en marché des bois* proposes in its auction design that an auction is cancelled if it receives 2 bids or less. In this context, the items for sale are

simply offered again during the next rounds of auctions. In many cases, such a constraint prevents collusion because the probability of collusion is lower when a bidder wants to have an item at a specific time and the bidder knows there is a chance of cancelling the auction. Therefore, the bidders prefer participating in the auction without collusion and winning the auction rather than collaborating with others and losing their winning chance. Similarly, mills belonging to the same corporate group or company are not allowed to bid separately. Another technique to avoid collusion is simply to increase the number of bidders. Because the auctioneer has a limited control over the number of bidders (there is indeed a limited number of forest products companies in the region), bids are allowed from outside of the region.

After selecting items to be auctioned, and for a given number of bidders, the auctioneer identifies the first highest price for each item, as long as the price is higher than the reserved price. Subsequently, the auctioneer announces the winner and the price paid by the winner. Hence, at the end of each period, all bidders know the winner of each item and its price. If an item does not have any winner, the auctioneer offers again that item at the next round of the auction.

4.2 Bidders

For each item i , bidder j chooses a value according to its bidding strategy. The value for the item could be either zero (not interested) or a number equal or larger than the reserve price RP_i . It is assumed the bidders know the reserve price of each item. In a single item auction, for each item i , bidder j has a private value $v_{i,j}$ for the item, as defined by equation (1).

$$v_{i,j} \in [RP_i, MP_{i,j}] \quad (1)$$

Equation 1 defines the value interval, where $MP_{i,j}$ shows the maximum price that bidder j is willing to pay for item i . This price reflects the price that the bidder is prepared to pay to have a minimum profit from the item. Obviously the farther the item is from the mill the smaller the maximum price, because of transport costs. According to the characteristics of the item (e.g., overall quality of the forest lot), bidders define their maximum prices. For each item i , V_i and R_i show respectively the volume and the type of item. Furthermore, the distance of the item i to bidder agent j is shown by $D_{i,j}$. Equation (2) defines the maximum price $MP_{i,j}$.

$$MP_{i,j} = V_i(MRP_i - HC_i - D_{i,j}TC_i - PC_{i,j} - PR_{i,j}) \quad (2)$$

MRP_i is the average revenue of the final product generated from item i . HC_i is the cost of harvesting item i . TC_i is the transportation cost. The average processing cost of converting the product i into the final product of bidder j is PC_{ij} . Finally, $PR_{i,j}$ is the minimum profit that the company is willing to gain from the item.

4.2.1 Bidders' items selection problem

At each round, bidders are offered many items. They might consequently be interested in more than one item simultaneously. They should therefore decide on which items they want to bid, considering the characteristics of the items, including their species, their size and their quality, as well as the bidders' need and the items' distance to their mills. The distance from a item to the processing facilities must be considered since transportation accounts for a significant share of procurement costs. Also, bidders must also make sure they have the capacity to process (i.e., harvest and transport) the items they bid on. Furthermore, in practice, bidders might be more interested on larger volume items. In other words, when companies have access to more volume in the same location, they need less coordination with other mills or entrepreneurs, who might be interested in buy undesirable species (in case of mixed species lots). Larger volume items might also involves scale economies with respect to harvest cost. Therefore, bidders must weight these parameters and constraints in order to find the best set of items to bid on. A set of such items is referred to as the solution of items to bid on at each round. This best solution represents the mills or entrepreneurs list of items that are the most profitable to bid on. It is expected that bidders bid according to it. In some cases, bidders may have several possible sets of items to bid on. Bidder agents try to find the best solution in their region. The challenge is to establish the option that yields the most profit for them. Two opposite problems may arise from the bidding process. On the one hand, obtaining more than needed induces unnecessary costs for the bidders, such as inventory related costs. On the other hand, bidders can bid on several items and win nothing because they have poorly estimated their value or bid too low. This decision problem is defined by the following binary integer programming model:

maximize:

$$\sum_{i \in I} \frac{V_i}{D_i} x_i \quad (3)$$

subject to:

$$MP_{i,j} x_i > RP_i \quad \forall i \in I \quad (4)$$

$$\sum_{i \in I} V_i x_i < ND_j \quad (5)$$

$$x_i = \{0,1\} \quad \forall i \in I \quad (6)$$

In this model, V_i , D_i , and RP_i are respectively the volume of item i , the distance of item i from the mill, and the reserve price of item i . $MP_{i,j}$ is the maximum price that bidder j is willing to pay for item i . The need of bidder j at time t (i.e., the total volume of wood to acquire) is ND_j . Binary decision variable x_i represents whether or not an item is selected.

In this formulation, each bidder assigns a weight to each possible item at each round. The weight is defined as the volume of the item over the distance of the item to the bidder's mill, in order to maximize the volume while minimizing the distance to obtain it. Therefore, the larger the volume and the smaller the distance, the more interesting the item is. Although it is rather simple, this interest indicator provides a good guide for bidders to identify interesting items.

The objective function aims to maximize the total interest of items. The first constraint (4) ensures that the bidder consider only the feasible items, for which the maximum price the bidder is willing to pay is higher than the reserved price of the item. Constraint (5) states that the sum of the selected items is less than its need, in order to avoid bidding on more items than needed. This may include a small buffer to account for lost bids. Equation (6) is the integrity constraint.

4.2.2 Approaches for the bidding strategy

In order to design realistic bidding strategies for simulation purposes, we first developed several strategies based on fundamental concepts from the literature that we adapted to the specific problem of forest auctions. Then, we compared them and analyzed their performance. To do so, we assumed that bidders have information concerning past auction outcomes, including winners and winning prices of all items. In other words, we generally assumed that the information known by bidders is limited to private information and public information concerning the results of the previous auctions. The behaviour of bidders during in auctions is called the bidding strategy. Five bidding strategy approaches are presented hereafter, respectively *random bidding*, *fixed behaviour*, *adaptive*, *learning*, and *adaptive learning*.

Random bidding approach

In the *random bidding* approach, bidders bid randomly a value between a minimum price (or the reserved price) and a maximum price. This approach is the most simple strategy as bidders are inattentive to past auctions or private information and do not follow any particular logic. Equation (7) describes the random bidding approach. In this equation, r is a random number between 0 and 1.

$$v_{i,j} = r * (MP_{i,j} - RP_{i,j}) + RP_{i,j} \quad (7)$$

Fixed behaviour approach

The second approach is a slight variation of the random bidding approach. Therefore, in the *fixed behaviour* approach, bidders systematically bid according to their risk averseness, as shown in equation (8):

$$v_{i,j} = K * (MP_{i,j} - RP_{i,j}) + RP_{i,j} \quad (8)$$

Here, k is a constant between 0 and 1, which is decided by the bidder before the auction as a fixed strategy. In other words, if $k=1$, then the bidder systematically bid its maximum value (i.e., risk averse). On the contrary, if $k=0$, then the bidder bids her lowest value (i.e., cool-headed).

Adaptive approach

Bidders using the random bidding and the fixed behaviour approaches are Zero Intelligent (ZI) agents (Mathieu *et al.* 2006). In other words, by using any of the first two approaches, bidders bid ignoring any internal and external information, such as past auctions wins. Although bidders do not know about other bidders' approaches, they can build a strategy line for themselves using private information. Wei *et al.* (2010) suggest a bidding strategy for multi-round auctions that considers the impact of time on the valuation function. Because we assume that bidders have an annual supply target to achieve in order to supply their mill with a specific type of wood, we exploited this idea to develop an approach that adjusts the valuation function according to the time remaining to achieve that target, but also to the remaining supply need of the bidder at the time of the auction. Indeed, the smaller the remaining time a bidder has to achieve her supply target, or the smaller the percentage of her target she achieved with previous auctions, the more concessions she is likely to make (i.e., the more risk averse she becomes). Consequently, we

develop the third approach, namely the *adaptive* approach, in which bidders bid according to their perception of the pressure of the remaining time and supply need. In other words, the adaptive approach is designed to keep the bid value low, and to only increase it when the pressure to achieve the supply target is high.

Therefore, we define the influence of the remaining time to achieve the supply target (t) and of the remaining supply need to achieve the target (n) as a linear increasing function, defined by equation (9)

$$v_{i,j} = \left(\frac{MP_{i,j} - NP_{i,j}}{2} \right) * \tanh \left(\alpha * \frac{f_1(y)}{f_2(d)} * \frac{f_{j3}(n)}{f_{j4}(c)} - 2 \right) + \left(\frac{MP_{i,j} + NP_{i,j}}{2} \right) \quad (9)$$

Here, $f_1(y)$ and $f_2(d)$ are functions representing respectively the whole year (or time period over which the bidder must achieve a specific supply target) and the remaining time until the end of the year (or time period). Next, $f_{j3}(n)$ and $f_{j4}(c)$ are functions representing respectively the remaining supply need at the specific time of the auction, and the overall supply target of the bidder. All of these functions are continuously increasing. This valuation reflects a number between the minimum price and the maximum price for an item i that a bidder j is willing to pay.

This type of behaviour can be interpreted in the system as adaptive to the perceived pressure of time and supply need, with respect to the bidder's supply target. In other words, at the start of the year, bidders offer near minimum price as they have time to achieve their target. However, if their target is high, their perception of the pressure to achieve it may force them to bid higher. As the auction progresses, bidders have less time and therefore have fewer possibility of winning in the next rounds of auctions. Therefore, while they initially accept the risk of losing and decide to bid less, as time passes without winning, they will choose to bid more, with less profit, in an attempt to increase the likelihood of winning a bid. Figure 1 shows some examples.

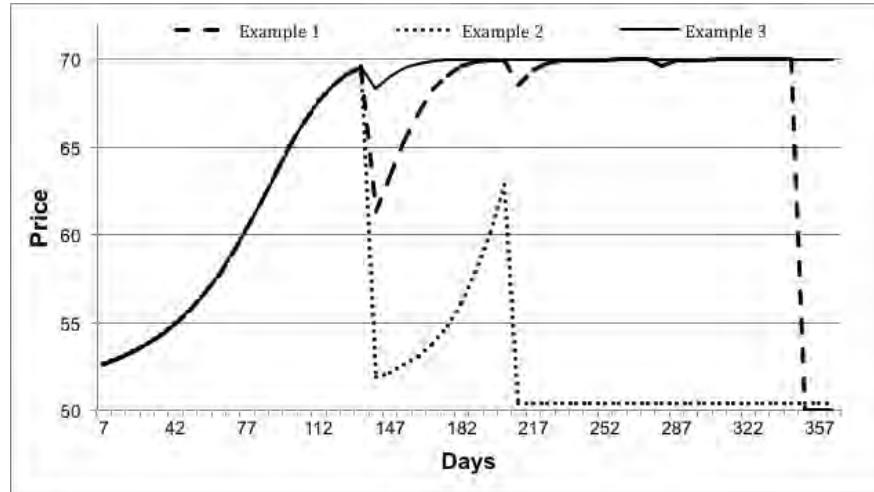


Figure 1: Examples of adaptive approach behaviour

In these examples, each point of each line shows a simulated bid at time t for an item with minimum and maximum prices of 50 and 70 respectively. At the start of the simulation, the bidder offers near minimum price as there are still many opportunities to win. As the time progresses, if the bidder does not win, she incrementally bids higher. Drop points represent a specific win. When the bidder wins an item which volume represents a higher portion of her supply target, the drop is larger. When the bidder wins, she starts bidding from drop point, and gradually increases her bid until it reaches her maximum price or she wins again. Similarly, an early win of an item has a larger drop point than a late win of the same item, unless the supply target is achieved by the win. For instance, example 2 shows a bidder that has stopped bidding because she won sufficient bids early on.

Learning approach

Recently, agent learning algorithms have reached remarkable outcomes (Vidal, 2007). The purpose of learning in multi-agent systems is to create some agents, which can use previous experience for their future bidding (Mitchell, 1997).

Learning theory leads to many valuable tools (Mathieu *et al.* 2006). These tools help multi-agent researchers to find the achievable equilibrium points of a system. At the design stage of a multi-agent system, designers do not know exactly every condition that agents will encounter during their operations. Therefore, by adding a learning capability to the agents, designers provide their agents with the capacity to adapt their behaviour to situations that happen at run time.

In sequential auctions, learning is a method that helps bidder agents to build their offers according to available information. Learning agents use algorithms to analyze available data to bid more carefully. In order to do this, we propose a learning approach, which aims to estimate the winning value of an item according to specific parameters, using the results from prior auctions, including sale prices, the items' lot sizes, and the winners' location. Using the distance between the winners' mill and the items, the price paid and the characteristics of the items, a learning bidder agent runs a regression model to estimate the likely value of new items to be auctioned. Such an approach allows bidder agents to identify an 'average' winning strategy based on past auctions. At each round, learning agents computes the coefficient of the regression function (10), in which $y_{i,j}$ is the price of item i estimated by bidder j , $d_{i,j}$ is the distance of bidder j to item i , and $x_{1,j}$, $x_{2,j}$, $x_{3,j}$ and $x_{4,j}$ are the volumes of the four species/quality combination considered in this study.

$$v_{i,j} = y_{i,j} = \beta_0 + \beta_1 d_{i,j} + \beta_2 x_{1,i} + \beta_3 x_{2,i} + \beta_4 x_{3,i} + \beta_5 x_{4,i} \quad (10)$$

Adaptive learning approach

Out of the four bidding strategies presented above, only the last two strategies proposed some sort of bidding behaviour that changes over time according to specific, yet different, information input. The adaptive approach adjusts the valuation function according to bidder's objectives and the time left to achieve it. The learning approach only adjusts the valuation function according to past winning conditions. If these two behaviours seem to follow reasonable bidding rationalities, a rational bidder can adopt any strategy that is between these two. Therefore, we introduced a fifth approach that is a hybrid of both the third (adaptive) and fourth (learning) approaches. More specifically, the valuation function of such a bidder is described by the equation (11).

$$v_{i,j} = \alpha \left(\frac{MP_{i,j} - NP_{i,j}}{2} \right) * \tanh \left(\frac{f_1(y)}{f_2(d)} * \frac{f_3(n)}{f_4(c)} - 2 \right) + \beta \left(\frac{MP_{i,j} + NP_{i,j}}{2} \right) + (1 - \beta)y_{i,j} \quad (11)$$

In this equation, α and β are coefficients defined within [0;1]. With such a hybridisation mechanism, the pure adaptive and learning bidding strategies can be reproduced. For instance, when $\alpha = \beta = 1$, the bidder behaves like a the pure adaptive bidder. When $\alpha = \beta = 0$, the bidder agent behaves like a the pure learning bidder. This mechanism also allows creating bidder agents that equally demonstrate both behaviours simultaneously. In other words, when $\alpha = 1$ and $\beta = 0$,

the valuation function of the pure learning agent is adjusted by the pressure of target achievement as with the adaptive behaviour. On the contrary, when $\alpha = 0$ and $\beta = 1$, the hybrid agent behave simply as a risk neutral bidder agent with a fixed behaviour.

5. Experiments

Several experiments were carried out in order to validate and analyze different aspects of the proposed model. The first two experiments were designed specifically to validate the programmed behaviors of the bidder agents. In the first experiment, we compared the four pure bidding strategies and analyse the results to make sure that the overall outcome of each strategies was coherent with their design specificities. Similarly, the second experiment was designed to validate the hybrid approach, and to compare, in a competitive setup, different configurations of hybrid bidder agents (from the pure adaptive to learning).

Next, in a second part, we carried out an experiment to better understand the impacts of various auction design parameters on the outcome of the auction. This experiment was specifically designed with randomly generated populations of hybrid bidder agents. A factorial design plan of 81 scenarios was implemented and simulated in order to understand the impacts of specific auction design parameters, including average lot size, periodicity, number of item sold and number of bidders. The results of this experiment were validated separately with experts from the *Bureau de mise en marché des bois* of the Québec government.

5.1 Methodology of experiments

The methodology we used to achieve the objectives of the study includes 3 experiments. In the first two experiments, four different scenarios were simulated. Each scenario is a combination of a number of potential bidders and an average lot size. Scenarios with different number of bidders are used to assess the impact of more or less competition on the auctions outcome, while scenarios with different average lot size are used to assess the impact of the average item size on supply target achievement. In each scenario of the first experiment, there are an equal number of bidder agents using each type of bidding strategies. In the second experiment, we simulated and compared the same four scenarios with a set of bidder agents containing an equal number of each five configurations of hybrid bidder agent, as described in Table 1. Finally, in the third experiment, a factorial design plan was used as the combinations of three levels of average lot

size, three levels of periodicity, three levels of number of item sold, and three levels of number of bidders.

Table 1: Defining five configurations by assigning different α and β

$\beta \backslash \alpha$	1	0.75	0.5	0.25	0
1	Adaptive approach (1)				
0.75				(4)	
0.5			(3)		
0.25		(2)			
0					Learning Approach (5)

5.2 Random parameters and common elements

For all experiments, the locations of bidders' mills and sold items are randomly generated. Transportation costs are calculated based on the Euclidian distance between items and mills. Other random parameters were generated by uniform distribution including lot size; volumes of hardwood and softwood of quality 1 and 2 in each item; process cost at each mills; annual production capacity of each bidder's mill; bidders' initial supply targets of each; and market price of each wood product made of hardwood and softwood of quality 1 and 2, for each bidder.

Because each bidder is generally interested in only one combination of species/quality, the market price is different for each combination of species/quality and for each bidder. Therefore, the market price is set to be lower for the species/quality the bidder does not want. It is equivalent to the price of the unprocessed wood in the market, plus transportation cost to the mill. Because market price affects the valuation function through equation 2, if an item contains a large volume of uninteresting species/quality, the resulting bid is lower. This assumption is realistic because we consider that in case of a win, the unused species/quality volumes are sold to other mills without any loss.

A simulation run consists in a 365 time period simulation, in which bidder agents have a unique bidding strategy according to the tested scenario. Each bidder agent is defined by specific public and private parameters including a mill location, a supply target to achieve and a set of cost and revenue functions. In order to obtain a relevant level of statistical significance, each simulation

of each experiment was repeated several times. Also, a simple Taboo Search application was programmed in the simulation platform to solve the Bidders' items selection problem described in Section 4.2.1. This algorithm was used by all bidder agents in every simulation.

Next, in order to analyse the influence of specific design parameters, the average sale price per m^3 and the average target achievement of each simulation runs were measured. In the context of public land, the designer of the auction process is interested in both aspects of the auction outcome. More specifically, the average target achievement is a criterion that measures how much bidders are able to fulfill their needs. In other words, it measures the impact of the auction process on the sustainability of mills' economic activities. Target achievement of a bidder is defined as the volumes of all items won by the bidder during the entire simulation over its supply target. Next, the average sale price represents public economical gain from the auction process. In other words, although the auction process must be designed to generate a large economical gain in the interest of the public, it cannot do so at the expense of local economic sustainability.

5.3 Synthesis of the experiments

As summarized in Table 2, the first part of the experiments focused on the validation of the agents' behaviours. A total of 280 simulation runs were carried out. The second part studies the impacts of various auction design parameters on the outcome of the auction. 1900 simulation runs were carried out and analyzed. The next section presents and discusses the results.

Table 2: Experimental design specificities

	# of scenarios	# of repetitions	# of simulation
Part 1: Behaviour validation experiments			
1	2 parameters, 4 scenarios	Each scenario was repeated 50 times	200
	• number of bidders	2 levels (100 and 200)	
	• average lot size	2 levels ($10,000 \text{ m}^3$ and $20,000 \text{ m}^3$)	
2	2 parameters, 4 scenarios	Each scenario was repeated 20 times	80
	• number of bidders	2 levels (100 and 200)	
	• average lot size	2 levels ($10,000 \text{ m}^3$ and $20,000 \text{ m}^3$)	
Part 2: Auction design experiments			
3	4 parameters, 81 scenarios	Each scenario was repeated 20 times	1620
	• auction periodicity	3 levels (7 days, 15 days and 30 days)	

	• average lot size	3 levels (10.000 m^3 , 15.000 m^3 and 20.000 m^3),	
	• number of items sold	3 levels (100, 250 and 500),	
	• number of bidders	3 levels (100, 150 and 200),	

6. Results and discussion

This section presents and discusses each experiment. Although more experiments were carried out during the development phases of the simulation platform, only the results of the mentioned experiments are analyzed in this paper.

6.1 Experiment 1

In this experiment, the average price is first considered to compare the four bidding strategies. **Figure 2** shows the average sale price per m^3 of the four approaches in all tested scenarios.

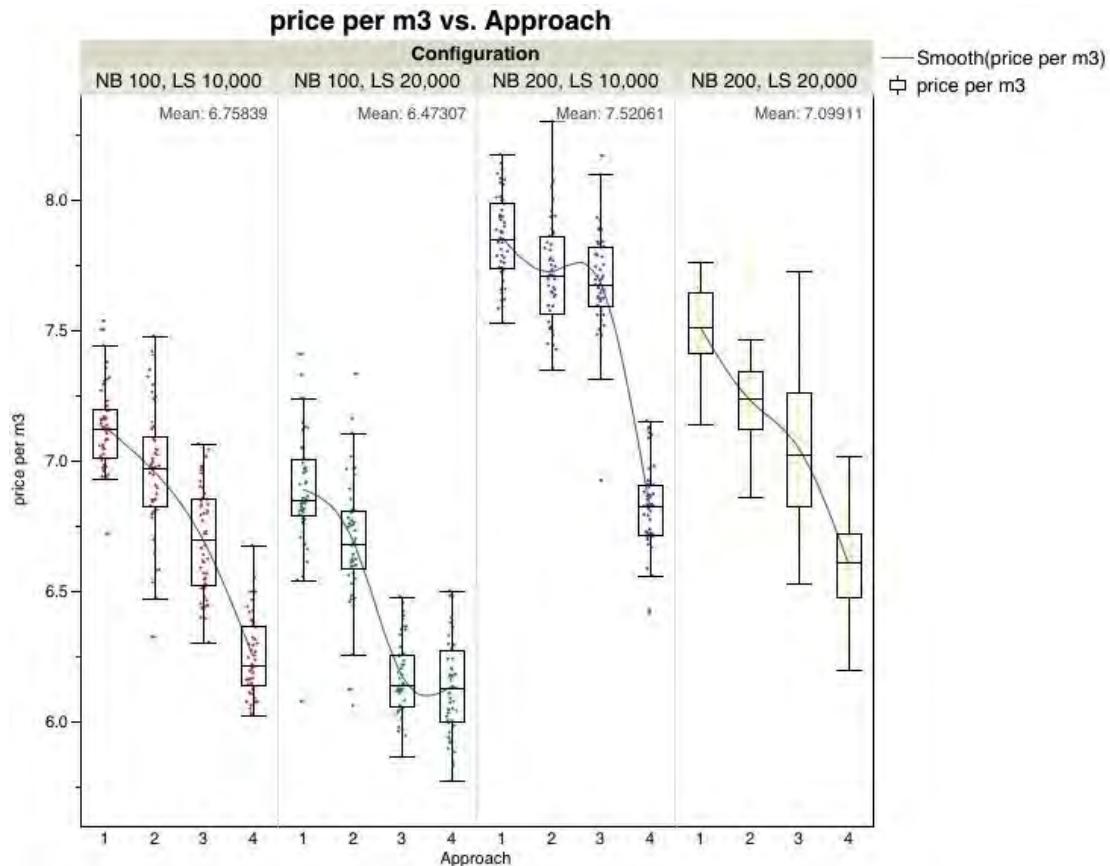


Figure 2: Average sale price per m^3 of four approaches in different setup configuration

First, we can observe that bidders with strategies 1 and 2 pay, on average, more than the last two strategies, which is consistent with the development objectives. Indeed, a bidder using strategy 3 (i.e., the adaptive approach) bids systematically low, unless it is under pressure of achieving its supply target. Similarly, a bidder using the fourth strategy only bids what is likely necessary to win, and not more. This strategy is also more adapted than the third to achieve lower buying prices. Indeed, as seen in Figure 2, bidders using approach 3 pay a higher price than the bidders using approach 4, except in the situation where the competition is lower and the lots are larger (NB 100, LS 20.000). In this specific case, when an adaptive bidder wins an item, because the large volume of the item represents a larger portion of its supply target, its next bid will be lower than if the item represented a smaller portion of its need. Therefore, based on the price paid per m³, the learning approach is better than all other approaches in almost every configuration. However, the price in the adaptive approach is almost equivalent to the price in the learning approach when the competition is low and the lots are bigger. This validates what we intended to program. Similarly, comparing the first scenario with the second, and the third scenario with the fourth, we can also observe that price paid seems to be less when items are bigger. This result, to be confirmed by the fourth experiments, is a first indicator on how to design the auction in order to maximize revenue from the seller point of view. The second aspect that we analyzed in this experiments is the average target achievement as seen in Figure 3.

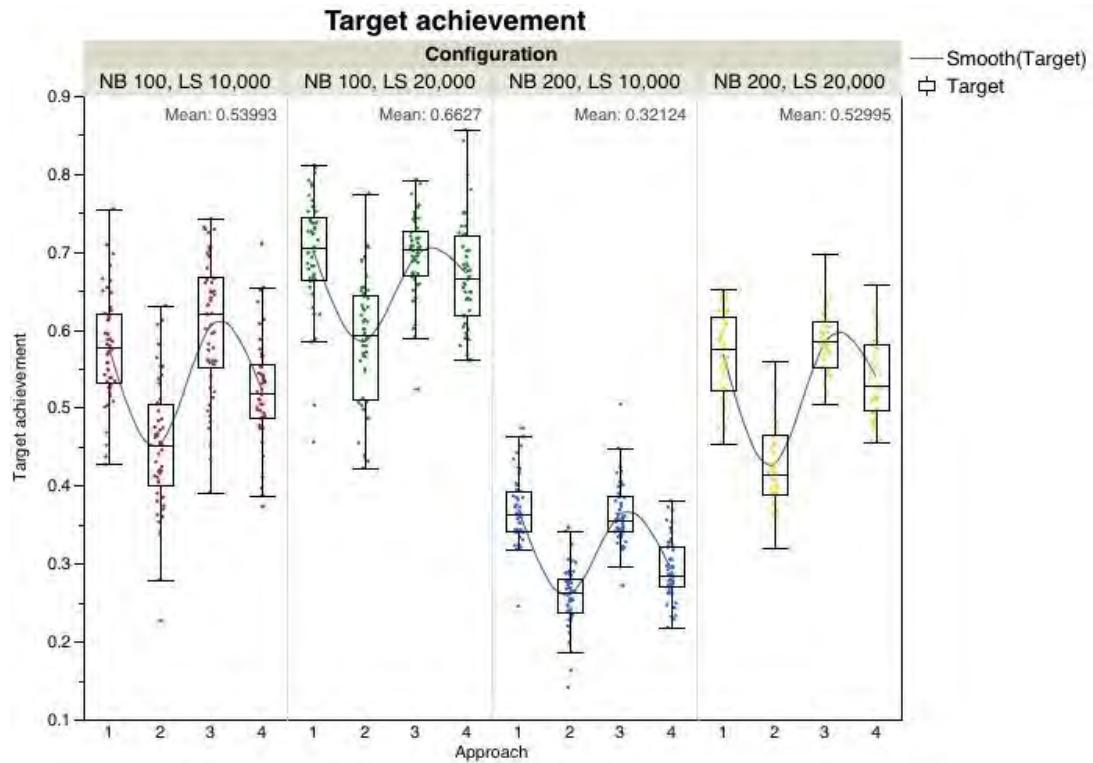


Figure 3: Average target achievement of four approaches in different setup configurations

Here, the average target achievement displays a similar general trend in all scenarios. Bidders using approach 2 have the lowest average target achievement. This is caused by the inability of their strategy to adapt the bid to win an item (not even by generating randomly a high bid like approach 1). Also, approach 1 and 3 are able to generate a better target achievement than the other approaches. Although the target achievement of approach 1 and approach 3 are equivalent, it seems that bidders using approach 1 are only able to obtain a good target achievement by sometime generating higher winning bids. Therefore, they do so at the expense of their average paid price, which is much higher than bidders using approach 3 (see Figure 2). On the same token, bidders using the adaptive approach 3 are able to achieve lower paid price because they only increase their bids when needed (pressure to achieve the target). These bidders also outperform the bidders using the learning approach 4, because their bidding strategy controls the bid so as to improve target achievement, while the learning approach is insensitive to target achievement. These results, again, validate what we intended to program. .

In general, as shown in **Figure 2** and Figure 3, bidders using approach 1 and 2 tend to pay higher for the items in comparison to bidders using approach 3 and 4. This occurs also while they do not necessarily achieve a better target achievement compared to bidder using approach 3 and 4. As a result, the learning approach bidders pay less for the item, while the adaptive approach bidders have a better target achievement. Therefore, according to their objective, bidders should use any combination of these two approaches. This is why, in the remaining experiments, approach 1 and 2 were abandoned, as they do not try to achieve any particular objective.

6.2 Experiment 2

In order to better understand the impacts of combining the adaptive and the learning bidding strategies, we carried out another experiment dedicated to validating this type of hybrid bidder agents. In order to see how this approach performs, we considered and compared 5 combination of the adaptive and learning approaches, with different α and β as explained in Table 1. Note that the pure adaptive and learning approaches were included in this experiment. As studied in experiment 1, we compared the average price per m^3 and the average target achievement. As shown in Figure 4 and Figure 5, there are no absolute best hybrid bidder agents. However, the performance of the approaches is different in each of the four simulated scenarios. For instance, target achievement (Figure 4) seems generally more correlated to the scenario, than to the type of hybrid combination. However, in more competitive scenarios, the more adaptive the bidder agent is, the (slightly) better its target achievement. In less competitive scenarios, this advantage of the adaptive behaviour seems to fade, especially with respect to the learning bidder agents, which actually perform well, which is rather different from the results of the first experiment. This can be explained by the nature of the competitive game. In other words, the auctions simulated in the second experiment are more competitive than the auctions in the first. Indeed, in the second experiment, all bidder agents present some more or less pronounced capacity to adapt to achieve their supply target objective. However, this was not the case in experiment 1, in which bidder agents using approach 1 and 2 were incapable of adapting to the situation. Therefore, these agents were more prone to loose against more intelligent agents. Consequently, pure adaptive agents were not necessarily better than the hybrid agents from that perspective.

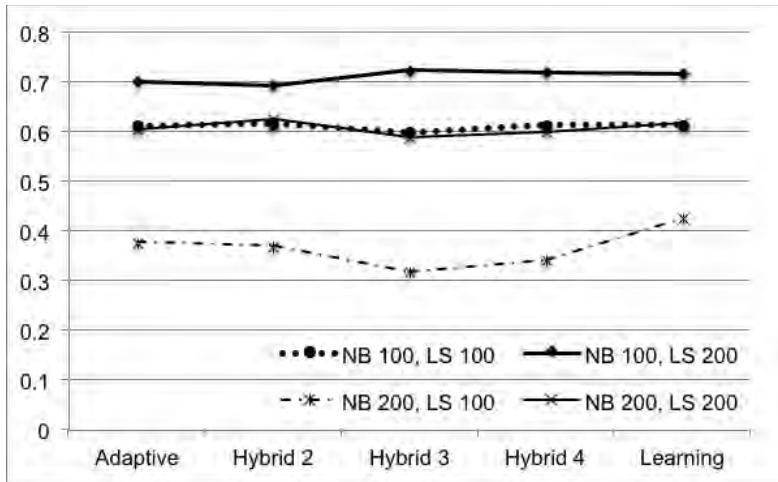


Figure 4: Average target achievement of five approaches in different setup configurations

Concerning the average sale price (Figure 5), several observations can be made. First, because an adaptive agent under pressure can offer bids that are higher than necessary to win an item, it is coherent to observe a poor performance of these agents to achieve a good sale price in a competitive game (BN 200, LS 10.000). However, when the game is less competitive (NB 100, LS 20.000), than adaptive agents actually perform well because they are designed to keep their bid as low as possible when not under pressure. This general result can be observed with hybrid agents as well. However, we can notice that because hybrid 2, 3 and 4 are respectively defined with an incremental decrease of α from equation (11), and therefore an incremental decrease of the influence of the adaptive behaviour, it is coherent to observe a performance of these agents that becomes also incrementally further to the performance of the pure adaptive behaviour. In other words, the less a hybrid agent is influenced by the adaptive behaviour, the less sensitive to competition it is to achieving good sale prices. This also confirms the findings of the first experiment, which, compared to the learning approach, the adaptive approach has a stronger negative impact on the sale price, than it has a positive impact on target achievement.

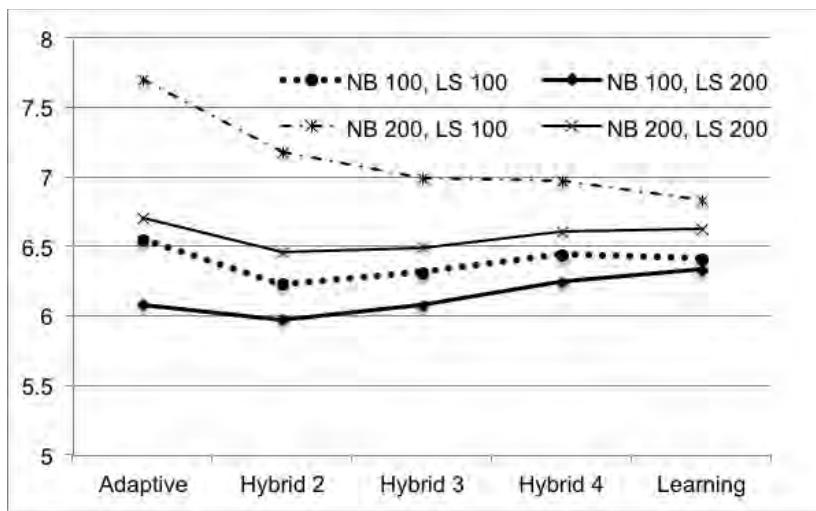


Figure 5: Average sale price of five approaches in different setup configuration

As expected, the observed performance of the different types of hybrid agent is generally correlated to how much of the pure behaviours they are made of. However, it seems that the influence of the adaptive behaviour is more significant than the influence of the learning behaviour, although they all display an almost equally good performance with respect to target achievement. Therefore we can safely assume that the generation of a population of randomly generated hybrid agents is representative of a population of rational bidders driven by any combination of both objectives.

6.3 Experiments 3

As discussed earlier, experiment 3 aims to better understand the influence of several auction design parameters on the outcome the auction. In order to design such an auction, the designer needs to define the number of auction per year, the average lot size of the items for sale, the periodicity of the auction, and, as far as possible, influence the number of bidders by, for instance, selecting lots' location in a strategic manner. These are referred to as the independent variable. From the seller's point of view, these design parameters should be defined in a way to maximize the revenue. However, because the seller is also the Québec government, it must make sure forest companies can operate at sufficient capacity to cover their fixed cost of operation by allowing them to be supplied with large enough quantity of timber. Therefore, as discussed previously, both the average price and the target achievement must be studied. These are referred to as the dependant variables.

As explained in the methodological section, experiment 3 is an extensive simulation of 81 different scenarios defined as the combination of various levels of number of bidders, item lot size, periodicity, and number of items sold as presented in Table 2. Using the data generated by the experiments, we carried out several analyses. First, an analysis of variance presented in the annexe (Table 3 and Table 4) validates statistically the resulted generated by the simulation model. For simplification purpose, this analysis only studies the influences of all combinations of just any two independent variables on both dependant variables. Both ANOVA studies present a R-square above .95, which indicates a high significant level of statistical confidence. Next, in order to better understand the combined influence of any two independent variables on the outcome of the auction, we systematically computed the average target achievement and sale price for the combination of all levels of all pairs of two design parameters.

First, from a general standpoint, all results show systematic opposite effects of all design parameters on both outcomes. For instance, lot size affects positively target achievement and negatively sale price (Figure 6). More specifically, when item size gets larger, the sale price per m³ decreases from \$9 to, sometime, \$4.5, which is rather considerable. This can be explained by the difference in the quantity of bidders interested in bidding. Indeed, large size items are not necessarily interesting to bidders with remaining supply needs smaller than the lot size. In other words, if items lot size are small, then the number of potential bidders for this item increases, which increases competition. Consequently, more participation in the auction causes more demand, which in turn affects the sale price. This result presents a limit of the model to be improved, as it is counter-intuitive for the experts who validated the model. Indeed, in this model, we consider a fix harvesting cost per m³, although in reality, a scale economy can be gained from harvesting larger items (e.g., less low bed transportation are needed to move harvesting equipment). Therefore, if a scale economy can be gained from larger items, then bidders might be willing to pay more to win these items. Although this shortcoming limits our ability to investigate properly the impact of item size, it does not affect the remaining of the study insofar as the lot sizes of each simulation configuration are within a limited range. In other words, within each round of auctions, because the lot sizes are similar, no scale economy is significantly higher for some items. Therefore, these items are not more interesting from that perspective. However, if both small and large items are simultaneously sold within one round of auctions, then a fix cost of harvesting should be considered.

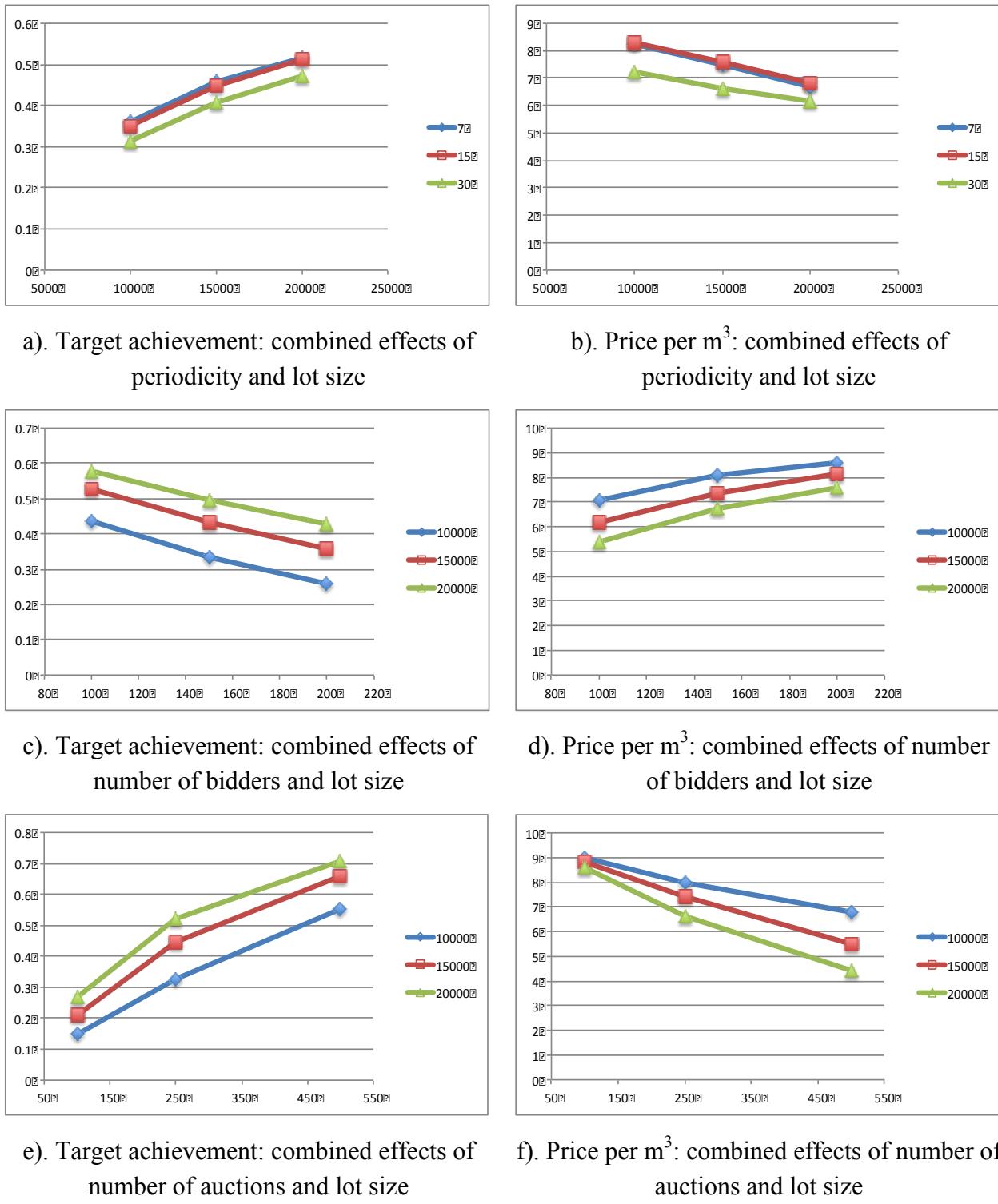


Figure 6: Comparative analysis of target achievement and sale price (Part 1)

Along the same line, if lot size has a rather clear general influence on both outcomes, this influence is mitigated to different extent by the other design parameters. For instance, periodicity (i.e., the delay between two rounds of auctions) has a rather limited influence for values below 15 days. Indeed, as verified in the statistical analysis, there is no significant statistical difference between the simulation results with a periodicity of 7 days and a periodicity of 15 (Figure 6, a., b. and Figure 7 a., b., c., d.). However, a periodicity of 30 days between two rounds of auctions tends to decrease both target achievement and sale price. Therefore, shorter periodicities tend to be generally more beneficial longer ones. This influence can be explained as follows. When periodicity increases, while the total number of auctions remains unchanged, the number of items for sale increases at each round. In other words, there are fewer rounds of auctions, but more auctions at each round. Consequently, the influence of periodicity on sale price target achievement can be explained by the fact that bidders can achieve lesser price if they can bid simultaneously on more items (i.e., more supply quantities per round).

Along the same line, and from a general standpoint, the number of auctions affects positively target achievement, and negatively sale price. Furthermore, it has a mitigating effect on both the lot size and the number of bidders. More specifically, as the number of auctions decreases, the influence of lot size (Figure 6, f.) and the influence of the number of bidders (Figure 7, f.) on sale price are reduced as well. For the same reason, this can be explained by the fact that reduced supply leads to a higher number of interested bidders, even if the items' lot size is large or the number of potential bidders is low. This result is interesting because it shows that higher competition, in other words, a higher number of interested bidders, with respect to a certain level of supply, leads to a market price that better represents the limit of forest companies to purchase items.

Differently, and as expected, the number of bidders has a positive impact on sale price, but a negative impact on target achievement, whatever the context ((Figure 6, c., d. and Figure 7 a., b., e., f.). This result can be explained as follow. As competition and demand increase, the number of bids received during each auction is similarly increased, which results in a higher probability of receiving high value bids. Along the same line, increased competition also reduces the probability of each forest company to win, and therefore reduces their ability to achieving their target.

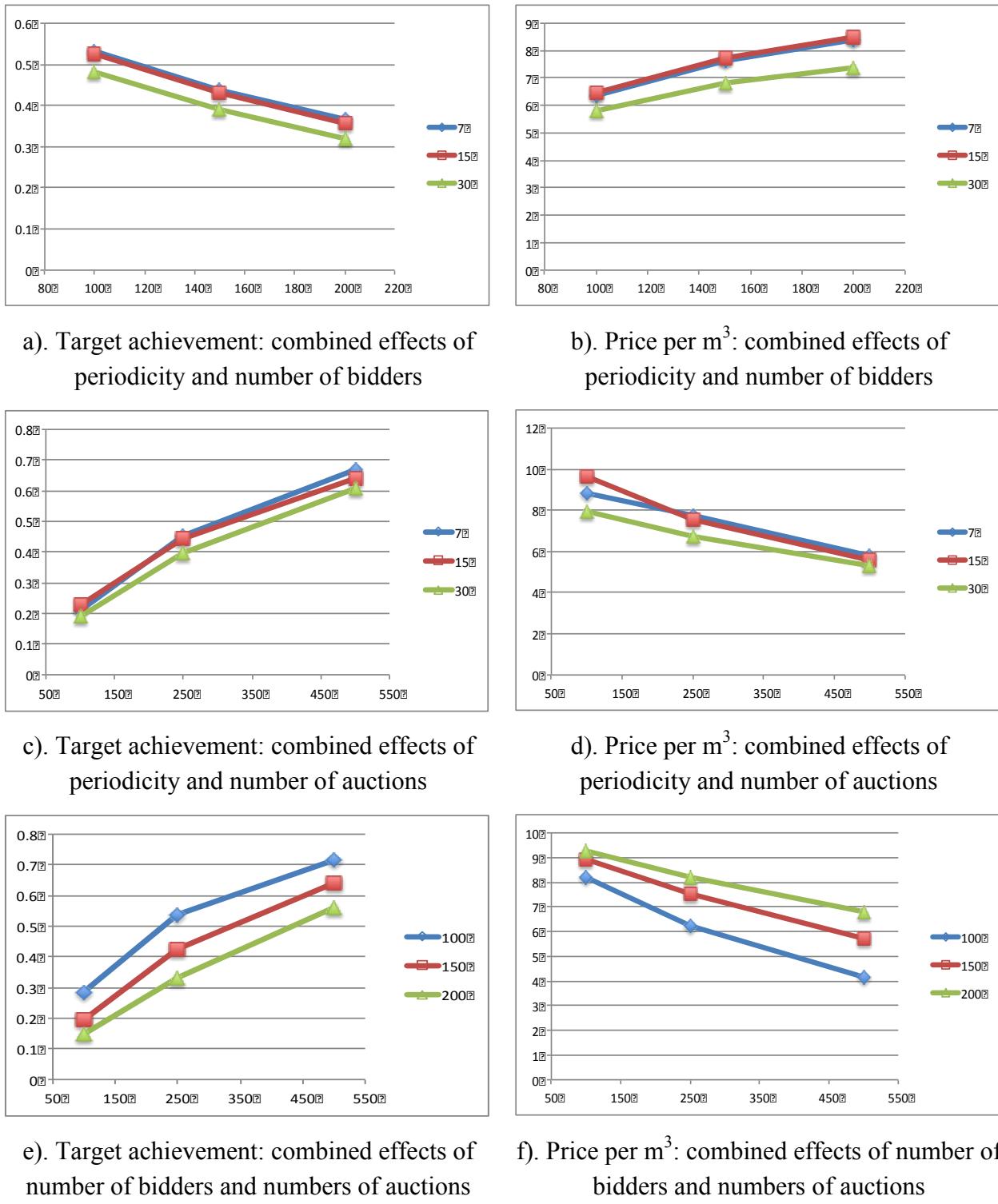


Figure 7: Comparative analysis of target achievement and sale price (Part 2)

Finally, we also studied the correlation between the target achievement and the sale price (results not presented). As it can easily be observed in Figure 6 and Figure 7, this correlation is negative, which tends to show that both objectives cannot be achieved simultaneously. In other words, it seems that the government that owns the forest has a dilemma as it can either maximize its revenue, or support the industry by allowing companies to better achieve their supply target, but not both.

7. Conclusion

This paper first proposed theoretical bidding strategies for the design of automated software agents in the context of natural resource auctions. These bidding strategies were then implemented into a multi-agent simulation platform, which was used in various simulation contexts in order to validate these models, as well as to better understand the impact of various auctions design parameters on the auction performance. This performance was measured through two main indicators illustrating, on the one hand, the forest companies' ability to achieve their supply needs (i.e., target achievement), and, on the other hand, the government's ability to generate revenue from the forest sales (i.e., sales price per m³).

The analysis of the results first shows that the adaptive and learning bidding strategies have the best results and achieve their design objectives. They can thus be used as general guidelines in designing a company's bidding strategy. Next, concerning the design of the auction process, the results tend to indicate that the government (i.e., the forest owner) cannot simultaneously achieve high revenue while providing an effective supply channel to forest companies. It is therefore necessary to find a compromise in order to maintain forest companies' activities, and generate descent revenue for the public. The results also demonstrate the intuitive impact of the number of potential bidder on the revenue generated. It also shows that target achievement is improved by the sales of larger forest lots, while it decreases the average sales price.

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Annexes

Table 3: Analysis of variance of Price per m³

<i>Number of observations =</i>	<i>1620</i>		<i>R-squared =</i>	<i>0.9687</i>
<i>Root MSE =</i>	<i>.320843</i>		<i>Adjusted R-squared =</i>	<i>0.9681</i>
Source	Partial SS	DF	MS	F
Model	5061.57087	32	158.17409	1536.56
Lot size	503.714321	2	251.85716	2446.63
Periodicity	262.610509	2	131.305255	1275.55
Lot size # Periodicity	13.0142071	4	3.25355179	31.61
Number of Bidders	999.679136	2	499.839568	4855.62
Lot size # Number of Bidders	21.4218789	4	5.35546972	52.02
Number of Auction	2855.00328	2	1427.50164	13867.27
Lot size # Number of Auction	172.841574	4	43.2103934	419.76
Periodicity # Number of Bidders	12.7947761	4	3.19869404	31.07
Periodicity # Number of Auction	103.995341	4	25.9988354	252.56
Number of Bidders # Number of Auction	116.495846	4	29.1239614	282.92
Residual	163.366382	1587	10294038	
Total	5224.93725	1619	3.22726204	

Table 4: Analysis of variance of Target Achievement

<i>Number of observations =</i>	<i>1620</i>		<i>R-squared =</i>	<i>0.9844</i>
<i>Root MSE =</i>	<i>.025587</i>		<i>Adjusted R-squared =</i>	<i>0.9841</i>
Source	Partial SS	DF	MS	F
Model	65.6814675	32	2.05254586	3135.01
Lot size	6.8359268	2	3.4179634	5220.52
Periodicity	.710617117	2	.355308558	542.69
Lot size # Periodicity	.003363833	4	.000840958	1.28
Number of Bidders	7.43414058	2	3.71707029	5677.37
Lot size # Number of Bidders	.039392754	4	.009848189	15.04
Number of Auction	49.977939	2	24.9889695	38167.57
Lot size # Number of Auction	.293985382	4	.073496345	112.26
Periodicity # Number of Bidders	.001303433	4	.000325858	0.50
Periodicity # Number of Auction	.13328915	4	.033322288	50.90
Number of Bidders # Number of Auction	.251509432	4	.062877358	96.04
Residual	1.0390365	1587	.000654717	
Total	66.720504	1619	.041210935	

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