CIRRELT

Centre interuniversitaire de recherche sur les réseaux d'entreprise, la logistique et le transport

Interuniversity Research Centre on Enterprise Networks, Logistics and Transportation

A Methodology to Form Product Families through Fuzzy Product Configuration

Marco Barajas
Bruno Agard

July 2009

CIRRELT-2009-30

Bureaux de Montréal :
Université de Montréal
C.P. 6128, succ. Centre-ville
Montréal (Québec)
Canada H3C 3J7
Téléphone : 514 343-7575
Télécopie : 514 343-7121

Bureaux de Québec :
Université Laval
Pavillon Palasis-Prince, local 2642
Québec (Québec)
Canada G1K 7P4
Téléphone : 418 656-2073
Télécopie : 418 656-2624

www.cirrelt.ca

A Methodology to Form Product Families through Fuzzy Product Configuration

Marco Barajas[1], Bruno Agard [1,*]

[1] Interuniversity Research Centre on Enterprise Networks, Logistics and Transportation (CIRRELT) and Department of Mathematics and Industrial Engineering, École Polytechnique de Montréal, P.O. Box 6079, Station Centre-ville, Montréal, Canada H3C 3A7

Abstract. More and more companies are designing product families with the aim of making mass customization a reality, offering a wider variety of products while at the same time reducing product cost by standardizing components and processes. This paper proposes a global methodology to form product families taking advantage of fuzzy product configuration. In this methodology, fuzzy logic is considered as a way to improve the decision-making process because of its ability to manage information more accurately than binary logic. This methodology is presented in three principal parts: market consideration, product family formation through product configuration, and product variety consideration. To achieve these parts, seven steps are proposed and explained through an illustrative application to demonstrate the applicability and practicality of the methodology.

Keywords. Product family, product configuration, fuzzy logic, market segmentation, mass customization.

Acknowledgement. The authors wish to acknowledge the support of the Natural Sciences and Engineering Research Council of Canada (NSERC). This research was also supported by the Fonds québécois de la recherche sur la nature et les technologies (FQRNT).

---

* Corresponding author: Bruno.Agard@cirrelt.ca

Dépôt légal – Bibliothèque et Archives nationales du Québec,
Bibliothèque et Archives Canada, 2009

1 Introduction

In recent decades, companies have applied various strategies in an attempt to be more competitive from a number of perspectives. Mass customization has played an important role in the improvement of product family design, allowing greater competitiveness with respect to product variety and cost by taking advantage of the benefits of product standardization. A powerful tool in product family design has been the product modularity; it makes possible the design of a variety of products using the same set of modules around a predefined platforms. In fact, according to Moon et al. (2006), a product family can be defined as a group of related products based on a product platform, which facilitates mass customization by providing a variety of products cost-effectively for different market segments.

The main objective of this paper is to propose a methodology for the design of product families, considering the customer preferences in different segments of the market from a fuzzy logic perspective. Fuzzy logic, principally fuzzy preference relation, has been applied in order to improve the decision making processes in most of the steps of the methodology. Product configuration is considered as one of the principal approaches for this methodology as well as other approaches and strategies such as mass customization, platforms, commonality and modularity are also significantly considered.

This paper differs from most prior studies, because they applied minimal and partially fuzzy logic tools in their processes. This research develops a global methodology with fuzzy logic-aided tools to design product families. These fuzzy logic-aided tools include: a procedure to perform the market segmentation, a procedure for the identification of modules, a procedure to identify alternatives of product configurations, and a procedure for the generic products configuration, all of them supported by fuzzy logic.

This paper is organized in the following sections. Section 2 presents a literature review of some interesting topics presented principally in three parts: market considerations, product considerations, and product family considerations, a summary and analysis part is presented as well. Section 3 presents a methodology for the formation of product families through product configuration by using fuzzy logic, and includes an illustrative application. Section 4 concludes the paper.

2 Literature review

This section is presented in three principal parts: market considerations, product considerations, and product family considerations. Market considerations include customer desires, and market segments. Product considerations such as: product development, and product configuration with fuzzy logic. Product development is divided in product definition, product design, process design, and product configuration. Product family considerations are classified in methodologies for product family design, and in some approaches and strategies for product family design. A summary and analysis part is presented at the end of this section as well.

2.1 Market considerations

2.1.1 Customer desires

Companies around the world generally aim to satisfy customer expectations. They try to avoid all the drawbacks inherent in failing to identify customer desires, such as the loss of a segment of the market and the shortening of the life cycle of a product.

During recent decades, Quality Function Development (QFD) has been a powerful tool used to translate customer needs and wants into product specifications. Lately, this tool has evolved through the application of fuzzy logic to its processes, and uses customer inputs to reveal the relative importance of their needs and to facilitate their implementation.

Several attempts have been made to simplify the application of QFD by using fuzzy logic. Such work considers: fuzzy inference techniques to accommodate possible imprecision and vagueness (Fung et al. 1999); fuzzy outranking to prioritize design requirements (Wang 1999); fuzzy numbers to represent the imprecise nature of judgments and to define the relationships between engineering characteristics and customer attributes (Vanegas and Labib 2001); and fuzzy regression to identify the relational functions between, and among, engineering characteristics and customer requirements (Chen et al. 2004).

2.1.2 Market segments

Market segmentation is a fundamental practice which makes possible the identification of different groups of customers with similar preferences and patterns of behavior with respect to some products and services. This aggregation allows the development of products and services that are closer to customer expectations and at the same time improve customer satisfaction. Interesting work on clustering techniques has been proposed with regard to market segmentation. In 1996, Tseng et al. applied clustering techniques to reveal optimal building blocks for the formulation of product family architectures by applying inductive learning software to identify clusters that may match the design parameters and the product's functional requirements. Also, clustering techniques have been used to analyze the relationship between product features and customer requirements and to analyze their changing trends (Chen and Wang 2008a).

Fuzzy logic has been applied in market segmentation. Chen et al. (1996) used fuzzy clustering to analyze company productivity, identifying clusters in training productivity patterns by using two methods, the fuzzy C-means algorithm and the fuzzy K-NN algorithm. Clustering analysis has been combined with fuzzy recognition to support product design, with a view to forming standard structural trees of products according to the design requirements (Lingling et al. 2006). Gao et al. (2008) combined similarity matrix fuzzy clustering to reengineer the product interfaces by identifying the relationships between them and attempting to reduce their redundancy. Also, fuzzy clustering approaches have been proposed in the context of product family design to identify groups of customers with similar preferences with the objective of designing the proper set of products in a product family by considering the engineering characteristics and by establishing the relationship between customer preferences and product attributes (Zhang et al. 2007). Also, fuzzy C-means clustering is applied to classify customer characteristics during the first stage of product definition, which is an essential issue in designing product families from a mass customization perspective (Yu and Wang 2007).

2.2 Product considerations

2.2.1 Product development

The product development process is an essential part of product family design. According to Jiao and Zhang (2005), it can be divided into three consecutive stages: product definition, product design, and process design. *Product definition* is characterized by the portfolio of products that represents the target of mass customization. *Product design* is an engineering process involving iterative and complex decision making. It usually starts with the definition of a need, proceeds through a sequence of activities to find an optimal solution to the problem, and ends with a detailed description of the product (Deciu et al. 2005). *Process design* is a very important issue to take into account during product development. A careful design of the product assembly sequence helps to create generic subassemblies which reduce subassembly proliferation and the cost of offering product variety (Gupta and Krishnan 1998). Also, *product configuration* is an important issue to product family design. It makes it possible to configure products more strongly closed to customer requirements and also it permits to develop a large variety of products taking into account company's constraints and limitations. A considerable number of tools have been developed to address the issue, among them an approach to find the perfect match between product configuration and industry requirements considering three principal steps: product configuration, bill of materials configuration, and routing configuration (Aldanondo et al. 1999). Another approach for evaluating product configurations from the sales point of view by applying a design structure matrix to show the interaction flow between configuration elements was designed by Helo (2006). Other attempts have been made to optimize the product configuration process based on a multi-objective genetic algorithm (Li et al. 2006).

Moreover, some models, including a decision model, have been proposed to select concepts in a product configuration by considering the interactions of those concepts caused by their constraints and functional couplings (Chen et al. 2002). Also, an interesting application of the case-based reasoning algorithm has been presented to reduce design time and cost, and generate an accurate bill of materials at the beginning of the product design process (Tseng et al. 2005).

In the same way, a methodology and an architecture for requirement and engineering configurations in the configuration design process have been developed integrating data mining approaches, such as fuzzy clustering, and association rule mining to link customer groups with clusters of product specifications (Shao et al. 2006). Another work offers a method for product configuration based on a multi-layer evolution model considering the customer requirements and the product configuration design analysis performed in three layers: function, qualification, and structure, and also addresses fuzzy and incomplete customer requirements (Yi et al. 2006). Even though fuzzy logic has been applied in some of the above work, these applications remain only partial.

2.2.2 Product configuration with fuzzy logic

Fuzzy logic has been increasingly applied during recent decades to issues related to product configuration, such as concept evaluation, design requirements, company capabilities, and customer requirements. Some of these applications are the following: an integrated approach to the design of configurable products developed based on multiple fuzzy models, such as fuzzy product specification, fuzzy functional network, fuzzy physical solution, and the fuzzy constraint model, all of them designed to translate customer specifications into physical solutions dealing with various forms of uncertainty, such as imprecision, randomness, fuzziness, ambiguity, and incompleteness (Deciu et al. 2005). Another approach to product configuration (Zhu et al. 2007) considered uncertain and fuzzy customer requirements by applying fuzzy multi-attribute decision making. More recently, this approach has been presented as a method which can be used in a product data management system and on e-commerce websites. With it, the preferred product can be obtained for the customer according to the utility value with respect to the whole set of product attributes (Zhu et al. 2008).

2.3 Product family considerations

A product family can be defined as set of products that share identical internal interfaces which must be standardized in each of the functional, technological, and physical domains to allow the full exchange of components (Erens and Verhulst 1997).

2.3.1 Methodologies for product family design

Product family design is a powerful tool which makes it possible to take advantage of product similarities to reduce design and manufacturing costs. In the current literature, some methodologies for product family design have been published, including a methodology for designing product families in order to manage product diversity, proposed by Agard and Tollenaere (2003a, 2003b). This methodology consists of eight principal points: (1) management of product diversity, (2) selection of indicators, (3) analysis of functional requirements, (4) creation of a functional structure, (5) creation of a technical structure, (6) process selection, (7) search for a valid solution, and (8) selection of the final solution. In the same way, Hsiao and Liu (2005) proposed a methodology for the design of product families by managing the variety of products. This methodology comprises three stages: (1) market planning, (2) application of Quality Function Deployment (QFD), and (3) application of the Interpretative Structural Model (ISM). More recently, Kumar et al. (2009) proposed a methodology to design product families integrating market considerations to examine the impact of increasing the product variety offered to different market segments, and to explore the cost savings associated with the application of commonality decisions. This methodology consists of four steps: (1) creation of the market segmentation grid, (2) estimation of the demand, (3) construction of models for product performance, and (4) application of the profit maximization model. Also, some interesting tools have been applied to improve the design of product families. Agard and Kusiak (2004) used data mining analysis to design families of products based on customer descriptions and requirements. This methodology consists of three steps: (1) analysis of functional requirements, (2) design of a functional structure, and (3) design of a technical structure.

2.3.2 Approaches and strategies for product family design

According to Simpson (2004), there are two approaches to product family design. The first is a top-down (proactive platform) approach, wherein the company's strategy is to develop a family of products based on a product platform and its derivatives. The second is a bottom-up (reactive redesign) approach, wherein a company redesigns and/or consolidates a group of distinct products to standardize components and thus reduce costs.

The key to a successful product family is the common product platform around which the product family is derived (Messac et al. 2002). An important number of works has been published for developing platforms. These works include methods for identifying a platform using data mining techniques and fuzzy clustering (Moon et al. 2006), methods for the platform development applying preference aggregation, optimization (Dai and Scott, 2006), and cluster analysis (Dai, 2005). Also, clustering and sensitivity analysis have been used to design multiple-platform configurations in an attempt to improve product family design (Dai and Scott 2007). Cluster analysis has also been applied to the design of product platforms by analyzing products designed individually and determining the optimal number of common values for each platform (Chen and Wang 2008b). Ninan (2007) presented a platform cascading method for scale-based product family design. This method is presented in three stages: (1) the single platform stage; (2) the evaluation stage; and (3) the cascading stage, aimed at reducing the poor performance of the product family due to the consideration of a single platform by instead taking into account multiple platforms.

According to Huang et al. (2005) commonality and modularity are two strategies successfully applied in the development of product platforms. A brief summary of the work carried out related to these strategies follows.

1. *Commonality*. The proper balance between product platform commonality and individual product performance is very important to the success of a product family. Two sources of commonality have been identified by Jiao and Tseng (2000): the component part and the process part. To model the commonality of components, two models were presented by Mishra (1999): the multiple product/multiple common component method, and the multiple product/single common component method. In the same vein, Dai (2005) proposed a method for making an appropriate commonality decision in order to achieve a meaningful trade-off between the technical and monetary aspects of the product family, and Fellini (2003) and Fellini et al. (2005) presented a methodology for performing commonality optimization by choosing the components of the product that are to be shared without exceeding user-specified

bounds on performance and allowing the maximization of commonality at different levels of acceptable performance. In order to cluster the attributes of the product family in a platform and its associated differentiating modules, Ye and Gershenson (2008) presented a methodology for identifying the appropriate commonality and variety trade-off at the product attribute level using market analysis and conceptual engineering knowledge. Three matrices are used for this purpose: one for the product attributes, one for the specification ranges, and one for the changes of the specification ranges.

2. *Modularity*. Modularity has also been applied successfully in product platform development. In this context, clustering analysis has been used to analyze the design matrix to identify modules by mapping the relationships between functional requirements and design parameters (Tseng and Jiao 1997). In 1999, Kusiak proposed different points of view for the modular design of products, processes, and systems. Another method, based on the simulated annealing algorithm that permits development of a modular product family, was proposed by Wang et al. (2005). Then, Sered and Reich (2006) proposed a method for modularity standardization, focusing the engineering effort on the product platform components, and Meng, X., et al. (2007) presented a methodology to identify the component modules for product families which includes four principles: (1) identification and isolation of individualized components into modules; (2) identification and isolation of components with a strong possibility of replacement by one module; (3) improvement of the functional independence of the modules; and (4) improvement of the structural independence of the modules. Da Cunha et al. (2007) proposed various heuristic algorithms for the design of modular elements in a mass customization context, focusing on minimizing the manufacturing and transportation cost in the supply chain.

2.4 Summary and analysis

Product family design is a challenge that considers taking advantage of product similarities to reduce design and manufacturing costs. Many processes into in the design of product families can be improved in different ways by the application of fuzzy logic.

Fuzzy logic allows input information to be provided in linguistic terms as colloquially expressed by people, for example to be moderately or highly interested to certain feature of a product such as the size or the weight, instead of crisp and non negotiable terms. This type of information permits to make better and more accurate decisions due to the wide range of possible answers that can be handled instead of just to be or not be interested to such product feature as permitted by traditional tools.

The publications considered in this paper were classified in different topics that include the market point of view (customer desires, market segments), the product point of view (product development, product definition, product design, process design, product configuration), and some methodologies and strategies for the product family design (platform, commonality, and modularity).

Fuzzy logic has not yet been applied to the entire process of design of product families, it has, however, been used in recent years to improve several specific tasks in that process. It is interesting to note that an important number of publications contain partial applications of fuzzy logic. Different fuzzy logic tools are used in one or more topics related to product family design. Customer desires, product definition, and product design are the topics the most frequently addressed. On the contrary, the topics that are less addressed with fuzzy logic applications are the design of processes, platforms, commonality, and modularity. Even if some works presented some application of fuzzy logic into the product family design process, these applications are very partial and still necessitate developing new tools for the entire product family design process.

This work aims at filling this lack and proposes to exploit the benefits of fuzzy logic to develop a global methodology to design families of products, it embrace all the related topics from a fuzzy logic perspective instead of partial applications to specific topics related to the design of product families.

3 Methodology for product family formation through product configuration using fuzzy logic and its application

Product family design can be improved in a wide range of areas by applying fuzzy logic, which allows opinions, knowledge, and expertise to be provided and managed in the linguistic terms commonly used by human beings. Fuzzy logic is increasingly used in decision aided systems, since it offers several advantages over other traditional decision making techniques. In this section, we propose a methodology for forming product families through product configuration applying fuzzy logic; in an attempt to improve customer satisfaction by offering the products that most closely meet to the expectations of different segments of the market (see Figure 1).

Figure 1: Product family formation methodology

The proposed methodology is presented in three principal parts: market considerations, product family formation through product configuration, and product variety consideration. These phases are achieved through the following seven steps: (1) market segmentation, (2) generic product configuration, (3) common features identification, (4) module identification, (5) alternative products configuration, (6) personalized products configuration, (7) listing of product variety (see Figure 1).

These steps are explained in greater detail through the following illustrative application.

Step 1. Market segmentation

First of all, we consider the application of fuzzy clustering techniques to identify groups of customers with similar needs and wants. According to Xu and Wunsch II (2005), fuzzy c-means (FCM) which was developed by Bezdek (1981), is one of the fuzzy clustering algorithms most often applied. The FCM function starts with an initial guess as to the cluster center, which is frequently incorrect. Then, the cluster centers are updated iteratively and the FCM moves the cluster centers, also iteratively, to the right location within the set of data. This iteration is based on minimizing an objective function, which represents the distance from any given data point to a cluster center. The output is a list of cluster centers and several membership grades for each data point that can be used to build a fuzzy inference system by creating membership functions to represent the fuzzy qualities of each cluster. There are other methods for estimating the number of clusters and their centers. According to Chiu (1996), the subtractive clustering method was first introduced by him in 1994 as a cluster estimation method to determine the number of clusters and their initial values that can be used to initialize other clustering algorithms such as FCM.

To perform the market segmentation, we propose the following five-phase procedure.

1. *Consider product features.* Let us assume that the design team found the most relevant features considered by customers in selecting a laptop. These include the processor (F_1), the operating system (F_2), the display (F_3), the memory (F_4), and the hard drive (F_5).
2. *Express customer preferences in linguistic terms.* In this application, we consider a case where a group of thirty customers has been surveyed about their preferences at the time of buying a laptop. The customer preferences for each feature are expressed in linguistic terms, such as: “highly important” (HI), “important” (I), “moderately important” (MI), “somewhat important” (SI), and “not important” (NI).
3. *Express customer preferences in numerical terms.* To represent these terms numerically, we use a five-level Liker scale with a range from 5 to 1, where 5 represents “highly important”, 4 “important”, and so on. Table 1 lists a portion of the customer preferences for each feature. The complete list appears in Appendix 1.

Table 1: Customer feature preferences

4. *Identify clusters using the FCM clustering method.* In this application, we apply the FCM clustering iterative method by using the Fuzzy Logic toolbox in Matlab to identify the clusters needed to represent different groups with similar preferences. Let us apply FCM to analyze the customer preferences listed in Appendix 1, evaluating three different scenarios: (a) four clusters, (b) three clusters, and (c) two clusters. Two interesting outputs of Matlab fuzzy clustering are: the membership matrix and the cluster centers. These are analyzed as follows.
   - *Membership matrix analysis.* A portion of the membership matrix obtained between clusters and customers for each scenario is presented in Figure 2. In this matrix, we may note that a customer can belong to different clusters with different membership degrees. For example, in case (a) with four clusters, customer 1 belongs 89% to cluster 4, 8% to cluster 3, 2% to cluster 2, and 1% to cluster 1.

Figure 2: Membership matrix for each scenario

Also, the entire membership matrix depicted in Figure 2 can be analyzed through some basic measures of a central tendency, such as: sum, average, and variance, where the highest sum and the highest average indicate that more customers belong to that cluster, and a low variance means that the customers are clustered more in the corresponding cluster than in the others. Figure 3 presents these measures for each cluster of all three scenarios, where the highest sum and highest average correspond to cluster 1 in scenario (c), with measures of 15.25 and 0.51 respectively, whereas that the lowest variance corresponds to cluster 1 in scenario (b).

Figure 3: Comparison of the membership matrices for the three scenarios

- *Cluster center analysis.* Because there is no scenario that satisfies both the above criteria, the designer could analyze the center of the clusters with respect to the product features. Figures 4 and 5 list and depict this information for each scenario respectively.

Figure 4: List of cluster center coordinates with respect to product features

Figure 5: Depiction of the cluster centers with respect to product features

5. *Selection of the best clusters scenario.* In selecting the best number of clusters, the scenario with the lowest variance is preferred. These variances are obtained from the analysis of the membership matrix. The lowest scenario variance means that the customers are better segmented into these clusters. It is important to consider that while greater the number of clusters is within the scenario its variance tends to decrease. But, it is better to identify the scenario with the smallest number of clusters looking for representing the principal segments of the market. According to the information presented in Figure 3, the three cluster scenario (b) is the best option, since it satisfies the lowest variance criteria. Figure 5 in scenario (b) shows how

cluster 1 includes customers moderately interested in almost all the laptop features, cluster 2 includes customers more interested in features such as the processor and the operating system, and cluster 3 includes customers more interested in storage capacity.

Step 2. Generic products configuration

To perform this step, we propose the following four-phase procedure, which is an adaptation from a method proposed by Barajas and Agard (2008). This method has been restructured and simplified in order to achieve the objective of this step. In the first phase, consideration of customer preferences, a rule has been added to permit the introduction of information from the previous step. This rule consists in round the information from the cluster centers to the nearest integer to represent the customer preferences. In the last phase, selection of product features, a simple comparison between $R(F_{ij},C_{ki})$ and 0.5 has been considered in order to identify the best features for the product instead of the calculation of the fuzzy indifference degree.

1. *Consideration of customer preferences.* For this application, these customer preferences correspond to the customers in the target scenario. In this case, the information can be obtained from the cluster centers listed in Figure 4(b) that correspond to the three cluster scenario. This information needs to be rounded to the nearest integer to represent the customer preference for each feature in each cluster (see Table 2(a)). This information could also be expressed in linguistic terms, as explained in the previous step (see Table 2(b)).

Table 2: Customer preferences for the three cluster scenario

2. *General prioritization of customer preferences*. Let us suppose that a team of specialists defined a general scale based on a customer survey to prioritize the set of features (see Table 3). Figure 6 shows how this prioritization is represented using fuzzy numbers.

Table 3: General prioritization of customer preferences represented by fuzzy numbers

Figure 6: Fuzzy number depiction of product feature general prioritization

3. *Technical evaluation of product features.* Generally, this evaluation can be obtained from specialized sources in the industry in question. If this information is not available, a survey designed by experts can be used as well. Once this information is available, it must be represented by fuzzy numbers in order to be used in this phase. To do that, we considered a work proposed by (Jarventausta et al. 1994) which includes a detailed explanation about how to represent uncertain situations by using fuzzy numbers through the determination of a proper membership function. Also, these authors considered that in uncommon situations where no statistics are available, an expert may be able to express degrees of confidence in various hypotheses. In this work, we assume that this information is available, and it has been represented in fuzzy numbers by applying fuzzy set theory as listed in Table 4 where each alternative of the product features are represented with trapezoidal fuzzy numbers. It is important to consider that other membership functions could be considered. For this application trapezoidal membership function better fits to represent the evaluation of the alternatives for the product features. More detailed information about fuzzy set theory can be found in Zimmermann, H.-J. (1991). Figure 7 presents a depiction of the available alternatives for feature 1 represented by fuzzy numbers.

Table 4: Technical evaluation of product features represented by fuzzy numbers

Figure 7: Fuzzy number depiction of the alternatives of feature 1

4. *Selection of product features.* As considered by Barajas and Agard (2008), if the fuzzy preference relation R(A,B) is equal to 0.5, then A and B are indifferent, where A represents the feature evaluation and B represents the customer preference for that feature.
   - *Fuzzy preference relation.* Let A and B be two normal and convex fuzzy numbers. Then, there exist two notions: dominance and indifference. If there exists an area of overlap between fuzzy numbers A and B (intersection between A and B), then

the overlap area is defined as the indifference area. Also, if there exist one or more non-overlap areas between fuzzy numbers A and B, then, for each non-overlap area, either A dominates B or B dominates A (see Figure 8). R(A,B) could be obtained using the following equation:

$$R(A,B) = [D(A,B) + I(A,B)]/[A(A) + A(B)] \quad (1)$$

Where: D(A,B) is the area where A dominates B, I(A,B) is the area where A and B are indifferent, and A(A) and A(B) are the areas of A and B respectively.

In this work, the fuzzy preference relation R(A,B) is denoted as $R(F_{ij},C_{ki})$, where $F_{ij}=\{F_{11}, F_{12}, \ldots, F_{nm}\}$ is the set of the evaluations of the feature (i) for each feature alternative (j) for all i=1, 2,…, n, and for all j=1, 2,…, m, and $C_{ki}=\{C_1, C_2,\ldots,C_{pn}\}$ is the set of customer preferences of cluster (k) for each feature (i) for all k=1, 2,…, p.

- *Example of fuzzy preference calculation.* Let`s calculate the fuzzy preference relation $R(F_{11},C_{11})$ which corresponds to the first alternative of feature 1 (F_{11}), and to the customer preference of the cluster 1 to such feature alternative (C_{11}). The corresponding fuzzy numbers for F_{11} and for C_{11} are [0 1 4 6] and [1 2 4 5] respectively (see Figure 8). By adapting equation (1) to adapted notation in this work, the fuzzy preference relation can be calculated as follows. $D(F_{11},C_{11}) = 0.5$, $I(F_{11},C_{11}) = 3.0$, $A(F_{11}) = 4.5$, $A(C_{11}) = 3.0$. Then, $R(F_{11},C_{11})= 0.4667$.

Figure 8: Fuzzy number depiction of F_{11} and C_{11}

Table 5 lists the fuzzy preference relation for all the relations in cluster 1. Appendices 2a and 2b present these preferences for cluster 2 and cluster 3 respectively.

Table 5: Fuzzy preference relation of Cluster 1

To identify the best product features for each cluster in this application, we consider that the $R(F_{ij},C_{ki})$ nearest to 0.5 corresponds to the feature that should be part of the generic product for each cluster. To do this, it is necessary to compare the absolute value of the difference between 0.5 and $R(F_{ij},C_{ki})$ to identify the features with the smallest differences (see Table 6).

Table 6: Product features for each cluster

Based on the previous statement and according to Table 6, the product configuration for each cluster is as follows: $F_{11} – F_{21} – F_{33} – F_{42} – F_{54}$ for cluster 1, $F_{13} – F_{22} – F_{33} – F_{42} – F_{52}$ for cluster 2, and $F_{11} – F_{21} – F_{33} – F_{43} – F_{56}$ for cluster 3.

Step 3. Common features identification

This step consists of identifying if one or more features are common to all the product configurations identified in step 2 for all the clusters. By analyzing the previous product configurations, it is possible to note that F_{33} is common to all the generic products for all the clusters (see Table 7). This alternative corresponds to option 3 of feature 3. For this application, this can be translated as a medium-sized laptop display being preferred by most of the customers. This alternative will then be considered as fixed in future product mass customization. For this feature, other alternatives will also be considered, but for personalized configuration instead of mass customization.

Table 7: Product features for each cluster

Step 4. Modules identification

In this work, a module is defined as the integration of two or more product features. To identify possible modules we propose the following four-phase procedure.

1. *Ranking of features preferences.* This can be achieved by analyzing the cluster centers with respect to the product features. To do that, we calculate the variance among the cluster centers for each product feature. The feature with the smallest

variance will be the first in the ranking. Based on the information in Table 8, the feature ranking is as follows: F_3, F_4, F_2, F_5, and F_1.

Table 8: Analysis of cluster centers with respect to product features.

2. *Availability of features alternatives.* Considering the information depicted in Table 6, it is possible to identify if there are feature alternatives that are not used in the generic product. According to this table, the availability for each feature alternative is as follows: for feature 1 (F_{12}); for feature 3 (F_{31}, F_{32}, F_{34}, F_{35}, and F_{36}); for feature 4 (F_{41} and F_{44}); and (F_{51}, F_{53}, and F_{55}) for feature 5. As can be noted, there is no alternative available for feature 2.
3. *Common features alternative consideration.* If there is/are an alternative/alternatives which is/are common to all the generic products, then this/these should be included in the modules. According to step 3, F_{33} is common to all the generic products, and so this will be included in all the modules.
4. *Modules formation.* The module will be formed according to the ranking of the feature preference obtained previously (F_3, F_4, F_2, F_5, and F_1), considering the common features and the features that are not available. For this application, feature 2 cannot be considered to form a module, because there is no alternative available for it. On the other hand, F_{33} is the alternative that should be common to all the modules. Figure 9 depicts this procedure.

Figure 9: Modules identification

Step 5. Alternative products configuration

To identify possible product configuration alternatives, we propose the following two-phase procedure.

1. *Features with no alternative availability.* If there exist one or more features with no available alternatives, then all the alternatives for these features will be considered in the alternative product configuration. According to step 4, there is no alternative

available for feature 2. That is, F_{21} and F_{22} will be part of the new product configuration (see Figure 10).

2. *Massive product configuration.* To form the alternative product configuration, the modules identified with feature alternatives which are not available must be combined. Table 9 lists the alternative product configuration for this application.

Figure 10: Alternative products configuration

Table 9: Features of the alternative product configuration

Step 6. Personalized products configuration

Let us suppose that a customer X is not satisfied with the customized products offered. This customer wants his product to be personalized. For him, all the product features are "highly important" (HI). This configuration can be obtained by performing step 2 considering his feature preferences. Appendix 3 lists the complete fuzzy preference relation for this case. As can be inferred, the product configuration for this customer (Px) is formed with the highest ranking alternative for each feature (F_{13}– F_{22} – F_{36} – F_{44} – F_{56}).

Step 7. Product variety listing

There are three types of product configuration: a generic product for each cluster, modular customized products, and a personalized product configuration (see Figure 11).

Figure 11: Product alternatives in the product family

Products 1 to 3 belong to clusters 1 to 3 respectively. But, it is important to identify which of the modular customized products are more closely associated with each cluster. From Table 2, it is possible to identify the most often preferred features for each cluster (see Table 10).

Table 10: Most often preferred features per cluster

According to the feature preferences for each cluster, we may note that P_4 to P_9 are more closely associated with cluster 1, P_{10} to P_{15} with cluster 2, and P_7 to P_9 and P_{13} to P_{15} with cluster 3 (see Figure 12 and Table 11).

Figure 12: Alternative products for each cluster

Table 11: Identification of product configuration for each cluster

4 Conclusions

A global methodology is proposed in this paper to form a product family through product configuration using fuzzy logic. It is aimed at contributing to increasing customer satisfaction by applying fuzzy preference relation in the various steps of the methodology to enrich the decision making process. This methodology unlike others published seeks to take advantage of fuzzy logic in all of its steps. The methodology is presented in three principal parts: market consideration, product family formation, and product variety consideration, and can be completed in seven steps. The output of the methodology is a family of products classified into three different types of products: a generic product for each segment of the market, a set of modular customized products associated with each segment of the market, and a personalized product for a specific customer. This methodology contributes to the possibility of offering both generic and standardized products for different segments of the market, and to reducing the costs of the product as a result of standardization of the components and the associated processes. It is also possible to form a personalized product, although at a higher cost, owing to the flexibility of using feature alternatives. Some future research directions could include study of a component-level instead of a feature-level methodology.

5 Acknowledgments

The authors wish to acknowledge the support of the Natural Sciences and Engineering Research Council of Canada (NSERC). This research was also supported by the Fonds Québécois de la Recherche sur la Nature et les Technologies (FQRNT).

6 References

Agard, B. and Kusiak, A. (2004). Data-Mining Based Methodology for the Design of Product Families, *International Journal of Production Research*, **42**, 2955-2969.

Agard, B. and Tollenaere, M. (2003a). Méthodologie de conception des familles de produits, *Journal Européen des Systèmes Automatisés*, **37**, 755-777.

Agard, B. and Tollenaere, M. (2003b). Design of product families: methodology and application, *14th International Conference on Engineering Design – ICED'03.*

Aldanondo, M., Veron, M., and Fargier, H. (1999). Configuration in manufacturing industry requirements, problems and definitions. *IEEE International Conference on Systems, Man and Cybernetics* **6**, 1009-1014.

Barajas, M. and Agard, B. (2008). Selection of products based on customer preferences applying fuzzy logic. *IDMME – Virtual Concept 2008.*

Chen, C. and Wang, L. (2008a). Integrating rough set clustering and grey model to analyze dynamic customer requirements. *Journal of Engineering Manufacture* **222**, 319-332.

Chen, C. and Wang, L. (2008b). Product platform design through clustering analysis and information theoretical approach. *International Journal of Production Research* **46**, 4259-4284.

Chen, L. -Ch. and Lin, L. (2002). Optimization of product configuration design using functional requirements and constraints. *Research in Engineering Design* **13**, 167-182.

Chen, L. -H., Kao, C., Kuo, S., Wang, T. -Y, and Jang, Y.-C. (1996). Productivity diagnosis via fuzzy clustering and classification: an application to machinery industry. *Omega* **24**, 309-319.

Chen, Y., Tang, J., Fung, R. Y. K., and Ren, Z. (2004). Fuzzy regression-based mathematical programming model for quality function deployment. *International Journal of Production Research* **42**, 1009-1027.

Chiu, S. (1996). Method and software for extracting fuzzy classification rules by subtractive clustering. *Annual Conference of the North American Fuzzy Information Processing Society - NAFIPS, New Frontiers in Fuzzy Logic and Soft Computing* 461-465.

Da Cunha, C., Agard, B., and Kusiak, A. (2007). Design for cost: Module-Based Mass Customization. *IEEE Transactions on Automation Science and Engineering* **4**, 350-359.

Dai, Z. (2005). Robust product family design. *Thesis (PhD).* University of Illinois.

Dai, Z. and Scott, M. J. (2006). Effective product family design using preference aggregation. *Journal of Mechanical Design, Transactions of the ASME* **128,** 659-667.

Dai, Z. and Scott, M. J. (2007). Product platform design through sensitivity analysis and cluster analysis. *Journal of Intelligent Manufacturing* **18**, 97-113.

Deciu, E. R., Ostrosi, E., Ferney, M. and Gheorghe, M. (2005). Configurable product design using multiple fuzzy models. *Journal of Engineering Design* **16,** 209-235.

Erens, F. and Verhulst, K. (1997). Architectures for product families. *Computers in Industry* **33**, 165-178.

Fellini, R., Kokkolaras, M., Papalambros, P. and Perez-Duarte, A. (2005). Platform selection under performance bounds in optimal design of product families. *Journal of Mechanical Design, Transactions of the ASME* **127,** 524-535.

Fellini, R. A. (2003). A model-based methodology for product family design. *Thesis (PhD).* University of Michigan.

Fung, R. Y. K., Law, D. S. T. and Ip, W. H. (1999). Design targets determination for inter-dependent product attributes in QFD using fuzzy inference. *Integrated Manufacturing Systems* **10**, 376-384.

Gao, F., Xiao, G., and Chen, J. -J. (2008). Product interface reengineering using fuzzy clustering. *CAD Computer Aided Design* **40**, 439-446.

Gupta, S. and Krishnan, V. (1998). Product family-based assembly sequence design methodology. *IIE Transactions* **30**, 933-945.

Helo, P. T. (2006). Product configuration analysis with design structure matrix. *Industrial Management + Data Systems* **106**, 997-1011.

Hsiao, S. -W. and Liu, E. (2005). A structural component-based approach for designing product family. *Computers in Industry* **56**, 13-28.

Huang, G.Q., Simpson, T.W. and Pine II, B.J. (2005). The power of product platforms in mass customisation. *International Journal of Mass Customisation* **1**, 1-13.

Jarventausta, P., Verho, P., and Partanen, J. (1994) Using fuzzy sets to model the uncertainty in the fault location process of distribution networks. *IEEE Transactions on Power Delivery,* **9**, 954-960.

Jiao, J. and Tseng, M. M. (2000). Understanding product family for mass customization by developing commonality indices. *Journal of Engineering Design* **11,** 225-243.

Jiao, J. and Zhang, Y. (2005). Product portfolio identification based on association rule mining. *CAD Computer Aided Design* **37**, 149-172.

Kumar, D., Chen, W., and Simpson, T. W. (2009). A market-driven approach to product family design. *International Journal of Production Research* **47**, 71-104.

Kusiak, A. (1999), *Engineering Design: Products, Processes, and Systems,* Academic Press, San Diego, CA, 427 pages.

Li, B., Chen, L., Huang, Z., and Zhong, Y. (2006). Product configuration optimization using a multiobjective genetic algorithm, *International Journal of Advanced Manufacturing Technology* **30**, 20-29.

Lingling, L., Quanming, Z., Zhigang, L., and Huijuan, Z. (2006). Product design based on clustering analyzing and fuzzy recognition. *IEEE Conference on Cybernetics and Intelligent Systems*.

Messac, A., Martinez, M. P. and Simpson, T. W. (2002). Introduction of a product family penalty function using physical programming. *Journal of Mechanical Design, Transactions of the ASME* **124**, 164-172.

Mishra, A. K. (1999). Optimization models for component commonality in assemble-to-order systems. *Thesis (PhD).* University of Pittsburgh.

Moon, S. K., Kumara, S. R. T., and Simpson, T. W. (2006). Data mining and fuzzy clustering to support product family design. *ASME International Design Engineering Technical Conferences and Computers and Information in Engineering Conference.*

Ninan, J. A. (2007). A platform cascading method for scale based product family design. *Ph.D. dissertation*, University of Oklahoma.

Sered, Y. and Reich, Y. (2006). Standardization and modularization driven by minimizing overall process effort. *CAD Computer Aided Design*, **38,** 405-416.

Shao, X. -Y., Wang, Z.-H., Li, P.-G., and Feng, C.-X. J. (2006). Integrating data mining and rough set for customer group-based discovery of product configuration rules. *International Journal of Production Research* **44**, 2789-2811.

Simpson, T. W. (2004). Product platform design and customization: Status and promise. *Artificial Intelligence for Engineering Design, Analysis and Manufacturing: AIEDAM* **18**, 3-20.

Tseng, H. -E., Chang, Ch. -Ch., and Chang, Sh. -H. (2005). Applying case-based reasoning for product configuration in mass customization environments. *Expert Systems with Applications* **29**, 913-925.

Tseng, M. M, Jiao, J., and Merchant M. E. (1996). Design for mass customization. *CIRP Annals – Manufacturing Technology* **45**, 153-156.

Tseng, M. M. and Jiao, J. (1997). A module identification approach to the electrical design of electronic products by clustering analysis of the design matrix. *Computers & Industrial Engineering* **33**, 229-233.

Tseng, T. Y. and Klein, C. M. (1989). New algorithm for the ranking procedure in fuzzy decision making, *IEEE Transaction Systems, Man and Cybernetics,* **19**, 1289-1296.

Vanegas, L. V. and Labib, A. W. (2001). A fuzzy quality function deployment (FQFD) model for deriving optimum targets. *International Journal of Production Research* **39,** 99-120.

Wang, J. (1999). Fuzzy outranking approach to prioritize design requirements in quality function deployment. *International Journal of Production Research* **37,** 899-916.

Wang, H. J., Zhang, Q. and Zhao, T. T. (2005). A SA-based method for developing modular product family. *International Conference on Machine Learning and Cybernetics, ICMLC 2005.*

Meng, X., Jiang, Z., and Huang, G. Q. (2007). On the module identification for product family development. *International Journal of Advanced Manufacturing Technology* **35**, 1-2, 26-40.

Xu, R. and Wunsch II, D. (2005). Survey of clustering algorithms. *IEEE Transactions on Neural Networks* **16**, 3, 645-678.

Ye, X. and Gershenson, J. K. (2008). Attribute-based clustering methodology for product family design. *Journal of Engineering Design* **19**, 571-586.

Yi, G., Zhang, S., and Tan, J. (2006). Product configuration design based on multi-layer evolution, *IEEE/ASME International Conference on Mechatronics and Embedded Systems and Applications*.

Yu, L. and Wang, L. (2007). Two-stage product definition for mass customization. *5th IEEE International Conference on Industrial Informatics*, 699-704.

Zhang, Y., Jiao, J., and Ma, Y. (2007). Market segmentation for product family positioning based on fuzzy clustering. *Journal of Engineering Design* **18**, 227-41.

Zhu, B., Wang, Z., Yang, H., and Li, H. (2006). Study on approach to fuzzy product configuration based on vague customer requirements. *Materials Science Forum* **532-533**, 1068-1071.

Zhu, B., Wang, Z., Yang, H., Mo, R., and Zhao, Y. (2008). Applying fuzzy multiple attribute decision making for product configuration. *Journal of Intelligent Manufacturing* **19**, 591-598.

Zimmermann H.-J. (1991) *Fuzzy set theory and its applications*. Kluwer Academic Publishers, Massachusetts.

Appendix 1 Customer preferences for each product feature

Customer	Product Features				
	F_1	F_2	F_3	F_4	F_5
1	5	4	3	4	2
2	1	2	2	3	4
3	4	3	2	3	2
4	1	2	3	4	5
5	5	5	3	4	1
6	5	4	3	3	2
7	4	4	3	5	2
8	2	2	2	3	4
9	5	4	3	2	1
10	5	4	2	2	2
11	1	3	3	3	4
12	2	2	3	3	3
13	1	1	3	4	5
14	2	3	2	3	4
15	1	3	3	3	5
16	5	4	3	2	1
17	5	4	3	3	2
18	1	2	3	4	4
19	2	2	3	3	3
20	5	4	3	3	1
21	3	3	2	3	2
22	5	5	3	4	1
23	1	2	2	2	5
24	5	5	3	4	2
25	1	2	2	4	5
26	1	1	2	5	5
27	3	2	3	3	2
28	5	4	2	1	1
29	1	2	3	4	5
30	5	4	3	3	2

Appendix 2a Fuzzy preference relation of Cluster 2

$F_{ij}\backslash C_{ki}$	C_{21}	C_{22}	C_{23}	C_{24}	C_{25}
	[7 9 10 10]	[5 6 8 9]	[3 5 5 7]	[3 5 5 7]	[1 2 4 5]
F_{11} [0 1 4 6]	0.0000				
F_{12} [2 4 6 8]	0.0208				
F_{13} [7 8 10 10]	0.4444				
F_{21} [0 4 5 7]		0.0952			
F_{22} [8 9 10 10]		0.9444			
F_{31} [0 1 2 3]			0.0000		
F_{32} [1 2 3 4]			0.0000		
F_{33} [3 4 5 7]			0.4444		
F_{34} [4 5 6 8]			0.6667		
F_{35} [6 7 8 9]			0.9167		
F_{36} [7 8 10 10]			1.0000		
F_{41} [0 2 4 6]				0.1875	
F_{42} [2 3 6 7]				0.4167	
F_{43} [4 6 7 9]				0.7750	
F_{44} [7 8 10 10]				1.0000	
F_{51} [0 1 2 3]					0.2000
F_{52} [1 2 4 5]					0.5000
F_{53} [2 3 5 6]					0.6667
F_{54} [3 4 6 7]					0.8333
F_{55} [5 6 8 9]					1.0000
F_{56} [7 8 10 10]					1.0000

Appendix 2b Fuzzy preference relation of Cluster 3

$F_{ij}\backslash C_{ki}$	C_{31}	C_{32}	C_{33}	C_{34}	C_{35}
	[0 0 1 3]	[1 2 4 5]	[3 5 5 7]	[5 6 8 9]	[7 9 10 10]
F_{11} [0 1 4 6]	0.7692				
F_{12} [2 4 6 8]	0.9792				
F_{13} [7 8 10 10]	1.0000				
F_{21} [0 4 5 7]		0.6429			
F_{22} [8 9 10 10]		1.0000			
F_{31} [0 1 2 3]			0.0000		
F_{32} [1 2 3 4]			0.0000		
F_{33} [3 4 5 7]			0.4444		
F_{34} [4 5 6 8]			0.6667		
F_{35} [6 7 8 9]			0.9167		
F_{36} [7 8 10 10]			1.0000		
F_{41} [0 2 4 6]				0.0000	
F_{42} [2 3 6 7]				0.1429	
F_{43} [4 6 7 9]				0.4167	
F_{44} [7 8 10 10]				0.8182	
F_{51} [0 1 2 3]					0.0000
F_{52} [1 2 4 5]					0.0000
F_{53} [2 3 5 6]					0.0000
F_{54} [3 4 6 7]					0.0000
F_{55} [5 6 8 9]					0.1333
F_{56} [7 8 10 10]					0.4444

Appendix 3 Fuzzy preference relation of customer X

$F_{ij}\backslash C_{ki}$	C_{x1} [7 9 10 10]	C_{x2} [7 9 10 10]	C_{x3} [7 9 10 10]	C_{x4} [7 9 10 10]	C_{x5} [7 9 10 10]
F_{11} [0 1 4 6]	0.0000				
F_{12} [2 4 6 8]	0.0208				
F_{13} [7 8 10 10]	0.4444				
F_{21} [0 4 5 7]		0.0000			
F_{22} [8 9 10 10]		0.5714			
F_{31} [0 1 2 3]			0.0000		
F_{32} [1 2 3 4]			0.0000		
F_{33} [3 4 5 7]			0.0000		
F_{34} [4 5 6 8]			0.0000		
F_{35} [6 7 8 9]			0.1667		
F_{36} [7 8 10 10]			0.4444		
F_{41} [0 2 4 6]				0.0000	
F_{42} [2 3 6 7]				0.0000	
F_{43} [4 6 7 9]				0.1000	
F_{44} [7 8 10 10]				0.4444	
F_{51} [0 1 2 3]					0.0000
F_{52} [1 2 4 5]					0.0000
F_{53} [2 3 5 6]					0.0000
F_{54} [3 4 6 7]					0.0000
F_{55} [5 6 8 9]					0.1333
F_{56} [7 8 10 10]					0.4444

Figures caption

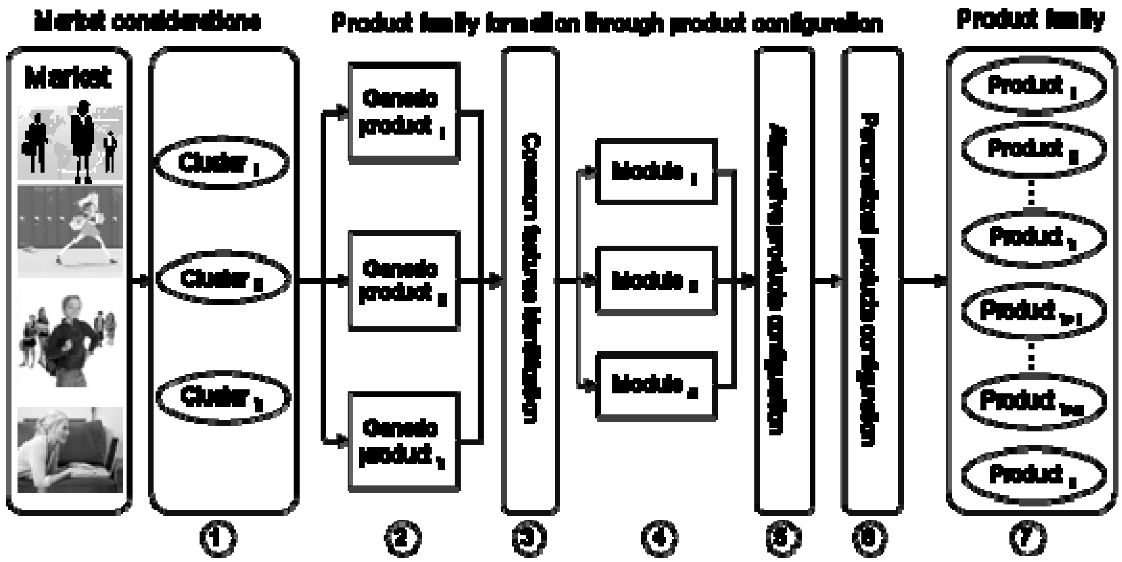


Figure 1: Product family formation methodology

Cluster	Customers			
	1	2	…	30
1	0.01	0.36	…	0.02
2	0.02	0.59	…	0.03
3	0.08	0.03	…	0.42
4	0.89	0.03	…	0.53

(a) Four clusters

Cluster	Customers			
	1	2	…	30
1	0.09	0.27	…	0.02
2	0.88	0.04	…	0.97
3	0.04	0.69	…	0.01

(b) Three clusters

Cluster	Customers			
	1	2	…	30
1	0.95	0.03	…	0.99
2	0.05	0.97	…	0.01

(c) Two clusters

Figure 2: Membership matrix for each scenario

Cluster	*Sum*	*Ave*	*Var*
1	7.83	0.26	0.10
2	7.37	0.25	0.08
3	7.17	0.24	0.08
4	7.63	0.25	0.09

(a) Four clusters

Cluster	*Sum*	*Ave*	*Var*
1	7.69	0.26	0.07
2	12.61	0.42	0.17
3	9.70	0.32	0.13

(b) Three clusters

Cluster	*Sum*	*Ave*	*Var*
1	15.25	0.51	0.18
2	14.75	0.49	0.18

(c) Two clusters

Figure 3: Comparison of the membership matrices for the three scenarios

Clus	F_1	F_2	F_3	F_4	F_5
1	1.04	1.86	2.68	3.89	4.81
2	1.82	2.29	2.52	3.00	3.59
3	4.86	3.91	2.64	2.21	1.39
4	4.83	4.39	2.95	3.79	1.68

(a) Four clusters

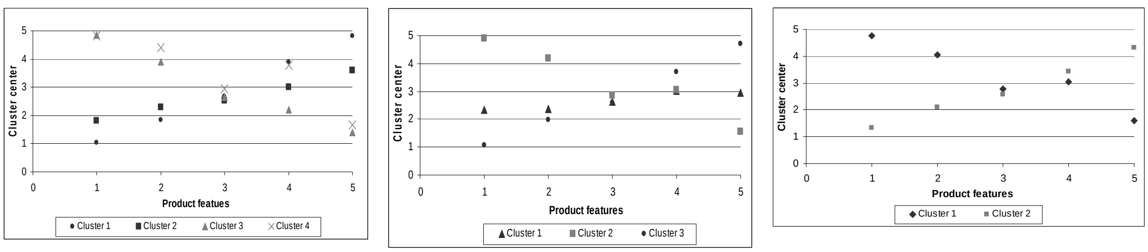
Clus	F_1	F_2	F_3	F_4	F_5
1	2.34	2.36	2.64	3.03	2.95
2	4.91	4.18	2.85	3.05	1.57
3	1.08	1.98	2.61	3.70	4.70

(b) Three clusters

Clus	F_1	F_2	F_3	F_4	F_5
1	4.76	4.06	2.79	3.05	1.61
2	1.33	2.09	2.57	3.43	4.31

(c) Two clusters

Figure 4: List of cluster center coordinates with respect to product features



(a) Four clusters (b) Three clusters (c) Two clusters

Figure 5: Depiction of the cluster centers with respect to product features

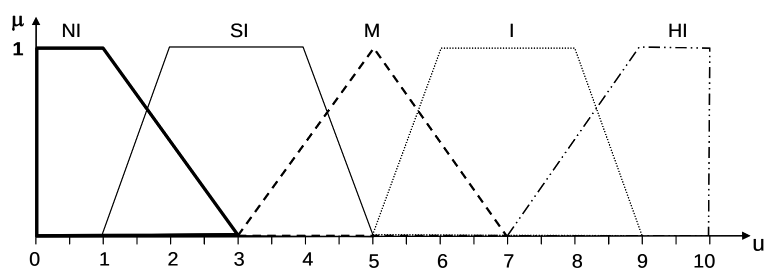


Figure 6: Fuzzy number depiction of product feature general prioritization

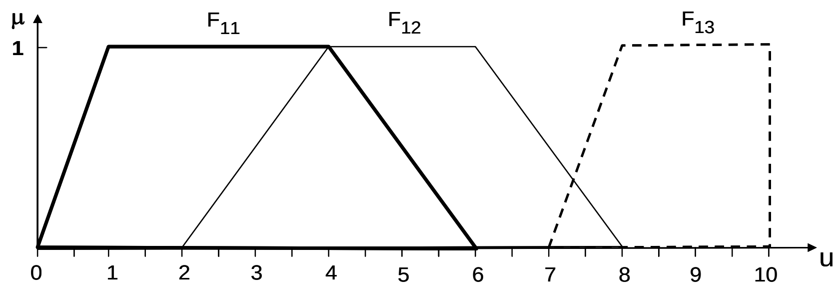


Figure 7: Fuzzy number depiction of the alternatives of feature 1

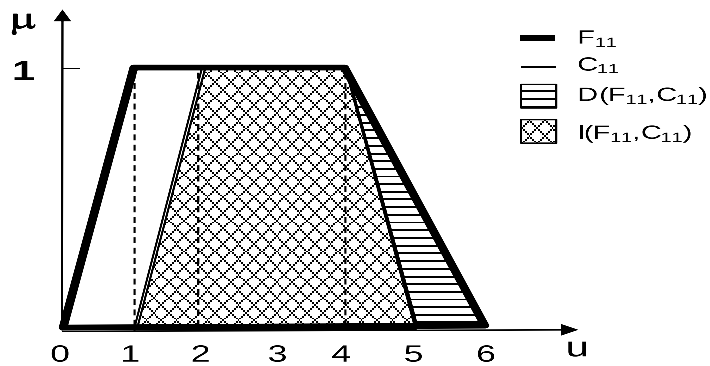


Figure 8: Fuzzy number depiction of F_{11} and C_{11}

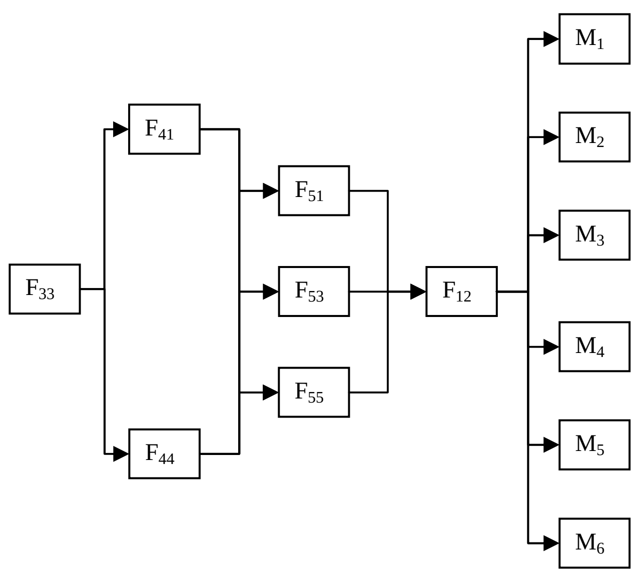


Figure 9: Modules identification

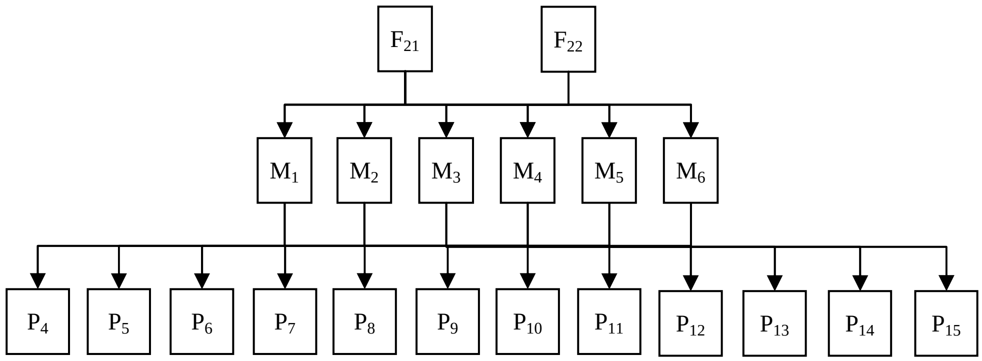


Figure 10: Alternative products configuration

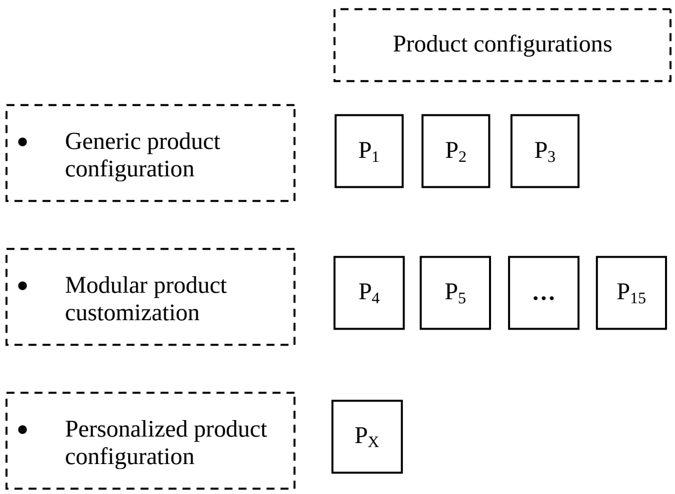


Figure 11: Product alternatives in the product family

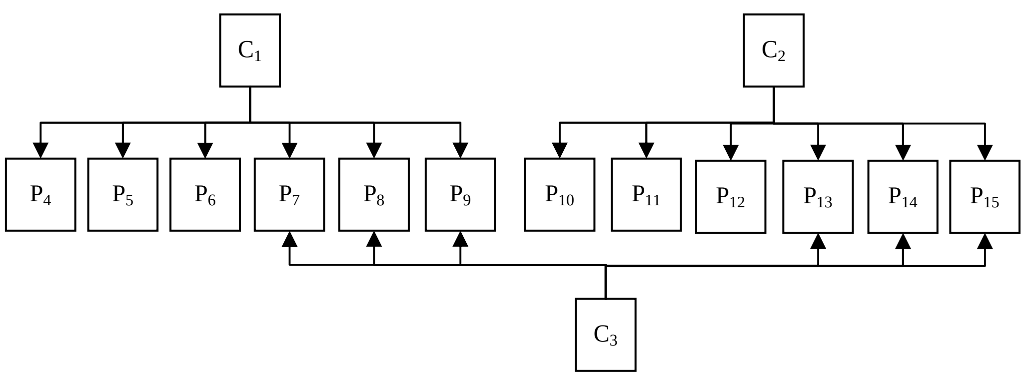


Figure 12: Alternative products for each cluster

Tables caption

Table 1: Customer feature preferences

Customer	Product Features				
	F_1	F_2	F_3	F_4	F_5
1	5	4	3	4	2
2	1	2	2	3	4
⋮	⋮	⋮	⋮	⋮	⋮
30	5	4	3	3	2

Table 2: Customer preferences for the three cluster scenario

Cluster	F_1	F_2	F_3	F_4	F_5
1	2	2	3	3	3
2	5	4	3	3	2
3	1	2	3	4	5

(a) Numerical terms

Cluster	F_1	F_2	F_3	F_4	F_5
1	SI	SI	MI	MI	MI
2	HI	I	MI	MI	SI
3	NI	SI	MI	I	HI

(b) Linguistic terms

Table 3: General prioritization of customer preferences represented by fuzzy numbers

Linguistic terms	Fuzzy numbers
HI – “Highly Important”	[7 9 10 10]
I – “Important”	[5 6 8 9]
M – “Moderately Important”	[3 5 5 7]
SI – “Somewhat Important”	[1 2 4 5]
NI – “Not Important”	[0 0 1 3]

Table 4: Technical evaluation of product features represented by fuzzy numbers

F_1	F_2	F_3	F_4	F_5
[0 1 4 6]	[0 4 5 7]	[0 1 2 3]	[0 2 4 6]	[0 1 2 3]
[2 4 6 8]	[8 9 10 10]	[1 2 3 4]	[2 3 6 7]	[1 2 4 5]
[7 8 10 10]	-------	[3 4 5 7]	[4 6 7 9]	[2 3 5 6]
-------	-------	[4 5 6 8]	[7 8 10 10]	[3 4 6 7]
-------	-------	[6 7 8 9]	-------	[5 6 8 9]
-------	-------	[7 8 10 10]	-------	[7 8 10 10]

Table 5: Fuzzy preference relation of Cluster 1

$F_{ij}\backslash C_{ki}$	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}
	[1 2 4 5]	[1 2 4 5]	[3 5 5 7]	[3 5 5 7]	[3 5 5 7]
F_{11} [0 1 4 6]	0.4667				
F_{12} [2 4 6 8]	0.7857				
F_{13} [7 8 10 10]	1.0000				
F_{21} [0 4 5 7]		0.6429			
F_{22} [8 9 10 10]		1.0000			
F_{31} [0 1 2 3]			0.0000		
F_{32} [1 2 3 4]			0.0000		
F_{33} [3 4 5 7]			0.4444		
F_{34} [4 5 6 8]			0.6667		
F_{35} [6 7 8 9]			0.9670		
F_{36} [7 8 10 10]			1.0000		
F_{41} [0 2 4 6]				0.1875	
F_{42} [2 3 6 7]				0.4167	
F_{43} [4 6 7 9]				0.7750	
F_{44} [7 8 10 10]				1.0000	
F_{51} [0 1 2 3]					0.0000
F_{52} [1 2 4 5]					0.0000
F_{53} [2 3 5 6]					0.3000
F_{54} [3 4 6 7]					0.5000
F_{55} [5 6 8 9]					0.8666
F_{56} [7 8 10 10]					1.0000

Table 6: Product features for each cluster

Features	**Clusters**		
	1	**2**	**3**
F_{11}	**0.0333**	0.5	**0.2692**
F_{12}	0.2857	0.4792	0.4792
F_{13}	0.5	**0.0556**	0.5
F_{21}	**0.1429**	0.4048	**0.1429**
F_{22}	0.5	**0.1667**	0.5
F_{31}	0.5	0.5	0.5
F_{32}	0.5	0.5	0.5
F_{33}	**0.0556**	**0.0556**	**0.0556**
F_{34}	0.1667	0.1667	0.1667
F_{35}	0.467	0.4167	0.4167
F_{36}	0.5	0.5	0.5
F_{41}	0.3125	0.3125	0.5
F_{42}	**0.0833**	**0.0833**	0.3571
F_{43}	0.275	0.2750	**0.0833**
F_{44}	0.5	0.5	0.3182
F_{51}	0.5	0.3	0.5
F_{52}	0.5	**0**	0.5
F_{53}	0.2	0.1667	0.5
F_{54}	**0**	0.3333	0.5
F_{55}	0.3666	0.5	0.3667
F_{56}	0.5	0.5	**0.0556**

Table 7: Product features for each cluster

Cluster	Product configuration
1	$F_{11} - F_{21} - \mathbf{F_{33}} - F_{42} - F_{54}$
2	$F_{13} - F_{22} - \mathbf{F_{33}} - F_{42} - F_{52}$
3	$F_{11} - F_{21} - \mathbf{F_{33}} - F_{43} - F_{56}$

Table 8: Analysis of cluster centers with respect to product features.

Feature	*Variance*
1	3.82
2	1.39
3	0.02
4	0.15
5	2.46

Table 9: Features of the alternative product configuration

Product alternative formation	Product configuration
$F_{21} + M_1 = P_4$	$F_{12} – F_{21} – F_{33} – F_{41} – F_{51}$
$F_{21} + M_2 = P_5$	$F_{12} – F_{21} – F_{33} – F_{41} – F_{53}$
$F_{21} + M_3 = P_6$	$F_{12} – F_{21} – F_{33} – F_{41} – F_{55}$
$F_{21} + M_4 = P_7$	$F_{12} – F_{21} – F_{33} – F_{44} – F_{51}$
$F_{21} + M_5 = P_8$	$F_{12} – F_{21} – F_{33} – F_{44} – F_{53}$
$F_{21} + M_6 = P_9$	$F_{12} – F_{21} – F_{33} – F_{44} – F_{55}$
$F_{22} + M_1 = P_{10}$	$F_{12} – F_{22} – F_{33} – F_{41} – F_{51}$
$F_{22} + M_2 = P_{11}$	$F_{12} – F_{22} – F_{33} – F_{41} – F_{53}$
$F_{22} + M_3 = P_{12}$	$F_{12} – F_{22} – F_{33} – F_{41} – F_{55}$
$F_{22} + M_4 = P_{13}$	$F_{12} – F_{22} – F_{33} – F_{44} – F_{51}$
$F_{22} + M_5 = P_{14}$	$F_{12} – F_{22} – F_{33} – F_{44} – F_{53}$
$F_{22} + M_6 = P_{15}$	$F_{12} – F_{22} – F_{33} – F_{44} – F_{55}$

Table 10: Most often preferred features per cluster

Cluster	F_1	F_2	F_3	F_4	F_5
1	SI	SI	MI	MI	MI
2	HI	I	MI	MI	SI
3	NI	SI	MI	I	HI

Table 11: Identification of product configuration for each cluster

Product	Product configuration
4	$F_{12} - F_{21} - F_{33} - F_{41} - F_{51}$
5	$F_{12} - F_{21} - F_{33} - F_{41} - F_{53}$
6	$F_{12} - F_{21} - F_{33} - F_{41} - F_{55}$
7	$F_{12} - F_{21} - F_{33} - F_{44} - F_{51}$
8	$F_{12} - F_{21} - F_{33} - F_{44} - F_{53}$
9	$F_{12} - F_{21} - F_{33} - F_{44} - F_{55}$
10	$F_{12} - F_{22} - F_{33} - F_{41} - F_{51}$
11	$F_{12} - F_{22} - F_{33} - F_{41} - F_{53}$
12	$F_{12} - F_{22} - F_{33} - F_{41} - F_{55}$
13	$F_{12} - F_{22} - F_{33} - F_{44} - F_{51}$
14	$F_{12} - F_{22} - F_{33} - F_{44} - F_{53}$
15	$F_{12} - F_{22} - F_{33} - F_{44} - F_{55}$

1

2

3