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Integration of Inventory Control in Spread Supply Chains

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Abstract. In spread supply chains, lack of visibility, considerable delivery delays and complex transportation networks make it difficult to integrate inventory control with other logistic activities. However, because of the impact of stock turnover rate on just-in-time operations, inventory control has to be considered for global optimization of the supply chain. In the literature, transportation and inventory control operations are seldom modeled altogether because minimizing transportation costs and increasing inventory turns are two contradictory objectives. This paper addresses this global optimization problem. Actually, it presents a decision support system (DSS) that allows simulating logistic activities in a spread supply chain by integrating inventory control and transportation operations. Delivery frequencies and phases are the decision variables used to study the behavior of the logistic system.

Keywords. Supply chain management, inbound logistics, inventory control, transportation operations, routing, global optimization, decision support system

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1 Introduction

Nowadays, global optimization of the supply chain is a major issue for companies of international scale. Low profit margins, diversity and flexibility requirements, excess production capacity, intense competition and market instability are many reasons that explain why these companies focus so much on their logistic operations. Indeed, a supply system that is market-responsive, efficient and well adapted to the company's needs can become a real competitive advantage.

One of the most important aspects of logistic operations management is supply chain integration. It aims at synchronizing every link of the chain, i.e. each trading partner involved. Many enterprises have achieved a high level of integration by applying lean manufacturing principles to optimize their entire supply chain. Actually, recently developed methods related to logistics management, like VMI (Vendor Managed Inventory), CPFR (Collaborative Planning, Forecasting and Replenishment) and CTM (Collaborative Transportation Management), are all built around the basic idea of lean thinking, i.e. to banish waste (Womack and Jones 2003).

However, these management techniques always consider transportation operations and inventory control separately, even if these two types of logistic activities are closely interrelated. The main reason that explains this situation is that minimizing transportation and inventory costs are usually considered as being two contradictory objectives. In the automotive industry, EOMs (Original Equipment Manufacturers) solved this problem by encouraging – if not forcing – their suppliers to create supplier parks close to their assembly plants. This allows EOMs to benefit from just-in-time supply without having to support high transportation costs. Of course, the automotive industry is particular. Most companies that assemble complex products have to deal with hundreds of suppliers spread in a wide area. Thus, minimizing total logistic costs by optimizing transportation operations and inventory control becomes a complex problem.

This paper exposes a decision support system (DSS) developed to integrate inventory control in a spread supply chain composed of multiple plants and suppliers. The purpose of this DSS is to help managers minimizing total logistic costs (transportation and inventory costs) and increasing inventory turnover rates throughout the supply chain.

This paper is divided as follows. First, the problem definition will be set out and a literature review will be presented to take a brief look at existing models related to global supply chain optimization. After determining the modeling approach, the DSS will be presented. Actually, the structure of this DSS will be exposed and the optimization model developed to integrate transportation operations and inventory control will be detailed. Then, some experiments will be carried out to study the behavior of the logistic system and the results will be analyzed. Finally, recommendations related to global supply chain optimization will be set out.

2 Problem Definition

To be able to show where this paper stands in the field of global supply chain research, it is important to define the issues addressed and the problem that has to be solved. The literature review will then be presented.

First of all, this paper focuses on the integration of the two most costly logistic operations, i.e. transportation and inventory control. It exposes a model built to minimize total costs related to these activities. As mentioned, the spread supply chain studied in this paper involves multiple suppliers and plants (assembly plants). Moreover, the transportation network is also composed of consolidation and transshipment centers. Thus, a multi-commodity and multi-level transportation and inventory control problem will be solved. It is important to specify that overseas operations are not considered and that truck transportation is the only transportation mode used to supply the plants.

In the field of supply chain management research, a lot of efforts are concentrated on supply chain design (Verter and Dincer 1992; Crainic 2000; Meixell and Gargeya 2005). However, the DSS exposed in this paper aims at helping logistic managers to make decisions related to transportation operations and inventory control. Indeed, this paper focuses on tactical and operational planning. Therefore, supply chain design problems like facility location, supplier selection and capacity determination will not be addressed.

3 Literature Review

In this literature review, two topics will be covered: global supply chain optimization and, more particularly, transportation operations modeling.

3.1 Global Supply Chain Optimization

Global supply chain optimization is called "global" because of two reasons (Vidal and Goetschalckx 1997). First, it studies the integration of multiple logistic activities, like transportation, inventory control, order processing and manufacturing. Second, it studies the integration of trading partners involved (suppliers, carriers, consolidation and transshipment centers and plants), which implies the development of efficient communication systems. Consequently, global models are, most of the time, very complex.

Many authors studied transportation and inventory control problems but only a few built models that integrate both logistic activities in a spread supply chain. As pointed out by Goetschalckx, Vidal and Dogan (2002), much of the research on global supply chain optimization ignores the inclusion of inventory control as part of the decision problem. Actually, in a literature review in which they surveyed 18 papers related to this subject, Meixell and Gargeya (2005) observed that only five of these papers were including inventory costs. Moreover, only two papers incorporated the impact of long transit times. However, many existing models contain interesting aspects for this paper.

Because of complexity reasons, researchers often choose to develop heuristics to solve multi-commodity problems. Indeed, Qu, Bookbinder and Iyogun (1999) and Van Norden and Van de Velde (2005) used a similar approach to simplify problem resolution. They divided their model into two entities: a master problem and a sub-problem. For example, the sub-problem can be a transportation problem and the master problem an inventory control problem. With an initial solution, the sub-problem is solved and, then, the master problem is solved with the results provided by the sub-problem. By iteration, it is possible to obtain a good solution. Even if these models are very theoretical, the modeling approach could be applied to many problems.

Goetschalckx, Vidal and Dogan (2002) developed an interesting but complex model to integrate production, transportation and inventory control operations. The authors solved the multi-commodity network flow problem for instances including 12 products and 3738 transportation channels. They observed savings of 2% (on a total cost of $401 000 000 per year) versus cases that did not include inventory control in the problem, showing the importance of integrating inventory control in global models.

Few authors built models in which the decision variables are delivery frequencies. Actually, these models contain many interesting aspects for this research project. Bertazzi, Speranza, Favaretto, Pesenti and Ukovich are five authors that specifically studied this type of problem. To minimize total logistic costs, their basic idea consists of building a network of the supply chain and to determine the delivery frequency on each arc of the network (Bertazzi and Speranza (1999); Bertazzi and al. (2000); Favaretto and al. (2001)). In the models studied, inventory levels at each node of the network are determined by the delivery frequencies and the supply chain is modeled with an integer linear program which is NP-hard. Thus, heuristics were developed to generate solutions. These models have two major drawbacks. First, only one supplier is considered, so the transportation networks are simplistic (no consolidation or transshipment operations). Second, the inventories in transit are not calculated, because instantaneous replenishment is supposed.

3.2 Transportation Operations Modeling

Since the beginning of the new millennium, a transportation management technique called Collaborative Transportation Management (CTM) is becoming more and more popular in the manufacturing industry. CTM is especially interesting for spread supply chains because it allows multiple plants to manage their logistic operations centrally (CTM Sub-committee 2004). The main advantage of this centralization is the reduction of transportation costs (Esper and Williams 2003). Actually, the objective of CTM is "to reduce or eliminate inefficiencies in the transportation process (for example time, inventory, space, errors and distance) through collaboration, in order to bring benefit to all trading partners[1]".

The literature related to CTM does not yet provide detailed examples showing how transportation operations are modeled. However, many researchers worked on minimum cost flow problems to model transportation operations and other logistic activities. Crainic (2002) presents, in a survey of optimization models for long-haul freight transportation, a minimum cost flow model developed for network design. Even if network design is not studied in this paper, the multi-commodity capacitated network design formulation exposed by Crainic is interesting because it is path-based, meaning that transportation decisions are based on pre-established paths for each commodity. This type of formulation allows to model complex transportation networks, but it is not flexible enough because the paths have to be pre-established. Moreover, Chen and al. (2006) present a minimum cost flow formulation to model the procurement of multiple plants by multiple suppliers. The plants are supplied via a consolidation center and the inventories at the consolidation center are calculated at each period of the planning horizon. This formulation is NP-hard. These two examples of minimum cost flow problems point out the potential of these types of models for transportation operations modeling.

In short, research related to integration of inventory control in spread supply chains has not yet provided models that can be applied to manage real supply chains.

[1] Sutherland, J. 2003. "Collaborative transportation management - creating value through increased transportation efficiencies". *Business Briefing: Pharmagenerics*. p.1.

4 Modeling Approach

In a spread supply chain, long distances between suppliers and plants increase the complexity of the procurement operations. First, transportation management is complex because multiple methods can be used (direct shipment, shipment with consolidation, transshipment, *etc*.). Second, inventories in transit cannot be neglected for the calculation of total logistic costs because shipments may be on the road for a few days. Thus, determining the right levels of inventories in the supply chain and the best transportation strategies in order to minimize the global cost is a very difficult task.

Because of the complexity of the problem, the choice of the modeling approach is important. In the literature, authors identify two main types of approaches: analytical methods (i.e. optimization) and simulation (Slats and al. 1995). Usually, optimization is used when the system to model is simple and the objective function can be defined analytically, whereas simulation is more appropriate for complex systems (Baptiste 2004). Therefore, it is important to analyze the problem to model before determining the right approach.

As mentioned, for spread supply chains, very few optimization models have been developed to optimize transportation operations and inventory control at the same time. Here are the main reasons that explain this situation:

- complex transportation networks;
- many components, suppliers and plants to deal with;
- determination of inventory levels throughout the supply chain;
- difficult integration of logistic operations.

It is important to work out on this last point. Actually, the integration of transportation operations and inventory control is difficult because these two logistic activities are not computed on the same time basis. For example, to calculate transportation costs, a transportation problem is solved for orders that have to be shipped on a certain day. Thus, the time basis is a period of the planning horizon. On the other hand, to compute inventory costs, it is necessary to calculate the average inventory, during the planning horizon, of every component taken into account. Consequently, the time basis is the duration of the planning horizon (a week for example). With two different time basis, it becomes more complicated to develop an optimization model that minimizes total costs. Authors usually bypass this difficulty by calculating the inventories at each period of the planning horizon (a day for example). However, this method increases considerably de number of variables of the problem and does not allow to compute inventories in transit.

Considering this analysis of the problem, it has been decided that a combination of optimization and simulation was an interesting approach to integrate transportation and inventory control operations. This approach is the basic idea of the DSS detailed in the next section.

5 Development of the DSS

This section presents the DSS developed to minimize transportation and inventory holding costs incurred to ensure the procurement of multiple plants in a spread supply chain.

5.1 Structure of the DSS

To study the behavior of spread supply chains, it is necessary to model this supply chain and to design a system that allows testing several procurement scenarios in a short time span. This is the purpose of the DSS presented in this paper.

The structure of this DSS is illustrated in figure 1. Actually, simulation is used to generate and to evaluate different procurement scenarios, whereas an optimization model has been developed to optimize transportation operations corresponding to a particular scenario. For the supply chain studied in this paper (see section 6), the planning horizon has been fixed to a week (7 days), because the assembly plants receive at least one shipment per week from each of their suppliers. Thus, to evaluate the relationship between input variables and total logistic costs, it is possible to simulate a typical week of production: the requirements of each plant are known and constant for every scenario tested.

Two types of decision variables are used: 1) delivery frequencies, 2) phases. A delivery frequency is the number of times a plant receives material from a particular supplier in a fixed planning horizon (for example, twice a week). A phase represents the days a plant has to be supplied (for example, Monday and Wednesday). These variables were chosen because they affect both transportation operations and inventory levels. Moreover, the DSS allows studying the impact, on logistic costs, of the variation of different parameters.

A scenario is generated by setting decision variables and parameters studied. Delivery frequencies and phases have to be determined for every couple (i,j), where (i,j) refers to the material provided by supplier i to plant j. Once all the inputs of the model have been set, it is possible to evaluate the scenario, i.e. to compute total logistic costs. It is important to note that inventory levels and transportation operations are modeled separately (see figure 1). The global optimization is made possible by evaluating multiple scenarios. The two modules of the DSS are detailed in sections 5.2 and 5.3.

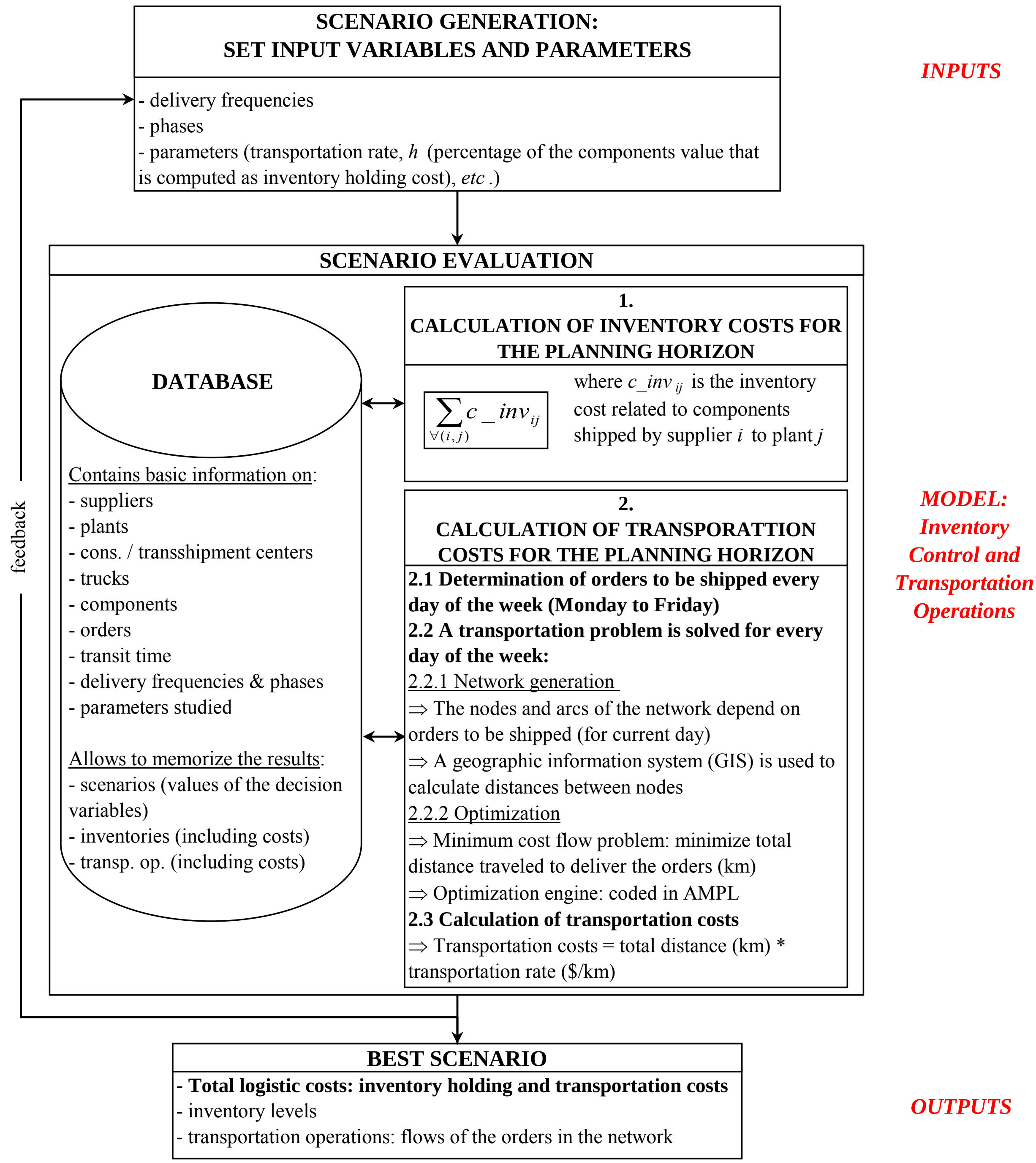

Figure 1 - Structure of the DSS

It is interesting to note that the DSS can be used for two interrelated purposes. First, by using a neighborhood search method to generate scenarios (like Tabu Search for example), the DSS can evaluate multiple scenarios and memorize the best solution. In that case, the DSS works as an optimization engine to support operational decisions. Second, generating scenarios in accordance with a design of experiment can help to quantify the impact of different parameters on logistic costs and, thus, to support tactical decisions. For example, the quantified effects of the distance separating suppliers and plants could help managers to figure out the best transportation strategies to minimize total costs and to ensure a good inventory turnover rate. Thus, this type of analysis can be useful to generate good starting solutions (or scenarios).

Before exposing the model that was integrated in the DSS, it is important to list some assumptions that were made in order to validate the results obtained with this DSS:

- the demand is considered to be constant during the planning horizon, meaning that the components are consumed by the plants at a constant rate during the week;
- the price of the components is constant, i.e. it does not depend on order processing costs;
- the components belong to the buyer as soon as they leave the supplier's plant (FOB Origin);
- the suppliers and the plants are opened from Monday to Friday.

5.2 Calculation of Inventory Holding Costs

As mentioned, the model of the DSS is composed of two modules. The first module calculates the inventory holding costs (C_{inv}) by determining the inventory levels in transit and at the plants (on hand). Since delivery frequencies and phases were fixed when the scenario was generated, it is possible to compute, for a couple (*i*,*j*), the value of the average inventories in the supply chain during the planning horizon ($\bar{I}_{ij}$):

$$\bar{I}_{ij} = \left[\frac{v_{ij}}{2f_{ij}} + \frac{v_{ij}}{f_{ij}} \alpha_{ij} \right] + \left[\frac{v_{ij}}{f_{ij}} \frac{f_{ij} t_{ij}}{7} \right] = \frac{v_{ij}}{f_{ij}} \left(\frac{1}{2} + \alpha_{ij} \right) + \frac{v_{ij} t_{ij}}{7} \tag{1}$$

where $\bar{I}_{ij}$ value ($) of the average inventories for couple (*i*,*j*) during the planning horizon

f_{ij} delivery frequency of couple (*i*,*j*): number of deliveries during the planning horizon

t_{ij} transit time for couple (*i*,*j*)

v_{ij} value ($) of the components provided by supplier *i* to plant *j* during the planning horizon

α_{ij} coefficient used to consider weekends and variability of inventory levels

In formula (1), $v_{ij}/f_{ij}*(0.5+\alpha_{ij})$ is the average value of on hand inventories (i.e. inventories at the plants), whereas $v_{ij}*t_{ij}/7$ is the average value of in transit inventories (i.e. inventories moving from the suppliers to the plants). The calculation of in transit inventories can be easily explained by presenting an example. Let us suppose that the total value (v_{ij}) of the components provided by supplier *i* to plant *j* during the week is $14 000 and that the transit time (t_{ij}) is two days. Thus, the average value of in transit inventories is: $14 000/week * 2 days / 7days/week = $4 000.

It is less obvious to determine the value of on hand inventories. For a plant that produces 365 days per year and 24 hours per day, the formula to calculate on hand inventories is $v_{ij}/f_{ij}*0.5$ (v_{ij}/f_{ij} is divided by two because constant demand is supposed during the planning horizon). However, many companies are closed on weekends and have to hold inventories during that period. Moreover, demand variability during the planning horizon and early or late deliveries also affect inventories on hand. Thus, a coefficient was added to formula (1) in order to consider these particularities. It corresponds to $v_{ij}/f_{ij}*\alpha_{ij}$. It is possible to estimate α_{ij} with the following formula:

$$\alpha_{ij} = \alpha_{ij}^1 + \alpha_{ij}^2 \tag{2}$$

where α_{ij}^1 coefficient used to consider the increase of average on hand inventory level created by weekends

α_{ij}^2 coefficient used to consider the random aspect of the variation of inventories (demand variability during the planning horizon, variations due to early/late deliveries, *etc*.)

Coefficient α_{ij}^2 can be estimated by analyzing historical data (demand variability, punctuality and regularity of deliveries, *etc*.). In this paper, α_{ij}^2 will be fixed to 0.1 for every couple (i,j) in order to simplify calculations.

Contrary to α_{ij}^2, coefficient α_{ij}^1 can be determined analytically. To show the effect of weekends on inventory levels, let us continue the example presented earlier. In that example, v_{ij} = \$14 000 and t_{ij} = 2 days. Adding f_{ij} = 2 deliveries/week and ρ_{ij} = {Tuesday; Friday}, it is possible to determine the average value of on hand inventories.

The following chart is showing the variation of on hand inventory for two weeks of production. The curve *Opened on weekends* represents a production system in which the supplier and the plant are opened on weekends (average value of on hand inventories = $v_{ij}/f_{ij}*0.5$), whereas the curve *Closed on weekends* exposes a production system in which the supplier can only ship from Monday to Friday and the plant is closed on weekends (average value of on hand inventories = $v_{ij}/f_{ij}*(0.5+\alpha_{ij}^1)$).

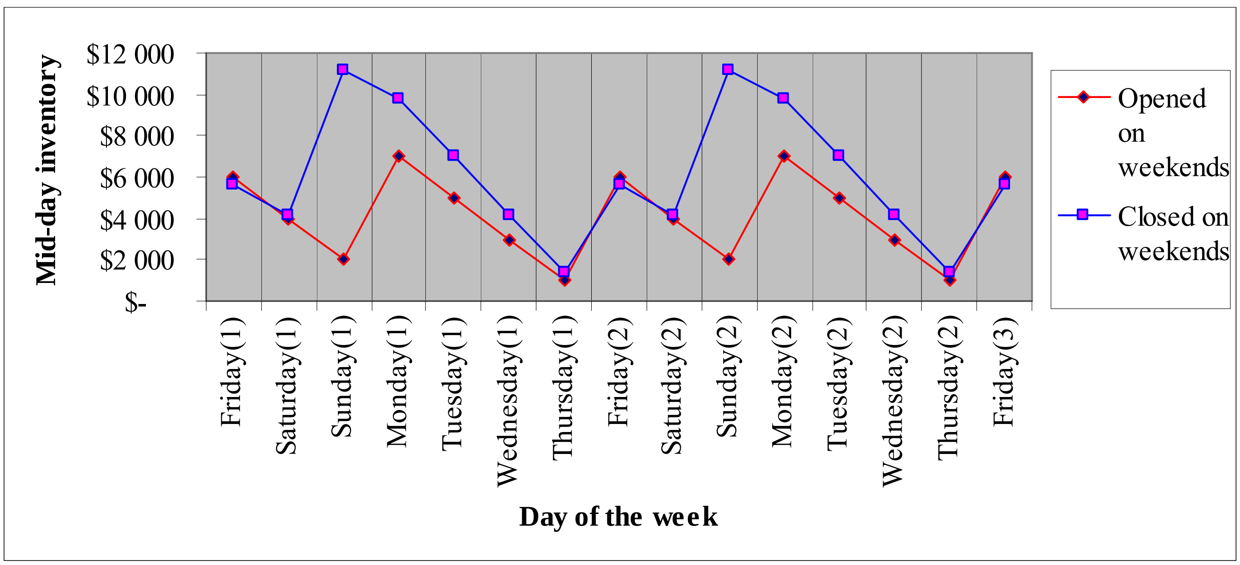

Figure 2 - Variation of on hand inventory[2]

As shown on the chart, weekends create an increase of the average on hand inventory. Two reasons explain that increase. First, inventories left from the weekly production have to be supported during the weekend. Second, the fact that suppliers can only ship orders on week days often causes a phase shift, so the orders are received earlier at the plants. In our example, the transit time is two days and the orders should be received on Tuesday and Friday, so the supplier should ship on Sunday and Wednesday. However, Sunday's shipment has to be shifted on Friday

[2] The calculation of mid-day inventory is the following: mid-day inventory of day *x* = mid-day inventory of day *(x-1)* - daily requirements (if applicable) + value of received shipment(s) on day *x* (it was supposed that the shipments are always received in the morning). For example, the mid-day inventory on Friday for the curve *Closed on weekends* is: \$1 400 - \$2 800 + \$7 000 = \$5 600. Actually, the value of the daily requirements is \$14 000 / 5 = \$2 800, and the value of a shipment is \$14 000 / 2 = \$7 000.

because the supplier is closed during the weekend. Thus, the shipment will be received on Sunday instead of Tuesday, so the components will be stored for two days, which increases average inventory.

The average inventory is \$3 500 for the *Opened on weekends* curve and it is \$6 200 for the *Closed on weekends* curve, which represents an 80% increase. Because $v_{ij}/f_{ij}*(0.5+\alpha_{ij}^{1})$ = \$6 200, α_{ij}^{1} = 0.39. Thus, α_{ij}^{1} is not negligible.

Once the average inventory is calculated, it is possible to estimate the cost of holding these inventories during the planning horizon. A coefficient *h* is used to represent the percentage of the components value that is computed as inventory holding cost ([*h*] = % of components value per year of storage). Consequently, the inventory holding cost per week for a couple (*i*,*j*) is:

$$c\_inv_{ij} = \frac{7}{365} h\bar{I}_{ij} \tag{3}$$

Finally, total inventory holding cost for a scenario is:

$$C_{inv} = \sum_{\forall(i,j)} c\_inv_{ij} \tag{4}$$

In short, delivery frequencies and phases determine the inventory levels throughout the supply chain. By generating multiple scenarios, it will be possible to analyze the variation of inventory holding costs.

5.3 Transportation Operations Modeling

The second module of the DSS is related to transportation operations (see figure 1). To calculate transportation costs (C_{transp}), a transportation problem has to be solved for every day of the week. First of all, the orders that have to be shipped from Monday to Friday have to be determined.

For a couple (*i*,*j*), the shipping days are deduced from the delivery phase (ρ_{ij}) and the transit time (t_{ij}), as shown in the previous section. In addition, value, volume and weight of the orders are calculated with the delivery frequency (f_{ij}). Actually, because the demand is considered to be constant during the planning horizon, it is supposed that each shipment has the same value. For example, if weekly requirements of plant *j* for components produced by supplier *i* are worth \$10 000, occupy 40 m^3 and weight 5 000 kg, a delivery frequency of two per week means that each shipment will be worth \$5 000, occupy 20 m^3 and weight 2 500 kg.

By memorizing the orders and the shipping days for every couple (*i*,*j*), it becomes possible to group the orders by shipping day and to solve a transportation problem. First, a graph modeling the movements of the orders from the suppliers to the plants is built. Then, a minimum cost flow problem is solved to minimize the total distance traveled by the trucks supplying the different plants. Finally, the transportation cost is calculated by multiplying the distance traveled by the transportation rate.

5.3.1 Minimum Cost Flow Problem

Because of the potential of minimum cost flow formulations to model transportation operations (see literature review, section 3.2), this type of model was incorporated to the optimization engine of the DSS.

An interesting way to model transportation operations is to build a network in which flows represent the movements of orders being shipped from suppliers to plants, whereas the nodes symbolize trading partners of the supply chain (suppliers, consolidation/transshipment centers and plants). Actually, for each day of the week, transportation operations are modeled with a minimum cost flow problem in which costs are distances traveled by trucks transporting the orders. The objective function is to minimize total distance traveled to supply the plants. It is worth noting that a geographic information system (GIS) is essential to determine the distances between the nodes of the network.

5.3.2 Network Generation

In a spread supply chain, multiple transportation strategies can be used to optimize transportation operations:

- direct shipment from a supplier to a plant;
- shipment via a consolidation or transshipment center;
- consolidation of orders for suppliers located in a particular region;
- consolidation of orders intended to different plants.

The model integrated in the DSS allows to build a network including a combination of these four transportation methods. To illustrate the problem, it is important to detail the network generation with an example. Let us consider a supply chain with the following characteristics:

- three plants have to be supplied (*P1*, *P2*, *P3*);
- three suppliers provide the components (*S1*, *S2*, *S3*);
- two transshipment centers are available (*C1*,*C2*).

Moreover, let us suppose that five orders have to be delivered:

1. *O1*: (*S1*, *P1*)[3];
2. *O2*: (*S1*, *P2*);
3. *O3*: (*S2*, *P1*);
4. *O4*: (*S3*, *P2*);
5. *O5*: (*S3*, *P3*).

Figure 3 illustrates the network corresponding to this example. It represents every path that an order (or a flow) can borrow to be delivered at the right plant. First of all, there is a node representing each order (*O1* to *O5*). For each of these nodes, a flow is generated. This flow contains the order's information: supplier, plant, value ($), volume ($m^3$) and weight (kg). To model the consolidation of orders between suppliers, arcs have been added between orders nodes. The costs related to these arcs are the distances separating the corresponding suppliers. For example, a cost of zero means that the corresponding orders are shipped from the same supplier.

Moreover, the flows leaving orders nodes (i.e. suppliers) can be dispatched to three other types of nodes:

1. plants (*P*): allows to model a direct shipment from the supplier to the plant;
2. artificial nodes (*U*): allows to model a shipment containing orders intended to different plants (*milk run*). For example, it may be advantageous to ship two orders in one truck that will visit two different plants instead of shipping these two orders separately. The *U*

[3] Order *O1* was made by plant *P1* and to supplier *S1*.

nodes are called "artificial nodes" because they are only used to model milk runs between plants. For example, a shipment entering node *U22* is transported by a truck that will visit both *P2* and *P3* plants. The cost related to an arc connecting an order to an artificial node corresponds to the total distance traveled from the supplier to the last plant visited. Thus, artificial nodes are connected to other artificial nodes (*U*) or plants (*P*) with arcs having a cost equal to zero;

3. transshipment centers (*C*): allows to model a shipment from a supplier to a transshipment center.

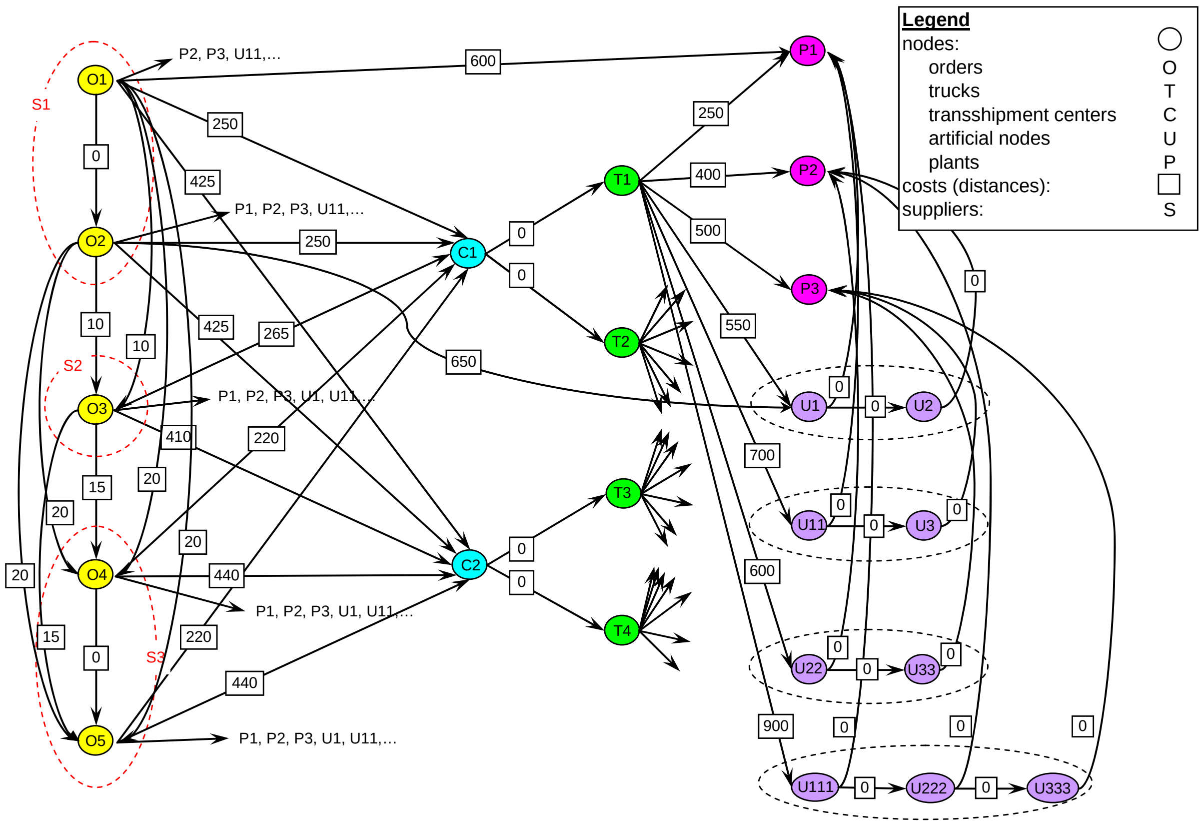

Figure 3 - Network example

Finally, to be able to form shipments with orders passing through a transshipment center, *C* nodes are connected to trucks (*T*). Actually, orders passing through an arc *C*-*T* have to respect the capacity of a trailer. The cost related to these arcs is equal to zero because no distance is traveled. Note that the number of trucks related to a transshipment center can be determined according to the expected volume of orders that will pass through this transshipment center.

In short, this example shows that the graph is generated according to the orders to be delivered and the number of trucks needed at each transshipment center. Thus, this type of graph can be used to model very complex transportation networks.

5.3.3 Mathematical Formulation

To be able to solve the problem modeled with the graph presented above, a mathematical formulation has to be developed. Here is the linear program corresponding to this minimum cost flow problem:

$\min \sum_{\forall (i,j)\in\beta} L_{ij} y_{ij}$		minimize total distance	(5)
s.t.			
$\sum_{\forall (i,j)\in\delta} x_{ij}^{i} = 1,$	$\forall i \in \gamma$	flow generation	(6)
$\sum_{\forall (i,j)\in\delta} x_{ij}^{k} = \sum_{\forall (j,l)\in\delta} x_{jl}^{k},$	$\forall j \in \gamma, \forall k \in \gamma \mid k \neq j$	flow conservation for orders nodes (*O*)	(7)
$\sum_{\forall (i,j)\in\delta} x_{ij}^{k} = \sum_{\forall (j,l)\in\delta} x_{jl}^{k},$	$\forall j \in \lambda, \forall k \in \gamma$	flow conservation for *C*, *T* & *U* nodes	(8)
$\sum_{\forall (i,j)\in\delta} x_{ij}^{k} = 1,$	$\forall j \in \mu, \forall k \in \gamma(j)$	delivering orders to the right plants	(9)
$\sum_{\forall k\in\gamma} r^{k} x_{ij}^{k} \le R,$	$\forall (i,j) \in \tau, \forall (i,j) \in \omega$	trailer capacity: volume	(10)
$\sum_{\forall k\in\gamma} w^{k} x_{ij}^{k} \le W,$	$\forall (i,j) \in \tau, \forall (i,j) \in \omega$	trailer capacity: weight	(11)
$\sum_{\forall k\in\gamma} x_{ij}^{k} \le M y_{ij},$	$\forall (i,j) \in \beta$	setting variables y_{ij}	(12)
$\sum_{\forall (i,j)\in\delta} y_{ij} = 1,$	$\forall i \in \gamma, \ \forall i \in \varphi$	prevents shipment division	(13)
$\sum_{\forall (i,j)\in\delta} y_{ij} \le 1,$	$\forall j \in \gamma$	only one shipment entering an order node	(14)
$x_{ij}^{k} \in \{0,1\},$	$\forall (i,j) \in \delta, \ \forall k \in \gamma$		(15)
$y_{ij} \in \{0,1\},$	$\forall (i,j) \in \beta$		(16)

where

r^k	volume occupied by order k
w^k	weight of order k
R	trailer capacity: volume
W	trailer capacity: weight
L_{ij}	distance between nodes i and j
M	represents a high number (ex.: 100)
γ	set of orders
μ	set of plants
$\gamma(j)$	set of orders made by plant j
φ	set of trucks
λ	set of nodes for which flow conservation applies (excluding *O* nodes): *C*, *T* & *U* nodes
δ	set containing every arc of the network
β	set of arcs with cost > 0 (i.e. arcs representing movements of orders)
τ	set of arcs related to the orders: *O-O*, *O-C*, *O-U*, *O-P*
ϖ	set of arcs entering truck nodes (*T*)
x_{ij}^{k}	binary variable equal to 1 if order k passes through arc (i,j), 0 otherwise
y_{ij}	binary variable equal to 1 if at least one order passes through arc (i,j), 0 otherwise

As mentioned, the objective of this linear program is to minimize total distance traveled to deliver the orders (5). The first series of constraints (6) is necessary to generate the flows. Constraints 7 and 8 are flow conservation constraints. Actually, two series of constraints were necessary because the flows are generated at *O* nodes. Thus, for these particular nodes, flow conservation does not apply to the flow generated. Moreover, constraints 9 were added to ensure that the orders are delivered to the corresponding plant. For example, if plant *U3* made the order *O2*, there must be one and only one variable x_{iU3}^{O2} equal to 1. In addition, constraints 10 and 11 ensure that trailer capacities (maximum volume and weight) are respected. Constraints 12 were added to determine the arcs borrowed by the flows (variables y_{ij}) and, thus, to be able to calculate the distance traveled. Constraints 13 were necessary to prevent shipment division. In other words, flows leaving an order or a truck node have to borrow the same arc. Finally, constraints 14 were added to ensure that only one shipment can enter an order node (*O*). Otherwise, it would be possible to consolidate different shipments at a supplier's plant, which is not a transportation strategy considered in this model.

Naturally, certain assumptions have to be made to validate this transportation model:

1. transit time of couple (i,j) is fixed, i.e. it does not depend on the type of transportation strategy used to deliver the shipments (direct transport, consolidation, …)[4];
2. time windows for pickups or deliveries are not considered;
3. the capacity of a trailer (*R*) is estimated according to the maximum space that can be occupied in a trailer, taking into account that the pallets can be stacked;
4. only one type of trailer is available;
5. in the network, the distance from node i to node j is equal to the distance from node j to node i ($L_{ij} = L_{ji}$);
6. only one transportation rate is considered (same rate for TL and LTL);
7. the arcs between the orders nodes (*O*) are unidirectional. For example, in the graph illustrated in figure 3, a flow cannot go from order *O5* to order *O1*.

Although these assumptions help to reduce the complexity of the model, the size of the transportation problem to solve is an important issue, as it will be discussed in the following section.

5.3.4 Size Reduction Process

The formulation presented in section 5.3.3 allows to model complex transportation networks because multiple transportation strategies are considered. However, the size of the problem to solve grows rapidly with the number of orders considered. For example, let us suppose that 70 orders have to be delivered to four different plants and that two transshipment centers are available (at each transshipment center, 10 trucks can be loaded). Table 1 details the size of this problem modeled with the linear program (5) to (16).

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[4] At first sight, this assumption may seem illogical. However, two reasons explain that assumption. First, in spread supply chains, transit times are calculated in days. Thus, even if, for example, an order from supplier A is consolidated with an order from supplier B located near by, the transit time related to a direct shipment from supplier A to plant C compared to the transit time required if the order from supplier A goes to supplier B before arriving at plant C may well be the same. Second, it is possible to determine the transit time by estimating, with fairly good accuracy, the transportation strategy that will be used.

Table 1 - Size of the transportation problem (example)

Problem Characteristics	Quantity
Orders	70
Nodes	121
Arcs	3875
Variables	**275 065**
x^k_{ij}	271 250
y_{ij}	3 815
Constraints	**19 555**

In this example, the linear program contains 275 065 variables and 19 555 constraints. Obviously, to be able to solve the problem to optimality, the size of the problem has to be reduced. Thus, a size reduction process composed of four steps has been applied to the model (see table 2).

Table 2 - Steps of the size reduction process

Step	Strategy	Implementation	Impact on the Experimental Error
1	Elimination of unnecessary *O-O* arcs	Arcs *O-O* between two suppliers that are far from each other will be eliminated	Negligible
2	Elimination of *O-U* & *O-P* arcs that are useless	Arcs *O-U* and *O-P* that do not represent a possible path for the orders will be eliminated	None
3	Elimination of unnecessary *P* nodes	Artificial nodes which imply long milk runs will be eliminated	None
4	Elimination of *T* nodes	Modification of the graph's structure	Depends on the volume of orders passing through the transshipment centers

The first step of the size reduction process aims at reducing the number of arcs between orders because, as noted in table 1, these arcs represent 63% of the network's arcs. Actually, it is possible to reduce the number of arcs between orders by more than 80%, depending on the geographical layout of the suppliers. Since these arcs model the consolidation of orders between suppliers, arcs linking suppliers that are not located in the same area can be eliminated. In a spread supply chain, only the arcs between suppliers that form a cluster will be created.

The second step of the size reduction process aims at eliminating *O-U* and *O-P* arcs. Regarding the network detailed in table 1, 700 arcs are *O-U* arcs and 280 arcs are *O-P* arcs. Actually, some of these arcs are useless. For example, if order *O3* has been made by plant *P1*, the arcs linking *O3* to *P2*, *P3* and *U22* are useless because they do not model a possible path for order *O3* (see figure 3). Thus, arcs *O3-P2*, *O3-P3* and *O3-U22* can be eliminated.

Even if the artificial nodes (*U*) are added to the network to model milk runs between plants, it is not necessary to add every combination of plants. For example, it may not be possible to visit the three plants *P1*, *P2* and *P3* because the distance to travel would increase considerably the transit time. Thus, the third step of the size reduction process consists of eliminating the artificial nodes that do not model realistic milk runs.

Finally one last step has been added to the size reduction process. This fourth step consists of eliminating the truck nodes. Figure 4 shows how the network has been modified (partial view of figure 3). As exposed on this figure, eliminating truck nodes divides the total number of *C*-*U* and *C*-*P* arcs by the number of trucks. However, with this modification, the truck nodes cannot be used anymore to form the shipments dispatched from the transshipment centers. Constraints related to trailer capacity (10-11) must then be relaxed. Moreover, the distance traveled by the trucks passing through a *C*-*U* or a *C*-*P* arc will be calculated by multiplying the distance of the arc by the number of trucks (nb_t) needed to ship the orders that have to go through the corresponding arc (nb_t = total volume of orders passing through the arc divided by trailer capacity). As the number of trucks (nb_t) may not be an integer, the elimination of truck nodes introduces an error in the objective function because, in reality, it is impossible to use a fraction of a truck to deliver a shipment. However, this error can either be neglected if the volume of orders passing through the transshipment center is high (for example, if 10.1 trucks are needed for an arc *C*-*P*, the error is less important than if 1.1 trucks are needed) or compensated by using a coefficient to increase the distance traveled to consider the fraction of a truck that cannot be included in the objective function. The revised formulation of the linear program considering the elimination of *T* nodes is presented in Appendix A.

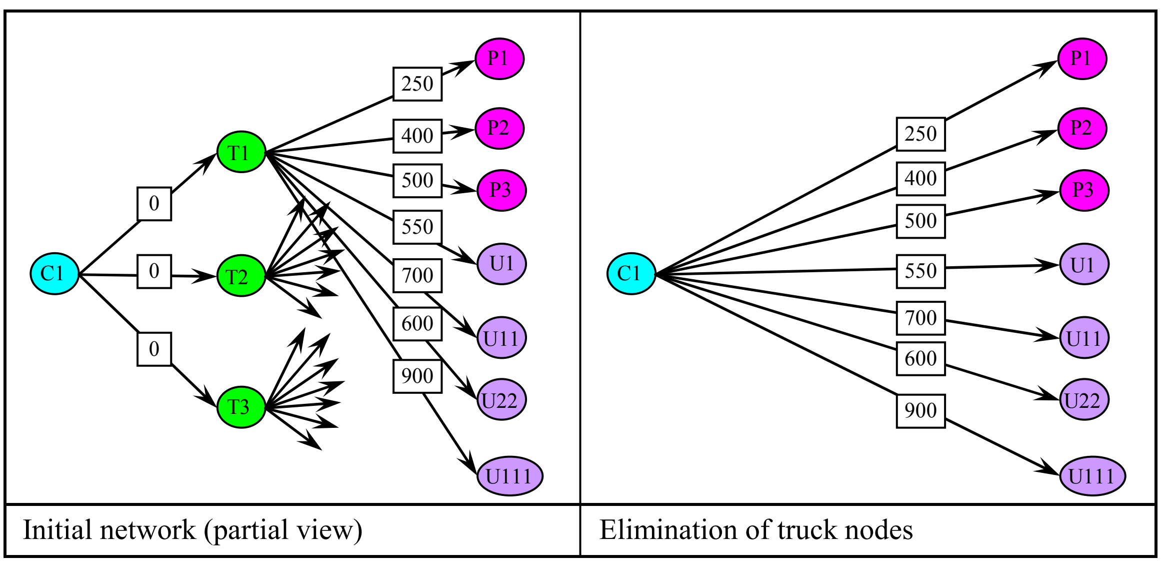

Figure 4 - Fourth step of the size reduction process: elimination of truck nodes

In short, these four steps allow reducing the size of the network considerably. By applying this process, the number of variables for the example detailed in table 1 could be reduced from 275 065 to around 40 000, depending on the geographical layout of the suppliers and the plants.

6 Experiments

The DSS detailed in the previous section was developed using different software. First, a Microsoft Access database was built to manage the data related to the supply chain studied (see figure 1), and to generate and evaluate the scenarios. Moreover, Microsoft MapPoint was used to generate the matrix giving the distances between the nodes (suppliers, transshipment centers and plants). Finally, the linear program modeling the transportation operations was coded with AMPL (A Mathematical Programming Language) and solved with a CPLEX solver for AMPL. This DSS was used to study the impact of delivery frequencies and phases on logistic costs.

6.1 Details on the Supply Chain Modeled

To carry out the experiments, a real supply chain was modeled. Actually, the data was gathered from Paccar's North-American supply chain. Paccar Inc is a multinational company that assembles trucks. In North-America, this company owns six assembly plants and deals with hundreds of suppliers spread around U.S.A., Canada and Mexico. Obviously, Paccar's North-American supply chain could not be modeled entirely for the purpose of this research. However, a representative sample of this supply chain was studied. This sample contains the following characteristics (see appendix B):

- 42 suppliers located in 19 different states (U.S.A.) and one Canadian province;
- two transshipment centers;
- four plants (three American plants and one Canadian plant);
- the average distance separating the suppliers and the plants is 1850 km (2570 km for the Canadian plant);
- transit time varies from one to seven days;
- 255 different components were included in the database.

Considering this supply chain, a "typical" week of production was simulated. For every scenario tested, around one million dollars worth of components had to be supplied to each plant during the week.

6.2 Experimentation Plan

To quantify the impact of delivery frequencies and phases on logistic costs, two experiments were elaborated. The first experiment (*E1*) consists of varying the average delivery frequency in the supply chain from 1.5 to 3 deliveries/week. For this experiment, the phases are not optimized. Thus, *E1* will show the impact of delivery frequencies on C_{inv}, C_{transp} and C_{TOT} (total logistic costs).

The second experiment (*E2*) consists of optimizing the phases for a given average delivery frequency. To optimize the phases, two different strategies will be tested:

1. Determining the phases in order to minimize inventories on hand;
2. Determining the phases in order to increase consolidation operations.

For a given delivery frequency, it is possible to determine analytically the phase that minimizes the average on hand inventory for a couple (i,j). Thus, scenarios optimized with strategy *1* were generated according to the phases presented in Appendix C. On the other hand, for scenarios optimized with strategy *2*, the phases were determined in a way that suppliers located in the same region ship their orders the same day(s).

6.3 Results

Table 3 presents the results obtained. This table details, for each scenario, the input variables, the distances traveled and the logistic costs ($U.S.). Seven scenarios were evaluated for experiment *E1* and six for experiment *E2*. It is important to note that a transportation rate of 1.00 $/km was used and that h = 10%.

Table 3 - Results obtained for the two experiments carried out

Scénario		*Average Delivery Frequency*	*Phase*	C_{inv}	Distance Traveled					C_{transp}	C_{TOT}
					Monday	Tuesday	Wednesday	Thursday	Friday		
		(deliveries/ week)	strategy	($)	(km)	(km)	(km)	(km)	(km)	($)	($)
E1	1	1.5	basic	6464	30083	32533	22292	14553	60325	159786	166250
	2	1.75	basic	5988	31919	28456	21545	14790	67898	164608	170596
	3	2	basic	5674	31910	29194	23813	14692	72320	171929	177603
	4	2.25	basic	5472	31216	30941	24856	14757	74547	176317	181789
	5	2.5	basic	5296	35283	29813	26762	20422	77532	189812	195108
	6	2.75	basic	5160	40621	29864	28548	21218	80034	200285	205445
	7	3	basic	5018	40732	31351	31562	23460	83923	211028	216046
E2	8	2.25	1	5472	30085	30430	32264	28066	68231	189076	194548
	9	2.25	1 +	5476	34297	31884	34383	29711	62362	192637	198113
	10	2.25	1 ++	5401	41365	25187	33647	29810	62142	192151	197552
	11	2.25	2	5400	40626	22262	34634	29508	64043	191073	196473
	12	2.25	2 +	5469	40935	16027	24063	13748	71411	166184	171653
	13	2.25	2 ++	5419	41073	14340	24063	12332	73522	165330	170749

6.3.1 Experiment *E1*

From scenario 1 to scenario 7, total logistic costs went from $166 250 to $216 046, which represents an increase of 30%. Actually, the results show that inventory holding costs represent less than 5% of total logistic costs. Thus, even if the increase of the average delivery frequency allowed reducing inventory holding costs by more than 20%, it wasn't enough to compensate for the increase of transportation costs. Figure 5 illustrates the increase of total costs according to the delivery frequencies.

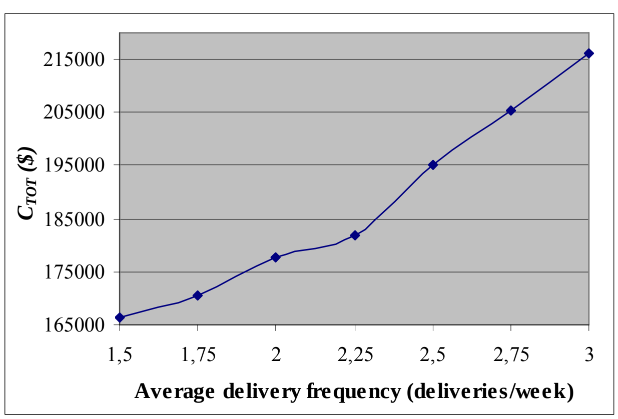

Figure 5 - Total logistic costs according to the average delivery frequency

This chart shows that from 1.5 to 2.25 deliveries/week, total costs only increased by 9%, whereas they increased by 19% from 2.25 to 3 deliveries/week. Thus, it seems that delivery frequencies can be increased until a certain point where transportation costs start increasing to fast.

6.3.2 Experiment *E2*

Scenarios 8 to 10 show that optimizing delivery phases according to strategy *1* is not advantageous. Actually, inventory holding costs only decreased by 1.3% (compared to scenario 4), whereas total logistic costs increased by 8.7% ($15 730 increase). On the other hand, strategy *2* generated impressive results. Indeed, from scenarios 10 to 13, this strategy allowed to reduce transportation costs by 14%, whereas the variation of inventory holding costs can be neglected.

In short, compared to scenario 4, the optimization of phases allowed to decrease total logistic costs by 6% (reduction of 11 040$).

6.4 Discussion

The results presented above show that it is possible to increase the inventory turnover rate in a spread supply chain without increasing total logistic costs too much. Actually, increasing delivery frequencies accelerates the flow of material throughout the supply chain and, thus, the inventory turnover rate.

As observed, delivery frequencies cannot be increased indefinitely. Here are two steps to follow in order to optimize the delivery frequencies in a spread supply chain:

1. Determine the ideal average delivery frequency in order to ensure a good material flow throughout the supply chain. This average frequency can be determined according to the company's needs and by simulating different scenarios in order to analyze the variation of total logistic costs.
2. Starting from the ideal average frequency, optimize the delivery frequency of each couple (i,j) according to the following rules:
    a. for a supplier i located near a plant j or a transshipment center, increase the delivery frequency if the value or the volume of the orders to be dispatched justifies this decision;
    b. if there is a cluster of suppliers in the transportation network, increase the delivery frequencies for these suppliers (because consolidation operations will contain the increase of transportation costs);
    c. for a supplier i located far from a plant j, reduce the delivery frequency if necessary.

These two steps will allow to increase the inventory turnover rate and to minimize logistic costs.

Moreover, experiment *E2* showed that combining strategies *1* and *2* allows to reduce, for given delivery frequencies, total logistic costs by more than 6%. Actually, strategy *1* has to be applied first and strategy *2* second, because strategy *2* has a direct impact on transportation costs. Even if the majority of companies only focus on delivery frequencies, the experiments carried out in this paper show that delivery phases can reduce logistic costs considerably.

Finally, it is important to note that the largest transportation problems solved in this paper contained around 100 orders. In real size problems, hundreds of orders could have to be delivered. Thus, the transportation model integrated in the DSS has to be improved in order to solve large size problems in a short time span.

7 Conclusion

Building a system that allows optimizing both inventory control and transportation operations is a hard task. It is even more difficult in the case of spread supply chains because of the complexity of the transportation networks involved. In fact, three major challenges had to be taken up. First, to be able to analyze the behavior of the supply chain, the right decision variables had to be determined. The DSS presented in this paper uses delivery frequencies and phases as variables to minimize total logistic costs. Second, it was necessary to find a way to integrate inventory control and transportation operations. The system uses a scenario generation and evaluation process to compute inventory and transportation costs. Third, a complex network had to be modeled. The optimization engine consists of solving a minimum cost flow problem for every period of the planning horizon. Thus, this DSS contains all the elements necessary to optimize globally the logistic operations in a spread supply chain.

On the other hand, further researches will have to be made to improve this DSS. Actually, more powerful tools will have to be used in order to solve real size problems. For example, to be able to manage a supply chain on a daily basis, specialized mathematical algorithms will have to be used to solve the transportation problems more quickly. Moreover, the database of the DSS will have to be web-based, in order to gather real-time information from suppliers, carriers, transshipment centers and plants. However, this DSS represents an interesting advancement in the domain of global supply chain optimization.

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Appendix A – Revised Formulation of the Linear Program (truck nodes eliminated)

$$\min \sum_{\forall(i,j)\in\beta^1} L_{ij} y_{ij} + \sum_{\forall(i,j)\in\beta^2} L_{ij} z_{ij} \qquad \text{minimize total distance} \qquad (17)$$

<u>s.t.</u>

(6) to (9), (14)

$$\sum_{\forall k\in\gamma} r^k x_{ij}^k \le R, \qquad \forall (i,j)\in\beta^1 \qquad \text{trailer capacity: volume} \qquad (18)$$

$$\sum_{\forall k\in\gamma} w^k x_{ij}^k \le W, \qquad \forall (i,j)\in\beta^1 \qquad \text{trailer capacity: weight} \qquad (19)$$

$$\sum_{\forall k\in\gamma} x_{ij}^k \le M y_{ij}, \qquad \forall (i,j)\in\beta^1 \qquad \text{setting variables } y_{ij} \qquad (20)$$

$$\sum_{\forall(i,j)\in\delta} y_{ij} = 1, \qquad \forall i\in\gamma \qquad \text{prevents shipment division} \qquad (21)$$

$$z_{ij} = \sum_{\forall k\in\gamma} r^k x_{ij}^k / R, \qquad \forall (i,j)\in\beta^2 \qquad \text{number of trucks necessary} \qquad (22)$$

$$y_{ij} \in \{0,1\}, \qquad \forall (i,j)\in\beta^1 \qquad (23)$$

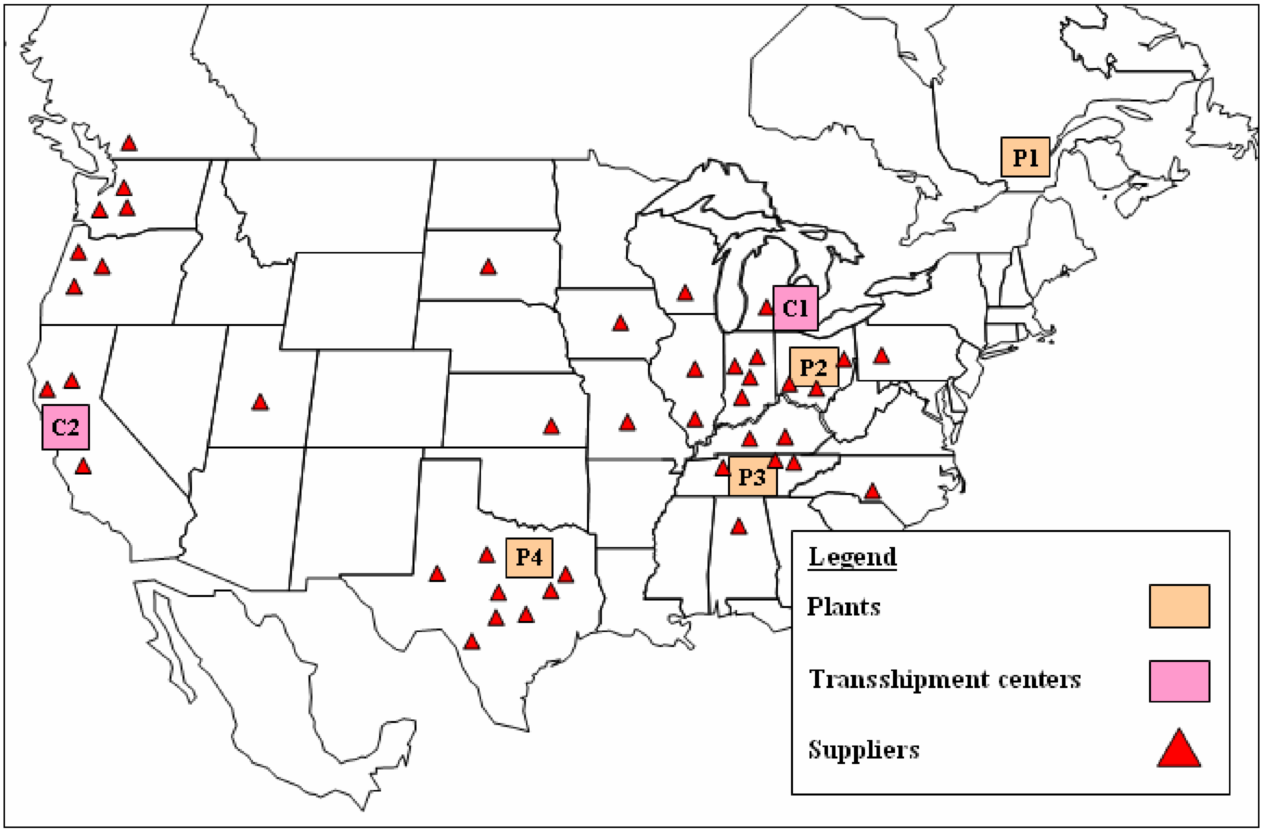
$$z_{ij} \in \Re^+, \qquad \forall (i,j)\in\beta^2 \qquad (24)$$

where β^1 set of arcs with cost > 0 (excluding *C-U* & *C-P* arcs)

β^2 set of *C-U* et *C-P* arcs

z_{ij} variable equal to the number of trucks needed to deliver the orders going through arc (*i,j*)

Appendix B – Paccar's North-American Supply Chain (sample)



Appendix C – Phase that Minimizes On Hand Inventory (according to the delivery frequency and the transit time)

Delivery Frequency	Transit Time (days)	Optimal Phase*	Average On Hand Inventory** (days)
1	1	Tuesday	2.43
1	2	Wednesday	2.71
1	3	Monday	2.14
1	4	Monday	2.14
1	5	Monday	2.14
1	6	Monday	2.14
1	7	Monday	2.14
2	1	Tuesday and Thursday	1.72
2	2	Monday and Wednesday	1.79
2	3	Monday and Thursday	1.57
2	4	Monday and Tuesday (or Wednesday or Thursday)	1.79
2	5	Monday and Wednesday	1.43
2	6	Monday and Wednesday	1.43
2	7	Monday and Wednesday	1.43
3	1	Tuesday, Wednesday and Friday	1.47
3	2	Monday (or Tuesday), Wednesday and Thursday	1.52
3	3	Monday, Tuesday (or Wednesday) and Thursday	1.43
3	4	Monday, Tuesday (or Wednesday) and Friday	1.62
3	5	Monday, Tuesday, Wednesday (or Thursday or Friday)	1.43
3	6	Monday, Tuesday and Thursday	1.19
3	7	Monday, Tuesday and Thursday	1.19
4	1	Tuesday, Wednesday, Thursday and Friday	1.35
4	2	Monday (or Tuesday), Wednesday, Thursday and Friday	1.46
4	3	Thursday, Friday and two days between Monday, Tuesday and Wednesday	1.39
4	4	Monday, Friday and two days between Tuesday, Wednesday and Thursday	1.32
4	5	Monday, Tuesday and two days between Wednesday, Thursday and Friday	1.25
4	6	Monday, Tuesday, Wednesday and Thursday	1.07
4	7	Monday, Tuesday, Wednesday and Thursday	1.07
5	1	Monday, Tuesday, Wednesday, Thursday and Friday	1
5	2	Monday, Tuesday, Wednesday, Thursday and Friday	1.14
5	3	Monday, Tuesday, Wednesday, Thursday and Friday	1.14
5	4	Monday, Tuesday, Wednesday, Thursday and Friday	1.14
5	5	Monday, Tuesday, Wednesday, Thursday and Friday	1.14
5	6	Monday, Tuesday, Wednesday, Thursday and Friday	0.85
5	7	Monday, Tuesday, Wednesday, Thursday and Friday	0.71

* The optimal phase is the one minimizing average on hand inventory.
** The daily inventory is calculated at the beginning of the day.