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The Impact of Operations Anticipations on the Quality of Supply Chain Network Design Models

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Abstract. This paper draws on the stochastic multi-period location-transportation problem (SMLTP) for studying the impact of various types of operations anticipations on the quality of the supply chain network (SCN) designs obtained. The problem solving approach used to design SCNs involves the modeling of facility location decisions and of network operations decisions. Most SCN design models are extensions of deterministic location-allocation models incorporating anticipations based on aggregate arc flow variables and demand zones. This is a crude approximation and it could lead to bad designs. In this paper, several alternative approximate anticipations are proposed and tested in order to analyze their impact on the quality of the designs obtained. More precise anticipations however yield more complex models, and the tractability and solvability of these models is an issue because of their size and their underlying combinatorial-stochastic structures. The paper explores various accuracy-solvability tradeoffs for the SMLTP and it makes recommendations on modeling issues leading to better SCN design methodologies.

Keywords. Location problems, transportation problems, stochastic demand, anticipation, scenario planning, stochastic programming, set partitioning.

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1. Introduction

Supply Chain Network (SCN) design problems deal with strategic decisions such as facility location, technology selection and capacity acquisition that involve important investments and are crucial value drivers for the firm. For example, by providing shorter delivery times, a SCN design improves market shares and, by providing a proper trade-off between production, inventory and transportation costs, it fosters operational effectiveness. In particular, the location of facilities and the specification of their mission in term of the downstream locations and/or customers to supply are two critical strategic SCN design decisions. Many approaches have been proposed to solve these design problems, such as the use of location-allocation models (Revelle *et al.*, 2008). However, in real world applications, several simplifications and assumptions must be made to use these models. For example, ship-to-points must be aggregated into demand zones, products into families and suppliers into supply sources, aggregate transportation cost functions must be estimated to compute unit arc-flow costs, etc. (Ballou, 2001).

Moreover, SCNs must be designed to cope with future business environments. Once a SCN design has been implemented, it is used on a daily basis to perform operations such as sales, warehousing, transportation, procurement, etc. In fact, the revenues and expenses generated by a SCN over its useful life are directly related to these user operations. Thus, SCN design decisions cannot be optimized without anticipating how operations will eventually be performed in the implemented network. The conceptual importance of such anticipations was discussed by Schneeweiss (2003). However, the concept of anticipation and its impact on SCN design quality were not investigated explicitly in the literature. In this paper, we study the impact of the fidelity and accuracy of anticipations on the tractability and solvability of SCN design models, and on the quality of the designs they provide. To do this, we investigate a typical SCN design problem: the stochastic multi-period location-transportation problem (SMLTP).

The paper is organized as follows. In section 2 the user anticipation concept, as it applies to SCN design problems, is defined more precisely. Section 3 reviews the existing literature on the problem. Section 4 presents and formulates the SMLTP, and discusses approaches to obtain approximate user anticipations. In section 5, several SMLTP models based on these approximate anticipations are formulated. Section 6 describes the methods used to generate and to solve these design models, and it also explains the approach proposed to evaluate and compare the SCN designs obtained. Section 7 presents our plan of experiment and the test results. Section 8 provides concluding remarks on the role of anticipations in SCN design models, as well as recommendations on the methodology to use to obtain good quality SCN designs.

2. Problem Statement

SCN design involves decisions on the acquisition, deployment and mission of production-distribution assets in order to create sustainable value for the firm. These decisions are made over a multi-period planning horizon, which may cover multiple decades, and therefore they have a lasting impact over an uncertain business environment. When the SCN designed or reengineered has been implemented, it becomes available for use during several *usage* periods. Consequently, design decisions cannot be made without evaluating the initial investments required, and without anticipating the revenues and expenses generated during the network usage periods. This clearly positions the SCN design problem as a hierarchical decision problem (Schneeweiss, 2003), the top level being associated to design decisions and the bottom level to user decisions. In such a decision-making framework, the design level must anticipate user decisions. The anticipation pertains to the reaction of the network's users to the resources provided by the design level. **Figure 1** shows a simplified depiction of the role of anticipations in a SCN design methodology under uncertainty.

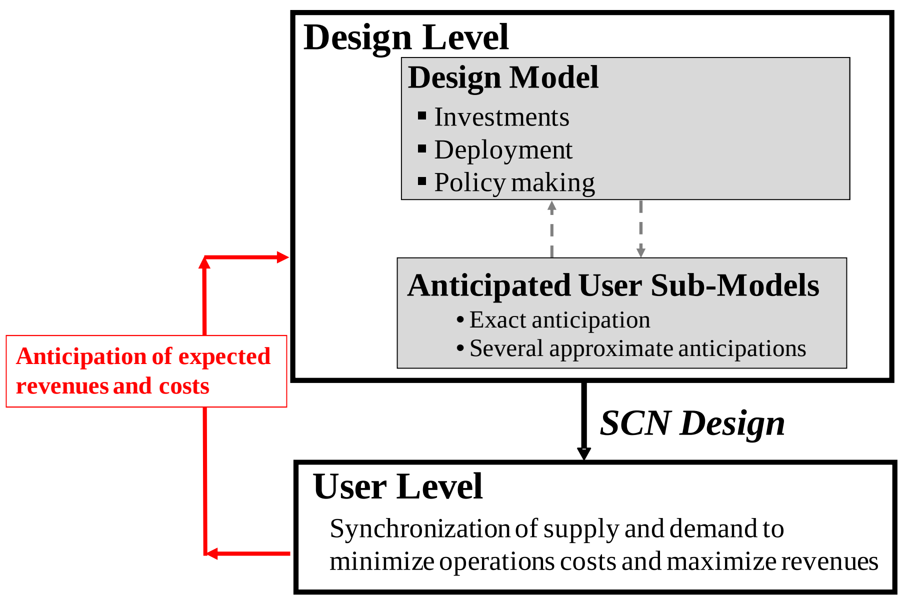

Figure 1- Anticipation-based SCN Design Methodology

In fact, the ideal design model that incorporates exact user anticipations for all possible futures is generally of no use because it is unsolvable for any non-trivial case. In practice, one must rely on approximate user anticipations. Depending on the approach favoured, the approximations made may concern decision variables, data parameters, probability distributions, evaluation functions, or a combination of these elements. However, the tractability and solvability of more realistic models are at issue because of the size of realistic SCN design models and their underlying combinatorial-stochastic structures. Therefore, it is legitimate to ask the following fundamental questions:

- What is the impact of the fidelity and accuracy of the models on the quality of the SCN design decisions?
- What are the compromises to be made between model accuracy and fidelity on one hand, and tractability and solvability on the other hand?

Model simplifications generally reduce complexity and lead to exact “optimal” solutions. Fidelity is linked to the extent to which the model is able to capture the essence of the real world problem (Pidd, 2010). Accuracy is related to the precision of the data embedded in the model through estimations and measurements (Roy, 2010). Clearly, higher fidelity and accuracy enhance the quality of the solution, but often it increases complexity and leads to possibly intractable and unsolvable models. Currently, with the increase in computational power and the improvement in exact and heuristic solution techniques, it is possible to solve more realistic models. Consequently, when faced with SCN design problems, should an analyst favour fidelity and accuracy over tractability and solvability? Or should he/she make model simplifications to improve solvability? In any case, what is lost by going one way or another? In other words, are *better* SCN designs obtained by finding the optimal solution of models with approximate user anticipations or by finding approximate (heuristic) solutions to models with exact user anticipations? Also, could a compromise between these two extremes be a better alternative?

In order to address these questions, we propose to examine the SMLTP business context. Through adequate approximations, several location model variants are proposed to obtain SCN designs. Some of these models are much more difficult to solve than others, and for realistic cases, some of them can be solved only heuristically. The SCN designs provided by the various pairs of (model, solution method) are compared, using a *user model*, for several plausible futures, in order to assess their performances. Note that our aim is not to find the best approach, but to investigate common user anticipation approximation practices, and to study the quality of resulting SCN design models in terms of the performance of their solutions.

3. Related Literature

The strategic design problem has been studied in the literature through the formulation of location-allocation models. The basic formulation of these models dates back to the 1960’s and much has been published since on variants of the problem, and on exact and heuristic methods to solve them. Comprehensive reviews are found in Klose and Drexl (2005) and Revelle *et al.* (2008). However, to use these models in real world circumstances, several problem reductions must be performed (Ballou, 2001). For instance, most of the available literature assumes that the problem is deterministic and involves a single planning period.

During the last few years major efforts have been devoted to the development of location models with a much more detailed anticipation of network users' operations decisions. A review on integrated location-routing and location-inventory models is found in Shen (2007). These models provide better anticipations, but they are more difficult to solve. Moreover, transportation capacity in contemporary SCNs is often provided by common or contract carriers, as opposed to private fleets, as implicitly assumed in location-routing problems (Nagy and Salhi, 2007), which offer more varied transportation options such as full truckloads (TL), multi-drop routes, and less-than-truckload (LTL) shipments, and these variants should be anticipated adequately. A Stochastic Multi-Period Location-Transportation Problems (SMLTP), taking these considerations into account, has been studied recently by Klibi et al. (2010b). The stochastic programming model proposed provides a better anticipation of the revenues and expenditures generated by the network user operational decisions, but it is solved with a heuristic method.

SCN design models can typically be partitioned into a design sub-model and a user anticipation sub-model. In classical location-allocation models (Geoffrions and Graves, 1974; Klose, 2000; Martel, 2005), design decisions are associated to binary variables and user anticipations to continuous aggregate throughput and flow variables. The anticipation variables are typically aggregates over several products, customers, means of transportation and usage periods. These decision variables are used to anticipate revenues and expenses, but they cannot be implemented. For example, in practice, flow decisions take the form of daily shipments in response to specific customer orders, and not the form of an annual quantity of products to ship between two locations. The later is used as a crude approximation of the former. In fact, such approximations are often so crude, that they raise issues about the validity of classical models. More recently, some authors proposed models with more accurate anticipations. A model incorporating detailed market response anticipations was proposed by Vila *et al.*, (2007). More elaborated transportation and inventory costs anticipations are proposed in the location-routing and location-inventory models reviewed in Shen (2007). Pomper (1976), Laporte *et al.* (1989) and Santoso *et al*. (2005) introduced stochastic programming with recourse formulations. In these models, design decisions are associated with first stage variables and anticipations with second stage variables. To the best of our knowledge, the investigation of the impact of various user anticipation sub-models on SCN design quality is yet to be published in the academic literature. Much of the existing literature on classical location-allocation models focus on solving approaches and algorithms. The solution methods proposed range from exact approaches to heuristic methods (see Klibi et al., (2010a) for a review). However, since all these models are based on approximate anticipations, it is difficult to appreciate the quality of the SCN designs they provide.

4. The Stochastic Location-Transportation Problem

A company purchases a family of similar products, considered as a single product, from a number of supply sources. This product is sold to customers located in a large geographical area and hence it must be shipped to a large number of ship-to-points. In order to provide a next day delivery service, the company must employ a number of uncapacitated depots. The customers order a varying quantity of product on a daily basis and the company delivers them on the next day, using common or contract carriers. For a given day τ, at each depot, when all the orders are in, the company plans its transportation for the next day and it requests from its carriers the trucks required to deliver products to ship-to points using truckload (TL) tariffs, or it specifies the loads to pick-up using less than truckload (LTL) transportation. The TL-trucks provided by the carriers can be used in three different ways: a full truckload (FTL) can be shipped to a destination, a partially loaded truck can be shipped to single ship-to-point (STL), or the truck can be assigned to a route including multiple ship-to-points (MTL). Let L be the set of depots considered, P the set of ship-to-points where orders can be delivered, $L_p \subset L$ the subset of depots which are able to provide next day delivery service to ship-to-point $p \in P$, and, conversely, $P_l \subset P$ the subset of ship-to-points which could be served by depot l. When a depot $l \in L$ is used, a fixed operating cost A_l is incurred, and the unit value of products shipped from that depot is v_l. The value v_l takes into account the product production/procurement costs, inbound shipment costs, warehousing costs and inventory holding costs. The unit price of products sold to ship-to-point p is u_p.

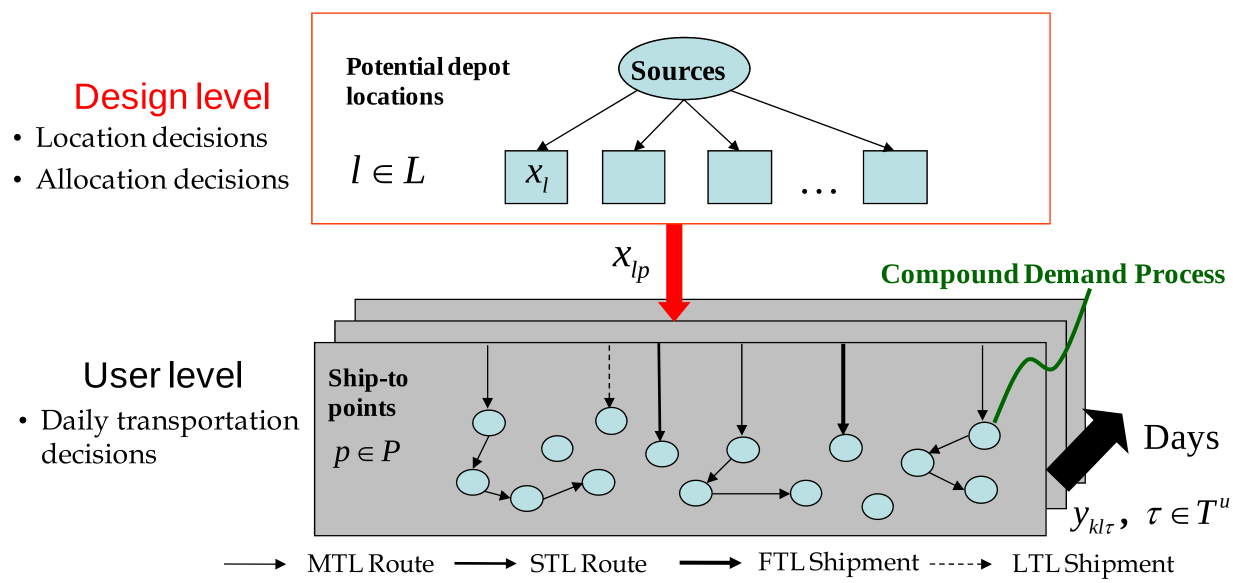

Figure 2- The SMLTP Structure

Given this context, the strategic decisions to make are the selection of a subset of the depots $\underline{L}^* \subset L$ to operate during the planning horizon T^u considered and the assignment of ship-to-points $\underline{P}_l^* \subset P_l$, $l \in \underline{L}^*$, to these depots, in order to maximize total expected profits. An important aspect of the problem is that the mission of the selected depots, defined by their customer sets, $\underline{P}_l^*$, $l \in \underline{L}^*$, must remain the same for each day $\tau \in T^u$ of the planning horizon. This stochastic multi-period location transportation problem is illustrated in **Figure 2**. A more detailed description of the problem is found in Klibi *et al.* (2010b). The SMLTP is as hierarchical decision problem due to the temporal hierarchy between the location decisions and the transportation decisions. Formulations of the design and user level problems over the planning horizon are presented next.

User Model

Let $\underline{L}$ be the subset of open depots in L, $\overline{L} = L \backslash \underline{L}$ its complement, and $\underline{P}_l \subset P_l$ the subset of ship-to-points assigned to depot $l \in \underline{L}$. On a daily basis, the depots $l \in \underline{L}$ receive orders from their customers $p \in \underline{P}_l$ and they make shipping decisions for the next day. It is assumed that the demands of the ship-to-points $p \in P$ follow a compound stationary stochastic process with a random order inter-arrival time q_p and a random order size o_p. The cumulative distribution functions of inter-arrival times and order sizes are denoted respectively by $F_p^q(.)$ and $F_p^o(.)$. A possible realization of these compound stochastic processes over planning horizon T^u constitute a demand scenario ω and the set of all demand scenarios associated to the compound demand processes considered is denoted by Ω. The probability that demand scenario $\omega \in \Omega$ will eventually be observed is denoted by $\pi(\omega)$. For a given scenario ω, the set of ship-to-points ordering products on day τ is denoted by $P_\tau(\omega)$, and the shipments to make on day τ at depot $l \in \underline{L}$ are defined by the loads $d_{p\tau}(\omega)$, $p \in \underline{P}_{l\tau}(\omega)$, where $\underline{P}_{l\tau}(\omega) = \underline{P}_l \cap P_\tau(\omega)$ is the set of depot *l* ship-to-points which order products on that day under scenario ω.

Given the loads $d_{p\tau}(\omega)$, $p \in \underline{P}_{l\tau}(\omega)$, to deliver on day τ, shipping decisions are made by the network depots users in two steps. First, for the loads that are larger than a truckload, a decision is made to ship as much as possible in full truckloads. Let $K_{pl\tau}^{FTL}(\omega)$ be the set of vehicle types (routes) selected to make full truckload shipments to point *p*. Then the residual loads to be inserted in STL, MTL or LTL shipments at depot *l* on day τ are:

$$\overline{d}_{p\tau}(\omega) = d_{p\tau}(\omega) - \sum_{k \in K_{pl\tau}^{FTL}(\omega)} b_k y_{kl\tau}^{FTL}(\omega), \quad p \in \underline{P}_{l\tau}(\omega) \tag{1}$$

where $y_{kl\tau}^{FTL}(\omega)$ is the number of truckloads shipped to point *p* from depot *l* on route *k*, and b_k is the capacity of route *k* vehicles.

Next, the best delivery routes must be constructed. Let,

K_l — Set of feasible STL, MTL and LTL delivery routes from depot $l \in L$ considering carrier offers and service requirements;

P_k Ordered set of ship-to-points in route $k \in K_l$ ($P_k \subset P$);

w_k: Tariff paid for route *k*;

$K_{l\tau}(\omega)$: Set of non-dominated feasible delivery routes from depot *l*, on day τ, under scenario ω (i.e. such that $P_k \subset \underline{P}_{l\tau}(\omega)$, $\sum_{p \in P_k} \bar{d}_{p\tau}(\omega) \le b_k$, and $k \in K_l$);

δ_{kp}: Binary coefficient taking the value 1 if ship-to point *p* is covered by route k (i.e. if $p \in P_k$), and to 0 otherwise;

y_k: Binary decision variable equal to 1 if route *k* is used for the depot, day and scenario considered, and to 0 otherwise.

Note that for a TL-route $k \in K_{l\tau}(\omega)$, the tariff is independent of the load carried in the vehicle. It is given by $w_k = \max(r_k m_k ; TL_l) + a_l(|P_k| - 1)$, where m_k is the total mileage of route *k*, r_k the transportation cost rate per mile, TL_l a minimum transportation charge, and a_l a drop charge for any additional stop. On the other end, for single destination LTL-routes in $K_{l\tau}(\omega)$, the tariff w_k depends on the load carried $\bar{d}_{p\tau}(\omega)$ between depot *l* and the ship-to-point $p \in P_k$ of route *k*.

For demand scenario ω, the best routes are obtained at depot *l* on day τ by solving the following transportation sub-problem:

$$C^u_{l\tau}(\omega) = \underset{\mathbf{y}}{Min} \sum_{k \in K_{l\tau}(\omega)} w_k y_k \qquad (2)$$

subject to

$$\sum_{k \in K_{l\tau}(\omega)} \delta_{kp}\, y_k = 1 \qquad p \in \underline{P}_{l\tau}(\omega) \qquad (3)$$

$$y_k \in \{0,1\} \qquad k \in K_{l\tau}(\omega) \qquad (4)$$

where $\mathbf{y}$ denotes the vector of all the transportation decisions, and $C^u_{l\tau}(\omega)$ is the cost of the optimal shipments made by depot *l* on day τ under scenario ω.

Furthermore, shipments made on a daily basis generate sales revenues. Taking these into account, as well as depots production/procurement, warehousing, inventory holding and customer shipment costs, the net revenues $R^u(\omega)$ generated at the distribution network user level for demand scenario ω are given by:

$$R^u(\omega) = \sum_{l \in \underline{L}} \sum_{\tau \in T^u} \left[\sum_{p \in \underline{P}_{l\tau}(\omega)} \left[\left(u_p - v_l \right) d_{p\tau}(\omega) \right] - \sum_{p \in \underline{P}_{l\tau}(\omega)} \sum_{k \in K^{FTL}_{pl\tau}(\omega)} w_k y^{FTL}_{kl\tau}(\omega) - C^u_{l\tau}(\omega) \right] \qquad (5)$$

These net revenues are an important element to anticipate in the design model.

Design Model

At the strategic level, the only decisions made *here and now* are the selection of the subset of facilities $\underline{L}^* \subset L$ to be used during the planning horizon, and the mission $\underline{P}_l^*$, $l \in \underline{L}^*$, of these facilities. However, adequate network design decisions cannot be made without anticipating the net revenues generated. The best possible anticipation involves the explicit inclusion, in the design model, of the transportation model (2)-(4) and of the net revenue expression (5), but with the information available at the time the network design decisions are made. This leads to the formulation of the SMLTP as a two-stage stochastic program with recourse (Ruszczynski and Shapiro, 2003). The following first stage decision variables are required to formulate the model:

x_l : Binary variable equal to 1 if depot *l* is opened, and to 0 otherwise;

x_{lp} : Binary variable equal to 1 if ship-to point *p* is assigned to depot *l*, and to 0 otherwise.

The notation $\mathbf{x}$ is used to denote the vector of all these decision variables, and $\mathbf{x}_l$ the vector of depot *l* ship-to-point assignment variables.

The stochastic programming model to solve is the following:

$$R = \underset{\mathbf{x}}{Max} \sum_{\omega \in \Omega} \pi(\omega) R^{du}(\mathbf{x}, \omega) - \sum_{l \in L} A_l x_l \tag{6}$$

subject to

$$\sum_{l \in L_p} x_{lp} = 1 \qquad p \in P \tag{7}$$

$$x_{lp} \le x_l \qquad l \in L,\ p \in P_l \tag{8}$$

$$x_l, x_{lp} \in \{0,1\} \qquad l \in L,\ p \in P_l \tag{9}$$

where, based on (2)-(5), the optimal value $R^{du}(\mathbf{x}, \omega)$ of the second stage program for design $\mathbf{x}$ and scenario ω is given by:

$$R^{du}(\mathbf{x}, \omega) = \sum_{l \in L} \sum_{\tau \in T^u} \left\{ \sum_{p \in P_\tau(\omega)} \left[(u_p - v_l) d_{p\tau}(\omega) - \sum_{k \in K_{pl\tau}^{FTL}(\omega)} w_k y_{kl\tau}^{FTL}(\omega) \right] x_{lp} - C_{l\tau}^{du}(\mathbf{x}_l, \omega) \right\} \tag{10}$$

with

$$C_{l\tau}^{du}(\mathbf{x}_l, \omega) = \underset{\mathbf{y}}{Min} \sum_{k \in K_{l\tau}(\omega)} w_k y_{kl\tau}(\omega) \tag{11}$$

subject to

$$\sum_{k \in K_{l\tau}(\omega)} \delta_{kp}\, y_{kl\tau}(\omega) = x_{lp} \qquad p \in P_\tau(\omega) \tag{12}$$

$$y_{kl\tau}(\omega) \in \{0,1\} \qquad k \in K_{l\tau}(\omega) \tag{13}$$

In the first term of objective function (6) the expected net revenues are calculated, based on (10), and in the second term the depots fixed costs are subtracted from net revenues to get expected profits. Constraints (7) in the first stage program enforce single depot assignments for ship-to-points, and constraints (8) limit ship-to-point assignments to opened depots. Note that (7)-(9) are classical location-allocation constraints. Constraints (12) in the second stage program are coupling relations ensuring that daily route selections respect depot mission decisions. This design model incorporates an exact anticipation of the user transportation decisions. Unfortunately, it is *intractable* due to the infinite number of possible scenarios and the extremely large number of possible transportation routes.

Approximate Anticipations for the SMLTP

Since a perfect anticipation (accurate and complete anticipation) is not possible, approximate anticipations are generally used to develop solvable network design models. Approximate user anticipations can be obtained through aggregations over transportation decisions, customer locations and/or time periods, and through scenario sampling. **Figure 3** distinguishes four modeling features on which approximate anticipations can be based for the SMLTP. For each feature, the option identified by an arrow (←) is the one used for the exact anticipation. An approximate anticipation is obtained by selecting at least one alternative option.

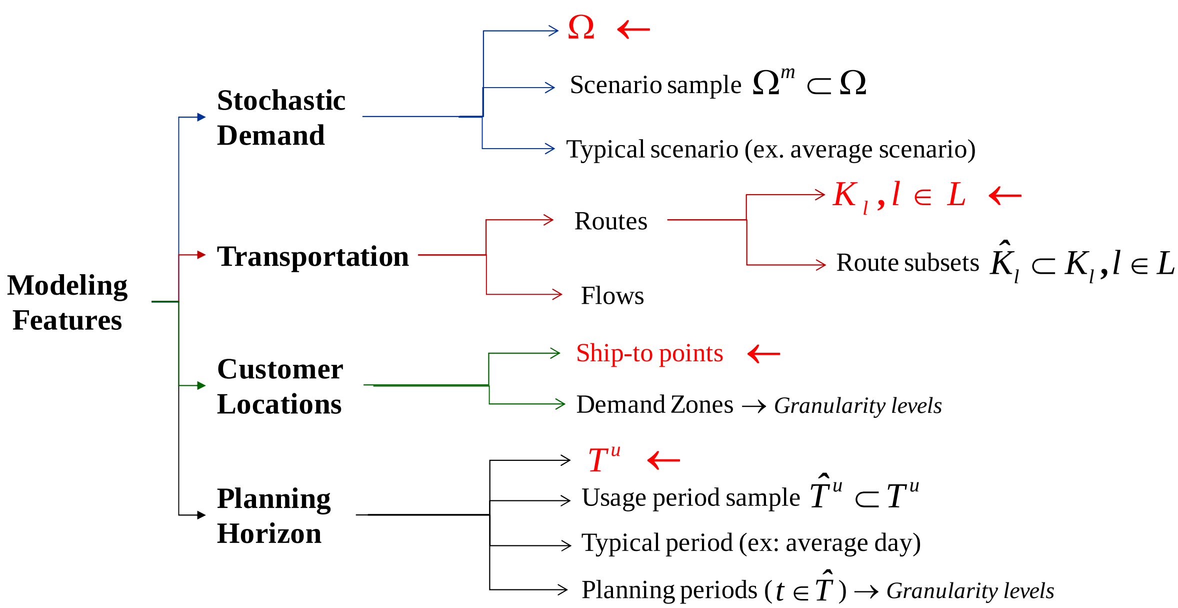

Figure 3 - User Sub-Model Reduction Options

The first modeling feature considered is stochastic demand. When the distribution function of inter-arrival times $F_p^q(.)$ and/or order sizes $F_p^o(.)$ is continuous, there is an infinite number of demand scenarios in Ω, and the resulting stochastic program cannot be solved. An approach which has been used to reduce the complexity of stochastic programming models is the sample average approximation (SAA) method presented in Shapiro (2003). This approach uses Monte Carlo methods to generate an independent sample of m equiprobable scenarios

$\{\omega^1,...,\omega^m\} = \Omega^m \subset \Omega$ from the probability distributions of the random variables, and it solves the model obtained by replacing Ω with Ω^m in the original program. This removes the necessity of explicitly computing the scenario probabilities $\pi(\omega)$, which is also a significant simplification. The SAA method has been successfully applied to SCN design problems by Santoso *et al.* (2005) and Vila *et al.* (2007). Another option is to select a typical scenario in Ω, which yields a deterministic model. When this is done, an average demand scenario is typically used. The large literature on deterministic SCN design models is implicitly based on this single-scenario option.

The second modeling feature considered is the anticipation of user transportation decisions. In the exact anticipation model, the transportation costs for a given design and scenario are provided by $C_{l\tau}^{du}(\mathbf{x}_l, \omega)$, and they depend on the optimal solution of the user transportation sub-model (11)-(13). This model is similar to the classical set partitioning formulation of the vehicle routing problem (VRP). Similar formulations were also used by Novoa *et al.* (2006) for the stochastic VRP, by Baldacci *et al.* (2002) for a capacitated location problem and by Berger *et al.* (2007) for an uncapacitated LRP. Toth and Vigo (1998) stress that this formulation has a very tight LP relaxation which helps reduce solution times significantly. Despite this, to obtain the optimal solution, all the non-dominated delivery routes $K_l = \cup_{\tau,\omega} K_{l\tau}(\omega)$, $l \in L$, must be used, and the size of these sets of routes grows exponentially with the number of ship-to-points. For this reason, in practice, pre-established subsets of routes $\hat{K}_{l\tau}(\omega) \subset K_{l\tau}(\omega)$, are usually used as input to the problem. These subsets are often built from historical data, but some authors (Novoa *et al.*, 2006) proposed approaches to obtain good route subsets. Clearly, the quality of the solution obtained depends on the quality of the candidate route subsets used. Another option available to simplify the problem is not to consider routing decisions explicitly, and to approximate the transportation costs in (10) by aggregate depot-to-customer flow costs. This requires the definition of unit flow costs g_{lp} between all depots $l \in L$ and ship-to points $p \in P_l$. These unit costs are typically provided by a regression function $g_{lp} = f(m_{lp}) + \varepsilon$, estimated from historical shipments, transportation rates data or simulated data, where m_{lp} is the mileage between depot l and ship-to points p and ε is a random error term (Ballou, 1991, 1994). Note that the precision of this function depends on the data used to estimate it. When it is estimated from historical data, it is linked to the status-quo design of the company and thus it may not yield good costs anticipations for other feasible designs. If the data used is generated by simulation, with the user model, for a sample of representative designs and scenarios, then as we shall see the resulting anticipation is much better.

The third modeling feature considered is related to ship-to locations granularity. To reduce the model size, ship-to-points can be aggregated into demand zones $z \in Z$. For example, aggregations can be based on a Zip Code proximity rules (Geoffrion, 1976). Typically, route-based formulations are considering deliveries to ship-to-points, but flow-based formulations can accommodate demand zones of any granularity level. The former reflect true transportation costs, but the later require the derivation of aggregate flow cost functions. Ballou (1994) studies the transport costing error associated with ship-to-point aggregation. Sankaran (2007) examines the error introduced by ship-to-point aggregation for large instances of the capacitated facility location problem. Prins *et al.* (2007) proposed a heuristic approach for the LRP that uses the concept of ship-to point's aggregation to solve iteratively the location problem. Practical recommendations have been made on the number of demand zones to use (Bender, 1985), but these depend largely on the problem breath and the computing power available.

The fourth modeling feature relates to the planning horizon granularity. An exact anticipation requires the consideration of all usage periods $\tau \in T^u$ of the user model which, for most practical problems, would lead to extremely large models. The number of periods considered can be reduced through sampling or aggregation. Sampling involves selecting a subset $\hat{T}^u \subset T^u$ of usage periods with a given frequency (ex: 1 day per week or month). Note that when the demand process is stationary, as assumed in this paper, the resulting design model includes $m|\hat{T}^u|$ usage periods with demands generated from the same probability distributions. Sampling periods in our case has therefore similar effect as sampling demand scenarios. When the demand is non-stationary, however, sampling scenarios and sampling periods lead to different anticipations. Aggregation involves summing usage period demands over larger periods (ex: weeks, seasons, years) to get a set $t \in \hat{T}$ of planning periods. This approach makes sense only under a flow-based formulation, and it raises serious capacity aggregation questions when the depots are capacitated. If a single aggregate period is used (ex: a year), this yields a static model. Much of the literature on location-allocation problems makes this assumption.

Another possibility is to represent demand by a typical usage period, an average day, for example. Most LRP formulations available in the literature make this assumption. Although, at first sight, this seems to be an attractive approach to get a good anticipation, it is not without problem. When the demand follows a compound process, as assumed here, the average usage period demand is not typical. If the mean time between arrivals is λ_p and the mean order size is μ_p then, on the average, the demand of customer *p* during a day is (μ_p/λ_p), which is not typical because, in reality, most customers do not order every day (i.e. $\lambda_p > 1$) and when they do their orders are larger (i.e. $\mu_p > \mu_p/\lambda_p$). Because of the drop charges added for MTL routes with several small orders, the best transportation option in this context is usually LTL transportation. Consequently, optimal average demand routes do not provide a good anticipation of transportation costs. Moreover, since MTL routes include several points, the sets of routes K_l, $l \in L$, to consider are extremely large, and the resulting model is very difficult to solve. For these reasons, this approach is not examined in more detail in what follows.

Based on these modeling features four approximate design models, corresponding to common location model variants found in literature, are described in **Table 1**. These models are formulated, solved and compared in the following sections.

Supply Network Design Models	*User Anticipation Approximation Features*			
	Stochastic Demand	**Transportation**	**Customer Locations**	**Planning Horizon**
Scenario Sample Average Approximation SMLTP Model (**M1**)	$\Omega^m \subset \Omega$	$\hat{K}_l \subseteq K_l$	P	T^u
Scenario-Period Sample Average Approximation SMLTP Models (**M2**)	$\Omega^m \subset \Omega$	$\hat{K}_l \subseteq K_l$	P	$\hat{T}^u \subset T^u$
Stochastic Location-Allocation Model (**M3**)	Ω	Flows	P	Planning period
Aggregate Location-Allocation Model (**M4**)	Typical scenario	Flows	Z	Planning period

Table 1-Supply Network Design Models Considered

5. Anticipation Based Models for the SMLTP

Scenario Sample Average Approximation SMLTP Model (**M1**)

Model **M1** is a sample average approximation of stochastic program with recourse (6)-(13) based on a sample $\Omega^m \subset \Omega$ of m scenarios. In addition, this model is based on adequately generated subsets of routes $\hat{K}_{l\tau}(\omega) \subset K_{l\tau}(\omega)$, $l \in L$, $\tau \in T^u$, $\omega \in \Omega^m$. The approach proposed to generate these routes is discussed in the following pages. This gives rise to the following binary integer program (BIP):

$$\hat{R}_{\mathbf{M1}} = \underset{\mathbf{x},\mathbf{y}}{Max} \; \frac{1}{m} \sum_{\omega \in \Omega^m} \sum_{l \in L} \sum_{\tau \in T^u} \left\{ \sum_{p \in P_\tau(\omega)} \left[\left(u_p - v_l\right) d_{p\tau}(\omega) - \sum_{k \in K^{FTL}_{pl\tau}(\omega)} w_k y^{FTL}_{kl\tau}(\omega) \right] x_{lp} - \sum_{k \in \hat{K}_{l\tau}(\omega)} w_k y_{kl\tau}(\omega) \right\} - \sum_{l \in L} A_l x_l \quad (14)$$

subject to

$$\sum_{k \in \hat{K}_{l\tau}(\omega)} \delta_{kp} \, y_{kl\tau}(\omega) = x_{lp} \qquad l \in L, \; p \in P_\tau(\omega), \; \tau \in T^u, \; \omega \in \Omega^m \quad (15)$$

$$y_{kl\tau}(\omega) \in \{0,1\} \qquad l \in L, \; k \in \hat{K}_{l\tau}(\omega), \tau \in T^u, \; \omega \in \Omega^m \quad (16)$$

and to the location-allocation constraints (7)-(9).

This model is still very complex to solve, especially when several usage periods and scenarios are considered. Consequently, in our experiments, it will be solved using an exact method for small and medium size problems but, for large cases, the heuristic proposed by Klibi *et al.* (2010b) will be used.

Scenario-Period Sample Average Approximation SMLTP Models (**M2**)

Model **M2** is similar to **M1,** but it considers only a subset $\hat{T}^u \subset T^u$ of usage periods. Given the original daily demands, one usage period per week (month) is randomly sampled to get the demands $d_{p\tau}(\omega), p \in P_\tau(\omega), \omega \in \Omega^m, \tau \in \hat{T}^u$, and the transportation sub-problems are solved only for these periods. A periodicity weight $|T^u|/|\hat{T}^u|$ is applied to net revenues to obtain an adequate approximation of the total expected profits. The models thus obtained are much smaller than **M1** and, consequently, they are much easier to solve. The resulting BIP has the following form:

$$\hat{R}_{\mathbf{M2}} = \underset{\mathbf{x},\mathbf{y}}{Max} \ \frac{|T^u|}{m|\hat{T}^u|} \sum_{\omega\in\Omega^m} \sum_{l\in L} \sum_{\tau\in\hat{T}^u} \left\{ \sum_{p\in P_\tau(\omega)} \left[\left(u_p - v_l\right) d_{p\tau}(\omega) - \sum_{k\in K_{pl\tau}^{FTL}(\omega)} w_k y_{kl\tau}^{FTL}(\omega) \right] x_{lp} - \sum_{k\in\hat{K}_{l\tau}(\omega)} w_k y_{kl\tau}(\omega) \right\} - \sum_{l\in L} A_l x \tag{17}$$

subject to

$$\sum_{k\in\hat{K}_{l\tau}(\omega)} \delta_{kp}\, y_{kl\tau}(\omega) = x_{lp} \qquad l \in L,\ p \in P_\tau(\omega),\ \tau \in \hat{T}^u,\ \omega \in \Omega^m \tag{18}$$

$$y_{kl\tau}(\omega) \in \{0,1\} \qquad l \in L,\ k \in \hat{K}_{l\tau}(\omega), \tau \in \hat{T}^u,\ \omega \in \Omega^m \tag{19}$$

and to the location-allocation constraints (7)-(9). Clearly, we would expect better results under weekly sampling than monthly sampling.

Stochastic Location-Allocation Model (**M3**)

Model **M3** is a stochastic location-allocation model obtained by aggregating usage period demands into a single planning period and by anticipating transportation costs using customer demand allocation variables. For a given scenario $\omega \in \Omega$, the annual demand $D_p(\omega)$ of ship-to-point $p \in P$ is obtained by summing daily demands over all usage periods, i.e. by computing $D_p(\omega) = \Sigma_{\tau\in T^u} d_{p\tau}(\omega)$. This model approximates transportation costs based on scenario specific depot to ship-to-point flows $D_p(\omega) x_{lp}, l \in L, p \in P_l$, using unit flow costs $g_{lp}(\omega)$, sampled from the Normal distribution $N(f(m_{lp}),\sigma_\varepsilon)$, where σ_ε is the standard deviation of the regression errors. This approximate anticipation yields the following stochastic location-allocation model:

$$\hat{R}_{\mathbf{M3}} = \underset{\mathbf{x}}{Max} \ \sum_{l\in L} \sum_{p\in P_l} \left\{ \sum_{\omega\in\Omega} \pi(\omega) \left(u_p - v_l - g_{lp}(\omega)\right) D_p(\omega) \right\} x_{lp} - \sum_{l\in L} A_l x_l \tag{20}$$

subject to location-allocation constraints (7)-(9).

In our case, because the assignment variables x_{lp} are first-stage variables (Birge and Louveaux, 1997), (20) reduces to:

$$\hat{R}_{\mathbf{M3}} = \underset{\mathbf{x}}{Max} \ \left|T^u\right| \sum_{l\in L} \sum_{p\in P_l} \left\{ \left(u_p - v_l - f\left(m_{lp}\right)\right) \frac{\mu_p}{\lambda_p} \right\} x_{lp} - \sum_{l\in L} A_l x_l \tag{21}$$

For the SMLTP, the model obtained with this approximate anticipation therefore has exactly the same form as a deterministic location-allocation model. For more complex SCN design problems, this type of anticipation yields models with random second stage variables, and the SAA method must be used to solve them (Santoso *et al.* 2005, Vila *et al.*, 2007). As indicated previously, the quality of this anticipation depends largely on the precision of the transportation cost function $g_{lp} = f(m_{lp}) + \varepsilon$ used.

Aggregate Location-Allocation Model (**M4**)

This approximate anticipation is a reduction of model **M3** based on the aggregation of ship-to-points into demand zones. The set of ship-to-points *P* is partitioned into demand-zone subsets $P_z, z \in Z$, based on the points geographical position. The geographical centroids of the demand zones are calculated in order to be able to measure the mileage m_{lz} between the depots $l \in L$ and the demand zones $z \in Z_l$ they can serve. The average annual demand for a demand zone $z \in Z$ is obtained by calculating $\hat{D}_z = |T^u| \Sigma_{p\in P_z} (\mu_p / \lambda_p)$. The ship-to-point allocation variables x_{lp} are replaced by demand-zone allocation variables x_{lz}, and it is understood that any point $p \in P_z$ will be supplied by the depot supplying zone *z*. In this model, approximate transportation costs are obtained by multiplying depot to demand-zone flows $\hat{D}_z x_{lz}, l \in L, z \in Z_l$, by average unit transportation costs. The later are obtained from a regression function $g_{lz} = f'(m_{lz}) + \varepsilon$ estimated from aggregate zone flow data. The unit price of products sold to zone *z* is denoted by u_z and is derived from u_p for ship-to-points *p* pertaining to zone *z*. This yield the following aggregate location-allocation model:

$$\hat{R}_{\mathbf{M4}} = \underset{\mathbf{x}}{Max} \ \sum_{l\in L} \sum_{z\in Z_l} \left\{ \left(u_z - v_l - f'\left(m_{lz}\right)\right) \hat{D}_z \right\} x_{lz} - \sum_{l\in L} A_l x_l \tag{22}$$

subject to

$$\sum_{l\in L} x_{lz} = 1 \qquad z \in Z \tag{23}$$

$$x_{lz} \le x_l \qquad l \in L, z \in Z_l \tag{24}$$

$$x_l, x_{lz} \in \{0,1\} \qquad l \in L, z \in Z_l \tag{25}$$

6. SCN Design Models Solution and Evaluation

Sampling and estimation procedures are needed to specify the different parameters used in the design models presented above. One must also elaborate solution methods for these models. Finally, an approach must be devised for the evaluation and comparison of the designs they provide. **Figure 4** summarizes the approach proposed here to generate, solve and evaluate the SCN design models formulated. The top box in the figure represents all the problem solving approaches examined. The SAA models require several scenario sample replications and the location-allocation (L-A) models are based on average demands and costs. In order to formulate these models, scenario samples $\Omega^m \subset \Omega$, route subsets $\hat{K}_l \subseteq K_l, l \in L$, a usage period sample $\hat{T}^u \subset T^u$ and/or a demand zones set Z must be generated. Truck load requirements $y^{FTL}_{kl\tau}(\omega)$ must also be calculated, and unit flow cost functions ($g_{lp} = f(m_{lp}) + \varepsilon$ or $g_{lz} = f'(m_{lz}) + \varepsilon$) estimated. The scenarios needed by the models are generated with a Monte Carlo procedure. The selection of a periodic sample of usage periods $\hat{T}^u$ is straightforward. The construction of demand zones based on zip or postal codes structures is also relatively simple. In the US, for example, 3-digit zip code zones are often used. Geographical coordinates are associated to each zone $z \in Z$ by calculating the weighted average of its ship-to-points latitude and longitude (Ballou, 1994). The details of the methods and procedures used to generate models and scenarios are presented in the Appendix.

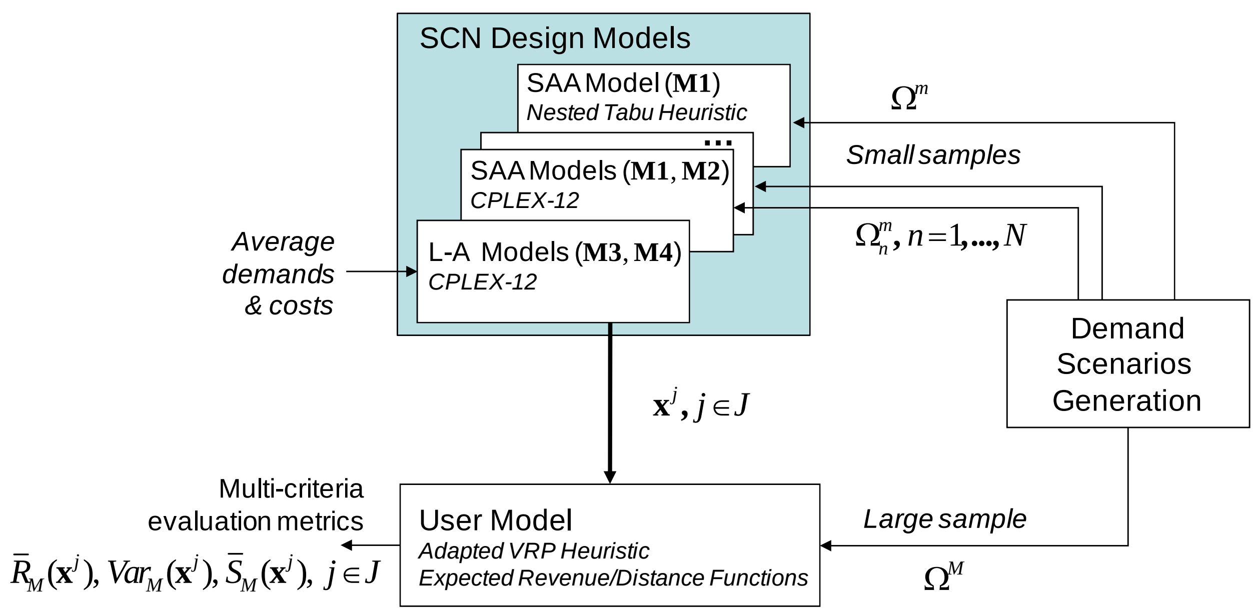

Figure 4- SCN Design Models Solution and Evaluation Approach

Once the models are generated, exact and heuristic solution methods are needed to solve them. As indicated in **Figure 4**, the SAA models **M1** and **M2** and the location-allocation models **M3** and **M4** are solved using CPLEX-12. **M1** is also solved using a metaheuristic that combines a modified Clarke and Wright savings procedure for transportation sub-problems with a Tabu-search location-allocation heuristic (Klibi *et al.*, 2010b). Altogether, the (model, solution method) pairs considered produce a set of alternative designs $\{\mathbf{x}^j, j \in J\}$ and it is these designs that must be evaluated and compared in order to assess the design models. The evaluation of SCN designs is an issue that has been neglected in the literature (Robinson and Swink, 1985; Ballou, 2001). Clearly, since each design model involves some approximations, using the objective function of one of the models formulated to evaluate all the designs obtained is completely inadequate. The approach proposed here to assess the performance of the SCN designs obtained using several criteria computed from the solution of the *user model* for a large sample Ω^M of demand scenarios.

The user model was presented in section 4 of the paper, however, to perform the evaluation of a design $\mathbf{x}^j$, the FTL shipments required must be calculated with (29) and transportation model (2)-(4) must be solved, for each usage period and each depot. The net revenue must then be calculated with (5), and this for all the scenarios $\omega \in \Omega^M$. In our evaluations model (2)-(4) is solved with the heuristic, based on perturbed Clarke and Wright savings and 2-opt improvements, proposed by Klibi *et al.*, (2010b). More specifically, the **UserEvaluation** procedure used to evaluate a design $\mathbf{x}^j$ for a scenario $\omega \in \Omega^M$ is found in **Figure 5**. The input vector $\mathbf{d}(\omega)$ provides all the demands associated to scenario ω. The set $\underline{L}^j$ in the procedure includes all the opened depots in design $\mathbf{x}^j$. The net revenues calculated with (5) for design $\mathbf{x}^j$ are denoted by $\hat{R}^u\left(\mathbf{x}^j, \omega\right)$. The last term in (5) is computed with $C^u_{l\tau}\left(\omega\right) = \sum_{k \in K_{l\tau}(\omega)} w_k y_{kl\tau}\left(\omega\right)$ for the solutions $\mathbf{y}_{l\tau}\left(\omega\right)$ provided by the transportation heuristic.

UserEvaluation$\left(\mathbf{x}^j, \mathbf{d}(\omega); \hat{R}^u\left(\mathbf{x}^j, \omega\right)\right)$

For all $(l, \tau) \in \underline{L}^j \times T^u$, do

- **S1:** Solve (29) by inspection to find the number of FTL shipments to make, and compute residual ship-to-point loads with (1);
- **S2:** Select the best direct delivery transportation modes (LTL vs STL) and compute their tariffs;
- **S3:** Solve the resulting transportation problem with the heuristic of Klibi *et al.*, (2010b);

End For

S4: Compute the net revenues $\hat{R}^u\left(\mathbf{x}^j, \omega\right)$ with (5).

Figure 5- Evaluation Procedure for Design $\mathbf{x}^j$ under Scenario $\omega \in \Omega^M$

Once the **UserEvaluation** procedure has been executed for all the scenarios $\omega \in \Omega^M$, expected value, mean semi-deviation and resilience measures can be computed to obtain a multi-criteria assessment of the design considered. The net revenues obtained for a design $\mathbf{x}^j$ can be used to estimate its expected value:

$$\bar{R}_M(\mathbf{x}^j) = \frac{1}{M}\sum_{\omega\in\Omega^M}\hat{R}^u\left(\mathbf{x}^j,\omega\right) - \sum_{l\in L} A_l(x_l)^j \qquad (26)$$

In order to evaluate the downside risk of a design, an adequate variability measure is its mean-semi-deviation:

$$MSD_M(\mathbf{x}^j) = \frac{1}{M}\sum_{\omega\in\Omega^M}\max\left[\frac{1}{M}\sum_{\omega\in\Omega^M}\hat{R}^u\left(\mathbf{x}^j,\omega\right) - \hat{R}^u\left(\mathbf{x}^j,\omega\right);0\right] \qquad (27)$$

In addition, another SCN quality sought by managers is resilience, i.e. the capability of the network to quickly recover from failures (Klibi *et al.*, 2010a). A measure of resilience is obtained by calculating the weighted average distance to a backup supply depot for each ship-to-point (i.e. its closest depot excluding the supply depot specified by $\mathbf{x}^j$):

$$\bar{S}_M\left(\mathbf{x}^j\right) = \frac{1}{\sum_{p\in P}\bar{D}_p}\sum_{p\in P} m_{l^2(p)}\bar{D}_p \qquad (28)$$

where $l^2\left(p\right) = \arg\min_{l\in \underline{L}_p\setminus\{l(p)\}}\left\{m_{lp}\right\}$ and $\bar{D}_p = {}^1\!/_M \sum_{\omega\in\Omega^M} D_p\left(\omega\right)$.

These three performance measures provide an adequate multi-criteria assessment of the SCN designs considered.

7. Computational Results

Test Cases

In order to test the design models proposed, several problem instances were generated based on four factors: the problem size, the cost structure, the network characteristics and the demand process. Problems of four different sizes were created as shown in **Table 2**. The networks incorporate about $3\%\left|P\right|$ potential depots, and ship-to points are realistically scattered in the geographical area covered. The distances between the network nodes based on existing roads are calculated with PC*MILER (www.alk.com). A one year planning horizon, with $\left|T^u\right|$ = 240 working days, is used. The next day delivery requirement is implemented through a 400 miles limit on the distance between depots and ship-to points. Exceptionally, when the number of incident lanes of a given ship-to-point is less than two, depots with distances larger than 400 miles are also considered. All the vehicles capacity (b^F and $b_k, k\in K$) are fixed to 400 cwt. Based on the cost structure of a real case, low level fixed costs (a) and high level fixed costs (b) were defined. The fixed cost for each depot A_l was randomly generated in the interval $[20K, 40K]$ for cost structure (a) and $[50K, 70K]$ for (b). The unit value of products, v_l, was selected randomly in $[\$20, \$21]$ for each depot and the products price, u_p, was fixed to \$23 for all ship-to-points.

Problem instance	Geographical Area	Potential depots	Number of ship-to-points
P_1	New York and Pennsylvania States	4	86
P_2	Central North Eastern US States	7	206
P_3	North Eastern & Midwest US States	15	706
P_4	North Eastern & Midwest US States	28	1206

Table 2- Test Problems Size

We assume that order inter-arrival times follow an exponential distribution $Exp(\lambda_p)$ with an expected time between orders λ_p. In order to study different types of ordering behaviour, two distributions are considered to characterize order quantities: a log-normal distribution $LN\left(\mu_p,\sigma_p^2\right)$ with mean μ_p and standard deviation σ_p, and a triangular distribution $Tr\left(\underline{\gamma}_p,\overline{\gamma}_p,\gamma_p\right)$ with minimum value $\underline{\gamma}_p$, maximum value $\overline{\gamma}_p$ and modal value γ_p. The two distinct order size distributions used lead to the generation of two types of demand structures: DS^1 and DS^2. DS^1 incorporates log-normal orders which can exceed a full truckload. DS^2 incorporates triangular orders which are always smaller than a truckload. The ship-to-points are partitioned into three customer size (Large, Medium and Small), and two types of network are generated: larger ship-to-points networks (*LN*) dominated by large and medium size customers and smaller ship-to-points networks (*SN*) dominated by small customers. The proportion of ship-to-points of different size in each network type is given in **Table 3**. The table also provides the probability distribution parameters used to generate the orders for each customer size.

Ship-to-point size		Large	Medium	Small
Larger ship-to points network (*LN*)		15%	65%	20%
Smaller ship-to-points network (*SN*)		10%	30%	60%
λ (days)		[2.5,4.5]	[5.5,15.5]	[20.5,35.5]
DS^1	μ (cwt)	[480,580]	[300,400]	[120,220]
	σ (% μ)	7%	10%	16%
DS^2	$\underline{\gamma}$ (cwt)	[200,250]	[125,175]	[80,100]
	$\overline{\gamma}$ (cwt)	[350,400]	[250,300]	[150,200]
	γ (cwt)	[275,325]	[200,250]	[100,150]

Table 3- Ship-to-Point Demand Structures

The combination of all these elements yields 32 problem instances. Each instance is denoted as follows: $(i,j,k,l),\quad i\in\{P_1,P_2,P_3,P_4\},\ j\in\{a,b\},\ k\in\{LN,SN\},l\in\{DS^1,DS^2\}$.

Models and Solution Methods Calibration

The solution methods proposed in this paper were implemented in VB.Net 2005, and the experiments reported in this section were performed on a 64-bit server with a 2.5 GHz Intel XEON processor and 16 GB of RAM. All the BIPs were generated with OPL Studio 6.3 and solved with CPLEX-12. As discussed previously, a number of parameters must be fixed for the models and solution methods proposed, namely: the various sample sizes (*m*) for the SAA models, the route set generation parameters ($\rho, y_{\max}$), the usage period sampling frequency for **M2**, the flow rate regression functions ($f(m_{lp})$, $f'(m_{lz})$) for **M3** and **M4**, the parameters of the metaheuristic used to solve **M1**, and the scenario sample size (*M*) to use for the evaluation of the designs obtained.

M1 and **M2** are stochastic programs and the issue of the adequate sample size to use for their SAA reductions arises. Several sample sizes have been tested (see **Table 8** in the Appendix) to study the tractability of these SAA models. Based on calibration tests, we concluded that even when a small sample size is used ($m = 6$) the SAA models **M1** are still extremely accurate and provide low statistical optimality gap values (Shapiro, 2003). This is due partly to the fact that, for the problem instance considered, the profits generated are relatively high and the objective function value has low variability (6.1%, 4.5%, 1.5% and 0.7% on average for P_1, P_2, P_3 and P_4, respectively), which could mean that the value of stochastic solutions (Birge and Louveaux, 1997) is relatively low. Note that two sampling frequency for **M2** are tested: the weekly period case **M2-w** where one day per week is sampled and the monthly period case **M2-m** where one day per month is sampled. The routes sets included in **M1** and **M2** were calibrated (with the **RouteGen** procedure described in the Appendix) based on the sample size, the number of periods and the number of ship-to points. Note also that when solving **M1** using the Tabu-search heuristic, samples of 6, 20 and 50 scenarios were tested for all the instances. The label **M1-H** is used to denote the designs obtained by solving **M1** with the heuristic. In the case of design model **M3**, two design solutions are evaluated depending on the approach employed to derive the observations used to estimate the transportation costs functions. **M3** refers to the basic approach where the observations come from an arbitrary status-quo design and historical scenario, and **M3-S** refers to the case where the observations are sampled from a set of heuristically generated designs and a subset of scenarios. The former uses a unique function aggregating all the transportation options, and the latter separates the FTL shipments from the other transportation options. In addition, in the case of **M4**, a 3 digits ZIP code rule has been used for the zones aggregation in order to guarantee the respect of the delivery distance rule. Thus, the percentage of zones over ship-to-points is about 60%, 51%, 43% and 28% for P_1, P_2, P_3 and P_4, respectively. The transportation functions were estimated using a sample of designs and scenarios as for **M3-S**, and hence we denote this model by **M4-S** in the following discussion on results. The calibration of all the parameters used is presented in more details in the Appendix.

Numerical Results

The design models solvability, the value of the designs obtained and the solution times are discussed for the 32 problem instances. Additional comparisons are provided in order to assess the quality of anticipations. First, the solvability of the models is examined in **Table 4** for problem instances with (a, *LN* and DS^2) attributes. The notation **M**(m,ρ) is used to describe the models, where ρ is the maximum number of segments in the routes generated. For each model, the number of variables, the number of constraints and the mean solution times (MST) in seconds are provided. As can be seen, there are a huge discrepancies between the models solved in terms of complexity and consequently of running times. The equivalent deterministic version of the stochastic location-allocation model (**M3**) is very easy to solve even for large instances: it has processing times that are less than 1 second. Also, the reduced version with zones aggregation (**M4**) is even easier to solve given the small size of the models obtained.

	Models	**M1**(50,5)	**M1-H**(50,5)	**M2-w**(50,5)	**M2-m**(50,5)	**M3**	**M4**
P_1	*Variables*	702 919		137 893	35 255	353	217
	Constraints	444 994		87 110	22 746	434	264
	MST (s)	3 164	190	14	1	< 1	<1
P_2	**Models**	**M1**(20,5)	**M1-H**(50,5)	**M2-w**(50,2+)	**M2-m**(50,5)	**M3**	**M4**
	Variables	1 535 418		765 690	184 229	1 454	754
	Constraints	741 427		363 119	92 758	1 652	852
	MST (s)	15 481	159	10 576	44	< 1	< 1
P_3	**Models**	**M1**(6,2)	**M1-H**(50,5)	**M2-w**(12,5)	**M2-m**(20,5)	**M3**	**M4**
	Variables	2 345 319		1 042 983	404 772	10 610	4 580
	Constraints	1 664 161		690 976	280 906	11 300	4 868
	MST (s)	24 666	3025	1 883	31	< 1	< 1
P_4	**Models**	**M1**	**M1-H**(50,5)	**M2-w**(6,2)	**M2-m**(12,2+)	**M3**	**M4**
	Variables			2 309 995	1 339 711	33 801	9 553
	Constraints			1 076 093	562 657	34 978	9 864
	MST (s)		3741	639	62	1	< 1

Table 4- (Model, Solution Method) Pairs Characteristics for a-*LN*-DS^2 Instance

As mentioned previously, models based on **M1** and **M2** have a very tight LP relaxation which helps reduce solution times significantly even with the large number of binary variables (larger than one million) involved. For this reason, it was possible to solve **M1** for P_1, P_2 and P_3. For P_3, **M1** gives the largest model solved with CPEX 12: it has 2 345 319 binary variables and it was solved in 6.8 hours. However, for P_4, although we were not able to solve **M1** with CPLEX, a heuristic solution for **M1-H** was obtained in 3741 seconds. For P_1 and P_2, **M2** was solved easily, which clearly shows that period sampling has a huge impact on solvability. However, for P_4, the number of ship-to points involved yield very large routing problems and enumerating all the MTL routes becomes impossible. Thus, for large problems, **M2** also becomes intractable: it can be solved only when using a small number of scenarios and a small subset of the possible routes. Note finally that some problem instances are more difficult to solve than others (*LN* and DS[1] instances are more difficult because the set of non-dominated routes to consider is much larger).

Now, let us examine the quality of the designs produced by all these models. As explained using **Figure 4**, the design generation phase produces several alternative designs for each problem instance. Given this set of designs, a large sample of scenarios ($M = 100$) is generated for each problem instance in order to evaluate each design in terms of expected value (i.e. mean net profit) using (26) and mean semi-deviation using (27). The average design value %-deviation and the average MSD %-deviation from the best-known solution, as well as the MST in seconds, are given in **Table 5** for each problem size. The mean design value of all the designs obtained for different subsets of problem instances are shown in **Table 6**. Note that for each model-solution method pair, only the result for the dominant design found is presented. The result of the best model is highlighted for each problem subset.

	$(P_1, ., ., .)$			$(P_2, ., ., .)$		
	Mean Value	**Mean Deviation**	**MST (s)**	**Mean Value**	**Mean Deviation**	**MST (s)**
M1	0.00%	0.49%	10 080	0.00%	0.27%	27 403
M1-H	-0.02%	0.43%	199	-0.02%	0.26%	378
M2-w	-0.02%	0.49%	60	-0.01%	0.27%	3 502
M2-m	-0.27%	0.00%	2	-0.35%	0.00%	216
M3	-0.54%	0.91%	1	-0.71%	1.13%	1
M3-S	-0.15%	0.96%	1	-0.21%	0.97%	1
M4-S	-0.33%	1.48%	1	-0.68%	0.49%	1
	$(P_3, ., ., .)$			$(P_4, ., ., .)$		
	Mean Value	**Mean Deviation**	**MST (s)**	**Mean Value**	**Mean Deviation**	**MST (s)**
M1	0.00%	0.44%	32 098			
M1-H	-0.10%	0.36%	1 816	-0.10%	0.91%	3 798
M2-w	-0.03%	0.36%	3 589	-0.08%	1.01%	2 529
M2-m	-0.51%	0.00%	1 457	-1.90%	0.00%	366
M3	-0.23%	0.66%	1	-0.13%	1.02%	1
M3-S	-0.09%	0.56%	1	0.00%	1.13%	1
M4-S	-0.29%	0.42%	1	-0.21%	0.82%	1

Table 5- Mean Value and MSD Deviations and Solution Times by Problem Size

These results indicate the superiority of **M1** in terms of design value and underline the tradeoffs between performance and solution time necessary to produce near optimal designs. The best MSD performance is given by **M2-m**. This is probably due to the fact that, since **M2-m** is the simplest location-transportation model, larger scenario samples and larger route subsets could be used for the instances solved. In terms of mean design value, **M1** produces superior designs for all P_1, P_2 and P_3 subsets. For P_3, even if smaller scenario and route sets had to be used, **M1** gives better designs than all the other models. The average design values obtained when **M1** is solved with the heuristic (**M1-H**) is always close to the best design found. Since the solution times of the heuristics are much smaller than those of CPLEX for the location-transportation models, using a heuristic solution provide a good compromise. On the other hand, the small % difference between the mean design value of **M1** and **M2-w** in **Table 5** proves that the later provides good quality designs with much smaller computation times. This confirms that the approximate anticipation based on weekly period sampling is accurate and, given the solvability of the model, it also yields a good compromise for larger instances. However, the designs obtained with **M2-m** are inferior to those found with **M1** and **M2-w**, which indicates that one should not be too aggressive when sampling periods.

P_1	(P_1, a, ., .)	(P_1, b, ., .)	(P_1, ., LN, .)	(P_1, ., SN, .)	(P_1, ., ., DS^1)	(P_1, ., ., DS^2)
M1	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
M1-H	-0.02%	-0.02%	-0.01%	-0.05%	0.00%	-0.06%
M2-w	-0.01%	-0.02%	-0.01%	-0.02%	-0.02%	-0.02%
M2-m	-0.14%	-0.43%	-0.16%	-0.46%	-0.16%	-0.55%
M3	-0.10%	-1.06%	-0.61%	-0.41%	-0.14%	-1.49%
M3-S	-0.04%	-0.28%	-0.03%	-0.37%	-0.02%	-0.45%
M4-S	-0.05%	-0.67%	-0.30%	-0.39%	-0.15%	-0.78%
P_2	(P_2, a, ., .)	(P_2, b, ., .)	(P_2, ., LN, .)	(P_2, ., SN, .)	(P_2, ., ., DS^1)	(P_2, ., ., DS^2)
M1	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
M1-H	-0.02%	-0.02%	-0.01%	-0.03%	-0.01%	-0.04%
M2-w	-0.01%	0.00%	-0.01%	0.00%	0.00%	-0.01%
M2-m	-0.40%	-0.29%	-0.19%	-0.59%	-0.27%	-0.51%
M3	-0.72%	-0.69%	-0.05%	-1.76%	-0.78%	-0.55%
M3-S	-0.16%	-0.27%	-0.05%	-0.47%	-0.06%	-0.54%
M4-S	-0.99%	-0.33%	-0.10%	-1.60%	-0.09%	-1.94%
P_3	(P_3, a, ., .)	(P_3, b, ., .)	(P_3, ., LN, .)	(P_3, ., SN, .)	(P_3, ., ., DS^1)	(P_3, ., ., DS^2)
M1	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
M1-H	-0.11%	-0.09%	-0.12%	-0.07%	-0.06%	-0.19%
M2-w	-0.03%	-0.03%	-0.01%	-0.06%	-0.01%	-0.06%
M2-m	-0.67%	-0.35%	-0.29%	-0.89%	-0.34%	-0.87%
M3	-0.31%	-0.16%	-0.16%	-0.36%	-0.27%	-0.14%
M3-S	-0.10%	-0.08%	-0.08%	-0.10%	-0.05%	-0.16%
M4-S	-0.33%	-0.26%	-0.32%	-0.25%	-0.29%	-0.30%
P_4	(P_4, a, ., .)	(P_4, b, ., .)	(P_4, ., LN, .)	(P_4, ., SN, .)	(P_4, ., ., DS^1)	(P_4, ., ., DS^2)
M1						
M1-H	-0.02%	-0.19%	-0.08%	-0.12%	-0.02%	-0.27%
M2-w	-0.03%	-0.15%	-0.10%	-0.06%	-0.08%	-0.08%
M2-m	-1.77%	-2.05%	-1.54%	-2.39%	-2.36%	-0.94%
M3	-0.06%	-0.21%	-0.02%	-0.28%	-0.18%	-0.03%
M3-S	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
M4-S	-0.23%	-0.19%	-0.18%	-0.25%	-0.15%	-0.35%

Table 6- Mean Design Values Deviations (%) for all Problem Types

When the location-allocation models **M3** and **M4** are tested, the quality of the designs produced for P_1, P_2 and P_3 is not as good as those obtained with **M1**, **M2-w** and **M1-H**, however, the difference is quite small. The performance of **M3-S** is surprisingly good and it gives the best designs for P_4. **M3-S** is superior to **M3** in about 80% of the instances solved, and in some cases by as much as 4%. This confirms that the precision of the transportation cost function used to generate the location-allocation models has a major impact on the quality of the designs obtained. It shows that it is much better to estimate transportation cost functions from (cost-distance) observations obtained by simulation, with the user model, for a sample of representative designs and scenarios, than to use historical data. In particular, it can be seen in **Table 6** that **M3-S** provides excellent results for DS[1] problems. This is due to the fact that these problems include a large proportion of single destination FTL transportation and that, under such conditions, as discussed in the Appendix, very precise transportation cost functions are obtained. The results also show that when demand zone aggregates are used, as in model **M4-S**, the quality of the designs obtained deteriorates, mainly when the ship-to-point density per zone is relatively high as in P_3 and P_4. Too much aggregation should therefore be avoided.

For each problem size and model, **Table 7** provides the percentage of *design* decisions (locations and allocations), and depot *location* decisions, that are identical to the best design obtained with all the models. For example, for P_1, it indicates that the designs obtained with **M1** and **M2-w** include identical depot locations for 87.5% of the instances, but exactly the same design only for 37.5%, of them. There is a high similarity in depot location decisions and a much lower similarity in allocation decisions. This indicates that few alternative depot locations are interesting but that allocation decisions are very sensitive to the anticipation of operational revenues and costs. It was observed by various authors (Verter and Dincer, 1995) that location decisions tend to be highly driven by the topology of the network. Our results confirm this characteristic. This is also probably why the location-allocation models perform so well.

Models	(P_1, ., .,.)		(P_2, ., .,.)		(P_3, ., .,.)		(P_4, ., .,.)	
	Design	**Location**	**Design**	**Location**	**Design**	**Location**	**Design**	**Location**
M1	100%	100%	87.5%	100%	100%	100%		
M1-H	37.5%	100%	0%	100%	0%	62.5%	50%	87.5%
M2-w	37.5%	87.5%	12.5%	100%	0%	87.5%	25%	87.5%
M2-m	25%	75%	0%	100%	0%	87.5%	0%	87.5%
M3	0%	50%	0%	25%	0%	25%	12.5%	50%
M3-S	37.5%	87.5%	0%	37.5%	0%	62.5%	62.5%	87.5%
M4-S	0%	75%	0%	37.5%	0%	37.5%	0%	37.5%

Table 7- Similarity (%) between Design Decisions for All Problem Size

Finally, when the resilience of the designs obtained is evaluated using (28), we find that for most instances, location-allocation models provide a better coverage in terms of the average distance to the second nearest depot. We observe average distance deviation gaps of 4.75%, 28.28%

and 2.75% between **M1** and **M3-S** for P_1, P_2 and P_3 respectively, which clearly indicates a better delivery service in case of disruptions for the later. This is due to the aggregate flow approximations used in these models which are highly correlated to depot to ship-to-point distances. These results stress the need to explicitly incorporate mathematical constructs in design models to take resilience strategies and measures into account.

8. Conclusions

We studied the role of anticipations using the business context of the SMLTP. Throughout several approximations related to uncertainty, transportation, planning horizons and ship-to locations were discussed, alternative design model formulations were proposed, and solution methods were investigated. We performed several experiments on models with different anticipations, with a focus on the solvability of these models and on the quality of the SCN designs they provide. Our results indicate significant differences between the designs obtained with the various models studied, and thus between the underlying anticipations. They demonstrate that there is a relationship between the quality of the operations anticipation used and the performance of the design obtained.

We also observed that the quality of the approximate anticipation used is negatively correlated with the solvability of the model. The more precise the anticipation, the more difficult the model obtained is to solve, and vice versa. Our empirical results indicate that there is a huge difference between the time required to solve stochastic location-transportation models (**M1** and **M2**) and stochastic location allocation models (**M3** and **M4**). This is due in part to the fact that, for the SMLTP, the deterministic equivalent program obtained for the stochastic location-allocation formulation is identical to the classical deterministic location-allocation model. For more complex SCN involving, for example, several products, non-stationary demands, production decisions, inventory decisions and/or capacitated facilities subject to environmental disruptions, the stochastic location-allocation model obtained may also be quite difficult to solve. Even with the power of current computers, and the sophistication of recent solution techniques, finding the optimal solution for large-scale models based on exact anticipations remains elusive. In order to solve larger stochastic models, one still needs to use specialised decomposition methods, or metaheuristics.

Another insight from our experiments is that complexity reduction approaches such as period sampling and ship-to-point aggregation work well when moderate sampling or aggregations are performed, but that it deteriorates when the sampling/aggregation is too aggressive. A comforting observation is also that classical location-allocation approximations based on aggregate depots to ship-to-points flows provide good results, provided that special care is taken to derive precise transportation costs functions. In other words, this approach works well if the empirical cost functions used to derive aggregate parameter values are precise. More precise functions can be obtained by estimating them from simulation data derived using the user model instead of from historical data. Also, when the SCN is dominated by full truckloads transportation, more precise transportation cost functions are naturally obtained.

Overall, our results show that in order to obtain the best possible SCN designs with the current state of the art, one should adopt a modeling-based methodology with a good compromise between model accuracy and solvability. In other words, the problem solving approach adopted should belong to the accuracy-solvability trade-off diagonal portrayed in **Figure 6**. Our results show that the quality of the operations anticipation used plays a key role in the model precision and thus in the effectiveness and robustness of the SCN designs obtained. The use of approximate anticipations is a necessary complexity reduction approach to solve real SCN design problems. In practice, in order to obtain better designs, one should explore various accuracy-solvability tradeoffs before adopting a decision support model for network design.

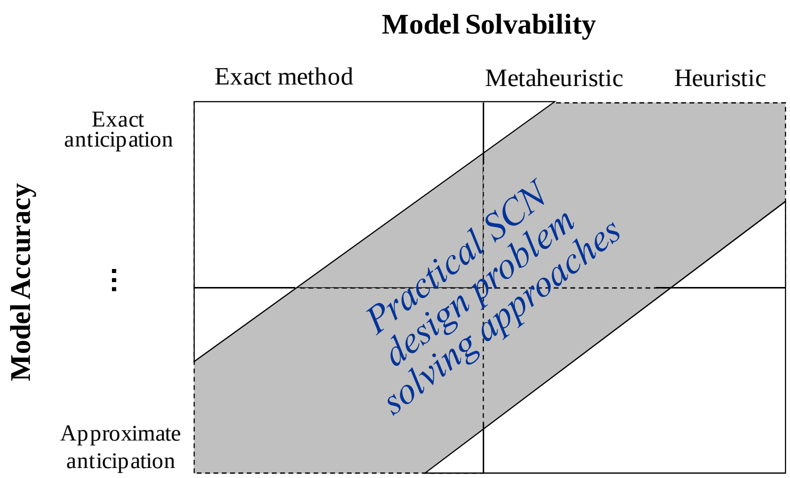

Figure 6- SCN Design Problem Solving Approaches

Our study is not without limitations. Our investigation was based on the SMLTP and therefore mainly on the anticipation of sales revenues and transportation costs. More comprehensive SCN design problems incorporating multiple products, multiple-echelons, capacitated production-distribution facilities, international factors and uncertain disruption-prone plausible futures should also be studied. These design problems are more complex than the one we considered: they will require more drastic anticipations and be more difficult to solve. Satisfactory anticipations of production and inventory decisions and costs, of exchange rates and tariffs… will therefore have to be elaborated. The added complexity will also require the development of more sophisticated exact and heuristic solution methods. The issue of the best compromise between these two fundamental axes will however remain.

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Appendix

In order to provide sufficient details to allow our work to be reproduced, this appendix gives complementary information on the procedures and methods used in the paper, and it also presents the parameter calibrations performed.

A. SCN Design Models Generation and Solution

In order to write the design models presented in the paper, scenario samples $\Omega^m \subset \Omega$, route subsets $\hat{K}_l \subseteq K_l, l \in L$, usage period samples $\hat{T}^u \subset T^u$ and/or demand zones sets Z must be elaborated. Truck load requirements $y_{kl\tau}^{FTL}(\omega)$ must also be calculated, and unit flow cost functions ($g_{lp} = f(m_{lp}) + \varepsilon$ or $g_{lz} = f'(m_{lz}) + \varepsilon$) estimated. The selection of a sample of usage periods $\hat{T}^u$ is straightforward. The construction of demand zones based on zip or postal codes structures is also relatively simple. In the US, for example, 3-digit ZIP code zones are often used. Geographical coordinates are associated to each zone $z \in Z$ by calculating the weighted average of its ship-to-points latitude and longitude (Ballou, 1994). The other sampling and estimation procedures needed to define the models are explained in the following paragraphs.

Scenario Generation Procedure

Demand scenarios are generated from the probability distributions $F_p^q(.)$ and $F_p^o(.)$ using Monte Carlo sampling methods. Assuming that the customer orders are independent of each other, to sample a scenario $\omega \in \Omega$, we generate independent pseudorandom numbers u_q and u_o uniformly distributed on the interval $[0;1]$, and we compute the inverse, $F_p^{q\text{-}1}(u_q)$ and $F_p^{o\text{-}1}(u_o)$, of the distributions of inter-arrival times and order sizes. Order arrivals are generated in the interval $]0,|T^u|]$ and mapped onto the corresponding daily periods $\tau \in T^u$. The **MonteCarlo** procedure used to generate the daily demands $d_{p\tau}(\omega)$, $p \in P$, $\tau \in T^u$, of the ship-to-points for a scenario ω is presented in **Figure 7**. Note that all the **Procedures** presented in the paper use the following syntax:

Procedure(*input_variable1*,…; *procedure_parameter1*,…; *output_variable1*,…)

MonteCarlo$((F_p^q(.), F_p^o(.), p \in P), T^u ; ; d_{p\tau}(\omega), p \in P, \tau \in T^u)$
For all $p \in P$, Do:
$\eta = 0$; $d_{p\tau}(\omega) = 0$, $\tau \in T^u$
    While $\eta \le |T^u|$, Do:
        Generate the Uniform [0,1] random numbers u_q and u_o
        Compute the next order arrival time $\eta = \eta + F_p^{q\text{-}1}(u_q)$ and $\tau = \lceil \eta \rceil$
        Compute the daily demand $d_{p\tau}(\omega) = d_{p\tau}(\omega) + F_p^{o\text{-}1}(u_o)$
    End While
End Do

Figure 7- Procedure MonteCarlo for the Generation of a Demand Scenario

In this procedure, the continuous variable η denotes order arrival times. Repeating sampling procedure **MonteCarlo** *m* times yields a sample of scenarios $\Omega^m \subset \Omega$.

Truck Requirements and Route Set Generation Procedures

Truck requirements for TL transportation and route sets are required for design models including an exact anticipation of the transportation sub-problem (**M1** and **M2**). More specifically, for usage period τ of scenario ω, the set partitioning formulation used requires the number $y_{kl\tau}^{FTL}(\omega)$ of truckloads that will be shipped to point *p* from depot *l* on route $k \in K_{pl\tau}^{FTL}(\omega)$ if point *p* is assigned to depot *l*, as well as the sets of non dominated STL, MTL and LTL routes $\hat{K}_{l\tau}(\omega)$, $l \in L$, as input to the problem. To obtain the FTL shipments, problem (29) below is solved by inspection for all ω, τ and *l*. The residual loads $\bar{d}_{p\tau}(\omega)$, $p \in \underline{P}_{l\tau}(\omega)$, to be shipped can then be computed with (1).

$$\left[y_{kl\tau}^{FTL}(\omega)\right]_{k \in K_{pl\tau}^{FTL}(\omega)} = \arg\left(\max_{\mathbf{y}} \sum_{k \in K_{pl\tau}^{FTL}(\omega)} b_k y_k \,\middle|\, \sum_{k \in K_{pl\tau}^{FTL}(\omega)} b_k y_k \le d_{p\tau}(\omega), y_k = 0,1,\ldots \right) \tag{29}$$

Next, given the residual loads, MTL, LTL and STL route selection variables $y_{kl\tau}(\omega)$, $k \in \hat{K}_{l\tau}(\omega)$, $l \in L$, $\tau \in \hat{T}^u$, $\omega \in \Omega^m$, must be generated. For realistic problems, enumerating all the possible routes usually leads to unsolvable models. A procedure is consequently required to generate comprehensive subsets of feasible and non-dominated routes $\hat{K}_{l\tau}(\omega) \subset K_{l\tau}(\omega)$. The procedure **RouteGen** presented in **Figure 8** was designed to do this. It is based on two parameters:

$y_{\max}$: Upper bound on the number of binary route selection variables to include in the model;

ρ: Upper bound on the number of segments to include in the routes generated.

The procedure can be used in two different ways. If the parameter $y_{\max}$ is unbounded (i.e. if $y_{\mathbf{max}} = \infty$), then it generates all the feasible non-dominated routes with ρ segments or less. If $y_{\max}$ is bounded, then it generates additional routes until $y_{\mathbf{max}}$ binary route variables are obtained. The selection of these additional routes is based on a route marginal cost given by

$$mc_{k\tau}(\omega) = w_k \Big/ \sum_{p \in P_k} m_{k(\to p)} \bar{d}_{p\tau}(\omega) \tag{30}$$

where $m_{k(\to p)}$ is the mileage of the route segment ending with ship-to-point *p* in route *k*. The routes with the smallest marginal costs are selected first to ensure that the more promising routes are considered. We assume that the parameters ρ and $y_{\max}$ are given values such that the number of binary route variables generated in step S2 does not exceed $y_{\max}$.

RouteGen$(P,L,\hat{T}^u,\Omega^m,(\bar{d}_{p\tau}(\omega)\forall p,\tau,\omega);\rho,y_{\max};(\hat{K}_{l\tau}(\omega)\forall l,\tau,\omega),(w_k,P_k,\forall k))$

S1: Generate the sets of STL, MTL and LTL routes K_l, $l \in L$, satisfying carrier offers and specified service conditions, and compute their tariff w_k, capacity b_k and points set P_k.

S2: Generate the sets $\hat{K}_{l\tau}(\omega) \subset K_l$, $l \in L$, $\tau \in \hat{T}^u$, $\omega \in \Omega^m$, of all the non-dominated routes with ρ segments or less satisfying the feasibility conditions

$$P_k \subset P_{l\tau}(\omega), \quad \sum_{p \in P_k} \bar{d}_{p\tau}(\omega) \le b_k \tag{31}$$

If $y_{\mathbf{max}} = \infty$, stop. Else:

S3: Generate the sets $\hat{K}^{+}_{l\tau}(\omega) \subset K_l$, $l \in L$, $\tau \in \hat{T}^u$, $\omega \in \Omega^m$, of all the non-dominated routes with more than ρ segments satisfying feasibility conditions (31).

S4: Compute the marginal costs $mc_{k\tau}(\omega)$ of the routes $k \in \hat{K}^{+}_{l\tau}(\omega), l \in L, \tau \in \hat{T}^u, \omega \in \Omega^m$, with (30), and include them in a *List* in increasing order of their $mc_{k\tau}(\omega)$.

S5: Add the routes in the *List* sequentially to their respective sets $\hat{K}_{l\tau}(\omega)$ until a total of $y_{\mathbf{max}}$ binary route variables is obtained.

Figure 8- RouteGen Procedure

Flow Cost Functions Estimation

Unit flow cost functions are required for the design models incorporating flow-based approximate anticipations of the transportation sub-problem (**M3** and **M4**). These functions can be estimated by regression using historical or simulated order delivery data, aggregated by planning period and by depot to ship-to-point (or demand zone) lane. Since several transportation means (FTL, STL, MTL and LTL) are used, it is possible to derive flow cost functions by transportation means, and then to combine them *a posteriori* to evaluate transportation costs; or to derive joint-functions *a priori* with combined means data. After some experimentation, we found that the best approach in our case was to derive two linear regression functions, one for FTL transportation and one for the other means (denoted by $\overline{\text{FTL}}$ and including STL, MTL and LTL), and to combine these two functions using a weight θ defined as the proportion of the network demand shipped by FTL. We also found that simulated order delivery data, generated simply by solving the user model for a set of feasible network designs $\{\mathbf{x}^j, j \in J^o\}$ and a few demand scenarios $\Omega^o \subset \Omega$ (obtained using procedure **MonteCarlo**), provided better unit flow cost functions.

Let L^o be the set of the depot to ship-to-point lanes (l, p) associated to the set of designs $\{\mathbf{x}^j, j \in J^o\}$, $\mathrm{K}^{\mathrm{FTL}}_{lp}(\omega)$ the set of all the FTL-shipments made to ship-to-point p from depot l during the planning period (year) considered under scenario $\omega \in \Omega^o$, and $d_{kp}(\omega), k \in \mathrm{K}^{\mathrm{FTL}}_{lp}(\omega)$, the associated shipments size. Then, the observations

$$g^{\mathrm{FTL}}_{(l,p)}(\omega) = \sum_{k \in \mathrm{K}^{\mathrm{FTL}}_{lp}(\omega)} w_k \Big/ \sum_{k \in \mathrm{K}^{\mathrm{FTL}}_{lp}(\omega)} d_{kp}(\omega), \quad ((l,p),\omega) \in \mathrm{L}^o \times \Omega^o \tag{32}$$

for non-empty lane sets $\mathrm{K}_{lp}^{\mathrm{FTL}}(\omega)$, are available to estimate the linear regression model for FTL transportation, namely:

$$g_{(l,p)}^{\mathrm{FTL}}(\omega) = \hat{\beta}_0^{\mathrm{FTL}} + \hat{\beta}_1^{\mathrm{FTL}} m_{lp} + \varepsilon_{(l,p)}^{\mathrm{FTL}}(\omega) \tag{33}$$

where $\hat{\beta}_0^{\mathrm{FTL}}$ and $\hat{\beta}_1^{\mathrm{FTL}}$ are the regression coefficients, and $\varepsilon_{(l,p)}^{\mathrm{FTL}}(\omega)$ is the error term. By using similar definitions, the following regression model is obtained for $\overline{\mathrm{FTL}}$ transportation:

$$g_{(l,p)}^{\overline{\mathrm{FTL}}}(\omega) = \hat{\beta}_0^{\overline{\mathrm{FTL}}} + \hat{\beta}_1^{\overline{\mathrm{FTL}}} m_{lp} + \varepsilon_{(l,p)}^{\overline{\mathrm{FTL}}}(\omega) \tag{34}$$

Note, however, that MTL-shipments $k \in \mathrm{K}_{lp}^{\mathrm{MTL}}(\omega)$ involve several ship-to-points. The tariff w_k of these routes must therefore be apportioned between their ship-to-points $p \in P_k$ before it is used to calculate flow rate observations using the revised formula:

$$g_{(l,p)}^{\overline{\mathrm{FTL}}}(\omega) = [\textstyle\sum_{k \in \mathrm{K}_{lp}^{\mathrm{STL}}(\omega) \cup \mathrm{K}_{lp}^{\mathrm{LTL}}(\omega)} w_k + \sum_{k \in \mathrm{K}_{lp}^{\mathrm{MTL}}(\omega)} w_{kp}] \Big/ \sum_{k \in \mathrm{K}_{lp}^{\overline{\mathrm{FTL}}}(\omega)} d_{kp}(\omega), \;\; ((l,p),\omega) \in \mathrm{L}^{\mathrm{o}} \times \Omega^{\mathrm{o}} \tag{35}$$

where, for $k \in \mathrm{K}_{lp}^{\mathrm{MTL}}(\omega)$, w_{kp} is calculated with the proportional distance-load apportionment formula $w_{kp} = w_k [\, m_{lp} d_{kp}(\omega) \big/ \Sigma_{p \in P_k} m_{lp} d_{kp}(\omega)]$.

Based on the regression lines (33) and (34), the expected unit flow costs required by model **M3** are given by:

$$f(m_{lp}) = \theta(\hat{\beta}_0^{\mathrm{FTL}} + \hat{\beta}_1^{\mathrm{FTL}} m_{lp}) + (1\text{-}\theta)(\hat{\beta}_0^{\overline{\mathrm{FTL}}} + \hat{\beta}_1^{\overline{\mathrm{FTL}}} m_{lp}), \quad l \in L,\, p \in P_l \tag{36}$$

The expected unit flow costs $f'(m_{lz})$ required by model **M4** can be estimated using a similar approach. The only difference for FTL-transportation is that the flow rate observations used must be derived from the sets $\mathrm{K}_{lz}^{\mathrm{FTL}}(\omega),\, ((l,z),\omega) \in \mathrm{L}' \times \Omega^{\mathrm{o}}$, of the FTL-shipments made from depot l to all the ship-to-points in demand zone z, where L' is the set of the aggregate depot to demand zone lanes (l, z) associated to designs set $\{\mathbf{x}^j, j \in J^o\}$. A similar adaptation is required to obtain the zone-based $\overline{\mathrm{FTL}}$ transportation regression line.

B. Models and Solution Methods Calibration

The procedures and solution methods proposed in this paper were implemented in VB.Net 2005, and the programs were executed on a 64-bit server with a 2.5 GHz Intel XEON processor and 16 GB of RAM. All the BIPs were generated with OPL Studio 6.3 and solved with CPLEX-12, using a *MIP Relative Tolerance* of 0.001. Location-allocation models **M3** and **M4** are relatively simple BIPs, and they were solved easily with CPLEX-12. SAA models **M1** and **M2** were much more difficult to solve. Their solvability depends largely on the cardinality of the subsets Ω^m, $\hat{K}_l$, $l \in L$, and $\hat{T}^u$ used. Our experiments showed that the current version of CPLEX can solve **M1** instances with about 2.5 million binary variables and 1.5 million constraints. To solve large instances of **M1**, we used a nested metaheuristic proposed by Klibi *et al.*, (2010b) that combines a modified Clarke and Wright savings procedure for transportation sub-problems with a Tabu-search location-allocation heuristic. The S_1 and S_2 neighborhood search strategies proposed in Klibi *et al.*, (2010b) were used in our experiments. We also set the parameter for the Tabu and User procedures to the values recommended by the authors, and used slightly increased values for the Reassign procedure.

Since the stochastic programs **M1** and **M2** were solved using the SAA method, the issue of the sample size to use had to be addressed (Shapiro, 2003). For **M1**, based on calibration tests, we found that a sample size $m = 6$ gives a very low statistical gap and thus is sufficient to provide near-optimal solutions for P_1 and P_2. Larger sample sizes were tested to study the tractability of the models (**Table 8**). Note that when weekly or monthly period sampling is used, it is clearly possible so solve problems with larger scenario samples. Note also that DS^1 instances are more difficult to solve because the transportation sub-problems are much larger.

<table>
<tr><th>Problem Attribute</th><th>M1</th><th>M2-w</th><th>M2-m</th></tr>
<tr><td>(P_1, ., .,DS^1)</td><td colspan="3" rowspan="2">6, 20, 50</td></tr>
<tr><td>(P_1, ., .,DS^2)</td></tr>
<tr><td>(P_2, ., .,DS^1)</td><td>6, 12</td><td>6, 12, 20</td><td rowspan="2">6, 12, 20, 50</td></tr>
<tr><td>(P_2, ., .,DS^2)</td><td>6, 12, 20</td><td>6, 12, 20, 50</td></tr>
<tr><td>(P_3, ., .,DS^1)</td><td>2, 4</td><td rowspan="2">6, 12</td><td rowspan="2">6, 12, 20</td></tr>
<tr><td>(P_3, ., .,DS^2)</td><td>6</td></tr>
<tr><td>(P_4, ., .,DS^1)</td><td rowspan="2"></td><td>2</td><td>2</td></tr>
<tr><td>(P_4, ., .,DS^2)</td><td>4, 6</td><td>4, 6, 8, 12</td></tr>
</table>

Table 8- Sample Sizes Tested for the SAA Models by Problem Type

For **M1** and **M2,** the complexity of the design models also depends on the number of segments ρ in the routes considered. In order to keep these models tractable we had to set $y_{\max}$, the maximum number of routes in $\hat{K}$, to 2 000 000. For smaller problems, a value of $\rho = 5$ was used which, in our case, yields all the potential routes and thus provides near-optimal solutions. However, for larger problems, a reduced set of routes had to be used. For these cases, ρ was set at 2 or 2+, the later meaning all the routes with 1 or 2 segments, and as may routes as possible within 2 000 000 with more than 2 segments, selected with the marginal cost rule in **RouteGen.** For P_4 the number of ship-to points to serve in each period yields very large routing problems, and enumerating all the MTL routes becomes intractable. For this case, we were not able to generate the non-dominated route set in 24 hours of running time.

The three transportation cost functions required for **M3** and **M4** also needed to be estimated for each problem type. Their regression coefficients were estimated using a sample of daily routes obtained with our VRP heuristic. **Table 9** gives the estimated parameters $\hat{\beta}_0$ and $\hat{\beta}_1$ of the linear regression functions of the *All Options*, FTL and $\overline{\text{FTL}}$ transportation options, the coefficient of determination (r^2) of the regression and the estimated proportions (θ) of FTL and $\overline{\text{FTL}}$ transportation, for a-LN-DS^1 instances. Based on the r^2 it can be seen that the FTL and

$\overline{\text{FTL}}$ functions needed for **M3-S** are a good fit, but that the *All Options* function needed by **M3**, is less precise. The estimates of demand proportions θ illustrate the dominance of the FTL mode in DS[1] problems, with more than 70% of the shipments. The functions required for **M4-S** were estimated in the same way, but from data based on 3 digits ZIP code demand zones.

	P_1			P_2			P_3			P_4		
	All Options	FTL	$\overline{\text{FTL}}$	*All Options*	FTL	$\overline{\text{FTL}}$	*All Options*	FTL	$\overline{\text{FTL}}$	*All Options*	FTL	$\overline{\text{FTL}}$
$\hat{\beta}_0$	0.0706	0.0415	0.1273	0.1077	0.0636	0.1478	0.1014	0.0432	0.1557	0.1166	0.0411	0.1853
$\hat{\beta}_1$	0.005	0.0036	0.0061	0.0048	0.0034	0.0059	0.0048	0.0035	0.0058	0.0043	0.0036	0.0051
r^2	0.6	0.99	0.78	0.63	0.98	0.77	0.68	0.99	0.82	0.73	0.99	0.86
θ	1	0.70	0.30	1	0.72	0.28	1	0.72	0.28	1	0.72	0.28

Table 9- Regression Parameters for the a-*LN*-DS[1] Problem Instances Solved with M3