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The Impact of Prudence on Optimal Prevention Revisited

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Abstract. This paper re-examines the link between absolute prudence and self-protection activities. We show that the level of effort chosen by a prudent agent is larger than the optimal effort chosen by a risk neutral agent if and only if the degree of absolute prudence is less than a threshold that is utility-independent. We explain this threshold by a trade-off between the variation of the variance and the level of the third moment of the loss distribution. We also discuss our result in terms of skewness. Our contribution extends the model of Eeckhoudt and Gollier (2005).

Keywords. Absolute prudence, moments of the loss distribution, self-protection, variance, skewness.

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1 Introduction

The optimality of self-protection activities was first examined by Ehrlich and Becker (1972). It is well known that increased risk aversion does not necessarily raise the optimal investment in prevention (see, e.g., Dionne and Eeckhoudt, 1985; Briys and Schlesinger, 1990; Briys et al., 1991). This negative result is still not well explained in the literature.

Recently, some papers studied the effect of prudence on optimal prevention. Jullien, et al. (1999) and Chiu (2000) show that prudence plays a role in the determination of thresholds for optimal prevention. However, their thresholds are utility-dependent, and thus vary from agent to agent. Eeckhoudt and Gollier (2005) propose as sufficient condition that the level of effort chosen by a prudent (imprudent) agent to be smaller (larger) than the optimal effort chosen by a risk-neutral agent.¹

We extend the analysis by linking optimal prevention and prudence. We prove that the level of effort chosen by a prudent agent is larger than the optimal effort chosen by a risk-neutral agent if and only if absolute prudence is less than a threshold that is utility-independent, and stays the same for all agents. This threshold is equal to the "marginal change in probability on variance per third moment of loss distribution." Intuitively, the level of effort chosen by a prudent agent is larger than the optimal effort chosen by a risk-neutral agent when the negative effect of self-protection on the variance is larger than the positive effect on the third moment of the loss distribution. We also show that our result can deliver unambiguous comparative static results for some self-protection problems. Our contribution extends the contribution of Eeckhoudt and Gollier (2005) by providing a necessary and sufficient condition.

The article is organized as follows. Section 2 presents the model and main result. Section 3 explains the result in terms of loss distribution moments and skewness. Section 4 shows how the main result applies to HARA utility functions. Section 5 provides some comparative static results. Section 6 concludes the paper.

¹Menegatti (2009) prove that, in a two-period framework, prudence has a positive effect on optimal prevention. Here we are limited to a single period model. Chiu (2005a) show that, when protection activities are mean-preserving, degree of absolute prudence plays an important role in determining the optimal choice of self-protection. We do not consider the mean-preserving condition in this paper. Eeckhoudt et al. (2010) and Dachraoui et al. (2004) consider background risk as well.

2 The Model and Main Result

We consider an expected utility maximizer who is endowed with wealth w_0 and faces the risk of losing the amount L with probability $p(e)$; e is the amount of money invested in prevention and $p(e)$ is differentiable with respect to e . We assume that the utility function u on final wealth is increasing and differentiable. The decision problem for a risk-averse individual can be written as

$$e^* \in \arg \max_{e \geq 0} V(e) = p(e)u(w_0 - e - L) + (1 - p(e))u(w_0 - e). \quad (1)$$

We assume that V is concave in e (see Arnott, 1991, and Jullien et al., 1999, for analyses of the different conditions). The optimal preventive investment e_n for the risk-neutral agent, assuming an interior solution, is given by

$$-p'(e_n)L = 1. \quad (2)$$

Condition (2) states that the marginal cost must equal the marginal benefit.

Define $p_n = p(e_n)$ as probability of loss of the risk-neutral agent, and $w_n = w_0 - e_n$ as the agent's final wealth in the no-loss state. We have the following main proposition.

Proposition 2.1 *Risk-averse agents exert more effort than the risk-neutral agent if and only if*

$$AP(w_n - L) \leq \frac{1 - 2p_n}{p_n} \frac{1}{L}, \quad (3)$$

where $AP(x) = -\frac{u'''(x)}{u''(x)}$ is the absolute prudence coefficient (Kimball, 1990).

Proof Because V is concave, the optimal effort, e^* , for a risk-averse agent, will be larger than e_n if and only if $V'(e_n)$ is positive:

$$V'(e_n) = -[p_n u'(w_n - L) + (1 - p_n)u'(w_n)] - p'(e_n)[u(w_n) - u(w_n - L)] \geq 0. \quad (4)$$

Using condition (2), we see that $V'(e_n) \geq 0$ if and only if

$$\frac{u(w_n) - u(w_n - L)}{L} \geq p_n u'(w_n - L) + (1 - p_n)u'(w_n) \quad (5)$$

or if and only if

$$u(w_n) - u(w_n - L) - L[p_n u'(w_n - L) + (1 - p_n)u'(w_n)] \geq 0. \quad (6)$$

i) Sufficiency:

Define $H(L) = u(w_n) - u(w_n - L) - L[p_n u'(w_n - L) + (1 - p_n)u'(w_n)]$. Then we have $H(0) = 0$ and

$$H'(L) = (1 - p_n)[u'(w_n - L) - u'(w_n)] + p_n L u''(w_n - L). \quad (7)$$

Hence $H'(0) = 0$ and

$$H''(L) = (2p_n - 1)u''(w_n - L) - p_n L u'''(w_n - L). \quad (8)$$

Therefore, $H''(L) \geq 0$ is a sufficient condition for $V'(e_n) \geq 0$. Because

$$\begin{aligned} AP(w_n - L) &\leq \frac{1 - 2p_n}{p_n} \frac{1}{L} \quad (9) \\ \Leftrightarrow -\frac{u'''(w_n - L)}{u''(w_n - L)} &\leq \frac{1 - 2p_n}{p_n} \frac{1}{L} \\ \Leftrightarrow H''(L) &\geq 0, \end{aligned}$$

we obtain

$$AP \leq \frac{1 - 2p_n}{p_n} \frac{1}{L} \Rightarrow V'(e_n) \geq 0. \quad (10)$$

ii) Necessity:

We prove the necessity by a contradiction. Suppose that $V'(e_n) \geq 0$ and $AP(w_n - L) > \frac{1 - 2p_n}{p_n} \frac{1}{L}$. Since

$$\begin{aligned} AP(w_n - L) &> \frac{1 - 2p_n}{p_n} \frac{1}{L} \quad (11) \\ \Leftrightarrow -\frac{u'''(w_n - L)}{u''(w_n - L)} &> \frac{1 - 2p_n}{p_n} \frac{1}{L} \\ \Leftrightarrow H''(L) &< 0, \end{aligned}$$

then, combining this result with the fact $H'(0) = 0$, we obtain that $H'(L) < 0$ for all $L > 0$. Finally, $H(0) = 0$ and $H'(L) < 0$ for all $L > 0 \Rightarrow H(L) < 0$ for all $L > 0$, which means $V'(e_n) < 0$. This is the desired contradiction! Q.E.D.

Proposition 2.1 shows that the threshold value, $\frac{1 - 2p_n}{p_n} \frac{1}{L}$, partitions the set of prudent agents into two subgroups: those who are hardly prudent and those who are very prudent. The hardly prudent agents produce more effort than the risk-neutral agent and the very prudent agents produce less effort than the risk-neutral agent.

When $p_n \geq \frac{1}{2}$, $\frac{1 - 2p_n}{p_n} \frac{1}{L} \leq 0$, and $AP \geq \frac{1 - 2p_n}{p_n} \frac{1}{L}$ for all risk-averse and prudent agents. Proposition 2.1 states that risk-averse and prudent agents exert less effort than the risk-neutral

agent. Dachraoui et al. (2004) showed that $p_n \leq \frac{1}{2}$ is a necessary condition for more mixed risk-averse agents to spend more effort. Proposition 2.1 is a complement of their result. If $p_n \leq \frac{1}{2}$, then $\frac{1-2p_n}{p_n} \frac{1}{L} \geq 0$, and $AP \leq \frac{1-2p_n}{p_n} \frac{1}{L}$ for all risk-averse and imprudent agents. Proposition 2.1 indicates that risk-averse and imprudent agents exert more effort than the risk-neutral agent. Eeckhoudt and Gollier's result (2005, Corollary 1) is recovered.

Proposition 2.1 also shows that, if $p_n \leq \frac{1}{2}$, in order to have more effort than the risk-neutral agent, we need an upper bound on prudence, that is $AP \leq \frac{1-2p_n}{p_n} \frac{1}{L}$.

Suppose $p_n < \frac{1}{2}$, then we have

$$\lim_{p_n \rightarrow 0} \frac{1-2p_n}{p_n} \frac{1}{L} = \infty \quad (12)$$

and

$$\lim_{L \rightarrow 0} \frac{1-2p_n}{p_n} \frac{1}{L} = \infty. \quad (13)$$

Hence, Proposition 2.1 suggests that, for small probability of loss of the risk-neutral agent ($p_n \rightarrow 0$), or small amount of loss ($L \rightarrow 0$), all risk-averse and prudent agents will exert more effort than the risk-neutral agent, which may appear to be a paradox. However there exist many applications where L is large and p_n is very low (catastrophe loss, environmental loss) and where L is small and p_n is quite high but less than $\frac{1}{2}$ (current life loss). In fact, there are very few accidents where $p_n \geq \frac{1}{2}$.

Before explaining our result in terms of second and third moments, we give another explanation for Proposition 2.1 in terms of relative prudence. We note that if $w_n - L > 0$, then

$$\begin{aligned} AP(w_n - L) &\leq \frac{1-2p_n}{p_n} \frac{1}{L} & (14) \\ \Leftrightarrow -\frac{u'''(w_n - L)}{u''(w_n - L)} &\leq \frac{1-2p_n}{p_n} \frac{1}{L} \\ \Leftrightarrow -(w_n - L) \frac{u'''(w_n - L)}{u''(w_n - L)} &\leq \frac{1-2p_n}{p_n} \frac{w_n - L}{L} \\ \Leftrightarrow RP(w_n - L) &\leq \frac{1-2p_n}{p_n} \frac{w_n - L}{L}, \end{aligned}$$

where $RP(x) = -x \frac{u'''(x)}{u''(x)}$ is the coefficient of relative prudence. Some contributions in the literature suggest that the benchmark value for RP is 2 (see, e.g., Hadar and Seo, 1990; Choi et al., 2001; Gollier, 2001, pp. 60-61; White, 2008). Because

$$\frac{1-2p_n}{p_n} \frac{w_n - L}{L} \geq 2 \Leftrightarrow p_n \leq \frac{w_n - L}{2w_n}, \quad (15)$$

we get the following corollary from Proposition 2.1:

Corollary 2.2 (i) *Risk-averse agents exert more effort than the risk-neutral agent if*

$$RP(w_n - L) \leq 2 \text{ and } p_n \leq \frac{w_n - L}{2w_n}; \quad (16)$$

(ii) *Risk-averse agents exert less effort than the risk-neutral agent if*

$$RP(w_n - L) \geq 2 \text{ and } p_n \geq \frac{w_n - L}{2w_n}, \quad (17)$$

where $RP(x) = -x \frac{u'''(x)}{u''(x)}$ is the relative prudence coefficient.

3 An Explanation for the Threshold based on Moments

In this section, we show that a close examination of second and third moments of the loss distribution explains Proposition 2.1 and the cost-benefit analysis of prevention for a risk-averse and prudent agent.

Define

$$\widetilde{Loss} = \begin{cases} L & \text{with } p_n \\ 0 & \text{with } 1 - p_n \end{cases}$$

as loss of the risk-neutral agent. The variance (or second central moment) of the loss is

$$Var(\widetilde{Loss}) = p_n(1 - p_n)L^2, \quad (18)$$

and its third moment is equal to

$$E(\widetilde{Loss}^3) = p_nL^3. \quad (19)$$

The marginal change in probability on variance is

$$\frac{dVar(\widetilde{Loss})}{dp_n} = (1 - 2p_n)L^2. \quad (20)$$

Hence the threshold can be rewritten as

$$\begin{aligned} \frac{1 - 2p_n}{p_n} \frac{1}{L} &= \frac{(1 - 2p_n)L^2}{p_nL^3} \\ &= \frac{\text{Marginal change in probability on variance}}{\text{Third moment of loss}}. \end{aligned} \quad (21)$$

Proposition 2.1 states that risk-averse agents exert more effort than the risk-neutral agent if and only if AP is less than the "marginal change in probability on variance per third moment of loss distribution."

Another way to see the result is to rewrite (8) as

$$H''L^2 = (2p_n - 1)L^2u''(w_n - L) - p_nL^3u'''(w_n - L) \geq 0. \quad (22)$$

More self-protection is desirable when (22) is positive. We see from (22) that our necessary and sufficient condition includes the variation of the variance and the level of the third moment. The variation of the variance and the level of the third moment multiply $u''(w_n - L)$ and $u'''(w_n - L)$ respectively. Hence, when $p < \frac{1}{2}$, the cost-benefit of self-protection for a risk-averse and prudent agent is equal to a reduction in the variance, which is desirable for a risk-averse agent, less the utility cost of spending money for protection against an uncertain event, which is not desirable for a prudent agent who prefers to save money in such circumstance (Kimball, 1990).

This trade-off was mentioned in a comment by Eeckhoudt and Gollier (2005):

When p_n is larger than $\frac{1}{2}$, the effect of risk aversion goes in the same direction as the effect of prudence to generate a smaller level of effort. In the more interesting case where p_n is less than $\frac{1}{2}$, these two effects go in opposite directions.

We now interpret the result in terms of skewness. In the statistics literature, skewness is often proposed to measure downside risk. It is well known that an increase of skewness will imply an increase in downside risk only under specific conditions (Chiu, 2005b; Menezes et al., 1980). The skewness of \widetilde{Loss} is defined as

$$S_{Loss} = \frac{E[\widetilde{Loss} - E(\widetilde{Loss})]^3}{[Var(\widetilde{Loss})]^{\frac{3}{2}}}. \quad (23)$$

Since

$$\begin{aligned} & E[\widetilde{Loss} - E(\widetilde{Loss})]^3 \\ &= p_n(L - p_nL)^3 + (1 - p_n)(-p_nL)^3 \\ &= p_n(1 - p_n)(1 - 2p_n)L^3, \end{aligned} \quad (24)$$

we have

$$S_{Loss} = \frac{1 - 2p_n}{\sqrt{p_n(1 - p_n)}}. \quad (25)$$

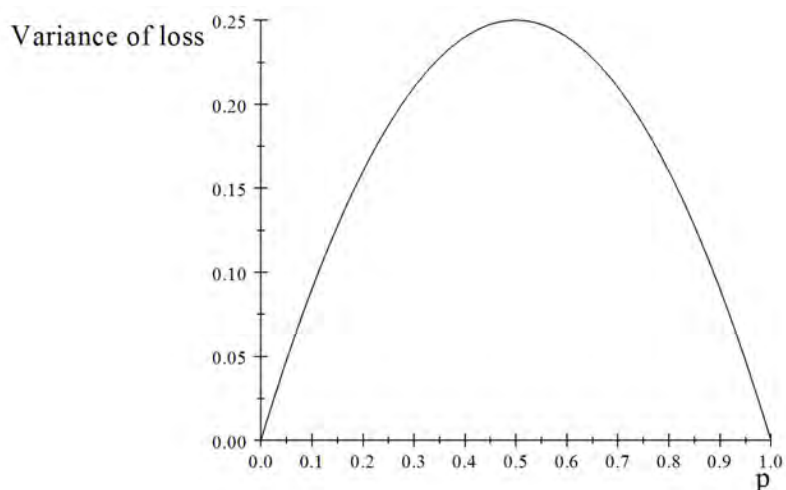


Figure 3.1 $Variance = p((1 - p))L^2$ with $L = 1$

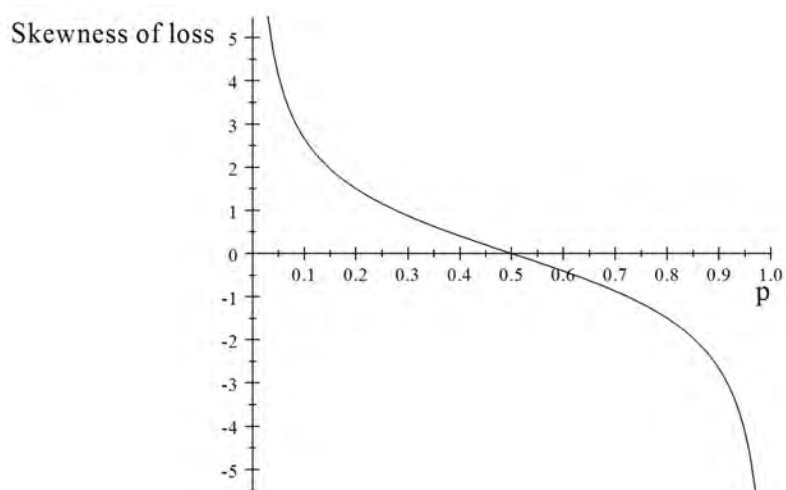


Figure 3.2 $Skewness = \frac{1-2p}{\sqrt{p(1-p)}}$

We observe that (25) is independent of L . Figures 3.1 and 3.2 illustrate the variance and skewness of loss distribution as a function of p . We observe that the variance first increases when $p < \frac{1}{2}$ and then decreases when p increases. The skewness always decreases when p increases but is positive when $p < \frac{1}{2}$ and negative otherwise.

When $p_n < \frac{1}{2}$, for the risk-averse and prudent agent, there is a trade-off between the variance and the skewness of the loss distribution. Spending more on self-protection (reducing p) decreases both the total expected loss and the variance of loss (see Figure 3.1), which is desirable. However, for the prudent agent, spending more on self-protection increases the positive skewness of loss (see

Figure 3.2), which is undesirable. Hence, when $p_n < \frac{1}{2}$, the cost-benefit analysis of preventive actions for a risk-averse and prudent agent depends on the trade-off between the decrease in the variance and the increase in the skewness. As shown by Chiu (2005b), prudence has an important role to play for characterizing this trade-off when both distributions have the same expected mean. Here we show that this is also the case even when self-protection changes the expected loss.

We propose a sufficient condition for more self-protection by a risk-averse and prudent agent in terms of the skewness of loss.

If $p_n < \frac{1}{2}$, then

$$S_{Loss} = \frac{1 - 2p_n}{\sqrt{p_n(1 - p_n)}} < \frac{1 - 2p_n}{p_n}. \quad (26)$$

From Proposition 2.1, we obtain the following corollary.

Corollary 3.1 *Suppose $p_n < \frac{1}{2}$, risk-averse and prudent agents exert more effort than the risk-neutral agent if*

$$AP(w_n - L) \leq \frac{S_{loss}}{L}. \quad (27)$$

Corollary 3.1 provides a short-cut sufficient condition for risk-averse and prudent agents to exert more effort than the risk-neutral agent: absolute prudence be less than skewness per loss.

4 Necessary and Sufficient Conditions for HARA

In the economics literature, the class of harmonic absolute risk aversion (HARA) utility functions is particularly useful to derive analytical results. HARA utility functions take the following form:

$$u(x) = \zeta \left(\eta + \frac{x}{\gamma} \right)^{1-\gamma}, \quad (28)$$

and the absolute prudence coefficient is equal to (see e.g., Gollier, 2001, p. 26)

$$AP(x) = \frac{\gamma + 1}{\gamma} \left(\eta + \frac{x}{\gamma} \right)^{-1}. \quad (29)$$

Hence Proposition 2.1 implies the following corollary.

Corollary 4.1 *HARA risk-averse agents exert more effort than the risk-neutral agent if and only if*

$$\frac{\gamma + 1}{\gamma} \left(\eta + \frac{w_n - L}{\gamma} \right)^{-1} \leq \frac{1 - 2p_n}{p_n} \frac{1}{L}. \quad (30)$$

The necessary and sufficient condition for some well known HARA utility functions is summarized in the following table.

Parameter	Utility	$AP(w_n - L)$	$AP(w_n - L) \leq \frac{1-2p_n}{p_n} \frac{1}{L}$
$\gamma = -1$	Quadratic	0	$p_n \leq \frac{1}{2}$
$\eta = 0$ and $\gamma = 1$	Logarithmic	$\frac{2}{w_n - L}$	$\frac{L}{w_n - L} \leq \frac{1-2p_n}{2p_n}$
$\eta = 0$ and $\gamma \neq 1$	Power	$\frac{\gamma+1}{w_n - L}$	$\frac{L}{w_n - L} \leq \frac{1-2p_n}{(1+\gamma)p_n}$
$\gamma \rightarrow \infty$	Negative exponential	$\frac{1}{\eta}$	$\frac{1}{\eta} \leq \frac{1-2p_n}{p_n} \frac{1}{L}$

Table 4.1: HARA utility functions

From Table 4.1, we observe that, for quadratic, logarithmic, power ($\gamma < -1$) and negative exponential ($\eta > 0$) functions, $p_n \leq \frac{1}{2}$ is a necessary condition for risk-averse and prudent agents to spend more on effort.

5 Comparative Static Results

In this section, we examine the effect of changes in the degree of absolute prudence and initial wealth on self-protection. One natural question is about the effect of an increase in prudence on self-protection. We can directly obtain the following result from Proposition 2.1.

Proposition 5.1 *If agent u is more prudent than v ($AP_u = -\frac{u'''(x)}{u''(x)} \geq -\frac{v'''(x)}{v''(x)} = AP_v$), then*

(i) *agent u exerts more effort than the risk-neutral agent \Rightarrow agent v exerts more effort than the risk-neutral agent.*

(ii) *agent v exerts less effort than the risk-neutral agent \Rightarrow agent u exerts less effort than the risk-neutral agent.*

Proof (i) Since

$$\begin{aligned}
 & \text{agent } u \text{ exerts more effort than the risk neutral agent} & (31) \\
 \Leftrightarrow & AP_u(w_n - L) \leq \frac{1-2p_n}{p_n} \frac{1}{L} \quad (\text{by Proposition 2.1})
 \end{aligned}$$

and

$$\begin{aligned}
 & \text{agent } u \text{ is more prudent than } v & (32) \\
 \Leftrightarrow & AP_u(x) = -\frac{u'''(x)}{u''(x)} \geq -\frac{v'''(x)}{v''(x)} = AP_v(x),
 \end{aligned}$$

we obtain $AP_v(w_n - L) \leq \frac{1-2p_n}{p_n} \frac{1}{L}$, and hence agent v exerts more effort than the risk-neutral agent. The negative weight of the third moment is less important.

(ii) We can prove this assertion by the same approach used in (i). Q.E.D.

If an increase in prudence implies less effort for the risk-averse agent than for the risk-neutral agent, it is natural that an increase in initial wealth w_0 implies more effort than the risk-neutral agent if absolute prudence is decreasing in wealth. This is shown in the following result.

Proposition 5.2 *Suppose AP_u is decreasing in wealth, then*

(i) *agent u exerts more effort than the risk-neutral agent under initial wealth $w_0 \Rightarrow$ she will exert more effort than the risk-neutral agent under initial wealth w'_0 with $w'_0 \geq w_0$.*

(ii) *agent u exerts less effort than the risk-neutral agent under initial wealth $w_0 \Rightarrow$ she will exert less effort than the risk-neutral agent under initial wealth w'_0 with $w'_0 \leq w_0$.*

Many utility functions commonly used in financial economics have derivatives with alternating signs showing positive odd derivatives and negative even derivatives. Caballé and Pomansky (1996) characterized the class of utility functions having this property which they called mixed risk aversion (*MRA*) (see also Eeckhoudt and Schlesinger, 2006). Dachraoui et al. (2004, p. 263) show that AP decreases in wealth for mixed risk aversion. Hence we get the following corollary.

Corollary 5.3 *Suppose u is mixed risk-averse, then*

(i) *agent u exerts more effort than the risk-neutral agent under initial wealth $w_0 \Rightarrow$ she will exert more effort than the risk-neutral agent under initial wealth w'_0 with $w'_0 \geq w_0$.*

(ii) *agent u exerts less effort than the risk-neutral agent under initial wealth $w_0 \Rightarrow$ she will exert less effort than the risk-neutral agent under initial wealth w'_0 with $w'_0 \leq w_0$.*

6 Conclusion

We have investigated the link between optimal prevention and prudence by providing a necessary and sufficient condition for risk-averse agents to exert more self-protection than a risk-neutral agent. We have formalized the intuition by using the second and third moment of the loss distribution. We have also interpreted our main result in terms of the skewness of the loss distribution. In addition, we have shown that our condition can deliver comparative static

results for some self-protection problems in terms of comparative prudence and variation in initial wealth.

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