



## SÉMINAIRE du CIRRELT SEMINAR

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## “ COPULA AND COMPOSITE LIKELIHOOD APPROACHES – ENHANCED TOOLS FOR TRAVEL BEHAVIOUR ANALYSIS ”



### Résumé / Abstract

In the fields of transportation and social sciences we often encounter situations that exhibit strong dependency structure across different interconnected choice dimensions for a particular individual or among different individuals for a single choice dimension. The dominant approach to modeling such dependency structures is to assume a bivariate (or multivariate as appropriate) normality assumption directly on the error terms, or on transformed error terms in the choice equations. Such an assumption can be restrictive and inappropriate, since the implication is a linear and symmetrical dependency structure between the error terms. In this research, we introduce and apply a flexible approach to model the distinct choice dimensions as a single choice bundle. The approach is based on the concept of a “copula”, which is a multivariate functional form for the joint distribution of random variables derived purely from pre-specified parametric marginal distributions of each random variable. The copula-based approach retains a parametric specification for the bivariate (multivariate) dependency, but allows testing of several parametric structures to characterize the dependency. The proposed approach is applied to model residential neighborhood choice and daily household vehicle miles of travel (VMT), using the 2000 San Francisco Bay Area Household Travel Survey (BATS). The results indicate that the VMT differences between households in different neighborhood types may be attributed to both built environment effects and residential self-selection effects. As importantly, the study indicates that use of a traditional Gaussian bivariate distribution to characterize the relationship in errors between residential choice and VMT can lead to misleading implications about built environment effects.

The copula approach, described above, allows us to accommodate flexible dependency structures in various choice scenarios. However, in terms of estimation of discrete choice models with a general dependency structure, the analyst still confronts a cumbersome multi-dimensional integral evaluation. For instance, consider accommodating spatial dependency across observational units in modeling teenager physical activity participation. In this context, the resulting likelihood function that accounts for spatial dependency would involve a multi-dimensional integral of the order of the number of individuals in the dataset. The existing approaches to accommodate spatial dependency are infeasible for moderate-to-large samples. We propose the composite marginal likelihood (CML) approach, an emerging inference approach in the statistics field, as an alternative. The CML estimation approach can be used when the full likelihood function is near impossible or plain infeasible to evaluate due to the underlying complex dependencies, as is the case with spatial discrete choice models. The combined copula-CML approach is applied to study the daily episode frequency of teenagers' recreational activity participation (both physically active and physically passive), a subject of considerable interest in the transportation, sociology, and adolescence development fields. The results of estimation of the combined copula-CML approach underscore the need to consider spatial effects in recreational activity participation to obtain consistent and efficient parameter estimates and elasticity effects. In fact, the CML approach should be very appealing for application to several other multivariate modeling contexts too because it is simple and flexible, and is easy to implement.

**VENDREDI / FRIDAY**

**28 octobre 2011 /  
October 28, 2011**

**10h30**

**Salle / Room 5441  
Pavillon André-Aisenstadt  
Université de Montréal**

**Bienvenue à tous / Welcome to all**

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