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December 2010

CIRRELT-2010-58

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Simulation Modeling Approaches for Automated Storage and Retrieval Systems – A Literature Review

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Abstract. Automated Storage and Retrieval Systems (AS/RS) are warehousing systems that use mechanized devices to accomplish the repetitive tasks of storing and retrieving parts in racks. Since these systems represent a significant investment and considerable operating costs, their use must be as efficient as possible. Still, AS/RS performance is the result of the interaction of many complex and stochastic subsystems. This reality creates a need for robust and efficient evaluation models. This article complements the recent survey on AS/RS by Roodbergen and Vis (*European Journal of Operational Research*, 194, 2009) by focusing on modeling assumptions of evaluation models for AS/RS, from analytical travel-time models, on which most research projects are based, to object-oriented simulation. We provide a review of AS/RS simulation techniques which represent one of the most appropriate ways to model the behaviour of complex stochastic environments.

Keywords. Logistics, automated storage and retrieval systems, warehousing, travel-time models, simulation.

Acknowledgements. This research was partially financed by grants [OPG 0293307 and OPG 0172633] from the Natural Sciences and Engineering Research Council of Canada (NSERC). This support is gratefully acknowledged.

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Dépôt légal – Bibliothèque et Archives nationales du Québec,
Bibliothèque et Archives Canada, 2010

1. Introduction

Automated Storage and Retrieval Systems (AS/RS) are now widely used in modern distribution centers (DC) as they provide fast, accurate and efficient handling of materials, 24 hours a day. In its most basic form, an AS/RS consists of storage racks where products are stored and retrieved automatically. This broad definition describes the wide variety of possible system implementations, ranging from carousels to systems designed for the storage and retrieval of maritime containers. In this paper, we are interested in one of the most common AS/RS type: the unit-load AS/RS (Figure 1). Unit-load AS/RS are fully automated systems in which storage and retrieval transactions involve a single unit-load. Although these systems are mostly seen in high-volume distribution centers, where the unit-loads are finished goods pallets, they can also exist in production environments for work-in-progress storage.

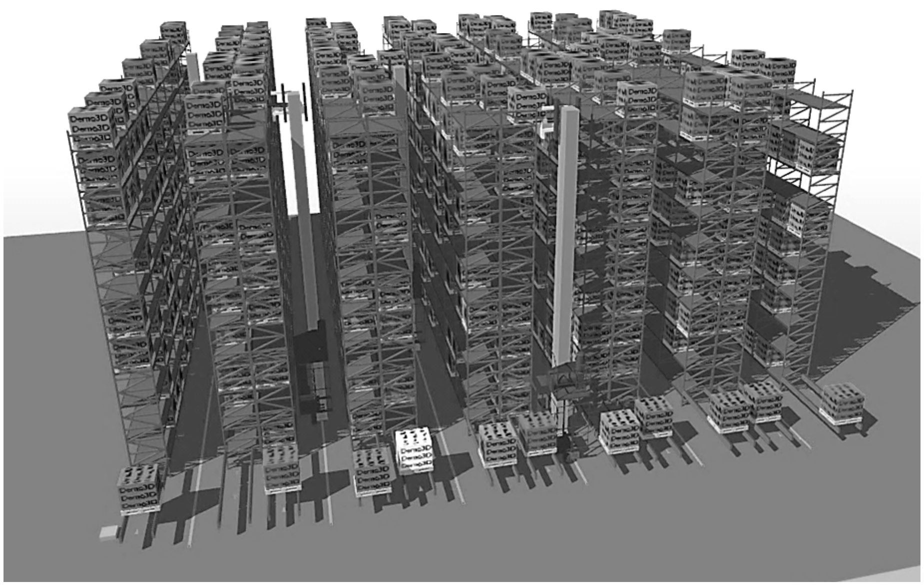

Figure 1: A unit-load AS/RS example

When conceptualizing a system like the one shown in Figure 1, numerous questions arise both on the design and control levels. Many of these questions have already been examined in the literature, and the recent survey by Roodbergen and Vis (2009) covers these articles very well. From a design perspective, the system configuration is challenging. Designers must first configure the storage racks in terms of their number, length, height and depth; these racks define the system's storage capacity and required floor space. In addition, the number of cranes and their degree of freedom must be set. In some systems, cranes are able to move from one aisle to another; in other systems, cranes don't have this ability (i.e., aisle-captive cranes). Another crucially important decision that designers have to make concerns the number and location of input and output stations, where the cranes can interface with upstream and downstream nodes of the material-handling system. All these design decisions greatly influence the required initial investment and, later, determine the maximum throughput that system can achieve.

From a very short-term perspective, control decisions must be made in order for the system to achieve its maximum throughput capacity for a given design. For example, these decisions concern storage assignment, dwell-point positioning and request sequencing. Storage assignment refers to the association of unit-load storage requests to empty storage locations. The dwell-point is the designated location for an idle crane while waiting for the next request. Request sequencing deals with the order in which the cranes execute storage and retrieval requests.

AS/RS design and control decisions are complex because AS/RS evolve in highly dynamic and turbulent environments, in which the demand (translated by retrieval requests) is known without much notice. Like in other logistics processes, demand is also stochastic and is often subject to seasonality. In addition, the system state can change within seconds. For instance, when the system sequences requests for a given moment, the set of available storage locations depends on the operations that have been previously executed and are currently being performed. These factors tend to favor approaches that formulate robust guidelines instead of approaches that search for the optimal solution.

In this paper, we survey the various models available for evaluating AS/RS performance, giving a lot of attention to simulation models. This paper may help academics and industrialists in their search for the best modeling approach for a specific problem. We also highlight the specific assumptions required by each model. The papers reviewed are first grouped according to the main issues examined (e.g., request sequencing, dwell point positioning, storage assignment, input/output configuration) and second, in chronological order to show the influence of past research on current papers.

The remainder of this paper is organized as follows. In order to help classify the approaches, section 2 presents the explicit assumptions found in most AS/RS papers. Each paper reviewed will be presented according to these assumptions, which will help to classify them. Section 3 presents analytical AS/RS models. Analytical models include the first travel-time studies of unit-load AS/RS, which are still considered as state-of-the-art benchmarks. It is impossible not to include analytical models because their influence can be seen even in modern AS/RS models. Section 4 surveys papers that incorporate a form of simulation in the decision-making process. Section 5 reports our conclusions and gives our prospects for future research.

2. General assumptions of AS/RS modeling

AS/RS are highly dynamic systems with many complex relationships between the decisions made and their underlying components. In order to obtain exact analytical information on a given system, there is no choice but to simplify those relationships (Law and Kelton, 2000). Since research into AS/RS began, models have been developed to answer very specific questions. Most of these models are travel-time models, following the pioneering works of Hausman et al. (1976), Graves et al. (1977) and Bozer and White (1984). Travel-time models give a good approximation of crane travel times under specific conditions. This approximation is of interest because the AS/RS throughput is inversely proportional to the crane travel time. Progressively, simulation has been introduced in the evaluation process essentially to capture the effect of stochasticity on travel-time performance. As will be shown in the next section, in order to be valid, analytic AS/RS models have very strict assumptions. This fact does not invalidate their scientific value but provokes uncertainty as to their validity for real-world applications in which some of these assumptions would probably be relaxed. Some authors combine both the analytical paradigm and the simulation paradigm by proposing analytical models that help analyze an AS/RS system prior to a simulation (e.g., see Malmborg and Altassan, 1998).

While each modeling approach is based on its own particular set of assumptions, the following assumptions are generally used in all unit-load AS/RS studies:

- H1. Each pallet holds only one part number or item type (i.e., the system is a unit-load AS/RS).
- H2. Cranes have independent drives on both axes, allowing them to travel horizontally and vertically simultaneously (i.e., travel time follows a Chebyshev distance metric).
- H3. All storage locations have the physical capability to store any item.
- H4. The turnover of each item is known in advance and does not change over time.
- H5. The distance (i.e., travel time) from rack location *i* to rack location *i*’ is symmetrical and does not change over time.
- H6. Crane acceleration and deceleration are assumed instantaneous and are ignored.
- H7. Pickup and deposit times are assumed constant and are ignored.

3. The analytical paradigm – the roots of AS/RS research

The analytical paradigm represents an AS/RS rack in such a way that it is possible to use a pure mathematical analysis in order to answer specific questions. We found three major models using this paradigm: i) the empirical travel-time model, with a discrete rack, ii) the continuous (normalized) travel-time model, in which the rack is represented in continuous space, and iii) the statistical travel-time model, with a continuous rack. All the models presented here compute crane travel times under very specific conditions.

The empirical travel-time model

To the best of our knowledge, Hausman et al. (1976) were among the first academics to study the unit-load AS/RS. Their objective was to examine storage assignment from a design perspective by defining optimal zones in storage racks where products or entire product classes could be stored. They used the following assumptions, in addition to those mentioned in section 2:

H8. A single crane serves a single two-sided aisle.

H9. Incoming and outgoing pallets are transferred at the same point. The I/O point is located at the corner of a rack.

H10. The system is “square in time”, meaning that the time required by the crane to reach the farthest column is equal to the time required to move to the farthest row from the I/O point.

H11. The crane executes single command cycles only, meaning that the crane moves back to the I/O point after a storage or retrieval has been accomplished.

Under these assumptions, they proposed an empirical rack model in which items are sorted in non-increasing order of λ_j, the empirical turnover of pallet j. The turnover represents the popularity of an item. In addition, they computed the one-way travelling distance (y_i) from the I/O point to each location i in the rack for N total locations. Using these two parameters, they derived equations that give the expected one-way travel time for *Pure Random Storage (PRS)* and *Turnover-Based Storage (TBS)* assignment policies, denoted respectively T_{PRS} and T_{TBS} in equations (1) and (2). Without loss of generality, the random storage policy implies that products are stored randomly inside the rack, while the turnover-based storage policy places fast moving items as close as possible to the I/O point.

$$T_{PRS} = \bar{y} = \frac{1}{N}\sum_{i=1}^{N} y_i \qquad (1)$$

$$T_{TBS} = \frac{\sum_{i=1}^{N} y_i \lambda_i}{\sum_{i=1}^{N} \lambda_i} \qquad (2)$$

In equation 1, the one-way travel time using a discrete rack representation and a PRS policy is based on the fact that each location has an equal probability of being visited. It is thus quite logical to estimate the travel distance within the rack using its midpoint, without taking product turnover into account. However, when products are sorted and positioned according to their turnover rate (equation 2), travel time depends on the number of trips to each location (λ_j), with one-way travel time equal to y_i. Using this discrete rack representation, Hausman et al. (1976) concluded that the TBS policy yields much better results than the PRS policy, producing an average decrease in one-way travel time of about 35%.

In our opinion, the discrete rack modeling approach relies on an implicit assumption concerning each item. In other words, one of these two scenarios must hold true: 1) there is, at most, a single pallet of each product stored in the system at all times, or 2) if there is more than one pallet of each item, each pallet must have the same turnover and be retrieved according to a *First In, First Out* policy. When one of these two conditions is met, it is possible to assume that the midpoint of the rack (PRS) or of a single zone (class-based storage) is a good estimator of the expected travel distance to reach item j. Thus, we conclude that the discrete model assumes a *Locations To Product Ratio* (LTPR) of 1 (i.e., each product has only one location). This assertion is confirmed by Graves et al. (1977), who mention that, when implementing the TBS policy, each location is considered as a distinct class. Many authors have used the discrete rack model as a benchmark to quantify their model's precision.

The continuous (normalized) travel-time model

Hausman et al. (1976) also introduced the continuous travel-time model in which the travel time to the i^{th} percentile of locations (y_i) and the turnover of item j (λ_j) are approximated by continuous functions in which i and j become continuous at the [0, 1] interval (see the discrete model, assumption H10). In this model, i is the proportion of locations closer to the I/O point than the location under consideration. The normalized travel time required for the crane to move to the i^{th} percentile of the continuous rack is given by $y(i) = \sqrt{i}$ for $0 < i \leq 1$, where the i percentage of locations are arranged in a square because the time to reach any point (x_1, x_2) follows a Chebyshev distance metric (see assumption H2).

In order to test different turnover scenarios, Hausman et al. (1976) also introduced the s factor to represent the ABC curve. Assuming that items are ranked in non-increasing order of their contribution to total turnover, the ABC function gives the ranked cumulative demand percentage instead of the percentage of inventoried items. The ABC function model proposed by these authors is given by $G(i) = i^s$, where $0 < s \leq 1$. In order to derive travel-time estimators for their continuous model, the authors formulated another assumption:

H12. Items are ordered following the standard Economic Order Quantity (EOQ) model.

Assumption H12 allowed Hausman et al. (1976) to compute the EOQ of each item, determine the average number of locations required by item *j*, and then the space required for the average inventory of items. The authors assumed that there could be more than one pallet of each item. Still, they assumed that LTPR = 1 because each pallet of an item has exactly the same turnover as the others. With this information, it is possible to group items into classes. Tests were performed for PRS, Class-Based Storage (CBS) with 2 and 3 classes, and TBS (see Van den Berg and Gademann, 2000). Based on these test results, the continuous model yields expected travel-time values that are, on average, 5% lower than those found for the discrete model. In addition, the gap between the model outputs increases with the skewness of the given ABC curve.

The continuous model proposed by Hausman et al. (1976) only considered one-way single-command travel time and was later completed by Graves et al. (1977), who added an interleaving function. The interleaving function allows the consideration of dual-command cycles in which a retrieval operation is performed immediately after a storage operation in order to minimize the travel time. By adding this functionality to Hausman's model, Graves et al. were trying to study the impact of dual-command cycles on travel-time performance. The model proposed by Graves et al. was used to test 3 factors: the storage assignment policy (i.e., random storage, class-based storage with 2 and 3 classes, and turnover-based storage), the system capacity to perform dual-command cycles, and the sequencing of retrievals (e.g., no sequencing, selection among *K* retrievals). From their results, it is clear that always performing dual-command cycles yields substantial time economies over a single-command system. Again, the results presented show that both the discrete and continuous models for estimating travel time yield the same nature of results, but the gap between both models' results increases with the skewness of the given ABC curve.

The statistical travel-time model

The two continuous modeling approaches proposed by Hausman et al. (1976) and Graves et al. (1977) have strict assumptions, such as the assumption that the rack is square-in-time. Bozer and White (1984) questioned those assumptions, stating that empirical experience frequently indicates that the optimum AS/RS design is not square-in-time, and furthermore most of the installed systems are not square-in-time. Their model derives expected travel times for racks in which the square-in-time assumption is relaxed. Bozer and White (1984) proposed a statistical travel-time model that, in addition to the assumptions H1-H9 presented above, holds true under the following assumptions:

H13. The rack is considered to be continuous and rectangular-in-time.
H14. The crane operates on either a single or a dual command.
H15. A pure random storage policy (PRS) is used.

In order to define the system shape as a continuous parameter to use in their model, Bozer and White introduced the rack shape factor, given by $\mathrm{b} = \mathrm{Min}\left[\frac{\mathrm{t_h}}{\mathrm{T}}, \frac{\mathrm{t_v}}{\mathrm{T}}\right]$ where T =Max$[t_h, t_v]$ and t_h and t_v are the time to reach, respectively, the farthest column and row in the rack from the I/O point. Based on this definition, a system with equal travel time on both axes yields b = 1 and so is square-in-time. Assuming b = 1, these authors show that the expected normalized single-command (SC) one-way travel time obtained is $E(SC) = \frac{4/3}{2} = 0.666$. Still assuming b = 1, the expected interleaving time between two rack points is $E(DC) = \frac{7}{15} = 0.466$. These values are identical to those previously obtained by Hausman et al. (1976) and Graves et al. (1977). Bozer and White also showed that, for various rack configurations defined by b, their statistical model gives results that are very close to the discrete rack model, with an average deviation of less than 1%. They extended the model to allow them to evaluate the different dwell-point policies and I/O locations by relaxing assumption H9 and computing crane travel time from the probability that the crane would terminate trips at certain locations, according to the studied configurations.

Bozer (2010) has proposed a simple enhancement to the pure random storage (PRS) policy, adjusting the model in order to take rack use into account. The adjusted PRS model (A-PRS) uses only the i^{th} locations closest to the I/O point, where i represents the average aisle space use ($0 < i \leq 1$). Using this adjustment, as explicitly shown by Fukurani and Malmborg (2008), the expected single-command cycle time becomes:

$$E(SC) = \sqrt{i}\,\frac{4T}{3}, \text{ for } i < b \qquad (3)$$

$$E(SC) = iT(1 + \frac{(b/i)^2}{3}, \text{ for } i \geq b \qquad (4)$$

The analytical paradigm has produced some of the best models, which are still used today by academics and industrialists. One of the reasons of this success is their focus on a top-level system abstraction, which makes them able to produce results efficiently. When a system uses the assumptions H1-H15, these models can produce very useful information for design decisions. However, these models have not yet been shown effective for multi-aisles environments.

4. The simulation paradigm

Real-world systems are often too complex to be evaluated analytically. Simulation is a numerical analysis technique designed to evaluate the responses of complex models. For instance, in order to evaluate storage assignment policies analytically, most models assume a rack utilization of 100%, which is not always true in real-world systems. In many AS/RS studies, the components behave stochastically and their operation could benefit from simulation analysis. The remainder of this section represents the core of this literature review, focusing on papers using a form of simulation to study AS/R systems. Since literature on AS/RS simulation is quite extensive, we will also add a level of taxonomy, grouping the papers according to the main issue that they examined (general, request sequencing, dwell-point positioning, storage assignment, input/output configuration, other similar types of AS/RS and specific applications). Section 4 is followed by two recapitulative tables showing the underlying assumptions for each paper as well as the issues examined.

General simulation studies

Schwarz et al. (1978) used a discrete rack simulation to examine and extend the analytical models of Hausman et al. (1976) and Graves et al. (1977). They studied the effect of storage assignment and interleaving on travel-time performance. Their analysis used the same two factors as Graves et al.: storage assignment policy (PRS, CBS with 2 and 3 classes, TBS) under different turnover distributions (20/60, 20/70, 20/80, 20/90, where α/β means that α % of items are responsible for β % of total turnover). They used the traditional assumptions (H1-H9, H11-H13) and simulated a single, two-sided aisle of 100 x 10 locations, in which the pallet arrivals follows a Poisson process with an average of 45 pallets per hour. Arriving pallets are placed in a storage queue of infinite capacity. At the moment of storage of item *i*, the item length of stay (LOS) is generated using $\mathrm{F(t)} = \mathrm{prob(LOS \leq t)} = {}^{\mathrm{st}}\!/_{\mathrm{u}}{}^{s/(1-s)}$, with $0 < t \leq u/s$ where *s* is the turnover distribution skewness factor of Hausman et al. (1976) and μ is the average LOS yielding a targeted rack utilization percentage. This percentage is given by $U_R = \frac{100\lambda\mu}{N}$, where λ is the mean pallet arrival rate and N is the number of locations in the rack.

Schwarz et al. (1978) also introduced the closest open location (COL) as a storage assignment rule. Applied within a storage assignment policy, this rule chooses the closest open location to the I/O point when processing a storage request. Using this formulation, these authors conclude that choosing an open location randomly or using COL are equivalent for a targeted 90% rack use when neither interleaving nor sequencing is allowed. Once again, it is obvious from these conclusions that the authors implicitly assume an LTPR=1. Still, it is not certain that the result would be the same with a different rack use level or with an LTPR $\neq$ 1.

Linn and Wysk (1984) presented a discrete simulation model implemented in the SLAM simulation language. The simulation model represents sequential pallet transactions in a system with up to 10 product types. In addition to the assumptions presented in section 2, the authors made assumptions that are very similar to those of Hausman et al. (1976). Since the number of product types is very low, there is more than one pallet of each product in the system. The size of the system remains unspecified. Even though the system is highly simplified in this example, to the best of our knowledge, it is the first simulation model for AS/RS that was designed according to a generic mindset, thus allowing more than one scenario to be studied.

An experimental plan was created for a single-aisle system served by a single crane and mainly considers modern AS/RS control issues, including: dwell-point positioning, request sequencing and storage assignment. This experiment was designed to compute total cost as the main performance measure. The cost component represents a mix of stock levels and crane response times, though inventories had little effect on the experimental outcome. Linn and Wysk (1984) drew the following conclusions from their experiment. First, a pursuit policy yields shorter travel times in all of the cases studied, meaning that the crane must remain where it is when it is idle. Second, most control decisions do not increase performance when system utilization is low; however, performance tends to increase with the utilization rate. Finally, the results obtained are aligned with Hausman et al. (1976), which suggest that a class-based storage assignment yields better results than a PRS policy.

Linn and Wysk (1987) used the model they proposed in 1984 to analyze a simulation of the different control policies of unit-load AS/RS under the assumption that demand varied with seasonal trends. They also took the crane's acceleration and deceleration into account. The system that they studied was a single-aisle system with a total of 40 × 10 locations and a single stacker crane. Two I/O points were located on the bottom corners on opposite sides of the aisle. They used the *Shortest Completion Time* (SCT) sequencing rule and some variants introduced in their previous work (Linn and Wysk, 1984). The SCT rule selects, among a list of requests, the one that has the shortest completion time. They also introduced a new concept of dynamic classes, in which, after a review, products may periodically change classes or classes may periodically gain or lose space; however, they did not discuss the implementation details. In their 1987 paper, the results are given for a maximum of 5 product types. For the parameters studied, the authors concluded that the pursuit dwell-point policy appears to be the best strategy when the storage and retrieval requests arrive randomly. They also concluded that, when storage and retrieval arrival rates are high, leading to high crane use (i.e., $\geq$ 50%), control decisions start to significantly affect system performance.

Van den Berg and Gademann (2000) presented one of the most complete simulation studies of AS/RS control policies. They compared several storage assignment policies as well as the sequencing of storage and retrieval requests. The simulated response used throughout the study is the tradeoff between crane travel time and retrieval requests completion time. It is assumed that the item turnover is known and constant (H4) and that the AS/RS is composed of a single two-sided aisle. The I/O point is located at a corner of the rack. The turnover rates, product mix and ordering policies are constant over time and the number of available unit-loads at period t is approximated using a normal distribution (see Van den Berg, 1996). It is also assumed that the system is balanced (i.e., the storage and retrieval requests queues are always of equal length), making dual command cycles possible. This model allows dynamic sequencing of retrieval requests; however, in order to maintain good-quality service levels, urgency rules were employed when a retrieval request approached its due date (see Han et al., 1987).

Their simulation model was implemented in C++, but no other details were given concerning its implementation, except for the fact that there is, at most, a single unit-load of each item in the aisle at all times ($LTPR = 1$). When a retrieval request is generated, a storage request for the same item is generated at time $t + TUNS_i$, where $TUNS_i$ is a random variable with constant distribution that represents the time between the arrival of a retrieval request and the arrival of a subsequent storage request for item i. In the same manner, when a storage request is generated, a retrieval request is generated at time $t + TUNR_i$.

Their extensive simulation results can help to choose the most suitable policy for specific cases. For example, choosing the storage location that minimizes the travel distance to the next retrieval location gives the best result for PRS and CBS storage. They also drew an interesting conclusion concerning storage assignment: when a storage request cannot be stored within its designated region in class-based storage (CBS), it is better to store it farther from the I/O point than its original designated position. Doing the opposite could result in longer mean travel times. They also concluded that, while request sequencing may reduce mean travel time and waiting time for each request, it is also highly likely to cause more waiting time for certain retrievals.

Request sequencing

Han et al. (1987) studied sequencing of storage and retrieval requests in unit-load AS/RS by evaluating a nearest-neighbor heuristic. They evaluated the heuristic using an analytic travel-between (i.e., interleaving) approximation for travel time derived from Bozer and White (1984). As a benchmark, the authors used the well-known FIFO rule for sorting retrieval requests. The model gives the travel-time savings compared to FIFO rule. These savings can be achieved by pairing storage and retrievals using the nearest-neighbor (NN) and shortest-leg (SL) heuristics in order to perform dual command cycles. The authors employed block sequencing, which consists of sequencing a block of retrieval requests. When a block is completed, the system continues with the next one.

They proposed their model under the following specific assumptions, in addition to the assumptions presented in section 2:

H16. Rack utilization is 100%.
H17. Storage of items follows the COL rule.
H18. Dual-command cycles are performed.

The NN heuristic selects a pair $P = (s \in S, r \in R)$ with the minimum interleaving time from the non-empty set of available retrieval requests (*R*) and the non-empty set of available storage requests (*S*). The authors also introduced the shortest-leg (SL) heuristic for sequencing retrievals, which considers both the travel time to a storage location *s* and the interleaving time from *s* to a retrieval location *r* as one leg, and executes the pair with the shortest leg. Both heuristics stop when the retrieval set is empty.

To evaluate their procedure, Han et al. (1987) used Monte-Carlo sampling, for which *n* retrieval points (*R*) and *m* random open locations (*S*) were randomly generated in the rack. A total of 1000 samples were tested for 30 *n*m* combinations. The main conclusions of these authors are first that pairing storage and retrieval requests can lead to a maximum reduction of 60% in interleaving time, which results in 10-15% improvement in throughput compared to the *First Come, First Served* (FCFS) treatment of retrieval requests. Second, the NN and SL heuristics yield results that are similar in nature, but in the long run, SL tends to distribute open locations farther away from the I/O point and thus results in a poorer performance.

Eben-Chaime and Pliskin (1997) questioned the fact that most AS/RS studies focus on single-aisle systems and ignore the other systems that are linked to the AS/RS. These authors extended their previously-proposed integrative model (Eben-Chaime and Pliskin, 1996) that models single-depth racks, cranes, storage and retrieval queues and a list of empty slot addresses. The authors essentially evaluate 3 crane operating modes: single command (SC), dual command (DC), and hybrid command (HC). The HC mode uses DC whenever possible and SC otherwise. Their experiment simulated 100 000 cycles to remove the need for a simulation warm-up. In order to keep the AS/RS inventory balanced, Eben-Chaime and Pliskin assumed that each retrieval request is followed by a storage request. They also explicitly stated that the model handles unit-load or mini-load operations. Their results indicate that, compared to strict SC or DC, the HC mode is the best strategy for reducing the length of storage request queues and maintaining good service levels.

Dwell-point positioning

Egbelu and Wu (1993) presented an interesting study on dwell-point policies for unit-load AS/RS. As a reminder, the crane dwell-point is the specific location where a crane must go when it is idle for a certain amount of time. The objective of a crane dwell-point policy is to minimize crane travel times. Egbelu and Wu studied two types of dwell-point policies: static and dynamic. Static dwell-point policies are derived from rules-of-thumb (e.g., Goetschalckx, 1983). These authors presented a dynamic dwell-point policy in the form of a stochastic linear program (LP) to be solved each time a crane is idle. Among the studied dynamic dwell-point policies (initially proposed by Egbelu in 1991) is the LP-expect policy, which solves an LP that minimizes the distance between the idle crane and the next request location. The authors proposed a simulation of 6 policies applied under PRS or TBS. The assumptions used were the same as those presented in section 2. The simulation model is represented as a flowchart and strictly focuses on pallet transactions without taking into account the AS/RS interfaces; retrievals are treated on a FIFO basis. The model was run for 5 replications for each combination. The results show that, under dedicated and random storage conditions, the LP-expect dynamic dwell-point policy minimizes crane travel time compared to static policies. These results show that dynamic AS/RS decision-making may have good potential for AS/RS research.

Meller and Mungwattana (2005) used simulation to evaluate the benefits of various dwell-point policies. The focus of their study is the relation between the dwell-point policy and system utilization. The main performance measures employed are crane use and the mean system time for retrieval requests. In addition to the general assumptions presented in Section 2, the authors assumed that distances are measured in normalized units (Hausman et al., 1976). Their model operates under the assumption that, if the crane receives a retrieval request while travelling to its dwell-point, it completes the trip before it processes the request. Dual-Command cycles were performed, with storage and retrieval requests being treated according to the FIFO rule. The model was simulated for

100 000 time units, in addition to 5 000 time units for a warm-up. The results indicate that, under the given assumptions, the dwell-point policy has an insignificant impact on system response time (i.e., between 2 and 5%) when the AS/RS has high utilization.

Storage assignment

Azadivar (1984) proposed a simulation model that makes it possible to both test the effect of assignment policies and optimize the system stochastically. A recursive simulation-optimization algorithm that is able to interact with a simulation model uses stochastic approximations to obtain optimum variable values. The model's objective is to optimize the average round-trip time needed by the crane to complete a storage or a retrieval request. Storage requests followed a given arrival rate and retrieval requests followed a given length-of-stay distribution (see Goetschalckx and Ratliff, 1990). Implementing the continuous rack representation (see Hausman et al., 1976), the author used class boundaries as decision variables (x_i) and the crane's average round-trip time was expressed as a function of x_i. A class boundary represents the spatial limits of a physical zone in the rack that is dedicated to a subset of items, grouped together as a class. This simulation-optimization model was run 50 times for 2 000 time units for each run, using a system identical to the one used by Schwarz et al. (1978), with a targeted rack utilization of 90%. His conclusions were similar to those of Schwarz et al. (1978) because similar assumptions were used in his simulation model. Nonetheless, the paper demonstrates one of the first simulation-optimization models in AS/RS research.

Kulturel et al. (1999) used simulation to investigate storage assignment policies in a 3-class unit-load AS/RS. These authors studied two criteria for selecting the items that are grouped in each class: item turnover and item length-of-stay (LOS). The *item turnover* criterion assumes that the items are grouped into classes based on their demand rate (i.e., turnover) and that the class containing the fast-moving products is located closest to the I/O point. Introduced by Goetschalckx and Ratliff (1990), the *LOS* criterion assumes that items are grouped according to the time they spend in storage. Thus, the product class with the shortest LOS is placed closer to the I/O point. The expected LOS of the ith unit in a lot is given by i/λ, if items are also replenished in a quantity Q of unit-loads and consumed at a rate of λ. The LOS criterion classifies all units according to each unit's expected length of stay and places the unit class with the shortest LOS closest to the I/O point. The simulation evaluated the crane travel time for a single two-sided aisle, using an (s,Q) replenishment model. The authors also assumed that all items in a particular class have the same constant replenishment lead time. The simulation results indicate that class-based storage according to the item turnover criterion outperforms the LOS criterion. However, the performance gap between the two methods becomes insignificant as the number of products increases.

Fukurani and Malmborg (2008) proposed an innovative method for estimating travel time in a single-aisle unit-load AS/RS. Their method used the queuing theory approach. They were interested in testing the Closest-Open-Location (COL) storage policy under single-class rack conditions when rack use is less than 100%. In this case, the assumption of equiprobable access to any rack location used by Hausman et al. (1976) is not satisfied. In fact, applying the COL policy tends to maximize storage location use close to the I/O point. Fukurani and Malmborg assumed that, when system use is less than 100%, COL may yield shorter travel cycles than a pure random storage (PRS) policy. Their method models each storage location as a single server with a M/M/N Poisson queuing model, where *N* represents the number of locations in the aisle. The *N* locations are sorted in the non-increasing order of their distance from the I/O point. The queuing model's state distribution is then used to approximate the probability that individual locations are occupied. These probabilities are then applied to estimate the storage and retrieval transaction times under the COL storage assignment rule. In order to validate the approach, they compared the results from their model to a discrete rack simulation model and to the Bozer's adjusted PRS (APRS) model (2010). The results indicate that, compared to APRS, the average response of their method is closer to the discrete rack model.

Gagliardi et al. (2010) were interested in the relative performance of pure random storage (PRS), class-based storage (CBS) and turnover-based storage (TBS) under the assumptions presented in section 2. To this end, the authors proposed a detailed discrete-event simulation model that allows the performance of the different storage assignment policies to be studied under realistic conditions. Their modeling approach is generic enough to test a wide variety of scenarios. The system is balanced, meaning that each retrieval request performed will trigger a subsequent storage request later in time. In order to validate the model, the authors first replicated the specific settings of Hausman et al. (1976) by reproducing assumptions H1-H11. The results of Gagliardi et al. are identical in nature as those of Hausman et al., thus validating their model. These results suggest that a TBS storage assignment policy yields shorter crane travel times than a PRS policy.

These authors also studied the situation in which the amount of storage space per product is superior to 1 (LTPR>1). In this type of situation, space allocation plays a central role in performance. By using different values of *s* (i.e., skewness parameter of the ABC distribution, as per Hausman et al., 1976), they showed that TBS does not always lead to the best performance, as is widely expected. They concluded that both PRS and CBS can achieve better performance than TBS if LTPR>1. These conclusions demonstrate the dangers associated with models that use assumptions that are not necessarily consistent with typical warehouse practices. These conclusions also show that simulation models, which are less restrictive in terms of the necessary assumptions, can give results that are much more precise than the approximations given by analytical models.

Input / Output configuration

Randhawa et al. (1991) used simulation to evaluate single and dual I/O-point configurations in unit-load AS/RS. The following assumption is made for a single one-sided aisle:

H19. Each I/O point is capable of serving storage and retrieval requests.

These authors proposed three performance criteria: the system throughput, the mean request waiting time, and the maximum request waiting time. They assumed that storage requests can only be processed using FIFO or COL rules. Retrieval requests can be arranged to minimize travel time by either working on a block composed of the first *K* requests or by dynamically updating the list each time a new retrieval request arrives. Their block sequencing is similar to the one presented by Han et al. (1987). Their simulation model is based on a discrete-event engine implemented in PCModel. The studied configurations all disposed of 5 rows x 20 columns (100 locations), and the initial rack space was 75% full. The simulation results were based on 5 replications, and common random numbers were used to reduce variance in the experimental outputs. The simulations showed that the dual-dock layout maximizes throughput. They also performed sensitivity analysis based on rack use, and they concluded that high rack use leads to throughput reductions because the crane needs to travel longer to process requests.

Other similar types of AS/RS and specific applications

Kaylan and Medeiros (1988) studied a different breed of AS/RS (Miniload AS/RS), which handles work in progress (WIP) using a simulation written in the SIMAN language. This AS/RS system was designed to store and dispatch WIP units to a set of workstations placed around the S/R system according to the job-shop concept. These authors simulated a single two-sided aisle. At time 0, the system is initialized with either 25%, 50%, 75%, 80% or 90% of its storage capacity, and WIP units are generated with random completion states. This approach was used to eliminate the very long transient state that would otherwise emerge. Three storage assignment policies were investigated: a PRS policy, a 4-class policy, and a 8-class policy. When a WIP unit arrives from a workstation, it is stored in the zone dedicated to the workstation that corresponds to the next operation in order to minimize travel time. One of the primary conclusions of these authors was that, compared to PRS, zoning with vertical boundaries is not a good idea for these particular settings because the crane's slow vertical travel speed makes the system's performance deteriorate. Still, the 8-class implementation results in good performance improvements, but the improvement decreases with the space used.

Guenov and Raeside (1992) proposed an analytical travel-time model to study the impact of zone shape on the optimal pick tour in a 3-class multi-command AS/RS. A multi-command AS/RS is able to transport more than one unit-load at once, making it possible for the crane to perform more than two operations before returning to the I/O point. The system studied was composed of a unique two-sided aisle. The authors used the band heuristic to define the first two classes in which fast-moving products are gathered. Bozer (1986) showed that, coupled with a 2-opt local search, the band heuristic can be very efficient under PRS conditions. In order to experiment with zone shapes, Guenov and Raeside presented the results of a full factorial experiment based on 6 factors: picks per cycle, routing method, crane velocity, rack length, turnover distribution and zone configuration. The experimental plan contained a total of 480 combinations, which were iterated 5 times each, yielding a total of 2 400 runs. These results show that the numbers of picks per cycle and zone configuration were the two main factors that affected the optimal tour. In addition, the authors observed that tours constructed with the assumption of a constant crane velocity were very different from those constructed using real travel parameters.

Taboun and Bhole (1993) proposed a simulation analysis that was validated against data obtained from an existing automated distribution center. The authors planned an experiment that studied two independent variables: system configuration and item-to-pallet assignment. Using a discrete-event engine implemented in the SIMAN language, their model was a multi-aisle system and required the following specific assumptions:

- H20. Each crane serves two double-sided aisles. A shuttle transfers the crane from one aisle to the other.
- H21. Two pallet sizes can be used to store items; large pallets can hold twice as much as small pallets. Each location is able to accommodate any pallet type.
- H22. A single pallet may store more than one item type.
- H23. Storage requests are processed FIFO.
- H24. Retrieval requests are processed FIFO.

The system was considered to be full at the beginning of each replication. Nine replications were executed for each combination. The test results indicate that, for each configuration studied, maximizing the number of item types on each pallet is a good approach for maximizing throughput.

Rosenblatt et al. (1993) were among the first to study AS/RS design and control simultaneously. In order to address the two issues, they proposed a sequential optimization-simulation approach that provides a system that can achieve the required throughput at the lowest cost. They assumed that the aisles have single-depth racks on both sizes. The cranes behaved like the ones presented by Taboun and Bhole (1993), with one added assumption:

H25. A crane can be transferred from one aisle to another if and only if there is at most one crane in each aisle at all times.

Based on several systems that they had worked on, these authors felt that an optimal AS/RS design generally required fewer cranes than aisles. The system in their study performed dual-command cycles and was used to feed a certain number of workstations interfaced by a closed loop conveyor. There was also the same input buffer space as output buffer space at each aisle's entrance. Their approach consisted of solving an integer optimization problem that attempts to build a system as inexpensively as possible. Once the problem has been solved, the procedure moves to the simulation phase. In addition to the general assumptions given at the beginning of this section, the simulation model also uses a specific set of assumptions:

H26. The number of pallets in the system is constant.
H27. Each pallet has a fixed address to which it returns at the end of every operation.
H28. Each pallet of an item has an equal probability of being selected.

Linn and Xie (1993) studied AS/RS from another perspective. They examined the effect of request sequencing on a Just-In-Time (JIT) production system. The system configuration was very different from systems presented previously. (For the sake of simplicity, its assumptions will not be detailed here. However, interested readers can consult Linn and Xie (1993) for more details.) The authors proposed a simulation model written in FORTRAN. This model is composed of 3 modules: a production schedule generator, a JIT assembly line and an AS/RS. The authors implicitly assumed that the number of loads of each type are balanced, meaning that when a load is retrieved, a storage request is generated with a certain lead time. Their results show that over a 55% reduction in production stockouts can be achieved by using a requests sequencing rule that favors the request that has the nearest due date.

Ekren and Eragu (2010) are also interested in AS/R systems where transactions are filled by autonomous vehicles that can transfer products from one tier to another, using elevators located at the entrance of each aisle. One of the main advantages of this design is that vehicles are not bound to a specific aisle, which makes it possible for the DC manager to transfer vehicles to other aisles if needed. The study, which took place in a French DC, was based on strict assumptions, for example: a PRS storage assignment policy is applied, requests are filled according to the FCFS rule and a pursuit dwell-point policy is also applied, which means that vehicles remain in place after a transaction has been completed.

The objective of their research was to propose the best system design that would take into account practical constraints of their industrial partner. They developed multiple regression models that explain 5 performance measures (i.e., average cycle time for storage and retrieval requests, average waiting time for transactions, average waiting time of autonomous vehicles at the elevator, average vehicle use, average elevator use) as function of the system design (i.e., number of levels, number of bays, number of aisles). They proposed a simulation model (developed in Arena 12.0) that is run for 10 replications for each of the 30 scenarios in order to formulate regression functions using MiniTab statistical software. The resulting non-linear functions are then optimized using Lingo for the practical constraints of the real warehouse. The solution retained is the one that was obtained from the greatest number of scenarios. These results suggest that it is better to design the aisles as long as possible.

Other simulation techniques for AS/RS analysis

Knapp and Wang (1992) introduced petri-nets for modelling AS/RS. These authors proposed a model composed of 32 bays used to store 4 item types in 4 classes. They implicitly assumed that the system is a unit-load AS/RS that applies a random storage assignment policy within the classes. Single-command cycles were also assumed, meaning that the crane always goes back to the I/O point after fulfilling each request. In their paper, Knapp and Wang (1992) explained Petri net principles extensively but drew few conclusions about the AS/RS model. Their main contribution is that they demonstrated that it is possible to use Petri Nets to study AS/RS. Later, Chincholar and Chetty (1996) used stochastic colored petri nets to study the effect of 5 AS/RS control factors on the makespan, the unproductive travel time, and the mean flow time. The AS/RS was modeled as a part of a Flexible Manufacturing System (FMS). These authors show that makespan, unproductive travel time and mean flow time of system items are most affected by the crane scheduling rules and crane operation mode.

Recapitulative tables

In this section, we have given a detailed and up-to-date portrait of simulation approaches applied in AS/RS research. A lot of research papers use simulation-derived approaches. Still, it is not easy to determine the assumptions used in these papers because often they are not mentioned. Table 1 below collects these assumptions and presents them in the perspective of helping researchers to find studies about a particular context. In addition, since many control rules have been mentioned in this review, we collected these rules and present them in Table 2.

Paper	Requests sequencing	Interleaving	Dwell-point positionning	Storage Assignment	IO configuration	Specific applications	Unit-load AS/RS	All locations have same dimensions	Single crane, Single aisle	Symetrical distances	Single IO Point per aisle, front corner	Square in time rack	Rectangular in time rack	Each IO can perform storages or retrievals	Non-aisle captive cranes	At most one crane in each aisle at all times	Chebyshev Travel	Crane Acceleration is constant	Pickup and deposit times are constant	More than one pallet size	LTPR = 1	Constant item turnover	Items ordered following EOQ model	Single command cycles	Single OR Dual command cycles	Dual-command cycles	FCFS Sequencing of Storage Requests	FCFS Sequencing of Retrieval Requests	Random Storage assignment	Closest open location storage assignment	Dedicated storage assignment	Rack utilization is 100%	Constant number of pallets in the system
	Issues under study						Modeling assumptions: Configuration										Crane			Products				Interleaving			Sequenc.		Storage				
Hausman et al. (1976)				x			x	x	x	x	x	x		x			x	x	x		x	x	x	x								x	x
Graves et al. (1977)	x	x		x			x	x	x	x	x	x		x			x	x	x		x	x	x		x		x					x	x
Bozer and White (1984)			x	x			x	x	x	x	x		x	x			x	x	x		x	x			x				x			x	x
Schwarz et al. (1978)	x	x		x			x	x	x	x	x	x		x			x	x	x		x	x			x		x						
Linn and Wysk (1984)	x	x		x			x	x	x	x	x		x				x		x								x						
Linn and Wysk (1987)	x	x		x			x	x	x	x	x		x				x		x								x						
Van den Berg and Gademann (2000)	x			x			x	x	x	x	x		x	x			x	x	x			x			x		x						x
Han et al. (1987)	x						x	x	x	x	x		x	x			x	x	x							x	x		x				
Eben Chaime and Pliskin (1997)		x					x	x		x	x	x		x			x	x	x						x				x				
Egbelu and Wu (1993)			x				x	x	x	x	x		x	x			x	x	x								x						
Meller and Mungwattana (2005)			x				x	x	x	x	x	x					x	x	x						x		x	x	x				
Azadivar (1984)				x			x	x	x	x	x		x	x			x	x	x			x			x		x						
Kulturel et al. (1999)				x			x	x	x	x	x		x	x			x	x	x			x			x		x	x					
Fukurani and Malmborg (2008)				x			x	x	x	x	x		x	x			x	x	x			x		x					x	x			
Gagliardi et al. (2010)				x			x	x	x	x	x		x	x			x	x	x			x			x		x						
Randhawa et al. (1991)	x				x		x	x	x	x		x		x			x	x	x			x				x	x		x				
Kaylan and Medeiros (1988)				x		x		x	x	x	x		x	x			x	x	x														
Guenov and Raeside (1992)	x			x		x		x	x	x	x		x	x			x		x			x										x	
Taboun and Bhole (1993)				x		x		x		x			x				x			x		x			x		x	x					
Rosenblatt et al. (1993)						x		x		x	x		x	x	x	x	x	x							x		x	x			x		x
Linn and Xie (1993)	x					x		x	x	x		x					x	x	x							x	x						
Ekren and Eragu (2010)						x	x	x		x	x			x								x					x	x	x				

Table 1: Recapitulative table of AS/RS modeling assumptions mentioned in this review

	Storage assignment						Dwell-point positionning						Sequencing				Interleaving		
	Pure Random Storage	Class-Based Storage	Turnover Based Storage	Continuous Storage	Length of Stay Storage	Closest Open Location	Mid-Point	Input	Output	Pursuit	LP-Expect	LP-Minmax	SCT	Longest-Waiting	NN	SL	SC	DC	HC
Hausman et al. (1976)	X	X	X																
Graves et al. (1977)	X	X	X																
Bozer and White (1984)	X						X	X	X	X									
Schwarz et al. (1978)	X	X	X																
Lynn and Wysk (1984)	X		X										X						
Lynn and Wysk (1987)													X						
Van den Berg and Gademann (2000)	X	X	X	X									X	X					
Han et al. (1987)															X	X			
Eben Chaime and Pliskin (1997)																	X	X	X
Egbelu and Wu (1993)							X	X	X	X	X	X							
Meller and Mungwattana (2005)							X	X	X	X	X								
Azadivar (1984)	X	X	X																
Kulturel et al. (1999)			X		X														
Fukurani and Malmborg (2008)						X													
Gagliardi et al. (2010)	X	X	X																

Table 2: Recapitulative table of the main control policies mentioned in this review

5. Conclusion

In conclusion to this literature survey on AS/RS modeling approaches, we would like to stress that, although they are good approaches and give insight into AS/RS performance, analytical models usually require strict assumptions (e.g., LTPR=1, single aisle, single crane, 100% space use) that are often static in nature. Thus, the validity of the conclusions drawn from those assumptions may be questionable for real-life scenarios (see Fukurani and Malmborg, 2008; Gagliardi et al., 2010). On the other hand, models incorporating some forms of simulation are often more precise and are quite naturally able to integrate stochasticity and the dynamic nature of decisions. Still, this precision often gives results that are scenario-specific, and thus are rarely generalizable. The best approach probably lies in between the paradigms presented in this paper. For example, a distributed decision-making framework (e.g., Schneeweiss, 2003) could pair a top-level analytical AS/RS design model with a basic simulation model used to evaluate the design under realistic conditions. This hybridization would take advantage of the strengths of the two complementary approaches.

We also note that very few papers deal with the interactions within and around the AS/RS. For example, there is little research about the multi-aisle context, which is much more complicated than its single aisle counterpart. The generalization of single-aisle AS/RS models to a multi-aisle context is difficult; in some cases, it is nearly impossible due to the interactivity between the individual aisles. We believe that a multi-aisle system cannot be seen as the sum of N independent aisles, at least from a macroscopic point of view. Let's imagine for a minute the storage assignment problem in a multi-aisle system. For example, it might not be necessarily optimal to store every item in each aisle, which adds complexity to the problem to be solved. And, since AS/RS can also suffer from failures, a potential question that must be answered is how should preventive maintenance be planned in order to maximize throughput.

We also noticed the almost complete absence of models dealing with the distribution network's upstream and downstream processes. These processes are of critical importance when studying AS/RS design and control and thus represent desirable research opportunities. In addition to the issues mentioned in this conclusion, many more could benefit from academic research. We refer interested readers to Roodbergen and Vis (2009) for a list of additional research opportunities.

While reading the AS/RS modeling literature, we observed a lack of coherency in the vocabulary choices made by the research community. Although some authors have made a good effort to make their vocabulary coherent with the vocabulary used in the literature, many more authors discuss the same aspects using different terms. Among other examples, we note the Shortest-Completion Time and Nearest-Neighbor sequencing rules. This situation makes it hard to find relevant papers dealing with specific issues. We also refer interested readers to Roodbergen and Vis (2009) for a good example of efforts in this direction.

As a final note to this paper, we remarked that most of the articles reviewed don't discuss their simulation model in detail; only very few do so. A direct consequence of this lack of detail is that it prevents other researchers from building upon past concepts and reduces the development speed in this domain. And, although simulation is the best choice to validate a specific model, we believe that validation should also come from the concerned communities (e.g., the industrial community) in order to ensure the acceptation and eventual propagation of the best models developed in the literature.

Acknowledgement

This research was partially financed by grants [OPG 0293307 and OPG 0172633] from the *Canadian Natural Sciences and Engineering Research Council* (NSERC). This support is gratefully acknowledged.

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