Short communication

A HEURISTIC FOR THE ROUTING AND CARRIER SELECTION PROBLEM

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Abstract

We consider the problem of simultaneously selecting customers to be served by external carriers and routing a heterogeneous internal fleet. Very little attention was devoted to this problem. A recent paper proposed a heuristic solution procedure. Our paper shows that better results can be obtained by a simple method and corrects some erroneous results presented in the previous paper.

Key words: Vehicle routing, external transportation carriers, heterogeneous limited fleet.

1. Introduction

The problem considered in this paper is the one where we have to deliver the demand of a set of customers where each customer should be served either by one of the vehicles of a heterogeneous internal fleet or by an external carrier. Thus, we have to decide which customers are to be served by external carriers and to route the vehicles of the internal fleet in order to serve the demand of the remaining customers. The objective is to minimize the sum of external carrier cost, variable and fixed cost of the internal fleet. Using external carriers is necessary if the capacity of the internal fleet is not sufficient. Also, it may be more economical to use external carriers instead of using an internal vehicle to serve one or very few customers.

Very little research effort has been devoted to deal with vehicle routing problems when external carrier services are available. Ball *et al.* (1983) considered the problem of determining optimal homogenous fleet size in the presence of an external carrier. Klincewicz *et al.* (1990) divide the area to service into a number of sectors, determines the private fleet size and the specific assignment of each sector to a private vehicle or to an outside carrier. Diaby *et al.* (1995) considered the problem where the company has only one vehicle.

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In a recent paper published in the *European Journal of Operational Research*, Chu (2005) introduced a heuristic to select customers to be served by external carriers and to route a limited number of heterogeneous trucks (internal fleet). To asses the performance of his heuristic, Chu (2005) solved 5 problems with 5, 10, 15, 22 and 29 customers respectively. Within these problems, external carrier cost was set equal to 6 times the distance between the depot and the corresponding customer. Chu provided solutions obtained by his heuristic and the corresponding optimal solutions. Unfortunately, solutions reported as optimal for problems 3, 4 and 5 are not.

The main steps of Chu's heuristic are the following:

1- Select customers to be served by the external transportation provider,
2- Use a modified version of Clarke and Wright's (1964) savings heuristic to construct the routes to serve the remaining customers,
3- Use classical local improvement heuristics (steepest descent heuristics) to improve the obtained solution.

Solutions provided by Chu indicate that he always assigned to external carriers the customer with the lowest external cost while having a demand larger than the capacity shortage. Also, we notice that he had used a sequential implementation of a modified version of the Clarke and Wright's heuristic to construct the initial solution. Within this implementation, he used a modified savings criteria that takes into account external transportation cost, fixed cost and variable cost of trucks. Finally, within the implemented improvement step, Chu uses three improvement procedures: (1) the 2-opt heuristic (Lin, 1965) to improve individual route separately, (2) moving one customer to another route, and (3) exchanging the position of two customers. The last two improvement strategies were suggested by Osman (1993) among many others. The used improvement procedures can be combined using 6 different permutations. Chu indicates that he used each and all of these permutations to improve each solution.

2. The proposed heuristic

The heuristic proposed in this paper, called the *SRI* (Selection, Routing and Improvement) heuristic, is composed of the following steps: (1) select customers to be served by the external carrier, (2) construct a first initial solution, (3) improve the obtained solution, (4) construct another initial solution, and (5) improve the second solution. Then, the best obtained solution is retained as a final solution. Using two initial solutions increases our chance to get good solutions within a very reasonable computation time.

Selection of customers to be served by the external carrier

Within the *SRI* heuristic, customers to be served by external carriers are selected as follows:

(1) Renumber customers in the ascending order of e_i/d_i, where e_i is the cost of delivering the demand d_i of customer *i* by external carriers.

(2) Determine *H* such that: $\sum_{i=1}^{H} d_i \geq D - C \geq \sum_{i=1}^{H-1} d_i$, where *D* is the sum of all demands and *C* is the total capacity of the owned fleet.

(3) Assign the first *H* customers to the external carrier.

Construction of initial solutions

As mentioned, two initial solutions are constructed and the improvement procedure described hereafter is applied to each of these solutions. The final solution to retain is the best among the two improved solutions. These initial solutions are constructed by modified versions of the sequential and parallel implementations of Clarke and Wright's heuristic. Within our versions, we use the original savings formula suggested by Clarke and Wright.

Individual routes of both initial solutions are improved by applying the 4-opt* improvement heuristic (Renaud, Boctor & Laporte, 1996). This heuristic requires as little computational time as the 2-opt heuristic but gives much better results. The 4-opt* improvement heuristic is a simplified version of the 4-opt heuristic (Lin, 1965) which test only 8 among the 48 ways to reconstruct a Hamiltonian cycle after removing 4 arcs from the initial cycle.

Like the original Clarke and Wright's heuristic, our modified sequential version constructs routes one by one except that it only constructs a number of routes equal to the number of available trucks. The total demand of a constructed route is limited to the capacity of the larger truck still unused (initially all trucks are considered as unused). Once a route is constructed, we assign it to the smallest truck having enough capacity to perform it. The selected truck is then removed from the list of unused trucks. Finally, if the resulting routes do not visit all those who should be served by the internal fleet, non served customers are added to the list to be served by the external carriers.

The parallel implementation of our modified version of Clarke and Wright's heuristic starts by assigning each customer to a temporary individual route and as it goes along, move some or all of them to permanent routes. At the end, all customers not assigned to permanent routes are moved to the list of those to serve by the external carriers. The number of permanent routes should be less than or equal to the number trucks and the corresponding demands should respect truck capacities. Without loss of generality, we present the heuristic for the case of symmetrical traveling cost. Extending it to the asymmetric case is almost straight forward. The following notation will be used:

n	number of the customers to be served by the internal fleet,
i or j	customer index,
p	customer pair index; $p \equiv (i,j)$ with i=1, …, n-1 and j=i+1, …, n,
k	truck index; k=1, …, K,
C_k	capacity of truck k,
d_{ij}	variable cost of traveling from i to j,
d_{0j}	variable cost of traveling from the depot to customer j,
s_{ij}	the saving obtained by merging a route having i as extremity (the last or first) customer and another one having j as extremity customer; $s_{ij}=d_{0i}+d_{0j}-d_{ij}$,
m	number of pairs such that $s_{ij} > 0$,
Q_i	the sum of the demand off all customers assigned to the route (permanent or temporary) where i is assigned.

Assuming that trucks are numbered in the descending order of their capacity C_k, our parallel version of the heuristic can be described as follows:

Initialization:
- Calculate s_{ij}; i=1, …, n-1 and j=i+1, …, n,
- Number all customer pairs in the descending order of their savings and determine m,
- Create n temporary routes such that each one visits only one of the n customers.

Iteration:
- Scan the list of ordered pairs, from the first down to the m^{th} pair, and:
- If either i or j (the customers of the considered pair p) is not one of the extremity customers (first or last) of its route, or if $Q_i + Q_j > C_1$, move to the next pair,

- Otherwise, merge the corresponding two routes, say (0, …, i, 0) and (0, j, …, 0) into one temporary route (0, …, i, j, …, 0), and calculate the corresponding total demand. Arrange the set containing permanent routes – excluding those used to create the temporary route – and the created temporary route in descending order of their total demand. Let K' be the number of routes in this set and D_k be the total demand of its k^{th} route,
- If $K' \leq K$ and $D_k \leq C_k$; k=1, …K' (i.e., we can assign k^{th} route to k^{th} truck without exceeding its capacity), then transform the temporary route into a permanent one and eliminate its composing routes. Otherwise, eliminate the created temporary route.

- Use the 4-opt* improvement heuristic to improve each of the permanent routes,
- Add all customers not assigned to permanent routes to the list to be served by external carriers.

We notice that the fixed cost of the internal fleet is not considered during the construction of initial solutions. However, it will be considered during the improvement phase.

Solution improvement procedure

Instead of applying the procedure suggested by Chu, we apply a restricted version of the λ-interchange procedure proposed by Osman (1993). This procedure takes two routes at a time and attempt all possible transfers and exchanges of up to λ vertices from each route, into all possible positions of the other route. Our implementation only considers up to 2 vertices of each route. These 2 vertices are either consecutive or separated by only one other vertex. Moves producing positive gain without violating truck capacities are implemented immediately.

Used moves are shown in Figure 1. We consider two routes and a chain of 5 consecutive customers on each route and try 25 moves. The chain may include the depot. However, we do not apply moves implying to transfer or to exchange the depot. Every chain of route 1 is tested against every chain of route 2. The first move on Figure 1 (transfer of 1 vertex) is to remove customer 3 from route 1 (represented by black nodes) and place it between customer 2 and customer 3 of route 2 (represented by white nodes). The third move, (exchange of 1 vertex) is done by replacing customer 3 of route 1 by customer 3 of route 2 and vice-versa.

This procedure is applied to every pair of routes until no further improvement can be reached. Afterwards, we apply the 4-opt* heuristic to improve the resulting individual routes separately.

If a route contains less than 3 customers, some moves may lead us to merge the two tested routes producing only one route. For example, this happens if route 1 serves only 2 customers and we evaluate the possibility (move) of transferring these 2 customers to route 2. In such a case, we save the fixed cost of the truck corresponding to route 1 and it should be considered in the calculation of cost improvement.

3. Computational results

The SRI heuristic was used to solve the 5 test problems solved by Chu. As we noticed that solutions obtained by *SRI* for problems 3, 4 and 5 are better than those reported by Chu as optimal, we decided to use the commercial mathematical programming code CPLEX to find optimal solution of all problems. We also generated five other problems with 5, 10, 15, 22 and 29 customers respectively. The coordinates of these customers are generated randomly in the same range as those of Chu's problems and the external carrier cost is set equal to 6 times the distance between the customer and the depot. Also, customers demand, truck capacities and costs are the same as those in Chu's problems. The data for these new problems are given in the Appendix. For these ten instances, CPLEX was able to obtained eight proven optimal solution. For the two larger instances we reported the best integer solution found after 150 hours of computation time.

Table 1 shows results obtained by the *SRI* heuristic as well as those obtained by Chu's heuristic. This table clearly shows that the proposed heuristic generated much better results. For original Chu's problems, the average deviation with respect to the optimal is 0.11 % for *SRI* while Chu's heuristic produced an average deviation of 14.20 %. For the five new instances, the average deviation is 1.27 % for *SRI* and 12.52 % for Chu's heuristic. The details of these solutions are given in Table 2. Notice that computational times are not reported here as they are less than 0.1 second.

For the whole set of ten instances, the average deviation with respect to the optimal is 0.69 % for *SRI* while Chu's heuristic produced an average deviation of 13.36 %.

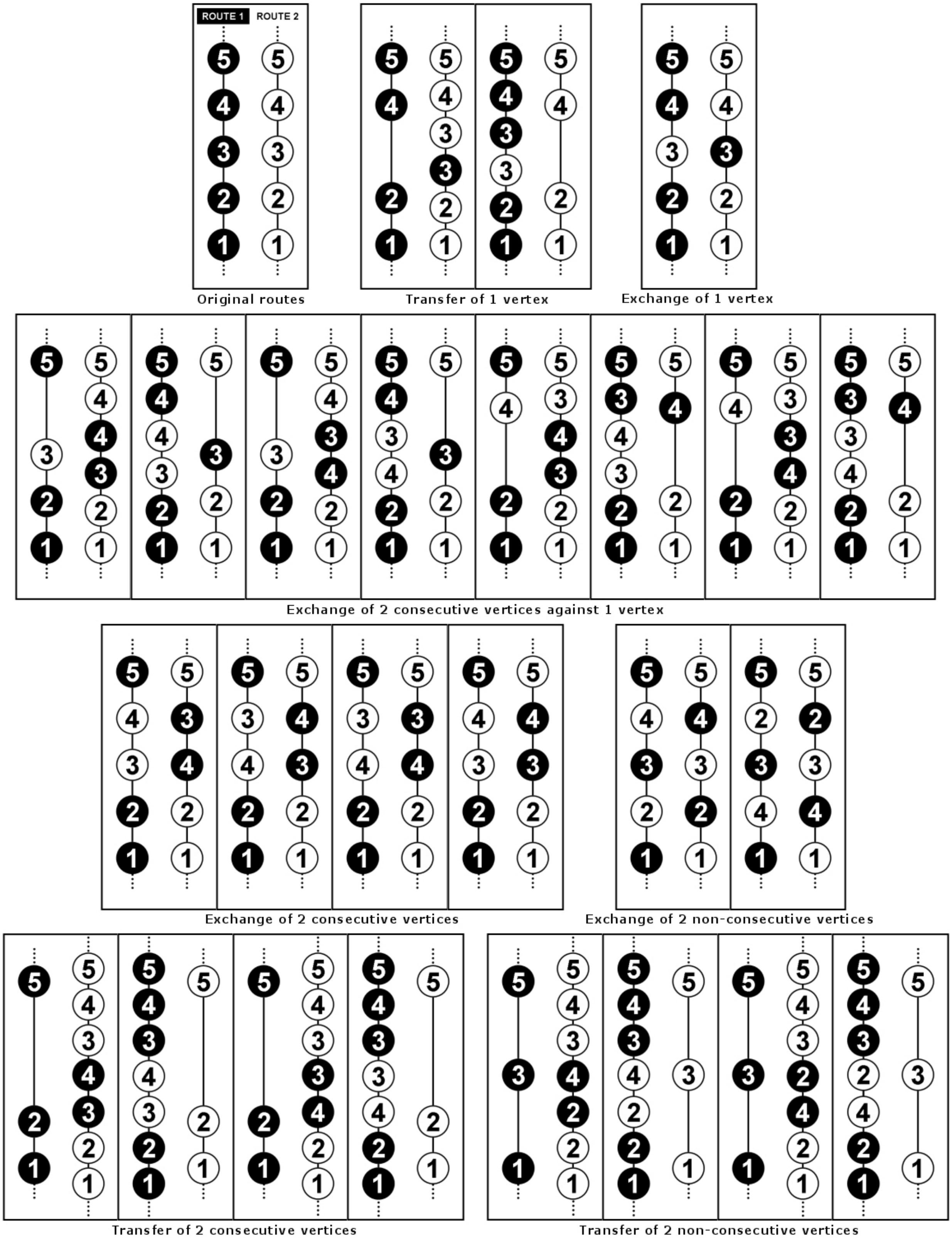

Figure 1. The tested 2-interchange moves.

Table 1. Solution values and percentage deviation from the optimum

Problem	Optimal solution	*SRI*		Chu	
		Solution	deviation	Solution	% deviation
Chu 1	387.5	387.5	0.00 %	387.5	0.00 %
Chu 2	586.0	586.0	0.00 %	631.0	7.68 %
Chu 3	823.5	826.5	0.36 %	900.0	9.29 %
Chu 4	1389.0	1389.0	0.00 %	1681.5	21.06 %
Chu 5	1441.5*	1444.5	0.21 %	1917.0	32.99 %
Average			0.11 %		14.20 %
New 1	423.5	423.5	0.00 %	503.0	18.77 %
New 2	476.5	476.5	0.00 %	476.5	0.00 %
New 3	777.0	804.0	3.47 %	884.0	13.77 %
New 4	1521.0	1564.5	2.86 %	1737.0	14.20 %
New 5	1609.5*	1609.5	0.00 %	1864.5	15.84 %
Average			1.27 %		12.52 %
Overall average			0.69 %		13.36 %

* Best obtained solution for this problem. CPLEX was stopped after 150 hours.

Table 2. Solution details

Problem	Optimal solution	*SRI* solution
Chu 1	Route 1: 1-3-5-4-1 Route 2: 1-6-1 External carrier: 2, Cost: 387.5	Route 1: 1-3-5-4-1 Route 2: 1-6-1 External carrier: 2, Cost: 387.5
Chu 2	Route 1: 1-4-3-10-11-5-1 Route 2: 1-2-9-8-7-1 External carrier: 6, Cost: 586.0	Route 1: 1-4-3-10-11-5-1 Route 2: 1-2-9-8-7-1 External carrier: 6, Cost: 586.0
Chu 3	Route 1: 1-14-16-6-3-2-7-1 Route 2: 1-8-12-15-9-1 Route 3: 1-4-10-11-13-1 External carrier: 5, Cost: 823.5	Route 1: 1-8-15-12-11-1 Route 2: 1-7-2-4-10-13-1 Route 3: 1-9-14-16-6-3-1 External carrier: 5, Cost: 826.5
Chu 4	Route 1: 1-8-22-5-6-9-10-14-12-13-1 Route 2: 1-7-2-3-4-17-16-15-18-23-21-20-19-1 External carrier: 11, Cost: 1389.0	Route 1: 1-8-22-5-6-9-10-14-12-13-1 Route 2: 1-7-2-3-4-17-16-15-18-23-21-20-19-1 External carrier: 11, Cost: 1389.0
Chu 5	Route 1: 1-21-23-3-6-5-4-20-1 Route 2: 1-16-17-14-8-18-10-15-9-13-12-11-24-19-1 Route 3: 1-27-29-28-30-26-25-2-7-1 External carrier: 22, Cost: 1441.5*	Route 1: 1-20-4-5-6-2-7-25-26-30-28-29-27-1 Route 2: 1-24-16-17-14-8-18-10-15-9-13-12-11-19-1 Route 3: 1-23-3-21-1 External carrier: 22, Cost: 1444.5
New 1	Route 1: 1-6-4-1 Route 2: 1-2-5-1 External carrier: 3, Cost: 423.5	Route 1: 1-6-4-1 Route 2: 1-2-5-1 External carrier: 3, Cost: 423.5
New 2	Route 1: 1-2-8-5-11-6-10-1 Route 2: 1-7-4-9-1 External carrier: 3, Cost: 476.5	Route 1: 1-2-8-5-11-6-10-1 Route 2: 1-7-4-9-1 External carrier: 3, Cost: 476.5
New 3	Route 1: 1-9-11-14-3-5-1 Route 2: 1-13-8-16-12-6-1 Route 3: 1-2-10-4-15-1 External carrier: 7, Cost: 777.0	Route 1: 1-10-4-14-11-9-1 Route 2: 1-13-8-16-12-6-1 Route 3: 1-5-3-15-1 External carrier: 2, 7, Cost: 804.0
New 4	Route 1: 1-16-2-9-7-21-18-5-8-1 Route 2: 1-23-6-20-19-4-22-10-13-3-12-15-14-17-1 External carrier: 11, Cost: 1521.0	Route 1: 1-16-8-5-18-21-7-9-2-15-14-12-3-17-1 Route 2: 1-13-10-22-4-19-20-6-23-1 External carrier: 11, Cost: 1564.5
New 5	Route 1: 1-23-26-4-22-16-19-5-7-9-24-14-11-1 Route 2: 1-18-12-29-2-17-28-1 Route 3: 1-25-10-8-15-30-21-27-13-20-6-1 External carrier: 3, Cost: 1609.5*	Route 1: 1-23-26-4-22-16-19-5-7-9-24-14-11-1 Route 2: 1-18-12-29-2-17-28-1 Route 3: 1-25-10-8-15-30-21-27-13-20-6-1 External carrier: 3, Cost: 1609.5*

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Appendix: Data for new problems.

Details of problem New 1				
No.	x	y	Q	LTL
1	35	35	0	0
2	28	59	10	150
3	35	42	7	42
4	45	53	13	124
5	54	22	19	138
6	68	63	26	260

Depot: no.1

Vehicle	Capacity	Fixed Cost
1	40	60
2	30	50

The variable cost for private vehicles is $1.5/per mile

Details of problem New 2				
No.	x	y	Q	LTL
1	30	40	0	0
2	40	33	7	73
3	26	43	30	30
4	47	60	16	157
5	46	26	9	128
6	67	25	21	240
7	52	57	15	167
8	38	31	19	72
9	36	59	23	120
10	50	39	11	120
11	47	22	5	149

Depot: no.1

Vehicle	Capacity	Fixed Cost
1	75	120
2	65	100

The variable cost for private vehicles is $1.5/per mile

Details of problem New 3				
No.	x	y	Q	LTL
1	40	40	0	0
2	44	43	18	30
3	42	59	26	115
4	47	57	11	110
5	29	55	30	112
6	29	30	21	89
7	46	39	19	36
8	67	22	15	195
9	64	65	16	208
10	45	52	29	78
11	65	68	26	225
12	33	29	37	78
13	64	24	16	173
14	55	66	12	180
15	44	54	31	87
16	41	22	8	108

Depot: no.1

Vehicle	Capacity	Fixed Cost
1	110	150
2	100	140
3	90	130

The variable cost for private vehicles is $1.5/per mile

Details of problem New 4				
No.	x	y	Q	LTL
1	266	235	0	0
2	227	276	125	340
3	303	243	84	227
4	312	196	60	362
5	258	196	500	239
6	286	195	300	268
7	204	186	175	474
8	249	212	350	172
9	209	268	150	395
10	323	212	1100	369
11	299	267	4100	276
12	300	254	225	234
13	312	225	300	282
14	300	268	250	284
15	305	278	500	348
16	251	238	150	92
17	294	238	100	169
18	256	192	250	225
19	293	202	120	55
20	291	202	600	278
21	221	186	500	302
22	317	181	175	324
23	265	206	75	215

Depot: no.1

Vehicle	Capacity	Fixed Cost
1	4500	250
2	4000	200

The variable cost for private vehicles is $1.5/per mile

Details of problem New 5									
No.	x	y	Q	LTL	No.	x	y	Q	LTL
1	162	354	0	0	21	138	403	300	270
2	111	354	300	306	22	194	409	1500	323
3	136	355	3100	156	23	177	361	100	309
4	183	401	125	309	24	163	336	300	260
5	214	374	100	334	25	158	387	500	282
6	131	371	200	212	26	180	392	800	108
7	215	332	150	344	27	119	399	300	140
8	141	385	150	225	28	128	352	100	246
9	173	334	450	137	29	116	346	150	466
10	156	388	300	207	30	140	407	1000	272
11	164	352	100	17					
12	126	341	950	230					
13	104	402	125	452					
14	171	346	150	72					
15	146	393	150	253					
16	201	393	550	331					
17	118	354	150	264					
18	149	337	100	310					
19	221	397	150	299					
20	117	385	400	99					

Depot: no.1

Vehicle	Capacity	Fixed Cost
1	4500	250
2	4000	200
3	3500	180

The variable cost for private vehicles is $1.5/per mile