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On the Determinants of the Implied Default Barrier

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Abstract. We use the maximum likelihood (ML) estimation approach to estimate the default barriers from market values of equities for a sample of 762 public industrial Canadian firms. The ML approach allows us to estimate the asset instantaneous drift, volatility and barrier level simultaneously, when the firm's equity is priced as a Down-and-Out European call (DOC) option. We find that the estimated barrier is positive and significant in our sample. Moreover, we compare the default prediction accuracy of the DOC framework with the KMV-Merton approach. Using probit estimation, we find that the default probability from the two structural models provides similar in-sample fits, but the barrier option framework achieves better out-of-sample forecasts. Regression analysis shows that leverage is not the only determinant of the default barrier. The implied default threshold is also positively related to financing costs, and negatively to liquidity, asset volatility and firm size. We also find that liquidation costs, renegotiation frictions and equity holders' bargaining power increase the implied default barrier level.

Keywords. Barrier option, default barrier, bankruptcy prediction, maximum likelihood estimation, strategic default.

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1. Introduction

One of the most important assumptions in the structural models of credit risk is that the firm defaults when its value reaches a minimal threshold, which is often called the default barrier or the default boundary. All the structural models in the credit risk literature specify assumptions about the default barrier and calibrate the level of asset value below which the firm defaults. Most of the empirical tests of these models compare the credit risk premia generated by the structural models with those actually observed on the credit derivatives or debt markets. Despite the large number of these studies, little attention has been paid to the underlying assumptions regarding the level of the default barrier, that is the value of the assets below which the default is triggered. One exception is Davydenko (2007), who studies whether default is triggered by low market asset value or by liquidity shortages.

Indeed, structural models often rely on parameters that are not directly observable; one example is the default barrier where the dynamics and the location are not visible. Researchers must then specify the default barrier based on indirect information. One of the vaguest aspects of structural models of default is the link between firm characteristics and the default barrier location. Because a better understanding of the determinants of the default threshold could be valuable for modeling the decision to default and for default prediction, this paper seeks to identify the firm-specific factors and the macroeconomic variables that have the potential to influence the location of the default barrier.

Using a sample of Canadian public companies, we compare the default prediction obtained from a Merton model-based approach, where the default barrier is set as a given fraction of the firm's debt, with that generated by the Debt-and-Out European Call option model (hereafter DOC option model) introduced by Brockman and Turtle (2003), where the firm defaults whenever its asset value reaches the estimated default barrier. Our results show that the estimated default barrier is significant on average for our sample of Canadian public firms. We also find that the Merton based approach and the DOC option model have similar in-sample fits in explaining default occurrence. However, the DOC option model provides superior out-of-sample forecasts of bankruptcies in our sample. Moreover, for the subsample of the defaulted firms, the estimated asset values with the DOC option model are much closer to the model-implied default barrier relative to the surviving firms. On average, the estimated default barrier gives a good measure of the value of the assets at the default time. We focus on these estimates of the default threshold and we perform a statistical analysis of the default barrier level on a set of firm characteristics. The results indicate that the DOC option model implied default barrier is affected not only by the level of leverage, but also by the liquidity of the firm and its debt cost, which underlines the importance of the liquidity shortage and external financing cost concerns. Moreover, the implied default barrier location is influenced by liquidation costs and bargaining power, which gives some support to models of strategic defaults.

The rest of the paper is organized as follows. Section 2 presents a brief literature review of structural models. Section 3 describes the methodology used to estimate the models' parameters. Section 4 analyses the results of the estimated barrier and compares the capacity to predict defaults of two models: the DOC option model and the Merton-KMV model. Section 5 discusses the choice of explanatory variables and the hypothesis regarding the default threshold, together with the regressions results. Section 6 concludes the paper.

2. Literature review

There are several structural models that propose default triggers. Most of them are first-passage-time approaches in the sense that they extend the seminal framework of Black and Scholes (1973) and Merton (1974) (hereafter BSM) by allowing default to occur whenever the value of a firm's assets crosses a pre-specified barrier, and not only at the debt's maturity (Black and Cox, 1976). The default triggering barrier can be given either endogenously or exogenously. For the endogenous default trigger, the equity holders choose to default (or to reorganize) strategically so as to maximize the value of equities (e.g. Leland, 1994; Leland and Toft, 1996; Acharya and Carpenter, 2002). Exogenous default trigger models, in contrast, impose a pre-specified default barrier and extend the basic framework to include stylized facts of bond markets, such as stochastic risk-free interest rates (Longstaff and Schwartz, 1995), stochastic default barrier (Hsu, Saá-Requejo, and Santa-Clara, 2002), and mean-reverting leverage (Collin-Dufresne and Goldstein, 2001).

Most often, the practical implementation of the structural models needs to specify some assumptions regarding the level of the default barrier. Usually, in the exogenous structural models, the default barrier is expressed as a fraction of the face value of the debt (less than or equal to 1). It is therefore assumed in these models that the default barrier depends solely on the level of the face value of debt. For instance, Longstaff and Schwartz (1995) consider a default barrier that equals the total principal value of debt. Nonetheless, such a default barrier seems unrealistic because many firms continue to operate with a negative net worth. To deal with this concern, Huang and Huang (2003) suppose that the default barrier equals 60% of the face value of debt, while Leland (1994) chose to calibrate the default barrier to match the observed recovery rates. This leads to a default barrier of 73% of the face value of the debt. Alternatively, what we refer to here as the Merton-KMV model assumes that default occurs only at debt maturity and the default point is set to the short-term debt plus one half of the long-term debt (Crosbie and Bohn, 2002; Vassalou and Xing, 2004).

The endogenous models pioneered by Black and Cox (1976), and extended by Leland and Toft (1996) and Acharya and Carpenter (2002) among others, offer a richer specification of the default barrier, because the equity holders/manager decide whether or not to default depending on the continuation value of the firm relative to current debt service payment. The default barrier corresponds to the cut-off point for the asset value below which it is more worthwhile for equity holders to default on the firm's debt. This setting makes the default barrier sensitive to other factors in addition to the principal value of the debt. For example, in Leland and Toft's (1996) model, the optimal barrier level is decreasing in debt maturity, asset volatility and the risk-free rate, whereas it is increasing in default costs and the book value of debt.

As mentioned by Davydenko (2007), this kind of model assumes the absence of either minimum cash-flow covenants or market frictions, which could limit the firm's ability to raise sufficient external financing. As a result, the firm will never fall into default for cash shortage reasons: If the firm faces a liquidity crisis, and the firm's value is above the default barrier, the equity holders will always be able to avoid default by raising new funds. Few models presented in the literature relax these assumptions (see Kim, Ramaswamy, and Sundaresan, 1993; Anderson and Sundaresan, 1996). Instead of setting the asset value as the default trigger, they assume that default occurs when the firm's cash-flow fails to meet the debt service payment.

Given the unavailability or the limitations of external financing, the default becomes exogenous and happens only in the case of a cash crisis. Fan and Sundaresan (2000) combine the endogenous value-based and exogenous liquidity-based defaults by assuming an exogenous covenant on the minimum cash flow for the former, and costly external fund raising for the latter.

Another trend in the literature considers the possibility of debt contract renegotiation and deviation from the absolute priority rule, allowing strategic debt service. Indeed, Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1996) stipulate that in the presence of liquidation costs and bargaining power of the equity holders, the firm creditors may accept a partial payment of the debt, which in turn may encourage opportunistic default by the equity holders. In addition, many firm-specific strategic factors were identified as having an effect on the default and recovery decisions. Asquith, Gertner and Scharfstein (1994), Frank and Torous (1994), and Betker (1995) document that the complexity of debt structure, managerial share ownership, and asset tangibility have an impact on the occurrence of the formal and informal reorganization and deviations from absolute rule. Therefore, we can expect the default barrier to depend on the strategic factors as well as other firm-specific factors.

François and Morellec (2004) make the distinction between default and liquidation of the firm, and explicitly account for the possibility of debt renegotiation under Chapter 11. In this setting, the firm is liquidated if its assets stay below the default barrier for a given period of time. Thus, the firm's equity is modelled as a Parisian down-and-out call option on the firm's assets. Moraux (2002) offers a rather different modelling by considering the cumulative time spent below the barrier (financial distress). Galai, Raviv and Wiener (2005) go further by considering not only the consecutive and the cumulative time spent in financial distress, but also the severity of this distress. Finally, Carey and Gordy (2007) develop a model where the default barrier is set mainly by private debt holders. They present evidence that the recovery rate increases sharply with the pre-bankruptcy share of private debt in all of the firm's debt.

To our knowledge, Davydenko (2007) is the only contribution that explicitly studies the value of assets at default, and investigates whether default is triggered by low asset values or by liquidity shortages. He uses a sample of low-grade US firms with observed market values of both debt and equity, which allows him to observe the asset value at default. He finds that the asset value at default varies largely in cross-section, and depends on balance sheet liquidity, asset volatility and tangibility. While on average a barrier of 72% of the face value of the debt correctly predicts the probability of default, the large cross-section variability regarding the default barrier of defaulted firms leads him to conclude that structural models based on a well-defined default trigger have a limited ability to predict defaults in cross-section.

In contrast with Davydenko (2007), we estimate the default barrier implied by Brockman and Turtle's (2003) model using a maximum likelihood estimation procedure for all the firms in our sample, and we don't limit our investigation to firms with directly observable asset value. We focus on the barrier level that is perceived by the market participants, as it is derived from the common equity price. Indeed, the default announcement could convey additional information about the defaulting firm's financial situation. Moreover, recovery rates may underestimate the asset value at which the firm defaults, due to the presence of bankruptcy costs and departures from the absolute priority rule. In other words, we focus on the set of information prior to default, namely the equity market prices.

3. Estimation of the implied default barrier

3.1 Estimation method

The Brockman and Turtle (2003) model is an extension of the basic BSM framework, where the firm's equities are viewed as a down-and-out call option and default is triggered when the value of the assets crosses the barrier level such that bond holders are able to receive the remaining value of the firm before it deteriorates further. In this setup, the default barrier can be seen as a debt covenant. One goal is to estimate the barrier level implied by the traded equity prices. The methodology relies on calibrating the barrier level such that the Down-and-Out European call option price formula matches the observed equity prices. To be able to do so, one needs to know the value of assets, the instantaneous drift and volatility of the assets' return, and the face value of debt. Because the actual value of the firm's assets is not observable, one can approximate the total value of the assets by summing the market value of equity and the book value of debt.

Brockman and Turtle (2003) use the time series of the quarterly market value obtained over a ten-year period to estimate the historical asset volatility.

They find an average barrier level significantly higher than zero, and the barrier to assets ratio is 69.2%, much larger than the average leverage ratio of their sample, which is equal to 44.7%. This result holds for the different industry sectors and for the nine first leverage deciles. These findings are counter-intuitive because many firms continue to operate with negative net worth, and it seems unrealistic that debt holders could get back more than their debt.

Wong and Choi (2004) point out this discrepancy and show that, in the down-and-out call framework, approximating the asset value by the market capitalization plus the book value of debt leads to a biased implied default barrier that is larger than the book value of corporate liabilities, regardless of the empirical data used. This underscores the necessity of using an alternative estimation procedure to measure the firm's asset value rather than a proxy.

The literature provides several ways of calibrating the firm's asset value, V_t, and the standard-deviation of the asset volatility, σ_V. In the framework of BSM, the first method, which is referred to here as the variance restriction method, makes use of Ito's lemma to obtain a system of two equations linking the unknown asset values and the asset volatility to the observed equity values and volatility (Jones, Mason and Rosenfeld, 1984; Ronn and Verma, 1986). However, several drawbacks of this method were identified. Indeed, Crosbie and Bohn (2002) point out that the equation relating the equity volatility to asset volatility holds only instantaneously. Furthermore, Duan (1994) criticizes the implicit assumption in the variance restriction method of constant equity volatility and its independence from the corporate asset value and time. He also points out the lack of statistical inference for the estimates of V_t and σ_V with the variance restriction method.

Duan (1994) developed a transformed data maximum likelihood estimation method in order to estimate V_t and σ_V from equity prices, which views the observed equity times series as a transformed data set where the theoretical equity pricing formula is used as a transformation. We will revisit this estimation method in greater detail below. In addition to the statistical inference provided by the maximum likelihood estimation, Ericsson and Reneby (2005) compare the three described estimation methods, and find that the transformed data maximum likelihood estimation method is superior. Wong and Choi (2004) also use this method in the down-and-out call option framework.

KMV developed an iterative method based on the variance restriction method, described in Crosbie and Bohn (2003). For the standard call approach, Duan, Gauthier and Simonato (2004) show that the KMV method estimates are identical to the maximum likelihood estimates for the Black-Scholes-Merton model. However, when more complex structural model involving unknown capital structure parameter is considered, such as in the DOC option model, the KMV method is unable to estimate the additional parameter involved, because only two equations are used. This contrasts with the transformed MLE method, which is able to estimate the capital structure parameter, namely the default barrier in the DOC option model. These features lead us to retain the MLE estimation as our preferred methodology for estimating the models' parameters.

3.2 Down-and-Out Call Option

As mentioned earlier, the DOC option model hinges on viewing the firm's equity as a DOC option on the firm's assets. We assume a geometric Brownian motion for the asset value, that is:

$$d\ln V_t = (\mu - \sigma^2/2)dt + \sigma dW_t$$

where V_t is the market value of the firm's assets at time *t*, σ is the asset value volatility, μ is the expected return on assets and W_t is a Wiener process. The down-and-out call option price is given by:

$$\begin{aligned} E_{DOC} &= VN(a) - Xe^{-rT}N(a-\sigma\sqrt{T}) - V(H/V)^{2\eta}N(b) + Xe^{-rT}(H/V)^{2\eta-2}N(b-\sigma\sqrt{T}) \\ &\quad + R(H/V)^{2\eta-1}N(c) + R(V/H)N(c-2\eta\sigma\sqrt{T}) \end{aligned} \tag{1}$$

where *T* is the time to maturity of the option, *H* is the default barrier, *R* is the rebate of the barrier option, that is the payment made to the equity holders if the value of the firm's assets breaches the barrier. *N*(.) is the cumulative distribution function for the standard normal distribution, and

$$a = \begin{cases} \dfrac{\ln(V/X) + (r+\sigma^2/2)T}{\sigma\sqrt{T}} & \text{if } X \geq H, \\ \dfrac{\ln(V/H) + (r+\sigma^2/2)T}{\sigma\sqrt{T}} & \text{if } X < H, \end{cases}$$

$$b = \begin{cases} \dfrac{\ln(H^2/VX) + (r+\sigma^2/2)T}{\sigma\sqrt{T}} & \text{if } X \geq H, \\ \dfrac{\ln(H/V) + (r+\sigma^2/2)T}{\sigma\sqrt{T}} & \text{if } X < H, \end{cases}$$

$$c = \frac{\ln(H/V) + (r+\sigma^2/2)T}{\sigma\sqrt{T}} \qquad \text{and} \qquad \eta = \frac{r}{\sigma^2} + \frac{1}{2}$$

where r is the risk-free rate.

It is interesting to note that the DOC framework includes the standard call option framework as a special case. Indeed if we set the barrier H to zero in equation (1) we obtain the pricing formula of a European call option. Moreover, in our setting we assume that the Absolute Priority Rule holds, that is, the equity holders receive nothing if the firm defaults. Thus, the last two terms in equation (1) become null, and equation (1) is reduced to:

$$\begin{aligned} E_{DOC} &= VN(a) - Xe^{-rT}N(a-\sigma\sqrt{T}) - V(H/V)^{2\eta}N(b) + Xe^{-rT}(H/V)^{2\eta-2}N(b-\sigma\sqrt{T}) \\ &= \beta(V,\sigma,H) \end{aligned} \tag{2}$$

The function relating the equity price to the asset value, $\beta(V,\sigma,H)$, is invertible for any given asset volatility. Thus, we can invert it and express V as a function of E_{DOC}, σ and H, that is $V^t = \beta^{-1}(E^t_{DOC},\sigma,H)$.

We apply the Wong and Choi (2004) likelihood function in the DOC framework.[1] In the first specification, V^t is the asset value at time t, and $f(\ln V^t \mid \ln V^{t-1};\mu,\sigma,H)$ is the conditional density function of $\ln V^t$, because the asset value should remain above the barrier between two

[1] See Duan, Gauthier and Simonato (2004) for an alternative likelihood function to estimate the Brockman and Turtle model with the maximum likelihood method. See Gharghori et al. (2006) for a comparison of an option-based model with an accounting-based one.

successive observation dates (respectively t and $t\text{-}1$), the density function should account for this feature. The corresponding density function is given by:

$$f(\ln V^t \mid \ln V^{t-1}; \mu, \sigma, H) = \varphi(\ln(V^t / V^{t-1})) - e^{2\eta(\ln H - \ln V^{t-1})}\varphi(\ln(V^t V^{t-1}) - 2\ln H) \tag{3}$$

where $\varphi(x) = \frac{1}{\sigma\sqrt{2\pi\Delta t}}\exp\left(-\frac{(x-(\mu-\sigma^2/2)\Delta t)^2}{2\sigma^2\Delta t}\right)$ and Δt is the time interval between two successive observation dates.

If the asset value was observable, the log-likelihood function would be:

$$L^V = \sum_{i=2}^{n} \ln f(\ln V^t \mid \ln V^{t-1}; \mu, \sigma, H).$$

However, because we do not observe V^t but rather E^t_{DOC}, we modify the model as follows:

$$\begin{aligned} L^E &= \sum_{t=2}^{T} \ln\left[f(\ln \hat{V}^t \mid \ln \hat{V}^{t-1}; \mu, \sigma, H) \times \left(\frac{\partial \beta(V,\sigma)}{\partial \ln V} \right)^{-1} \right] \\ &= \sum_{t=2}^{T} \ln\left[f(\ln \hat{V}^t \mid \ln \hat{V}^{t-1}; \mu, \sigma, H) \times \left(\hat{V}^t \frac{\partial \beta(V,\sigma)}{\partial V} \right)^{-1} \right] \end{aligned} \tag{4}$$

where $\hat{V}^t = \beta^{-1}(E^t_{DOC}, \sigma, H)$ and [2]

$$\begin{aligned} \frac{\partial \beta(V,\sigma)}{\partial V} &= N(a) + \frac{1}{\sigma\sqrt{T}} n(a)(1-\frac{X}{H}) + \frac{X}{V} e^{-rT} (\frac{H}{V})^{2\eta-2} N(b-\sigma\sqrt{T})(2-2\eta) \\ &\quad - (1-2\eta)(\frac{H}{V})^{2\eta} N(b) + \frac{1}{\sigma\sqrt{T}} n(b)(1-\frac{X}{H})(\frac{H}{V})^{2\eta} \end{aligned} \qquad \text{if } H \geq X,$$

and

$$\begin{aligned} \frac{\partial \beta(V,\sigma)}{\partial V} &= N(a) - (\frac{H}{V})^{2\eta} N(b)(1-2\eta) \\ &\quad + (2-2\eta)\frac{X}{V} e^{-rT} (\frac{H}{V})^{2\eta-2} N(b-\sigma\sqrt{T}) + \frac{1}{\sigma\sqrt{T}} n(b)(1-\frac{X}{H})(\frac{H}{V})^{2\eta} \end{aligned} \qquad \text{if } H < X,$$

[2] See Hao (2006).

We conduct simulations to check for the estimation's ability to retrieve the model parameters. We also estimate the Merton model, in order to compare the performance of the DOC option model in predicting default probabilities with the standard European call framework. In this setting the pricing equation becomes:

$$E_{SC} = VN(d) - Xe^{-rT}N(d - \sigma\sqrt{T}) \text{ where } d = \frac{\ln(V/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \tag{5}$$

The corresponding transformed maximum log likelihood function as derived by Duan et al (2004) is given by:

$$L^E = -\frac{n}{2}\ln(2\pi\sigma^2\Delta t) - \frac{1}{2}\sum_{t=2}^{n}\left[\frac{\ln(\hat{V}^t/\hat{V}^{t-1}) - (\mu - \sigma^2/2)\Delta t}{\sigma\sqrt{\Delta t}}\right]^2 - \sum_{t=1}^{n}\ln\hat{V}^t - \sum_{k=1}^{n}\ln(\Phi(\hat{d})).$$

Here the $\hat{V}_t$ is obtained by inverting (5). Given its high non-linearity, the likelihood function in both cases is maximized using the Nelder-Mead Simplex Algorithm (Fminsearch in Matlab).

Once the models' parameters are estimated, the default probabilities are given by:

$$DP_{Barrier} = N\left(\frac{-(\ln(V_0/H) - \mu\tau)}{\sigma\sqrt{\tau}}\right) + e^{-\frac{2\mu\ln(V_0/H)}{\sigma^2}} N\left(\frac{-(\ln(V_0/H) + \mu\tau)}{\sigma\sqrt{\tau}}\right)$$

for the DOC option model while for the standard call option this probability takes the following form:

$DP_{SC} = N(-d)$ where d is defined in (5).

3.3 Simulations

In order to assess the maximum likelihood function's ability to recover the asset drift, volatility and barrier level we use Monte-Carlo simulations. The performance of the estimation method is examined in this subsection. The procedure for the simulations is described below:

1. We begin by generating the time series of the asset value between the time 1 and T, where Δt is the time interval between two successive observation dates. We refer to this

time series by $\{V^0, V^1, .., V^t, ..., V^T\}$. Because the return on assets is assumed to be normal, the value of the assets follows a lognormal distribution. Thus, the value of V^t is given by : $V^{t+1} = V^t \exp\left((\mu - \frac{1}{2}\sigma^2)\Delta t + \sigma\sqrt{\Delta t}\varepsilon^t\right)$ where $\{\varepsilon^0, \varepsilon^1, .., \varepsilon^t, ..., \varepsilon^T\}$ is a sequence of independent and identically distributed standard normal random variables.

2. The second step is to compute the time series of the equity prices $\{E^0_{DOC}, E^1_{DOC}, .., E^t_{DOC}, ... E^T_{DOC}\}$ from the simulated asset values using equation (2).

3. Finally, we use the log-likelihood function given in equation (4) to estimate the asset drift (μ), volatility (σ) and barrier level (*H*) from the obtained equity prices.

We conduct maximum likelihood estimation and compute the point estimates for each quantity. We repeat the simulation 1,000 times. We retain 200 daily observations of equity prices, that is *N*=200 and Δt =1/250. We assume that the capital structure remains unchanged through the 200-day observation period, hence we do not take into account the survivorship consideration as did Duan, Ghautier, Simonato and Zaanoun (2003). Furthermore, we assume that the initial value of the assets is V^0 =\$10,000 and the face value of the debt is *F*= \$6000. The true barrier level is set to *H*=5000, and the drift and volatility of the assets are set to $\mu = 0.1$ and $\sigma = 0.3$. The retained risk-free rate is *r* = 5% and the maturity of the barrier option retained is *T*=20 years. To allow a better comparison with previous studies we express the barrier level as a fraction of the nominal value of total liabilities. That is:

$$H = \alpha F .$$

Note that there is no difference between the estimates of *H* or α . When the barrier level is nil, we obtain an α equal to zero. Moreover, a barrier level *H* higher than the level of debt will lead to an α higher than 1, and when the barrier level is below the nominal debt, α will be less than 1. In this case, the true α is 0.833. Moreover, we assume that the debt of the firm is a zero coupon bond, and is rolled over for all the option maturity. We report the simulation results in Table 1.

<Insert Table 1 here>

The average estimates of the three parameters are all close to their true value. To test whether the difference between the mean of the estimates and their corresponding true values are significant, we report in Table 1 the *t-statistic* and the related *p-value* for each parameter. None of the means is significantly different from the real value.

Bharath and Shumway (2004) find that the asset drift parameter is important in estimating the default probabilities. That is, in out-of-sample results, they find that distance to default (DD) computed with estimated continuously compounded return on assets, $\hat{\mu}$, outperforms the DD measure where this parameter is set to the risk-free rate. Therefore, we decided to use real probabilities of default instead of risk-neutral ones.

3.4 Data

The study covers the public Canadian industrial firms listed in the Toronto Stock Exchange from January 1988 to December 2004. To be able to obtain default probabilities for the first year we needed market and accounting data on the previous year, because we needed to obtain one year of daily observations. Thus, in order to estimate the structural models we gathered data starting from January 1987.

Firms that went bankrupt or were in reorganization were identified using Financial Post Predecessors & Defunct, CanCorp Financials (Corporate Retriever), and Stock Guide. Between 1988 and 2004, 130 firms were identified as being in default: 112 were bankrupt and 18 were undergoing reorganization. After merging the accounting data with the daily market data, 77 firms remained in the intermediary database of defaults. This attrition is mostly attributable to the fact that, for some firms, we had only incomplete market data and, for others, only one year of accounting data, rendering the data unusable for our study in both cases. In fact, application of the structural model requires at least 200 consecutive daily market prices coupled with available accounting data on the book value of debt for defaulted firms.

As in Vassalou and Xing (2004), we use the book value of debt for the new fiscal year starting only four months after the end of the previous fiscal year. The goal is to ensure that we utilize only the data available to investors at the time of calculation. As a result, we needed at least two successive financial statements to obtain the 200 estimation observations required.

We examined the lags separating the default dates from the last financial statements of some defaulted firms in greater detail. Many firms do not publish financial statements during the years prior to their bankruptcy. We felt obliged to withdraw from the database defaults for which these lags exceeded 18 months. For the others, i.e., those that had defaulted between 12 and 18 months after their final financial statement, we moved the date of the default up to reconcile it with the last available year of financial statements. This filtering reduced the number of defaults retained in our sample to 60 companies.

The data on daily market capitalization of equities for both defaulting and surviving firms were obtained from DataStream. The accounting data for the non-default sample came from the Stock Guide database, while accounting data for the defaulted firms were gathered from various sources, including Stock Guide, CanCorp Financials, and the companies' financial statements from SEDAR. We end up with 4916 observations (year-firm), representing 762 single firms, of which 56 are defaults. For further details on the data used see Dionne et. al. (2008).

4. Analysis of the results

4.1 Estimated default barriers

For the estimation of both models considered here, we use a one-year window, which is equivalent to an average of 261 daily market value observations. Except for the market value of equities and the risk-free rate, which are the same for the two models, we should mention here some differences in the parameter choice, given that the two models have different definitions of variables. In the standard call option, the default point retained is the same as in Crosbie and Bohn (2002) and Vassalou and Xing (2004), namely the sum of current liability and 50% of the long-term debt. The time to maturity is adjusted in consequence, and is set to one year for the standard call option, because this amount of debt is supposed to mature one year later. The parameters of the DOC option model differ according to the underlying assumptions. Here the level of debt retained is the total liability because the option time to maturity is set to 20 years, which represents the life interval of the firm's equity. Brockman and Turtle use a 10-year interval. However, Reisz and Perlich (2004) try different time horizons between 5 and 20 years and find that this choice does not affect the default barrier. Moreover, Brockman and Turtle (2003) contend that varying the option maturity from 3 to 100 years has a minor effect on the barrier level estimates. In our case we assume a time to maturity of 20 years for the European DOC option.

<Insert Table 2 here>

The barrier estimates are presented in Table 2. The estimated barrier to implied assets for the pooled sample in Panel B is around 29% and has a standard-deviation of 27%, which is significant at all the usual confidence levels. The median of this ratio is 25%. Thus, the first finding of this study is that the Canadian public firm's equities can be seen as a down-and-out call option on their assets, because, on average, the implied default barrier is not nil. A closer look at the barrier estimates shows that the percentage of observations with barriers greater than zero attains 77%. Moreover, the average leverage ratio in our sample is 54% of the book value of assets, as shown in Panel A of Table 2. Compared with 29.38% for the implied barrier/estimated asset value, it seems that the default barrier is far below the face value of debt.

Our estimate of an average of 29.38% (median of 24.93%) is in line with the results of Reisz and Perlich (2004), where the average barrier to implied asset value is 30.53% (median of 27.58%). This result contrasts with the average barrier of 69.2% found by Brockman and Turtle (2003). As mentioned earlier and as reported by Wong and Choi (2004) and Reisz and Perlich (2004), this discrepancy comes primarily from using the sum of the market value of equity plus the book value of debt as proxies for the market value of assets. This approximation overstates the default barrier estimate.

When we compare the ratio of the default barrier to the implied asset value between defaulting and surviving firms, we observe an obvious difference between the two sub-samples. Indeed, while for non-defaulting firms the median is as low as 25% of the asset value, the median for defaulting firms is 76% of the implied asset value, and the third quartile reaches 91%. Therefore, we can conclude that for defaulting firms the barrier level is much closer to the estimated asset value.

Regarding the asset drift estimate in Panel C of Table 2, we observe a difference between surviving and bankrupt firms. While the average asset drift for the former attains 22%, it is -25% for the defaulted group. This seems to corroborate Bharath and Shumway (2004), who found that the asset drift parameter contains valuable information about the default likelihood of a firm. This also confirms the use of physical probabilities instead of risk-neutral ones in order to compare the models' performance in predicting defaults. In Panels E and F of Table 2, we report the average implied barrier to market value estimates by year and by debt load respectively. This ratio varies substantially from year to year, going from a minimum of 19% in 1996 to a maximum of 43% in 1998. Further, the average barrier increases in debt load.

4.2 Comparison of models' capacity to predict defaults

In this section we compare the estimates from the DOC option model and the Merton-KMV model. We also compare these models' capacity to predict default occurrence in our sample of public Canadian firms. The aim here is to see whether the DOC option is able to predict the defaults more accurately than the Merton-KMV. The descriptive statistics of the estimated default probabilities with both models are reported in Table 3. The average default probability for non-defaulting firms obtained from the DOC option model is 10.88% versus 61.05% for defaulting firms, while for the Merton-KMV model those average probabilities are respectively 13.66% and 48.51%.

<Insert Table 3 here>

As a first comparison of the models' performance in predicting defaults, we report in Table 4 the number and the percentage of default and non-default observations in each decile of default probabilities, where the defaults are grouped into deciles of the estimated default probabilities with the DOC option and Merton-KMV models, and the 10th decile represents the highest default probabilities. The advantage of this classification is that it is not affected by the calibration technique. That is, the overall level of the estimated default probabilities has no effect on the number of hits and false alarms in each decile. We notice that for the 10th decile the DOC option model captures 68% of the defaults while the Merton-KMV model predicts only 55% of the defaults in the whole sample. This difference in the number of hits in the 10th decile between the two models is not outweighed by a proportional increase in the number of misclassified observations for the DOC option model, because there is the same number of observations in the 10th decile for the two models. The same figure appears when we consider the first quintile, where 75% of defaults are captured with the DOC option model compared with 66% for that of Merton-KMV. The DOC option default probability quintiles appear to classify default risk across firms much more effectively than those of Merton-KMV. This finding is in line with those of Reisz and Perlich (2004) and Hao (2006) where the DOC approach attains higher accuracy than the standard call approach.

<Insert Table 4 here>

In order to better compare the ability of these models to forecast bankruptcy one year in advance, we perform two probit regressions where the dependent variable is a dummy equal to 1 if the firm went bankrupt in a given year, and 0 otherwise. The only independent variable is the estimated default probability using the DOC option model for the first regression, and the Merton-KMV default probability for the second. The fit and association statistics for these regressions are reported in Table 5.

<Insert Table 5 here>

The estimated coefficients when the whole period is used as the estimation period in Panel A, are 1.91 for the first model and 1.65 for the second model, respectively, and both are statistically significant at all the usual confidence levels, with *p-values* below 0.001. The maximum rescaled R square of Nagelkerke (1991) is a generalization of the coefficient of determination to a more general linear model. It shows that the default probabilities from the DOC option model have more explanatory power than their Merton-KMV counterpart in explaining default occurrence. However, the percentage of concordant observed values with the models' predictions are similar (75.4 % for the DOC option model compared with 75.2% for that of KMV-Merton).

The Receiver Operating Characteristic curves (ROC) are used extensively for binary response models. Furthermore, the ROC curve accounts not only for Type I errors but also for Type II errors. Indeed, the ROC curve reports the percentage of 'hits', (i.e., defaults correctly classified) as a function of the 'false alarm' (i.e., non-defaults erroneously classified as defaults), for every cutoff point between 0 and 1. A better model would have an ROC curve closer to the upper left corner, while a perfectly random classification of observations would have the main diagonal as an ROC curve. We report the ROC curves for the two competing models in Figure 1. It appears from this figure that the DOC option model ROC curve dominates that of Merton-KMV for most of the cut-off points.

The area under the ROC curve summarizes the model's ranking ability, and ranges from 0.5 to 1, 1 being the best achievable value, corresponding to a perfect model. We retain the AUC measure as our primary statistic to compare the two models. Moreover, DeLong, DeLong and Clarke-Pearson (1988) offer a nonparametric test for the difference of AUC for correlated ROC curves. This statistic follows a Chi-square distribution with 1 degree of freedom.

We report the AUC in Table 5, as well as the Chi-square statistic for the difference in the AUC. The DOC option model achieves a higher AUC than the KMV-Merton model, 0.86 versus 0.824. However, the statistical test of no difference between the two AUC is not rejected. Indeed, the Chi-square statistic attains 1.2, corresponding to a *p-value* of 0.273. Thus, we cannot conclude that the DOC option model is superior in explaining default incidence.

We now assess the ability of the two models to forecast defaults in out-of-sample estimations. For the out-of-sample validation, we split the full data set into a training sample, which is used to estimate the coefficient on the structural models' default probabilities in the probit model. These estimates are then used to compute the scores for the remaining unused data (i.e., out-of sample data). The out-of-sample validation allows evaluation of the ability of these scores to predict future defaults. Sobehart, Keenan and Stein (2000), suggest that quantitative models of credit risk should be developed and validated using an out-of-sample, in order to avoid embedding undesirable sample dependency. In Panels B and C of Table 5, we report the out-of sample estimation results. In Panel B, the training sample goes from 1988 to 1995, for a total of 1310 observations containing 23 defaults, while the out-of sample contains 3606 observations, of which 33 are defaults. The probit estimation on the first subsample shows a higher AUC for the Merton-KMV, 0.883 versus 0.845. However, the difference between these two performance measures is not significant at all the usual confidence levels. The opposite figure appears in the out-of-sample validation, the AUC measure in out-of-sample is in favor of the DOC option model. In fact, the Chi-square statistic value is 3.224, which is significant at the 10% confidence level. Thus, it seems that the DOC option model achieves better bankruptcy prediction in out-of-sample.

In Panel C of Table 5, we check for the robustness of the previous result to the choice of the cutoff year for the separation of the training and the out-of-sample data. We perform another out-of-sample test. The training sample expands from 1988 to 1996, for a total of 1567 observations including 26 bankruptcies and reorganizations. The remaining sample contains 3349 firm-year observations, of which 30 are defaults. Here again, while the in-sample estimation shows no statistically significant difference in the AUC between the two models, the out-of-sample validation shows that the area under the curve for DOC option is significantly higher than that of the Merton-KMV. Indeed, the Chi-square statistic, testing for no difference between the areas under the two ROC curves, is 6.907 and rejects the null with a *p-value* below 1%.

Hence, even if the DOC option and Merton-KMV models are equivalently accurate in predicting bankruptcy occurrence in in-sample probit estimation, the former seems to achieve better predictive power in out-of-sample tests.

5. Barrier determinants

5.1 Independent variables

The theoretical financial literature identifies several firm-specific factors that can explain both the decision to default and the output of the reorganization process. These factors can therefore explain the barrier level. Roughly, we can group them in two broad categories: Strategic and non-strategic factors. In the following subsections we discuss these factors and justify the choice of the proxies.

5.1.1 Non-strategic factors

Most of the exogenous default models, including the basic Merton (1974) and Longstaff and Schwartz (1995) models, specify the barrier level as a fraction of the debt. It is therefore natural that our first barrier determinant is the leverage of the firm. We expect a positive relation between the firm's leverage and our implied barrier measure. We measure the leverage as the ratio of total liabilities to book asset value.

In opposition to value-based models, where default depends on the value of the assets, we find cash-based models where the default is assumed to happen whenever the firm's cash flows are insufficient to cover its debt payments.[3] Cash-based models include Ross (2005), Anderson and Sundaresan (1996) and Kim, Ramaswamy, and Sundaresan (1993). These models assume that the firm has no access to external financing, implying that default can occur due to a cash shortage, even if the company has a positive net worth. However, this assumption is restrictive because external financing could be accessible at a given cost, depending on the financial soundness of the firm and its debt capacity. The existence of financing costs raises the issue of liquidity management. Indeed, firms may accumulate a cash cushion to avoid external financing during downturns. Asvanunt, et al. (2007), Acharya et al (2006) and Anderson and Carverhill (2007) account for liquidity management and financing costs. If a cash shortage can cause default, or at least accelerate default occurrence, we could expect an adjustment of the implied barrier level to the cash holding of the firm. More cash in the firm's assets should be associated with a lower barrier to the estimated asset market value. We measure the liquidity as the ratio of cash and equivalents to the book value of assets. The same reasoning applies to external financial constraints, if the firm can contract new debt at low cost in order to avoid cash shortage defaults, its implied default barrier should account for this effect, and we should observe a lower barrier level when the debt costs are low. Hence, we expect a positive relation between the debt cost, measured by the ratio of interest expenses to total liabilities, and the implied barrier.

While credit risk models are scale free, we include the size, as measured by the logarithm of assets, to account for information availability. Indeed, Yu (2005) finds that accounting transparency is associated with lower credit spreads. Because large firms typically have lower information asymmetries than smaller firms, the latter may have higher uncertainty regarding the barrier location. We could reasonably hypothesize that market participants presume a higher barrier level when this uncertainty is greater, therefore we expect a negative relation between

[3] Davydenko (2007) compares the value-based and cash-based models and their different assumptions

the firm's size and the default barrier.[4] Finally, we control for the asset volatility because it is related to firm risk. The firm risk is measured by the estimated asset volatility. We also control for the state of the economy as measured by the growth rate of the real GDP of the Canadian economy.

5.1.2 Strategic factors

Strategic factors are specific to the endogenous default models. They fall into three categories: 1) Costs of liquidation, 2) Relative bargaining power and 3) Renegotiation friction. Betker (1995), Franks and Torous (1994), among others, find that these factors have an effect on the occurrence and the outcome of reorganizations. In addition, models put forth by Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997), Hart and Moore (1998) and Fan and Sundaresan (2000) allow for strategic defaults. In contrast with liquidity default, when the firm defaults due to insufficient cash flows, strategic default happens when equity holders decide to forgo debt payment even if they have the necessary funds. Indeed, if deadweight costs associated with liquidation of the firm's assets are high, it could be beneficial for the debt holders to concede some of their debt in order to allow the firm to survive. Therefore, equity holders may be interested in defaulting opportunistically to benefit from such debt cutback. As the likelihood of strategic default is higher when the liquidation costs and equity holders' bargaining powers are more pronounced, we can anticipate a positive relation between these variables and the level of default barrier. Moreover, Davydenko and Strebulaev (2007) find that the credit spread is positively sensitive to liquidation costs and bargaining power, and negatively related to renegotiation friction.

As a proxy for the liquidation costs we use the percentage of fixed assets. Fixed assets are measured by the total value of capital assets including land, buildings, computers, factories, office equipment, leasehold improvements, and assets under capital leases, net of accumulated depreciation and amortization. Therefore, fixed assets are the physical assets of the firm which

[4] The size of the firm can be seen as a strategic factor. Indeed, large/mature firms are often associated with high bargaining power in debt renegotiation with debt holders, whereas small/young firms are considered weak firms in renegotiation. See for instance Hackbarth et al (2007). Moreover, Houston and James (1996), Johnson (1997), Krishnaswami et. al. (1999) and Denis and Mihov (2003) report evidence that the proportion of public debt in total debt increases with the size and the age of the firm, while Carey and Gordy (2007) observe a higher recovery rate for firms with more bank debt. This could be another way in which size affects the default barrier.

are easiest to sell in case of liquidation. As a result we can expect a negative relation between the proportion of fixed assets and the default barrier. As an additional measure of the asset specificity we use the R&D expenditures to the book value of assets. Indeed the research expenditures could be a good proxy for the asset specificity of the firm. We anticipate a positive relationship between these asset specificity measures and the level of the default barrier. Indeed, more specific assets are generally harder to liquidate in case of bankruptcy and imply higher liquidation costs. To account for the equity holders' bargaining power we use the percentage of votes attached to the voting shares of a company held by the directors and other individuals or companies that own more than 10% of all voting rights. Here we choose to retain the percentage of votes instead of shareholding because we believe that it better reflects the control held by the manager and major equity holders over the firm's assets.

Finally, renegotiation frictions could prevent debt renegotiation, but also reduce recovery rates ex-post. Hart and Moore (1998) and Fan and Sundaresan (2000), argue that renegotiation frictions could prevent strategic defaults, but they also render the liquidation costs harder to avoid in liquidity default, and thus decrease recovery rates. Our proxy for renegotiation friction is the portion of current liabilities relative to total liabilities. Indeed, Berglöf and von Thadden (1994) point out that for financially distressed firms, short-term creditors rarely forgive debt, while concessions often are made by subordinated long-term claim holders. Thus, if the strategic default effect prevails, as higher short-term debt indicates more renegotiation friction, it could prevent strategic defaults and therefore lower the default barrier level. However, when liquidity default risk effect is more pronounced, short-term debt holders may prevent debt renegotiation and force bankruptcy with the associated liquidation costs, which have the potential to increase the ex ante default threshold. Therefore, the overall effect of the renegotiation friction is ambiguous, as measured by the current to total debt ratio on the default barrier location.

However, as short-term liabilities are paid first, they have de facto higher seniority relative to long-term unsecured debt. Davydenko and Strebulaev (2007) observe that a larger proportion of current debt relative to long-term debt increases the liquidity shortage risk, because more cash flows are used for day-to-day debt service. Hence, we conjecture that more short-term debt could push the default barrier upward. We use the ratio of short-term liabilities to total liabilities as a proxy of renegotiation frictions.

Because the short term debt proxy for renegotiation friction is contaminated by liquidity default risk, we also use the proportion of the outstanding public debt to the book value of total debt as an alternative renegotiation friction proxy. In fact, Davydenko and Strebulaev (2007) find a negative relationship between credit spread and the proportion of public debt. Therefore, the public debt seems to have the potential to deter strategic default of equity holders, because it is more difficult for firms with multiple dispersed creditors to renegotiate their debt, as argued by Hege and Mella-Barral (2004), Gertner and Scharfstein (1991) and Berglöf and von Thadden (1994), among others. Moreover, Carey and Gordy (2007) allege that private debt holders (banks) endogenously set the asset value threshold below which firms declare bankruptcy, and find evidence of a strongly increasing recovery rate in the share of private debt. This could be another possible explanation for the negative relation between the proportion of public debt and the default barrier.

In order to compute the outstanding amount of public debt for companies in our sample, we start by scanning the new Canadian bond issues lists of FISD and the SDC Platinum databases to compile a list of companies in our dataset that are active on the bond market. For each identified issuer, we manually collect information on outstanding public debt for each fiscal year from the long-term debt section of printed Moody's/Mergent international manuals. Moreover, because the FISD and SDC Platinium databases are not exhaustive for Canadian issuers, we also look for the remaining Canadian firms in the Moody's/Mergent international manuals to check if they have public debt in their capital structure. We end up with 104 unique bond issuers out of 575 single firms in our dataset, for a total of 867 firm-year observations between 1988 and 2004. The remaining firms are assumed to have only private debt in their capital structure.

5.2 Regression analysis results

5.2.1 Descriptive statistics

Our dependent variable is the ratio of the implied barrier estimated from market price to the estimated asset value. Here we choose to standardize the implied barrier by the estimated asset value instead of the book value of debt or the book value of assets because these accounting measures can diverge substantially from their corresponding market values. We believe that the estimated market prices give a better measure of the value of the assets under management.

For our regression analysis we drop observations with nil implied barrier estimates, which reduce our initial sample of 4916 to 3609 firm-year observations.[5] After dropping observations with insufficient data on independent variables we end up with 3232 firm-year observations for 575 single firms, covering 17 years from 1988 to 2004. The average number of years by firm is hence 5.6 years.

<Insert Table 6 here >

Our final sample of 127 firm-year observations comes from our default database, and includes 50 observations of defaults or reorganizations. The descriptive statistics for both dependent and independent variables are reported in Table 6. The average default barrier attains 40.1% of the estimated asset value, while the average leverage is 48.2% of the book value of assets. Thus on average the ratio of barrier to estimated assets is below the leverage ratio in our sample. For regression analysis we discuss in the following subsections the regression results for strategic and non-strategic factors.

5.2.2 Non-strategic factors

The regression analysis results of the implied barrier on the non-strategic factors are presented in Table 7. The objective here is to test whether the implied default barrier estimated from equity price, viewed as a down-and-out call option, is adjusted by market participants to account for the possibility of cash shortages and impossibility of contracting new debts.

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[5] We also performed the regression analysis on the whole sample; the results found are similar to the restricted sample.

<Insert Table 7 here>

Regression (1) in Table 7 shows the result of regressions of the default barrier on the non-strategic factors. As expected, the leverage ratio is positively associated with the implied default barrier. The coefficients on the leverage ratio are positive and significant at the 1% level in all the regressions. The question that we seek to answer in this study is whether indebtedness is the only driver of the ex-ante perceived default barrier. Our regression results demonstrate clearly that this is not the case. The liquidity measure is negatively related to the implied barrier. These coefficients are also highly significant in all regressions. Thus, the implied default barrier associated with firms with more cash holdings accounts for the fact that they may eventually default at lower asset value, because they can handle debt payments and avoid default due to liquidity constraints. This result supports the underlying assumptions of cash-based models, such as Ross (2005), Anderson and Sundaresan (1996) and Kim, Ramaswamy, and Sundaresan (1993). The debt cost also has a significant positive impact on the location of the default threshold. Financing frictions seem to be a major determinant of the value at which the firm is expected to default. A firm's higher ability to contract new debt at a low cost decreases its default threshold. This result, in combination with the liquidity concern results, shows that credit risk models with endogenous cash management in the presence of external financing costs seem to better describe the reality of the firm, despite their complexity.

The negative relation of the ex-ante default barrier with volatility is concordant with endogenous barrier models like that of Leland and Toft (1996). Higher volatility makes the option to wait more valuable, and decreases the level of default barrier. The size of the firm has the expected sign. Larger firms benefit from the lower perceived ex ante default barrier level, probably for informational reasons. Large firms have greater visibility and are followed more closely by analysts. This helps reduce the uncertainty regarding the asset level below which firms default. Consequently, the default barrier for large firms is lower than that of smaller companies. Finally, the GDP growth rate is positively and significantly related to the default barrier. This result may seem surprising, because one could expect lower barrier levels in economic expansion. However, a possible explanation lies in the expectations of investors regarding future economic conditions. If firms set the default barrier in accordance with these expectations instead of with actual economic conditions, and given the cyclical aspect of the real GDP growth rate, a positive relationship between GDP growth and barrier level may result.

To check the robustness of our results, we perform a panel regression with random and fixed effect using the same specifications as in regressions (2) and (3) of Table 7. Our results are robust to the inclusion of both firm-specific constants and random error terms. Moreover, the R square of the model achieves 29.9% for the random effect regression and 26.8% with fixed effect panel regression.

We also test whether our results for the liquidity and debt cost are driven by the presence of outliers in our data. To detect the presence of such outliers we applied the method of Hadi (1992, 1994) for outlier detection in multivariate data to these variables. This leads to the exclusion of 62 observations. Columns (4) and (5) in Table 7 report the random and fixed effect panel regressions results for the remaining observations. Here again our results are not altered either in terms of the coefficient estimates sign or their statistical significance. The debt cost becomes even significant at the 1% level in the fixed effect panel regression.

Moreover, a closer look at the liquidity variable shows that 78 observations have a ratio of cash and equivalents to total assets above 50%. Among them, only two observations come from our defaulted firms' data set. These observations may represent firms in asset liquidation process. To ensure that our results are not driven by observations where the assets are liquidated or reorganized, we drop them from the sample in regression (6), and find that our results are not driven by observations where firms undertake asset liquidation. As an additional check of the liquidity effect on the barrier level, we try another measure of asset liquidity, namely the current ratio. The current ratio is measured as current assets to current liabilities, and is a proxy for the ability of the firm to meet its short-term obligations with its short-term assets. In unreported results, this alternative measure of liquidity also has a negative and significant coefficient estimate, both in random and fixed effect panel regressions.

On the debt cost side, we observe that a fairly high proportion (around 14%) of firms in our sample do not use long-term debt, and rely solely on accounts payable to suppliers. Because suppliers generally allow a 90–day grace period for payment, these firms have zero or low interest expenses. We test whether the positive relation between debt cost and the location of the default barrier are driven by these observations. Regression (7) in Table 7 shows the contrary. The coefficient on the debt cost variable is higher for the remaining sample and is significant at all the usual confidence levels.

The overall results show the importance of liquidity shortages and costs of external finance as drivers of the default barrier level location. It seems clear from the regression analysis that market participants adjust, through equity prices, the level of assets at which the firm is expected to default for liquidity shortage concerns and for the difficulty to raise new debt financing. This result holds, even after controlling for the leverage, asset volatility, firm size and economic conditions, and is robust to the presence of potential outliers.

5.2.3 Strategic factors

We now turn to the results of the regression analysis of the implied default barrier on strategic factors. We report these results in Table 8. All of the regressions include the non-strategic factors. We do not report them in Table 8 for the sake of brevity. All the non-strategic factors are significant and have the same signs as in Table 7. It should be noted here that regressions are restricted to observations where we were able to find data on the voting rights of the directors and major shareholders. The final sample contains 3085 observations for 509 single firms. Our estimates in Table 7 are robust to the exclusion of firms without data on voting rights.

Liquidation costs are an important strategic factor in endogenous default models. Firms' creditors should be more willing to forgive a part of their debt when the asset values for going concerns are much higher than their liquidation value, and when the liquidation costs are high. This gives equity holders more incentives for strategic default in order to benefit from these debt concessions. If equity market participants are aware of such strategic default effects, the implied barrier should increase with default costs. Regression (1) of Table 8 supports this hypothesis. As expected, the coefficient on the fixed effect is negative and significant at the 1% confidence level in regression (1) of Table 8. This liquidation cost effect is also supported by Davydenko and Strebulaev (2007), who find that the proportion of non-fixed assets is positively correlated with credit spread.

<Insert Table 8 here>

As an alternative liquidation cost proxy, we use the ratio of research and development expenses to asset book value. Regression (3) of Table 8 shows, however. that R&D is not relevant and does not have the expected sign, with a *t-statistic* of –0.33. We obtain the same figure after including random effects to account for the panel pattern in our data. Given that a large number of firms in our data set do not undertake R&D programs, in unreported results we used a dummy variable set to 1 if the R&D expenses are non-nil, and 0 otherwise. We found that the R&D dummy is positive as expected but it is not significant, with a *t-statistic* of 1.44. It seems from these results that the proportion of fixed assets better approximate liquidation costs in our data.

Regarding the renegotiation friction, we use two alternative proxies; the short-term debt to total debt ratio and the public debt to total debt ratio. Regressions (5) to (6) show that the short term debt coefficient is positive and statistically significant at all the usual confidence levels. This supports the higher liquidity default risk in the presence of more short-term debt rather than the strategic default explanation. Indeed, the effect of the liquidity risk seems to dominate the strategic default effect on the ex ante default barrier in our data. Further, short-term debt is a noisy measure of renegotiation friction because it is related to liquidity default risk.

In order to better isolate the effect of renegotiation friction on the estimated default barrier, we use the public debt variable in specifications (1) to (4) of Table 8. In all four regressions, the estimated default barrier decreases significantly with the proportion of public debt in the firm's capital structure. The coefficient on the renegotiation friction proxy is negative and significant at the 1% level in these regressions. This result underlines the role of renegotiation frictions in discouraging potential opportunistic shareholders from defaulting despite the liquidation costs that renegotiation could avoid.

Finally, our proxy for the bargaining power of CEOs and major shareholders is positively and significantly related to the implied default barrier. This result indicates that more shareholder bargaining power implies a higher default barrier. This is consistent with strategic default effect: higher bargaining power of equity holders encourages the occurrence of strategic defaults, as equity holders could gain more in renegotiation. Moreover, once the firm defaults, higher shareholder bargaining power implies greater deviation from the absolute priority rule. This result is consistent with Betker (1995), who finds that deviations from APR in chapter 11 increase sharply with CEO shareholding. In addition, Davydenko and Strebulaev (2007) find that CEO shareholding increases the credit spread.

The overall regression results lend strong support to the strategic default effect on the implied default threshold. This evidence is in line with endogenous default models, in the spirit of Leland (1994) and Fan and Sundaresan (2000), for instance, where the shareholders deliberately choose to default in order to benefit from debt cutback.

6. Robustness tests

It is intuitive that more leverage implies a higher implied default barrier level, and we find evidence of this relationship in the non-strategic factor regressions. However, one could argue that this level of default threshold could also influence borrowing decisions. Therefore, we can reasonably suspect a simultaneous relationship between default barrier and indebtedness, where the optimal leverage ratio would be the result of equilibrium.

To be able to account for the endogenous relationship linking leverage and implied default barrier, we need to specify the rest of the leverage determinants. The financial literature identifies several determinants of capital structure choice. Definitely, the first motive for companies to contract debt is to benefit from the debt tax shield. We therefore include the actual tax rate, defined as the ratio of the tax payment to the earnings before tax as an explanatory variable for the debt equation. As firms also benefit from a non-debt tax shield, we add the depreciation and amortization scaled by the book value of total assets as an additional independent variable. We expect a positive sign for both *tax rate* and *depreciation and amortization* variables.

Moreover, Titman and Wessels (1988) argue that the firm's collateral value increases its ability to contract more debt. Our proxy for collateral value is the *book-to-market* ratio. Indeed, we conjecture that value companies, those with high book-to-market ratio, have more productive assets in place than do low book-to-market firms, whose value is primarily driven by less pledgeable growth options. Thus, we expect a positive sign for the coefficient on the book-to-market ratio. In the same spirit, we added the ratio of *R&D* expenses to the book value of assets, to account for the fact that firms operating in technology intensive sectors have more specific assets. Because these kinds of assets are less valuable in case of liquidation, we could expect a negative relation with leverage. The *R&D* expenses can be interpreted as a measure of the firm's uniqueness. As an additional proxy for firm uniqueness we use selling, general and administrative expenses scaled by net sales; the same logic makes us anticipate a negative sign. Finally, the profits generated by the firm's operations decrease its need for external financing and should be associated with less debt. Our measure for profitability is the EBITDA divided by net sales; here again we expect a negative relation with leverage.

<Insert Table 9 here>

We report the results of the three-stage least-square estimation in Table 9, the dependent variables being the estimated default barrier to estimated asset value and the leverage ratio (book value of debt to book value of assets). We notice that all the independent variables in the leverage equation have the expected signs and are significant at the 1% confidence level. Moreover, in regard to the default barrier equation, the previous results hold for both strategic and non-strategic factors. The endogeneity between the leverage and default barrier does not bias our regression results.

Furthermore, in the DOC framework, we have to assume a lifespan of the firms. In other words, to model the firm's equity as a DOC option, we have to set the option maturity which represent the lifetime of the company. In the previous analysis, we assumed a lifespan of 20 years. To ensure that our findings are not affected by the choice of this input parameter, we estimated the default barrier assuming lifespans of 5 and 10 years. The results in Table 10 show that the average estimated barrier is not particularly sensitive to the choice of the firm's lifespan assumption. Indeed, changing the option maturity from 20 to 5 years moves the mean of estimated barrier from 0.29 to 0.26. Thus, the economic importance of the default barrier estimates is robust to the alternative maturity choice. Moreover, the correlation of the default barriers estimated with 20 years maturity and those estimated with 10 years maturity attains 0.99, whereas the correlation between the 20 years and the 5 years default barriers achieve 0.96. Finally, the estimates of the implied barrier on the non strategic and strategic factors keep the same signs and significance when the assumed firm's lifespans are changed. We can therefore conclude that our findings are robust to the choice of the maturity parameter.

7. Conclusion

In the structural models of credit risk, default is often assumed to happen when the market value of assets falls below a given barrier. The financial theory stipulates different assumptions regarding the default barrier, ranging from Merton (1974), whose threshold is simply the debt value at maturity, to more sophisticated settings where the default barrier is determined endogenously by stakeholders, as in Leland and Toft (1996). However, due to the unobservability of the firm's asset value, these assumptions were not directly tested, but rather the overall model performance is assessed in predicting either defaults or credit spreads. In this paper, we use the maximum likelihood estimation method of Duan (1994) in order to infer the implied default barrier of the DOC option (2003) model from equity prices. We use a sample of public Canadian firms to compare the KMV-Merton model with that of DOC option model in terms of default prediction accuracy. We find that the KMV-Merton and the DOC option models perform equally well for in-sample fitting. However, the DOC option default probability estimate attains higher accuracy in out-of-sample default forecasting. Moreover, the implied barrier for defaulting firms is close to their estimated asset market value at default. We also use regression for the implied default barrier against firm-specific and macro-economic factors, and find that not only does the capital structure influence the default barrier location, but firm-specific factors also do. Further, the ex ante implied default barrier is adjusted to both non-strategic factors and strategic factors. It is sensitive to asset liquidity, debt cost and liquidation costs, renegotiation friction and equity holders' bargaining power. Thus, it seems that the market adjusts the implied default threshold, through equity prices, to account for firm-specific determinants that go beyond the level of debt. Our results give new insights for modeling the decision to default and for default predictions.

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Figure 1: Comparison of ROC curves of DOC option and Merton-KMV models

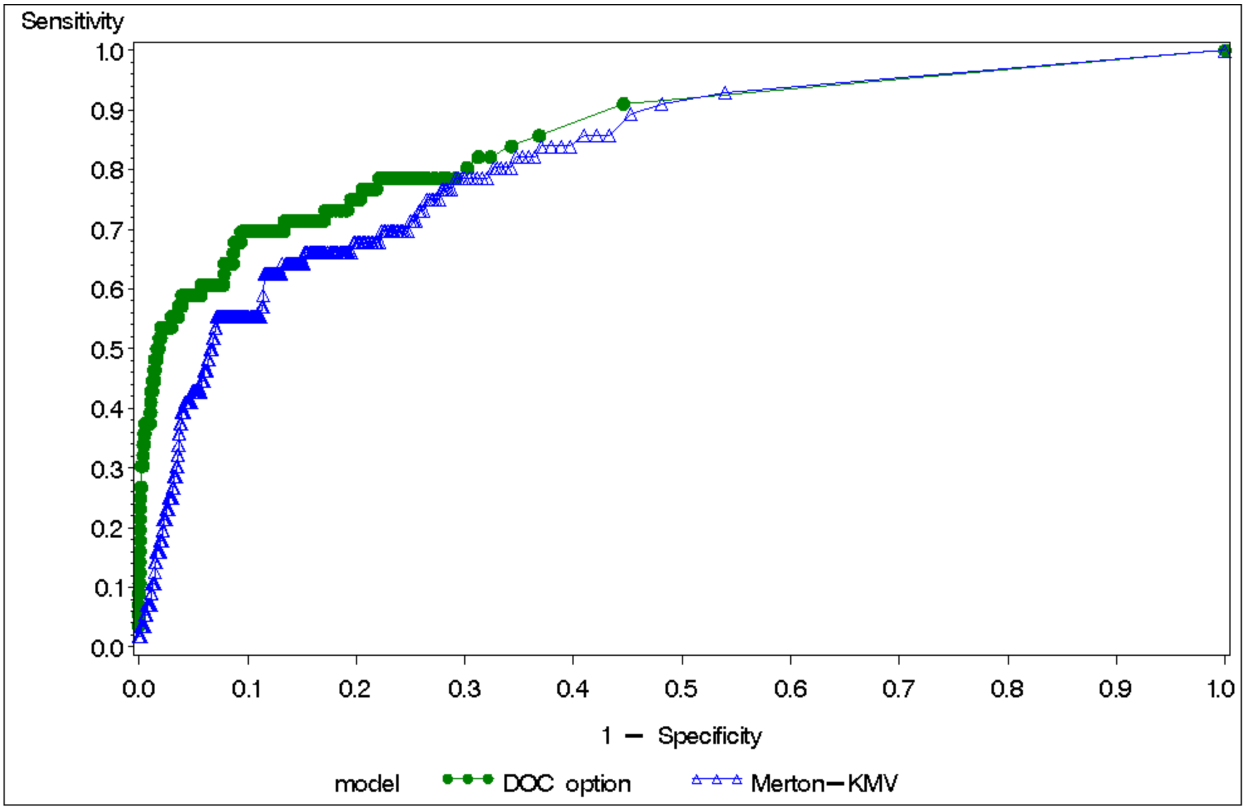

Table 1: Monte Carlo study of the MLE estimation

The true values used in the Monte Carlo simulation for μ, σ and α are respectively 0.1, 0.3 and 0.833. Mean, Median, Standard-deviation, Min and Max are the sample statistics of the estimates from the 1,000 simulations. The values used in the simulation are as follows: V_0=10,000, F=6,000, α =0.833 (H=5000), r = 0.05, Δt =1/250 and N =200 is the number of daily observations. We use The Wong and Choi (2004) maximum likelihood function as given in equation (4).

	$\hat{\mu}$ True=0.1	$\hat{\sigma}$ True=0.3	$\hat{\alpha}$ True=0.833
Mean	0.103	0.301	0.824
Median	0.095	0.296	0.849
Std	0.321	0.068	0.350
Min	-0.866	0.109	0
Max	1.144	0.548	1.840
t-stat	0.34	0.42	-0.86
p-value	0.73	0.68	0.39

Table 2: Estimated barrier, asset drift and volatility

This table presents the MLE estimates of the default barrier. The results are presented for firm-year observations. Panel A presents the leverage ratio for the sample, the ratio is the book value of total liabilities divided by the book value of total assets. Panel B presents the barrier as a fraction of the estimated asset market value. Panels C and D presents the estimated asset drift and volatility respectively with the DOC option model. Panels E and F presents average implied barrier to assets by year and debt load respectively.

	N.	Mean	Std	Min	Q1	Median	Q3	Max
Panel A: Leverage ratio (total liabilities/ total assets)								
Overall sample	4916	0.54	0.70	0	0.30	0.49	0.66	19.5
Non-defaulted firms	4860	0.53	0.69	0	0.30	0.49	0.66	19.5
Defaulted firms	56	1.12	1.16	0.26	0.62	0.85	1.23	8.36
Panel B: Barrier to implied asset value								
Overall sample	4916	0.29	0.27	0	0	0.25	0.5	0.99
Non-defaulted firms	4860	0.29	0.27	0	0	0.25	0.5	0.99
Defaulted firms	56	0.65	0.31	0	0.47	0.76	0.91	0.99
Panel C: Estimated asset drift								
Overall sample	4916	0.22	0.45	-4.90	-0.01	0.12	0.35	8.2
Non-Defaulted firms	4860	0.22	0.44	-0.57	-0.01	0.12	0.36	8.2
Defaulted firms	56	-0.25	0.85	-4.90	-0.56	-0.09	0.13	1.1
Panel D: Estimated asset volatility								
Overall sample	4916	0.53	0.40	0.00	0.24	0.41	0.70	3.98
Non-defaulted firms	4860	0.53	0.40	0.00	0.24	0.41	0.70	3.98
Defaulted firms	56	0.57	0.36	0.05	0.26	0.58	0.82	1.77

Table 2 (continued)

Year	# Obs.	Average	Std	Student t-statistic	t-statistic p value
Panel E: Average implied barrier/ MVA estimate by year					
1988	100	0.38	0.27	14.07	0.001
1989	113	0.29	0.26	11.95	0.001
1990	138	0.42	0.29	16.84	0.001
1991	155	0.35	0.27	16.38	0.001
1992	155	0.33	0.26	15.96	0.001
1993	173	0.21	0.24	11.83	0.001
1994	190	0.23	0.23	13.60	0.001
1995	230	0.30	0.26	17.04	0.001
1996	257	0.19	0.22	14.03	0.001
1997	304	0.26	0.29	15.69	0.001
1998	370	0.43	0.31	25.99	0.001
1999	420	0.36	0.28	26.16	0.001
2000	443	0.31	0.29	22.74	0.001
2001	497	0.25	0.23	23.88	0.001
2002	534	0.28	0.25	25.54	0.001
2003	567	0.25	0.24	24.18	0.001
2004	270	0.21	0.24	14.38	0.001
Panel F: Average barrier/implied MVA estimate by debt load					
Debt proportion ≤ 0.1	334	0.21	0.24	22.08	0.001
$0.1 <$ Debt proportion ≤ 0.2	399	0.24	0.25	19.17	0.001
$0.2 <$ Debt proportion ≤ 0.3	472	0.28	0.28	21.72	0.001
$0.3 <$ Debt proportion ≤ 0.4	611	0.28	0.26	26.62	0.001
$0.4 <$ Debt proportion ≤ 0.5	691	0.32	0.27	31.15	0.001
$0.5 <$ Debt proportion ≤ 0.6	719	0.31	0.27	30.79	0.001
$0.6 <$ Debt proportion ≤ 0.7	743	0.31	0.27	31.29	0.001
$0.7 <$ Debt proportion ≤ 0.8	406	0.31	0.27	23.13	0.001
$0.8 <$ Debt proportion ≤ 0.9	199	0.34	0.29	16.53	0.001
Debt proportion > 0.9	342	0.33	0.31	19.68	0.001

Table 3: Comparison of default probabilities between the Merton-KMV and the DOC option models

This table presents the default probabilities estimate obtained from the Merton-KMV model and the DOC option model for a 1-year horizon. The probabilities presented here are real probabilities and not risk neutral.

	# Obs.	Mean	Std	Min	Q1	Median	Q3	Max
Panel A: DOC option-Wong and Choi								
Overall sample	4916	10.88%	21.79%	0%	0%	0%	8.36%	100%
Non-Defaulted firms	4860	10.30%	20.82%	0%	0%	0%	7.77%	99.5%
Defaulted firms	56	61.05%	39.49%	0%	16%	81.37%	96.93%	100%
Panel B: Merton-KMV								
Overall sample	4916	13.66%	22.36%	0%	0%	0.6%	19.7%	99.9%
Non-Defaulted firms	4860	13.26%	22.36%	0%	0%	0.53%	19.15%	99.5%
Defaulted firms	56	48.51%	32.01%	0%	15.96%	58%	74.06%	99.99%

Table 4: Default probabilities by decile

Performance of the DOC option and Merton-KMV models in predicting bankruptcy probabilities within one year. We report the number and percentage of defaults after classifying observations into deciles based on the estimated default probabilities. The 10th decile is the largest one and the 1st decile is the smallest. The DOC option and Merton-KMV models are estimated using the Maximum Likelihood method. We use the Wong and Choi maximum likelihood function to estimate the DOC option model's parameters.

	DOC option model						Merton-KMV model					
Decile	N	Defaults	Cumulative sum	Non-defaults	Cumulative sum	Average DP	N	Defaults	Cumulative sum	Non-defaults	Cumulative sum	Average DP
10	491	38	38	453	453	68.06%	491	31	31	460	446	68.34%
		68%	68%	9%	9%			55%	55%	9%	9%	
9	492	4	42	488	941	29.99%	492	6	37	486	918	38.64%
		7%	75%	10%	19%			11%	66%	10%	19%	
8	492	2	44	490	1431	9.18%	492	7	44	485	1388	20.10%
		4%	79%	10%	29%			13%	79%	10%	29%	
7	491	5	49	486	1917	1.57%	491	3	47	488	1863	7.57%
		9%	88%	10%	39%			5%	84%	10%	39%	
6	492	2	51	490	2407	0.09%	492	5	52	487	2337	1.80%
		4%	91%	10%	50%			9%	93%	10%	50%	
5	492	1	52	491	2898	0.00%	492	2	54	490	2812	0.23%
		2%	93%	10%	60%			4%	96%	10%	60%	
4	491	1	53	490	3388	0.00%	491	0	54	491	3289	0.01%
		2%	95%	10%	70%			0%	96%	10%	70%	
3	492	2	55	490	3878	0.00%	492	0	54	492	3766	0.00%
		4%	98%	10%	80%			0%	96%	10%	80%	
2	412	0	55	412	4290	0.00%	492	0	54	492	4244	0.00%
		0%	98%	8%	88%			0%	96%	10%	90%	
1	571	1	56	570	4860	0.00%	491	2	56	489	4720	0.00%
		2%	100%	12%	100%			4%	100%	10%	100%	
Total	4916	56	56	4860	4860	10.88%	4916	56	56	4860	4860	13.66%

Table 5: Comparative performance in predicting bankruptcy

This table reports the comparative performance in predicting bankruptcy one year ahead. The estimated default probability from structural models is used as a regressor in probit regression including an intercept. In sample estimation we use the full data set to estimate probabilities, while out-of-sample uses the coefficient estimated from the in-sample estimation to evaluate the predictive performance of the resulting probabilities on the remaining unused out-of-sample data. For each probit regression considered, we report the maximum rescaled R^2 of Nagelkerke (1991), and the percentage of bankruptcy concordant with the model, and the area under the receiver operating characteristic curve (AUC) measures of the ability of the model to correctly rank observations. In the last column we add the nonparametric test of difference between areas under correlated ROC curves of Delong et al (1988).

Model	DOC option	KMV-Merton	$\chi^2_{(1)}$ Statistic (*p-value*)
Panel A: In-sample estimation: estimation period 1988-2004			
# obs.	4916	4916	
# Defaults	56	56	
Default probability	1.99	1.65	
	(0.001)	(0.001)	
Max Rescaled R square	0.256	0.152	
Percent concordant	75.4	75.2	
Area under ROC	0.860	0.824	1.200
			(0.273)
Panel B: Out-of-sample validation; estimation period 1988-1995; evaluation period 1996-2004			
In sample			
# obs.	1310	1310	
# Defaults	23	23	
Max Rescaled R square	0.292	0.22	
Percent concordant	71.7	86.8	
Area under ROC	0.845	0.883	0.614
			(0.433)
Out-of-sample			
# obs.	3606	3606	
# Defaults	33	33	
Max Rescaled R square	0.283	0.107	
Percent concordant	73.5	62.2	
Area under ROC	0.871	0.787	3.224
			(0.072)
Panel C: Out-of-sample validation; estimation period 1988-1996; evaluation period 1997-2004			
In sample			
# obs.	1567	1567	
# Defaults	26	26	
Max Rescaled R square	0.303	0.23	
Percent concordant	71.7	83.4	
Area under ROC	0.837	0.884	0.722
			(0.395)
Out-of-sample			
# obs.	3349	3349	
# Defaults	30	30	
Max Rescaled R square	0.274	0.095	
Percent concordant	74.1	61.4	
Area under ROC	0.883	0.780	6.907
			(0.009)

Table 6: Descriptive statistics of dependent and independent variables

Default barrier is the implied default barrier divided by the estimated asset value *Leverage* is the book value of total liabilities divided by the book value of total assets. *Liquidity* is the cash and cash equivalents divided by the book value of total assets. *Debt Cost* is the ratio of interest expenses to the book value of total liabilities. *Asset volatility* is the estimated asset volatility from the DOC option model. *Size* is the logarithm of the total book assets in millions of dollars. *GDP Growth* is the real GDP growth rate. *Fixed assets* is the total value of capital assets including land, buildings, computers, factories, office equipment, leasehold improvements and assets under capital leases, net of accumulated depreciation and amortization to the book value of total assets. *R&D* is the ratio of research and development expenses to total assets. *Short term debt* is the ratio of current liabilities to total liabilities. *Voting* is the percentage of votes attached to the voting shares of a company held by the directors and other individuals or companies that own more than 10% of all voting rights. *Public Debt* is the ratio of the amount of outstanding public debt to the book value of total debt.

Variable	N	Mean	Median	Std. Dev.	Minimum	Maximum
Default barrier	3232	0.401	0.383	0.242	0	0.992
Leverage	3232	0.482	0.488	0.293	0.002	8.361
Liquidity	3232	0.093	0.034	0.137	0	0.988
Debt Cost	3232	0.034	0.031	0.031	0	0.284
Volatility	3232	0.458	0.359	0.334	0.007	3.983
Size	3232	11.67	11.38	2.18	4.63	17.54
GDP Growth	17	0.031	0.031	0.018	-0.021	0.055
Fixed Assets	3232	0.426	0.401	0.278	0	1
R&D	3232	0.029	0.000	0.09	0	1.855
Short-term debt	3232	0.565	0.553	0.284	0	1
Voting	3085	0.304	0.242	0.257	0	0.961
Public Debt	3232	0.15	0	0.311	0	1

Table 7: Regression analysis of the implied default barrier on non-strategic factors

This table reports the results of regression analysis of the implied default barrier on non-strategic variables. The dependent variable is the implied default barrier divided by the estimated asset value. The sample consists of all firm-year observations with non-nil estimated barriers. *Leverage* is the book value of total liabilities divided by the book value of total assets. *Asset volatility* is the estimated asset volatility from the DOC option model. *Liquidity* is the cash and cash equivalents divided by the book value of total assets. *Debt Cost* is the ratio of interest expenses to the book value of total liabilities. *Size* is the logarithm of the total book assets in millions of dollars. *GDP growth* is the annual growth rate of the real GDP. Values of *t-statistics* are reported in parentheses. Coefficients marked ***, ** and * are significant at the 1%, 5% and 10% significance levels, respectively. *Hausman test* is the Hausman test for random effect following a chi square distribution with *k* degrees of freedom, where *k* is the number of independent variables, the corresponding p value is reported below.

	All			Without outliers			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Leverage	0.106***	0.103***	0.07***	0.102***	0.069***	0.069***	0.08***
	(8.16)	(6.84)	(3.71)	(6.69)	(3.50)	(3.49)	(3.91)
Liquidity	-0.112***	-0.142***	-0.188***	-0.135***	-0.202***	-0.242***	-0.189***
	(-3.97)	(-4.47)	(-5.16)	(-3.66)	(-4.69)	(-5.03)	(-3.76)
Debt Cost	0.277**	0.237*	0.3**	0.399**	0.472***	0.316**	0.444***
	(2.26)	(1.81)	(2.06)	(2.48)	(2.57)	(2.12)	(2.75)
Volatility	-0.438***	-0.522***	-0.611***	-0.522***	-0.613***	-0.612***	-0.595***
	(-34.88)	(-37.98)	(-38.94)	(-37.64)	(-38.41)	(-38.47)	(-35.44)
Size	-0.044***	-0.05***	-0.036***	-0.051***	-0.036***	-0.036***	-0.032***
	(-22.45)	(-17.16)	(-6.71)	(-17.24)	(-6.49)	(-6.43)	(-5.20)
GDP Growth	1.44***	1.38***	1.3***	1.466***	1.38***	1.36***	1.25***
	(7.06)	(7.36)	(6.75)	(7.34)	(7.10)	(6.99)	(6.01)
Const.	1.02***	1.15***	1.04***	1.14***	1.03***	1.04***	0.967***
	(35.43)	(29.59)	(15.76)	(29.37)	(15.09)	(15.07)	(12.48)
Fixed effect	No	No	Yes	No	Yes	Yes	Yes
Random effect	No	Yes	No	Yes	No	No	No
R^2	0.299	0.299	0.268	0.298	0.269	0.269	0.257
Observations	3232	3232	3232	3170	3170	3154	2716
Single firms	575	575	575	574	574	571	508
Hausman test	-	-	147.3	-	141.91	144.74	106.74
p-value	-	-	0.00	-	0.00	0.00	0.00

Table 8: Regression analysis of the implied default barrier on strategic factors

This table reports the results of regression analysis of implied default barrier on strategic variables. The dependent variable is the implied default barrier divided by the estimated asset value. The sample consists of all firm-year observations with non-nil estimated barriers. *Fixed assets* is the total value of capital assets including land, buildings, computers, factories, office equipment, leasehold improvements and assets under capital leases, net of accumulated depreciation and amortization to the book value of total assets. The *R&D* is the research and development expenses to total assets. *Short term debt* is the ratio of current liabilities to total liabilities. *Voting*, is the percentage of votes attached to the voting shares of a company held by the directors and other individuals or companies that own more than 10% of all voting rights. *Public Debt* is the ratio of the amount of outstanding public debt to the book value of total debt. *Leverage, Liquidity, Debt cost, Volatility, Size, GDP growth,* and the constant are included in all specifications. Values of *t-statistics* and *z-statistic* are reported in parentheses. Coefficients marked ***, ** and * are significant at the 1%, 5% and 10% significance level, respectively.

		(1)	(2)	(3)	(4)	(5)	(6)
Liquidation costs	*Fixed assets*	-0.044***	-0.035*				
		(-3.09)	(-1.91)				
	R&D			-0.137	0.001	-0.053	0.001
				(-0.33)	(0.02)	(-1.27)	(0.02)
Renegotiation frictions	*Short term debt*					0.088***	.082***
						(6.16)	(4.95)
	Public debt	-0.048***	-0.064***	-0.057***	-0.068***		
		(-3.44)	(-3.58)	(-4.10)	(-3.84)		
Bargaining power	*Voting*	0.079***	0.08***	0.086***	0.086***	0.076***	0.079***
		(7.05)	(3.86)	(6.00)	(4.15)	(5.22)	(3.78)
Random effect		No	Yes	No	Yes	No	Yes
R^2		0.30	0.30	0.31	0.33	0.30	0.30
Observations		3085	3085	3085	3085	3085	3085
Single firms		509	509	509	509	509	509

Table 9: Three stage least square estimation of barrier and leverage equations

This table reports the results of the three stage least square regressions for panel data with fixed assets where the endogenous variables are the implied default barrier divided by the estimated asset value and the leverage ratio defined as the book value of total liabilities divided by the book value of total assets. The sample covers all firm-year observations with non-nil estimated barriers and sufficient data. *Asset volatility* is the estimated asset volatility from the DOC option model. *Liquidity* is the cash and cash equivalent divided by the book value of total assets. *Debt Cost* is the interest expenses to the book value of total liabilities. *GDP growth* is the annual growth rate of the real GDP. *R&D* is the research and development expenses scaled by the book value of assets. Profitability is the EBITDA to net sales ratio. *Tax rate* is the tax payment of the year divided by the earnings before tax. *Dep & Amt* is the depreciation and amortization scaled by the book value of total assets at the end of the year. *Selling & Adm* is the selling and corporate expenses divided by the net sales. Values of *z-statistics* are reported in parentheses. Coefficients marked ***, ** and * are significant at the 1%, 5% and 10% significance level, respectively.

	Barrier equation	Leverage equation
Constant	0.426***	0.250***
	(8.86)	(17.59)
Default barrier		0.26***
		(8.86)
Leverage	0.191**	
	(2.53)	
Liquidity	-0.187***	
	(-4.35)	
Debt Cost	0.823**	
	(5.21)	
Volatility	-0.334***	
	(-27.01)	
GDP Growth	1.252***	
	(6.51)	
R&D	0.132***	-0.235***
	(2.86)	(-4.99)
Fixed assets	-0.075***	
	(-4.94)	
Public debt	-0.06***	
	(-4.03)	
Voting	0.07***	
	(4.80)	
Profitability		-0.22**
		(-2.18)
Book-to-Market		0.028***
		(16.42)
Tax rate		0.099***
		(5.37)
Dep & Amt		1.158***
		(10.01)
Selling & Adm		-0.062***
		(-2.50)
N	3085	3085
R^2	0.21	0.08
Chi2 stat	1160.33***	549.75***

Table 10: Robustness results for implied average barriers

	N.	Mean	Std	Min	Q1	Median	Q3	Max
Panel A: Implied default barrier to estimated asset value for various option lives								
5 years	4916	0.26	0.26	0	0	0.19	0.42	0.99
10 years	4916	0.27	0.26	0	0	0.22	0.46	0.99
20 years	4916	0.29	0.27	0	0	0.25	0.5	0.99