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Vehicle Routing Models and Algorithms for Winter Road Spreading Operations

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Abstract. Winter road maintenance operations involve challenging vehicle routing problems that can be addressed using operations research (OR) techniques. Three key problems involve routing trucks and specialized vehicles for spreading chemicals and abrasives on roadways, snow plowing, and snow disposal, all of which are undertaken in a very difficult and dynamic operating environment with stringent level of service constraints. This chapter provides a survey of recent optimization models and solution methodologies for the routing of vehicles for spreading operations. We also present a detailed classification scheme for spreader routing models developed over the past 40 years. Key trends in recent model developments include the inclusion of more details of the practical operating constraints, the use of more sophisticated hybrid solution strategies and consideration of more comprehensive models that integrate vehicle routing with models for other related strategic winter maintenance problems. We highlight some factors that may be limiting the application of OR models in practice and discuss promising future research trends.

Keywords. Winter road maintenance, snow removal, arc routing problem.

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1. INTRODUCTION

There are many challenging and expensive winter road maintenance decision problems that can be addressed using operations research techniques. A key operation is spreading of chemicals and abrasives on the road network, which is conducted on a regular basis in almost all rural and urban regions that experience significant snowfall or roadway icing. The importance of winter road maintenance operations is obvious from the magnitude of the expenditures required to conduct winter road maintenance operations, as well as the indirect costs from both the lost productivity due to decreased mobility and from the effects of chemicals (especially salt) and abrasives on infrastructure, vehicles and the environment. In the US alone, 70% of the population and 74% of the roads are in snowy regions and state and local government agencies spend over US $2.3 billion (US) per year for snow and ice control activities (Federal Highway Administration [FHWA], 2010; Pisano, Goodwin, & Stern, 2002). Indirect costs (e.g., for environmental degradation, economic losses and mobility reductions) are thought to be several times larger; for example, the costs for weather-related freight delays in the US have been estimated at US $3.4 billion (US) per year (Nixon, 2009).

Recent developments in winter road maintenance technologies and operations improve efficiency, reduce resource (materials, equipment and personnel) usage, and minimize environmental impacts (Shi et al., 2006; Transportation Research Board, 2005, 2008; Venner Consulting and Parsons Brinkerhoff, 2004). These developments include use of alternative deicing materials, anti-icing methods, improved snow removal equipment, more accurate spreaders, better weather forecasting models and services, road weather information systems, vehicle-based environmental and pavement sensors, etc. These new technologies, and their growing use by state and local government agencies, have improved the effectiveness and efficiency of winter maintenance operations, benefiting government agencies, users, and the general public.

While new winter road maintenance technologies are being developed and deployed on a broad basis, implementations of optimization models for the winter road maintenance vehicle routing remain very limited. Most agencies continue to design vehicle routes based on manual approaches derived from field experiences and most agencies rely on static weather forecasts (Fu, Trudel, & Kim, 2009; Perrier, Langevin, & Campbell, 2007a, 2007b). As Handa, Chapman and Xin (2006) note, “In practice [route] optimization has traditionally been a manual task and is heavily reliant on local knowledge and experience.” The limited deployment of optimization models for winter road maintenance vehicle routing is especially surprising given the documented successes in other areas of arc routing, perhaps most notably for waste management (Sahoo, Kim, Kim, Kraas, & Popov, 2005). Thus, winter road maintenance vehicle routing optimization would appear to offer the promise of significant cost savings, along with a reduction in negative environmental and societal impacts.

There are probably many reasons for the limited field use of vehicle routing optimization. In large part, this has been due to the complexity of the problems studied, which in turn is derived from the difficult operating environment. However, it also results from the unique organizational characteristics of the winter road maintenance agencies. Winter road maintenance decisions problems, including vehicle routing for spreading (and plowing), problems are more complex than most other arc routing problems because of unique characteristics of each site and agency, and the tremendous diversity in operating conditions such as geographical location, climatic and weather conditions, demographics, economics, technological innovations (for materials application, mechanical removal, and weather monitoring), legislative requirements, interagency agreements, variations of traffic rate, and information on the status of personnel, equipment and materials. Also, real-life vehicle routing problems facing winter road maintenance planners should be studied in a dynamic context, where the arrival of new information, such as meteorological forecasts received in real time, can lead to dynamic modifications to the current vehicle routes. Furthermore, political and operational constraints and policies depend on the specific level of service policies and expectations, the characteristics of the transportation network, the strategic and tactical decisions related to design of the operational sectors, choice of chemicals and abrasives, depot and material stockpile locations, vehicle fleet compositions, and driver rules. Differences in these conditions and constraints necessitate differences in the planning and operation of winter road maintenance across agencies.

One important theme in recent winter road maintenance modeling efforts is the inclusion of real-world characteristics of the problems arising in applications. These models offer greater potential for implementation as they better capture more of the complexities from the field. These improved models, together with the increasing budget pressures on state and local agencies, continuing expectations for high levels of service, and desires for reduced environmental impacts, all motivate a greater role for vehicle routing optimization in winter road maintenance.

The aim of this chapter is to provide a review of recent contributions dealing with the routing of vehicles for winter road spreading operations. We will cover models and solution algorithms developed over the last decade or so. Earlier models will not be treated here but the interested reader is referred to recent work by Perrier, Langevin, and Campbell (2007a). In this chapter, we describe the important characteristics, model structure and algorithmic aspects for vehicle routing models in spreading operations. In addition to extending the earlier review, the contributions of this chapter include a detailed classification scheme for spreader routing models developed over the past 40 years, discussion of application issues, and identification of key opportunities and needs in future research.

The chapter is organized as follows. The operations of spreading chemicals and abrasives and the vehicle routing problems related to those operations are presented in Section 2. Recent models dealing with the routing of vehicles for spreading operations are reviewed in Section 3. An analysis of existing research on vehicle routing problems for spreading operations is presented in Section 4. Conclusions along with some promising future research opportunities are presented in the last section.

2. OPERATIONS CONTEXT AND DECISION PROBLEMS

This section contains a brief description of spreading operations for winter maintenance and a discussion of associated problems of vehicle routing. More detailed information on the state of the practice in managing winter road maintenance operations is presented in the Transportation Research Board reports (2005, 2008).

2.1 Spreading Operations

Spreading operations for winter maintenance are directed at achieving three specific goals: anti-icing, deicing, and traction enhancement. Anti-icing is the timely application of a chemical freezing-point depressant before or during the initial stages of a precipitation event, to attempt to prevent the bonding of snow and ice to the pavement. Deicing is a similar process used to remove snow and ice from the pavement, often requiring destruction of bond between pavement and snow/ice to eliminate the frozen layer. Traction enhancement is the spreading of abrasive materials, such as sand, cinders, ash, tailings, or crushed stone and rock, to improve traction on thick snow-packed and ice-covered roadways. The selection of the appropriate spreading operation is based on economics, environmental constraints, climate, desired level of service, material availability, and application equipment availability. The level of service policies determine the extent of the resource investment. The environmental constraints e.g., current and forecast weather conditions) influence the choice of a chemical or nonchemical material to spread.

2.2 Vehicle Routing Problems for Spreading

The routing of vehicles for spreading operations is the problem of designing a set of routes such that all required road segments of a transportation network are serviced by a fleet of spreaders, which may be heterogeneous vehicles (e.g., trucks of different capacities) based at multiple depots. The transportation network is generally described through a graph, whose arcs and edges represent the one-way streets and two-way streets to be serviced, respectively, and whose nodes correspond to the road junctions and to vehicle and materials depot locations. Not every road segment may need to be serviced, and road segments with positive demands (amounts of chemicals and abrasives) are called *required* road segments.

This section presents a comprehensive classification scheme for vehicle routing models in winter road spreading operations (see Table 1). The first level of the classification is composed of six categories: (1) Problem type, (2) Planning level, (3) Problem characteristics, (4) Model structure, (5) Solution method, and (6) Instance data. The first category, Problem type, identifies whether the model is limited to vehicle routing only, or includes vehicle routing along with another problem such as facility location or sector design. The second category, Planning level, taken from the work of Perrier et al. (2007a), classifies vehicle routing models for spreading operations according to the planning horizon considered. Decisions concerning the location of vehicle or materials depots may be viewed as strategic or tactical, while decisions relating to the routing of vehicles for spreading operations usually belong to the operational planning level. In “static” models, all inputs required to solve the problem are known in advance for the duration of the period covered by the routing process, such as a winter, (although the input may vary over time, as in the time-dependent variant of the spreader routing problem). In “dynamic” models, the input (that is, which road segments actually require service) varies over time in a fashion that is revealed to the router vey shortly before the routes are constructed. This may occur when new routes are created for each storm or precipitation event based on each unique forecast and weather conditions. We distinguish “real-time” routing from dynamic routing, by using real-time routing to refer to cases where the routes are (re)computed during the vehicles traversal of the route, because of new inputs received in real-time.

The third category, Problem characteristics, includes numerous factors that are part of the problem environment or constraints embedded into the solution. This category is an extended version of the work of Perrier et al. (2007a). Typical characteristics of this category include:

- road network characteristics;
- service hierarchy constraints, including linear precedence relations between classes of road segments in a route that require higher level roadways (e.g., based on level of traffic) to be served prior to lower level roadways, and class upgrading, which allow servicing of lower-class roads in a route servicing higher-class roads (in order to reduce the service completion time of this class and/or the total completion time);
- service costs associated with each road segment, possibly dependent on the time of beginning of service;
- limits on the maximum time or distance of routes and of service completion;
- time windows for servicing road segments, possibly by road class or road segment;
- minimum road service frequencies, possibly by road class or road segment;
- basic units of analysis used to design sectors (for example, small geographic zones);

- operational constraints regarding the number of lanes covered in a single pass and the number of passes per road segment;

- road segment-specific vehicles, which require that a road segment be serviced by a specific type of vehicle (for instance, because of possible access limitations);

- route constraints to ensure load balancing (approximately equal workloads, lengths or durations across routes), class continuity (each route services road segments with the same priority class), turn restrictions, etc.;

- vehicle and materials depot characteristics;

- service connectivity or route continuity, which requires that the subgraph induced by the set of road segments serviced by a spreader be connected; and

- the objective (e.g., minimize route costs, fleet size, constraint violations, etc.

Table 1: Characteristics of vehicle routing problems for spreading

1. Problem type
   - 1.1. Spreader routing only
   - 1.2. Combined Spreader routing + other problem(s)
     - 1.2.1. Combined routing and sector design
     - 1.2.2. Combined routing and depot location
2. Planning level
   - 2.1. Strategic
   - 2.2. Tactical
   - 2.3. Operational
     - 2.3.1. Static routing
     - 2.3.2. Dynamic routing
     - 2.3.3. Real-time routing
3. Problem characteristics
   - 3.1. Transportation network
     - 3.1.1. Undirected network
     - 3.1.2. Directed network
     - 3.1.3. Mixed network
     - 3.1.4. Rural network
   - 3.2. Service hierarchy
     - 3.2.1. Linear precedence relation imposed
     - 3.2.2. Class upgrading allowed
   - 3.3. Time (or distance) limit for service completion
     - 3.3.1. Restriction on routes
     - 3.3.2. Restriction on road classes
     - 3.3.3. Restriction on road segments
   - 3.4. Road segment service costs
     - 3.4.1. Independent of service start time
     - 3.4.2. Dependent on service start time
   - 3.5. Service time window type
     - 3.5.1. Restriction on road classes
     - 3.5.2. Restriction road segments
   - 3.6. Service frequency type
     - 3.6.1. Restriction on road classes
     - 3.6.2. Restriction on road segments
   - 3.7. Number of passes per road segment
   - 3.12. Number of routes per spreader
     - 3.12.1. One route
     - 3.12.2. Multiple routes
   - 3.13. Route configuration
     - 3.13.1. Load balancing imposed
     - 3.13.2. Class continuity imposed
     - 3.13.3. Both-sides service imposed
     - 3.13.4. Turn restrictions imposed
     - 3.13.5. Service connectivity or route continuity imposed
     - 3.13.6. Sector boundaries imposed
   - 3.14. Objectives
     - 3.14.1. Min variable or routing costs
     - 3.14.2. Min sum of fixed and variable costs
     - 3.14.3. Min time-dependent service costs
     - 3.14.4. Min fleet size
     - 3.14.5. Min alternations between deadheading and servicing
     - 3.14.6. Min operational constraints violations
4. Model structure
   - 4.1. Integer programming models
     - 4.1.1. Linear 0-1 IP model
     - 4.1.2. Linear MIP model
     - 4.1.3. Nonlinear MIP model
   - 4.2. Arc routing problems
     - 4.2.1. Directed Chinese postman problem
     - 4.2.2. Capacitated arc routing problem
     - 4.2.3. Location-arc routing problem
   - 4.3. Capacitated vehicle routing problem
   - 4.4. Spanning tree problems
     - 4.4.1. Capacitated minimum spanning tree problem
5. Solution method
   - 5.1. Exact methods
     - 5.1.1. Column generation
   - 5.2. Constructive methods
     - 5.2.1. Sequential constructive methods
     - 5.2.2. Parallel constructive methods

3.7.1. One pass
3.7.2. Multiple passes
3.8. Number of lanes in a single pass
3.8.1. One lane
3.8.2. Two lanes
3.9. Sectors
3.9.1. Compactness or shape
3.9.2. Balance in sector size or workload imposed
3.9.3. Basic units defined
3.10. Vehicle and materials depots
3.10.1. Single depot
3.10.2. Multiple depots
3.10.3. Centrally located depots relative to sectors
3.11. Vehicle (capacities)
3.11.1. Similar vehicles
3.11.2. Road segment-specific vehicles
3.11.3. Heterogeneous vehicles
5.2.3. Cluster first, route second methods
5.2.4. Optimization-based methods
5.3. Composite methods
5.4. Adaptation of metaheuristics
5.4.1. Simulated annealing
5.4.2. Tabu search
5.4.3. Elite route pool
5.4.4. Genetic algorithms
5.4.5. Memetic algorithms
5.4.6. Variable neighbourhood search
5.5. Simulation used
5.6. Solution method implemented
6. Instances data
6.1. Real world instances
6.2. Randomly generated instances
6.3. No instance used

The fourth category describes the basic mathematical model structure and the fifth categories provide the solution method. The last category, Instances data, classifies the type of instances solved. In Section 4, the categories presented in Table 1 are applied to classify the spreader routing models developed during the last four decades.

3. VEHICLE ROUTING MODELS FOR SPREADING

Vehicle routing problems related to spreading operations are generally formulated as arc routing problems. Corberan and Prins (2010) presented an annotated bibliography on recent results on arc routing problems. In this section, our purpose is to survey the more recent solution approaches for the routing of vehicles for spreading operations. We first discuss the exact algorithm proposed by Tagmouti, Gendreau, and Potvin (2007), followed by metaheuristics applied to the routing of vehicles for spreading operations during the last decade. Earlier models for the routing of vehicles for spreading operations will not be treated in this section; we instead refer the interested reader to recent survey by Perrier et al. (2007a).

3.1 Exact algorithms

In the classical version of the problem, the cost associated with servicing each road segment is fixed. However, in the *time-dependent* variant of the vehicle routing problem for spreading operations, the timing of each service pass is of prime importance. That is, the cost to service a road segment depends on the time of beginning service. Recently, Tagmouti et al. (2007) proposed a nonlinear, mixed integer program and a column generation algorithm for a salt spreader routing problem with capacity constraints and time-dependent service costs. In this problem, the service cost on each required road segment is a piecewise linear function of the time of beginning of service. We clarify here that all of the problem inputs are known in advance. Hence, the problem studied is a static problem, even though the term “time-dependent” might be interpreted as synonymous to “dynamic”. To present the formulation, let $G = (V, A)$ be a directed graph where V is the vertex set and A is the arc set. First, the arc routing problem in graph $G = (V, A)$ is transformed into an equivalent node routing problem in a transformed graph $G' = (V', A')$. The depot is duplicated into an origin depot o and a destination depot d in V'. Let also N' be the set of nodes that must be serviced ($N' = V' \setminus \{o, d\}$). Each required arc in graph G corresponds to a node i in graph G' with demand d_i, service time st_i and time-dependent service cost $sc_i(T_i)$, where T_i is time of beginning of service on node i. Each pair of distinct nodes i and j in G' is connected by an arc $(i, j) \in A'$ with travel time tt_{ij} and travel cost tc_{ij}. Let K be the set of identical spreader trucks with capacity Q. For each arc $(i, j) \in A'$ and for each spreader truck $k \in K$, let x_{ij}^k be a binary variable equal to 1 if and only if spreader k

travels on arc (i, j) to service node j. For every node $i \in V'$ and for every spreader truck $k \in K$, let Q_i^k be a nonnegative real variable representing the load of spreader k just after servicing node i and let also T_i^k be a nonnegative real variable specifying the time of beginning of service of spreader k at node i. The formulation is given next.

Minimize

$$\sum_{k\in K}\left(\sum_{(i,j)\in A'} tc_{ij}x_{ij}^k + \sum_{i\in N'} sc_i\left(T_i^k\right)\sum_{j\in N'\cup\{o\}} x_{ji}^k\right) \quad (1)$$

subject to

$$\sum_{k\in K}\sum_{i\in N'\cup\{o\}} x_{ij}^k = 1 \qquad (j \in N') \quad (2)$$

$$\sum_{k\in K}\sum_{j\in N'} x_{oj}^k \le m \quad (3)$$

$$\sum_{j\in N'\cup\{d\}} x_{oj}^k = 1 \qquad (k \in K) \quad (4)$$

$$\sum_{j\in N'\cup\{d\}} x_{ij}^k - \sum_{j\in N'\cup\{o\}} x_{ji}^k = 0 \qquad (k \in K, j \in N') \quad (5)$$

$$\sum_{i\in N'\cup\{o\}} x_{id}^k = 1 \qquad (k \in K) \quad (6)$$

$$x_{ij}^k\left(T_i^k + st_i + tt_{ij} - T_j^k\right) \le 0 \qquad (k \in K, (i, j) \in A') \quad (7)$$

$$x_{ij}^k\left(Q_i^k - d_j - Q_j^k\right) \le 0 \qquad (k \in K, (i, j) \in A') \quad (8)$$

$$0 \le T_i^k \le T \qquad (k \in K, i \in V') \quad (9)$$

$$0 \le Q_i^k \le Q \qquad (k \in K, i \in V') \quad (10)$$

$$0 \le x_{ij}^k \le 1 \qquad (k \in K, (i, j) \in A') \quad (11)$$

$$x_{ij}^k \in \{0,1\} \qquad (k \in K, (i, j) \in A') \quad (12)$$

The objective function (1) minimizes the sum of travel costs and time-dependent service costs. Constraint set (2) requires that each node (except the depot node) be serviced exactly once. Constraint set (3) imposes an upper bound m on the number of spreader trucks. Flow conservation is guaranteed by constraint sets (4)-(6). Constraint sets (7) and (8) ensure the feasibility of the time schedule and loads, respectively Constraint set (9) ensures that the time that service begins at every node is a nonnegative value that does not exceed the deadline T. Similarly, constraint set (10) requires nonnegative load values that do not exceed the spreader salting capacity Q. Tagmouti et al. (2007) proposed to decompose the model into a master problem and a set of $|K|$ different independent subproblems. The master problem corresponds to constraints (2) and (3) in the original formulation (1)-(12). Let Ω be the set of all feasible paths from the origin depot o to the destination depot d. For each path $p \in \Omega$, let u_p be a binary variable equal to 1 if and only if path p is selected and define C_p as the total cost of path p (sum of travel costs and service costs on all arcs and nodes along the path). The model for the master problem can be stated as follows:

Minimize

$$\sum_{p\in\Omega} C_p u_p \tag{13}$$

subject to

$$\sum_{p\in\Omega} a_{ip} u_p = 1 \qquad (i \in N') \tag{14}$$

$$\sum_{p\in\Omega} u_p \le m \tag{15}$$

$$u_p \ge 0 \qquad (p \in \Omega) \tag{16}$$

where the binary constant a_{ip} is equal to 1 if and only if node i is in path p. Moreover, for every spreader truck $k \in K$, the subproblem is of the following form:

Minimize

$$\sum_{(i,j)\in A'} \overline{tc}_{ij} x_{ij}^k + \sum_{i\in N'} sc_i\left(T_i^k\right) \sum_{j\in N'\cup\{o\}} x_{ji}^k \tag{17}$$

subject to

$$(4) - (12)$$

where $\overline{tc}_{ij}$ is the reduced travel cost on arc $(i, j) \in A'$. The master problem, solved with CPLEX, is a linear relaxation of a set covering problem with an additional constraint on the total number of spreader trucks. Columns (paths) of the master problem are generated by solving, for each spreader truck $k \in K$, the corresponding subproblem with an objective that is iteratively updated to reflect the new values of the dual variables. The subproblem for each spreader truck is an elementary shortest path problem with resource constraints that is solved using an extension of the algorithm of Feillet, Dejax, Gendreau, and Gueguen (2004) to take into account the time-dependent service costs. The resource constraints are the capacity constraint and the time deadline for the return of the spreader to the depot. To obtain an integer solution, the column generation approach is embedded in a previously reported branch-and-bound algorithm (Feillet, Dejax, Gendreau, & Gueguen, 2004). Computational results were presented on problems derived from a set of instances of the vehicle routing problem with time windows (Solomon, 1987). The largest instances solved contained 40 customers.

3.2 Metaheuristics

In a previous survey, Perrier et al. (2007a) described a linear, mixed integer programming model developed by Qiao (2002) for routing salt spreader trucks. The model, which is an extension of a previous formulation proposed by Haghani and Qiao (2002), incorporates service connectivity and vehicle capacity. The model will not be presented here but we instead refer the reader to the work by Perrier et al. (2007a). The model is solved with a classical tabu search algorithm and an elite route pool procedure. The *elite route pool* procedure is similar to the technique of genetic algorithms. The population is formed by a pool of good routes found in the best solutions, called the elite route pool. Associated with every route in the elite route pool is a weight corresponding to the frequency with which the route appears in the best solutions. New offspring routes are produced by selecting the routes with the highest weights in the elite route pool while avoiding duplications of serviced required arcs. Mutations are then obtained by applying the multiroute improvement methods developed by Haghani and Qiao (2002). Qiao (2002) provided an interesting comparison of the various multiroute improvement methods, the tabu search algorithm, the elite route pool procedure and four popular constructive methods for the capacitated arc routing problem from Pearn (1984), Golden, DeArmon, and Baker (1983) and Christofides (1973). Computational tests on 23 networks derived from the test problems used by Pearn (1984) showed that the elite route pool procedure obtained the largest number of best solutions on sparse networks with $7 \leq |V| \leq 27$ and arc densities between 13% and 40%. On dense networks, the algorithm in Pearn (1984) produced the best solutions in most cases.

Toobaie and Haghani (2004) studied the problem of designing spreader routes in a multi-depot network so as to minimize the number of vehicles and the deadhead distance, while satisfying vehicle capacities (all the same), route continuity and workload balance. Also, some two-lane highways require servicing in both directions (one lane in a single pass), whereas others can be serviced in a single pass. The problem is solved using a three-stage procedure. The first stage decomposes the road network into subnetworks, one for each vehicle, by solving a minimal arc partitioning problem with vehicle capacities and service connectivity constraints. The objective of the first stage is to minimize the number of subnetworks (vehicles). Given a connected network in which costs are associated with links, the minimal arc partitioning problem consists of partitioning the network into a minimum number of connected subnetworks so that the overall cost of each subnetwork does not exceed the budget limit for the subnetwork. In the salt spreader routing problem, the link cost corresponds to the link salt requirement and budget corresponds to the spreader's salt capacity. The minimal arc partitioning problem is similar to the arc partitioning problem studied by Bodin and Levy (1991) in the context of postal delivery. To solve the minimal arc partitioning problem, Toobaie and Haghani (2004) developed a genetic algorithm in which each solution, or collection of subnetworks, is represented as a string of n real numbers (a chromosome with n genes), where n is the number of links in the network. The genetic algorithm can be described as follows.

1. *Initialization*. Generate the initial population by assigning a random real number to each gene from a uniform distribution between 0 and 1.
2. Generate initial routes using the first-fit heuristic.
3. *Evaluation*. Evaluate the population and update the best solution on the basis of the maximization of the fitness function $F = k \times e^{\alpha \times N + \beta}$, where α and β are coefficients, α is negative, N is the number of subnetworks, and k is a positive number.
4. *Selection*. Apply the roulette wheel selection method to generate a new population.
5. *Elitism*. Randomly replace a chromosome with the best solution.
6. *Crossover*. Apply two-point crossover on the basis of crossover probability.
7. *Mutation*. Select and replace genes with random numbers between 0 and 1 on the basis of the mutation probability.
8. Repeat Steps 2 to 7 until the convergence criteria are met (elitism guarantees the convergence of the algorithm).

In Step 1, if each chromosome consists of n genes and each population consists of P chromosomes, then $n \times P$ random real numbers are generated. In Step 2, the first-fit heuristic is a greedy procedure that starts by sorting the links in a given order, and then constructs subnetworks one at a time by repeatedly adding the next unassigned link that preserves the route continuity and vehicle capacity constraints to the current subnetwork. In Step 4, the roulette wheel mechanism is adopted for the selection procedure. In this method, the cumulative fitness ratio f_j for chromosome j is computed as

$$f_j = \frac{\sum_{i=1}^{j} F_i}{\sum_{i=1}^{P} F_i}$$

where F_i is the fitness value of chromosome *i*. To select *P* chromosomes for the new generation, *P* random numbers between 0 and 1 are chosen from a uniform distribution. For each random number *r*, chromosome *j* is selected such that $f_{j-1} < r < f_j$. After the selection process, elitism is applied in Step 5 to migrate the best individual to the new generation. Elitism consists of randomly selecting and replacing one chromosome with the best chromosome. In Steps 6 and 7, the two classical genetic algorithm operators, crossover and mutation are adapted for the reproduction phase. The two-point crossover is adapted for the crossover step, while the mutation operation is achieved by randomly selecting a gene and changing its value to another random real number. Once the network is partitioned into connected subnetworks, the second stage of the procedure tries to balance the subnetwork workloads (salt demands) by swapping links between neighbouring subnetworks so as to reduce the imbalance between the two subnetworks, while satisfying the route continuity and vehicle capacity constraints. The following conditions must be satisfied to move a link l_i from subnetwork S_1 to subnetwork S_2:

1. Link l_i has common nodes with at least a link in subnetwork S_2.
2. Link l_i can be removed from subnetwork S_1 without violating the route continuity constraint.
3. The salt demand criteria, $D(S_1) - D(l_i) < D(S_2)$, is satisfied, where $D(S_1)$, $D(l_i)$ and $D(S_2)$ are the salt demands for subnetwork S_1, link l_i and subnetwork S_2, respectively.

Finally, in the last stage, spreader routes are obtained by solving a Chinese postman problem for each subnetwork using Edmonds and Johnson's algorithm (Edmonds & Johnson, 1973). The three-stage procedure was tested on data from Calvert County, Maryland. The instance contained 42 nodes, including 2 depots, and 52 edges grouped into four subnetworks. The procedure reduced the number of vehicles, the distance covered by deadheading trips and the workload imbalance by 14%, 27% and 67%, respectively, over the solution in use by the County with short computing times (in the order of seconds).

Spreader routing problems are often studied in a static context, where all data input are assumed to be given in advance. However, in real-life applications, some information might not be readily available when the vehicles start their routes. In an attempt to address the dynamic nature of the problem in which road surface temperature data and condition across the road network are revealed over a 24 hour period, Handa, Chapman, and Xin (2005) developed a prototype system that combines a memetic algorithm with Road Weather Information Systems (RWIS) to solve a dynamic salt spreader routing problem. The problem is modeled as a dynamic capacitated arc routing problem where the set of required road segments and their demands (amount of salt) are defined based on the predicted temperature provided by the RWIS. Typically, a road segment is defined as required if there is at least one RWIS point with a predicted temperature less than a predefined threshold. Thus, the amount of required salt on the same required road segment for two days can be different. Moreover, the amount of salt required can vary with road width (type), e.g. motorway, high-class road segments, medium-class road segments, etc. The memetic algorithm is based on a hybrid algorithm of evolutionary algorithms and local search methods. The main steps of the memetic algorithm include: selecting parents, reproducing offspring, applying local search to offspring, and replacing the resultant offspring if the offspring is better than the worst individual in the population. The permutation representation of a chromosome details the order of required edges in which a vehicle must spread salt. Symbols are also used in the chromosome to indicate the beginning of the route for each vehicle. The authors used the edge assembly crossover operator proposed by Nagata and Kobayashi (1997) and Nagata (2004). However, since this operator is designed for solving traveling salesman problems, it can yield infeasible solutions where the vehicle capacity is exceeded. In order to fix these infeasible solutions, a repair operator for offspring individuals is incorporated in the memetic algorithm. As with the memetic algorithms presented by Lacomme, Prins, and Ramdane-Cherif (2004), some initial individuals are generated using the path-scanning algorithm developed by Golden et al. (1983) for the capacitated arc routing problem. Also, three local search methods are used in the memetic algorithm: move one or two edges from one route to another and swap two edges among two routes. Finally, the following fitness function is used to evaluate a set of routes:

$$F = \sum_{i=0}^{m} \left[C_i + \left(p \times E_i \right) \right]$$

where C_i denotes the total distance traveled by vehicle i, E_i is the amount of salt by which vehicle i is above its capacity and p is the corresponding penalty parameter. Results on two instances (two nights) of the South Gloucestershire network, UK, with 385 and 97 required road segments, respectively, showed that the proposed system is effective at finding dynamic salting routes. In a follow-up paper, Handa, Chapman, and Xin (2006) discussed extensions to the case where a robust solution is required. This is an important practical consideration since it may confuse the highway agency and truck drivers if every different road temperature gave rise to a different set of salting routes. Therefore, a robust solution is desirable. The memetic algorithm is adapted to address this version of the problem by placing emphasis on “thermally ranking” salting routes so that the “warmer” routes could be left untreated on marginal nights. Comparisons on real data from the South Gloucestershire Council, UK, for various values of environmental parameters showed that the memetic algorithm reduced the total distance traveled by the vehicles by more than 10% over the routes in use by the Council.

Omer (2007) proposed a model for a salt spreader truck routing problem in which maximum route length and duration, fleet size, vehicle capacity, and service frequency constraints are considered, with an objective of minimizing the total distance traveled. The model is based on the formulation proposed by Golden and Wong (1981) for the undirected capacitated arc routing problem (later modified by Haghani & Qiao, 2001). Let $G = (V, A)$ be a directed graph where $V = \{v_1, \ldots, v_n\}$ is the vertex set and $A = \{(v_i, v_j) : v_i, v_j \in V \text{ and } i \neq j\}$ is the arc set. The depot is represented by the node v_1. With every arc $(v_i, v_j) \in A$ are associated a nonnegative length c_{ij} and a deadheading time t_{ij}. Define $R \subseteq A$ as the set of required arcs. With each arc $(v_i, v_j) \in R$ are associated a demand q_{ij}, expressed as the total amount of chemicals required for servicing the arc, and a time g_{ij} corresponding to the difference between the time for servicing arc (v_i, v_j) and the time for deadheading arc (v_i, v_j). Define also $A_1 \subseteq R$ as the set of counterpart arcs in opposite directions between intersection nodes that can be serviced only once from one direction. Associated with every arc $(v_i, v_j) \in R \setminus A_1$ is a positive number of times n_{ij} arc (v_i, v_j) should be spread. Let K be the set of vehicles. For every arc $(v_i, v_j) \in A$ and for every vehicle $k \in K$, let x_{ijk} be a binary variable equal to 1 if and only if arc (v_i, v_j) is either serviced or traversed while deadheading by vehicle k and let f_{ijk} be a nonnegative real variable representing the flow on arc (v_i, v_j) in the route associated with vehicle k. For every arc $(v_i, v_j) \in R$ and for every vehicle $k \in K$, let y_{ijk} be a binary variable equal to 1 if and only if arc (v_i, v_j) is serviced by vehicle k. Finally, let D, T and W be the maximum distance a vehicle can cover in a route, the maximum time a vehicle can take to cover a route, and the vehicle capacity, respectively. The formulation is then as follows.

Minimize

$$\sum_{k \in K} \sum_{(v_i, v_j) \in A} c_{ij} x_{ijk} \tag{18}$$

subject to

$$\sum_{v_j \in V, v_j \neq v_i} x_{jik} - \sum_{v_j \in V, v_j \neq v_i} x_{ijk} = 0 \qquad (v_i \in V, k \in K) \tag{19}$$

$$\sum_{k \in K} y_{ijk} + y_{jik} = 1 \qquad ((v_i, v_j) \in A_1) \tag{20}$$

$$\sum_{k \in K} y_{ijk} = n_{ij} \qquad ((v_i, v_j) \in R \setminus A_1) \tag{21}$$

$$\sum_{(v_i, v_j) \in A} c_{ij} x_{ijk} \leq D \qquad (k \in K) \tag{22}$$

$$\sum_{(v_i, v_j) \in A} t_{ij} x_{ijk} + \sum_{(v_i, v_j) \in R} g_{ij} y_{ijk} \leq T \qquad (k \in K) \tag{23}$$

$$\sum_{(v_i, v_j) \in R} q_{ij} y_{ijk} \leq W \qquad (k \in K) \tag{24}$$

$$x_{ijk} \geq y_{ijk} \qquad ((v_i, v_j) \in R, k \in K) \tag{25}$$

$$\sum_{v_r \in V, v_r \neq v_i} f_{irk} - \sum_{v_r \in V, v_r \neq v_i} f_{rik} = \sum_{v_j \in V} y_{ijk} \qquad (v_i \in V \setminus \{v_1\}, k \in K) \tag{26}$$

$$f_{ijk} \leq |V|^2 \cdot x_{ijk} \qquad ((v_i, v_j) \in A, k \in K) \tag{27}$$

$$f_{ijk} \geq 0 \qquad ((v_i, v_j) \in A, k \in K) \tag{28}$$

$$x_{ijk} \in \{0,1\} \qquad ((v_i, v_j) \in A, k \in K) \tag{29}$$

$$y_{ijk} \in \{0,1\} \qquad ((v_i, v_j) \in R, k \in K) \tag{30}$$

The objective function (18) minimizes total distance traveled. Constraints (19) ensure route continuity. Constraints (20) and (21) state that each arc is serviced as required. Maximum route length, maximum route duration and vehicle capacity are not violated on account of constraints (22), (23) and (24), respectively. Constraints (25) guarantee that an arc can be serviced by a vehicle only if the vehicle covers that arc. Constraints (26)-(28) prohibit the formation of illegal subtours. The flow variable f_{ijk} can take on positive values only if $x_{ijk} = 1$. For details, see Golden and Wong (1981). The problem is solved using a Greedy Randomized Adaptive Search Procedure (GRASP) heuristic. In each iteration of the GRASP, an initial solution is built using a constructive method, and local search is performed on the solution obtained using simulated annealing. The best overall solution obtained from several iterations of the GRASP is considered as the final solution. The constructive method builds a route starting at the depot node and incrementally inserts an arc until a feasible route is completed. At each iteration of the constructive method, a list of candidate arcs is created by considering all possible arcs that satisfy a greedy evaluation function and that can be added to the current partial solution without violating operational constraints. The greedy evaluation function calculates the incremental increase in total cost due to addition of the candidate arc to the partial solution and considers only the candidate arcs whose incremental cost lies below a threshold value. A candidate arc is randomly selected from the list and inserted into the current partial solution. The simulated annealing algorithm starts with the solution obtained from the constructive method and searches for better solutions by moving arcs between pairs of routes. Two types of moves can be performed: one-arc move and *m*-*n* exchange. The one-arc move involves moving a single arc from one route to another. The *m*-*n* exchange consists in moving *m* arcs from one route A to another route B and moving *n* arcs from route B to route A, without exceeding the vehicle capacity. Arcs that are removed or inserted in a route may separate the route into disconnected components. The author proposed an improvement algorithm to combine these disconnected components into a new feasible low cost route.

The algorithm first builds a route starting and ending at the depot and connecting all the disconnected components in the route. The route is then again divided into multiple disconnected components and the sequence of disconnected components in the route is modified by interchanging the disconnected components within the route. This results in multiple sequences or routes. Finally, the route (sequence of disconnected components) with the lowest cost among the evaluated sequences of disconnected components is chosen. Comparisons with both the CARPET heuristic (Hertz, Laporte, & Mittaz, 2000), and memetic algorithm (Lacomme, Prins, & Ramdane-Cherif, 2004) on four sets of problem instances (total of 115 instances) obtained from the literature (DeArmon, 1981; Belenguer & Benavent, 2003; Prins, Belenguer, Benavent, & Lacomme, 2006) showed that the proposed GRASP heuristic improved the best-known solution on 18 of the 115 instances, and matched the results on 89 of those instances.

The location of depots related to spreading operations is usually given as an input in spreader routing models (Haghani & Qiao, 2002). Since the quality of the vehicle routes is highly dependent on the location of the depots, this sequential approach obviously leads to suboptimal decisions, at both the strategic and operational levels. A better approach consists in treating simultaneously the vehicle routing problem and the depot location problem. Cai, Liu, and Cao (2009) used this approach for designing routes for spreading operations. The author proposed a tabu search algorithm to help planners in constructing combined depot location and spreader routing plans. The algorithm takes into account the materials depot capacities, spreader capacities (all the same) and the maximum number of routes. The tabu search algorithm first finds an initial solution to the depot location problem and then tries to improve this solution by applying two types of moves. The first move involves opening a depot and closing another depot, while the second move consists in increasing the number of depots. After every move, the current vehicle routes are optimized. The author did not, however, provide a detailed description of the tabu search algorithm. Computational experiments carried out on real-data from the central part of the Changchun city, China, involving 321 road sections, 20 candidate depots and 13 spreaders allowed an improvement of 6% over the solution obtained from the traditional sequential approach.

Tagmouti (2008) studied a salt spreader truck routing problem with capacity constraints and time-dependent service costs. The problem is modeled as a variant of the capacitated arc routing problem, where a time-dependent piecewise linear service cost is associated with each required arc in a directed graph. The authors proposed a variable neighborhood descent heuristic for solving the problem. An initial solution is first obtained by means of either the parallel version of the Clarke and Wright (1964) savings procedure for the capacitated vehicle routing problem, or a sequential insertion heuristic, where the routes are constructed one by one. Then, a variable neighborhood descent is applied to the initial solution to improve it. During a local descent, three different neighborhoods are explored: arc move, cross exchange and block exchange. Each neighborhood is explored using a first-improvement local descent. The arc move neighborhood structure removes a required arc from one route and inserts it between two other required arcs in the same route or in another route. Given a pair of routes in the current solution, the cross exchange neighborhood structure exchanges two sequences of arcs. Each sequence must contain the same number of required arcs, with up to five required arcs, plus the arcs on the shortest path between them in the route. Similarly, the block exchange neighborhood structure identifies sequences made of consecutive required arcs with no deadhead arcs in-between, called blocks, and exchanges them between two routes. However, the number of required arcs in a block is not limited and two blocks can be exchanged even if they do not contain the same number of required arcs. An improvement procedure, called *shorten* (Hertz et al., 2000), is also used for attempting to reduce the total travel cost of the routes by inverting the service and travel on a given arc, when this arc is crossed twice. The variable neighborhood descent heuristic proposed by Tagmouti (2008) can be summarized as follows:

1. *Initialization*. Let M be the maximum number of required arcs in a sequence of arcs in a route. Define N_1, N_2 to N_{M+1} and N_{M+2} as the arc move neighborhood, the M cross exchange neighborhoods, and the

block exchange neighborhood, respectively. Define also $s(N_j)$ as a local optimum solution based on neighborhood N_j, $j = 1,\ldots, M + 2$. Find an initial solution $s(N_0)$.

2. Set $j = 1$. Until $j = M + 2$, repeat the following steps:
   (a) *Exploration of neighborhood*. Perform a local descent based on neighborhood N_j with $s(N_{j-1})$ as initial solution. Denote with $s(N_j)$ the local optimum obtained.
   (b) *Move or not*. If the solution thus obtained $s(N_j)$ is different from $s(N_{j-1})$, set $s(N_0) := s(N_j)$ and go to Step 2.
3. Apply the *shorten* procedure to $s(N_{M+2})$ to obtain $s(short)$. If $s(short)$ is better than $s(N_{M+2})$, then set $s(N_0) := s(short)$ and go to Step 2. Otherwise, the best solution is $s(N_{M+2})$.

In this procedure, Step 3 is reached and the *shorten* procedure is applied only when the $M + 2$ neighborhoods are explored without any improvement to the starting solution. If the *shorten* procedure improves the solution, Step 2 is restarted with the improved solution. Otherwise the best solution found is returned. The variable neighborhood descent algorithm is executed twice, using the savings and the insertion heuristics to generate an initial solution, and the best solution is returned at the end. Tested on problems derived from classical capacitated arc routing problem instances (Golden et al., 1983; Li, 1992; Li & Eglese, 1996), the algorithm appeared to be fast and competitive when compared with the recent adaptive multi-start local search algorithm of Ibaraki et al. (2005) for solving vehicle routing problems with soft time window constraints.

In a follow-up paper, Tagmouti, Gendreau, and Potvin (2010) adapted the variable neighborhood descent heuristic to address the dynamic version of the problem where weather report updates lead to real-time modifications to the current routes. In this dynamic variant, a starting solution is first computed with the variable neighborhood descent heuristic using service time cost functions based on some initial forecast. As spreader trucks execute their routes, regular weather report updates lead to modifications to the optimal service time interval associated with each required arc. Basically, each time a weather report is received, the variable neighborhood descent heuristic is applied on a new problem, called the *static problem*, defined with updated service time functions for unserved required arcs based on the new storm location and speed. Since the computation times with the heuristic are not negligible, when a new report is received, the current solution is followed for an additional Δt time units using the updated service time functions. During that time, the solution is optimized with the heuristic based on the projected state of the system at time $t + \Delta t$. The new solution obtained can then be implemented as soon as it is available. Computational results were presented for three types of generated instances with 25, 49 and 100 vertices and with 36, 76, and 162 required arcs, respectively. Comparisons with both the *a priori* solutions obtained with the initial service cost functions (based on some initial storm speed forecast), but evaluated in the dynamic setting, and the *a posteriori* solutions computed with the true service cost functions (namely those obtained at the end of the dynamic process when everything is known), showed that the dynamic solutions lie within 10% of the *a posteriori* solutions on the instances with 25 vertices. However, on the 49-vertex instances, the gap jumps to 50%. This gap stabilizes on the largest 100-vertex instances, due to a smaller time step between two updates on these instances, which leads to more frequent calls to the reoptimization procedure.

4. ANALYSIS OF EXISTING RESEARCH ON VEHICLE ROUTING PROBLEMS FOR SPREADING OPERATIONS

In this section, we provide a classification of important optimization models developed over the past 40 years for the routing of vehicles for spreading operations. We utilize the categories and characteristics presented in Table 1 as a basis to classify the research works in Tables 2 and 3 at the end of the section. These tables are arranged chronologically with the oldest works near the top. The categories from Table 1 are listed across the top of Tables 2 and 3 and are included in parentheses where appropriate throughout this section. The remainder of this section follows the classification categories in Tables 1-3 to highlight key research contributions and gaps.

The vast majority of the operations research literature on vehicle routing for spreading considers the routing problems alone (1.1). Only a few works combine vehicle routing and other tactical or strategic problems in winter road maintenance, such as sector design of the location of depots for vehicles or materials (1.2). Typically, these different problems are treated sequentially in a hierarchy with the lower level more operational problems subject to the conditions resulting from solving the higher level more strategic problems. The integrated models that combine routing and other problems generally adopt a longer-term perspective at the strategic or tactical level (2.1 and 2.2), although some pure routing models also incorporate a more strategic view (Evans(1990) and Evans and Weant (1990)).The dominance of static routing models (2.3.1) is clear in Table 2, with dynamic or real-time routing models being quite rare. An example of dynamic spreader routing (2.3.2) is Handa, Chapman, & Xin (2005), which allows new routes to be developed each day as conditions change. This is in contrast to the real-time routing (2.3.3) in Tagmouti, Gendreau, & Potvin (2010), where vehicle routes may change during the traversal of the route as new information (e.g., weather conditions) becomes available. Recent technological developments, such as road weather information systems, weather forecasting services, geographic information systems, global positioning systems, electronic data interchange, and intelligent vehicle-highway systems, enhance the possibilities for efficient dynamic and real-time vehicle routing for spreading operations by providing new instructions directly to vehicle operators in response to sudden changes in weather and road surface conditions.

Tables 2 and 3 link the many and varied problem characteristics (category 3 of Table 1) to the key research. Some very rarely observed attributes are: service hierarchy (3.2), service time windows (3.5), number of passes per road segment (3.7), load balancing (3.13.1), class continuity (3.13.2), both-sides service (3.13.3), turn restrictions (3.13.4), service connectivity (3.13.5), and sector boundaries (3.13.6). These problem characteristics appear less than five times in Tables 2 and 3. Also, one empty column related to problem characteristics denotes “road segment-specific vehicles” (3.11.2), which was not addressed by any contribution. Not surprisingly, the earlier works considered relatively few, more straightforward characteristics, and it was only after introduction of faster heuristics that more complex and realistic arc routing problems with a larger variety of constraints and possibilities were considered. Examples include modeling of service hierarchy constraints (Dror, Stern, & Trudeau, 1987), time windows (Labadi, Prins, & Reghioui, 2008), and turn penalties (Corberán, Martí, Martínez, & Soler, 2002). Note that routing with both-sides service constraints and multiple passes per road segment usually arises in plowing operations, which are limited to one lane at a time. In spreading operations, chemicals and abrasives are often spread onto the road segment through a spinner which can be adjusted so that two lanes are treated on a single pass. Also, the impact of undesirable turns, such as U-turns and turns across traffic lanes, is generally lower in routing spreaders as compared to plowing operations.

The model structures in Tables 2 and 3 (category 4 of Table 1) show that vehicle routing problems for spreading operations are generally formulated as capacitated arc routing problems, where the capacity of the spreader is expressed as the maximum quantity of chemicals or abrasives the spreader can discharge. Vehicle capacities can also be taken into consideration during the sector design process (Liebling, 1973; England, 1982a,b), where the service region (e.g., city) is first divided into a number of geographically disjoint sectors, so that a route satisfying the vehicle capacity can be constructed for each sector. The capacities of the vehicles can also be given as time limits (Soyster, 1974) or as maximum distances which can be spread in one route (Soyster, 1974; Eglese, 1994; Li & Eglese, 1996). Very little work has been reported concerning the possibility for spreader vehicles to refill with materials at intermediate facilities (materials depots) without returning to the original starting point. Hayman and Howard (1972) proposed a model to determine the spreader truck fleet size based at each depot with this condition. Li and Eglese (1996) proposed a three-stage heuristic with decision rules to determine if the spreader should head back towards the vehicle depot or the nearest materials depot to refill with salt. Qiao (2002) showed how multiple salt depots can be taken into account. One interesting line of research would be the further development of more realistic models where vehicles may refill at materials depots by exploiting the research on arc routing problems with intermediate facilities (Ghiani, Improta, & Laporte, 2001; Zhu, Li, Xia, Deng, & Liu, 2009; Ghiani, Laganà, Laporte, & Mari, 2010).

Because of the inherent difficulties of vehicle routing problems for spreading operations, most solution methods that have been developed are heuristics. Much early work (1970-1990) adapted or extended simple capacitated arc routing models with little consideration of operational constraints. These simplified models were generally solved with simple constructive heuristics (5.2), such as sequential constructive methods (5.2.1), parallel constructive methods (5.2.2), cluster first, route second methods (5.2.3), or optimization-based methods (5.2.4), for undirected networks (3.1.1). These constructive heuristics gradually build a feasible solution while giving attention to solution cost, but they do not contain an improvement phase. Some of these heuristics were embedded into discrete event simulation models (5.5) to evaluate benefits and to model spreader movements and interactions (Soyster, 1974; Cook & Alprin, 1976; England, 1982a,b). Recently proposed models are solved with more sophisticated local search techniques, such as composite methods (5.3), which blend route construction and improvement algorithms, and metaheuristics (5.4), which have proven to be very effective for several classes of discrete optimization problems. However, even though recent models tend to incorporate a larger variety of practical characteristics, very few implemented solution methods can be found in the literature (Evans, 1990; Evans & Weant, 1990; Li & Eglese, 1996; Benson, Bander, & White, 1998). In fact, implementation details are rarely considered. This contrasts with the frequency of papers with real world data (6.1), which seems relatively high (see Table 3).

Authors	1.1.	1.2.1.	1.2.2.	2.1.	2.2.	2.3.1.	2.3.2.	2.3.3.	3.1.1.	3.1.2.	3.1.3.	3.1.4.	3.2.1.	3.2.2.	3.3.1.	3.3.2.	3.3.3.	3.4.1.	3.4.2.	3.5.1.	3.5.2.	3.6.1.	3.6.2.	3.7.1.	3.7.2.	3.8.1.	3.8.2.	3.9.1	3.9.2	3.9.3	3.10.1	3.10.2	3.10.3	3.11.1.	3.11.2.	3.11.3.	3.12.1.	3.12.2.
Liebling (1973)		X		X		X				X								X					X						X	X		X		X			X	
Soyster (1974)	X					X			X						X			X									X				X					X	X	
Cook and Alprin (1976)	X					X			X									X								X					X					X	X	
England (1982a,b)		X		X		X			X									X					X					X	X			X					X	
Reinert, Miller, and Dickerson (1985)			X		X	X																										X					X	
Ungerer (1989)	X					X			X						X			X													X					X	X	
Xin and Eglese (1989)	X					X			X			X			X		X	X			X									X		X				X		X
Evans (1990) , Evans and Weant (1990)	X			X		X			X			X			X			X								X	X					X				X	X	
Eglese (1994)	X					X			X				X	X	X			X		X							X			X		X				X	X	
Li and Eglese (1996)	X					X					X	X	X		X	X		X									X					X	X			X	X	
Lotan, Cattrysse, Oudheusden, and Leuven (1996)			X	X		X			X				X			X		X									X					X		X			X	
Benson, Bander, and White (1998)	X					X				X		X			X			X						X	X							X				X	X	
Haghani and Qiao (2001)	X					X			X			X	X		X			X		X	X		X		X		X				X					X	X	
Haghani and Qiao (2002)	X					X				X		X						X								X	X				X			X			X	
Qiao (2002)	X					X			X			X						X			X				X		X				X	X				X	X	
Toobaie and Haghani (2004)	X					X			X									X					X			X	X					X		X			X	
Handa et al. (2005, 2006)	X						X		X			X						X													X			X			X	
Omer (2007)	X					X			X			X			X			X					X								X			X			X	
Tagmouti et al. (2007)	X					X				X		X					X		X												X			X			X	
Tagmouti (2008)	X					X				X		X							X												X			X			X	
Cai et al. (2009)			X	X		X				X							X	X														X				X	X	
Tagmouti et al. (2010)	X							X		X		X							X												X			X			X	

Table 2: Summary of the classifications of spreader routing problems (attributes 1 to 3.12)

Authors	3.13.1.	3.13.2.	3.13.3.	3.13.4.	3.13.5.	3.13.6.	3.14.1.	3.14.2.	3.14.3.	3.14.4.	3.14.5.	3.14.6.	4.1.1.	4.1.2.	4.1.3.	4.2.	4.2.1.	4.2.2.	4.2.3.	4.3.	4.4.	4.4.1.	5.1.	5.1.1.	5.2.	5.2.1.	5.2.2.	5.2.3.	5.2.4.	5.3.	5.4.1.	5.4.2.	5.4.3.	5.4.4.	5.4.5.	5.4.6.	5.5.	5.6.	6.1.	6.2.	6.3.
Liebling (1973)	X					X	X										X											X										X	X		
Soyster (1974)							X					X	X					X									X		X								X			X	
Cook and Alprin (1976)	X		X				X											X									X										X		X		
England (1982a,b)				X		X	X				X					X												X									X	X	X		
Reinert, Miller, and Dickerson (1985)[1]							X						X										a																X		
Ungerer (1989)							X													X							X														X
Xin and Eglese (1989)							X			X								X												X									X		
Evans (1990), Evans and Weant (1990)							X											X								X												X	X		
Eglese (1994)										X											X										X								X		
Li and Eglese (1996)						X		X										X								X												X	X		
Lotan, Cattrysse, Oudheusden, and Leuven (1996)		X					X			X									X						X														X		
Benson, Bander, and White (1998)							X					X			X																	X						X			X
Haghani and Qiao (2001)							X							X																X									X		
Haghani and Qiao (2002)			X		X		X			X												X							X										X		
Qiao (2002)	X				X		X							X																		X	X							X	
Toobaie and Haghani (2004)	X				X		X			X								X																X					X		
Handa et al. (2005, 2006)							X											X																	X				X		
Omer (2007)					X		X							X																	X									X	
Tagmouti et al. (2007)							X		X						X									X																X	
Tagmouti (2008)							X		X									X																		X				X	
Cai et al. (2009)								X											X													X							X		
Tagmouti et al. (2010)							X		X									X																		X				X	

[a]The problem was solved using IBM's MPSX mathematical programming package.

Table 3: Summary of the classifications of spreader routing problems (attributes 3.13 to 6)

5. CONCLUSION AND FUTURE RESEARCH DIRECTIONS

The research described in this chapter for optimizing the routing of spreader vehicles builds on earlier, more idealized, operations research models by better addressing a variety of practical considerations. While the new models demonstrate impressive capabilities to include more issues important to the operating agencies, there is still a large gap between state-of-the-art models and actual implementations. Some reasons for this gap include the difficulty of the problems, the unfamiliarity in the practitioner community with the advantages and benefits of OR models, and problems of technology transfer to a decentralized area such as winter maintenance.

Arc routing research remains a rich area within OR, and winter road maintenance vehicle routing problems will likely remain an important subarea of arc routing. However, vehicle routing for winter road maintenance is different than other arc routing applications for a variety of reasons. First, the demand for service (i.e., the current and forecast road conditions) can vary dramatically over small geographic regions and change quickly over time. Second, the timing of operations is crucial to achieve the desired level of service: spreading a roadway too early is ineffective, and spreading a roadway too late increases costs and reduces the level of service achieved. Third, winter road spreading operations are often conducted in a very difficult and dynamic operational environment characterized by limited visibility, poor traction, and unexpected obstacles (e.g., parked, stalled or abandoned vehicles), all of which can change very rapidly.

In spite of the wealth of theoretical research in winter road maintenance, and the prominent contributions from OR to practice in seemingly similar areas such as emergency services and waste removal (Green & Kolesar, 2004; Sahoo et al., 2005), OR has not yet reached its potential in winter road maintenance operations. A recent “synthesis report on winter highway operations” (Transportation Research Board 2005) surveyed 22 prominent winter road maintenance agencies in North America including 19 US states or Canadian provinces and there municipalities. They found a widespread and increasing level of technology being deployed, with all but two agencies using pavement temperature sensors and all but a single agency using computerized spreader controls on vehicles. However, the report includes only a short paragraph on “route optimization” with one mention of using sensor data and automatic vehicle location (AVL) to “optimize plowing and spreading activities”. There is no mention of route optimization as commonly used in the OR (e.g,. arc routing models) and it seems that a somewhat different language is being spoken regarding route optimization in the practitioner and OR communities.

Another reason for the limited linkages between OR and public works agencies may be the decentralized nature of winter maintenance operations, where individual regional and local agencies have responsibility for snow and ice control over small regions (cities or towns). This differs from some other public works and emergency service systems (e.g., waste removal or fire protection) that have become more centralized on regional or national levels across political jurisdictions, in part to exploit efficiencies of scale and standardization. It may be that winter road maintenance operations are less amenable to standardization and consolidation of neighboring regions with the potential benefits from improved routings.

There remain a number of important areas for future research on vehicle routing for winter maintenance operations, including: (1) the development of new models, especially dynamic routing models, that exploit the availability of more accurate and timely information from new technologies, (2) the development of better models that integrate vehicle route optimization with other winter maintenance decision problems, (3) studies that use optimized routing to quantify the tradeoffs between cost and level of service, especially in light of rising environmental concerns, and (4) implementations of OR models in the field. Each of these areas is briefly described below.

1. *Models that exploit the availability of more accurate and timely information from new technologies*. An impressive array of technologies are in use and in development to provide better information for winter maintenance operations. Two major categories of technologies involve improved weather forecasting tools, such as RWIS and now-casting, and fixed or vehicle-based sensors. The availability of better weather data that is more accurate, more timely, and at a finer geographic scale is important for improved winter maintenance operations and in case studies has been show to generate a very favourable benefit-cost ratio (Ye, Shi, & Strong, 2009b). A recent scan for the “best practices” in winter road maintenance describes the range of technologies to sense road and environmental conditions, improve safety and effectiveness of spreading operations, and reduce environmental impacts (Pletan, 2009).

Sensor technologies for winter maintenance may be mobile (on vehicles) or fixed, and include automatic vehicle location (AVL), roadway surface sensors (for temperature, freezing, and ice-presence), salinity measuring devices, radar, other visual and multi-spectral sensors, etc. (See Shi et al., 2006 and Transportation Research Board, 2008, for details.) Some agencies also deploy a variety of fixed assets for roadway snow and ice control, such as FAST (fixed automated spray technology), road warming, snow fences, etc. While such technologies are expensive, they can be used at selected critical locations on roadways. Naturally, this affects vehicle routing plans and research is needed to assess the integration of such fixed technologies into optimal vehicle routes.

One challenging opportunity for vehicle routing research is to develop real-time routing models that can exploit the real-time winter maintenance information. Real-time routing may be needed to respond dynamically not just to changing ice and snow conditions and forecasts, but also to equipment breakdowns, traffic congestion and accidents, all of which are more common in winter driving. Real-time route changes can also be used to respond to citizen complaints (e.g., streets not cleared) in a more timely manner, thereby increasing customer service. The survey from the “synthesis report” mentioned above (Transportation Research Board 2005) highlighted the use of dynamic routes in practice as 72% of the agencies responding indicated that they dynamically change routes. A few researchers have begun to address some dynamic and real-time issues in winter road maintenance as noted earlier in the chapter, but more research is needed on dynamic, real-time, and stochastic winter maintenance arc routing problems. One avenue for this research may be to exploit the growing research on challenging dynamic *node* routing problems (Ghiani, Guerriero, Laporte, & Musmanno, 2003; Ichoua, Gendreau, & Potvin, 2000; Psaraftis, 1995).

2. *Models that integrate vehicle route optimization with other winter maintenance decision problems*. There are a range of winter maintenance problems beyond vehicle routing that are amenable to operations research approaches, such as depot and materials stockpile locations, fleet sizing, sector design, personnel and vehicle scheduling, etc. (Perrier, Langevin, & Campbell, 2006a, 2006b). The traditional approach has been to solve these problems separately and sequentially, which is likely to be suboptimal. Noble, Jang, Klein, & Nemmers (2006) adopt a broad perspective and consider sector design, depot location and route design in an iterative sequential approach – along with fleet assignment. This more integrative perspective is promising and it may provide new opportunities for implementing route optimization in conjunction with strategic or tactical planning activities. However, coupling a very difficult arc routing (operational) problem with other difficult more strategic OR problems creates a host of challenges for researchers. The recent effort by Cai et al. (2009) is also a step in this direction, though a truly integrated model that optimizes several decision areas simultaneously remains a promising area for future research.

3. *Studies that use optimized vehicle routing to quantify the tradeoffs between cost and level of service*. Vehicle routes play an important role in determining the cost and the level of service provided by winter road maintenance and it is essential to have a good routing model to assess accurately the tradeoff between cost and level of service. Improved vehicle routes could lead to higher levels of service at the same cost, or lower costs to achieve the same level of service. Tradeoff analysis can be at a strategic level

to determine the level of service for a whole season – or more dynamic, as to assess different levels of service that may be deployed in response to individual storms. Furthermore, because improved vehicle routing provides environmental benefits from reduced materials (especially chemicals) usage and broader social benefits from improved mobility, along with the direct cost benefits from reduced use of vehicles and drivers, winter maintenance routing models that incorporate environmental and social costs and benefits are an important area of future research. Tradeoff analyses are also needed to assess the benefits from different weather and sensor technologies, many of which have proven difficult and expensive to implement and to integrate into the winter maintenance decision-making process (Shi et al., 2006; Ye et al. 2009a).

4. Implementation of OR tools to optimize vehicle routing.
While engineers from several disciplines have long played a prominent role in working with public works agencies to develop, test and deploy new technologies for winter road maintenance, operations researchers have been much less successful and reported implementations of sophisticated OR models for winter road maintenance remain rare. Campbell and Langevin (2000) describe in detail three implementations of sophisticated OR models (in the US, Canada and the UK). There is certainly interest from many operating agencies in optimizing the vehicle routes, and we are aware of a variety of projects funded by state, provincial or municipal agencies that include vehicle routing. These projects generally develop models for "optimizing" vehicle routing and then test the models with real-world data from the associated agency. However, subsequent actual implementations of the new optimized routing approach seem very rare, in spite of the generally positive results from testing with "real" data.

There are likely a variety of reasons for the lack of implementation success stories, including those noted earlier, but it does *not* seem that the slow pace of adoption of vehicle routing optimization is due ineffective models or an inability to generate timely results. Nor is a "fear" of technology a likely reason, as agencies are becoming increasingly reliant of sophisticated technologies and computerized controls for operations (and communications). One approach to increase implementation successes might be to integrate vehicle routing optimization with other technologies being implemented, such as the ongoing development and deployment of the winter MDSS (maintenance decision support system) in the US (Pisano, Hoffman, & Stern, 2009; Ye, Shi, & Strong, 2009a). This is a multistate effort to develop a system that can "provide weather and road condition forecasts and real-time treatment recommendations (e.g., treatment locations, types, times, and rates) for specific road segments, tailored for winter road maintenance decision makers." (Ye et al., 2009a). Although the MDSS includes real-time treatment recommendations, it does not include real-time routing optimization at this point.

In summary, there remain many challenging research opportunities (theoretical and applied) in winter road maintenance vehicle routing. One strong trend in practice has been to exploit technological advances and move away from traditional *reactive* static snow and ice control such as de-icing, to more *proactive* approaches, such as anti-icing and dynamic operations based on localized, accurate and timely forecasts. As the technology and operations continue to evolve, rich opportunities for applied arc routing research will continue to emerge – and we hope that optimized vehicle routing will become a "best practice" for winter road maintenance.

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KEY TERMS & DEFINITIONS

Winter road maintenance - Operations conducted to create safe roadways (and sometimes sidewalks) for winter travel. The primary operations include: spreading of chemicals and abrasives to melt snow and ice or to prevent snow and ice from bonding to the pavement, snow plowing to mechanically remove snow and ice from roadways, and snow disposal to load snow into trucks and haul it to disposal sites.

Snow removal - Clearing of a roadway of snow and ice by chemical (e.g., spreading salt to melt snow and ice), mechanical (e.g., plowing), or thermal (e.g., roadway warming).

Snow disposal - Physical removal of snow and ice to designated disposal sites following plowing operations. This is usually accomplished by loading the snow in to trucks and hauling it to disposal sites that may be vacant land, waterways, or openings into a sewer system.

Snow plowing - Mechanical removal of snow and ice from pavement, generally using a truck or other maintenance vehicle equipped with a metal blade.

Spreading operations - Dispersal of chemical or abrasive materials onto a roadway (or sidewalk) usually by means of spreader equipment on trucks for winter maintenance operations. The materials may be in dry form, liquids or brines. Chemicals, commonly salt (sodium chloride) or calcium magnesium acetate, are used to lower freezing-points and change the chemical properties of snow and ice, while abrasives, such as sand, crushed stone, or cinders, are used to improve traction on roadways.

Arc routing - Arc routing problems determine a set of tours that covers a predefined subset of edges or arcs of a transportation network at minimum cost, while satisfying some side constraints. Arc routing problems arise in contexts where road segments require treatments. Practical examples include the routing of street sweepers, snow plowing, salt spreading, postal delivery, meter reading, school bus routing, garbage collection and road maintenance.

Chinese postman problem - The Chinese postman problem consists of determining a minimum cost tour that traverses every edges or arcs of a network at least once.

Capacitated arc routing problem - The capacitated arc routing problem consists of designing a set of routes performed by vehicles of restricted capacity such that a set customers represented by the edges or arcs of a network are serviced and the total cost is minimized.