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# Multi-Class Demand Matrix Adjustment

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**Abstract.** In this paper, the gradient method for adjusting a single class origin-destination matrix by using observed flows (see Spiess, 1990) is extended to adjusting simultaneously the origin-destination matrices of several classes of traffic. The algorithm is developed in detail and computational tests demonstrate the efficiency of the method. A comparison is carried out with two other ways of sequentially adjusting origin-destination matrices of several classes by using the transportation planning network of the Montreal region.

**Keywords.** Multi-class equilibrium assignment, demand adjustment, gradient method.

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## 1. Introduction

The adjustment of an origin-destination (O-D) matrix by using observed flows (counts) on the links and turns of a transportation planning network has attracted the attention of many researchers. The methods proposed may be subdivided into two categories depending whether the network considered is assigned constant travel times or flow-dependent travel times.

Some of the contributions made for O-D matrix adjustment on uncongested networks include those of Van Zuylen and Willumsen (1980), Maher (1983), Cascetta (1984), Bell (1984), Spiess (1987), Tamin and Willumsen (1989), Willumsen (1984), Bell (1991) and Bierlaire and Toint (1994).

When the network considered for the O-D matrix adjustment is subject to congestion the underlying route choice method is an equilibrium assignment. Some of the numerous contributions made for this version of the problem are those of LeBlanc and Farhangian (1982), Nguyen (1984), Fisk (1988, 1989), Spiess (1990), Kawakami et al (1992), Florian and Chen (1995), Yang et al (1992) and Yang et al (1994). In this case the O-D matrix adjustment method may be formulated as a bi-level optimization problem or, as others denote such problems a mathematical programming problem with equilibrium constraints (MPEC).

Several of these methods have been implemented in practice (see Van Vliet, 1982, and Spiess, 1990), and are used on a regular basis for the adjustment of an out-of-date O-D matrix for the evaluation of contemplated supply changes in a short term planning horizon.

The consideration of several classes of traffic, such as private cars and different types of trucks, has led to the common use of multi-class assignments to predict the use of the transportation infrastructure. In turn, this has created the need to adjust simultaneously the demand for several classes of traffic by using link and turn counts for each class. Most of the applications resorted to the sequential use of a single class O-D adjustment method. The one exception is the paper by Wong et al (2005) which considers the adjustment of O-D matrices by several classes by using an entropy based method. In the context of freight flows, Crainic et al (2001) propose a simultaneous multi-class O-D adjustment method but do not present an implementation or computational results.

In this article the gradient method developed by Spiess (1990) for the adjustment of a single class O-D matrix is extended to multiple classes of traffic. The method is developed and implemented for computations in the Emme transportation planning software package (see INRO, 2007). Then comparisons are carried out with two ways of adjusting O-D matrices for several classes sequentially by using the transportation planning network of the Montreal Region.

The paper is organized as follows. The next section contains the formulation of the model and the development of the gradient based algorithm. The implementation of the method is presented in Section 3 and the computational results are presented in Section 4. A short conclusion ends this article.

## 2. The model formulation and the solution algorithm

In this section, the notation used is introduced in order to state the mathematical formulation for the problem.

The nodes of the road network are denoted  $n$ ,  $n \in N$  and the links are denoted  $a$ ,  $a \in A$ , where  $N$  is the set of nodes and  $A$  is the set of links. The set of O-D pairs is denoted by  $I$  and it is convenient to refer to the O-D pair with index  $i = (p, q)$ ,  $i \in I$ , where  $p, q \in N$ .  $\hat{A} \in A$  is the set of links where counts are available. The demand for travel by user class  $m \in M$  for the origin-destination pair  $i$  is denoted as  $g_i^m$  where  $M$  is the set of classes. These demands may use paths  $K_i^m \in K$  where  $K$  is the set of all routes  $K = \bigcup_{(i)m} K_i^m$  and  $k$  is a path index. The path flow of class  $m$  on the route  $k$  is denoted  $h_k^m$  and gives rise to link flows  $v_a^m$  of class  $m$ ,  $m = 1, \dots, |M|$  on link  $a$ ,  $a \in A$ ; the total link flow  $v_a$  on link  $a$  is the sum of the class flows  $v_a = \sum_m v_a^m$ , for  $m = 1, \dots, |M|$ . The counts by class are denoted as  $\hat{v}_a^m$ . And finally,  $s_a(v_a)$  is the travel time on link  $a$  for total link flow  $v_a$ .

The compact formulation of the bi-level (or MPEC) multi-class O-D adjustment problem is given by

$$\text{Min } Z(g) = \frac{1}{2} \sum_{m \in M} \sum_{a \in \hat{A}} (v_a^m - \hat{v}_a^m)^2 \quad (1)$$

Subject to

$$v = \text{assign}(g) \quad (2)$$

where  $\text{assign}(g)$  is the notation used to indicate that the vector of flows  $v$  is the result of the multi-class equilibrium assignment of demand  $g$ , This assignment problem is:

$$\text{Min } F(v) = \sum_{a \in A} \int_0^{v_a} s_a(v) dv \quad (3)$$

Subject to

$$v_a = \sum_{m \in M} v_a^m \quad a \in A \quad (4)$$

$$v_a^m = \sum_{i \in I} \sum_{k \in K_i^m} \delta_{ak}^m h_k^m \quad a \in A, \quad m \in M \quad (5)$$

$$\sum_{k \in K_i^m} h_k^m = g_i^m \quad i \in I, \quad m \in M \quad (6)$$

$$h_k^m \geq 0 \quad k \in K_i^m, \quad m \in M \quad (7)$$

$$\delta_{ak}^m = \begin{cases} 1 & \text{if } a \in k \text{ for mode } m \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

In order to develop the solution algorithm it is necessary to derive the local gradient of the objective function by assuming that the path proportions  $p_k^m$  are fixed. This is the simplification that was used in the gradient computation in Spiess (1990).

Using the path proportions  $p_k^m$ :

$$p_k^m = \frac{h_k^m}{g_i^m} \quad k \in K_i^m, \quad m \in M \quad (9)$$

$v_a^m$  can be rewritten as:

$$\begin{aligned} v_a^m &= \sum_{i \in I} \sum_{k \in K_i^m} \delta_{ak}^m p_k^m g_i^m \\ &= \sum_{i \in I} g_i^m \sum_{k \in K_i^m} \delta_{ak}^m p_k^m \quad a \in A, \quad m \in M \end{aligned} \quad (10)$$

Considering that the path proportions are locally constants:

$$\frac{\partial v_a^m}{\partial g_i^m} = \sum_{k \in K_i^m} \delta_{ak}^m p_k^m \quad a \in A, \quad i \in I, \quad m \in M \quad (11)$$

Hence the gradient may be computed as:

$$\begin{aligned} \frac{\partial Z(g^m)}{\partial g_i^m} &= \frac{\partial Z(g^m)}{\partial v_a^m} * \frac{\partial v_a^m}{\partial g_i^m} \\ &= \sum_{a \in \hat{A}} (v_a^m - \hat{v}_a^m) \sum_{k \in K_i^m} \delta_{ak}^m p_k^m \\ &= \sum_{k \in K_i^m} p_k^m \sum_{a \in \hat{A}} \delta_{ak}^m (v_a^m - \hat{v}_a^m) \end{aligned} \quad (12)$$

In order to obtain the optimal step length by class,  $\lambda^{m*}$ , the following problem must be solved:

$$\text{Min}_{\lambda^m} Z(g_i^m (1 - \lambda \frac{\partial Z(g^m)}{\partial g_i^m})) \quad (13)$$

Subject to

$$\lambda \frac{\partial Z(g^m)}{\partial v_a^m} \leq 1 \quad i \in I, \quad m \in M, \quad g_i^m > 0 \quad (14)$$

The derivative is computed as:

$$\frac{dZ(\lambda^m)}{d\lambda^m} = \sum_{a \in \hat{A}} \frac{dv_a^m}{d\lambda^m} * \frac{\partial Z(\lambda^m)}{\partial v_a^m} \quad (15)$$

Since

$$v_a^{m'} = \frac{dv_a^m}{d\lambda^m} = \sum_{i \in I} \frac{dg_i^m}{d\lambda^m} * \frac{\partial v_a^m}{\partial g_i^m} = - \sum_{i \in I} g_i^m ( \sum_{k \in K_i^m} p_k^m \sum_{a \in \hat{A}} \delta_{ak}^m (v_a^m - \hat{v}_a^m) ) ( \sum_{k \in K_i^m} \delta_{ak}^m p_k^m ) \quad (16)$$

and

$$\begin{aligned} \frac{\partial Z(g^m)}{\partial v_a^m} &= \frac{\partial}{\partial v_a^m} ( \frac{1}{2} \sum_{a \in \hat{A}} (\tilde{v}_a^m - \hat{v}_a^m)^2 ) \\ &= \frac{\partial}{\partial v_a^m} ( \frac{1}{2} \sum_{a \in \hat{A}} (v_a^m + \lambda v_a^{m'} - \hat{v}_a^m)^2 ) = v_a^m + \lambda v_a^{m'} - \hat{v}_a^m \end{aligned} \quad (17)$$

then

$$\frac{dZ(\lambda^m)}{d\lambda^m} = \sum_{a \in \hat{A}} v_a^{m'} (v_a^m + \lambda v_a^{m'} - \hat{v}_a^m) \quad (18)$$

and by annulling (18) the optimal step size is then:

$$\lambda^{m*} = \frac{\sum_{a \in \hat{A}} v_a^{m'} (\hat{v}_a^m - v_a^m)}{\sum_{a \in \hat{A}} (v_a^{m'})^2} \quad m \in M \quad (19)$$

It is worthwhile to note that the optimal step size is different for each class of traffic.

The statement of the algorithm is given next.

**The Multi-Class O-D Adjustment Algorithm**

Step 0. *Initialization.* Iteration  $l = 0$

Step 1. *Multi-class assignment.* Multi-class assignment of demand  $g^{m,l}$  ( $\forall m \in M$ ) to obtain link volumes  $v_a^{m,l}$  for  $a \in A$ ,  $m \in M$

Step 2. *Link derivatives and objective function.* Computation of the link derivatives  $(v_a^{m,l} - \hat{v}_a^{m,l})$  for  $a \in \hat{A}$ ,  $m \in M$  and the objective function  $\frac{1}{2} \sum_{m \in M} \sum_{a \in \hat{A}} (v_a^{m,l} - \hat{v}_a^{m,l})^2$

If the maximum number of iterations  $L$  is reached go to Step 7.

Step 3. *Assignment to compute the gradient matrix.* Multi-class assignment with path analysis to compute the gradient matrices:

$$\nabla Z(g)^{m,l} = \frac{\partial Z(g^{m,l})}{\partial g_i^{m,l}} = \sum_{k \in K_i^{m,l}} p_k^{m,l} \sum_{a \in \hat{A}} \delta_{ak}^{m,l} (v_a^{m,l} - \hat{v}_a^{m,l})$$

Step 4. *Assignment to obtain the derivatives.* Multi-class assignment with path analysis to obtain the derivatives:

$$v_a^{m,l \prime} = - \sum_{i \in I} g_i^{m,l} \nabla Z(g)^{m,l} \sum_{k \in K_i^{m,l}} \delta_{ak}^{m,l} p_k^{m,l}$$

Step 5. *Update of the demand matrices.* For each class  $m \in M$  :

Computation of the maximal gradient as  $\max g^{m,l} = \max(\nabla Z(g)^{m,l} / g_i^{m,l})$

Computation of the optimal step length as  $\lambda^{m,l*} = \frac{\sum_{a \in \hat{A}} v_a^{m,l \prime} (\hat{v}_a^{m,l} - v_a^{m,l})}{\sum_{a \in \hat{A}} (v_a^{m,l \prime})^2}$

Update of the demand matrix:

$$g_i^{m,l+1} = g_i^{m,l} + \min(\lambda^{m,l*}, 1) * \nabla Z(g)^{m,l} / \max g^{m,l}$$

Step 6. *Iteration counter.* Update the iteration counter  $l = l + 1$  and return to Step 1.

Step 7. *End.*

### 3. Implementation of the Algorithm

The implementation of the algorithm was done in the Emme 3 (INRO, 2007) transportation planning software. Three approaches were tested. The first approach is the multi-class adjustment where the demand of all classes is adjusted simultaneously. The second approach adjusts the demand for one class at a time, but letting the flows of all the other classes variable during the assignments. In the third approach, the demand of one class is adjusted at a time, but in this variant the volumes of the other classes are fixed.

For the first approach, the already existing EMME/2 macro `demadj22.mac` (by Heinz Spiess) was modified to consider multiple classes of vehicles. The new macro, `demadjmc.mac`, uses the « generalized cost multi-class assignment with path analysis » option, which has become available recently. The use of this option makes possible to solve the problem carrying out only  $|M|+2$  multi-class assignments per iteration. In fact, this kind of assignment allows the user to analyse the paths for all the classes at the same time and to save the class specific volumes. However,  $|M|$  assignments are required in order to specify the particular class link attribute for the path analysis (Step 3). The total number of multi-class assignments becomes  $L*(|M|+2)$ , where  $L$  is the maximum number of iterations.

As the macro `demadjmc.mac` allows to select the classes for which the demand must be adjusted, this macro was modified slightly to implement the second approach. The new macro, called `demadj-seq.mac`, has an additional external loop which allows it to adjust sequentially the demand for one class of vehicles at the time, leaving the other demands fixed but the flows variable. The procedure is as follows:

*For*  $i = 1$  to  $|M|$   
     *Call* `demadjmc.mac` (class  $i$ )

The number of multi-class assignments by iteration decreases to 3, but the total number of iterations increases  $|M|$  times, so one can expect that this macro will be more time consuming. The total number of multi-class assignments goes up to  $3*L*|M|$ . Note that this amount considers that the same maximum number of iterations is chosen for all the classes.

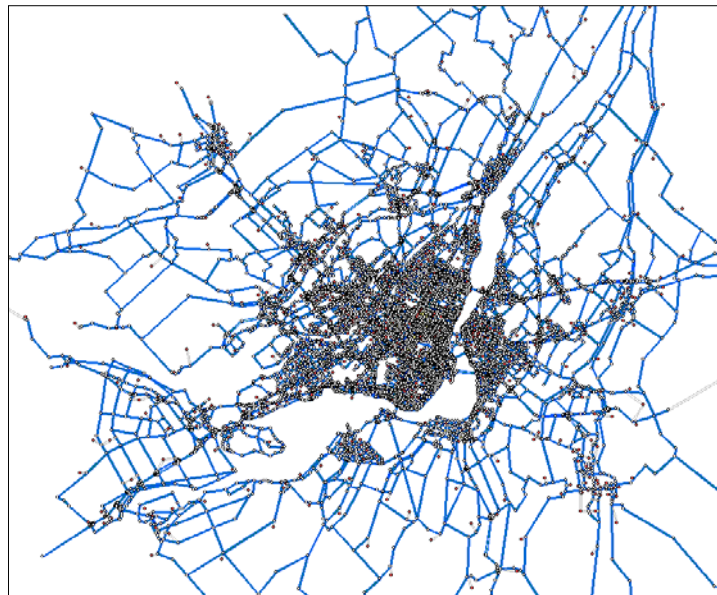
For the third approach, a new macro was developed. This macro, called `demadj-fix.mac`, implements an iterative sequence of calls to the single class demand adjustment macro `demadj22.mac`. A multi-class assignment is carried out before calling `demadj22.mac` in order to calculate the volumes of the classes that are not adjusted. These volumes are fixed as background flows. The total number of multi-class assignments is  $|M|$ , and the total number of single class assignments is  $3*L*|M|$ .



All the macros were adapted for the Montreal Region data set described below but could be easily customized for any other application. demadjmc.mac was already generalized to work with any data set.

## 4. The Computational Tests

The computational tests were carried out by using one network data set originating from the Region of Metropolitan Montreal. The corresponding network is displayed in Figure 1. The considered Montreal network uses 3 classes of traffic: private car, regular trucks and heavy trucks.



**Figure 1. The Montreal Network**

The network characteristics are given in Tables 1 to 3. The demand is given in Table 4.

	<b>AM peak</b>	<b>PM peak</b>	<b>Off peak</b>
Zones (centroids)	1 425	1 425	1 425
Regular nodes	12 771	12 756	13 019
Links	30 681	30 663	32 284
Classes	3	3	3

**Table 1. The network parameters**

<b>Class</b>	<b>Characteristics</b>
Regular truck	One unit, 2 or 3 axles
Heavy truck	One unit, 4 axles, or more than one unit

**Table 2. The Truck characteristics**

Code	Period	Hours	Counts
NI	Night	0 :00 to 6 :00	457
AM	AM Peak	6 :00 to 9 :00	534
OPD	Off Peak Day	9 :00 to 15 :30	536
PM	PM Peak	15 :30 to 18 :30	535
OPN	Off Peak Night	18 :30 to 24 :00	457

**Table 3. The Time Periods and the Number of Link Counts**

Period	Auto	Regular Truck	Heavy truck
NI	246,212	7,048	7,542
AM	976,715	26,631	15,367
OPD	1'905,037	84,091	42,157
PM	1'259,606	24,305	14,804
OPN	1'007,921	47,703	27,111

**Table 4. The Demand**

A set of logistic volume delay functions have been calibrated for all the links of the network <sup>1</sup>. The logistic functions are continuous positive non decreasing functions of the flow. Their functional form is:

$$s_a(v_a) = t_a^0 * \left(1 + \frac{\eta_a}{1 + \alpha_a / ((v_a + \theta_a) / c_a)^{\beta_a}}\right) \quad a \in A$$

where  $s_a(v_a)$  is the travel time on link  $a$ ;  $t_a^0$  is the travel time in the link at free flow speed;  $\eta_a$ ,  $\alpha_a$ ,  $\theta_a$  and  $\beta_a$  are the parameters for the link  $a$ ;  $c_a$  is the capacity of the link

and  $v_a = \sum_m v_a^m$  as previously mentioned.

The original demand was perturbed to test the performance of the three solution methods. For every class, the total demand was decreased approximately 20 percent and only the largest O-D pair values were changed. The total demand “To be adjusted” by type of vehicle and period of the day is listed in Table 5.

Period	Auto	Regular Truck	Heavy truck
NI	205,673	5,785	6,086
AM	805,512	21,619	12,486
OPD	1'566,806	68,808	33,445
PM	1'021,659	19,812	11,986
OPN	822,865	38,798	22,303

**Table 5. The Demand “To be adjusted”**

<sup>1</sup> Service de la modélisation des systèmes de transport. Ministry of Transportation of Quebec.

For the second and the third adjustment approaches the vehicle classes were considered in the order of decreasing total demand: auto, regular truck and then heavy truck.

The stopping criterion for the assignment routines is shown on Table 6. The number of iterations for the adjustment process was fixed to 5 for all the tests and all the classes.

<b>Period</b>	<b>Number of iterations</b>	<b>Relative Gap</b>	<b>Normalized Gap</b>
NI	10	0.01	0.01
AM	100	0.05	0.05
OPD	80	0.01	0.01
PM	100	0.05	0.05
OPN	50	0.01	0.01

**Table 6. The Assignment Parameters**

All three methods worked as expected. The results are very satisfactory. In all the cases the objective function and the  $R^2$  coefficient (from the regression between the observed and the simulated link flows) were improved. Moreover, the slope of the regression,  $B$ , is also improved, especially for the trucks; showing that the missing demand is partially recovered.

In the following, the first approach, in which all the classes are adjusted at the same time is referred as MC Adj.; the second approach, where the adjustment is done sequentially class by class, leaving all the flows variable, is identified as SEQ Adj.; and the third approach, in which one class is adjusted at the time considering the flows of the rest of the classes as fixed is called FIX Adj.

The regression coefficients  $R^2$  and  $B$  are listed in Tables 7 and 8, respectively. The objective function values are listed in Table 9 and graphically presented in Figures 2 to 4 for the three congested periods of the day.

Some explanations are needed to understand Figures 2 to 4. In the case of the multi-class adjustment (MC Adj.) even if 5 iterations were requested, 7 values are computed; the first one corresponds to the initialization iteration and the last one is the value at the end of the adjustment. The same procedure is used in the sequential adjustment (SEQ Adj.), but in this case the last value of the objective function for a class corresponds to the initial value for the next class. In the fixed flows adjustment case (FIX Adj.) the global objective function is computed only four times; before each class demand adjustment and at the end of the adjustment process.

		To be adjusted	MC Adj.	SEQ Adj.	FIX Adj.
NI	Auto	0.96	0.99	0.99	0.99
	Regular truck	0.93	0.96	0.96	0.96
	Heavy truck	0.91	0.98	0.98	0.98
AM	Auto	0.93	0.97	0.97	0.97
	Regular truck	0.87	0.93	0.94	0.93
	Heavy truck	0.88	0.95	0.95	0.94
OPD	Auto	0.92	0.98	0.98	0.98
	Regular truck	0.92	0.96	0.96	0.95
	Heavy truck	0.94	0.97	0.97	0.97
PM	Auto	0.92	0.97	0.97	0.96
	Regular truck	0.87	0.92	0.93	0.90
	Heavy truck	0.83	0.94	0.95	0.94
OPN	Auto	0.94	0.98	0.99	0.99
	Regular truck	0.93	0.96	0.97	0.97
	Heavy truck	0.92	0.97	0.97	0.97

Table 7. The  $R^2$  regression coefficient

		To be adjusted	MC Adj.	SEQ Adj.	FIX Adj.
NI	Auto	0.91	0.98	0.98	0.98
	Regular truck	0.72	0.94	0.95	0.94
	Heavy truck	0.75	0.96	0.96	0.96
AM	Auto	0.99	1.01	1.00	1.00
	Regular truck	0.79	0.94	0.94	0.99
	Heavy truck	0.66	0.95	0.97	1.02
OPD	Auto	0.91	1.01	1.00	0.98
	Regular truck	0.77	0.96	0.97	1.05
	Heavy truck	0.76	0.99	1.00	1.03
PM	Auto	1.07	1.00	0.98	0.98
	Regular truck	0.72	0.97	0.99	0.95
	Heavy truck	0.60	0.96	0.97	1.00
OPN	Auto	1.02	1.02	1.03	1.03
	Regular truck	0.80	0.98	0.97	0.97
	Heavy truck	0.84	0.96	0.96	0.95

Table 8. The B regression coefficient

	To be adjusted	MC Adj.	SEQ Adj.	FIX Adj.
NI	24 208	5 865	5 845	5 834
AM	604 984	219 303	217 630	232 279
OPD	1 727 603	354 702	331 645	476 588
PM	908 776	256 309	248 067	324 978
OPN	380 069	98 205	93 582	92 794

Table 9. The Objective function

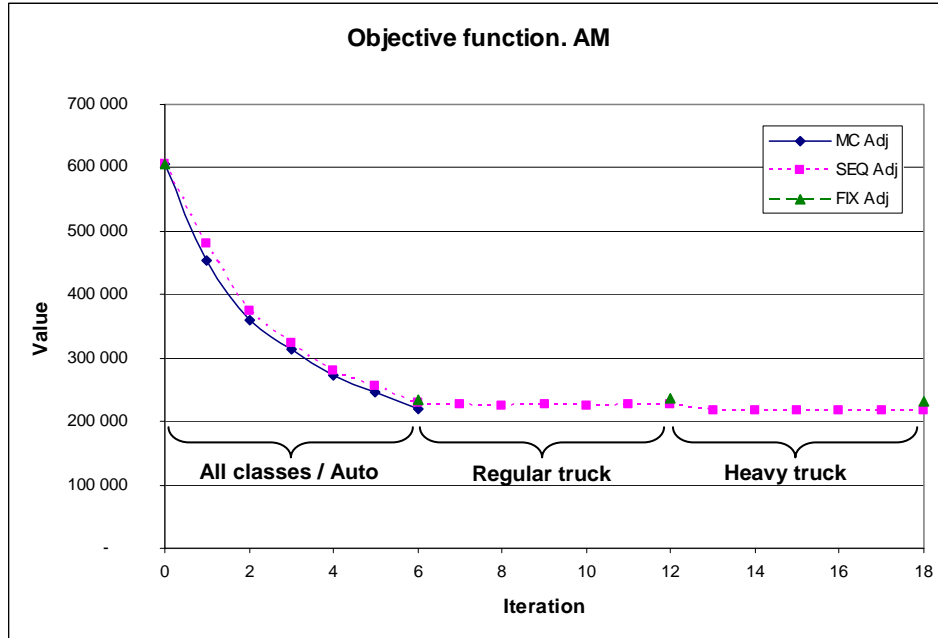


Figure 2. Improvement of the objective function value. AM

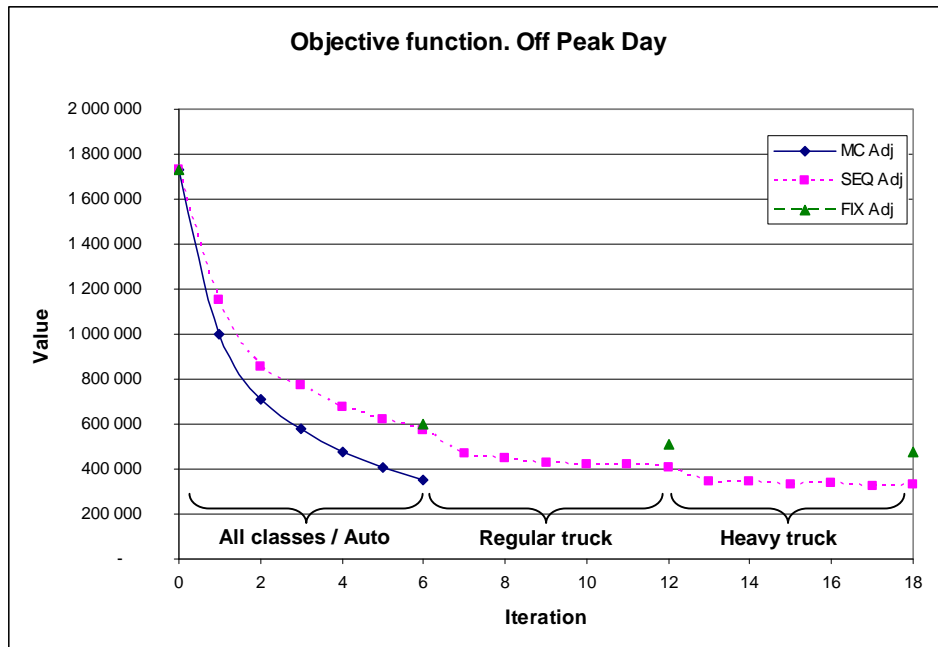
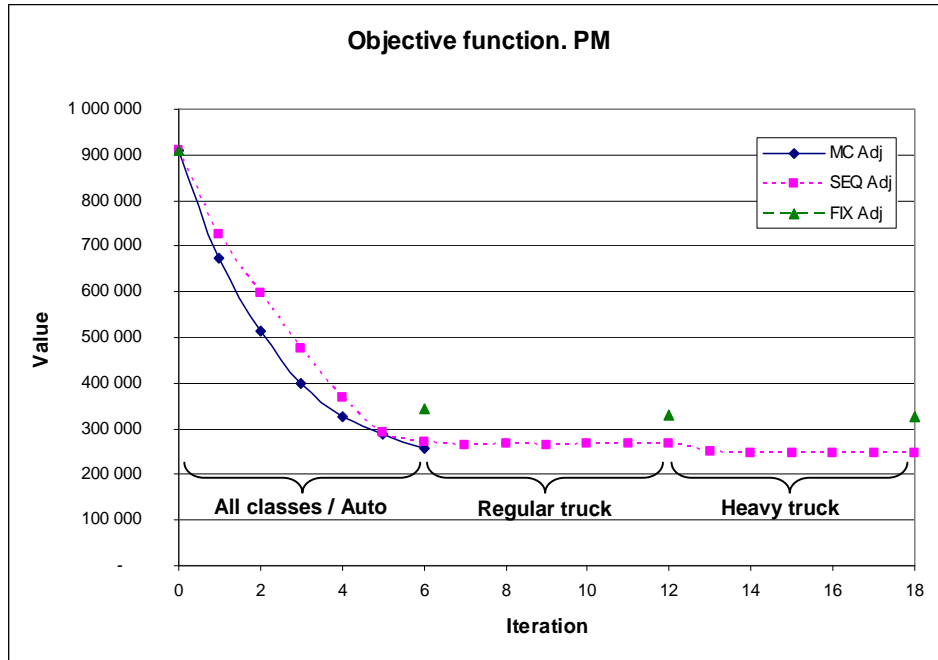


Figure 3. Improvement of the objective function value. OPD



**Figure 4. Improvement of the objective function value. PM**

Some conclusions result from the study of Figures 2 to 4. The improvement of the objective function with the second and the third approaches is significant for the autos (note that the demand of trucks is always less than 10% of the auto demand). In the case of the SEQ Adjustment for trucks, after the first iteration, the improvement is negligible; however, if only 1 iteration is done for these classes of vehicles, the fit between the simulated and the observed flows will not be good enough.

The adjusted demand is shown in Table 10 and a summary of the computation times is given in Table 11. The tests were carried out on an Intel® Core (TM) 2 CPU 6400 @ 2.13GHz, 3.25 Gb of RAM.

		Original	To be adjusted	MC Adj.	SEQ Adj.	FIX Adj.
NI	Auto	246 212	205 673	214 667	214 681	214 692
	Regular truck	7 048	5 785	6 832	6 813	6 791
	Heavy truck	7 542	6 086	6 916	6 914	6 918
AM	Auto	976 715	805 512	841 179	840 747	844 152
	Regular truck	26 631	21 619	25 574	25 512	26 942
	Heavy truck	15 367	12 486	14 808	14 723	15 024
OPD	Auto	1 905 037	1 566 806	1 720 990	1 750 499	1 741 977
	Regular truck	84 091	68 808	89 154	89 782	92 642
	Heavy truck	42 157	33 445	39 802	39 564	39 878
PM	Auto	1 259 606	1 021 659	1 073 093	1 081 084	1 062 908
	Regular truck	24 305	19 812	24 399	24 649	24 855
	Heavy truck	14 804	11 986	14 722	14 690	15 170
OPN	Auto	1 007 921	822 865	850 881	853 489	854 037
	Regular truck	47 703	38 798	46 290	46 559	46 712
	Heavy truck	27 111	22 303	23 642	23 614	23 593

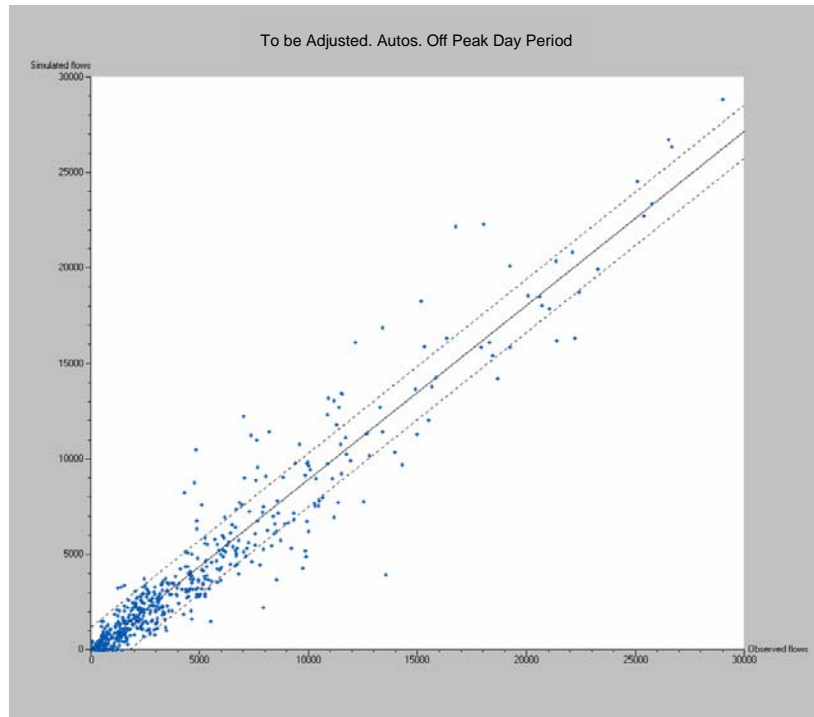
**Table 10. The Adjusted Demand**

<b>Period</b>	<b>MC Adj.</b>	<b>SEQ Adj.</b>	<b>FIX Adj.</b>
NI	30	58	21
AM	548	1 018	189
OPD	426	835	177
PM	524	882	170
OPN	163	319	67

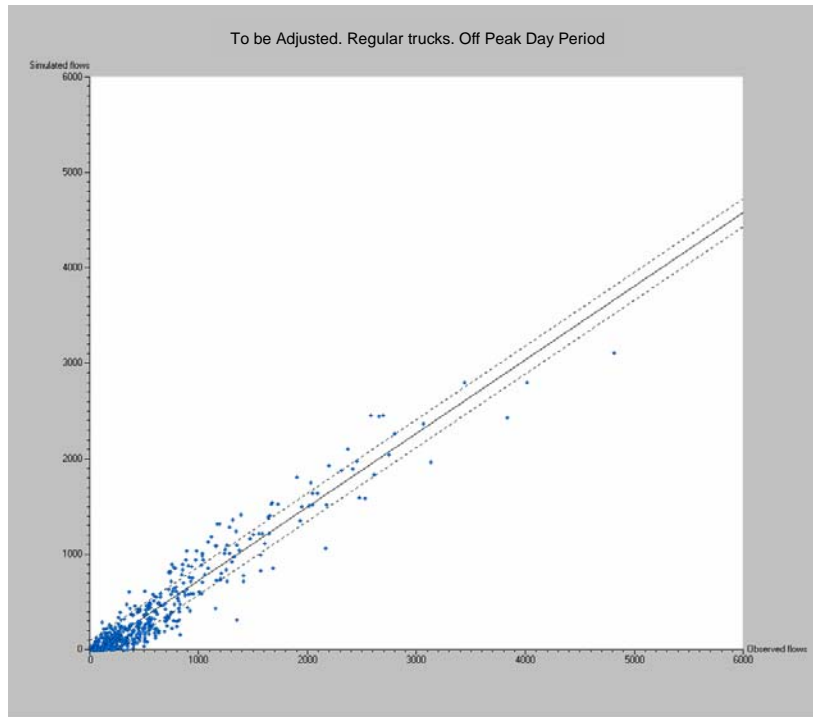
**Table 11. The Computation Times (min.)**

Based on the computation times, the first choice would be the FIX Adjustment, then the MC Adjustment. The SEQ Adjustment is very time consuming and the results are almost the same as the others.

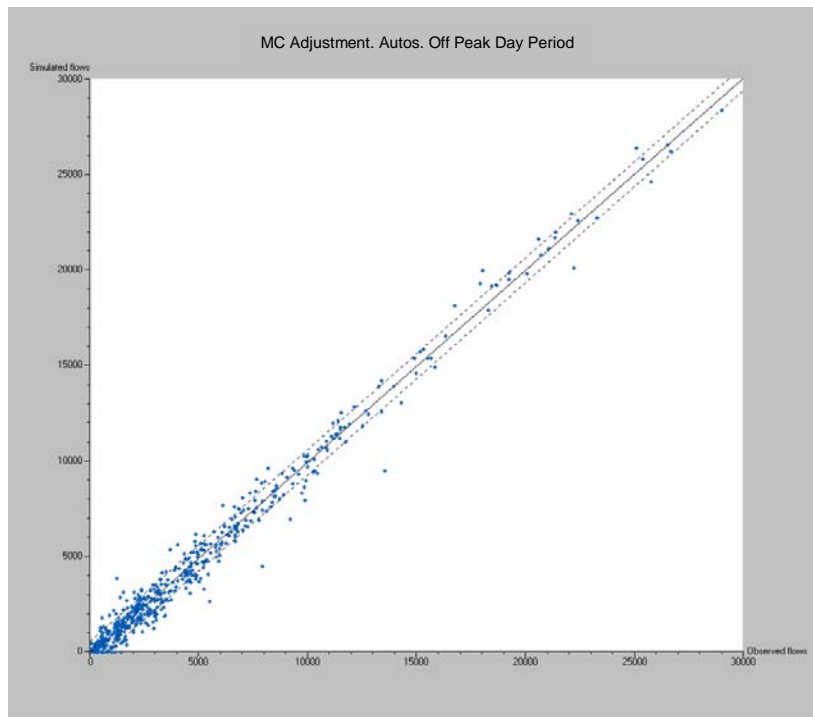
Four scattergrams which compare the observed vs. the simulated flows are presented in Figures 5 to 8. They correspond to the Off Peak Day period of the day, for cars and regular trucks. The first two scattergrams show the fit of the demand “To be adjusted”. The two last scattergrams show the fit after the MC Adj. was carried out. As one can see, the fit is always significantly improved after the adjustments.



**Figure 5. Link flow scattergram. Observed vs. simulated autos before the adjustment**

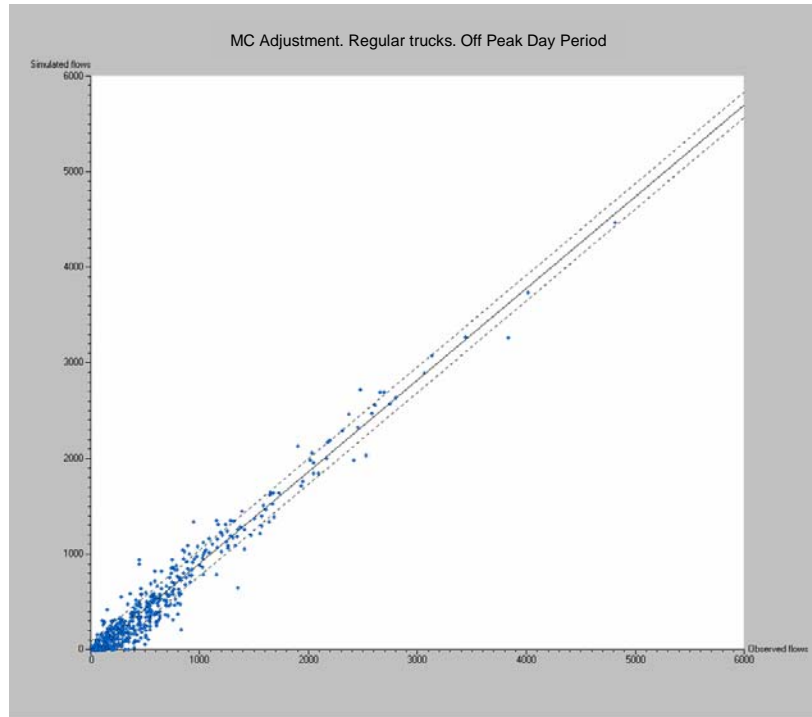


**Figure 6. Link flow scattergram. Observed vs. simulated regular trucks before the adjustment**



**Figure 7. Link flow scattergram. Observed vs. simulated autos after the adjustment**





**Figure 8. Link flow scattergram. Observed vs. simulated regular trucks after the adjustment**

Results indicate that using the MC Adjustment is the best option. The SEQ Adjustment takes much more time, and the results are not much better. The FIX Adjustment is the fastest but the fit is not the best one can get. With this method there is not continuous feedback among classes.

## Conclusions

The simultaneous adjustment of several origin-destination matrices, for several classes of traffic, is feasible by using the method presented in this article. The simultaneous method is preferable to the sequential adjustment of the origin-destination matrix of each class. The generalization of the gradient method of Spiess (1990) for several classes is an efficient algorithm.

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