CIRRELT

Centre interuniversitaire de recherche sur les réseaux d'entreprise, la logistique et le transport

Interuniversity Research Centre on Enterprise Networks, Logistics and Transportation

Tabu Search Heuristic for a Two-Echelon Location-Routing Problem

Teodor Gabriel Crainic
Antonio Sforza
Claudio Sterle

January 2011

CIRRELT-2011-07

Bureaux de Montréal :
Université de Montréal
C.P. 6128, succ. Centre-ville
Montréal (Québec)
Canada H3C 3J7
Téléphone : 514 343-7575
Télécopie : 514 343-7121

Bureaux de Québec :
Université Laval
2325, de la Terrasse, bureau 2642
Québec (Québec)
Canada G1V 0A6
Téléphone : 418 656-2073
Télécopie : 418 656-2624

www.cirrelt.ca

Université Laval · UQÀM · HEC Montréal · École Polytechnique Montréal · Université de Montréal

Tabu Search Heuristic for a Two-Echelon Location-Routing Problem

Teodor Gabriel Crainic[1,*], Antonio Sforza[2], Claudio Sterle[2]

[1] Interuniversity Research Centre on Enterprise Networks, Logistics and Transportation (CIRRELT) and Department of Management and Technology, Université du Québec à Montréal, P.O. Box 8888, Station Centre-Ville, Montréal, Canada H3C 3P8

[2] Universitá degli Studi di Napoli “Federico II”, Dipartimento di Informatica e Sistemistica, Via Claudio 21, Napoli, Italy

Abstract. In this paper we tackle a two-echelon location-routing problem (2E-LRP) by a tabu search metaheuristic. The aim is to define the location and number of two kinds of capacitated facilities, the size of two different vehicle fleets and the related routes on each echelon. The proposed Tabu Search (TS) for the 2E-LRP is based on the decomposition of the problem in two location-routing sub-problems, one for each echelon. Then each subproblem is decomposed into a capacitated facility location problem (CFLP) and a multidepot vehicle routing problem (MDVRP). The heuristic uses an iterative-nested approach to combine the solutions of the four subproblems. TS has been experienced on a wide set of small, medium and large instances and the obtained results have been compared with the ones of 2E-LRP models on small size instances and with upper bounds, obtained with a simple sequential approach, on medium and large size instances. Experimental results prove that proposed TS is effective in terms of quality of solutions and computation times in most of the solved instances.

Keywords. Tabu search, two-echelon location-routing.

Acknowledgements. Partial funding for this project has been provided by the Universitá degli Studi di Napoli “Federico II”, the Natural Sciences and Engineering Council of Canada (NSERC), through its Industrial Research Chair and Discovery Grants programs, by the partners of the Chair, CN, Rona, Alimentation Couche-Tard and the Ministry of Transportation of Québec.

---

* Corresponding author: TeodorGabriel.Crainic@cirrelt.ca

Dépôt légal – Bibliothèque et Archives nationales du Québec
Bibliothèque et Archives Canada, 2011

1 Introduction

The basic location-routing problem (*LRP*) or *single-echelon LRP* refers to a system composed by one kind of facilities (eventually of limited capacity) and final customers. It is aimed at determining the number and locations of the facilities over a set of feasible locations, the number of vehicles to use for the distribution and related routes, in order to satisfy the demand (of a single representative product) of the final customers at minimum total cost. Total cost of the system is given by three components: location, routing and vehicle costs.

In this paper we consider an extension of the basic *LRP* where the aim is to determine the locations and the number of two kinds of capacitated facilities (referred as *primary* and *secondary* facilities), the size of two different vehicle fleets and the related routes on each echelon, with the aim of satisfying demands of final customers at minimum total cost (location, routing and vehicle costs on the two echelons). Products flow from primary to secondary facilities and then from secondary facilities to final customers. This problem can be defined as a two-echelon location-routing problem *2E-LRP*, since we have location and routing decisions involving two echelons. At the best of our knowledge, even if several papers treat location-routing problems arising for two-echelon system, just few papers consider facility and routing decisions simultaneously on more than one level. Two-echelon location-routing problems arise in several contexts, as freight distribution in urban areas, express delivery service, large distribution to groceries and stores, etc., where products available at primary facilities pass through intermediate facilities before being delivered to final customers.

In the just cited applications, generally, the number of customers to be served is high. Hence, given the *2E-LRP* NP-hardness, the usage of heuristic methods is required to solve the problem on medium and large size instances. To this aim a Tabu Search (*TS*) metaheuristic has been proposed and implemented. This paper provides a comprehensive report of the *TS* algorithm proposed in the short paper by Boccia et al. (2010), explaining in more detail the features of the metaheuristics and providing additional results. It builds on the two-phase iterative approach, proposed by Tuzun and Burke (1999) and on the nested approach of Nagy and Salhy (1996), hence it can be defined as a *multi-phase iterative-nested approach*. The proposed *TS* decomposes the *2E-LRP* in two single-echelon location-routing sub-problems. Each sub-problem, in turn, is decomposed into a capacitated facility location problem (*CFLP*) and a multi-depot vehicle routing problem (*MDVRP*). The *TS* starts with an initial feasible solution and tries to improve it in two phases, *location* and *routing phases*, where simple moves on location and routing variables of the two-echelons are performed. Swap and add moves are performed for location variables, whereas swap and insert moves are considered for routing variables. Then sub-problems solutions are opportunely combined to obtain a solution which is globally good. Tabu Search heuristic has been tested on several sets of small, medium and large instances (up to 5 primary facilities, 20 secondary facilities and 200 customers).

The sets differ for the spatial distribution of secondary facilities. Each instance has been solved with different settings of the tabu search parameters. Results have been compared with bounds obtained solving a *2E-LRP* model on small size instances and using a simple sequential approach on medium and large size instances. The models used for the bounds are reported in appendix of the paper. The obtained results show that the proposed Tabu Search is able to find good solutions with limited computation time.

The paper is structured as follows. In section *2* a brief description of the tackled problem is provided. In section *3* a description of the solving approaches present in literature for single and multi-echelon *LRP* is presented. Then a wide literature review of *LRP* papers is provided. In section *4* the proposed tabu search heuristic is presented, focusing on its main features. Section *5* is devoted to computational results.

2 Problem statement

In this paper we tackle the basic *2E-LRP* problem proposed in Boccia et al.(2011) and arising in the context of two-echelon freight distribution system design. The system is composed by three interacting levels, linked by two or more vehicle fleets performing distribution:

1. *Primary facilities*: high capacitated facilities generally located far from final customers. At this locations, freight is broken and charged on first echelon vehicles, which perform distribution to successive levels.

2. *Secondary facilities*: low capacitated facilities devoted to transshipment operations. Freights arriving from primary facilities on first echelon vehicles are transferred on smaller vehicles (referred as *second echelon vehicles*), which perform the final distribution.

3. *Customers*: end points of distribution. Each customer is served by at least one vehicle coming from upper levels.

Given this structure, the *2E-LRP* consists in defining number and locations of primary and secondary facilities; assign customers to open secondary facilities and open secondary facilities to open primary ones, satisfying capacity constraints; define the number of two different vehicle fleet to use for the distribution on the two echelons and related routes. A three-index mixed-integer formulation for the *2E-LRP* has been proposed in Crainic et al.(2009) and is reported in appendix. The assumptions defining the basic *2E-LRP* can be summarized as follows:

- A single substitute product is considered (*single-commodity*). Demand of each customer is known in advance (*deterministic*) and referred to a *single planning period.*
- All freight starts at primary facilities. Distribution cannot be managed by direct shipping from primary facilities to final customers, but freight must be first consolidated at secondary facilities. In particular: 1^{st} echelon routes start from a primary facility, serve one or more secondary facilities and ends to the same primary facility; 2^{nd} echelon routes start from a secondary facility, serve one or more customers and ends to the same secondary facilities.
- Platforms and satellites are capacitated. Platform capacity is much higher than satellite capacity. Facilities belonging to the same echelon can have different capacity and location costs.
- Each secondary facility has to be served by a single primary facility and by a single first echelon vehicle. Each customer has to be served by a single secondary facility and by a single second echelon vehicle (single sourcing at both echelons).
- Vehicles operating in the same echelon have the same capacity value. Capacity of each first echelon vehicle is higher than capacity of each second echelon vehicle and of each secondary facility. Capacity of each first echelon vehicle is much higher than demand of each customers.
- No time windows and synchronization constraints are considered.

3 Literature review on *LRP*

The idea of combining location and routing decisions dates back to Maranzana (1964). Generalized *LRP* is a very hard problem since it arises from the combination of two NP-hard problems, facility location (*FLP*) and vehicle routing (*VRP*). NP-hardness of location-routing problems has been demonstrated in several papers, among which we cite Karp (1972) and Lenstra and Rinnooy Kan (1981). For this reason *LRP*s have been scarcely treated in literature if we compare with huge number of exact and heuristic approaches proposed for *FLP* and *VRP*.

Anyway in the last ten years, many researchers have tackled *LRP*s and related variants. *LRP* surveys have been proposed by Balakrishnan et al. (1987), Laporte (1988b), Laporte(1989) and Min et al. (1998). The most recent one is by Nagy and Salhi (2007), who provide a deeply focused discussion of problems and methods present in literature and future perspectives for *LRP*s. To the best of our knowledge the only exact methods for *LRP* dates back at the beginning of '80s and have been proposed by Laporte et al. (details in the following), whereas most of the recent literature is devoted to heuristic approaches, based on the decomposition of the problem in its three main subproblems, customer allocation, facility location and vehicle routing. In some cases location and allocation decisions are treated together. Subproblems are then solved by exact or heuristic methods and their solutions are opportunely combined.

In literature four basic heuristic approaches can be distinguished, differing in the way they combine subproblem solutions:

1. *Sequential approach*: a hierarchical relation between the two problems is considered. The main problem is the location and secondary one is the routing. The set of facilities to be opened is determined approximating routing costs by direct distance or by an estimator and then a *VRP* for each open facility or a *MDVRP* over all the open facilities is solved.

2. *Iterative approach*: the two components of the problem are considered as on equal term. They are solved iteratively exchanging information at each iteration, until a stopping criterion is verified.

3. *Nested approach*: a hierarchical structure of the general problem is recognized and *LRP* is considered as a location problem, where routing aspects have to be taken into account. The difference with the sequential approach is in the fact that in this case the routing sub-problem is solved for more solutions of the location problem.

4. *Clustering approach*: customers are clustered using proximity functions or solving a traveling salesman or minimum spanning tree problems (TSP or STP). Then a facility is located in each cluster and a capacitated vehicle routing problem for each depot is solved (the two steps can be also performed in opposite order). Generally in this approach the two problems are solved just once. Hence it can be seen as sequential approach where routing measures are used in the definition of clusters to have better results.

Iterative and nested approaches explore more combinations of location and routing solutions. This means that to evaluate a global *LRP* solution they have to evaluate several time routing cost components. This operation is the most demanding part from the computation time point of view. For this reasons they generally foresee mechanisms to minimize the number of times that routing component has to be solved.

In the following a literature review of *LRP* is provided. For each paper the related Laporte's notation (1988b), enhanced in Boccia et al.(2011), and the used solution approach are specified.

Laporte et al. (1981), Laporte et al. (1983), Laporte et al. (1986) and Laporte et al.(1988) are the only ones approaching $2/\overline{T}$ location-routing problems by exact methods.

In Laporte and Nobert (1981) a single depot has to be selected and a fixed number of vehicles has to be used. A branch-and-bound algorithm is proposed. The authors note that the optimal depot location rarely coincides with the node closest to the center of gravity. Laporte et al. (1983) consider the problem of locating several depots, with or without depot fixed costs and with or without an upper limit on the number of depots. For the special case with one vehicle for each depot, it was found to be more efficient to first reintroduce subtour elimination constraints and then use Gomory cuts to achieve integrality. Otherwise, the authors recommend using Gomory cuts first and then reintroducing subtour and chain barring constraints. On the other hand, the method of Laporte et al. (1986) applies a branching procedure where subtour elimination and chain barring constraints are reintroduced. Laporte et al. (1988) use a graph transformation to reformulate the LRP into a traveling salesman type problem. They apply a branch-and-bound algorithm, where in the search tree, each subproblem is a constrained assignment problem and can thus be solved efficiently.

Or and Pierskalla (1979) treat a $2/\overline{T}$ problem for the location of regional blood banking in the area of Chicago. They propose a non-linear integer programming model for the related LRP and a sequential algorithm, based on the decomposition of the problem in four sub-problems, opportunely merged and solved.

Jacobsen and Madsen (1980) and Madsen (1983) treat a $3/T/\overline{T}$ location-routing problem. The aim is to optimize the newspapers deliveries in Denmark. They solve two routing problem among three layers, and they locate facilities in the intermediate level. They propose three sequential heuristics for the problem. The Tree-Tour Heuristic (TTH), which exploits the property that by the deletion of an arc for each defined tour, the solution of the problem is a spanning tree with the characteristics that only first and second layer facilities have multiple successor; the ALA-SAV heuristic, which is a three stage sequential procedure composed of the Alternate Location Allocation model (ALA) and the Savings method (SAV); the SAV-$DROP$ heuristic, which is a three stage sequential heuristic composed of the Saving method (SAV) and the Drop method ($DROP$).

Perl and Daskin (1985) treat a $3/R/\overline{T}$ warehouse location-routing problem, with constraints on facility and vehicle capacity and on the maximum length route. They solve the problem decomposing it in its three sub-components which are solved by exact or heuristic methods in a sequential way. The three problems are: complete multi-depot vehicle-dispatch problem; warehouse location-allocation problem; multi-depot routing-allocation problem.

Srivastava and Benton (1990) and Srivastava (1993) present three heuristics for the $2/\overline{T}$ location-routing problem with capacitated vehicles. They propose three sequential heuristics. A "save drop" heuristic, where at each iteration they consider simultaneously dropping depots and assigning customers to routes developed from open depot. A "saving-add" heuristic, which is based on a similar scheme of the "save drop", but it opens the depots one by one, considering all the feasible sites closed at the beginning. A "cluster-routing" approach, which identifies the desired number of cluster and customers, and a depot is located in the site nearest to the centroid of each cluster. The routing in each cluster is achieved solving a traveling salesman problem (TSP) for a subset of customers, defined on their polar coordinates. They also perform a statistical analysis on the parameters affecting the solutions obtained with the three heuristics.

Chien (1993) propose a heuristic procedure for the $2/\overline{T}$ uncapacitated location-capacitated routing problems, i.e. capacitated vehicles and uncapacitated facilities. The heuristic is based on two sequential steps. In the first step a feasible solution to the location/allocation problem is generated, where the routing costs are evaluated through two different estimators. Then the routing problem is solved with the generated solution of the location/allocation problem. The improvement of the routing solution is then based on the use of four operations: consolidation/change of vehicle, insertions, swapping and change of facility. Different combinations of these operations are performed using the two estimators.

Hansen et al. (1994) extend the work of Perl and Daskin (1985) for a $2/\overline{T}$ problem. In fact they propose a modified formulation for the warehouse location routing problem presented by Perl and Daskin, and they use the same decomposition of the problem, but improving the results of each single component, and consequently the quality of the final solution.

Bruns and Klose (1996) propose a heuristic for a $3/T/\overline{T}$ *LRP* with limitations on the length of the routes. They used a location first-route second clustering iterative approach, where the costs to serve the customers are updated at each iteration. The location phase is solved with a Lagrangian relaxation, whereas the routing phase with a local search heuristic.

Nagy and Salhi (1996) treat the $2/\overline{T}$ location-routing problem with a nested heuristic. They propose an approach aimed at of avoiding the classical hierarchical decomposition location-first route-second. All customer sites are potential depot sites. The solution space consists of all possible combinations of customer sites. A first feasible solution is determined using a subset of the potential sites. For the location phase, the neighborhood structure is defined by the three moves add, drop and shift. Add means opening a closed depot, drop means closing an open depot and shift refers to the simultaneous opening of a closed depot and closing of an open depot. The most improving one is selected. For the routing phase customers are divided in two subsets: the nearest ones, which are directly assigned to an open depot to create the initial routes and the farthest ones, which are instead inserted in a route in function of the capacity constraints. The determined routes are the improved by a local search which include several tabu search features.

Tuzun and Burke (1999) propose a two-phase tabu search iterative-nested heuristic for the $2/\overline{T}$ location-routing problem with no capacity constraints on the depots. The heuristic starts with the opening of just one depot. Performing location and routing moves it tries to find the best solution to serve the customers with just one depot. Then when no improvement is obtained for a given number of iterations, it adds another depot to the location solution and repeat the same operations. After a given number of add moves without improvement, the heuristic stops. The proposed approach foresee several moves for the definition of the neighborhood solutions. More precisely for the location moves, add and swap moves are considered. Whereas for the routing phase, insert and swap moves of customers are taken into account. These moves are performed in an efficient way, avoiding to explore all the possible insertions or exchanges of customers. In fact insertion moves are limited to routes assigned to nearest depots and swap moves are limited to the nearest customers.

Wu et al. (2002) solve the $2/\overline{T}$ location-routing with capacity constraints for heterogeneous vehicles and depots. The problem is solved with a sequential metaheuristic approach. They first solve location-allocation problem and the general vehicle routing problem, then they are combined with a simulated annealing approach, integrated with "tabu list" concept, in order to prevent the cycling.

Lin et al. (2002) treat a $2/\overline{T}$ problem, where vehicles are allowed to take multiple trips by a sequential approach. First, the minimum number of facilities required is determined. Then, the vehicle routing solution is completely evaluated for all combinations of facilities. Vehicles are allocated to trips by completely evaluating all allocations. If the best routing cost found is more than the setup cost for an additional depot, the algorithm moves on to evaluating all sets of facilities that contain one more depot. The applicability of this method is limited as it relies on evaluating what may well be a large number of depot configurations.

Albareda-Sambola et al. (2005) solve a $2/\overline{T}$ location-routing problem where they have a single vehicle for each depot. They define an auxiliary compact formulation of the problem, which transform the problem in finding a set of paths in the auxiliary network that fulfill additional constraints. They propose upper and lower bounds and they solve the problem through a tabu search heuristic, based on an initial rounding procedure of the LP solution.

Melechovsky et al. (2005) propose for the first time a two-index formulation for the $2/\overline{T}$ location-routing problem, based on the two-index *VRP* formulation. They propose an iterative algorithm which starting from an initial feasible solution, searches for better solutions with a hybrid metaheuristic, which merge Variable Neighborhood Search (VNS) and Tabu Search (*TS*) principles. Therefore the key element of their approach is the integrated use of the two methods, which they realize replacing the local search procedure in the VNS framework with a Tabu Search algorithm.

Wang et al. (2005) propose a two-phase sequential hybrid heuristic for the $2/\overline{T}$ location-routing problem. They decompose the problem in the location/allocation phase and routing phase. In the first phase a tabu search is performed on the location variables to determine a good configuration of facilities to be used in the distribution. In the second phase ant colony algorithm is run on the routing variables in order to obtain a good routing for the given configuration. In the second phase, the routing problem is also decomposed in smaller sub-problems.

Ambrosino and Scutellá (2005) study a complex distribution network design problem $4/R/\overline{T}/\overline{T}$. They consider a problem where two different kinds of facilities have to be located in hierarchically ordered layers. The products are delivered from the first layer to the second one with R trips and different vehicles perform the distribution of products among second layer and third layer, and third layer and customers on T trips. Therefore, differently from the previous treated problems, they have to solve two-location routing problems. They propose different formulations for the problem in static ha and dynamic scenarios, extending the three-index arc formulations proposed by Perl and Daskin (1985) and the three index flow formulation proposed by Hansen et al. (1994). They solve the problem on small instances with a general optimization software.

Prins et al. (2006) solve the $2/\overline{T}$ capacitated location problem with a GRASP approach integrated with learning process and path relinking. They use a two index formulation for the problem, which differs from the one used by Melechovsky et al. (2005) for the definition of the arc-variables. Their approach is based on two phases. A first phase executes a GRASP based on an extended and randomized version of Clarke and Wright algorithm. This phase is implemented with a learning process on the choice of depots. In a second phase, new solutions are generated by a post-optimization using path relinking.

Barreto et al. (2007) propose a sequential clustering distribution-first and location-second heuristic for the $2/\overline{T}$ problem. The method is based on customer clustering. They perform a huge experimentation with seven different proximity measures.

Chen and Ting (2007) propose a three phase sequential heuristic approach for the $2/\overline{T}$ multi-depot location-routing problem. They start solving the location/allocation problem through a Lagrangian heuristic, then they solve a *VRP* for each selected facility location through a simulated annealing procedure (route construction), and finally they run the simulated annealing for all the routes.

Özyurt and Aksen (2007) propose a nested Lagrangian relaxation-based method for the $2/\overline{T}$ uncapacitated multi-depot location-routing problem. They consider the possibility of opening new facilities or closing existing ones. The problem is decomposed in two subproblems. The first is solved exactly by a commercial MIP solver, and the second resembles a capacitated and degree constrained minimum spanning forest problem, which is tackled with an augmented Lagrangian relaxation. The solution of the first subproblem reveals a depot location plan. As soon as a new distinct location plan is found in the course of the subgradient iterations, a tabu search algorithm is triggered to solve the multi-depot vehicle routing problem associated with that plan, and a feasible solution to the parent problem is obtained. Its objective value is checked against the current upper bound on the parent problems true optimal objective value.

Prins et al. (2007) propose a cooperative iterative metaheuristic to solve a $2/\overline{T}$ problem with capacitated routes and depots. The approach is composed of two phases. In the first phase a Lagrangean relaxation is proposed for the main facility-location problem. In the second phase, the routes from the resulting multidepot vehicle-routing problem (VRP) are improved using a granular tabu search (GTS) heuristic. The two phases exchange information about most often used edges. The method is evaluated on three sets of randomly generated instances and compared with other heuristics and a lower bound.

Ambrosino et al. (2009) deal with a $2/T/\overline{T}$ problem arising in distribution network design problem, involving location, fleet assignment and routing decisions. The system under investigation is characterized by one central depot, a set of customers split into regions, and a heterogeneous fleet of vehicles. The aim is to locate one regional depot in each region, to assign some vehicles to each region, and to design the vehicles routes, each starting and ending at the central depot satisfying capacity constraints and minimizing the overall distribution cost is minimized. They propose a two-phase heuristic for this problem which first determines an initial feasible solution, and then improves it by using very large neighborhood search techniques, based on path and cyclic exchanges of customers among routes. Then they use a classic relocation mechanism to perform depot location adjustments. The heuristic has been experienced on instances of different size and provide very good quality solutions in a limited amount of time.

Salhi and Nagy (2009) tackles a $2/\overline{T}$ problem on a plane by an iterative heuristic. They highlight the benefits in considering routing aspects when solving continuous location problems and suggest possible research avenues on the topic.

Boccia et al. (2010) tackles a $2/\overline{T}/\overline{T}$ by a multi-phase iterative-nested tabu search. The aim is to define the structure of a system optimizing the location and the number of two different kinds of facilities, the size of two different vehicle fleets and the related routes on each echelon. The proposed tabu-search heuristic is based on the decomposition of the whole problem in four subproblems, one FLP and one VRP for each echelon. Tabu Search has been experienced on three set of instances of varying dimensions and the obtained results have been compared with the available bounds. Experimental results prove that the proposed TS is effective in terms of quality of solutions and computation times in most of the solved instances.

Duhamel et al. (2010) tackle a capacitated $2/\overline{T}$ problem by a iterative metaheuristic. The proposed solution method is a greedy randomized adaptive search procedure (*GRASP*), calling an evolutionary local search (*ELS*) and searching within two solution spaces: giant tours without trip delimiters and true *CLRP* solutions. Giant tours are evaluated via a splitting procedure that minimizes the total cost subject to vehicle capacity, fleet size and depot capacities. The approach is evaluated on benchmark instances present in literature and show that the approach outperforms all previously published methods and provides numerous new best solutions.

Boccia et al. (2011) provide a modeling framework for the $2/\overline{T}/\overline{T}$ problem arising in context of multi-echelon freight distribution system design. They present the main features of the system and define the basic two-echelon location-routing problem (*2E-LRP*). Three mixed-integer programming models are proposed extending and/or adapting classical *VRP* and *MDVRP* formulations present in literature. Then an instance generator is presented and results of proposed models on a wide set of small and medium instances are reported, comparing their performances in terms of quality of solution and computation times.

From the previous literature review, we can say that most of *LRP* literature is devoted to single-echelon case, where locations of just one kind of facilities and routing on a single echelon have to be determined. On the contrary, literature about multi-echelon *LRP* and in particular about $2/\overline{T}/\overline{T}$ is very limited. The only contributions on this topic are the ones of Ambrosino and Scutellá (2005), Boccia et al. (2010) and Boccia et al. (2011).

4 A tabu search heuristic for *2E-LRP*

The presented *TS* heuristic is based on the integration of the nested approach of Nagy and Salhy (1996) and the two-phase iterative approach of Tuzun and Burke (1999). Hence it can be defined as an "*iterative-nested approach*". The problem is decomposed in its two main components, i.e. two location-routing problems. Each component, in turn, is decomposed in a capacitated facility location problem (*CFLP*) and a multi-depot vehicle routing problem (*MDVRP*). A bottom-up approach is used, i.e. first echelon solution is built and optimized on second echelon solution. *TS* operates on each echelon in two coordinate and integrated phases (*location and routing phases*).

The heuristic is structured in two phase for each echelon. Indeed it starts with an initial feasible solution and try to improve it performing the following phases:

1. Location phase: a tabu search is performed on the location variables in order to determine a good configuration of facility locations. The passage from a configuration to another is obtained through the usage of add and swap moves. The two moves are performed sequentially, first swap moves and then add moves. The swap

moves keep the number of facilities unchanged but locations change. They are performed until a maximum number is reached. Then an add move is performed, until a stopping criterion is satisfied.

2. Routing phase: for each location solution determined during the location-phase, a tabu search is performed on the routing variables. The initial routes are built with Clarke and Wright algorithm and then improved by local searches. Finally a tabu search based on insert and swap moves is performed.

The key element of the heuristic resides in the combination and integration of the location and routing solutions on each echelon and of the location-routing solutions of the two echelons, in order to obtain a solution that is globally good. Concerning the combination of single echelon solutions, in the location phase of the algorithm, a *TS* is performed on the location variables, starting from the configuration with the minimum number of open facilities. For each location configurations, another *TS* is run on the routing variables in order to obtain a good routing for the given configuration. Therefore each time a move is performed on the location phase, the routing phase is started in order to update the routing according to the new configuration. Concerning instead the combination of location and routing solutions of the two echelons, each time a change of the demand assigned to a set of open secondary facilities occurs and a pre-defined condition on facility and vehicle capacity is satisfied, then the location-routing problem of the first echelon should be re-solved in order to find the best location and routing solution to serve the new demand of the secondary facilities.

TS has been applied to both the facility location type problems and various forms of the *VRP*, and the results are encouraging. This suggests that *TS* can be used to solve the *LRP* which combines the two problems in an integrated model.We use random tabu tenure values because randomness generally guarantee better solutions.

In the following the main issues of the proposes tabu search heuristic will be described: initialization and evaluation criteria; location and routing moves, tabu attributes and stopping criteria; combination of subproblems solutions; diversification criteria.

4.1 First feasible solution and evaluation criteria

The *TS* starts with a fast heuristic for the construction of a first feasible solution of the two-echelon capacitated facility location problem (*2E-FLP*), where no routes are defined on the two echelons but each customer and each secondary facilities are served on dedicated routes. It starts with the definition of a feasible solution for the location and allocation problem on the second echelon and then it repeats the same operations on the first echelon.

The aim is to open the minimum number of facilities on both echelons. At first secondary facilities are sorted in function of their capacity. Then the minimum number of them (S^*), able to satisfy the total demand of the customers, is opened. In particular we impose that the total capacity of open secondary facilities, decreased of a given percentage α ($\alpha \in [90\% \div 95\%]$), has to exceed the total customer demand. This condition, together with the assumption that demand values are much smaller than facility capacities, should guarantee that a feasible assignment of the customers to the secondary facilities could be determined. Then customers are sorted in decreasing order of their demand and are assigned to secondary facilities considering three different rules:

1. *random*: each customer is assigned randomly to one of the open secondary facilities;
2. *min-distance*: each customer is assigned to the nearest open secondary facility with enough residual capacity;
3. *residual capacity*: each customer is assigned to the secondary facility with higher residual capacity value.

Once determined the demand assigned to each secondary facility, i.e. the sum of the demands of all customers assigned to the facility, the same procedure is repeated to find the minimum number (P^*) of primary facilities to open.

The application of this simple heuristic to both echelons returns a solution for the *2E-FLP*, whose structure is reported in Figure 1:

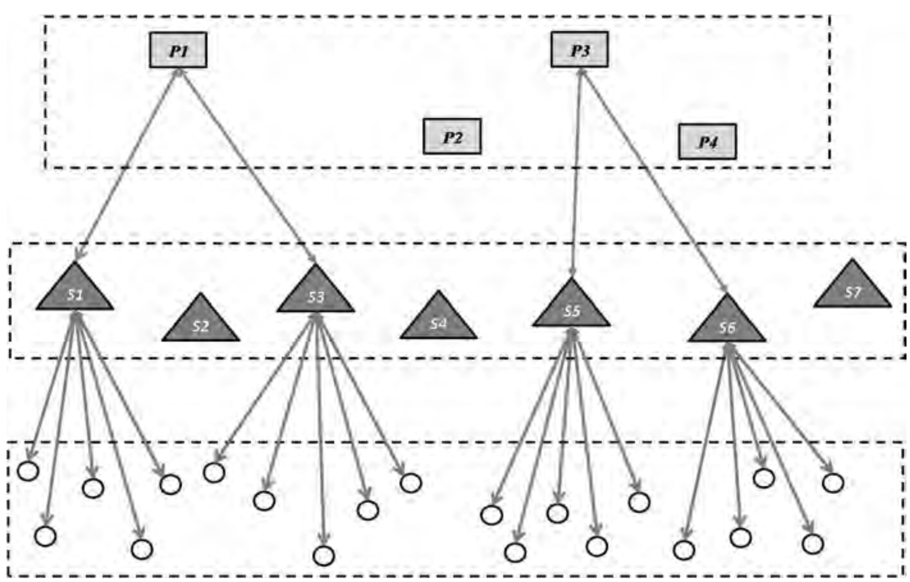

Figure 1: First feasible solution.

Two ways to evaluate a solution during the tabu search are considered: *estimated cost* and *actual cost*. Both are given by the sum of two components, location and routing (including also vehicle costs) on the two echelons. The difference is in the way routing costs are computed. In estimated costs, the routing component is approximated with the double of direct distances among the nodes, whereas, in the actual cost, the routing component is given by the sum of transportation costs of each route. The estimated cost is used to evaluate the first feasible solution and the goodness of a location move, whereas the actual cost is computed each time a routing move is performed.

4.2 Solution neighborhood definition

In the following, the *TS* moves will be presented. For sake of clarity location and routing moves and the related parameters will be presented separately and then we will focus on the combination of sub-problem solutions. Moreover for sake of brevity the moves will be presented referring just to the second echelon, since the extension to first echelon is straightforward. We will use the notation P and S for the set of all possible primary and secondary facility locations and respectively with P^* and S^* for the set of open primary and secondary facilities at a given iteration of the *TS*.

4.3 Location moves

The location phase of the heuristic has to define a good location configuration for the two echelons and affects the solution in terms of number and location of the facilities. Two simple moves are performed: *swap* and *add* moves. The two moves are applied sequentially and iteratively on each configuration. More precisely, for a given number of open facilities, we try to change the configuration of the solution by swap moves, then, when no improvements are obtained and a stopping criterion is met, we increase the number of facilities with an add move and repeat swap moves for the increased number of open facilities.

Swap moves. With this move the status of two facilities is exchanged, i.e. an open facility is closed and a closed facility is opened. Hence the number of the open facilities is kept constant. The key element of these moves is the selection of the facilities to be swapped. With reference to second echelon, the secondary facility to be removed from solution set S^* is chosen with one of the following criteria:

1. *Rand-sel-out*: random selection of a facility belonging to the solution set S^*;
2. *Max-loc*: facility of S^* associated with the highest location cost.

3. *Max-cost*: facility of S^* associated with the highest location and routing cost (weighted with the number of served customers):

4. *Max-route*: facility of S^* associated with the highest cost for a single route.

The set of possible secondary facilities to be opened is defined considering just the ones able to satisfy the total demand of the customers. Once determined the set of candidate facilities, the entering one is selected by two possible criteria:

1. *Rand-sel-in*: random selection of a facility in the candidate set.

2. *Min-cost*: introduction in the solution of the facility associated with the minimum estimated total cost

A move is performed only if it is not tabu (no aspiration criteria are defined). Then the two facilities are declared *tabu* for a number of iteration depending on the number of open facilities. More precisely tabu tenure, *tabu-swap-loc-s*, for secondary facilities location moves is:

$$tabu - swap - loc - s == \alpha \left| S^* \right|, \alpha \in [\alpha_{min} \div \alpha_{max}]$$

Swap moves are performed until a max number of not-improving iterations, *max-swap-loc-s* is met.

Concerning swap moves for primary facilities, the same criteria defined for secondary facilities are used. Hence tabu tenure values is *tabu-swap-loc-p* $= \alpha \left| P^* \right|$, $\alpha \in [\alpha_{min} \div \alpha_{max}]$ and *max-swap-loc-p* is the fixed number of allowed iterations without improvement.

Add move. Once the maximum number of swap moves without improvement is met, we perform an add move, i.e. we increase the number of open facilities. The increased number of open facilities could provide a reduction of the transportation costs, which overcomes the additional location cost and moreover allows to open smaller facilities, characterized by lower location costs.

The node to be added to solution set is the one associated with the minimum estimated total cost. A move is performed just if it is not tabu. The added node is declared tabu for a number of iterations depending on the overall number of available locations of primary and secondary facilities ($|P|$ and $|S|$). In particular tabu tenures are *tabu-add-loc-s* $= \alpha \left| S \right|$ for secondary facilities and *tabu-add-loc-p* $= \alpha \left| P \right|$ for primary ones, $\alpha \in [\alpha_{min} \div \alpha_{max}]$. Add moves are performed until max number of not-improving iterations, $max - add - loc - s$ and $max - add - loc - p$, respectively for satellites and platforms, are met.

4.4 Routing moves

Starting from a first feasible assignment solution, we perform sequentially several operations to improve the routing cost component, acting locally on each route and then on multiple routes. These operations are performed in three steps:

1. Define and improve multi-stop routes: sequential application of Clarke and Wright (C&W) algorithm and 2-opt/3-opt algorithms.
2. Optimize *multiple* routes assigned to a *single* facility: insert and swap moves.
3. Optimize *multiple* routes assigned to *multiple* facilities: insert and swap moves.

The three phases have different effects on the global solution. Indeed first and second phases do not affect the assignment of customer to secondary facilities and consequently first echelon routing cost does not change. On the contrary, third phase can provide significant changes of demand assigned to secondary facilities and therefore the assignment of secondary facilities to primary ones can change, affecting also first echelon routing cost. The used approach to face this issue will be explained in next section.

We focus on the explanation of second and third phases. The main issue for the used moves is the neighborhood definition. Three selection criteria are used to restrict the sizes of neighborhoods:

1. *One-select*: select one node and evaluate the related neighborhood;
2. *Perc-sel*: select a percentage of all the nodes composing an echelon and evaluate the related neighborhoods;
3. *Path-select*: select randomly a path and evaluate for all the nodes the related neighborhood.

For second phase, the nodes to be selected are the ones assigned to a single facility. Instead, for third phase, the nodes to be selected are the ones composing an echelon. For both phases a simple aspiration criterion is used, i.e. a move is performed, even if tabu, but it provides an improvement of the best solution.

4.4.1 Inter-route improvements for a single facility.

These moves are feasible only if vehicle capacity constraints are satisfied and unfeasible moves are not allowed (no penalty criteria have been defined). Once performed an insert or swap move, a local search is run to re-optimise locally the routes of the involved facility, performing 2-opt and 3-opt moves.

Insert move. a customer is deleted from one route and is assigned to another route belonging to the same facility. The neighborhood is defined evaluating the insertions of selected customer (customers) in all routes assigned to the facility under investigation. If a neighbor solution provides an improvement, then the move is performed and added node is declared tabu, otherwise we choose the best not-tabu deteriorating move and added node is declared tabu. A tabu move is performed if it provides an improvement of the solution (aspiration criterion).

Tabu tenure value, $tabu-r-ins-single-s$ depends on the number of nodes assigned to a facility. Being Z_s the total number of customers assigned to a secondary facility, the value is computed with the following relation:

$$tabu - r - ins - single - s = \lceil \alpha \; Z_s \rceil \, , \alpha \in [\alpha_{min} \div \alpha_{max}] \tag{1}$$

These moves are performed until max number of not-improving moves, *max-r-ins-single*, is met.

The extension to first echelon is straightforward. Being S_p the number of satellites assigned to a primary facility , *tabu-r-ins-single-p* $= \lceil \alpha \; S_p \rceil$, $\alpha \in [\alpha_{min} \div \alpha_{max}]$.

It is important to underline that in the evaluation of the routing costs deriving from an insert move, we consider also the possibility that if we have a route with a single customer, then its insertion in another route provides a saving equal to the cost of a vehicle. Therefore in this way we could also obtain a minimization of the number of used vehicles.

Swap moves. The positions of two customers belonging to two routes assigned to the same facility are exchanged. The neighborhood is defined evaluating the exchanges of the selected customer (customers) with all the customers assigned to the facility under investigation. If the move is not-tabu and it provides a saving on the routing cost, it is performed. Otherwise we perform the best not-tabu deteriorating move. Then customers are both declared tabu. Tabu tenure values for first and echelon are computed with the same relation introduced for insert moves, i.e. *tabu-r-swap-single-s* $= \lceil \alpha \; Z_s \rceil$ and *tabu-r-swap-single-p* $= \lceil \alpha \; S_p \rceil$. These moves are performed until the max number, *max-r-swap-single*, of not-improving moves is met.

4.4.2 Inter-route improvements for multiple facilities.

The moves presented for a single facility are extended to multiple facilities. In this case, a move to be feasible has to satisfy capacity constraints for both facilities and vehicles and unfeasible moves are not allowed (no penalty criteria have been defined). Once performed an insert or swap move, a local search is run to re-optimise locally the routes of the involved facilities, performing 2-opt and 3-opt moves and insert and swap moves for a single facility.

Insert move. A customer is deleted from its route and inserted in another route belonging to another open facility. The move can be performed if and only if the facility and the vehicle to which the customer will be re-assigned have still enough residual capacity. Therefore no dedicated route can be used to serve the inserted customer. If a move is not-tabu and it provides an improvement, it is performed, otherwise, the best deteriorating not-tabu one is performed. A tabu move is performed if it provides an improvement of the solution (aspiration criterion).

In this case the neighborhood of a solution has to be restricted. Indeed considering all the possible insertions would provide and exponential growth of computation time of routing phase and would not be effective. Hence the neighborhood is defined considering the insertions of the selected customer (customers) in all the routes belonging to the "closest" open facility. The *closest secondary facilities*, *near-ins-s*, are a percentage of the number of open facilities and is computed as follows: *near-ins-s* $= \lceil \beta \ S^* \rceil$, $\beta \in [0 \div 1]$.

Tabu tenure value for this move depends on the total number of customers Z:

$$tabu - r - ins - single - s = \lceil \alpha \ Z \rceil \,, \alpha \in [\alpha_{min} \div \alpha_{max}]$$

The same relations can be extended to the first echelon for which we have *near-ins-p* $= \lceil \beta \ P^* \rceil$, $\beta \in [0, 1]$ and, being S the total number of secondary facility locations, *tabu-r-ins-multi-p* $= \lceil \alpha \ S \rceil$, $\alpha \in [\alpha_{min} \div \alpha_{max}]$. This move is performed until the max number, $max - r - ins - multi$ of not-improving moves is reached.

Swap move. The position of two nodes belonging to routes assigned to different facilities are exchanged. The neighborhood is defined considering the exchanges of the selected customer (customers) with all the "closest" customers. If a move is not tabu and it provides an improvement, then it is performed, otherwise the best deteriorating not-tabu move is performed. A tabu move is performed if it provides an improvement of the solution (aspiration criterion).

Also in this case it is important to efficiently define the neighborhood of a solution. Therefore we restrict our search trying to swap two nodes just if they are "close". The *closest customers*, *near-swap-s*, are a percentage of all customers and is computed as follows: *near-swap-s* $= \lceil \beta\ Z \rceil$, $\beta \in [0 \div 1]$.

Tabu tenure value for this move is given by the same relation introduced for insertion move, i.e.*tabu-r-swap-multi-s* $= \lceil \alpha\ Z \rceil$, $\alpha \in [\alpha_{min} \div \alpha_{max}]$.

The same relations are extended to the first echelon, for which we have *near-swap-p* $= \lceil \beta\ S^* \rceil$, ,$\beta \in [0 \div 1]$ and *tabu-r-swap-multi-p* $= \lceil \alpha\ S^* \rceil$, $\alpha \in [\alpha_{min} \div \alpha_{max}]$. This move is performed until the max number, *max-r-swap-multi* of not-improving moves is met.

4.5 Combining sub-problems: The *TS* algorithm

The application of the previous location and routing moves on each echelon locally optimise the four sub-components of the *2E-LRP*. At this point the key element is the mechanism to combine location and routing solutions on each echelon and location-routing solutions of the two echelons. The four sub-problems are solved separately, but not in a pure sequential way. Indeed our approach foresees their resolution several times, in order to explore different location-routing solution combinations of first and second echelon.

In particular, concerning a single echelon, the idea proposed in Tuzun and Burke (1999) is adopted, i.e. each time a move is performed in the location phase, then the routing phase is run for the new location configuration in order to obtain better routes on the two echelons. The used mechanism is shown in Figure 2)

Concerning instead the two echelons, each time a change of the demand assigned to a set of open secondary facilities occurs, i.e. each time a routing move for multiple secondary facilities is performed, then the location-routing problem of the first echelon could be re-solved in order to find the best location and routing solution to serve the new demand configuration. Three criteria have been defined to control the return on the first echelon:

1. *Always*: return to first echelon each time an intra satellite move is performed on the second echelon.

2. *Imp-CR2*: each time a better solution for the second echelon routing problem has been determined.

3. *Violated-cap*: each time an improvement of second echelon routing cost is obtained and capacity constraints of the best determined first echelon routing solution are violated by the new demand configuration.

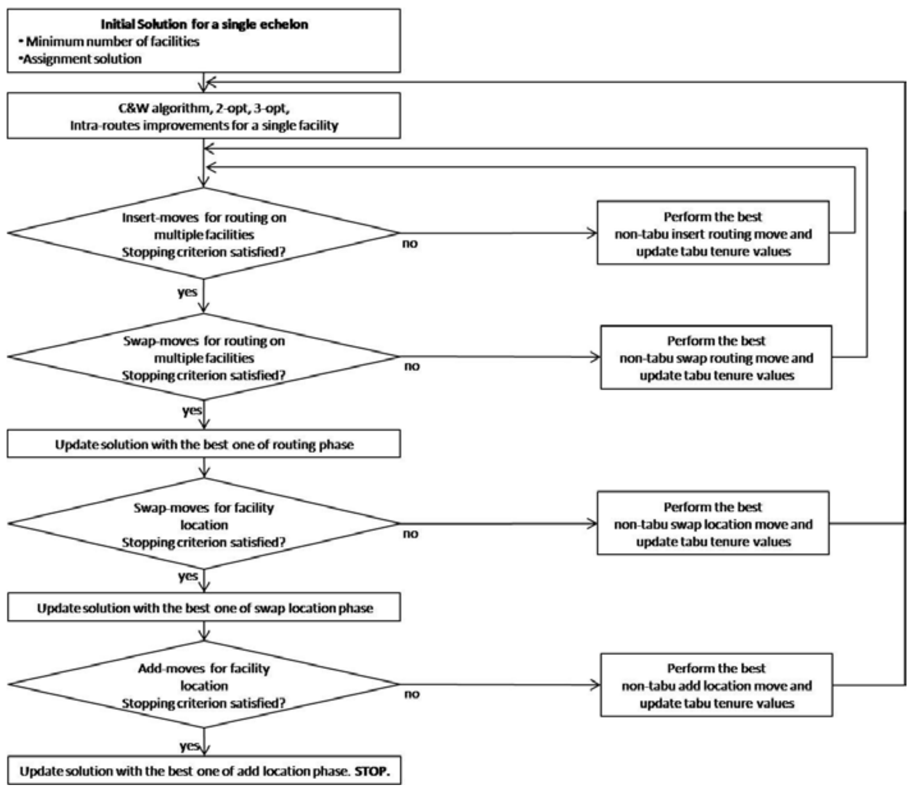

Figure 2: Combining sub-problems on a single-echelon.

The main steps of the Tabu Search to combine the four sub-problems can be summarized as follows and are sketched in Figure 3:

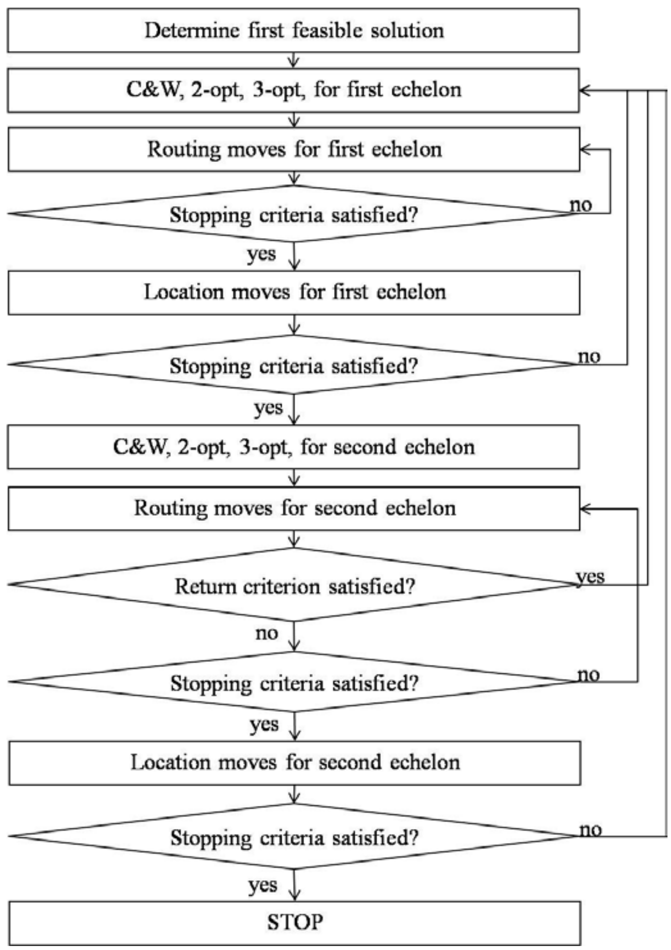

Figure 3: TS scheme

- *Step 0*: determine the first feasible solution.
- *Step 1*: define and optimize multi-stop routes on first echelon with C&W, 2-opt and 3-opt algorithms and go to *Step 2*.
- *Step 2*: perform sequentially insert and swap moves for a single platform. If max number of not improving moves is met, then update solution with the best determined one and go to *Step 4*. Otherwise repeat *Step 2*.
- *Step 3*: perform sequentially insert and swap moves for multiple platforms. If max number of not improving moves is met, then update solution with the best determined one and go to *Step 4*. Otherwise repeat *Step 3*.

- *Step 4*: perform sequentially swap and add location moves for first echelon and return to *Step 2*. If max number of not improving moves is met, then update solution with the best determined one and go to *Step 5*. Otherwise repeat *Step 4*.
- *Step 5*: define and optimize multi-stop routes on the second echelon with C&W, 2-opt and 3-opt algorithms and go to *Step 6*.
- *Step 6*: perform sequentially insert and swap moves for a single open satellite. If max number of not improving moves is met, then update solution with the best determined one and go to *Step 7*. Otherwise repeat *Step 6*.
- *Step 7*: perform sequentially insert and swap moves for multiple satellites. If one of the criteria *Imp-CR2* or *Violated-cap* is satisfied, return to *Step 1*, otherwise repeat *Step 7*. If max number of not improving moves is met, then update the solution with the best determined one and go to *Step 8*.
- *Step 8*: perform sequentially swap and add location moves for the second echelon and return to *Step 1*. If max number of not improving moves is met, then update solution with the best determined one and *STOP*. Otherwise return to *Step 1*.

4.6 Diversification criteria

A simple diversification criterion has been defined in order to better explore the solution space of the *2E-LRP*. The diversification is applied on the location variables of both echelons during the swap moves of the location phase. The criterion works as follows. When a prefixed number of swap location moves without improvement is reached, $div-val-s$ and $div-val-p$ respectively for secondary and primary facilities, we force a change in the set of open facilities on both echelons. We close the facilities which appear more frequently in the explored solutions and we open the least used ones. It is important to note that $div-val-s$ has to be lower than $max-swap-locs$ and $div-val-p$ lower than $max-swap-loc-p$, otherwise the diversification will not be performed.

5 Computational results of *TS* for *2E-LRP*

In this section the results of the Tabu Search heuristic on three sets of random generated instances are presented. The three sets differ for the spatial distribution of secondary facilities. For each instance set, different combinations of numbers of customers, secondary and primary facilities were used:

- Customers: $\{8, 9, 10, 12, 15, 20, 25, 50, 75, 100, 150, 200\}$;

- Secondary facilities: $\{3, 4, 5, 8, 10, 15, 20\}$;
- Primary facilities: $\{2, 3, 4, 5\}$

Customer demands were randomly generated in the range $[1, 100]$. Facility and vehicle capacity values and related costs vary with the size of the instances:

- secondary facility capacity: for instances up to 100 customers, their capacity is randomly generated in the range $[300, 600]$, whereas for instances with 150 and 200 customers their capacity is randomly generated in the range $[800, 2500]$. Location costs vary in the range $[40, 80]$.
- primary facility capacity: for instances up to 100 customers, their capacity is randomly generated in the range $[1000, 2000]$, whereas for instances with 150 and 200 customers their capacity is randomly generated in the range $[4000, 8000]$. Location costs vary in the range $[150, 250]$.
- first echelon vehicle capacity: for instances up to 25 customers, their capacity is equal to *500*; for instances up to 100 customers their capacity is equal to *1500*; for instances up to 200 customers, their capacity is equal to *3000*. No cost is considered for the usage of a vehicle.
- second echelon vehicle capacity: for instances up to 25 customers, their capacity is equal to *100*; for instances up to 100 customers their capacity is equal to *200*; for instances up to 200 customers, their capacity is equal to *500*. No cost is considered for the usage of a vehicle.

In the following, instances are indicated with a notation composed by instance set name and cardinality of $\boldsymbol{P}$, $\boldsymbol{S}$ and $\boldsymbol{Z}$. For example $I1/2-8-20$ stands for an instance of set I1 with 2 primary facilities, 8 secondary facilities and 20 customers.

TS heuristic was developed in C++ and instances were run on an Intel(R) Pentium(R) 4(2.40 GHz, RAM 4.00 GB). At first results of four different *TS* settings for each instance will be presented and compared in terms of quality of solution and computation time. Then the *TS* results have been compared with available bounds obtained by Xpress-MP. In particular for small instances, *TS* results have been compared with those obtained by the three-index formulation proposed in Boccia et al. (2011) within 2 hours computation time. For medium and large instances, the comparison has been done with the results obtained with a decomposition approach, sequentially solving one *2E-FLP* and two *MDVRP* (one for each echelon).

5.1 *TS* settings

TS heuristics require an important tuning phase for the parameters in order to be effective. The number of parameters of the proposed *TS* is huge and they are summarized in Tables 1 and 2.In the following we will not report the results obtained with all the experienced parameter settings of the *TS*, but we will concentrate on four of them which provided good results in terms of quality of solutions and computation times. The four considered *TS* settings will be respectively referred as *TS1*, *TS2*, *TS3*, *TS4*.

These settings differ for the size of the explored neighborhood determined by the choice of tabu tenure values reported in Tables 3 and 4.

Rand-sel-out	random selection of the node to be swapped
Max-cost	selection of the node to be swapped associated to max estimated cost
Max-route	selection of the node to be swapped associated to max route cost
Max-loc	selection of the node to be swapped associated to max location cost
Rand-sel-in	random selection of the entering node
Min-cost	selection of the entering node associated to min estimated cost
One-select	neighborhood of a single randomly selected node
Path-select	neighborhood of the nodes of a single randomly selected path
Perc-sel	neighborhood of a percentage of the total number of nodes for each echelon
Always	return on the first echelon every time a routing move is performed on second echelon
Imp-CR2	return on the first echelon just if an improvement of second echelon routing is found
Violated-cap	return on the first echelon just if an improvement of second echelon routing is found and capacity constraints are violated

Table 1: Tabu Search criteria.

TS results for three set of instances of varying dimensions are reported in Tables 5, 6, 7. For each setting the related best solution values (*TS1*, *TS2*, *TS3*, *TS4*) and computation times in seconds (*CPU-1*, *CPU-2*, *CPU-3*, *CPU-4*) are reported.

From Tables 5, 6, 7 we can observe that from *setting 1* to *setting 4* results are characterized by increasing computation time and increasing quality of solutions. Results of *setting 2* and *3* are very similar, whereas results of *setting 1* and *4* present opposite characteristics in terms of quality of solutions and computation times. In any case computation times are lower than *3600* seconds and just for instances with more than 150 customers they increase until about *7200* seconds.

In the following we will concentrate on *Setting 1* and *Setting 4* to evaluate the goodness of the *TS* in the worst and the best case.

tabu-swap-loc-s	tabu tenure value for satellite swap moves $\alpha\,\lvert S^*\rvert\,, [\alpha_{min} \div \alpha_{max}]$
max-swap-loc-s	maximum number of iterations without improvement for satellite swap move
tabu-swap-loc-p	tabu tenure value for platform swap moves $\alpha\,\lvert P^*\rvert\,, [\alpha_{min} \div \alpha_{max}]$
max-swap-loc-p	maximum number of iterations without improvement for platform swap move
tabu-add-loc-s	tabu tenure value for satellite add moves $\alpha\,\lvert S*\rvert\,, [\alpha_{min} \div \alpha_{max}]$
max-add-loc-s	maximum number of iterations without improvement for satellite add move
tabu-add-loc-p	tabu tenure value for platform add moves $\alpha\,\lvert P*\rvert\,, [\alpha_{min} \div \alpha_{max}]$
max-add-loc-p	maximum number of iterations without improvement for satellite add move
tabu-r-ins-single-s	tabu tenure value for single satellite routing insertion moves $\lceil \alpha\ Z_s \rceil\,, [\alpha_{min} \div \alpha_{max}]$
tabu-r-ins-single-p	tabu tenure value for single platform routing insertion moves $\lceil \alpha\ S_p \rceil\,, [\alpha_{min} \div \alpha_{max}]$
max-r-ins-single	maximum number of iteration without improvement for single facility routing insert moves
tabu-r-swap-single-s	tabu tenure value for single satellite routing swap moves $\lceil \alpha\ Z_s \rceil\,, [\alpha_{min} \div \alpha_{max}]$
tabu-r-swap-single-p	tabu tenure value for single platform routing swap moves $\lceil \alpha\ S_p \rceil\,, [\alpha_{min} \div \alpha_{max}]$
max-r-swap-single	maximum number of iteration without improvement for single facility routing swap moves
near-ins-s	percentage of all the open satellites for insert routing moves for multiple facilities $\lceil \beta\ S^* \rceil\,, \beta \in [0 \div 1]$
tabu-r-ins-multi-s	tabu tenure value for multiple satellites routing insertion moves $\lceil \alpha\ Z \rceil\,, [\alpha_{min} \div \alpha_{max}]$
near-ins-p	percentage of all the open platforms for insert routing moves for multiple facilities $\lceil \beta\ P^* \rceil\,, \beta \in [0 \div 1]$
tabu-r-ins-multi-p	tabu tenure value for multiple satellites routing insertion moves $\lceil \alpha\ Z \rceil\,, [\alpha_{min} \div \alpha_{max}]$
max-r-ins-multi	maximum number of insertion moves for multiple facilities
near-swap-s	percentage of all the open satellites for insert routing moves for multiple facilities $\lceil \beta\ Z \rceil\,, \beta \in [0 \div 1]$
tabu-r-swap-multi-s	tabu tenure value for multiple satellites routing swap moves $\lceil \alpha\ Z \rceil\,, [\alpha_{min} \div \alpha_{max}]$
near-swap-p	percentage of all the open satellites for swap routing moves for multiple facilities $\lceil \beta\ S \rceil\,, \beta \in [0 \div 1]$
tabu-r-swap-multi-p	tabu tenure value for multiple platforms routing swap moves $\lceil \alpha\ S \rceil\,, [\alpha_{min} \div \alpha_{max}]$
max-r-swap-multi	maximum number of insertion moves for multiple facilities
div-val-s	diversification criterion for satellites
div-val-p	diversification criterion for satellites
max-freq-s	max frequency value in diversification for satellites
max-freq-p	max frequency value in diversification for satellites

Table 2: Tabu Search parameters.

TS set 1		TS set 2	
Rand-sel-out	true	*Rand-sel-out*	true
Min-cost	true	*Min-cost*	true
Perc-sel	0.10	*Perc-sel*	0.50
Violated-cap	true	*Violated-cap*	true
tabu-swap-loc-s	[25% ÷ 50%]	*tabu-swap-loc-s*	[30% ÷ 90%]
max-swap-loc-s	4	*max-swap-loc-s*	4
tabu-swap-loc-p	[25% ÷ 50%]	*tabu-swap-loc-p*	[30% ÷ 80%]
max-swap-loc-p	2	*max-swap-loc-p*	4
tabu-add-loc-s	[15% ÷ 30%]	*tabu-add-loc-s*	[20% ÷ 50%]
max-add-loc-s	3	*max-add-loc-s*	3
tabu-add-loc-p	[15% ÷ 30%]	*tabu-add-loc-p*	[10% ÷ 30%]
max-add-loc-p	3	*max-add-loc-p*	3
tabu-r-ins-single-s	[30% ÷ 80%]	*tabu-r-ins-single-s*	[20% ÷ 50%]
tabu-r-ins-single-p	[30% ÷ 80%]	*tabu-r-ins-single-p*	[20% ÷ 50%]
max-r-ins-single	3	*max-r-ins-single*	3
tabu-r-swap-single-s	[30% ÷ 80%]	*tabu-r-swap-single-s*	[30% ÷ 60%]
tabu-r-swap-single-p	[30% ÷ 80%]	*tabu-r-swap-single-p*	[20% ÷ 50%]
max-r-swap-single	3	*max-r-swap-single*	3
near-ins-s	0.10	*near-ins-s*	0.30
tabu-r-ins-multi-s	[10%; 15%]	*tabu-r-ins-multi-s*	[10%; 30%]
near-ins-p	0.10	*near-ins-p*	0.30
tabu-r-ins-multi-p	[10% ÷ 15%]	*tabu-r-ins-multi-p*	[10% ÷ 30%]
max-r-ins-multi	5	*max-r-ins-multi*	5
near-swap-s	0.10	*near-swap-s*	0.15
tabu-r-swap-multi-s	[10% ÷ 15%]	*tabu-r-swap-multi-s*	[10% ÷ 30%]
near-swap-p	0.10	*near-swap-p*	0.25
tabu-r-swap-multi-p	[10% ÷ 15%]	*tabu-r-swap-multi-p*	[10% ÷ 30%]
max-r-swap-multi	3	*max-r-swap-multi*	5

Table 3: Tabu Search setttings 1 and 2.

TS set 3		TS set 4	
Rand-sel-out	true	*Rand-sel-out*	true
Min-cost	true	*Min-cost*	true
Perc-sel	0.25	*Perc-sel*	0.50
Violated-cap	true	*Violated-cap*	true
tabu-swap-loc-s	[50% ÷ 75%]	*tabu-swap-loc-s*	[30% ÷ 80%]
max-swap-loc-s	5	*max-swap-loc-s*	7
tabu-swap-loc-p	[30% ÷ 60%]	*tabu-swap-loc-p*	[30% ÷ 50%]
max-swap-loc-p	3	*max-swap-loc-p*	5
tabu-add-loc-s	[30% ÷ 50%]	*tabu-add-loc-s*	[10% ÷ 30%]
max-add-loc-s	3	*max-add-loc-s*	5
tabu-add-loc-p	[10% ÷ 30%]	*tabu-add-loc-p*	[10% ÷ 30%]
max-add-loc-p	3	*max-add-loc-p*	5
tabu-r-ins-single-s	[30% ÷ 100%]	*tabu-r-ins-single-s*	[20% ÷ 50%]
tabu-r-ins-single-p	[30% ÷ 80%]	*tabu-r-ins-single-p*	[20% ÷ 50%]
max-r-ins-single	5	*max-r-ins-single*	5
tabu-r-swap-single-s	[30% ÷ 100%]	*tabu-r-swap-single-s*	[30% ÷ 80%]
tabu-r-swap-single-p	[30% ÷ 80%]	*tabu-r-swap-single-p*	[30% ÷ 80%]
max-r-swap-single	5	*max-r-swap-single*	5
near-ins-s	0.25	*near-ins-s*	0.50
tabu-r-ins-multi-s	[10% ÷ 15%]	*tabu-r-ins-multi-s*	[5% ÷ 25%]
near-ins-p	0.30	*near-ins-p*	0.50
tabu-r-ins-multi-p	[10% ÷ 15%]	*tabu-r-ins-multi-p*	[5% ÷ 25%]
max-r-ins-multi	7	*max-r-ins-multi*	7
near-swap-s	0.25	*near-swap-s*	0.25
tabu-r-swap-multi-s	[10% ÷ 15%]	*tabu-r-swap-multi-s*	[5% ÷ 25%]
near-swap-p	0.25	*near-swap-p*	0.50
tabu-r-swap-multi-p	[10% ÷ 15%]	*tabu-r-swap-multi-p*	[5% ÷ 25%]
max-r-swap-multi	5	*max-r-swap-multi*	7

Table 4: Tabu Search settings 3 and 4.

Instance	TS1	CPU-1	TS2	CPU-2	TS3	CPU-3	TS4	CPU-4
I1/2-3-8	591.83	0.39	591.83	0.55	591.83	0.42	591.83	0.50
I1/2-3-9	902.45	0.59	902.45	0.49	902.45	0.43	878.69	1.08
I1/2-4-8	625.96	0.85	625.96	0.94	625.96	0.08	625.96	1.67
I1/2-4-10	862.91	0.85	862.91	1.53	862.91	0.80	862.91	4.07
I1/2-4-15	1121.50	1.92	1115.98	1.68	1116.95	1.31	1105.67	4.15
I1/3-5-10	952.86	1.25	932.67	1.83	952.89	1.34	829.25	5.29
I1/3-5-15	1068.00	2.21	1068.00	3.07	1070.93	2.05	1019.57	6.16
I1/2-8-20	1114.41	5.28	1059.41	23.77	1051.59	26.81	1055.20	48.22
I1/2-8-25	1021.69	4.00	1024.98	14.14	1024.98	31.14	979.85	35.91
I1/2-10-15	754.63	1.53	732.48	7.79	732.48	11.40	732.48	10.81
I1/2-10-20	1008.17	3.40	1003.94	15.27	982.56	50.90	947.65	51.94
I1/2-10-25	1085.67	6.81	1085.67	29.04	1071.63	68.82	1084.26	86.17
I1/3-8-10	604.37	1.24	606.68	3.68	604.37	5.72	604.37	11.26
I1/3-8-15	730.36	1.61	730.36	4.51	730.36	7.10	730.36	11.12
I1/3-8-20	968.59	5.54	947.54	28.77	892.05	63.38	898.08	154.62
I1/3-8-25	943.25	7.65	948.64	62.49	961.13	68.06	896.99	171.55
I1/3-10-15	744.57	2.35	744.57	9.97	735.38	9.65	731.77	28.49
I1/3-10-20	979.07	4.55	860.25	43.13	881.31	111.61	851.18	189.97
I1/3-10-25	1131.59	3.94	1105.91	28.03	1122.54	72.05	1105.91	113.48
I1/4-10-20	1287.14	9.49	1258.91	69.53	1224.35	13.14	1158.92	243.36
I1/4-10-25	1588.95	45.01	1588.95	64.07	1588.95	72.15	1582.01	308.68
I1/5-8-50	1236.65	15.57	1226.24	100.34	1252.40	231.45	1210.27	521.72
I1/5-10-50	1256.59	30.26	1300.78	274.83	1280.79	439.94	1279.02	853.57
I1/5-10-75	1669.67	61.06	1679.84	342.48	1649.91	429.38	1591.60	1026.12
I1/5-15-75	1780.32	32.82	1739.81	204.83	1754.44	828.39	1708.79	2614.13
I1/5-10-100	2458.50	121.66	2392.56	558.87	2401.88	621.73	2257.35	1906.17
I1/5-20-100	2124.69	249.70	2087.29	1340.48	2089.68	2517.46	2071.76	3780.61
I1/5-10-150	2220.47	345.53	2105.09	1115.53	2095.48	1662.28	2097.81	3740.38
I1/5-20-150	2098.87	538.99	2076.23	2243.41	2029.99	2858.92	1919.35	3271.92
I1/5-10-200	2761.73	440.27	2708.19	1697.46	2751.71	1944.08	2601.33	2239.09
I1/5-20-200	2546.74	473.41	2457.04	3125.20	2446.01	3844.64	2407.33	6037.38

Table 5: Experimental results of TS settings on test instances I1.

Instance	TS1	CPU-1	TS2	CPU-2	TS3	CPU-3	TS4	CPU-4
I2/2-3-8	589.38	0.42	589.38	0.45	589.38	0.42	589.38	0.66
I2/2-3-9	413.54	0.54	413.54	0.49	413.54	0.49	413.54	1.01
I2/2-4-8	605.40	0.56	605.40	0.69	605.40	0.58	605.40	1.61
I2/2-4-10	629.38	0.91	629.38	1.17	629.38	1.26	629.38	2.34
I2/2-4-15	943.35	1.76	938.04	1.59	947.41	1.40	912.73	3.71
I2/3-5-10	551.45	1.13	551.45	2.74	551.45	1.06	551.45	5.87
I2/3-5-15	1214.31	6.05	1210.45	9.98	1201.30	6.05	1170.83	32.12
I2/2-8-20	867.41	2.67	829.82	20.14	842.02	32.00	822.85	37.72
I2/2-8-25	959.13	6.07	993.02	20.98	955.03	29.17	956.34	37.48
I2/2-10-15	749.17	3.97	741.73	18.33	731.54	30.50	727.77	39.36
I2/2-10-20	856.57	4.17	813.80	21.57	813.80	40.75	790.57	54.55
I2/2-10-25	1017.53	4.79	1052.18	14.17	1012.04	52.19	961.74	61.29
I2/3-8-10	583.73	1.06	504.20	8.51	504.20	11.35	504.20	13.04
I2/3-8-15	688.68	1.74	672.42	5.56	688.68	15.06	685.48	20.06
I2/3-8-20	769.04	4.57	769.04	32.35	762.25	42.29	765.01	82.03
I2/3-8-25	1055.80	4.73	1069.02	20.69	1037.59	52.89	1026.36	38.65
I2/3-10-15	813.52	2.15	791.13	27.01	791.13	73.87	777.49	82.22
I2/3-10-20	843.23	5.39	827.15	30.14	821.75	134.35	794.58	153.01
I2/3-10-25	1015.10	6.56	1021.21	43.59	1013.32	71.00	1010.51	152.30
I2/4-10-20	868.03	20.29	868.03	53.04	856.40	9.57	802.60	433.90
I2/4-10-25	1193.23	20.35	1193.23	43.31	1193.23	10.50	1185.31	320.03
I2/5-8-50	1207.39	21.02	1185.75	198.69	1180.46	145.55	1185.75	665.22
I2/5-10-50	1350.55	18.08	1348.33	157.06	1133.05	210.83	1335.81	390.62
I2/5-10-75	1813.01	68.34	1784.81	370.43	1772.13	525.61	1756.88	1252.88
I2/5-15-75	1710.38	53.43	1843.75	280.29	1809.01	568.79	1644.79	944.35
I2/5-10-100	2411.03	60.60	2320.13	381.25	2299.03	265.96	2290.64	769.24
I2/5-20-100	2051.39	257.63	2078.37	817.05	2049.21	2053.00	2041.13	2608.40
I2/5-10-150	2018.49	302.78	1937.35	1674.89	1931.36	1171.31	1907.71	4852.92
I2/5-20-150	1772.90	631.48	1764.34	2650.43	1806.03	3618.01	1707.73	4540.74
I2/5-10-200	2435.05	101.01	2522.22	476.34	2542.03	797.35	2407.88	1078.87
I2/5-20-200	2260.65	1237.99	2343.11	3656.93	2265.41	5921.24	2223.72	7850.52

Table 6: Experimental results of TS settings on test instances I2.

Instances	TS1	CPU-1	TS2	CPU-2	TS3	CPU-3	TS4	CPU-4
I3/2-3-8	589.80	0.39	589.80	0.38	589.78	0.92	589.78	1.00
I3/2-3-9	466.01	0.41	466.01	0.41	486.08	0.48	454.63	1.01
I3/2-4-8	451.62	1.61	451.62	0.89	504.70	0.92	451.62	1.61
I3/2-4-10	546.36	0.92	546.36	0.97	546.36	1.08	546.36	2.29
I3/2-4-15	805.46	1.25	805.46	1.47	787.45	2.18	718.16	4.73
I3/3-5-10	747.37	1.76	747.37	1.61	747.37	2.86	745.85	5.49
I3/3-5-15	1071.98	2.48	1054.23	3.42	1071.98	3.61	1033.79	7.08
I3/2-8-20	893.36	2.59	892.21	20.98	858.09	11.44	829.20	32.24
I3/2-8-25	1004.86	4.89	977.81	29.29	967.11	16.24	959.97	41.69
I3/2-10-15	620.86	1.98	620.86	13.39	620.86	5.86	620.86	15.05
I3/2-10-20	757.21	2.54	757.21	28.10	756.71	8.22	756.51	38.06
I3/2-10-25	879.83	5.74	895.54	53.61	879.83	21.34	867.60	49.18
I3/3-8-10	490.78	1.00	490.78	9.32	490.78	2.95	412.91	14.88
I3/3-8-15	626.84	1.39	637.22	18.41	624.55	14.45	624.55	22.35
I3/3-8-20	732.83	3.22	710.09	110.21	732.63	37.98	707.57	187.18
I3/3-8-25	860.26	4.10	833.43	71.69	830.26	29.28	806.71	133.70
I3/3-10-15	624.73	1.70	586.94	19.66	613.72	18.34	574.26	40.61
I3/3-10-20	781.39	3.69	775.29	137.71	770.77	30.63	745.85	256.91
I3/3-10-25	913.31	2.70	897.71	126.28	891.83	21.48	860.81	91.11
I3/4-10-20	1301.56	10.49	1216.79	7.58	1234.44	79.17	1204.57	274.56
I3/4-10-25	1141.80	20.28	1125.74	29.49	1110.40	55.73	1089.40	467.69
I3/5-8-50	1351.27	16.67	1292.54	75.23	1233.59	107.61	1240.80	474.94
I3/5-10-50	1297.51	24.68	1256.68	145.49	1253.78	109.82	1243.87	919.53
I3/5-10-75	1937.27	45.20	1911.85	293.33	1881.52	314.64	1839.38	806.94
I3/5-15-75	1602.72	42.19	1653.44	299.98	1635.61	875.87	1590.00	1910.94
I3/5-10-100	2420.47	37.79	2366.36	272.50	2323.13	329.78	2294.44	546.61
I3/5-20-100	2278.57	75.72	2197.11	283.22	2185.37	431.00	2170.45	696.22
I3/5-10-150	1398.69	182.22	1387.54	1132.39	1414.37	1067.02	1342.18	2635.06
I3/5-20-150	1454.31	232.34	1357.09	862.33	1382.10	2150.06	1343.72	3379.30
I3/5-10-200	2030.30	351.31	1910.32	979.62	1914.98	2194.54	1893.68	2633.48
I3/5-20-200	2737.23	343.97	2728.95	1081.43	2706.50	2375.41	2692.31	2765.42

Table 7: Experimental results of TS settings on test instances I3.

5.2 Comparisons of *TS* with exact methods

In this section the results of Tabu Search are compared with the solutions obtained with the commercial solver and reported, in order to evaluate the effectiveness of our *TS* method in terms of quality of solution and computation times with reference to the available bounds.

As said above, for small instances, *TS* results have been compared with those obtained by the three-index formulation proposed in Boccia et al. (2011) within 2 hours computation time. For medium and large instances, the comparison has been done with the results obtained with a decomposition approach, sequentially solving one *2E-FLP* and two *MDVRP* (one for each echelon).

The evaluation of the gap $\Delta(z)$ between *TS* and bounds, for a generic instance *I*, is computed as $\Delta(z) = [1 - z(TS_I)/z(BS_I)]$, where $z(TS_I)$ and $z(BS_I)$ are respectively the solution value obtained by the *TS* heuristic and the bound value. A positive gap value indicates that *TS* solution improves available bound.

Results on small instances are shown in Tables 8, 9, 10, reporting, for each instance, the model solution value (*MS*) and *TS* solution value (*TS1* and *TS4*) with related CPU time and gap. From these Tables we can observe that in all cases, where the optimal solution for an instance was known (marked with *), *TS* was able to determine it at least with one setting. More precisely, concerning *setting TS1*, the gap varies between $+0.206$ and -0.208. In the worst cases it is equal to -0.208 for set *I1*, -0.158 for set *I2* and -0.189 for set *I3*. On the other side computation time is always lower than *45* seconds. Concerning instead *setting TS4*, the gap is, in the most of the cases, positive and it varies between $+0.256$ and -0.051. Computation times increase with respect to*setting 1*, but they are significantly lower than the ones of the solver (less than *360* seconds).

Results on medium and large size instances are shown in Tables 11, 12 and 13. *TS* solution values are compared with those of the decomposition approach (*DA*). Concerning *setting TS1* we can observe that *TS* results are very close to the ones of the decomposition approach, but the saving in terms of computation time is meaningful. The gap varies between $+0.283$ and -0.094 and computation times are generally lower than *600* seconds. Concerning instead *setting TS4*, it outperforms decomposition approach in most of the instances, but the saving in terms of computation time is not so large as for *setting TS1*. In particular the gap varies between $+0.295$ and -0.008 and computation time varies between 390.62 and 7850.52 seconds.

Instance	MS	CPU	TS1	CPU-1	GAP-1	TS4	CPU-4	GAP-4
I1/2-3-8	591.83	10.23	591.83	0.39	0.000	591.83	0.50	0.000
I1/2-3-9	878.69	9.87	902.45	0.59	-0.027	878.69	1.08	0.000
I1/2-4-8	625.96	175.60	625.96	0.85	0.000	625.96	1.67	0.000
I1/2-4-10	862.91	582.90	862.91	0.85	0.000	862.91	4.07	0.000
I1/2-4-15	1105.67	1469.90	1121.50	1.92	-0.014	1105.67	4.15	0.000
I1/3-5-10	829.25	2194.70	952.86	1.25	-0.149	829.25	5.29	0.000
I1/3-5-15	1019.57	3893.50	1068.00	2.21	-0.048	1019.57	6.16	0.000
I1/2-8-20	1055.65	7200.00	1114.41	5.28	-0.056	1055.20	48.22	0.000
I1/2-8-25	992.08	7200.00	1021.69	4.00	-0.030	979.85	35.91	0.012
I1/2-10-15	732.48	7200.00	754.63	1.53	-0.030	732.48	10.81	0.000
I1/2-10-20	951.01	7200.00	1008.17	3.40	-0.060	947.65	51.94	0.004
I1/2-10-25	1170.72	7200.00	1085.67	6.81	0.073	1084.26	86.17	0.074
I1/3-8-10	604.37	4982.30	604.37	1.24	0.000	604.37	11.26	0.000
I1/3-8-15	730.36	7200.00	730.36	1.61	0.000	730.36	11.12	0.000
I1/3-8-20	898.75	7200.00	968.59	5.54	-0.078	898.08	154.62	0.001
I1/3-8-25	1141.26	7200.00	943.25	7.65	0.173	896.99	171.55	0.214
I1/3-10-15	699.11	7200.00	744.57	2.35	-0.065	731.77	28.49	-0.047
I1/3-10-20	810.26	7200.00	979.07	4.55	-0.208	851.18	189.97	-0.051
I1/3-10-25	1291.68	7200.00	1131.59	3.94	0.124	1105.91	113.48	0.144
I1/4-10-20	1208.72	7200.00	1287.14	9.49	-0.065	1158.92	243.36	0.041
I1/4-10-25	1615.33	7200.00	1588.95	45.01	0.016	1582.01	308.68	0.021

Table 8: Tabu Search vs. models on small instances I1.

Instance	MS	CPU	TS1	CPU-1	GAP-1	TS4	CPU-4	GAP-4
I2/2-3-8	589.38	6.45	589.38	0.42	0.000	589.38	0.66	0.000
I2/2-3-9	413.54	8.31	413.54	0.54	0.000	413.54	1.01	0.000
I2/2-4-8	605.40	182.50	605.40	0.56	0.000	605.40	1.61	0.000
I2/2-4-10	629.38	834.30	629.38	0.91	0.000	629.38	2.34	0.000
I2/2-4-15	912.73	1525.30	943.35	1.76	-0.034	912.73	3.71	0.000
I2/3-5-10	551.45	2281.50	551.45	1.13	0.000	551.45	5.87	0.000
I2/3-5-15	1170.83	4365.50	1214.31	6.05	-0.037	1170.83	32.12	0.000
I2/2-8-20	822.85	7200.00	867.41	2.67	-0.054	822.85	37.72	0.000
I2/2-8-25	947.84	7200.00	959.13	6.07	-0.012	956.34	37.48	-0.009
I2/2-10-15	727.77	7200.00	749.17	3.97	-0.029	727.77	39.36	0.000
I2/2-10-20	801.28	7200.00	856.57	4.17	-0.069	790.57	54.55	0.013
I2/2-10-25	1263.54	7200.00	1017.53	4.79	0.195	961.74	61.29	0.239
I2/3-8-10	504.20	6412.23	583.73	1.06	-0.158	504.20	13.04	0.000
I2/3-8-15	685.48	7200.00	688.68	1.74	-0.005	685.48	20.06	0.000
I2/3-8-20	805.38	7200.00	769.04	4.57	0.045	765.01	82.03	0.050
I2/3-8-25	1026.36	7200.00	1055.80	4.73	-0.029	1026.36	38.65	0.000
I2/3-10-15	812.13	7200.00	813.52	2.15	-0.002	777.49	82.22	0.043
I2/3-10-20	806.67	7200.00	843.23	5.39	-0.045	794.58	153.01	0.015
I2/3-10-25	1254.62	7200.00	1015.10	6.56	0.191	1010.51	152.30	0.195
I2/4-10-20	1093.34	7200.00	868.03	20.29	0.206	802.60	433.90	0.266
I2/4-10-25	1380.86	7200.00	1193.23	20.35	0.136	1185.31	320.03	0.142

Table 9: Tabu Search vs. models on small instances I2.

Instance	MS	CPU	TS1	CPU-1	GAP-1	TS4	CPU-4	GAP-4
I3/2-3-8	589.80	8.13	589.80	0.39	0.000	589.78	1.00	0.000
I3/2-3-9	454.63	7.10	466.01	0.41	-0.025	454.63	1.01	0.000
I3/2-4-8	451.62	164.70	451.62	1.61	0.000	451.62	1.61	0.000
I3/2-4-10	546.36	416.80	546.36	0.92	0.000	546.36	2.29	0.000
I3/2-4-15	718.16	1225.30	805.46	1.25	-0.122	718.16	4.73	0.000
I3/3-5-10	745.85	2674.20	747.37	1.76	-0.002	745.85	5.49	0.000
I3/3-5-15	1033.79	3065.50	1071.98	2.48	-0.037	1033.79	7.08	0.000
I3/2-8-20	829.20	7200.00	893.36	2.59	-0.077	829.20	32.24	0.000
I3/2-8-25	1100.31	7200.00	1004.86	4.89	0.087	959.97	41.69	0.128
I3/2-10-15	620.86	7200.00	620.86	1.98	0.000	620.86	15.05	0.000
I3/2-10-20	790.99	7200.00	757.21	2.54	0.043	756.51	38.06	0.044
I3/2-10-25	944.84	7200.00	879.83	5.74	0.069	867.60	49.18	0.082
I3/3-8-10	412.91	3376.23	490.78	1.00	-0.189	412.91	14.88	0.000
I3/3-8-15	624.55	7200.00	626.84	1.39	-0.004	624.55	22.35	0.000
I3/3-8-20	707.57	7200.00	732.83	3.22	-0.036	707.57	187.18	0.000
I3/3-8-25	977.10	7200.00	860.26	4.10	0.120	806.71	133.70	0.174
I3/3-10-15	574.26	7200.00	624.73	1.70	-0.088	574.26	40.61	0.000
I3/3-10-20	789.49	7200.00	781.39	3.69	0.010	745.85	256.91	0.055
I3/3-10-25	1038.58	7200.00	913.31	2.70	0.121	860.805	91.11	0.171
I3/4-10-20	1287.23	7200.00	1301.56	10.49	-0.011	1204.57	274.56	0.064
I3/4-10-25	1089.40	7200.00	1141.80	20.28	-0.048	1089.40	467.69	0.000

Table 10: Tabu Search vs. models on small instances I3.

Instance	DA	CPU	TS1	CPU-1	GAP-1	TS4	CPU-4	GAP-4
I1/5-8-50	1226.24	4421.30	1236.65	15.57	-0.008	1210.27	521.72	0.013
I1/5-10-50	1783.60	6134.90	1279.02	30.26	0.283	1256.59	853.57	0.295
I1/5-10-75	1591.60	7512.60	1669.67	61.06	-0.049	1591.60	1026.12	0.000
I1/5-15-75	1783.60	6134.90	1780.32	32.82	0.002	1708.79	2614.13	0.042
I1/5-10-100	2247.32	8033.80	2458.50	121.66	-0.094	2257.35	1906.17	-0.004
I1/5-20-100	2055.88	10218.10	2124.69	249.70	-0.033	2071.76	3780.61	-0.008
I1/5-10-150	2177.77	8407.10	2220.47	345.53	-0.020	2097.81	3740.38	0.037
I1/5-20-150	1933.82	7786.60	2098.87	538.99	-0.085	1919.35	3271.92	0.007
I1/5-10-200	2625.11	10119.50	2761.73	440.27	-0.052	2601.33	2239.09	0.009
I1/5-20-200	3140.17	12750.30	2546.74	473.41	0.189	2407.33	6037.38	0.233

Table 11: Tabu Search vs. decomposition approach on medium-large instances I1.

Instance	DA	CPU	TS1	CPU-1	GAP-1	TS4	CPU-4	GAP-4
I2/5-8-50	1185.75	2023.34	1207.39	21.02	-0.018	1185.75	665.22	0.000
I2/5-10-50	1325.61	5039.50	1350.55	18.08	-0.019	1335.81	390.624	-0.008
I2/5-10-75	1768.88	7061.00	1813.01	68.34	-0.025	1756.13	1252.878	0.007
I2/5-15-75	1644.79	9499.40	1710.38	53.43	-0.040	1644.79	944.352	0.000
I2/5-10-100	2391.17	10379.60	2411.03	60.60	-0.008	2290.64	769.242	0.042
I2/5-20-100	2051.39	12405.60	2051.39	257.63	0.000	2041.13	2608.404	0.005
I2/5-10-150	2111.97	14060.90	2018.49	302.78	0.044	1907.71	4852.92	0.097
I2/5-20-150	1800.89	10134.50	1772.90	631.48	0.016	1707.73	4540.74	0.052
I2/5-10-200	2430.93	8871.80	2435.05	101.01	-0.002	2407.88	1078.866	0.009
I2/5-20-200	2274.29	15602.10	2260.65	1237.99	0.006	2223.72	7850.52	0.022

Table 12: Tabu Search vs. decomposition approach on medium-large instances I2.

Instance	DA	CPU	TS1	CPU-1	GAP-1	TS4	CPU-4	GAP-4
I3/5-8-50	1298.89	7741.90	1351.27	16.67	-0.040	1240.80	474.94	0.045
I3/5-10-50	1256.68	4929.60	1297.51	24.68	-0.032	1243.87	919.53	0.010
I3/5-10-75	1879.56	13720.00	1937.27	45.20	-0.031	1839.38	806.94	0.021
I3/5-15-75	1704.65	12903.90	1602.72	42.19	0.060	1590.00	1910.94	0.067
I3/5-10-100	2601.44	20599.60	2420.47	37.79	0.070	2294.44	546.61	0.118
I3/5-20-100	2261.36	15724.50	2278.57	75.72	-0.008	2170.45	696.22	0.040
I3/5-10-150	1470.77	243.90	1398.69	182.22	0.049	1342.18	2635.06	0.087
I3/5-20-150	1508.07	21240.50	1454.31	232.34	0.036	1343.72	3379.30	0.109
I3/5-10-200	2193.32	41145.10	2030.30	351.31	0.074	1893.68	2633.48	0.137
I3/5-20-200	2784.47	23319.40	2737.23	343.97	0.017	2692.31	2765.42	0.033

Table 13: Tabu Search vs. decomposition approach on medium-large instances I3.

6 Conclusions

In this paper a two-echelon location-routing problem (*2E-LRP*) has been tackled. This problem is NP-hard and mixed integer programming formulations proposed in literature cannot solve instance arising in real situations, characterized by high numbers of customers and facilities. For this reason the usage of a heuristic approach is required. In this work a tabu search metaheuristic has been proposed and implemented. It is based on the decomposition of the whole problem in four subproblems, one *FLP* and one *MD-VRP* for each echelon. The four sub-problems are sequentially and iteratively solved and their solutions are opportunely combined in order to determine a good global solution. Tabu Search has been experienced on three set of small, medium and large instances and the obtained results have been compared with the results of the models. Experimental results prove that the proposed *TS* is effective in terms of quality of solutions and computation times in the most of the solved instances. The proposed Tabu Search presents a modular structure which makes it very flexible. Therefore it could be easily integrated with intensification criteria, extended with other constraints (such as maximum length of the routes, more fleets of vehicles for each echelon) and adapted to the asymmetric case. The problem is still new and the research field is unexplored. Therefore future research work should move towards exact approaches in order to test the heuristics, which seem to be anyway the more effective way to approach this problem.

Acknowledgments

While working on this project, T.G. Crainic was the NSERC Industrial Research Chair in Logistics Management, ESG UQAM, and Adjunct Professor with the Department of Computer Science and Operations Research, Université de Montréal, and the Department of Economics and Business Administration, Molde University College, Norway. Partial funding for this project has been provided by the Universitá degli Studi di Napoli "Federico II", the Natural Sciences and Engineering Council of Canada (NSERC), through its

Industrial Research Chair and Discovery Grants programs, by the partners of the Chair, CN, Rona, Alimentation Couche-Tard and the Ministry of Transportation of Québec, .

References

[1] AMBROSINO, D., SCIOMACHEN, A., SCUTELLA', M.G.: A heuristic based on multi-exchange techniques for a regional fleet assignment location-routing problem. Computers & Operations Research, 36, 442 - 460, (2009).

[2] AMBROSINO, D., SCUTELLÁ, M.G.: Distribution network design: New problems and related models. European Journal of Operational Research, 165, 610624, (2005).

[3] ALBAREDA-SAMBOLA, M., DIAZ, J., FERNÁNDEZ, E.: A compact model and tight bounds for a combined location-routing problem. Computers and Operations Research, 32 , 407-428, (2005).

[4] BALAKRISHNAN, A., WARD, J.E., WONG, R.T.: Integrated facility location and vehicle routing models: Recent work and future prospects. American Journal of Mathematical and Management Sciences, 7, 35-61, (1987).

[5] BARRETO, S., FERRIERA, C., PAIXAO, J., SANTON, B.S.: Using clustering analysis in a capacitated location-routing problem, European Journal of Operational Research, 179, 3, 968-977, (2007).

[6] BOCCIA, M., CRAINIC, T.G., SFORZA, A., STERLE, C.: A Metaheuristic for a Two Echelon Location-Routing Problem. Proceeding of SEA2010. Lecture Notes in Computer Science, 6049/2010, 288-301, (2010).

[7] BOCCIA, M., CRAINIC, T.G., SFORZA, A., STERLE, C.: Location-routing models for designing a two-echelon freight distribution system. *CIRRELT*, CIRRELT-2011-XX, (2011).

[8] BRUNS, A., KLOSE, A.: A location first-route second heuristic for a combined location-routeing problem. In U. Zimmermann, U. Derigs, W. Gaul, R. Mohring, et K. Schuster (Eds.), Operations Research Proceedings. Springer, (1996).

[9] CHEN, C.H., TING, C.J. : A hybrid lagrangian heuristic/simulated annealing algorithm for the multi-depot location routing problem. Proceedings of the Eastern Asia Society for Transportation Studies, 6, (2007).

[10] CHIEN, T.W.: Heuristic Procedures for Practical-Sized Uncapacitated Location-Capacitated Routing Problems. Decision Sciences, 24, 5, 993-1021, (1993).

[11] DUHAMEL C., LACOMME P., PRINS, C., PRODHON, C.: A GRASPxELS approach for the capacitated location-routing problem. Computers & Operations Research - Metaheuristics for Logistics and Vehicle Routing, 37, 11, 1912-1923, (2010).

[12] HANSEN, P.H., HEGEDAHL, B., HJORTKJAER, S., OBEL, B.: A heuristic solution to the warehouse locationrouting problem, European Journal of Operational Research, 76, 111127, (1994).

[13] JACOBSEN, S.K., MADSEN, O.B.G.: A comparative study of heuristics for a two-level locationrouting problem. European Journal of Operational Research, 5, 378-387, (1980).

[14] KARP, R.:Reducibility among combinatorial problems. In Miller, R. and Thatcher, J. (eds.), Complexity of Computer Computations. Plenum Press, New York, 85-104, (1972).

[15] LAPORTE, G., NOBERT, Y.: An exact algorithm for minimizing routing and operating costs in depot location. European Journal of Operational Research 6, 224-226, (1981).

[16] LAPORTE, G., NOBERT, Y., PELLETIER, P.: Hamiltonian location problems. European Journal of Operational Research 12, 82 89, (1983).

[17] LAPORTE, G., NOBERT, Y., ARPIN, D.: An exact algorithm for solving a capacitated location-routing problem. Annals of Operations Research 6, 293-310, (1986).

[18] LAPORTE, G., NOBERT, Y., TAILLEFER, S.,: Solving a family of multi-depot vehicle routing and location-routing problems. Transportation Science 22, 161172, (1988).

[19] LAPORTE, G.: Location-routing problems. In B.L. Golden & A.A. Assad (Eds), Vehicle routing: Methods and studies. Amsterdam: North-Holland, (1988b).

[20] LAPORTE, G.: A survey of algorithms for location-routing problems. Investigación Operativa, 1, 93123, (1989).

[21] LENSTRA, J. K., RINNOY KAN, A.H.G.: Complexity of vehicle routing and scheduling problems, Networks, 11, 2, 221-227, (1981).

[22] LIN, C.K.Y., CHOW, C.K., CHEN, A.: A location-routing loading problem for bill delivery services. Computers and Industrial Engineering 43, 525, (2002).

[23] MADSEN, O.B.G.: Methods for solving combined two level location-routing problems of realistic dimensions. European Journal of Operational Research, 12, 3, 295-301, (1983).

[24] MARANZANA, F.E.: On the location of supply points to minimise transport costs. Operational Research Quarterly 15, 261270, (1964).

[25] MELECHOVSKÝ, J., PRINS, C., CALVO, R.W.: A metaheuristic to solve a location-routing problem with non-linear costs. Journal of Heuristics, 11, 375-391, (2005).

[26] MIN, H., JAYARAMAN, V., SRIVASTAVA, R.: Combined location-routing problems: A synthesis and future research directions. European Journal of Operational Research, 108, 1-15, (1998).

[27] NAGY, G., SALHI, S.: Nested Heuristic Methods for the Location-Routeing Problem. The Journal of the Operational Research Society, 47, 9, 1166-1174, 1996.

[28] NAGY, G., SALHI, S.: Location-routing: Issues, models and methods. European Journal of Operational Research, 177, 649672, (2007).

[29] OR, I., PIERSKALLA W.P.: A transportation location-allocation model for regional blood banking. AIIE Transactions, 11, 86-95, (1979).

[30] ÖZYURT, Z., AKSEN, D.: Solving the Multi-Depot Location-Routing Problem with Lagrangian Relaxation. In Book Series Operations Research/Computer Science Interfaces, 37, 125-144, (2007)

[31] PERL, J., DASKIN, M.S.: A warehouse location-routing problem. Transportation Research Part B, 19, 5, 381-396, (1985).

[32] PRINS, C., PRODHON, C., WOLFER-CALVO, R.: Solving the capacitated location-routing problem by a GRASP complemented by a learning process and a path relinking. 4OR, 4, 221-238, (2006).

[33] PRINS, C., PRODHON, C., RUIZ, A., SIRIANO, P., WOFLER-CALVO, R.: Solving the Capacitated Location-Routing Problem by a Cooperative Lagrangean Relaxation-Granular Tabu Search Heuristic. Transporation Science, 41, 4, 470-483, (2007).

[34] SALHI S., NAGY G.: Local improvement in planar facility location using vehicle routing. Annals of Operations Research, 167, 287-296, (2009).

[35] SRIVASTAVA, R.: Alternate solution procedures for the location-routing problem. Omega International Journal of Management Science 21, 4, 497-506, (1993).

[36] SRIVASTAVA, R., BENTON, W.C.: The location-routing problem: Considerations in physical distribution system. Computers and Operations Research 17, 5, 427-435, (1990).

[37] TUZUN, D., BURKE, L.I.: A two-phase tabu search approach to the location routing problem, European Journal of Operational Research, 116, 8799, (1999).

[38] WANG, X., SUN, X., FANG,Y.: A two-phase hybrid heuristic search approach to the location-routing problem. IEEE International Conference on Systems, Man and Cybernetics, 4, 3338-3343, (2005).

[39] WU, T.H., CHINYAO, L., BAI, J.W.: Heuristic solutions to multi-depot location-routing problems. Computers & Operations Research, 29, 1393-1415, (2002).

A Location and location-routing models

The two models used to compare the result of the TS are reported. First model is a three index formulation proposed for *2E-LRP* in Boccia et al. (2011) and used to solve the problem on small instances. Second model is a two index formulation proposed for the *2E-FLP* used for the sequential approach of medium and large instances.

The following sets and parameters are common to the two formulations:
- $\mathcal{P} = \{p\}$: set of the possible primary facility locations;
- $\mathcal{S} = \{s\}$: set of the possible secondary facility locations;
- $\mathcal{Z} = \{z\}$: set of customers whose positions are fixed and known in advance;
- $\mathcal{T} = \{t\}$: set of *first echelon vehicles*;
- $\mathcal{V} = \{v\}$: set of *second echelon vehicles*;
- $K_i, i \in P \cup S$: facility capacity values;
- $k^i, i \in T \cup V$: vehicle capacity values;
- $H_i, i \in P \cup S$: facility location costs;
- $h^i, i \in T \cup V$: cost for using a vehicle;
- $D_z, z \in Z$: demand of each client.
- C_{ij}: transportation cost from node i to j.

A.1 Three-index formulation for *2E-LRP*

$$\begin{aligned} \text{Minimize} \quad & \sum_{p\in\mathcal{P}} H_p\ y_p + \sum_{s\in\mathcal{S}} H_s\ y_s + \sum_{t\in\mathcal{T}} h^t\ u_t + \sum_{v\in\mathcal{V}} h^v\ u_v + \\ & + \sum_{v\in\mathcal{V}} \sum_{i\in\mathcal{S}\cup\mathcal{Z}} \sum_{j\in\mathcal{S}\cup\mathcal{Z}} C_{ij}\ x^v_{ij} + \sum_{t\in T} \sum_{i\in\mathcal{P}\cup\mathcal{S}} \sum_{j\in\mathcal{P}\cup\mathcal{S}} C_{ij}\ r^t_{ij} \end{aligned} \tag{2}$$

subject to

$$\sum_{v\in\mathcal{V}} \sum_{j\in\mathcal{S}\cup\mathcal{Z}} x^v_{zj} = 1 \qquad \forall z \in \mathcal{Z} \tag{3}$$

$$\sum_{l\in\mathcal{S}\cup\mathcal{Z}} x^v_{lj} - \sum_{l\in\mathcal{S}\cup\mathcal{Z}} x^v_{jl} = 0 \qquad \forall j \in \mathcal{Z}\cup\mathcal{S}, \forall v \in \mathcal{V} \tag{4}$$

$$L_i - L_j + (|S| + |Z|) \sum_{v\in\mathcal{V}} x^v_{ij} \leq (|S| + |Z| - 1) \qquad \forall i, j \in \mathcal{Z}\cup\mathcal{S}, i \neq j \tag{5}$$

$$\sum_{l\in\mathcal{S}\cup\mathcal{Z}} \sum_{j\in\mathcal{S}} x^v_{lj} \leq 1 \qquad \forall v \in \mathcal{V} \tag{6}$$

$$\sum_{t\in\mathcal{T}} \sum_{j\in\mathcal{P}\cup\mathcal{S}} r^t_{lj} = y_l \qquad \forall l \in \mathcal{S} \tag{7}$$

$$\sum_{l \in \mathcal{P} \cup \mathcal{S}} r^t_{lh} - \sum_{l \in \mathcal{P} \cup \mathcal{S}} r^t_{hl} = 0 \qquad \forall h \in \mathcal{P} \cup \mathcal{S}, \forall t \in \mathcal{T} \tag{8}$$

$$L_i - L_j + (|P| + |S|) \sum_{t \in \mathcal{T}} r^t_{ij} \leq (|P| + |S| - 1) \qquad \forall i, j \in \mathcal{S} \cup \mathcal{P}, i \neq j \tag{9}$$

$$\sum_{l \in \mathcal{P} \cup \mathcal{S}} \sum_{j \in \mathcal{P}} r^t_{lj} \leq 1 \qquad \forall t \in \mathcal{T} \tag{10}$$

$$\sum_{h \in \mathcal{S} \cup \mathcal{Z}} x^v_{zh} + \sum_{h \in \mathcal{S} \cup \mathcal{Z}} x^v_{sh} - w_{zs} \leq 1 \qquad \forall z \in \mathcal{Z}, \forall v \in \mathcal{V}, \forall s \in \mathcal{S} \tag{11}$$

$$\sum_{s \in \mathcal{S}} w_{zs} = 1 \qquad \forall z \in \mathcal{Z} \tag{12}$$

$$\sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} f^t_{ps} - \sum_{z \in \mathcal{Z}} D_z \; w_{zs} = 0 \qquad \forall s \in \mathcal{S} \tag{13}$$

$$\sum_{s \in \mathcal{S}} f_{ps} - K_p \; y_p \leq 0 \qquad \forall p \in \mathcal{P} \tag{14}$$

$$\sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} f^t_{ps} - K_s \; y_s \leq 0 \qquad \forall s \in \mathcal{S} \tag{15}$$

$$k^t \sum_{h \in \mathcal{S} \cup \mathcal{P}} r^t_{sh} - f^t_{ps} \geq 0 \qquad \forall t \in \mathcal{T}, \forall s \in \mathcal{S}, \forall p \in \mathcal{P} \tag{16}$$

$$k^v \sum_{h \in \mathcal{S} \cup \mathcal{P}} r^t_{ph} - f^t_{ps} \geq 0 \qquad \forall t \in \mathcal{T}, \forall s \in \mathcal{S}, \forall p \in \mathcal{P} \tag{17}$$

$$\sum_{i \in \mathcal{Z}} D_z \sum_{j \in \mathcal{S} \cup \mathcal{Z}} x^v_{zj} \leq k^v \; u_v \qquad \forall v \in \mathcal{V} \tag{18}$$

$$\sum_{p \in \mathcal{P}} \sum_{s \in \mathcal{S}} f^t_{ps} \leq k_t \; u^t \qquad \forall t \in \mathcal{T} \tag{19}$$

$$\begin{aligned}
r^t_{ij} &= \{0,1\} && \forall i, j \in \mathcal{P} \cup \mathcal{S}, t \in \mathcal{T} \\
x^v_{ij} &= \{0,1\} && \forall i, j \in \mathcal{S} \cup \mathcal{Z}, v \in \mathcal{V} \\
w_{zs} &= \{0,1\} && \forall z \in \mathcal{Z}, s \in \mathcal{S} \\
y_p &= \{0,1\} && \forall p \in \mathcal{P} \\
y_s &= \{0,1\} && \forall s \in \mathcal{S} \\
u_t &= \{0,1\} && \forall t \in \mathcal{T} \\
u_v &= \{0,1\} && \forall v \in \mathcal{V} \\
f^t_{ps} &\geq 0 && \forall p \in \mathcal{P}, s \in \mathcal{S}, t \in \mathcal{T}
\end{aligned} \tag{20}$$

Model variables

- $y_i = \{0,1\}, i \in \mathcal{P} \cup \mathcal{S}$: *1*, if a facility is open at node i, *0* otherwise;

- $r_{ij}^{g} = \{0, 1\}, i, j \in \mathcal{P} \cup \mathcal{S}, t \in T$: *1*, if i precedes j in first echelon route, performed by a first echelon vehicle g, *0* otherwise;
- $x_{ij}^{v} = \{0, 1\}, i, j \in \mathcal{S} \cup \mathcal{Z}, v \in \mathcal{V}$: *1*, if i precedes j in second echelon route, performed by a second echelon vehicle v, *0* otherwise;
- $w_{zs} = \{0, 1\}, s \in \mathcal{S}, z \in \mathcal{Z}$: *1*, if customer z is assigned to a secondary facility s, *0* otherwise;
- $u_i = \{0, 1\}, i \in \mathcal{T} \cup \mathcal{V}$: *1*, if a vehicle i is used on an echelon, *0* otherwise;
- $f_{ps}^{t} \geq 0, p \in \mathcal{P}, s \in \mathcal{S}, t \in \mathcal{T}$: flow from primary facility p to secondary facility s on a first echelon vehicle t;
- L_i are continuous variables used in the subtour breaking constraints.

The objective function 2 minimize the sum of location costs for primary and secondary facilities, fixed cost of vehicles and transportation costs on the two echelons. Constraints (3) impose that each customer z, $z \in \mathcal{Z}$, is served by exactly one second echelon vehicle v, $v \in \mathcal{V}$. Constraints (4) impose that for each vehicle v, $v \in V$, number of arcs entering in a node i, $i \in \mathcal{Z} \cup \mathcal{S}$, is equal to number of arcs leaving the node. Constraints (5) are subtour elimination constraints. Constraints (6) impose that each second echelon vehicle v, $v \in \mathcal{V}$, has to be assigned unambiguously to one secondary facility s, $s \in \mathcal{S}$. Constraints (7), (8), (9), (10) are routing constraints imposing on first echelon the same conditions that constraints (3), (4), (5) and (6) impose on second echelon. Constraints (13) are flow conservation constraints at secondary facilities. Constraints (14) impose that flow leaving a primary facility p, $p \in \mathcal{P}$, has to be less than its own capacity if the facility is open. Constraints (15) impose that flow entering in a secondary facility $s \in \mathcal{S}$ has to be less than its own capacity if the facility is open. Constraints (18) impose that demand satisfied by second echelon vehicle v, $v \in \mathcal{V}$ has to be less than its own capacity if the vehicle is used. Constraints (19) impose that amount of flow transferred by a first echelon vehicle t, $t \in \mathcal{T}$, has to be less than its own capacity if the vehicle is used. Constraints (11) link allocation and routing variables. Constraints (12) imposes that each customer z has to be assigned to a secondary facility s. These constraints are redundant, but allow to slightly improve the bounds of linear relaxations. Constraints (16) and (17) guarantee that amount of flow on a vehicle t, $t \in \mathcal{T}$, from a primary facility p, $p \in P$, to a secondary facility s, $s \in \mathcal{S}$, is positive if and only if both facilities are visited by the same vehicle t. In end constraints (20) express integrality constraints and non-negativity constraints.

A.2 Two-index formulation for *2E-FLP*

$$\text{Minimize} \quad \sum_{p \in \mathcal{P}} H_p \; y_p + \sum_{s \in \mathcal{S}} H_s \; y_s + \sum_{s \in \mathcal{S}} \sum_{z \in \mathcal{Z}} C_{ij} \; x_{zs} + \sum_{p \in \mathcal{P}} \sum_{s \in \mathcal{S}} C_{sp} \; r_{sp} \tag{21}$$

subject to

$$\sum_{s \in S} x_{zs} = 1 \qquad \forall z \in \mathcal{Z} \tag{22}$$

$$\sum_{p \in \mathcal{P}} r_{sp} = y_s \qquad \forall s \in \mathcal{S} \tag{23}$$

$$\sum_{z \in \mathcal{Z}} D_z \; x_{zs} - U_s \; y_s \leq 0 \qquad \forall s \in \mathcal{S} \tag{24}$$

$$\sum_{s \in \mathcal{S}} f_{sp} - U_p \; y_p \leq 0 \qquad \forall p \in \mathcal{P} \tag{25}$$

$$\sum_{p \in \mathcal{P}} f_{sp} - \sum_{z \in \mathcal{Z}} D_z \; x_{zs} = 0 \qquad \forall s \in \mathcal{S} \tag{26}$$

$$U_p \; r_{sp} - f_{ps} \geq 0 \qquad \forall p \in \mathcal{P}, \forall s \in \mathcal{S} \tag{27}$$

$$\begin{aligned} r_{sp} &= \{0,1\} && \forall p \in \mathcal{P}, s \in \mathcal{S}, \\ x_{zs} &= \{0,1\} && \forall s \in \mathcal{S}, z \in \mathcal{Z} \\ y_p &= \{0,1\} && \forall p \in \mathcal{P} \\ y_s &= \{0,1\} && \forall s \in \mathcal{S} \\ f_{ps} &\geq 0 && \forall p \in \mathcal{P}, s \in \mathcal{S} \end{aligned} \tag{28}$$

Model variables
- $y_i = \{0,1\}, i \in \mathcal{P} \cup \mathcal{S}$: *1*, if a facility is open at node i, *0* otherwise.
- $r_{sp} = \{0,1\}, p \in \mathcal{P}, s \in \mathcal{S}$: *1*, if secondary facility s is assigned to primary facility p, *0* otherwise;
- $x_{zs} = \{0,1\}, s \in \mathcal{S}, zin\mathcal{Z}$: *1*, if customer z is assigned to secondary facility s, *0* otherwise;
- $f_{ps} \geq 0, p \in \mathcal{P}, s \in \mathcal{S}$: flow from primary facility p to secondary facility s.

The objective function (21) minimize the sum of facility location costs and transportation costs on the two echelons. Constraints are the classical ones of facility location problems. Constraints (22) and (23) assign respectively each customer to an open secondary facility and each open secondary facility to an open primary one. Constraints (24) and (25) are capacity constraints for secondary and primary facilities. Constraints (26) are flow conservation constraints. Constraints (27) are consistency constraints between flow variables and assignment variables for the first echelon. Finally constraints (28) are integrality and non-negativity constraints for the used variables.