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# The CAT Metaheuristics for the Solution of Multi-Period Activity-Based Supply Chain Network Design Problems

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**Abstract.** This paper proposes an agent-based metaheuristic to solve large-scale multiperiod supply chain network design problems. The generic design model formulated covers the entire supply chain, from vendor selection, to production-distribution sites configuration, transportation options and marketing policy choices. The model is based on the mapping of a conceptual supply chain activity graph on potential network locations. To solve this complex design problem, we propose CAT (Collaborative Agent Team), an efficient hybrid metaheuristic based on the concept of asynchronous agent teams (A-Teams). Computational results are presented and discussed for large-scale supply chain networks, and the results obtained with CAT are compared to those obtained with the latest version of CPLEX.

**Keywords**. Supply chain network design, activity graph, location, facility configuration, vendor selection, transportation options, market offers, metaheuristic, A-teams

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#### **1** INTRODUCTION

In recent years, the emphasis on trade globalization as well as the emergence of new economic powers such as the BRICs (Brazil, Russia, India, and China) brought forth new competitive challenges as well as new opportunities for growth and cost reductions. The ensuing mergers, acquisitions as well as supply chain reconfigurations involve a large number of complex interrelated supply chain network (SCN) design decisions that heavily impact company's competitive position, debt and profitability. Moreover, the large investments associated with these decisions require the consideration of a planning horizon covering several years. In such a context, companies seek to improve their profitability by generating economies of scale as well as making efficient use of capital while improving customer service (Cooke, 2007). Given the complexity and interdependence of supply chain network design decisions, it has been shown that the use of operations research techniques and tools such as mixed-integer programming models can result in significant returns (Geoffrion and Powers, 1995; Shapiro, 2008). Unfortunately, the problems to be modeled are so large and complex that even the best-of-breed commercial solvers are seldom able to solve real instances to optimality in a reasonable amount of time. Thus, the need for an efficient and flexible heuristic solution method arises.

A typical SCN design problem sets the configuration of the network and the missions of its locations. Some facilities may be opened, others closed, while others can be transformed using different capacity options. Each selected facility is assigned one or several production, assembly and/or distribution activities depending on the capacity options available at each location. The mission of each facility must also be specified in terms of product mix and facilities/customers to supply. Key raw-material suppliers must be selected. For each product-market, a marketing policy setting service and inventory levels, as well as maximum and minimum sales levels, must also be selected. The objective is typically to maximize net profits over a given planning horizon. Typical costs include fixed location/configuration costs, fixed vendor and market policy selection costs, as well as some variable production, handling, storage, inventory and transportation costs (Amrani *et al.* 2010).

The objective of this paper is, first, to propose a generic formulation of the multi-period SCN design problem based on the mapping of a conceptual supply chain activity graph on potential network locations, and, second, to propose an efficient hybrid metaheuristic based on a collaborative agent team (CAT) to solve large instances of this model. The rest of the paper is

organized as follows. In Section 2, a general review of the relevant literature is provided. Section 3 defines the activity-based concepts required to model SCNs. Section 4 formulates the mathematical programming model to be solved. Section 5 outlines the solution approach developed to tackle the problem. Computational results are presented and discussed in Section 6, and Section 7 concludes the paper.

## 2 LITERATURE REVIEW

Several modeling approaches can be used to formulate the supply chain network design problem. The simplest models available are appropriate to solve facility location problems (FLP), which can be either capacitated (CFLP) or uncapacitated (UFLP). Some formulations also impose single-sourcing (CFLPSS), i.e. they require that demand zones are supplied from a single facility. Since the publication of the original formulation published by Balinski (1961), several exact approaches and heuristics have been proposed to solve these single-echelon, single-product network design problems. Hansen *et al.* (2007) tackle very large instances of the CFLPSS with a primal-dual variable-neighborhood search metaheuristic that yields near-optimal solutions with an optimality gap not exceeding 0.04%. Several extensions or variants of the CFLP and CFLPSS have been proposed. Multi-product as well as multi-echelon models have been formulated and solved, usually by Benders decomposition (Geoffrion & Graves, 1974) or Lagrangean relaxation (Klose, 2000). These extended models are more difficult to solve than basic CFLP or CFLPSS models, yet they are simpler than the problem tackled in this paper. A recent and thorough review of the literature on facility location problems and their extensions is found in Klose & Drexl (2005).

In facility location models, the capacity of potential facilities is assumed to be predetermined. As capacity acquisition is a rather fundamental aspect of supply chain design problems, several authors investigated capacity expansion and relocation alternatives. Verter and Dincer (1992) discuss the relationship between facility location, capacity expansion and technology selection problems. Paquet *et al.* (2004) and M'Barek *et al.* (2010) consider several discrete facility capacity options for each location, while others such as Eppen *et al.* (1989) and Amrani *et al.* (2010) consider alternative site configurations (platforms), an approach also used in this paper. Following the observation by Ballou (1992) that the throughtput-inventory relation in facilities is not linear but rather concave, due to risk-pooling effects, some recent papers such as Martel (2005) and

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Amrani *et al.* (2010) also consider economies of scale in inventory costs. Variable costs are generally assumed to be linear.

In several recent applications found in the literature (Elhedhli and Goffin, 2005; Romeijin *et al.*, 2007), it is assumed that the type of activities that can be performed over a given location are predetermined (such as production, assembly or warehousing). Lakhal *et al.* (1999) introduced the concept of activity graph to map the succession of sourcing, manufacturing, warehousing and transshipment activities that constitutes the company's supply chain. In these models, the actual mapping of activities on locations is determined by the model. Supply chain network design models based on activity graphs were subsequently proposed by Vila *et al.* (2006) and M'Barek *et al.* (2010). Although several applications consider a single period, some authors included multiple production and demand seasons in their model (Arntzen *et al.*, (1995); Dogan and Goetschalckx, (1999)). According to Martel (2005), multi-season models anticipate variations in demand and activity levels during a planning horizon. An integrated multi-season model is found in Martel (2005), while a multi-period model is proposed in Paquet *et al.* (2007).

For the sake of simplicity, our model does not include modeling components related to international dimensions such as the inclusion of transfer prices, import/export duties and income taxes. International adaptations of supply chain network design models have been proposed by Arntzen *et al.* (1995), Vidal and Goetschalckx (2001), Martel (2005), Vila *et al.* (2006) and M'Barek *et al.* (2009). The modifications required to adapt the model presented in this text to the international context are straightforward. A review of the literature on global supply chain network design is found in Meixell and Gargeya (2005).

Several solutions approaches have been proposed and tested to solve supply chain network design models. Some of the most popular methods are Benders decomposition (Geoffrion and Graves, 1974; Dogan and Goetschalckx, 1999; Paquet *et al.*, 2004; Cordeau *et al.*, 2006), Lagrangean-based methods (Klose, 2000; Elhedhli and Goffin, 2005; Amiri, 2006), successive linear programming or mixed-integer linear programming with valid cuts (Vidal and Goetschalckx, 2001; Martel, 2005; M'Barek, 2010), and Dantzig-Wolfe decomposition (Liang and Wilhelm, 2008). In the last few years, several authors proposed metaheuristic solution procedures such as variable-neighborhood search or tabu search (Amrani *et al.*, 2010), iterated local search (Cordeau *et al.*, 2008), as well as simulated annealing (Jayaraman and Ross, 2003) and genetic algorithms (Syarif *et al.*, 2002; Zhou *et al.*, 2002; Altiparmak *et al.*, 2006). It should be noted that

all of those metaheuristic procedures assume single sourcing or single assignment constraints for all locations in the network. While this kind of formulation is harder for MIP-based approaches to solve, it circumvents the well-known weakness of most metaheuristics in dealing with continuous variables (which are used to model flows).

The model proposed in this paper can be seen as a reformulation and generalization of Amrani *et al.* (2010), based on concepts and modeling constructs introduced in Martel (2005), Vila *et al.* (2006) and M'Barek *et al.* (2010), as well as on original extensions such as transportation options. It covers the entire supply chain, from vendor selection to site configuration and market policy selection. A variant of the model (Martel *et al.*, 2010) is implemented in the SCN design software commercialized by Modellium Inc. The method proposed to solve the model is based on the A-Team paradigm introduced by Talukdar *et al.* (2003), and it incorporates several specialized metaheuristics.

*Notational Conventions* – In the following Sections:

- Labels are used to refer to concepts associated with the modeling formalism (ex: activity types, movement types, transportation modes). Labels are denoted by capital letters and they do not change from a business context to another. They are specified using lists and they are incorporated as *superscripts* in the notation. A summary of the labels found in the paper is provided in *Appendix A*.
- Indexes are used to define application specific instances of a concept (ex: activities, movements, products). They are denoted by italic lowercase letters and defined using sets. They are incorporated as *subscripts* in the notation.
- To distinguish *concept lists* from *index sets*, we use bold capital letters to denote lists and capital italic letters to denote sets. For example: A = [V,C,F,W,D] versus A = {1,2,...,8}. Arbitrary elements of a list are denoted by the corresponding lower case letter (for example: a ∈ A), and arbitrary elements of a set by the corresponding italic lower case letter (for example: a ∈ A).
- Sets are partitioned into subsets using concept superscripts. For example:  $A^{\rm F} = \{3,4,5,6\}$ ,  $A^{\rm W} = \{2,7\} \subset A$ . The union of type subsets is denoted using sub-list superscripts. For example,  $A^{\rm S}$ , with  ${\bf S} = [{\rm C},{\rm F},{\rm W}]$ , denotes  $A^{\rm C} \cup A^{\rm F} \cup A^{\rm W}$ .
- The arrow → is used as a superscript to represent outbound flows or successors and the arrow
   ← to represent inbound flows or predecessors.
- Decision variables are denoted by capital italic letters.
- Parameters are denoted by lower-case italic or Greek letters.

### **3** ACTIVITY-BASED VIEW OF THE SUPPLY CHAIN NETWORK DESIGN PROBLEM

We consider a supply chain network (SCN) composed of external *vendors* (or vendor clusters), internal production-distribution *sites*, possibly including third-party facilities (subcontractors, public warehouses...), and external *demand zones* (clusters of ship-to-points located in a given geographical area). In order to be as generic as possible, several modeling concepts are introduced. In this section, these concepts are explained, associated variables and parameters are introduced, and related constraints are formulated.

#### 3.1 Planning Horizon and Time Representation

Our aim is to design the best possible SCN over a planning horizon incorporating several planning cycles  $h \in H$ , each covering several planning periods  $t \in T_h$   $(T = \bigcup_{h \in H} T_h)$ . We use h(t) to denote the planning cycle of period t. Strategic decisions, related to facility location and configuration, to vendor contracts, to market policies and to transportation options, are made at the planning cycle level, which may encompass one or more planning periods, as shown in *Figure 1*. On the other hand, aggregate operational decisions related to activity levels, inventories and network flows are made at the planning period level.





#### 3.2 Products, Activities and Locations

A product  $p \in P$  corresponds to a family of items requiring the same type of production capacity, or supplied by similar vendors, and having the same type of demand process. A product can be a raw material, an intermediate component used in an assembly activity or a final product that is sold to a customer.

The SCN design policies adopted by a company and its manufacturing processes can be defined conceptually by a directed *activity* graph,  $\Gamma = (A, M)$ , such as the one illustrated *Figure 2*. The graph incorporates a set A of internal and external activities. Two generic external activities are always present, namely a supply activity (a = 1) and demand activity ( $a = \overline{a} = |A|$ ). Three types of internal activities can be defined: fabrication-assembly ( $a \in A^F$ ), warehousing-storage ( $a \in A^W$ ) and consolidation-transshipment ( $a \in A^C$ ) activities. Fabrication-assembly activities are restricted to many-to-one production processes, i.e., for a transformation activity  $a \in A^{\mathsf{F}}$ , output products  $p \in P_a^{\rightarrow}$  are manufactured with a specified quantity  $g_{ap'p}$  of each input products  $p' \in P_a^{\leftarrow}$  (this quantity can be zero for some input products). The arrows between activities define possible product *movements*  $(a, a') \in M$ . Movements are associated with a set of products  $P_{(a,a')} \subset P$ , and they can be restricted *a priori* to inter-location moves  $M^{\mathsf{T}} \subset M$  (transportation) or intra-location moves  $M^{\mathsf{H}} \subset M$  (material handling). Some movements  $m \in M$  may also be unrestricted.



Figure 2: Directed Activity Graph Example

The following parameters are defined:

 $g_{app'}$ :Quantity of product  $p \in P_a^{\leftarrow}$  needed to make one product  $p' \in P_a^{\rightarrow}$  in activity  $a \in A^F$  $q_{pa}$ :Capacity consumption per unit of product  $p \in P_a^{\rightarrow}$  flowing through activity  $a \in A$  $s_{pa}$ :Space required per unit of product  $p \in P_a^{\rightarrow}$  stored in activity  $a \in A^W$ 

Vendors, facility sites and demand zones are associated with geographical *locations*  $l \in L$ , and, accordingly, we distinguish three types of locations: vendor locations (V), site locations (S) and demand zones (D). Vendor  $l \in L^{\vee} \subset L$  can supply products  $P_l \subset P$ , and demand zone  $l \in L^{\mathbb{D}} \subset L$  requires products  $P_l \subset P$ . The demand zones to serve may change from period to period, and  $L_{pt}^{\mathbb{D}} \subseteq L^{\mathbb{D}}$  is the subset of zones requiring product p in period  $t \in T$ . The facility site locations  $L^{\mathbb{S}} \subset L$  considered correspond to existing company or third-party facilities, or to locations where a facility could be operated.

## 3.3 Transportation Options

Transportation between locations can be performed using different shipping means  $s \in S^{T}$ , subdivided according to their transportation mode: air ( $S^{A}$ ), ocean ( $S^{O}$ ), railway ( $S^{R}$ ), driveway ( $S^{D}$ ) or intermodal ( $S^{T}$ ), with T=[A,O,R,D,I]. The network capacity of a shipping mean  $s \in S^{T}$  during a time period is provided by a set of transportation options  $o \in O$ . These options may be associated with an internal fleet, a long term 3PL contract or short term for-hire transportation. It is assumed that a transportation mean is not based at a particular facility site and that it can be used anywhere in the network provided that the required infrastructures are available. There is a variable cost associated with the use of a transportation mean and a fixed cost is incurred when an option is selected. This fixed cost covers fleet terminal, replacement and repair costs, or external contract costs. Some options may already be in place at the beginning of the planning horizon. Intra-location moves can be performed using different handling means  $s \in S^{H}$  with distinct variable costs. Collectively, transportation and handling means define a set of *transfer* means  $S = S^{T} \cup S^{H}$ .

The following sets, variables and parameters are required to consider transportation options:

- $O_{sh}$  Capacity options available for shipping mean  $s \in S^{T}$  during planning cycle h.
- $Z_{oh}:$  Binary variable equal to 1 if transportation capacity option  $o \in O$  is selected at the beginning of planning cycle  $h \in H$ .
- $q_p^t$ : Capacity consumption (in handling units) per unit of product *p* flowing through reception/shipping facilities for transportation mode  $t \in \mathbf{T}$ .
- $\tau_{ll's}$ : Traveling time consumed per trip (one way if it is a one-time for-hire mean and round-trip otherwise) when transportation mean  $s \in S^{T}$  is used on lane  $(l,l') \in L \times L$ .
- $u_{ps}$ : Transportation capacity consumed (number of vehicle load required) per shipping unit of product  $p \in P$  when transportation mean  $s \in S^{T}$  is used.
- $\overline{\beta}_{ot}$ : Transportation capacity available (in standard traveling time units) for shipping mean s(o) in period t when option  $o \in O$  is selected (the capacity provided by some options may be unbounded).
- $\underline{\beta}_{ot}$ : Minimal usage (in standard traveling time units) of shipping mean s(o) needed in planning period *t* to be able to use transportation capacity option  $o \in O$ .
- $z_{ot}$ : Fixed cost of using transportation capacity option  $o \in O$  during time period  $t \in T$ .

#### 3.4 Platforms

The facilities already in place are characterized by a *platform* specifying their capacity for each of the activities they perform, as well as their fixed and variable costs. Alternative platforms  $c \in C_l$  (facility configurations) can however be considered for each site  $l \in L^S$ . These alternative platforms may correspond to current layouts, to a reengineering of current layouts or equipments, to the addition of new space and/or equipment to expand capacity, to different facility specifications for new sites, or to alternative third-party facilities for a potential location. Alternative platforms may be associated with different equipment size to capture economies of scale. For each potential site, a set of possible platforms can thus be considered. For a site  $l \in L^S$  and planning period  $t \in T$ , a platform  $c \in C_l$  is characterized by:

- A set of activities  $A_{lc} \subset A^{\mathbf{S}} = A^{\mathbf{C}} \cup A^{\mathbf{F}} \cup A^{\mathbf{W}}$  supported by the platform.
- A capacity, b
   <sub>(l,a)ct</sub>, for each activity a ∈ A<sub>lc</sub>, expressed in terms of an upper bound on a standard capacity measure (production time, storage space...). It is assumed that all the output products p ∈ P<sup>→</sup><sub>a</sub> of an activity a ∈ A<sub>lc</sub> share the capacity provided by the platform for this activity. A capacity consumption rate q<sub>pa</sub> is used to convert the throughput of product p ∈ P<sup>→</sup><sub>a</sub> in the standard capacity measure.
- When platform *c* is implemented at the beginning of planning cycle *h*, if  $t \in T_h$ , a part  $\delta_{(l,a)ct} \leq \overline{b}_{(l,a)ct}$  of the capacity available for activity *a* in period *t* is lost.
- A minimum throughput,  $\underline{b}_{(l,a)ct}$ , for each activity  $a \in A_{lc}$ , required to implement the platform.
- A reception and shipping capacity  $b_{let}^{t}$ , for each transportation mode  $t \in \mathbf{T} = [A, O, R, D, I]$ .
- An alternative platform *c*'(*c*) which could be used as an upgrade. Upgrade-platform *c*'(*c*) can be implemented only when platform *c* is in place. Some platforms cannot be upgraded.
- A fixed exploitation cost  $y_{clt}$  for the planning period. This cost includes fixed operating costs as well as a *rent* paid for using the platform during period *t*. When the facility is rented, or a third-party facility is used, this rent corresponds to the payments made to the facility owners. When the platform is owned, built, reconfigured or acquired by the company, then the rent is the amount that would be obtained if the company was renting the facility on the market. Normally, this rent would cover financial charges and market value depreciation, and possibly an opportunity cost, and it would take into account the asset economic life and the financial horizon of the company.
- An implementation cost  $y_{clt}^+$  if the platform is installed at the beginning of planning cycle h(t). Normally, this cost is positive if the planning period *t* considered is the first period of

cycle h(t), and close to zero otherwise. It is an opening or upgrade project cost paid during the period and it does not include any capital expenditure. It may include costs related to the initial provisioning of safety stocks, personnel hiring costs, support activity set-up costs, etc.

- A disposal cost (return) y<sub>clt</sub><sup>-</sup> if the platform is closed at the beginning of planning cycle h(t). This would cover any cash flow incurred in period t following a shutdown in the first period of cycle h(t). It may include costs/returns associated with the repositioning or disposal of material, equipment and personnel. Closing platform c ∈ C<sub>l</sub> results in the permanent closing of site l, i.e. when a platform is closed on a site, the site cannot be reopened during the horizon.
- A variable throughput cost  $x_{p(l,a)ct}$ , for each output product  $p \in P_a^{\rightarrow}$  of activity  $a \in A_{lc}$ , covering relevant reception, production, handling and shipping expenses.

The set of activities  $A_l$  that could be performed on a potential site  $l \in L^S$  depends on the platforms considered for that site, i.e.  $A_l = \bigcup_{c \in C_l} A_{lc}$ .

In the model, the following sets, variables and parameters are required:

$C_{lh}$	Platforms that can	be used for	site l during	cycle h.
10				2

- $C_{(l,a)h}$  Platforms that can be used to perform activity *a* in site *l* during cycle *h*.
- $Y_{clh}^+, Y_{clh}, Y_{clh}^-$ : Binary variable equal to 1 if, respectively, opening, using or closing platform  $c \in C_l$ at site  $l \in L^S$  at the beginning of planning cycle  $h \in H$ .  $Y_{cl0}, c \in C_l$ , are binary parameters providing the state of site  $l \in L^S$  at the beginning of the horizon.
- $\overline{b}_{(l,a)ct}$ : Maximum capacity available for activity  $a \in A_{lc}$  when platform  $c \in C_{(l,a)h(t)}$  is used at site  $l \in L^{S}$  during period  $t \in T$ .
- <u> $\underline{b}_{(l,a)ch}$ </u>: Minimum activity level for activity  $a \in A_{lc}$  when platform  $c \in C_{(l,a)h}$  is used at site  $l \in L^{S}$  during planning cycle  $h \in H$ .
- $b_{lct}^{t}$  Reception and shipping capacity (in handling units) at site  $l \in L^{s}$  for transportation mode  $t \in \mathbf{T}$  when platform  $c \in C_{lh(t)}$  is used during period  $t \in T$  (taking location  $l \in L^{s}$  transportation infrastructure capabilities into account).
- $\delta_{(l,a)ct}$ : Capacity lost for activity *a* in period *t* when platform *c* is implemented at the beginning of planning cycle *h*(*t*).
- $x_{p(l,a)ct}$ : Unit cost of processing product  $p \in P$  on platform  $c \in C_{(l,a)h(t)}$  in node (l,a) during period  $t \in T$ .

 $y_{clt}^+$ ;  $y_{clt}$ ;  $y_{clt}^-$ : Respectively, unit cost of opening, using and closing platform  $c \in C_{lh(t)}$  at site  $l \in L^{\mathbf{S}}$  during period  $t \in T$ .

Internal location configurations are specified by the platform selection variables  $Y_{clh}^+$ ,  $Y_{clh}$  and  $Y_{clh}^-$ , which must respect the following conditions. Constraint (1) states that no more than a single platform can be implemented on a site in any given planning cycle. Constraints (2) and (3) ensure that a site cannot be closed, or opened, more than once during the planning horizon.

$$\sum_{c \in C_{lh}} Y_{clh} \le 1 \qquad \qquad l \in L^{\mathsf{S}}, h \in H \qquad (1)$$

$$\sum_{h\in H} \sum_{c\in C_{lh}^{\circ}} Y_{clh}^{+} \le 1 - Y_{cl0} \qquad l \in L^{\mathbb{S}}$$

$$(2)$$

$$\sum_{h \in H} \sum_{c \in C_{lh}} Y_{clh}^{-} \le 1 \qquad \qquad l \in L^{\mathbb{S}} \qquad (3)$$

Constraint (4) specifies precedence relations for the upgrade of platforms. An upgrade platform can only be installed if its preceding platform is already in place and if it is not closed at the beginning of the cycle. Constraint (5) ensures that platform states are accounted for correctly, i.e. that a platform can be closed only if it was used during the previous planning cycle, and that a platform cannot be opened and closed during the same planning cycle.

$$Y_{c'(c)lh}^{+} \le Y_{clh-1} - Y_{clh}^{-} \qquad l \in L^{S}, h \in H, c \in C_{lh} \qquad (4)$$

$$Y_{clh} + Y_{c'(c)lh}^{+} + Y_{clh}^{-} - Y_{clh}^{+} - Y_{clh-1}^{+} = 0 \qquad l \in L^{S}, h \in H, c \in C_{lh} \qquad (5)$$

#### 3.5 Vendor Contracts

A vendor may offer different pricing conditions related to guaranteed minimum sales volumes for each period of a planning cycle. These offers are considered as alternative supply contracts. To simplify the notation, we consider that alternative contracts offered by a given vendor define distinct supply sources, and they are all incorporated in the set  $L^{v}$  of potential vendors. To model vendor contracts selection, the following variables and parameters are required:

- $V_{lh}$ : Binary variable equal to 1 if vendor contract  $l \in L^{\vee}$  is selected for planning cycle  $h \in H$ .
- $U_{lt}$ : Penalty paid to the vendor under contract  $l \in L^{V}$  if the minimum sales value specified in the contract is not reached for period  $t \in T$  (decision variable).
- $\overline{b}_{plt}$ : Upper bound on the quantity of product family  $p \in P$  which can be supplied by the vendor under contract  $l \in L^{\vee}$  during period  $t \in T$ .

- <u> $\underline{b}_{lt}$ </u>: Lower bound on the value of products purchased in period  $t \in T$  specified in contract  $l \in L^{\vee}$ .
- $\pi_{plt}$ : Unit procurement price of product family  $p \in P$  from vendor  $l \in L^{\vee}$  during period  $t \in T$ .
- $v_{li}$ : Fixed cost of using vendor contract  $l \in L^{\vee}$  during period  $t \in T$ .

#### 3.6 Product-Markets and Marketing Policies

It is assumed that products are sold in a set of distinct product-markets  $k \in K$ . A productmarket k is defined by a geographical region covering a set of demand zones,  $L_k^{\rm D} \subset L^{\rm D}$ , in which a set of product-families,  $P_k \subset P$  having similar marketing conditions are sold. Three types of markets can be distinguished: inventory-based replenishment markets (I), made-to-order markets (O) and vendor managed inventory (VMI) markets (V). The set of product-markets can thus be partitioned in three subsets  $K^k$ ,  $k \in \mathbf{K} = [I, O, V]$ . We assume that a demand zone is associated with a single market type, i.e. if a geographical location has customers in more than one market type, a distinct demand zone l is defined for each market type. k(l) denotes the market type of location l, and  $L^{Dk} \subseteq L^{D}$  the set of demand zones in the markets of type k. For a market type  $k \in \mathbf{K}$ , a given product-zone pair (p,l) thus belongs to a unique product-market  $k(p,l) \in K^{k(l)}$ . In order to win orders on these product-markets, the company develops different offers to satisfy potential customers better than its competitors. It is assumed that these offers must be defined in terms of delivery response, fill rates and product prices. These offers can be formalized through the marketing *policy* concept. We assume that a set  $J_k$  of policies is considered for each productmarket  $k \in K$ , and that a policy j is associated to a single product-martet k(j). A policy  $j \in J_k$ for a product-market  $k \in K^k$  is characterized by:

- Product prices  $p_{ipt}, p \in P_{k(i)}$ , when the policy is used in period *t*.
- A maximum delivery time, if the product-market is of the *inventory-based replenishment* type, or a minimum fill rate, if it is a *VMI* product-market. Since it may not be possible to satisfy the delivery time, or to provide an adequate fill rate, from all the sites in the network, using any transportation means, because some sites are too far, or some transfer means are too slow or for any other reason, this leads to the association of a set of admissible (location-transportation mean) pairs to the policy (defined in the next section as the sets  $NS_{jpl}^{\leftarrow}$ ).
- A fix marketing and logistics cost  $w_{jt}$  when the policy is used in period  $t \in T$ . For VMI product-markets, this cost would include the inventory holding cost incurred at the customer

location to provide the specified fill rate.

- A minimum market penetration sales quantity  $\underline{d}_{jplt}$  for product  $p \in P_k$  in demand zone  $l \in L_k^{D}$ during period  $t \in T$ .
- A maximum demand quantity  $\overline{d}_{jplt}$  for product  $p \in P_k$  in demand zone  $l \in L_k^{D}$  during period  $t \in T$ .

The following variables and parameters are required to model marketing policies:

- $W_{jh}$ : Binary variable equal to 1 if policy  $j \in J$  is selected for product-market k(j) during planning cycle  $h \in H$
- $\underline{d}_{jplt}, \overline{d}_{jplt}$ : Minimum market penetration quantity and maximum demand quantity for product family  $p \in P$  in demand zone  $l \in L^{D}$  when marketing policy  $j \in J$  is selected during period  $t \in T$
- $p_{jpt}$ : Unit sales price of product family  $p \in P_{k(j)}$  during period  $t \in T$  when marketing policy  $j \in J$  is selected for cycle h(t)
- $w_{it}$ : Fixed cost of using marketing policy  $j \in J$  during period  $t \in T$

Since market policies represent long-term commitments and strategies rather than sales planning tactics, a marketing policy is enforced for a planning cycle rather than for a single time period. The following condition, stating that no more than one market policy can be selected for each market in each planning cycle, must be respected:

$$\sum_{j \in J_k} W_{jh} \le 1 \qquad \qquad h \in H, k \in K \qquad (6)$$

When no policy is selected, it implies that the product-market k will not be serviced by the company during planning cycle h.

## 3.7 Supply Chain Network

When the activity graph  $\Gamma = (A, M)$  is mapped onto the potential locations  $l \in L$ , the supply chain network represented in *Figure 3* is obtained. In this network, the nodes correspond to feasible location-activity pairs  $n = (l, a) \in N$ , and the arcs to feasible product flows between nodes with a given transfer mean in a given time period  $t \in T$ . In what follows, we use l(n) and a(n) to denote, respectively, the location and the activity of node *n*. A location-activity pair (l, a) is feasible if  $a \in A_l$ . A flow between nodes n = (l, a) and n' = (l', a') is not feasible if  $[l = l'] \land [(a, a') \in M^T]$  or if  $[l \neq l'] \land [(a, a') \in M^H]$ . For a given node *n*, the set of destinations of feasible outbound arcs is denoted by  $N_n^{\rightarrow}$ , and the set of origins of feasible inbound arcs by  $N_n^{\leftarrow}$ . Note also that, for *internal* origin-destination pairs (n,n') = ((l,a),(l,a')), parallel arcs exist for all feasible pairs  $(p,s) \in P_{(a,a')} \times S_{pnn'}$ , where  $S_{pnn'}$  is the set of transfer means which can be used for product *p* between origin *n* and destination *n'*. Similarly, for *supply* origin-destination pairs (n,n') = ((l,1),(l',a')), parallel arcs exist for all  $(p,s) \in (P_{(1,a)} \cap P_l) \times S_{pnn'}$ , and for *demand* origin-destination pairs (n,n') = ((l,1),(l',a')), parallel arcs exist for all  $(p,s) \in (P_{(1,a)} \cap P_l) \times S_{pnn'}$ , and for *demand* origin-destination pairs  $(n,n') = ((l,a),(l',\overline{a}))$ , parallel arcs exist for all the products *p* required by demand zone *l'*, and for all transportation means *s* and policies *j* which can be implemented from node *n*.



Figure 3: Supply Chain Network Representation for a Time Period  $t \in T$ 

To model activity levels and flows, the following sets, variables and parameters are required:

- $N^{a}$ : Feasible nodes for activity type  $a \in \mathbf{S} = [\mathbf{C}, \mathbf{F}, \mathbf{W}] (N^{a} \subset L^{S} \times A^{a}).$
- $N_t^{\rm D}$ : Feasible demand nodes in period  $t (N_t^{\rm D} = \{(l, \overline{a})\}_{l \in I^{\rm D}})$ .
- $N_{pn}^{\rightarrow}$ : Destinations of feasible outbound arcs from node *n* for product  $p \in P_{a(n)}^{\rightarrow}$ , i.e. such that  $p \in P_{(a(n),a(n'))}$
- $N_{pn}^{\leftarrow}$ : Origins of feasible inbound arcs to node *n* for product  $p \in P_{a(n)}^{\leftarrow}$ , i.e. such that  $p \in P_{(a(n),a(n))}$

- $NS_{jpl}^{\leftarrow}$ : Set of (node-transportation mean) pairs  $(n, s) \in N^{\mathbf{S}} \times S_{pn(l,\overline{a})}^{\mathbf{T}}$  the company could use to provide product  $p \in P_l$  to demand zone  $l \in L_{pk(j)}^{\mathbf{D}}$  when marketing policy  $j \in J_{k(p,l)}$  is selected.
- $X_{pnct}$ : Activity level in node *n* for product  $p \in P_{a(n)}^{\rightarrow}$  when platform  $c \in C_{nh(t)}$  is used in period *t* (quantity produced when  $a(n) \in A^{\text{F}}$  and throughput when  $a(n) \in A^{\text{W}} \cup A^{\text{C}}$ ).
- $F_{pnn'st}$ : Flow of product  $p \in P_{(a(n),a(n'))}$  from node *n* to node *n'* with transfer mean  $s \in S$ during period  $t \in T$  (transportation if  $s \in S^{T}$  and handling if  $s \in S^{H}$ ).
- $F_{jpn(l,\overline{a})st}$ : Flow of product  $p \in P_{a(n)}^{\rightarrow}$  from node  $n \in N^{\mathbf{S}}$  to demand-node  $(l,\overline{a}), l \in L_{pt}^{\mathbf{D}}$ , with transportation mean  $s \in S^{\mathbf{T}}$ , under policy  $j \in J_{k(n,l)}$  during period  $t \in T$ .
- *I*<sub>pnct</sub>: Level of strategic inventory of product family  $p \in P$  for storage node  $n \in N^{W}$  held with platform *c* at the end of period  $t \in T$ .
- $f_{p(l,a)(l,a')t}^{h}$ : Unit material handling cost of product  $p \in P_{(a,a')}$  between node (l,a) and node (l,a') during period *t*.
- $f_{pnn'st}^{o}$ : Unit cost of the flow of product *p* between node *n* and node *n'* when using transportation mean *s*, paid by the *origin n* during period *t* (this cost includes the customer-order processing cost, the shipping cost, the variable transportation cost and the inventory-in-transit holding cost).
- $f_{pn'nst}^{d}$ : Unit cost of the flow of product p between node n' and node n when using transportation mean s, paid by *destination* n during period t (this cost includes the supply-order processing cost and the reception cost for all  $n \in N^{s}$ , as well as the variable inbound transportation cost when the origin is a vendor, i.e. when  $l(n') \in L^{v}$ ).

Vendors' capacity and pricing contracts are expressly embedded in the model. Constraint (7) specifies that under contract  $l \in L^{\vee}$  the vendor can supply a limited quantity of each product per time period. Constraint (8) ensures that the minimum sales volume per period required to benefit from a contract prices are reached or otherwise that a penalty  $U_{lt}$  is paid.

$$\sum_{n \in N_{p(l,1)}^{\rightarrow}} \sum_{s \in S_{p(l,1)n}} F_{p(l,1)nst} \leq V_{lh(t)} \overline{b}_{plt} \qquad \qquad l \in L^{\vee}, p \in P_l, t \in T$$
(7)

$$V_{lh(t)}\underline{b}_{lt} \leq \sum_{n \in N_{(l,1)}^{\rightarrow}} \sum_{p \in P_l \cap P_{(l,a(n))}} \sum_{s \in S_{p(l,1)n}} \pi_{plt} F_{p(l,1)nst} + U_{lt} \qquad l \in L^{\vee}, t \in T \qquad (8)$$

For variable throughput costs to be modeled adequately, the node activity levels in period t,

 $X_{pnct}$ , must be associated to the platform  $c \in C_{nh(t)}$  used. Equation (9) defines the node's throughput for a given product and time period as the sum of outflows to other internal nodes and to customers.

$$\sum_{c \in C_{nh(t)}} X_{pnct} = \sum_{n' \in N_{pn}^{\rightarrow} \cap N^{\mathbf{S}}} \sum_{s \in S_{pm'}} F_{pnn'st} + \sum_{(l,\overline{a}) \in N_{pn}^{\rightarrow} \cap N^{\mathbf{D}}_{t}} \sum_{(j,s) \mid (n,s) \in NS_{jpl}^{\leftarrow}, j \in J_{k(p,l)}} F_{jpn(l,\overline{a})st} \ n \in N^{\mathbf{S}}, p \in P_{a(n)}^{\rightarrow}, t \in T$$
(9)

Throughputs must also be related to inflows. Constraint (10) is required to ensure that production levels do not exceed what can be done with incoming components. For consolidation-transshipment nodes, (11) ensures flow equilibrium. For storage nodes, (12) provides strategic inventory accounting constraints. Strategic inventories are passed from period to period to smooth operations or to prepare for network structure modifications at the end of planning cycles.

$$\sum_{c \in C_{nh(t)}} \sum_{p' \in P_{a(n)}^{\rightarrow}} g_{app'} X_{p'nct} \leq \sum_{n' \in N_{pn}^{\leftarrow}} \sum_{s \in S_{pn'n}} F_{pn'nst} \qquad n \in N^{\mathsf{F}}, p \in P_{a(n)}^{\leftarrow}, t \in T$$
(10)

$$\sum_{c \in C_{nh(t)}} X_{pnct} = \sum_{n' \in N_{pn}^{\leftarrow}} \sum_{s \in S_{pn'n}} F_{pn'nst} \qquad n \in N^{\mathbb{C}}, p \in P_{a(n)}^{\leftarrow}, t \in T$$
(11)

$$\sum_{c \in C_{nh(t)}} (I_{pnct} + X_{pnct} - I_{pnct-1}) = \sum_{n' \in N_{pn}^{\leftarrow}} \sum_{s \in S_{pn'n}} F_{pn'nst} \qquad n \in N^{\mathsf{W}}, p \in P_{a(n)}^{\leftarrow}, t \in T$$
(12)

Platforms capacity and implementation conditions must also be enforced. Constraints (13) state that for a given platform to be opened, a minimum throughput must be achieved.

$$\underline{b}_{(l,a)ch}Y_{clh} \leq \sum_{t\in T_h} \sum_{p\in P_a^{\rightarrow}} q_{pa}X_{p(l,a)ct} \qquad (l,a)\in N^{\mathbf{S}}, h\in H, c\in C_{(l,a)h}$$
(13)

Capacity constraints (14) set an upper bound on maximum throughput per period for a given node, taking into account the fact that, when the platform is opened in the planning cycle of the period considered, a portion of its capacity may be lost.

$$\sum_{p \in P_a^{\to}} q_{pa} X_{p(l,a)ct} \le \overline{b}_{(l,a)ct} Y_{clh(t)} - \delta_{(l,a)ct} Y_{clh(t)}^+ \qquad (l,a) \in N^{\mathbf{S}}, t \in T, c \in C_{(l,a)h(t)}$$
(14)

Reception and shipping capacity limits (in handling units) imposed by the transportation infrastructure capabilities of a platform must also be considered. Constraint (15) imposes these restrictions. Constraint (16) ensures that the network transportation capacity provided by the capacity options selected for a given shipping mean is not exceeded.

$$\sum_{n\in\mathbb{N}_{l}}\sum_{s\in\mathbb{S}^{t}}\left[\sum_{p\in\mathbb{P}_{a(n)}^{\leftarrow}}q_{p}^{t}\sum_{n'\in\mathbb{N}_{pn}^{\leftarrow}}F_{pn'nst}+\sum_{p\in\mathbb{P}_{a(n)}^{\rightarrow}}q_{p}^{t}\left(\sum_{n'\in\mathbb{N}_{pn}^{\rightarrow}\cap\mathbb{N}^{S}}F_{pnn'st}+\sum_{n'\in\mathbb{N}_{pn}^{\rightarrow}\cap\mathbb{N}^{D}}\sum_{j\in J_{k(p,J(n'))}|(n,s)\in\mathbb{N}S_{jpl(n)}^{\leftarrow}}F_{jpnn'st}\right)\right]$$

$$\leq\sum_{c\in\mathcal{C}_{lh(t)}}b_{lct}^{t}Y_{clh(t)} \qquad t\in\mathbf{T}, l\in\mathbb{L}^{S}, t\in\mathbf{T}$$

$$(15)$$

$$\sum_{o \in O_{sh(t)}} \underline{\beta}_{ot} Z_{oh(t)} \leq \sum_{n \in \mathbb{N}^{S}} \sum_{p \in P_{a(n)}^{\leftarrow}} u_{ps} \sum_{n' \in \mathbb{N}_{pn}^{\leftarrow}} \tau_{l(n')l(n)s} F_{pn'nst} + \sum_{n \in \mathbb{N}_{t}^{D}} \sum_{p \in P_{a(n)}^{\leftarrow}} u_{ps} \sum_{n' \in \mathbb{N}_{pn}^{\leftarrow}} \sum_{j \in J_{k(p,l(n))} \mid (n',s) \in NS_{jpl(n)}^{\leftarrow}} \tau_{l(n')l(n)s} F_{jpn'nst} \leq \sum_{o \in O_{sh(t)}} \overline{\beta}_{ot} Z_{oh(t)} \qquad s \in S^{T}, t \in T$$

$$(16)$$

Finally, market conditions must also be respected. Constraints (17) state that we must comply with the market penetration targets and maximum demands associated to the marketing policies selected.

$$W_{jh(t)}\underline{d}_{jplt} \leq \sum_{(n,s)\in NS_{jpl}^{\leftarrow}} F_{jpn(l,\overline{a})st} \leq W_{jh(t)}\overline{d}_{jplt} \qquad t \in T, l \in L_t^{\mathsf{D}}, p \in P_l, j \in J_{k(p,l)}$$
(17)

#### 3.8 Order Cycle and Safety Stocks

In addition to strategic inventories, order cycle inventories and safety stocks must also be considered in the model since they depend on storage activity throughputs and on the transfer means used. The level of these stocks also depends on the operations management policies of the company and on the ordering behavior of customers. It can be shown (Martel, 2003) that, when sound inventory management and forecasting methods are used, the relationship between the throughput  $X_{pn}$  of product  $p \in P_{a(n)}^{\rightarrow}$  in storage node  $n = (l,a) \in N^{W}$ , the procurement lead time  $\tau_{pn}$  associated with the location of the supply source, the transfer mean used, and the average cycle and safety stock  $\overline{I}_{pa}(X_{pn}, \tau_{pn})$  required to support this throughput takes the form of the following power function  $\overline{I}_{pa}(X_{pn}, \tau_{pn}) = \alpha_{pa}(X_{pn})^{\beta_{pa}}(\tau_{pn})^{\chi_{pa}}$ , with  $\beta_{pa}, \chi_{pa} \leq 1$  to reflect economies of scale. The parameters  $\alpha_{pa}$ ,  $\beta_{pa}$  and  $\chi_{pa}$  of this function are obtained by regression, from historical or simulation data (Ballou, 1992). We assume here that the throughput  $X_{pn}$  used as an argument in this function is the sum of all product p shipments from node  $n \in N^{W}$  to feasible destinations  $n' \in N_n^{\rightarrow}$ .

If the historical throughput level, average lead time and average inventory level observed for a period (for product p in node n) are  $X_{pn}^{o}$ ,  $\tau_{pn}^{o}$  and  $\overline{I}_{pa}(X_{pn}^{o}, \tau_{pn}^{o})$ , respectively, then the ratio  $X_{pn}^{o}/\overline{I}_{pa}(X_{pn}^{o}, \tau_{pn}^{o})$  is the familiar inventory turnover ratio, and its inverse  $\rho_{pn}^{o} = \overline{I}_{pa}(X_{pn}^{o}, \tau_{pn}^{o})/X_{pn}^{o}$  is the number of periods of inventory kept in stock. Assuming that the relationship between inventory level and throughput is linear boils down to approximating  $\overline{I}_{pa}(X_{pn}, \tau_{pn})$  by  $\rho_{pn}^{o}X_{pn}$ . Since the facilities' throughputs, the sourcing location and the transfer mean are not known before the network design model is solved, and since they can be far from historical values (mainly if new facilities are opened or existing ones closed), calculating inventory levels with historical inventory turnover ratios can be completely inadequate. An effort is therefore made in this paper to take risk

pooling effects into account explicitly. Starting from the inventory-throughput function just defined, and taking into account the average unit inventory holding cost  $r_{pnct}$  of products  $p \in P_{a(n)}^{\rightarrow}$  when platform  $c \in C_{nh(t)}$  is used at site l(n) during period  $t \in T$ , the following inventory cycle and safety stock cost function results, when the product is supplied from node  $n' \in N_{pn}^{\leftarrow}$  using transfer mean  $s \in S_{pn'n}$ :

$$H_{pnct}(X_{pnct},\tau_{pn'ns}) = r_{pnct}\overline{I}_{pa(n)}(X_{pnct},\tau_{pn'ns}) = r_{pnct}\alpha_{pa(n)}(X_{pnct})^{\beta_{pa(n)}}(\tau_{pn'ns})^{\chi_{pa(n)}}$$
(18)

where  $\tau_{pn'ns}$  is the procurement lead-time of product  $p \in P_{a(n)}^{\rightarrow}$  in node *n* when supplied by node  $n' \in N_{pn}^{\leftarrow}$  using transfer mean  $s \in S_{pn'n}$ .

Since (n', s) is to be optimized, the lead-time  $\tau_{pn'ns}$  is not known beforehand but, for period *t*, it can be approximated by the average lead-time  $T_{pnt}/X_{pnct}$ , to get the simplified inventory-throughput function:

$$\overline{I}_{pa(n)}(X_{pnct}, T_{pnt}) = \alpha_{pa(n)}(X_{pnct})^{\beta_{pa(n)}}(T_{pnt}/X_{pnct})^{\chi_{pa(n)}}, \quad T_{pnt} = \sum_{n' \in N_{pn}^{\leftarrow}} \sum_{s \in S_{pn'n}} \tau_{pn'ns} F_{pn'nst}$$
(19)

This is still a complex non-separable concave function and additional assumptions can be made to simplify it further.

First, we can assume that the lead-time  $\tau_{pn'ns}$  does not depend on procurement flows so that it can be estimated empirically from historical data to get

$$\overline{I}_{pa(n)}(X_{pnct}) = \alpha_{pa(n)}(\tau_{pn}^{o})^{\chi_{pa(n)}}(X_{pnct})^{\beta_{pa(n)}}$$
(20)

where  $\tau_{pn}^{o}$  is the empirically estimated lead-time. When this is done, the function still captures economies of scale but it is separable and the model obtained can be solved more easily using separable or successive linear programming techniques. The impact of sourcing and transfer mean selection decisions on safety stocks is not considered, however. Under this assumption, the following relations must be included in the model:

$$\sum_{c \in C_{nh(t)}} \overline{I}_{pnct} = \overline{I}_{pa(n)}(X_{pnct}) \qquad \qquad n \in N^{\mathsf{W}}, p \in P_{a(n)}^{\rightarrow}, t \in T$$
(21)

where,

$$\overline{I}_{pnct}$$
: Average level of cycle and safety stocks of product family  $p$  held in period  $t$ , using platform  $c$ , for storage node  $n \in N^{W}$ .

An alternative is to assume that the lead-time and throughput terms are linear (i.e. that  $\beta_{pa(n)} = \chi_{pa(n)} = 1$ ). Then the inventory-throughput function reduces to:

$$\overline{I}_{pnt} = \sum_{n' \in N_{pn}^{\leftarrow}} \sum_{s \in S_{pn'n}} \rho_{pn'ns} F_{pn'nst} \qquad (\text{with } \rho_{pn'ns} \equiv \alpha_{pa(n)} \tau_{pn'ns})$$
(22)

where  $\rho_{pn'ns}$  is the average number of period of product  $p \in P_{a(n)}^{\rightarrow}$  cycle and safety stock kept at node  $n \in N^{W}$ , when supplied from node  $n' \in N_{pn}^{\leftarrow}$  using transfer mean  $s \in S_{pn'n}$ . This takes the impact of sourcing and transfer mean selection decisions into account, but it neglects economies of scale. Under this assumption, constraint (21) is replaced by (23), which simplifies the model considerably.

$$\sum_{c \in C_{nh(t)}} \overline{I}_{pnct} = \sum_{n' \in N_{pn}^{\leftarrow}} \sum_{s \in S_{pn'n}} \rho_{pn'ns} F_{pn'nst} \qquad n \in N^{\mathsf{W}}, p \in P_{a(n)}^{\rightarrow}, t \in T$$
(23)

Capacity for storage nodes is usually expressed in terms of storage space available, rather than maximum platform throughput. For storage nodes, if there is no throughput constraint, the capacity  $\bar{b}_{nct}$ ,  $n \in N^{W}$ , in (14) can be set to an arbitrary large number. The constraints are still required, however, to ensure that the relationship between throughput variables and platform selection variables is properly defined. The following storage space constraints are also required for each platform:

$$\sum_{p \in P_{a}^{\rightarrow}} s_{pa}(\eta_{pa}\overline{I}_{p(l,a)ct} + I_{p(l,a)ct}) \leq \overline{b}_{(l,a)ct}Y_{clh(t)} - \delta_{(l,a)ct}Y_{clh(t)}^{+} \qquad (l,a) \in N^{W}, t \in T, c \in C_{(l,a)h(t)}$$
(24)

where

 $\eta_{pa}$ : Order cycle and safety stocks (maximum level)/(average level) ratio for product  $p \in P$  for activity  $a \in A^{W}$ .

## 4 MATHEMATICAL PROGRAMMING MODEL

This Section completes the formulation of the optimization model proposed to design supply chain networks. The objective of the model is to maximize the value added by the network over the planning horizon. Expenses can be split in two categories: general costs that are paid across the network, such as market policy and vendor contract fixed costs, and expenses that are linked to a specific site. *Table 1* lists the network costs for each period *t*. *Table 2* lists the revenues and expenses associated with each site for each period *t*. The revenues and expenses in these tables provide the elements necessary to prepare site and corporate financial statements. The modeling of the revenues and expenses in the tables is based the following assumptions:

- All outbound transportation costs on the network arcs, except those coming from vendors, are paid by the origin and they are linear with respect to flows.

- Order processing costs, reception costs and shipping costs are independent of the platform used.

	Period $t \in T$							
Expenses	(a) Transportation capacity options	$\sum_{o \in O} z_{ot} Z_{oh(t)}$						
	(b) Marketing policies	$\sum_{j\in J} w_{jt} W_{jh(t)}$						
	(c) Vendor contracts	$\sum_{l \in L^{V}} \left( v_{lt} V_{lh(t)} + U_{lt} \right)$						

Table 1: Network Expenses

		Site $l \in L^{S}$ , period $t \in T$
Expenses	(d) Raw material procurement	$\sum_{l' \in L^{V}} \sum_{a \in A_{t} \mid (1,a) \in \mathcal{M}} \sum_{p \in P_{(1,a)} \cap P_{l}} \sum_{s \in S_{p(l',1)(l,a)}^{T}} \pi_{pl't} F_{p(l',1)(l,a)st}$
	(e) Inbound flows from all locations	$\sum_{a \in A_l} \sum_{n \in N_{(l,a)}^{\leftarrow}} \sum_{p \in P_{(a(n),a)}} \sum_{s \in S_{pn(l,a)}^{\mathbf{T}}} f_{pn(l,a)st}^{d} F_{pn(l,a)st}$
	(f) Platforms	$\sum_{c \in C_{lh(t)}} (y_{clt}^{+} Y_{clh(t)}^{+} + y_{clt} Y_{clh(t)} + y_{clt}^{-} Y_{clh(t)}^{-})$
	(g) Activity processing	$\sum_{a \in A_l} \sum_{c \in C_{(l,a)h(t)}} \sum_{p \in P_a^{\rightarrow}} x_{p(l,a)ct} X_{p(l,a)ct}$
	(h) Material handling	$\sum_{a \in A_l} \sum_{a' \in A_l \setminus \{a\}} \sum_{p \in P_{(a,a')}} \sum_{s \in S^{\mathrm{H}}} f_{p(l,a)(l,a')t}^{h} F_{p(l,a)(l,a')st}$
	(i) Inventory holding cost	$\sum_{a \in A_l \cap A^{W}} \sum_{c \in C_{(l,a)h(t)}} \sum_{p \in P_a^{\rightarrow}} r_{p(l,a)ct} [\overline{I}_{p(l,a)ct} + I_{p(l,a)ct}]$
	(j) Outbound flows to all locations	$\sum_{a \in A_l} \Big[ \sum_{n' \in N_{p(l,a)}^{\rightarrow} \cap N^{\mathbf{S}}} \sum_{p \in P_{(a,a(n'))}} \sum_{s \in S_{p(l,a)n'}^{\mathbf{T}}} f_{p(l,a)n'st}^{\mathbf{o}} F_{p(l,a)n'st} + \sum_{(l',\overline{a}) \in N_{p(l,a)}^{\rightarrow} \cap N^{\mathbf{D}}} \sum_{p \in P_{(a,\overline{a})} \cap P_{l'}} \sum_{s \in S_{p(l,a)(l',\overline{a})}^{\mathbf{T}}} f_{p(l,a)(l',\overline{a})st}^{\mathbf{o}} \Big( \sum_{j \in J_{k(p,l')} \mid ((l,a),s) \in NS_{jpl'}^{\leftarrow}} F_{jp(l,a)(l',\overline{a})st} \Big) \Big]$
Revenues	(k) Sales to demand zones	$\sum_{a \in A_l} \sum_{(l',\overline{a}) \in N_{p(l,a)}^{\rightarrow} \cap N^{\mathrm{D}}} \sum_{p \in P_{(a,\overline{a})} \cap P_{l'}} \sum_{s \in S_{p(l,a)(l',\overline{a})}^{\mathrm{T}}} \sum_{j \in J_{k(p,l')} \mid ((l,a),s) \in NS_{jpl'}^{\leftarrow}} p_{jpt} F_{jp(l,a)(l',\overline{a})st}$

Table 2: Site Revenues and Expenses

- Financial statements are produced for third-party locations even if they are not controlled by the company.
- All financial charges, assets market value depreciation and opportunity costs are covered by the annual rent charged for a platform.
- Income taxes are not taken into account.

Let:

- $E_t$ : Total general network expenditures for period t
- $R_{lt}$ : Total site *l* revenues for period *t*
- $E_{lt}$ : Total site *l* expenses for period *t*

Using the expressions in *Tables 1* and 2, revenues and expenditures are calculated as follows:

 $E_t = (a) + (b) + (c)$   $t \in T$  (25)

$$E_{lt} = (d) + (e) + (f) + (g) + (h) + (i) + (j) \qquad l \in L^{S}, t \in T$$
(26)

$$R_{lt} = (\mathbf{k}) \qquad \qquad l \in L^{\mathrm{S}}, \ t \in T \tag{27}$$

In our context, the value added by the SCN in period *t* is given by net operating profits:

$$NOP_{t} = \sum_{l \in L^{S}} R_{lt} - \left(\sum_{l \in L^{S}} E_{lt} + E_{t}\right) \qquad t \in T$$

$$(28)$$

The objective of the company is to maximize the sum of discounted net operating profits over the planning horizon:

$$Max \sum_{t \in T} \left[ \frac{NOP_t}{(1+\alpha)^t} \right]$$
(29)

where  $\alpha$  is a discount rate based on the weighted average costs of capital of the company.

Based on the previous discussion, the mathematical programming model obtained for the multiperiod activity-based supply chain network design problem considered is the following:

Maximize objective function (29)

subject to the following constraints:

- Platform selection constraints (1) (5)
- Vendor capacity and contract condition constraints (7) and (8)
- Platform throughput calculation and flow equilibrium constraints (9) (12)
- Platform throughput capacity constraints (13) and (14)

- Reception, shipping and transportation capacity constraints (15) and (16)
- Market policy selection and sales constraints (6) and (17)
- Order cycle and safety stock definition constraints (21) or (23)
- Storage capacity constraints (24)
- Revenue and expenditure definition constraints (25) (28)
- Non-negativity and binary variable definition constraints:

$$F_{pnn'st} \ge 0 \qquad p \in P, n \in N \setminus N^{\mathsf{D}}, n' \in N^{\mathsf{S}}, s \in S, t \in T$$
(30)

$$F_{jpnn'st} \ge 0 \qquad p \in P, n \in N^{\mathsf{S}}, n' \in N^{\mathsf{D}}, j \in J_{k(p,l(n'))}, s \in S, t \in T$$

$$(31)$$

$$X_{pnct} \ge 0 \qquad \qquad n \in N^{\mathbf{S}}, p \in P_{a(n)}^{\rightarrow}, c \in C_{nh(t)}, t \in T$$
(32)

$$I_{pnt}^{\max}, \mathsf{T}_{pnt} \ge 0 \qquad \qquad p \in P, n \in N^{\mathsf{W}}, c \in C_{nh(t)}, t \in T$$
(33)

$$I_{pnct} \ge 0 \qquad p \in P, n \in N^{w}, t \in T \qquad (34)$$
$$U_{t} \ge 0 \qquad l \in L^{v}, t \in T \qquad (35)$$

$$Y_{clh}^{+}, Y_{clh}, Y_{clh}^{-} \in \{0, 1\}$$

$$l \in L^{S}, h \in H, c \in C_{lh}$$

$$i \in J, h \in H$$
(36)
$$i \in J, h \in H$$
(37)

$$V \in \{0,1\}$$

$$I \in I^{V} h \in H$$
(38)

$$V_{lh} \in \{0,1\} \qquad l \in L, n \in H \qquad (38)$$

$$Z_{oh} \in \{0,1\} \qquad \qquad o \in O, \ h \in H \tag{39}$$

## 5 SOLUTION APPROACH

In this Section, we propose an agent-based metaheuristic in order to tackle this SCN design problem. The algorithm proposed is called CAT (Collaborative Agent Team) and it is based on the A-Team paradigm. According to Talukdar *et al.* (2003), "an asynchronous team is a team of software agents that cooperate to solve a problem by dynamically evolving a shared population of solutions." A-Teams have been successfully developed for production planning in the paper industry (Murthy *et al.* 1999), for the problem (Ratajczak-Ropel 2010), among others. CAT is a hybrid distributed solution approach encompassing several types of optimization techniques. The implementation presented here includes mixed-integer linear programs, classical heuristics and metaheuristics.

*Figure 4* displays the main components of our CAT approach. The CAT system is composed primarily of several optimization agents. Each agent has its own methods and rules for deciding when to work, what to work on and when to stop working. An optimization agent can embed one or more optimization algorithms. Four types of agents are defined and used in our system:

- Construction agents create new solutions without referring to any of the existing solutions in the pool. Greedy algorithms are a good example of heuristics used by a typical construction agent.
- Improvement agents start with an existing solution and try to improve it using one or more algorithms. Tabu search is a good example of a typical improvement agent method.
- Destruction agents control the size of the population by eliminating solutions. They remove solutions of least quality and help prevent early convergence by removing solutions that are almost identical.
- Integration agents create new solutions by combining different features from several solutions in the population, instead of working from a single solution.



**Figure 4: CAT Components** 

The blackboard acts as a memory and a hub for all communications. It consists of two components: the population of solutions and a repository of statistics. As shown in *Figure 4* agents communicate solely through the blackboard interface and do not exchange information directly. New solutions, or partial solutions, are put on the blackboard and existing solutions are retrieved

when necessary. Support agents are also provided to assist the user or the other agents. The user interacts with CAT through a web application agent.

One of the main advantages of a distributed approach such as CAT is that each agent may have its own representation of the problem to be solved. For example, one agent may focus on location decisions while another optimizes annual product flows over the entire supply chain network. This allows us to decompose the supply chain network design problem over three dimensions:

- The functional dimension refers to the interrelations between different supply chain decisions such as purchasing and vendor selection decisions, production-distribution facility location and platform selection decisions, marketing policy choices, and transportation capacity options selection.
- The spatial dimension refers to the geographical positioning of business entities such as sales territories, national divisions or subsidiaries.
- The temporal dimension refers to the nature of the multi-period problem. One could focus on periodic decisions related to flows, throughputs and inventories, or on strategic options that span over a specific planning cycle.

Each agent can have either an integrated or decomposed view of each dimension. As a result, most agents work on different subproblems instead of working on the complete formulation. The CAT implementation presented here hosts 16 different agents. Table 3 presents the most important features of each agent; its name, its type, the number of different heuristics it implements, as well as whether the agent has an integrated (full) or decomposed (partial) view over each of the problem dimensions. Since CAT uses 40 different heuristics, it is not possible to provide the pseudo-code for each algorithm. Instead, a general outlook of the approach used by each agent is provided, along with references to similar heuristics. All heuristics and agents are coded in C# and VB.NET 2005, and each agent is an executable program.

The FPump agent implements generic MIP heuristics of the "feasibility pump" type, based on the variants proposed by Bertacco *et al.* (2007) and Achterberg and Berthold (2007). Additional heuristic solutions are obtained by adding redundant valid inequalities in the model such as global capacity cuts (Paquet *et al.* 2004): using a different problem formulation yields a different solution. The Greedy agent uses several greedy heuristics in order to construct complete solutions; each algorithm has a different starting point and uses different priority systems. RIRSS is a generic MIP heuristic that uses progressive variable fixing strategies similar to those found in Thanh (2008). The BasicNet agent constructs partial networks using only the network representation of the problem and simple methods such as basic facility location algorithms and minimal cost network flow models.

Agent	Туре	Heuristics	Functional	Spatial	Temporal
Fpump	Construction	6	Full	Full	Full
Greedy	Construction	8	Full	Full	Full
RIRSS	Construction	2	Full	Full	Full
BasicNet	Construction	3	Partial	Partial	Full
TSV	Improvement	1	Partial	Full	Full
TSI	Improvement	2	Partial	Full	Full
TSD	Improvement	1	Partial	Full	Full
TransOpt	Improvement	1	Partial	Full	Full
RegionalTS	Improvement	2	Full	Partial	Full
FlowOpt	Improvement	1	Full	Full	Partial
CPLEX-SP	Improvement	1	Full	Full	Full
ILS	Improvement	1	Full	Full	Full
CBLS	Improvement	1	Partial	Partial	Partial
Terminator	Destruction	3	Full	Full	Full
Integrate	Integration	6	Full	Full	Full
PIRSS	Integration	1	Full	Full	Full

**Table 3: CAT Agents Implemented** 

TSV and TSI are tabu search agents that focus on the vendor contract selection variables and the production and production-distribution facility location and configuration variables, respectively. TSD also uses tabu search but focuses its work on distribution facility location, configuration and marketing policy selection variables. RegionalTS is also a tabu search which operates on all decision variables relevant to a small portion of the territory covered by the company's supply chain network; this portion usually refers to one of the sales territories or a zone dynamically constructed by the agent itself. TransOpt uses a similar mechanism to optimize transportation options selection and transportation mean usage across the whole network. All tabu search algorithms have a similar structure to the tabu search found in Sörensen (2002) and the variable neighbourhood search heuristic of Amrani *et al.* (2010). FlowOpt solves a network flow problem over the supply chain network; the heuristic fixes the value of all binary variables and then runs the resulting pure linear programming model with the CPLEX® solver. CPLEX-SP uses the same mathematical formulation as the FPump agent but implements the solution polishing feature available in CPLEX® 12.1. ILS is an iterated local search type heuristic whose implementation is similar to the ILS found in Cordeau *et al.* (2008). CBLS is a local search heuristic whose main objective is to explore new solution spaces rather than finding near-optimal solutions to the optimization problem. As such, it constructs a special tabu list which is composed of the variables that have the same value across most of all solutions in the population. Although the solutions it yields are not of exceptional quality, it is very effective for diversification purposes. This agent starts whenever two phenomena are observed simultaneously: solution quality ceases to improve within the solution pool and solution diversity decreases.

The Integrate agent combines features from different solutions into a single solution. For example, vendor selection options from a solution can be integrated with facility configurations and marketing policy selections from another solution. Improvements are then made until a strong local optimum is reached. This agent also uses solution combination heuristics inspired from the crossover operators found in genetic algorithms. PIRSS is an agent that uses a scatter-search type algorithm; it effectively models the solution space formed by the union of two complete solutions as a restricted MIP then explores it thoroughly using CPLEX®.

### **6** COMPUTATIONAL RESULTS

In order to validate and assess our solution approach, a set of 15 benchmark problem instances were generated. These instances are based on the supply chain network structure of the Usemore case presented originally in Ballou (1992) and extended in Amrani *et al.* (2010). The case represents a typical B2B company manufacturing and selling products through the United States. Product demands and prices, transportation costs as well as the fixed and variable costs of each platform, vendor offer and transportation options are randomly generated but are based on realistic parameter value ranges found in Ballou (1992).

The potential supply chain network comprises 6 to 12 potential production-distribution facilities, 40 to 48 potential distribution centers, 192 demand zones representing clusters of customers in the vicinity of major U.S. cities, and 50 to 300 vendor offers. For the production-distribution facilities, 8 alternative base platforms are considered, and up to 4 potential upgrades are available per base platform. For the distribution facilities, 5 alternative base platforms are considered, with a maximum of 2 upgrades per base platforms. The upgrades are mutually exclusive. Up to 5 product families are sold to the customers while 10 products are used primarily as components. Various transport capacity options are modeled; TL and LTL shipping is

considered, both in the form of a limited-size private fleet, long-term truck leasing as well as the use of a common carrier. Five marketing policies are defined for each product.

Of the 15 benchmark instances, 5 are modeled with linear inventory-throughput relationships using equation (23); they are labeled as PL-01 to PL-05. The remaining instances (PC-06 to PC-15) have concave inventory-throughput functions (using equation (21)). For those instances, when the model is solved with the CPLEX solver, the concave functions are approximated by 3-segments piecewise linear functions using the procedure described in Amrani *et al.* (2010). For each of the benchmark instances, the performance of our heuristic is compared to the best solution found by IBM's ILOG CPLEX 12.1 solver. All default CPLEX parameters were used. Experiments were performed on a dual 2.0 GHz 64-bit Intel Xeon® QuadCore computer with 16 GB of RAM. Both CPLEX and CAT were allowed to use the eight processor cores as needed.

Since the benchmarks presented here are very challenging problems, neither our heuristic nor CPLEX 12.1 reaches a provable global optimum in a reasonable amount of time. We thus present two sets of results obtained respectively with 1-hour and 8-hour computational time limits. Interestingly, CPLEX 12.1's performance varies considerably from run to run while executing in the parallel mode. When enforcing a fixed time limit, variations on the solution value obtained by CPLEX are thus observed. Each solution method was run 10 times and both the average of all runs and the value of the best run are listed.

*Table 4* presents the computational results for our 15 benchmarks with a time limit of one hour. The instances are sorted in increasing order of computational complexity. For each instance, the distance between the best solution found for a run (*BSol*) and the best solution found over all 8-hour CPLEX and CAT runs (*BSol\**) is computed using  $100 \times |BSol*-BSol|/|BSol*|$ . Avg(CAT) indicates the average distance obtained over 10 runs of our heuristic, while Avg(CPLEX) indicates the average distance obtained over 10 runs of CPLEX. CV(CAT) and CV(CPLEX) indicate the coefficient of variation over the 10 runs, and Best(CAT) and Best(CPLEX) indicate the distance obtained in the best of 10 runs, for CAT and CPLEX respectively. GAP(CAT) and GAP(CPLEX) indicate the average gap between the best solution found in each run (*BSol*) and the best (lowest) upper bound found by all the CPLEX runs (*BUB*), using  $100 \times |BUB - BSol|/|BUB|$ . This gap provides an estimation of the maximum distance between the solution found and the optimal solution. However, for the benchmarks with concave holding cost functions, it must be interpreted with care because the *BUB* values are obtained from CPLEX when solving the problems with a polygonal approximation.

Instance	Avg(CAT)	CV(CAT)	Best(CAT)	GAP(CAT)	Avg(CPLEX)	CV(CPLEX)	Best(CPLEX)	GAP(CPLEX)
PL-01	0,70	54,17%	0,47	0,80	0,05	152,21%	0,00	0,15
PL-02	2,19	55,34%	1,82	2,66	1,32	25,63%	0,96	1,79
PL-03	1,48	60,84%	1,18	2,13	1,63	30,88%	1,46	2,28
PL-04	0,79	44,85%	0,64	1,34	2,11	29,00%	1,97	2,65
PL-05	2,03	54,17%	1,29	2,90	0,93	38,25%	0,47	1,81
PC-06	1,26	79,10%	0,98	1,69	5,66	39,13%	3,62	6,08
PC-07	1,12	58,72%	0,68	1,71	6,23	38,28%	4,46	6,79
PC-08	1,80	53,82%	1,25	2,83	2,33	31,13%	1,33	3,36
PC-09	0,57	51,49%	0,13	2,04	2,04	23,99%	1,10	3,49
PC-10	2,91	53,71%	2,35	4,64	4,07	22,88%	2,35	5,78
PC-11	1,55	53,47%	0,91	3,35	2,95	26,75%	1,50	4,73
PC-12	1,55	54,17%	0,87	3,36	3,17	24,00%	2,23	4,95
PC-13	1,22	56,39%	0,79	3,20	2,10	12,63%	1,52	4,06
PC-14	2,91	29,24%	2,49	5,54	3,39	26,50%	1,42	6,01
PC-15	3,21	37,98%	1,99	5,95	6,78	19,88%	5,18	9,42
Average	1,69	53,16%	1,19	2,94	2,98	36,07%	1,97	4,22

Table 4: Performance Obtained for a 1-Hour Time Limit for CAT and CPLEX

When using a 1-hour time limit, we see that for 11 out of 15 instances, CAT yields both the best average value and the best solution found. CPLEX yields the best average solution value for 3 instances, and it found the best solution for 4 instances. Furthermore, the average gap across all the instances favors CAT over CPLEX by a margin of 1.30%. However, CAT's performance is more variable over a 1-hour time limit than CPLEX, since CAT's coefficient of variation over 10 runs yields an average of 53,16% compared to 36,07% for CPLEX. One may notice that the gaps shown here are fairly high compared to those reported in the literature for cost minimization problems. Since our model maximizes net profits (Revenues - Costs), the objective function value represents a small fraction of the company's actual revenues and costs. For example, reducing costs by 1% while maintaining revenues could yield an increase in objective function profits of up to 20%.

*Table 5* presents the results on the same set of instances with a time limit of 8 hours. This time limit seems long enough to allow for the CAT algorithm to converge. Furthermore, after 8 hours of CPU time, CPLEX uses all the physical memory available on the computer without reaching any provable optimum. With 7 more hours of computation, the average distance over all instances drops by 1,52% for CAT and 1.22% for CPLEX. The average distance and gap provided by CAT is smaller than its CPLEX counterpart for 13 out of 15 instances, while CPLEX yields a smaller distance and gap in 1 out of 15. Furthermore, the best known solution is provided by CAT for 13 of the instances. For 11 instances, CAT yields an average gap that is at least 1% smaller than

CPLEX's. It is interesting to note that CAT's best solutions are, on average, at most 1,45% worse than the optimal solution, while this gap is at most 3,02% for CPLEX. CAT's coefficient of variation over all instances is 35,53%, while CPLEX's is still smaller at 25,53%. We believe that these results show the method's relevance and effectiveness for the problem studied, mainly when concave inventory holding cost functions are used.

Instance	Avg(CAT)	CV(CAT)	Best(CAT)	GAP(CAT)	Avg(CPLEX)	CV(CPLEX)	Best(CPLEX)	GAP(CPLEX)
PL-01	0,09	35,88%	0,07	0,19	0,05	15,03%	0,00	0,15
PL-02	0,08	54,02%	0,01	0,56	0,08	12,50%	0,00	0,56
PL-03	0,07	76,80%	0,00	0,73	1,39	27,93%	1,13	2,04
PL-04	0,19	36,72%	0,00	0,74	1,71	20,62%	1,47	2,25
PL-05	0,24	36,96%	0,00	1,13	0,67	59,05%	0,16	1,55
PC-06	0,05	22,68%	0,00	0,49	3,71	36,58%	3,40	4,13
PC-07	0,07	17,58%	0,00	0,67	4,22	35,11%	3,91	4,79
PC-08	0,06	13,56%	0,00	1,11	1,27	22,21%	1,04	2,31
PC-09	0,18	33,12%	0,00	1,66	0,94	18,22%	0,60	2,41
PC-10	0,21	39,60%	0,00	1,99	2,18	19,95%	1,98	3,92
PC-11	0,16	27,72%	0,00	1,99	1,43	14,76%	1,18	3,23
PC-12	0,19	33,60%	0,00	2,03	2,67	23,41%	2,23	4,46
PC-13	0,30	44,64%	0,00	2,29	1,81	31,12%	1,40	3,77
PC-14	0,27	23,16%	0,00	2,97	1,95	30,99%	1,42	4,61
PC-15	0,33	36,96%	0,00	3,15	2,43	15,43%	2,24	5,19
Average	0,17	35,53%	0,01	1,45	1,77	25,53%	1,48	3,02

Table 5: Performance Obtained for an 8-Hour Time Limit for CAT and CPLEX

## 7 CONCLUSIONS

This paper proposed a novel modeling approach for activity-based multi-period supply chain network design problems. It effectively integrates design and modeling concepts found in previous papers into a generic model that can be efficiently used to reengineer real-world supply chain networks. An agent-based metaheuristic (CAT), grounded in the A-Teams paradigm, was also proposed to solve this model effectively. Comparisons with CPLEX indicate that our algorithm performs better than CPLEX on the vast majority of the instances solved. Furthermore, the CAT metaheuristic can easily be extended and improved by adding new agents as needed.

There are two main avenues to extend this work. From a CAT implementation perspective, much could be done to increase the efficiency of agents and reduce the time spent on nonproductive tasks such as writing and reading solutions. From the SCN modeling point of view, the model presented could be extended to incorporate financial constraints, international factors and reverse logistics structures. Finally, in order to account for the uncertainty inherent in these

multi-period problems, a scenario-based stochastic programming version of the model could and should be studied.

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## **Appendix A: Concept Lists Notation Summary**

A = [V, S, D] = [V, C, F, W, D]: Activity types list with

V: Supply (vendors)

S=[C,F,W]: Internal site activity types list.

C: Consolidation and transshipment

F: Fabrication (transformation)

W: Warehousing (storage)

D: Demand

**M**=[T,H,B]: Movement types list with

T: Transportation movement

- H: Handling movement
- B: Both (inter or intra location movement)

L=[V,S,D]: Location types list with

- V: Vendors
- S: Site locations
- D: Demand zones

T = [A,O,G,I] = [A,O,R,D,I]: Possible transportation *modes* list with

A: Air

O: Ocean

G = [R,D]: Ground transportation

R: Railway

D: Driveway (trucking)

I: Intermodal

**K**=[I,O,V]: Market types list with

- I: Inventory-based replenishment
- O: Made-to-order
- V: Vendor managed inventory (VMI)