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E-DEA: Enhanced Data Envelopment Analysis

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Abstract. Data Envelopment Analysis (DEA) has enjoyed a wide range of acceptance by researchers and practitioners alike as an instrument of performance analysis and management since its introduction in 1978. Many formulations and thousands of applications of DEA have been reported in a considerable variety of academic and professional journals all around the world. Almost all of the formulations and applications have basically centered at the concept of “relative self-evaluation”, whether they are single or multi-stage applications. This paper suggests a framework for enhancing the theory of DEA through employing the concept of “relative cross-evaluation” in a multi-stage application context. Managerial situations are described where such Enhanced-DEA (E-DEA) formulations had actually been used and could also be potentially most meaningful and useful.

Keywords. Data envelopment analysis (DEA), enhanced data envelopment analysis (E-DEA), relative performance, multi-stage DEA, multi-stage E-DEA, consensus formation, project selection, mathematical programming.

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1. INTRODUCTION

Since the appearance of the original and seminal DEA article by Charnes, Cooper and Rhodes in 1978, known as the CCR model, there has been a rapid growth in the field, both in terms of methodological developments and practical applications. It was reported that between the years 1978 and 2001 more than 1,800 articles published in refereed journals (Gattoufi *et al*, 2004a) worldwide. This number must have now exceeded 4,000. For a cyber-bibliography of a complete current listing of DEA writings as of 2005, the reader is referred to Seiford (2005).

Given this kind of rapid success and popularity of DEA as a performance evaluation method, there has been some need for and effort to classify the DEA literature. Gattoufi, Oral and Reisman, 2004b) suggested a particular taxonomy which is labeled as DEAN (D=Data, E=Envelopment Type, A=Approach to Analysis, N=Nature of the Article). Each of these four attributes are further subdivided to obtain a detailed description of each article comprising this rather wide-ranging field of knowledge as represented by the articles appeared in journals such as *European Journal of Operational Research* (where the first DEA article appeared), *Management Science, Journal of Productivity Analysis, Applied Economics, Journal of Econometrics, Journal of Banking and Finance, Journal of Socio-Economic Planning Sciences*. After presenting their taxonomy scheme, they classified some of the well known articles in the field according to their suggested scheme of categorization. See also Gattoufi *et al* (2004c) for a content analysis of DEA and Gattoufi *et al* (2004d) for an epistemological treatment of DEA.

The other comprehensive and more explained coverage of DEA literature is due to Cooper, Seiford and Tone (2007). In their book titled *Data Envelopment Analysis: A Comprehensive Text with Models, Applications, References and DEA-Solver Software*, the authors start with the basic CCR Model (the formulation given by Charnes, Cooper, and Rhodes in 1978) and then they discuss alternative DEA models and their theoretical properties in full detail. Their collection of alternative DEA models has been mostly based on the applications reported in the literature covering hundreds of journals representing different fields, ranging from finance and banking to health care, education to resource re-allocation, project selection to managerial performance measurement. In its essence, the book of Cooper, Seiford and Tone (2007) is an excellent source for those who are DEA researchers and/or practitioners.

For more effective and improved use of DEA models in dealing with a variety of practical problems, many new formulations and approaches have been suggested in the DEA literature. One of them is the “multi-stage approach”, which combines some use of parametric (mostly statistical methods) and nonparametric (basically DEA formulations) models. Cooper, Seiford and Tone (2007), for instance, discuss a particular three-stage DEA model that was employed to study Japanese Banking. The first stage is devoted to define the inputs and outputs of credit risk management, on the basis of which the authors conducted the conventional static DEA analysis to estimate efficiency scores and the output slacks. The second stage deals with the data adjustment through a set of environmental variables to explain the variation in the output slacks. Here a regression model in logorithmic form is used. The third stage is again a typical static DEA application, but this time the “adjusted data” obtained in the second stage are used in the formulations, rather than the raw original data, to estimate the efficiency scores of the same decision making units. Said differently, the first stage is a DEA work, the second stage is a statistical study, and the third stage is again DEA. Such a process is depicted in a summarized form in Figure 1. There are also theoretical studies that relate DEA to other models. In this regard, the work of Cooper (2005) establishing relationships between goal programming and DEA, and the article of Wang and Chin (2009) integrating AHP (analytical hierarchy process) with DEA are just two examples for multi-stage approach. For an industrial application of DEA with AHP, the reader is referred to, for instance, Sevkli *et al* (2007).

More recently, Eilat, Golany, Shtub (2008) discuss a model, basically a multi-stage approach, that integrates the Balanced Scorecard (BSC) of Kaplan and Norton (1996) with DEA and name it as “extended DEA model”. The input and output scores for the integrated DEA–BSC model are grouped in “cards” which are associated with a “BSC for R&D Projects”, the BSC model. The authors then embed the BSC in the DEA model through a hierarchical structure of constraints that reflect the BSC balance considerations. They illustrate the proposed methodology with a case study involving an industrial research laboratory that selects and executes dozens of R&D projects every year. Interestingly, the methodology is able to handle the R&D project evaluation process corresponding to different stages of project life cycle: initiation, planning, execution, and closure. Because of this very characteristic of the methodology, it provides a basis for decision-making with respect to which ongoing projects should be continued, how the resources should be allocated to selected or ongoing projects. This process seems to create a knowledge base of “best practices” and “lessons learned” that contributes to organizational learning.

Given the variety and extensive use of DEA formulations in practice, there was a need for identifying and discussing some pitfalls in DEA applications and their remedies. Dyson *et al* (2001) identified 18 pitfalls in DEA and suggested remedial protocols for each: 3 pitfalls in *homogeneity assumptions,* 3 pitfalls with respect to *the set of inputs/outputs* used, 4 pitfalls regarding *factor measurement*, 4 pitfalls in relation to *weights*, and 4 pitfalls in formulating *weight restrictions.* This set of pitfalls and protocols are extremely useful for DEA practitioners and it will be even more so as the field of DEA progresses. In this paper, we will add three more pitfalls and protocols to the list of Dyson *et al* (2001) when using the cross-efficiency concept of DEA in theory and practice.

When looked from a different classification perspective, however, one can observe that almost all DEA models are based on the concept of "relative self-evaluation" for each decision making unit (DMU). There are some exceptions to this statement though. The concept of "cross-efficiency" or "peer-appraisal", in addition to "self-efficiency" or "self-evaluation", has also been used in DEA formulations, albeit in rather limiting forms. We shall return to this issue later in Section 2. The works of Sexton *et al* (1986), Oral *et al* (1991), Doyle and Green (1994), Green *et* al (1996), Oral *et* al (2001), Adler *et al* (2002), Wu *et* al (2008), Liang *et al* (2008a), Liang *et al* (2008b), and Wu *et al* (2009) are some examples using the concept of "cross-efficiency" in one way or another.

The primary objective of this paper is to emphasize the usefulness and importance of using both self-evaluations and cross-evaluations properly, called Enhanced-DEA, or shortly E-DEA. The structure of this paper is as follows. The next section, Section 2, introduces the concept of Enhanced-DEA and explains it fully in connection with DEA, whether it is a single-stage or multi-stage modeling process. Section 3 presents one example where an E-DEA model has been constructed and used in practice. Section 4 suggests two more areas where E-DEA models can be potentially meaningful and useful. The last section, Section 5, includes some concluding remarks.

2. DEA VERSUS E-DEA

In this section, we first present the original DEA formulation that is basically the classical "self-efficiency" model, and its use in decision making with other non-DEA models. Then we provide a formulation of E-DEA, a formulation that integrates both "self-efficiency" and "cross-efficiency" scores. Also to be discussed is the decisional context that motivates and justifies the use of E-DEA formulation, along with some non-DEA models. Another point to be made is the way the concept of "cross-efficiency" defined and used in the literature.

DEA: The basic relative performance model of DMU – *i*, as perceived by DMU – i itself, can be formulated, following the CCR model (Charnes, Cooper, Rhodes,1978 and 1981), as

Model A: Self-Evaluation Model (DEA)

$$\left\{ \begin{array}{l} E_{ii} = Max \quad \left(\sum_k u_{ik} y_{ik}\right) / \left(\sum_r v_{ir} x_{ir}\right) \\ \text{subject to} \\ \qquad \left(\sum_k u_{ik} y_{jk}\right) / \left(\sum_r v_{ir} x_{jr}\right) \le 1, \quad \forall j \\ \qquad u_{ik} \ge 0, \forall k \text{ and } v_{ir} \ge 0, \forall r \end{array} \right\}$$

where

E_{ii} = the efficiency of DMU–*i, (i=1,2,…,n)* as "most favorably" evaluated by DMU–*i,*

y_{jk} = the quantity of output *k* produced by DMU-*j, k=1,2,…,m* and *j=1,2,…,n*

u_{ik} = the coefficient of y_{ik}, the value of which is to be optimally determined,

x_{jr} = the quantity of input *r* used by DMU-*j, r=1,2,…,q* and *j=1,2,..,n,*

v_{ir} = the coefficient of x_{ir}, the value of which is to be optimally determined.

Model A, in the presence of n different DMUs, needs to be used n times to estimate the self-evaluation scores of all DMUs, implying that the above optimization is to be performed n times. The self-evaluation scores, E_{ii}, are then used, either by themselves alone or combined with other methods, as it is done in the case of multi-stage applications, to make decisions. See Figure 1.

DEA Model A provides the "most favorable" efficiency score E_{ii} for DMU-*i* ; that is, the efficiency of DMU-*i* is most favorably perceived or optimistically estimated by DMU-*i* itself. If Model A is repeated for all DMU-*i, i*=1,2,,…, n, then we have n number such optimistic estimates: E_{ii}, *i*=1,2,,…, n. The Model A and the self-evaluation scores obtained from it in fact define a decision making context with the following characteristics:

- There are two sets of criteria used to estimate relative performance: one set includes *inputs* and the other *outputs* and the ratio of *outputs* to *inputs* is called *efficiency* as the measure of performance*.* This definition of efficiency is the basis of decision making and therefore suggests a context where performance is defined in linkage with the concept of “system”. In this system, a set of inputs is transformed into a set of outputs.

- The *efficiency* score E_{ii} is *most favorable* because of the maximization, and *relative* because of the constraints in Model A. There might be situations however where the concept of *most favorable* needs to be replaced by *least favorable* one*.* (See, for instance, Oral, Kettani, Yolalan, 1992 and Despotis, 2002). But this need or preference does not change the very nature of DEA models.
- The set of outputs are linked non-parametrically to the set of inputs through the concept of efficiency that is expressed in ratio form. In the efficiency expression of Model A, the output coefficients (u_{ik}) and the input coefficients (v_{ir}) are to be optimally found from the perspective of DMU-*i.* If DMU-*i* is found to be *inefficient* using Model A*,* then managerial measures are formulated according to the efficient DMUs that are in the *reference set* of DMU-*i.* These efficient DMUs in the reference set can be also called *local leaders* for DMU-*i.* If one needs to find the efficiency of DMU-*i* with respect to not only to the local leaders but also with respect to a “global leader”, then the formulation of Model A can be slightly modified. (See, for instance, Oral and Yolalan 1990, Oral *et al* 1992, and Despotis, 2002.)
- The input and output coefficients, (v_{ir}) and (u_{ik}) respectively, are more than being only “weights”. They play two roles at the same time: (1) they convert incommensurate units into commensurate ones, and (2) they indicate the importance of inputs and outputs – only in this case they correspond to the term “weights” as used in the literature
- Letting each DMU-*i* determine their own optimal coefficient values, with which none of the DMUs could have an efficiency score higher than 1, in fact, defines a particular decision making context, a context in which each DMU is allowed to have a “say” or “voice” with respect to its own relative performance. This is an important feature that DEA models are able to offer. Thus subjectivity, favoring itself optimally, is an accepted feature and applies to every DMU equally. In a

sense we can even term the efficiency scores E_{ii} as *model-based behavioral relative self-evaluations.*

- Although each DMU is allowed to have a voice with respect to its own relative performance, no DMU however is permitted to have a "say" or "voice" when it comes to the performance evaluations of the other DMUs in the observation set. This is rather a limiting feature of DEA models, especially for those decision contexts where one DMU's perception of the other DMUs is important and needs to be taken into consideration. In other words, DEA models produce and use only the E_{ii} values and ignore the other possible values E_{ij} of the matrix **E**, when $i \neq j, \forall i, j$. Here, E_{ij} is the cross-efficiency score of DMU-*j* from the perspective of DMU-*i.*

Model A implies that only the diagonal elements of a possible complete matrix **E** = $\lfloor E_{ij} \rfloor$ are being used in "conventional" DEA decisions. In fact, on the other hand, decision making process can be "enhanced" by using all the elements of matrix **E.** For this purpose however the "enhancing" elements E_{ij}'s for all $i \neq j$ of matrix **E** need to be computed. This is done using Model B: Cross-Evaluation Model below. Moreover, there are decisional contexts where such "enhanced" efficiency scores are most meaningful, useful, and even necessary.

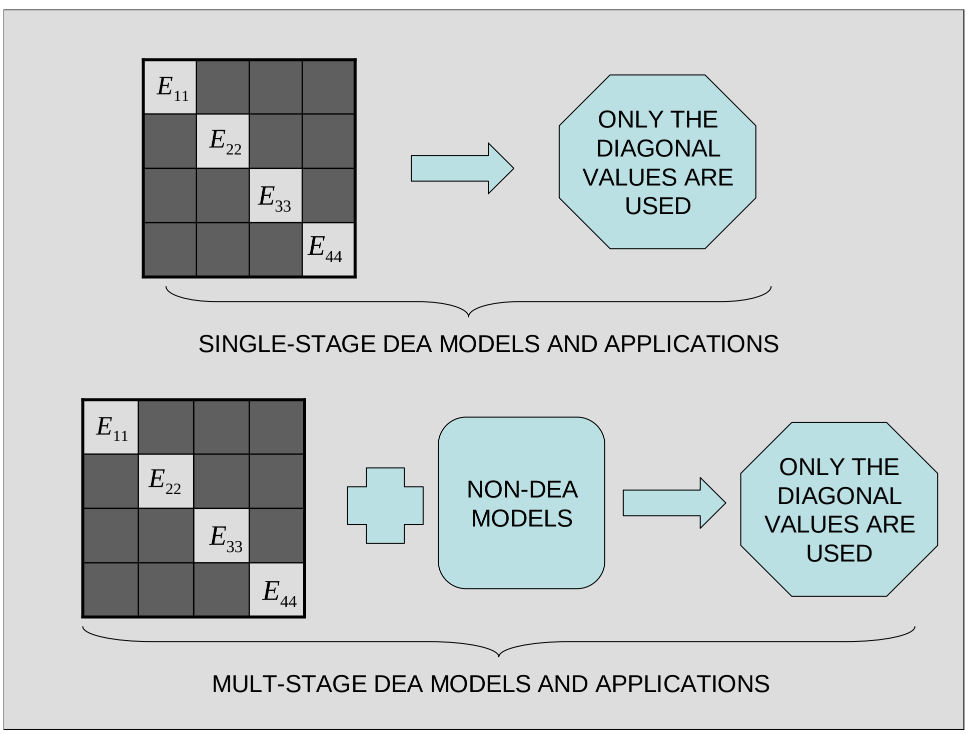

Figure 1: Single-Stage and Multi-Stage DEA Models

E-DEA: The concept of E-DEA refers to the use of the entirety of the information included in the nxn matrix **E**, the diagonal elements of which are obtained from Model A and the rest from Model B below. There are decisional contexts where all of the values of the matrix **E** are needed or required. Before discussing such decisional contexts we shall first describe how the matrix **E** is formed. For this purpose, we define E_{ij} as the relative efficiency of DMU-*j* as evaluated by DMU-*i.* Also assume that we have already obtained the efficiency score E_{ii} from Model A above, thus the diagonal elements of matrix **E** are already available. For the non-diagonal elements, now consider the following DEA model:

Model B: Cross-Evaluation Model

$$\left\{ \begin{array}{l} E_{ij} = Max \left(\sum_k u_{ijk} y_{ik}\right) / \left(\sum_r v_{ijr} x_{ir}\right) \\ \text{subject to} \\ \quad \left(\sum_k u_{ijk} y_{tk}\right) / \left(\sum_r v_{ijr} x_{tr}\right) \leq 1, \ \forall t \\ \quad \left(\sum_k u_{ijk} y_{jk}\right) / \left(\sum_r v_{ijr} x_{jr}\right) = E_{ii} \\ \quad u_{ijk} \geq 0 \text{ and } v_{ijr} \geq 0, \ \forall i, j, k, r, t \end{array} \right\}$$

where

E_{ij} = the efficiency score of DMU-*j* estimated by using those "coefficients" of DMU-*i* that maintain the efficiency level of DMU-*i* at the previously estimated value E_{ii}, or DMU-*i* evaluates DMU-*j,*

u_{ijk} = the coefficient of output *k* produced by DMU-*j* that maintains the efficiency level of DMU-*i* at E_{ii},

v_{ijr} = the coefficient of input *r* used by DMU-*j* that maintains the efficiency level of DMU-*i* at E_{ii},

Model B produces cross-evaluation scores that are basically *relative cross-evaluations* in the sense that each and every DMU in the observation group, but while doing this they maintain their *relative self-evaluation* scores unchanged that are obtained from Model A. Therefore, Model B needs to be repeatedly used ($n^2 - n$) times to produce the non-diagonal elements of the matrix **E**. The diagonal elements were already obtained from Model A. So we have formed the matrix **E** in its entirety.

The information content of the matrix **E** is much more representative of the collective values of DMUs, for it includes (i) each DMU's own perception of itself with respect to relative performance and (ii) each and every DMU's perception of the others again with respect to relative performance. In other words, using the entirety of the matrix **E**, without losing any information, "enhances" decision making processes. The lost of any information included in the matrix **E**, such as using some central tendency measures instead of the entirety of the matrix **E**, is against the very nature of collective decision making. This issue will be discussed later in more detail. Let us now list some of the decisional contexts where the entirety of the matrix **E** is most meaningful, and perhaps even required.

Full Participation: Suppose there are n different DMU's. Each DMU wants to have a "say" not only in its own case, but also in every DMU's case. This "right" of each DMU could have been given formally and openly. Budget discussions in large organizations, be private or public, sets an example for a decisional context where some sort of full participation of its members is expected or required. In such cases, we need the entirety of the matrix **E.**

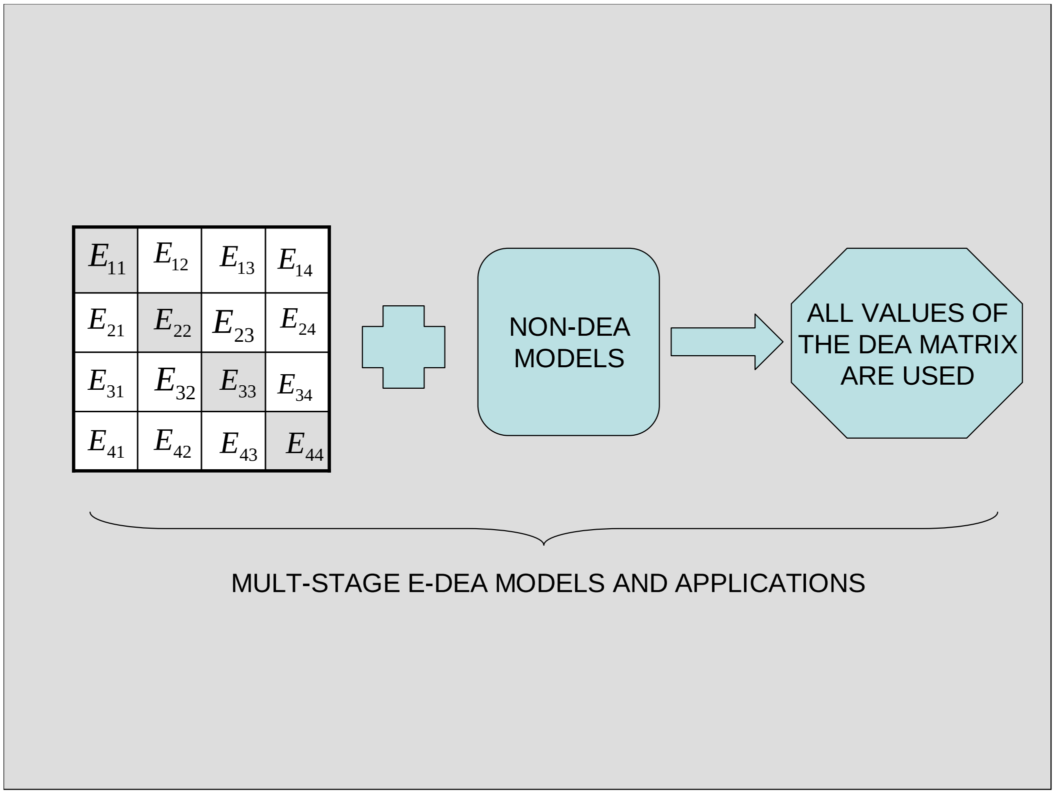

Figure 2: Multi-Stage E-DEA Models

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Different Value Systems: Each DMU might have its own different value system as to importance or "weight" of each criterion used for evaluation. It is very natural and normal that each DMU uses its own "coefficients", which reflect its own value system in a sense, for the evaluation of the other DMU's as well. Any zero-sum type of decisional context is an example where everybody evaluates everybody according to one's own value system. Such decisional contexts imply that we need to use the entirety of the matrix **E** again.

Transparency: There are decisional contexts where the presence of "transparency" matters considerably for their stakeholders, mostly in international organizations such as World Bank, IMF, NATO, NAFTA, UNIDO, UNICEF as well as in all sorts of national government bodies. The members of such organizations would like know how the decisions are made and what the rules are. In such decisional contexts again the nature of "vote" and "appraisal" of a member about the other members in the group are crucial. Once again this requires the entirety of matrix **E.**

At this juncture, it is appropriate to discuss the concept of "cross-efficiency" as used in the literature and compare and contrast them with the one given by Model B above. In DEA literature, the concept of "cross-efficiency" is based on a central tendency measures, but mostly on an "average" estimate. See, for instance, Sexton *et al* (1986), Doyle and Green (1994), Green *et al* (1996), Lins *et al* (2003), Liang *et al* (2008), and Wu *et al* (2009), More specifically, "cross-efficiency" score is given by, using our notation,

$$\overline{E}_j = \frac{1}{n}\sum_{i=1}^{n} E_{ij} \qquad (1)$$

where E_{ij} is the efficiency of DMU-*j* according to DMU-*i* and *n* is the number of DMUs under consideration. The implicit assumption is that the "cross-efficiency" scores E_{ij} s are obtained through the constraints of Model A.

There are some pitfalls of using Equation 1 to find "cross-efficiency" scores. We shall discuss three of them in the context of E-DEA. The reader is referred to Dyson *et al* (2001) for a set of pitfalls and protocols in DEA.

<u>Pitfall 1: Multiple Optimal Solutions:</u> The non-uniqueness of the DEA optimal coefficients obtained from Model A creates a confusion in applying Equation 1 as to which set of the optimal coefficients of DMU-*i* is to be used in evaluating DMU-*j*. More specifically, let $S_{Ai} = \{S_{i1}, S_{i2}, ..., S_{ip}\}$ be the set of p number of optimal solutions obtained from Model A as the coefficients yielding the same value of E_{ii}, where $S_{ip} = (\vec{u}_{ip}, \vec{v}_{ip})$ and $\vec{u}_{ip}$ is the vector of p-th output optimal coefficients for DMU-*i* and $\vec{v}_{ip}$ is the vector of p-th input optimal coefficients. Which of these p optimal solutions then will be used in finding the cross-efficiency of DMU-*j* ? Is it the same optimal solution to be used for every DMU or the one that gives the best result for DMU-*j* ?

<u>Remedy 1: Use the Optimal Solution Giving the Best Result:</u> The basic spirit behind DEA is to give the benefit of doubt to every DMU in estimating self-efficiencies in a most favorable manner. This very spirit of DEA needs to be maintained in the case of cross-efficiency estimations as well. Model B does exactly this by finding the optimal solution for DMU-*j* while maintaining the optimal solution previously obtained from Model A for DMU-*i.*

<u>Pitfall 2: Central Tendency versus Pareto Estimation:</u> Using Equation 1 to find cross-efficiency scores results in considerable loss of information that could or should have been used in decision making process. Rather than using the entirety of the matrix **E**, which has n^2 elements, Equation 1 reduces this number to $2n$, *n* number of averages plus *n* number of self-efficiency scores*,* thus limiting the use of information considerably. Perhaps it is a good idea to define a measure called "*degrees of lost information."* Let us define it as the number of elements of matrix **E not used** in decision making. Then the consequence of using Equation 1 is $n^2 - 2n = n(n-2)$ as the degrees of lost information. The very philosophy of DEA is to work with a concept related to efficiency frontier, not with average or any other central tendency measure. This fact has also been recognized by some of the users of cross-efficiency concept and they tried to remedy the weakness of Equation 1 by suggesting some procedures. Doyle and Green (1994) proposed aggressive/benevolent formulations, Wu *et al* (2008) used cooperative game approach, Liang *et al* (2008) the Nash equilibrium to decrease the degrees of lost information.

Another way interpreting Equation 1 is that DMUs do not really matter as individual units, only their "average" counts, rather a very limiting way of considering DMUs in decision making process.

<u>Remedy 2: Use the Entirety of Matrix **E**:</u> To maximize the use of available information one needs to include all the elements of matrix **E** as obtained from Model A and Model B in decision making process. Ideally, the degree of lost information should be equal to zero. This might however require the development of a non-DEA model that really utilizes all the information, as will be seen later in the next section.

<u>Pitfall 3: Differences in the Sets of Optimal Solutions:</u> The set of optimal solutions S_{Ai} obtained from Model A might not be the same as the set of optimal solutions S_{Bj} found from Model B. However, we know that there is at least one optimal solution in set S_{Bj} that is also in set S_{Ai}, because of the second constraint in Model B, implying $S_{Ai} \cap S_{Bj} \neq \emptyset$. It is also possible that some optimal solutions in S_{Bj} might not be in S_{Ai}. If one is restricted to use only those optimal solutions in S_{Ai} in finding cross-efficiency scores then one is violating the very principle of DEA; that very principle is to favor the DMU being evaluated. Perhaps there are some optimal

solutions in S_{Bj}, but not in S_{Ai}, that might give a higher cross-efficiency score E_{ij} for DMU-*j* while maintaining the optimal self-efficiency score E_{ii} for DMU-*i.*

Remedy 3: Use Model B: To favor DMU-*j* in finding its cross-efficiency from the perspective of DMU-*i*, use Model B because it produces optimal coefficients for DMU-*j* while assuring the optimal efficiency score E_{ii} for DMU-*i.*

In what follows, we discuss a case where the entirety of matrix **E** is properly used and also suggests two more areas of possible applications.

3. AN APPLICATION OF E-DEA

In this section, we shall summarize a real-life application of E-DEA Methodology in multistage form: Collective Evaluation and Selection of Industrial R&D Projects.

Collective Evaluation and Selection of Industrial R&D Projects: An E-DEA methodology in a multistage form, was first used, although not under the E-DEA label, in evaluating and selecting R&D projects in the Turkish iron and steel industry (Oral, Kettani, and Lang, 1991). The stages of the methodology used are summarized in Figure 3. In the first stage, as can be observed from Figure 3, the relative "self-efficiency" E_{ii} of each R&D project was found using a conventional DEA formulation – Model A. The R&D project "outputs" were "direct economic contribution", "indirect economic contribution", "technological contribution", "scientific contribution", and "social contribution". There was one "input" considered and it was the "budget" of each R&D project. The reader is referred to Oral, Kettani, and Lang (1991) for the details of input and output criteria used and how the scores were obtained with respect to each criterion. There were 37 R&D projects proposed. In the second stage, the relative "cross-efficiency" scores E_{ij}'s were obtained from Model B. In this context, E_{ij} is the efficiency of R&D Project *j* from the viewpoint of R&D Project *i.* In the third stage, we have a non-DEA model, but based on the entire matrix **E,** for the selection of R&D projects. Here the elements of the matrix **E** were first converted into what is called *concordance* matrix **C** = $[C_{ij}]$, where $C_{ij} = (1/n)\sum_k \phi_{ijk}$ and $\phi_{ijk} = 1$ if $E_{ik} \geq E_{jk}$ and $\phi_{ijk} = 0$ otherwise. With these definitions, C_{ij} is then the ratio of the superiority of R&D Project *i* over R&D Project *j* as perceived from the view points of all R&D projects. For example, if $C_{ij} = 0.75$, then according to 75% of the R&D projects, R&D Project *i* is superior to R&D Project *j.* Or, in multiple criteria analysis terminology, R&D Project *i* outranks R&D Project *j* at the concordance level of 0.75. In a sense, the matrix **C** provides pair wise comparisons between the R&D projects. Now the question is what minimum level of concordance one would like to accept for selecting projects? This question leads to definition of *consensus* level, denoted by θ. R&D Project *i* is said to outrank R&D Project *j* at the consensus level θ if $C_{ij} \geq \theta$. If R&D Project *i* outranks R&D Project *j* then we define an indicator variable α_{ij} as $\alpha_{ij} = 1$ if $C_{ij} \geq \theta$, and $\alpha_{ij} = 0$ otherwise. This definition permits us to identify the pairs of R&D projects between which there is an outranking relationship at the consensus level of θ. In other words, for a given value of θ, α_{ij}'s completely determine all the existing outranking relationships between the R&D projects under consideration for selection. The totality of these outranking relationships is given by the following expressions:

$$\theta + \alpha_{ij} \leq C_{ij} + 1, \qquad \forall i, j \text{ , } i \neq j$$
$$\theta + \alpha_{ij} \geq C_{ij} + \varepsilon \text{ ,} \qquad \forall i, j \text{ , } i \neq j$$

where ε is a sufficiently small positive number, used to actually enforce a strict inequality. Since θ could take on values only in the discrete set of {0, 1/n, 2/n,…,1} it is readily verified that any value in the interval of (0, 1/n) is appropriate for ε. The important point behind the above expressions is that outranking relationships between R&D projects can be analytically formulated and analyzed.

The "transparency" principle of consensual decision making context suggests that we must obey some resentment avoiding rules in selecting R&D projects. The *internal* and *external consistencies* of Roy and Vincke (1981) were used as the resentment avoiding rules. The *internal consistency* requires that the selected set of projects should include only those projects that are not outranked by any selected project. The *external consistency*, on the other hand, holds when the set of rejected projects include only those projects each of which is outranked by at least one of the selected projects. Then the *internal* and *external consistencies* can be mathematically expressed as

$$\sum_{i \neq j} \alpha_{ij} \beta_i + \beta_j \geq 1, \forall j$$

$$\sum_{i \neq j} \alpha_{ij} \beta_i + (n-1)\beta_j \leq n-1, \forall j$$

where $\beta_i = 1$ if R&D Project *i* is selected for funding and $\beta_i = 0$ otherwise.

SELF-EVALUATION MODEL – MODEL A

$$E_{ii} = Max \left(\sum_k u_{ik} y_{ik}\right) / \left(\sum_r v_{ir} x_{ir}\right)$$

$$\left(\sum_k u_{ik} y_{jk}\right) / \left(\sum_r v_{ir} x_{jr}\right) \leq 1$$

$u_{ik} \geq 0, \forall k$ and $v_{ir} \geq 0, \forall r$

CROSS-EVALUATION MODEL – MODEL B

$$E_{ij} = Max \left(\sum_k u_{ijk} y_{ik}\right) / \left(\sum_r v_{ijr} x_{ir}\right)$$

$$\left(\sum_k u_{ijk} y_{tk}\right) / \left(\sum_r v_{ijr} x_{tr}\right) \leq 1$$

$$\left(\sum_k u_{ijk} y_{jk}\right) / \left(\sum_r v_{ijr} x_{jr}\right) = E_{ii}$$

$u_{ijk} \geq 0$ and $v_{ijr} \geq 0 \quad \forall i, j, k, r, t$

PROJECT SELECTION MODEL – NON DEA MODEL

$$Max\ \theta$$

$$\theta + \alpha_{jk} \leq C_{jk} + 1, \forall j, k \ldots j \neq k$$

$$\theta + \alpha_{jk} \geq C_{jk} + \varepsilon, \forall j, k \ldots j \neq k$$

$$\sum_{i \neq j} \alpha_{ij} \beta_i + \beta_j \geq 1, \forall j$$

$$\sum_{i \neq j} \alpha_{ij} \beta_i + (n-1)\beta_j \leq n-1, \forall j$$

$$\sum_j E_j \beta_j \leq B$$

Figure 3: An E-DEA Model for R&D Project Selection

The last constraint in the "Project Selection Model – Non-DEA Model" in Figure 3 corresponds to a budgetary constraint in funding R&D projects. This constraint states that the optimal selection of R&D projects must be done within the available budget *B,* where E_j is the budget required for R&D Project *j.*

The multistage E-DEA model presented in Figure 3 needs to be used repeatedly or in an iterative manner until the available budget *B* for funding R&D projects is exhausted. This is what was exactly done in the case of the Turkish iron and steel industry to evaluate and select R&D projects for funding. Out of 37 candidates, only 16 were selected and took 9 iterations to complete the evaluation and selection process. The first 8 iterations suggested sets of R&D projects for funding, each at a consensus level of 100%, whereas the last iteration with a consensus level of only 73%. The reader is referred to Oral, Kettani, and Lang (1991) for the details of this application of E-DEA model.

The "Project Selection Model - Non-DEA Model" in Figure 3 appears to be a complicated model to obtain a solution from. This is due to the presence of quadratic integer constraints

$$\sum_{i \neq j} \alpha_{ij} \beta_i + \beta_j \geq 1, \forall j \text{ and } \sum_{i \neq j} \alpha_{ij} \beta_i + (n-1)\beta_j \leq n-1, \forall j .$$

and therefore one might need to use of a linearization method to transform the quadratic constraints into a set equivalent linear constraints. Although such a linearization can be achieved using the method of Oral and Kettani (1992), there is no need to go through such a linearization process by simply observing that α_{ij} s are in fact a function of θ, because α_{ij} s are defined as $\alpha_{ij} = 1$ if $C_{ij} \geq \theta$, and $\alpha_{ij} = 0$ otherwise. If one wishes to write the definition of α_{ij} in more detailed form, we have $\alpha_{ij}(\theta) = 1$ if $C_{ij} \geq \theta$, and $\alpha_{ij}(\theta) = 0$ otherwise. Given this detailed definition, we can easily conclude that the values of α_{ij} s are known once a value of θ is given. On the other hand, we know that θ can take on values only in the discrete set of {0, 1/n, 2/n,…,1} because of the outranking possibilities that exist between projects. Then the procedure to be used in project selection becomes rather straightforward. First, set $\theta = 1$ and find the corresponding $\alpha_{ij}(\theta)$ s. Then substitute these values of $\alpha_{ij}(\theta)$ in the constraints of the project selection model. Any feasible solution in β_j s is an optimal solution. If there is no feasible solution then set $\theta = (n-1)/n$, the next maximum value of θ. And repeat the same procedure until a feasible solution is found.

The approach Oral *et al* (1991) has been taken as a benchmark by some others. For instance, Doyle and Green (1994), Green *et al* (1996), Liang *et al* (2008), and Wu *et al* (2009). They have developed their own cross-efficiency formulations, mostly central tendency measures, and applied them to the raw data provided in Oral *et al* (1991). The data set of 37 project proposals (coded with numbers 1,2,3,…,37) prepared in the Turkish Iron and Steel Industry has the following properties. Each project is valued with respect to five output criteria

Table 1: Cross-Efficiency Formulations and Their Impact on Project Selection

Project Code	Oral *et al* (1991)	Doyle and Green (1994)	Green *et al* (1996)	Liang *et al* (2008)	Wu et al (2009)
35	Yes	Yes	Yes	Yes	Yes
17	Yes	Yes	Yes	Yes	Yes
31	Yes	Yes	Yes	Yes	Yes
16	Yes	Yes	Yes	Yes	Yes
36	Yes	Yes	Yes	Yes	Yes
18	Yes	Yes	Yes	Yes	Yes
27	Yes	Yes	Yes	Yes	Yes
34	Yes	Yes	Yes	Yes	Yes
37	Yes	Yes	Yes	Yes	Yes
23	Yes	Yes	Yes	Yes	Yes
26	Yes	Yes	Yes	Yes	Yes
1	Yes	Yes	Yes	Yes	Yes
14	Yes	Yes	Yes	Yes	Yes
32	No	Yes	Yes	Yes	Yes
21	Yes	Yes	Yes	Yes	Yes
29	Yes	Yes	Yes	No	Yes
12	No	Yes	Yes	Yes	Yes
5	Yes	No	No	No	No
15	No	No	No	Yes	No
Number of Projects	16 Projects	17 Projects	17 Projects	17 Projects	17 Projects
Budget Required	956.3	982.9	982.9	994.7	982.9
Budget Available	1,000	1,000	1,000	1,000	1,000

(direct economic contribution, indirect economic contribution, technological contribution, scientific contribution, and social contribution) and one input criterion (project budget). The reader is referred to Oral *et al* (1991) for the details of the data set of 37 R&D projects and how these scores were obtained with respect to each output and input criterion through a series of participative workshops. The total available budget to be allocated to the selected projects is 1,000 Monetary Units.

Table 1 summarizes the project selection results of the five studies that used the same data set but employing different cross-efficiency formulations.

A comparison of the results in Table 1 suggests some conclusions. First, the method of Oral *et al* (1991) selects 16 projects out of 37 for funding whereas the other studies favor 17 projects. The smaller number might be due to the fact that the Oral *et al* (1991) approach respect the resentment avoiding principles at the highest level of consensus possible. The other studies, however, do not take such principles into consideration. Second, the procedures suggested by Doyle and Green (1994), Green *et al* (1996), and Wu *et al* (2009) produced exactly the same set of projects for funding. The sets of Oral *et al* (1991) and Liang *et al* (2008) are slightly different in this regard. Third, the highest utilization of the available budget is realized by the R&D program of Liang *et al* (2008), which is an amount of 994.7 monetary units. The lowest utilization is given by the R&D program of Oral *et al* (1991). This lower budget utilization might be again due to the fact of respecting the resentment avoiding principles.

For a recent application of the same approach in the context of an international organization, the reader is referred to Oral, Kettani, Çınar (2001) where a consensual investment decisions in a network of collaboration were to be made. The reader is also referred to Lee *et al* (2009) for a comparative evaluation of performance of national R&D programs using a DEA approach.

4. POTENTIAL APPLICATIONS AREAS FOR E-DEA

We shall present two potential applications areas for E-DEA model, again in multistage forms. Both areas are of great importance to those who are in a position to formulate public policies. The first one falls in the area of country risk rating, and the second in country competitiveness.

Country Risk Rating: The way the international financing is done, especially between sovereign borrowers and international creditors, profoundly influences the relationships between countries, not to mention its effect on world peace. As Oral *et al* (1992) indicated, the role of certain financial institutions and agencies in this process is more than negligible. For instance, the process of country risk ratings that are periodically produced by Institutional Investor (biannually), International Country Risk Guide (monthly), Euromoney (annually), and Business Environment Risk Index (quarterly) is a good example where sovereign borrowers are put in a position of being judged by some group of experts and consultants as to their economic capacity and credit-worthiness. These ratings are most influential on determining the interest rates to be charged to sovereign borrowers by the international creditors, thus leaving the borrowing countries to face the consequences of such transactions.

Moreover, the country risk ratings produced by the above institutions are not usually revealed to the public. Because it is not transparent, the researchers in international finance try to “guess” how these ratings are produced mostly by relating the ratings with a certain set of country-specific characteristics. The existing literature is full of articles aiming at finding the determinants of country risk so that the ratings, especially the ones produced by Institutional Investor, can be explained or described, mostly through a statistical model. Works of Feder and Raiffa (1985), Brewer and Rivoli (1990), and Cosset *et al* (1993) are typical examples in this category. One can question the merits of such an approach for at least two reasons: (1) why the transparency of country risk ratings is being avoided, and (2) why all sovereign borrowers are not fully participating in the rating process.

To remedy the shortcomings of the existing rating procedures, “real” or “guessed” ones, it is possible to suggest an alternative methodology for country risk rating. The suggested methodology has the following characteristics:

- It acknowledges the fact that creditors and sovereign borrowers are the principal stakeholders who have to live with the consequences of an evaluation process of credit-worthiness. Then it is most natural to have them directly involved in this process. Therefore, a country has a “say” about its credit-worthiness as well as in the credit-worthiness of other countries. However, the weight of a “say” of a given country might be different than the weights of the other countries in the group of credit seeking countries.
- It assumes that a country is a “socio-economic system” which consumes human, financial, and natural resources to produce the goods and services needed for the

country. The relative efficiency and effectiveness of such a system within a group of countries is taken as the measure for the country's socio-economic achievement and hence is employed as the basis of its credit-worthiness.

- It complies with a certain set of resentment-avoiding rules so that the credit-seeking countries could have confidence in the process, even in the case of their unfavorable credit-worthiness classification.
- It assumes that there is a certain amount of funds available for the credit-seeking countries.

The mathematical formulations of the suggested methodology are, in appearance, exactly the same as the one given in Figure 3. However, there are some differences in the meanings attached to the parameters and variables. When the models in Figure 3 are to be used for country risk rating we need to be aware of the following differences:

- The first difference is that the definitions of C_{ij}'s are not the same. Recall that the definition of C_{ij} in Figure 3 was $C_{ij} = (1/n)\sum_k \phi_{ijk}$, implying that equal "voting weight" for everyone. However, within the context of country risk rating, we need to recognize big differences between the countries. For instance, USA (one of the most powerful economies in the world) and Swaziland (a tiny little kingdom in South East Africa with almost relatively no economic presence) cannot carry the same weight when it comes to rating the credit worthiness of other countries. To make a distinction between the countries, we need to assume each country could have different "voting power" or "voting weight", say λ_k for Country *k.* Then the definition of C_{ij} becomes $C_{ij} = \sum_k \lambda_k \phi_{ijk}$, $\lambda_k \geq 0$ and $\sum_k \lambda_k = 1$. This definition assumes that each country could have a different "voting weight or power" that reasonably reflects her role in international finance arena. Now the question becomes how the values of λ_k's should be determined? One way could be the country's share in the world population. In this case, $\lambda_k = P_k / \sum_i P_i$, where P_i is the population of Country *i, i*=1,2,...*k*,..N. Another way could be the country's share in the gross world product; that is, $\lambda_k = G_k / \sum_i G_i$, where G_i is the gross national product of Country *i*. Or, any other formulation that will justify the "voting powers" of the countries.

- The notations E_j and B stand, respectively, for the amount of credit sought by sovereign borrower *j* and the totality of international availability of funds.
- The other implicit difference of the models in Figure 3, when used for country risk rating, is that a "class of countries" with respect to credit-worthiness is identified. For instance, at the end of iteration one, the most credit-worthy countries are identified (we might label them as "AAA countries"); and at the end of iteration two, the second most credit-worthy countries (we might call them "AA countries") are found, and so on. In other words, the models in Figure 3 can be used as a clustering approach for countries according to their credit-worthiness.

Here are the general guidelines for using the models in Figure 3 for the purpose of country risk rating. *First*, assuming a country is a "socio-economic system", we need to identify the "inputs" and "outputs" of such a system that make sense within the context of country risk rating. Just to be suggestive, we can consider the following as inputs: business efficiency, government efficiency, infrastructure, domestic economic performance, science and technology, and the like. On the output side, we can include, for instance, domestic consumption, exports, savings, and foreign debt services. What is needed here is to develop a sort of accounting system in the sense of "national accounting" that is used in economics, but appropriate for the concepts employed in the proposed methodology above. *Second*, we form the set of sovereign borrowers for which the country risk rating will be performed. Depending on the potential users of the results, one can form different sets of sovereign borrowers for different public policies. For instance, if countries like Argentina, Brazil, Thailand, or Turkey would like to position themselves in a particular group of credit-seeking sovereign borrowers, they may do so by deciding which countries to include and which countries to exclude from the "observation set." *Third*, we need to find a meaningful method to find the values of λ_k's. A couple of suggestions were already made above. Another method could be to reflect the viewpoints of the existing country risk evaluators (Fitch Ratings, Institutional Investor, Euromoney, etc.) in a similar way. For instance, $\lambda_k = R_k / \sum_i R_i$, where R_i is the risk rating of Country *i* by, say, Institutional Investor. *Fourth*, there are four types of data needed to use the suggested methodology: (1) inputs used, (2) outputs produced, (3) amount of credit sought by each country, and (4) information needed for estimating λ_k's. The main sources for the data and information could be

IMD's World Competitiveness Yearbooks, WEF's Global Competitiveness Reports, World Bank, OECD, and IMF.

Country Competitiveness: Firms are at the front line in the battle of international competition. Although this is true, the competitive environment of firms also plays an important role in and contributes considerably to the competitiveness of firms. A country with its natural resources, human capabilities, research and educational institutions, government organizations, financial and banking system, and cultural and social values provides a competitive environment in which firms are created, organized, and managed. There is no doubt that the national competitive environment in a country considerably influences the performance of its firms at home and abroad. Therefore, it is of prime importance for both governments and firms to study the competitive environment of a country in comparison with those of others, especially within the context of globalization of business, politics, and culture.

The competitiveness of countries has been made a subject of research since the early 1980s. Porter (1990), in his book titled *Competitive Advantage of Nations,* employs a framework, named "the National Diamond", to study the competitiveness of 10 countries (Britain, Denmark, Germany, Italy, Japan, S. Korea, Singapore, Switzerland and the United States.) Applying "the National Diamond" framework to these countries, he suggests an agenda for each country to pursue to become internationally more competitive. The basic idea behind "the National Diamond" framework is to analyze the economy of a country, historically, industry by industry, in terms of (1) *factor conditions,* (2) *demand conditions,* (3) *supporting and related industries,* (4) *firm strategy, structure, and rivalry,* (5) *government role,* and (6) *chance factor.* The results of the analysis are then translated into a set of policy recommendations for each country that is included in the study. "The National Diamond", in terms of methodology, favors basic statistical techniques to understand the characteristics of a country's competitive advantage, industry by industry, over a long period of time, 20-25 years. There is however no mathematical formula or model that describes adequately the "National Diamond" and the way it can be used and therefore it remains as a "user-specific" framework.

The other major studies dealing explicitly with the competitiveness of countries are due to the Institute for Management Development (IMD) and the World Economic Forum (WEF), both are located in Switzerland. Since 1980, they produce independent annual reports, sometimes jointly though, titled now *World Competitiveness Yearbook* (IMD) and *Global Competitiveness*

Report (WEF). Both IMD and WEF basically use the same methodology, multiple criteria approach, in rating and ranking countries. For instance, IMD's *World Competitiveness Yearbook* uses more than 300 criteria in rating the competitiveness of a country. A summary of IMD's methodology is given in Figure 5. As can be observed form Figure 5, the competitiveness score of a country is a function of four "factors": *economic performance, government efficiency, business efficiency,* and *infrastructure.* Then each "factor" is defined by a set of "sub-factors." The competitive factor "Economic Performance" consists of the following "sub-factors": *domestic economy, international trade, international investments, employment,* and *prices.* And each "sub-factor" is, in return, defined by a set of "criteria".

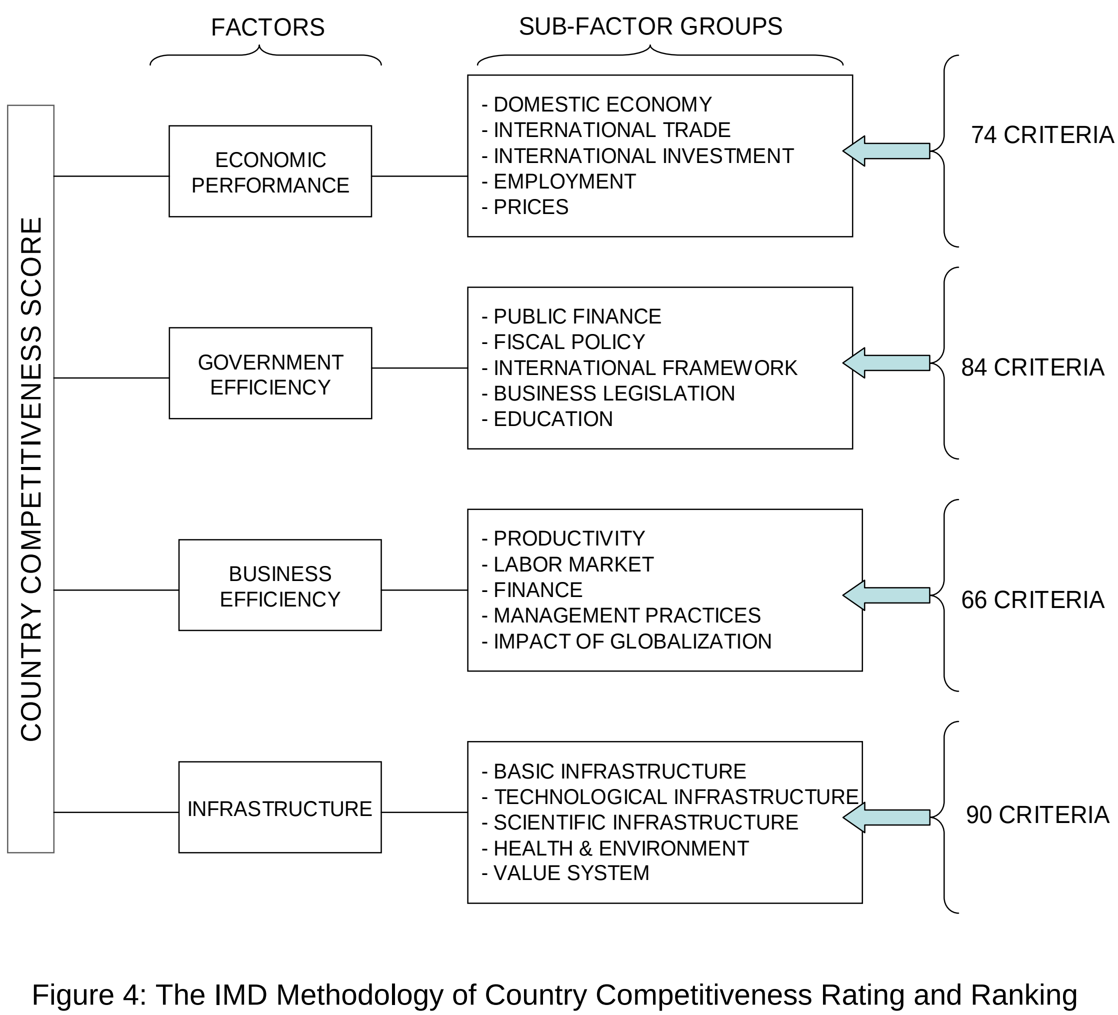

Figure 4: The IMD Methodology of Country Competitiveness Rating and Ranking

The methodology of IMD is more explicit when compared with that of Porter's 1990 framework. In fact, the IMD methodology can be expressed mathematically. Let L_k be the competitiveness rating of Country *k.* Then

$$L_k = F_{k1} + F_{k2} + F_{k3} + F_{k4}$$

where F_{kr} is the score of Country *k* with respect to "Factor" *r, r*=1,2,3, and 4,. The scores F_{kr}'s are given by

$$F_{kr} = f_{kr1} + f_{kr2} + f_{kr3} + f_{kr5} + f_{kr5}$$

where f_{kri} is the score of Country *k* with respect to "Sub-Factor" *i* of "Factor" *r,* and *i*=1,2,3,4, and 5. And at the lowest level of aggregation,

$$f_{kri} = \sum_{j \in C_i} w_j S_{krij}$$

where S_{krij} is the standardized score of Country *k* with respect to Criterion *j* of "Sub-Factor" *i* of "Factor" *r* , w_j is the importance of Criterion *j,* and C_i is the set of criteria used to define "Sub-Factor" *i.*

Given this methodology of IMD, the countries are ranked at four levels: (1) criterion level ranking based on S_{krij}'s, (2) sub-factor level ranking based on f_{kri}'s, (3) factor level based on F_{kr}'s, and (4) overall country ranking based on L_k's. In addition to these country rankings, *The World Competitiveness Yearbook* (WCY) is also useful for different analyses. First, the 5-year competitiveness trends of countries at the overall and factors levels are provided, so that countries can study their performance patterns. Second, WCY also includes a "competitiveness balance sheet" in which the strengths and weaknesses of each country are indicated. Third, WCY permits to examine the impact of "factors" and "sub-factors" so that the competitive structure of a country can be understood. For some comments on the earlier version of IMD methodology, see Oral and Chabchoub (1996, 1997)

A comparison of Porter's framework with IMD's methodology reveals that the approach of IMD is much more explicit as to what is being done in competitiveness rating and ranking of countries. Put differently, Porter's framework serves as a sort of guideline for the "case study" to be done, whereas IMD's method is a formal structure that explicitly describes how the ratings and rankings are determined. With respect to usability and usefulness, Porter's framework can produce a country-specific agenda as to what is to be done to become more competitive, for it is based on the needs of the "case" country. IMD's method, on the other hand, gives a more general picture within the group of over 50 or so countries. Although it is a general picture the IMD methodology nevertheless depicts the relative position of each country among the countries included in the yearbook. This feature of IMD's WCY is lacking in the case of Porter's framework.

Although the above approaches might have some advantages and disadvantages against one another, there are certain conceptual elements that are absent in both. The world is *a network of economic, political, and cultural collaboration*. Friedman (2006) offers many examples of such collaborations in his book titled *The World is Flat.* Given the globalization process that is expanding with the advances in communication and logistics, almost exponentially, we need to think of and conceptualize country competitiveness within the context of a network of collaborating countries. We observe that at least the following characteristics in such a network of collaborating countries exist:

- Each country is trying to do her best in a global competitive environment, according to her preferences, values, and goals. This implies that each country would like to be perceived as a worthy partner in the network of collaborating countries and at the same time her preferences, values and goals are respected. In our context of E-DEA, this characteristic translates into a "self-evaluation" model.

COUNTRY SELF-EVALUATION – MODEL A

$$E_{ii} = Max\left(\sum_k u_{ik} y_{ik}\right) / \left(\sum_r v_{ir} x_{ir}\right)$$

$$\left(\sum_k u_{ik} y_{jk}\right) / \left(\sum_r v_{ir} x_{jr}\right) \leq 1$$

$$u_{ik} \geq 0, \forall k \quad \text{and} \quad v_{ir} \geq 0, \forall r$$

COUNTRY CROSS-EVALUATION – MODEL B

$$E_{ij} = Max\left(\sum_k u_{ijk} y_{ik}\right) / \left(\sum_r v_{ijr} x_{ir}\right)$$

$$\left(\sum_k u_{ijk} y_{tk}\right) / \left(\sum_r v_{ijr} x_{tr}\right) \leq 1$$

$$\left(\sum_k u_{ijk} y_{jk}\right) / \left(\sum_r v_{ijr} x_{jr}\right) = E_{ii}$$

$$u_{ijk} \geq 0 \text{ and } v_{ijr} \geq 0 \quad \forall i, j, k, r, t$$

COUNTRY COMPETITIVENESS RANKING MODEL – NON DEA MODEL

$$Max \ \theta$$

$$\theta + \alpha_{jk} \leq C_{jk} + 1, \forall j, k \ldots j \neq k$$

$$\theta + \alpha_{jk} \geq C_{jk} + \varepsilon, \forall j, k \ldots j \neq k$$

$$\sum_{i \neq j} \alpha_{ij} \beta_i + \beta_j \geq 1, \forall j$$

$$\sum_{i \neq j} \alpha_{ij} \beta_i + (n-1)\beta_j \leq n-1, \forall j$$

$$\sum_j \beta_j = R$$

Figure 5: An E-DEA Model for Country Competitiveness Ranking

- Each country has a perception of other collaborating countries, and such perceptions shape the nature of networked collaboration. The perception a country forms of another country is shaped by the preferences, values, and goals of the perception forming country. In a community of collaborating countries it does matter what a member country think of others and vice versa. This characteristic is nothing but “cross-evaluation” in the context of E-DEA in multi stage format.
- World trade is to be done according to a set of agreed rules and regulations, thus enforcing some degree of transparency among collaborating countries. Such a transparency is needed to reduce, if possible avoid, the likely resentments that might occur among collaborating countries. The implication of this characteristic is that any kind of competitiveness rating and ranking should be done in such a way that lower ranked countries should not question the positions of higher ranked

> countries. In other words, lower ranked countries have no resentments at all as to their positions in the ranking and the positions of higher ranked countries are justified. This implies that the competitiveness ranking of countries needs to be done with the highest level of consensus possible and without any resentment. In the parlance of E-DEA, this corresponds to "Country Competitiveness Ranking Model" given in Figure 5.

There are many conceptual similarities between the models in Figure 3 and the models in Figure 5. The only major conceptual difference is the last constraint. In the "Country Competitiveness Ranking" Model in Figure 5, the last constraint is $\sum_j \beta_j = R$ whereas it is $\sum_j E_j \beta_j \leq B$ in the Selection Model in Figure 3. The meaning of constraint $\sum_j \beta_j = R$ is that ranking is to be done by a group of *R* countries. If $R = 1$, then the ranking will be done one by one, which is nothing but conventional ranking.

The general guidelines to apply the "Country Competitiveness Ranking" Model in Figure 5 can be listed as the following. *First,* again assuming that a country is a "socio-economic system" competing in a global context, we need to identify the "inputs" and "outputs" to be used in "self-evaluation" DEA model and "cross-evaluation" E-DEA model. For instance, as "outputs" we might consider "exports", "domestic consumption" and "investments abroad". These three criteria suggest that the totality of a country's output is consumed internally (domestic consumption) and externally (exports and investments abroad). On the inputs side, one might consider "imports", "foreign direct investments", "domestic investments", "production factors", "government efficiency", and "infrastructure." *Second,* we form the set of countries that will be included in competitiveness ranking. Again depending on the likely users of country ranking results, one can form different sets of countries. For instance, we can use the same set of countries included in the annual reports of IMD or WEF. Or, we can include only those countries that are in developing stage. *Third*, as in the case of "Country Risk Rating", here also we define C_{ij}'s as $C_{ij} = \sum_k \lambda_k \phi_{ijk}$, $\lambda_k \geq 0$ and $\sum_k \lambda_k = 1$. A meaningful definition of λ_k could be $\lambda_k = G_k / \sum_i G_i$, where G_i is the gross national product of Country *i*. *Fourth*, there are three types of data needed to use the suggested methodology: (1) inputs used, (2) outputs produced, (3) information needed for estimating λ_k's. The main sources for the data and information could be again IMD's World Competitiveness Yearbooks, WEF's Global Competitiveness Reports, World Bank, OECD, and IMF.

5. CONCLUDING REMARKS

The paper has formally introduced the concept of E-DEA in a multi-stage form and how it distinguishes itself from the conventional DEA approach has been offered. The multi-stage E-DEA approach, because of its rich information content, is most appropriate for many complex and participative decision making contexts. To illustrate the usefulness and usability of the approach, one real-life application has been summarized in the area of industrial R&D project evaluation and selection. Moreover, another reference was given that describes a real application in the area of investment decision making in an international organization. Also explained were two potential consensual decisions making contexts where a multi-stage E-DEA approach could be most useful: one in the area of country risk rating which is a global concern for both sovereign borrowers and international creditors, and the other in the area of country competitiveness rating and ranking which is a concern for all national policy makers. The list of likely applications of multi-stage E-DEA approach can be increased. For instance, performance evaluation in human resource management is an important task and this area presents itself as a good candidate. This is even more so if the human resources department wishes to apply a 360 degree performance evaluation method to find the best candidate for a higher position in the organization.

Also pointed out was that the concept of cross-efficiency as defined and used in the DEA literature is a limiting version. In this regard, the notion of “degrees of lost information” was introduced and when a central tendency measure like “average” is used as a surrogate for the totality of matrix **E**, it was found that the degree of lost information is $n(n-2)$. Not using the entirety of matrix **E** as obtained from E-DEA Model; that is Model B in this paper, three pitfalls are identified in the sense of Dyson *et al* (2001) and remedies were suggested.

In summary, multi-stage E-DEA methodology is particularly appropriate for decisional contexts with the following characteristics: (1) each and every DMU has a “say” in its own evaluation, (2) each and every DMU has also a “say” in the evaluation of other DMUs, (3) transparency and democratic principles are to be respected, (4) resentments among DMUs are to be avoided, and (5) decisions are to be made to achieve a highest level of consensus possible. There is no doubt that there are many decisional contexts in different functional areas where the above characteristics prevail. Existence of such areas implies that there is really great potential for DEA researchers to expand and extend their expertise further.

REFERENCES

Adler, N., L. Friedman, S. Sinuany-Stern, Review of Ranking Methods in the Data Development Analysis Context, *European Journal of Operational Research,* 2002, Vol.140, pp.249-265.

Brewer,T., Rivoli. P. Politics and Perceived Country Credit-Worthiness in International Banking. *Journal of Money, Credit, and Banking,* 1990, Vol.22, p. 357-369.

Cooper, W.W. Origins, Uses of, and Relations Between Goal Programming and Data Envelopment Analysis, *Journal of Multi-Criteria Decision Analysis,* 2005, Vol.13, p.1-9.

Cooper, W.W., Seiford, L.M., Tone, K. *Data Envelopment Analysis: A Comprehensive Text with Models, Applications, References and DEA-Solver Software,* Springer. 2007.

Charnes, A., Cooper, W.W., Rhodes, E.L. Measuring the Efficiency of Decision Making Units, *European Journal of Operational Research* 1978; 2; p. 429-444.

Charnes, A., Cooper, W. W., Rhodes, E. L. Evaluating Program and Managerial Efficiency: An Application of Data Envelopment Analysis to Program Follow Through, *Management Science* 1981, Vol.27, p. 668-697.

Cosset, J-C., Daouas, M., Kettani, O., Oral, M., Replicating Country Risk Ratings, *Journal of Multinational Financial Management*, 1993, Vol. 3 (1/2) pp.1-29.

Despotis, D.K., Improving the Discriminating Power of DEA: Focus on Globally Efficient Units, *Journal of the Operational Research Society,* 2002, Vol.53, pp.314-323.

Doyle, J.and R. Green, (1994), Efficiency and Cross Efficiency in DEA: Derivations, Meanings, and Uses," *Journals of the Operational Research Society,* 1994*,* Vol.45, No.5, pp.567-578.

Dyson, R.G., Allen, R., Camanho, A.S., Podinovski, V.V., Sarrico, C.S., Shale, E.A., Pitfalls and Protocols in DEA, 2001, *European Journal of Operational Research,* Vol.132, pp.245-259.

Eilat, H., Golany, B., Shtub, A., R&D Project Evaluation: An Integrated DEA and Balanced Scorecard Approach. *Omega,* 2008, Vol.36, Issue 5, p.895-912.

Feder, R.L., Raiffa, H. The Determinants of International Credit-Worthiness and Their Policy Implications. *Journal of Policy Modeling,* 1985; 7; p.133-156.

Friedman, T.L., *The World is Flat: The Globalized World in the Twenty-First Century,* Penguin Books, London, England; 2006.

Gattoufi, S., Oral, M., Reisman, A. Data Envelopment Analysis Literature: A Bibliography Update 1996-2001. *Journal of Socio-Economic Planning Sciences*, 2004a; 38; p.159-229.

Gattoufi, S., Oral, M., Reisman, A. A Taxonomy for Data Envelopment Analysis. *Journal of Socio-Economic Planning Sciences*, 2004b; 38; p. 141-158.

Gattoufi, S., Oral, M., Kumar, A., Reisman, A. Content Analysis of Data Envelopment Analysis Literature and Comparison with that of OR/MS Fields. *Journal of the Operational Research Society*. 2004c; 55; Issue 9, p.911-935.

Gattoufi, S. Oral, M., Kumar, A., Reisman, A., Epistemology of Data Envelopment Analysis and Comparison with Other Fields of OR/MS for Relevance to Applications", *Journal of Socio-Economic Planning Sciences*, 2004d; 38; p.123-140.

Green, R.H, Doyle, J.R., Cook, W.D. Preference Voting and Project Ranking Using DEA and Cross-Evaluation, *European Journal of Operational Research,* 1996, Vol.90, pp. 461-472.

Kaplan, R., Norton, D. *The Balanced Scorecard,* Harvard College, 1996, Boston, USA.

Lee, H., Park, Y., Choi, H., Comparative Evaluation of Performance of National R&D Programs with Heterogeneous Objectives: A DEA Approach, *European Journal of Operational Research,* 2009, Vol.196, pp.847-855.

Liang, L., Cook, W.D., Zhu, J., The DEA Game Cross-Efficiency Model and Its Nash Equilibrium. *Operations Research,* 2008, Vol.56, No.5, pp.1278-1288.

Liang, L., J.Wu, W.D. Cook, J. Zhu, "Alternative Secondary Goals in DEA Cross-Efficiency Evaluation," *International Journal of Production Economics,* 2008, Vol.113, pp.1025-1030.

Lins, M.P.E., Gomes, E.G., Soares de Mello, J.C.C.B., Soares de Mello, A.J.R., Olympic Ranking Based on a Zero Sum Gains DEA Model, *European Journal of Operational Research,* 2003, Vol.148, pp.312-322.

Oral, M., Kettani, O., Çınar, Ü. Project Evaluation and Selection in a Network of Collaboration: A Consensual Disaggregation Multi-Criterion Approach. *European Journal of Operational Research,* 2001; 130; p.332-346.

Oral, M., Chabchoub, H. On the Methodology of the World Competitiveness Report. *European Journal of Operational Research*, 1996; 90; p.514-535.

Oral, M., Chabchoub, H. An Estimation Model for Replicating the Rankings of the World Competitiveness Report. *International Journal of Forecasting*, 1997; 13; p.527-537.

Oral, M., Kettani, O., Lang, P. A Methodology for Collective Evaluation and Selection of Industrial R&D Projects. *Management Science*, 1991; 37; No.7, p.871-885.

Oral, M. and O. Kettani, (1992), "A Linearization Procedure for Quadratic and Cubic Mixed-Integer Problems," *Operations Research*, Vol.40, Supplement 1, pp.S109-S116.

Oral, M., Yolalan, R. An Empirical Study on Measuring Operating Efficiency and Profitability of Bank Branches. *European Journal of Operational Research*, 1990; 46; No.3, p.282-294.

Oral, M., Kettani, O., Yolalan, R. An Empirical Study on Analyzing the Productivity of Bank Branches," *IIE Transactions*, 1992; 24; No.5, p.166-176.

Oral, M., Kettani, O., Cosset, J-C., Daouas, M., An Estimation Model for Country Risk Rating," *International Journal of Forecasting,* 1992; Vol.8, No.4, pp.583-593.

Porter, M. E. *Competitive Advantage of Nations,* Free Press, New York, N.Y. 1990.

Roy, B., Vincke, P., Multicriteria Analysis: Survey and New Directions. *European Journal of Operational Research,* 1991; Vol.8, p.207-218.

Seiford, L.M., “A Cyber-Bibliography of Data Envelopment Analysis (1978-2005),” CD included in Cooper, W.W., L.M. Seiford, and K. Tone, (2007), *Data Envelopment Analysis: A Comprehensive Text with Models, Applications, References and DEA-Solver Software,* Springer.

Sevkli, M., Koh, S.C.L., Zaim, S., Demirbag, M., Tataoglu, E., An Application of Data Envelopment Analytic Hierarchy for Supplier Selection: A Case Study of BEKO in Turkey, *International Journal of Production Research,* 2007, Vol.45, No.9, 1973-2003.

Sexton, T.R., Silkman, R.H. Hogan, A.J., Data Envelopment Analysis: Critique and Extensions. In Silkman, R.H. (Ed.), *Measuring Efficiency: An Assessment of Data Envelopment Analysis,* 1986, Vol.32, Jossey-Bass,San Francisco, pp.73-105.

Wang, Ying-Ming, and Chin, Kwai-Sang, “A New Data Envelopment Analysis Method for Priority Determination and Group Decision Making in the Analytical Hierarchy Process,” *European Journal of Operational Research*, 2009, Vol.195, pp.239-250,

Wu, J., L. Liang, F. Yang, H. Yan, Bargaining game model in the evaluation of decision making units, *Expert Systems with Applications,* 2009, Vol.36, pp.4357-4362.

Wu, J., L. Liang, Z. Ying-chun, Determination of Weights of Ultimate Cross Efficiency based on the Solution of Nucleolus in Cooperative Game, *Systems Engineering,* 2008, Vol.28, No. 5, pp.92-97.