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Location-Routing Models for Two-Echelon Freight Distribution System Design†

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Abstract. In this paper, we address the decision problem of designing a two-echelon freight distribution system. The problem is modeled as a two-echelon location-routing problem, which arises when considering, within the same decision process, the location of facilities on two adjacent echelons of a distribution system, together with the routing of vehicles at both echelons. We describe the issues, set up the general problem class, and study its basic variant, proposing and comparing three mixed-integer programming formulations, aimed at defining the locations and numbers of two types of capacitated facilities, the sizes of two different vehicle fleets, and the related routes. The computational evaluation and comparisons are performed on a large set of instances inspired by two-tiered City Logistics system settings with various numbers and relative distributions of potential locations for the two types of facilities.

Keywords. Two-echelon location-routing, City Logistics, distribution systems.

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1 Introduction

Multi-echelon systems are well-known in industrial and retail logistics, where material flows circulate from suppliers and producers to final customers, through chains made up of processing and handling facilities (plants, warehouses, distribution centers, and so on), and transportation activities (e.g., Ambrosino and Scutellá, 2005; Goetschalckx et al., 2002; Vidal and Goetschalckx, 1997). The planning of such systems, particularly at the strategic level, selects the location, number, and characteristics of facilities, the customer-to-facility assignments, the transportation modes among facilities and, sometimes, the routes of the vehicles providing the transportation services. The importance of the topic is reflected by a rich literature on single and multi-echelon facility location problems (FLP) (see, e.g., the books and surveys by Aikens, 1985; Drezner and Hamacher, 2002; Daskin, 1995; Daskin and Owen, 2003; Klose and Drexl, 2005; Melo et al., 2009; ReVelle and Eiselt, 2005; Sahin and Sural, 2007).

Equally well-acknowledged in practice and academic research is the strong interplay between the location of facilities and the efficiency of the transportation activities, and the need to adequately represent this relation when planning the design of the system. The explicit representation of this relation is particularly important when "transportation" corresponds to vehicles undertaking tours to pick up or deliver (or both) loads, due to the difficulty to properly estimate the transportation costs without generating representative tours. *Location-routing* models and methods were proposed to address this important and challenging issue, characterized by the interplay between the combinatorial nature of the two processes, i.e., selecting facilities and routing vehicles. The literature is again rich, as illustrated by several surveys (e.g. Balakrishnan et al., 1987; Laporte, 1988, 1989; Min et al., 1998; Nagy and Salhi, 2007).

It is noticeable, however, that only one level of routing is considered in most of this literature, even when several echelons of facilities and activities are present. This probably follows from the fact that, historically, research within the field focused on the last echelon and the interplay between the location of warehouses (or distribution centers) and customers. The evolution of the world economy and commerce changes the research requirements, however. Thus, for example, supply chains, production, retail, and distribution networks are deployed over broad geographical and political spaces, involving several participating firms, including several pick up and delivery activities (supply, components, finished goods, etc.). This makes the integration within the same methodology of several levels of location and routing a relevant and challenging research topic with broad practical applications. Our goal with this paper is to contribute to the emergence and progress of the field.

We are particularly motivated by the two-tiered City Logistics systems which are increasingly proposed to address the considerable challenges of freight transportation within large urban areas (Crainic et al., 2004). Such systems are characterized by consolidation of loads of different shippers and consignees within the same ("green") vehicles and by the coordination of carrier operations, through the deployment of intermediate facilities, named satellites, and the coordination of two fleets of vehicles performing tours between external platforms (or city distribution centers) and satellites, and between these and the actual customers. Setting up or evaluating the expected performance of a two-tiered City Logistics system thus requires to simultaneously determine the locations of new external platforms (or selection of those where work will be performed) and those of the satellites, while also determining typical vehicle routes at both levels.

The goal of this paper is to introduce the *two-echelon location-routing* problem, *2E-LRP*, and to gain insight into how it behaves and could be addressed. We describe the issues, set up the general problem class, and study its basic variant, which already displays a high level of complexity. Three MIP formulations are proposed and compared. The computational evaluation and comparisons are performed on a large set of instances inspired by various settings of two-tiered City Logistics systems in terms of number and relative distribution of potential locations for the two types of facilities.

To sum up, the contributions of the paper are: 1) describing the two-echelon location-routing problem class; 2) proposing three mathematical formulations for the basic 2E-LRP and computationally comparing them on a set of realistic and diverse problem instances. The proposed models unify two areas of study, addressed separately up to now, the two-echelon facility location and two-echelon vehicle routing problems..

The paper is structured as follows. The two-echelon freight distribution system is described in Section 2, together with the assumptions leading to the basic *2E-LRP*. The relevant literature is reviewed in Section 3. Section 4 is dedicated to the three MIP models. Section 5 describes an instance generator and presents the computational results for the proposed models on a wide set of small and medium size instances.

2 Problem Setting

We describe in this section the general two-echelon location-routing problem and identify a number of major problem classes in this area. We then proceed to state the basic version we address in this paper. For simplicity's sake, we start from the vocabulary of two-tiered City Logistics systems. This does not reduce the scope of the paper, however, the problem description and mathematical formulations being relevant for 2E-LRP in any setting.

2.1 Two Echelon Location-Routing Problems

Platforms, satellites, and customers interact in two-tiered City Logistics systems, linked by two or more vehicle fleets performing distribution activities. Briefly (see Crainic et al., 2004, for more details), shipments with destination within the city (under City Logistics "control") are delivered at platforms, or *primary facilities*, where they are sorted and consolidated into first-echelon *urban vehicles*. These vehicles deliver the freight at satellites, or *secondary facilities*, visiting one or more on their routes, and return to platforms. Freight delivered at satellite is transferred (often through cross-dock operations) into second-echelon vehicles, or *city freighters*, appropriate for operations in the controlled areas. Each city freighter then performs a route to serve several customers and generally travels back to a satellite (it could return to the garage) to start a new route. Figure 1 illustrates these elements (time aspects are not considered).

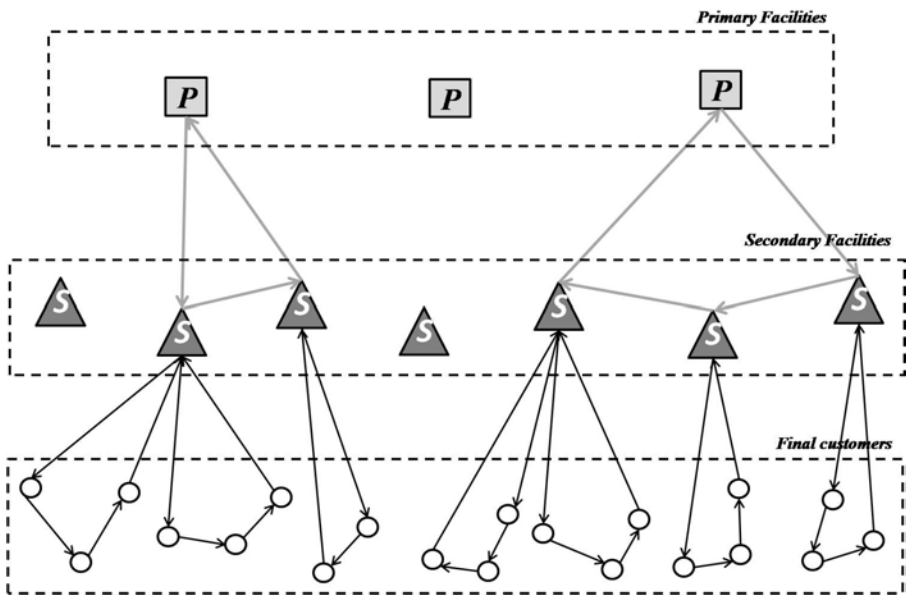

Figure 1: Two-echelon distribution system representation.

The goal of City Logistics is to reduce the generalized cost of distribution, which accounts for the impact of freight vehicles on the city (congestion/mobility, environment, safety, etc.) and the cost of providing the service. Similarly to any multi-echelon system, efficiency has to be achieved through adequately planned and executed system layout and operations (Benjelloun and Crainic, 2008; Crainic et al., 2004). The layout of the system, i.e., the location of the primary and secondary facilities, actually constitutes a major factor in the smooth and efficient operations of the system. In multi-echelon systems this implies not only the "absolute" position of facilities within the transportation network relative to customers, but also the "relative" position of primary and secondary facilities, one level with respect to the other and to the final customers. It is the routing of the first and second echelon vehicles that transforms these relative positions into efficient and productive operations.

The two-echelon location-routing problem 2E-LRP may therefore be described as follows. Given a two-echelon distribution system with its physical, operational, and economic characteristics, possible location sites for primary and secondary facilities, and an estimate of customer demands, select the facilities to open at each level, and the corresponding typical vehicle routes, to minimize the total cost of the system satisfying the demand.

Many problem variants may be defined out of this general setting, based on the characteristics of the case at hand and the aim of the study. Such major characteristics, in terms of their impact on the type and subsequent difficulty of the formulation and solution process, are:

Demand definition and requirements. Two main issues should be considered with respect to demand, homogeneity and *substitutability*. The simplest case with respect to the former is to consider that all freight is delivered in the same type of box and, thus, the problem becomes *single commodity*. When *multi-commodity* settings are contemplated, several other issues must be addressed, e.g., the compatibility of particular products and vehicles.
In many problem settings, demand is assumed *substitutable*, meaning that the flow required to reach a particular destination may come from any origin in the network with sufficient capacity. This is the case, for example, in most vehicle routing problems (VRP) and, thus, in most location-routing problems in the literature. *Origin-to-destination* demand(by product in multi-commodity settings), *OD*, is assumed in the opposite case, generally encountered in network flow-based problems, e.g., network and service network design in transportation, logistics, telecommunications, and so on.

Time dependency. A freight distribution system could be designed considering a single period, *static* problem, assuming its behavior is homogeneous for the length of the planning horizon. Alternatively, a multi-period planning horizon, *time-dependent* problem, provides the means to take into account variability of system parameters, e.g., demand and travel times.

Timing. Two main characteristics should be considered in this respect. The first relates to the usual *time-windows* restrictions on the operations of facilities and service to customers. The second is proper to multi-level systems and addresses the issue of synchronization of operations at facilities. Thus, for City Logistics, vehicles operating on different levels but being involved in trans-dock transfers at a satellite should be time-coordinated to avoid waiting delays at the respective satellite (Crainic et al., 2004).

Data uncertainty. *Deterministic* problems assume all relevant data and parameters will not change for the length of the planning horizon, and no "unexpected" external event, man made or natural, will disturb the normal activities of the system. *Stochastic* problems are defined when this hypothesis can no longer be assumed. Unexpected events might be relevant for long and medium-term planning to provide insight into alternate operating strategies when part of the urban network becomes unavailable (due to, e.g., traffic accident, infrastructure failure, and so on.), but are very difficult to estimate and they are not the first ones to be addressed in the planning literature. Similarly to other problem classes in transportation and logistics, demand uncertainty will probably be the first uncertain element to be addressed. Travel-time uncertainty also appears interesting, but more so for operational planning and control. Indeed, particularly in urban zones, one can have a pretty accurate statistical prediction for the travel times for given days during the week and times of day.

2.2 The basic 2E-LRP

We can now specify the basic 2E-LRP addressed in this paper. We consider a single-commodity, substitutable, static, and deterministic problem setting. Facilities are limited in the quantity of freight they can handle, the capacity of platforms being higher than that of the satellites. Facilities on the same echelon may have different capacities. Freight is assumed to be at the platforms and to flow through satellites to customers. No direct shipping is allowed.

Two fleets of vehicles provide transportation services, one at each echelon. All vehicles have capacity limitations, those belonging to the same fleet (on the same echelon) sharing the same capacity value. The capacity of first-echelon vehicles is higher than the largest satellite capacity, while that of second-echelon vehicles is higher than the largest customer demand. Each 1^{st} echelon route starts from a platform, serves one or more satellites, and returns to the same platform. Similarly, a 2^{nd} echelon route starts from a satellite, serves one or more customers, and returns to the same satellite. Single sourcing is assumed for satellites and customers. Thus, each satellite/customer has to be served from a single platform/satellite by a single vehicle of the appropriate fleet.

This basic *single and substitutable commodity, single sourcing, capacitated, static, deterministic* 2E-LRP targets decisions on:

Facility location. Determine the location and number of platforms and satellites;

Allocation. Assign each customer to one of the open satellites, and each satellite to one of the selected platforms;

Fleet size and routing. Determine the number of vehicles to use in each fleet and the associated routes at each echelon.

Figure 1 illustrates this basic problem setting, and Section 4 introduces the corresponding formulations. Notice that the 2E-LRP is *NP-hard*, as shown via a reduction to facility location and vehicle routing problems. Indeed, requiring customers and satellites to be directly linked to a facility (no routing), reduces the problem to a standard multi-level FLP. If, on the other hand, we fix facility locations, the 2E-LRP reduces to a multi-level VRP. We now proceed to a brief review of relevant literature.

3 Literature Review

At the best knowledge of the authors, there are no contributions in the literature for the 2E-LRP as defined above. We briefly review related fields, however, focusing on two-echelon vehicle routing and location-routing problems.

Contributions directly targeting two-echelon vehicle routing problems (*2E-VRP*s) are recent. Crainic et al. (2004) introduced the two-echelon, synchronized, scheduled, multi-depot, multiple-tour, heterogeneous vehicle routing problem with time windows (*2SS-MDMT-VRPTW*), proposed arc and path-based formulations and discussed algorithmic perspectives. Addressing this general problem setting is still an open problem, however.

Perboli et al. (2010b) introduced the basic 2E-VRP with hypotheses similar to the ones described above for the 2E-LRP we propose. The authors proposed a flow based model, several valid inequalities (see also Perboli et al., 2010a), and two matheuristics. Clustering-based heuristics for this problem were proposed by Crainic et al. (2008). The heuristic proceeds by decomposing the 2E-VRP into two routing sub-problems, one for each echelon, and thus addresses rather large instances. The authors then used this heuristic within multi-start approaches (Crainic et al., 2010, 2011) and improved the quality of the solutions obtained.

The strong relations between distribution routes and costs, on the one hand, and location of facilities, on the other hand, as well as the need to model and optimize them simultaneously have been observed early on (e.g., Webb, 1972; Christophides and Eilon, 1969; Eilon et al., 1971; Wren and Holliday, 1968; Perl and Daskin, 1985; Salhi and Rand, 1989; Chien, 1992)). The idea of combining two decision levels, strategic and tactical, for a transportation system dates back to 1960 (Maranzana, 1964), but papers on LRP started to appear in large numbers in the '80s, as reflected in the surveys mentioned in the Introduction.

Laporte (1988) provided a definition of location-routing problems, together with a classification based on the number of interacting levels and the type of routes connecting them. The author introduced expression $\lambda/M_1/\ldots/M_{\lambda-1}$ to represent a location-routing problem, where λ is the number of layers and M_i indicates the type of tour one needs to build between two consecutive layers i and $i+1$, R for customer-dedicated routes (back and forth) and T for multi-customer routes. An LRP is then characterized by location decisions and T routes at least for one level.

We propose a simple enhancement of this notation to clearly indicate the number of levels where location decisions have to be performed: mark the route identifier letter (R or T) with an *overline* if location decisions have to be taken on the same echelon. By default, location decisions concern always the starting point of the routes. To illustrate, the expression $3/\overline{R}/\overline{T}$ refers to a problem with three layers, location decisions for primary and secondary facilities, R routes between the first and second levels (first echelon) and T routes between the second and the third levels (second echelon). The expression $3/R/\overline{T}$ indicates that location decisions involve just secondary facilities.

Literature on multi-echelon location-routing problems is extremely limited. Actually, we are aware of only two contributions, Ambrosino and Scutellá (2005) and Boccia et al. (2010), addressed below. Otherwise, even when the two echelon location-routing setting is mentioned, location takes place on one level only. Thus, Jacobsen and Madsen (1980), Madsen (1983), and Bruns and Klose (1996) propose heuristics to address the $3/T/\overline{T}$ problem. Taniguchi et al. (1999) address a $3/T/\overline{T}$ problem in the context of City Logistics. A mathematical model is proposed aimed to determine the location and the size of satellites, referred to as public logistics terminals. Satellites are considered as small platforms and not just transshipment points. Exact and heuristic approaches are proposed.

Ambrosino and Scutellá (2005) address $4/R/\overline{T}/\overline{T}$ problems represented as mixed-integer programming models. Different formulations are proposed for a distribution network design problem with location decisions for primary and secondary facilities in static and time-dependent settings, extending the three-index arc and flow formulations proposed in Perl and Daskin (1985) and Hansen et al. (1994), respectively. In the other contribution directly targeting the 2E-LRP, Boccia et al. (2010) propose a tabu search meta-heuristic for the $3/\overline{T}/\overline{T}$ problem described in this paper. The authors decompose the 2E-LRP into four subproblems, one FLP and one VRP for each echelon. The four sub-problems are then sequentially and iteratively solved and their solutions are opportunely combined to determine a good global solution.

4 Two-Echelon Location-Routing Formulations

We propose three formulations for the 2E-LRP, differing by the type and number of routing variables. The first and third models are derived from classical VRP formulations

(Toth and Vigo, 2002), while the second is inspired by the multi-depot vehicle-routing formulation (*MDVRP*) proposed in Dondo and Cerdá (2007). We first introduce the notation common to all three formulations.

4.1 Notation

The $3/\overline{T}/\overline{T}$ 2E-LRP we address is defined on a multi-level network $\mathcal{G} = \{\mathcal{N}, \mathcal{A}\}$. The node set $\mathcal{N} = \{\mathcal{P} \cup \mathcal{S} \cup \mathcal{Z}\}$ is made up of the sets of possible platform locations $\mathcal{P} = \{p\}$ (first level), possible satellite locations $\mathcal{S} = \{s\}$ (second level), and customers $\mathcal{Z} = \{z\}$ (fixed positions on the third level). Arcs in $\mathcal{A} = \{\mathcal{A}^1 \cup \mathcal{A}^2\}$ link consecutive node levels and build up routes, where $\mathcal{A}^1 = \{(i, j),\ i, j \in \mathcal{P} \cup \mathcal{S}\}$ is the set of first-echelon arcs from platforms to satellites, between satellites, and back to the starting platform, while arcs in $\mathcal{A}^2 = \{(i, j),\ i, j \in \mathcal{S} \cup \mathcal{Z}\}$ is the set of second-echelon arcs from satellites to customers, among customers, and back to satellites.

Fixed costs $H_i,\ i \in \mathcal{P} \cup \mathcal{S}$ are associated to locating facilities. These costs are related to the contemplated facility capacity $K_i,\ i \in \mathcal{P} \cup \mathcal{S}$. Customer demand is noted $D_z,\ z \in \mathcal{Z}$

Two sets of vehicles are considered, urban vehicles $\mathcal{T} = \{t\}$ on the first level and city freighters $\mathcal{V} = \{v\}$ on the second.. Each vehicle is characterized by an utilization cost h^i and a capacity $k^i,\ i \in \mathcal{T} \cup \mathcal{V}$. The cost of operating a vehicle on a link of the network is noted $C_{ij},\ i, j \in \mathcal{N}$.

The set of facility-location decision variables

$$y_i = 1, \text{ if the facility at node } i,\ i \in \mathcal{P} \cup \mathcal{S} \text{ is open, and } 0, \text{ otherwise,}$$

are common to all proposed formulations. Additional variables and, in particular, routing and vehicle-utilization variables are particular to each formulation and are defined in the appropriate sub-section.

4.2 Three index formulation: *3i-2E-LRP*

This model has been inspired by the multi-echelon LRP formulation proposed by Ambrosino and Scutellá (2005). Differences derive from problem setting, e.g., two different vehicle fleets with specified territories, and the subtour elimination constraints.

Let's define the additional sets of variables

- 1st *echelon routing*: $r^t_{ij} = 1$, if node i precedes node j, $i, j \in \mathcal{P} \cup \mathcal{S}$, in the first echelon route performed by urban vehicle $t, t \in \mathcal{T}$, and 0, otherwise;

- 2^{nd} *echelon routing*: $x_{ij}^v = 1$, if node i precedes node j, $i, j \in \mathcal{S} \cup \mathcal{Z}$, in the second echelon route performed by city freighter $v, v \in \mathcal{V}$, and 0, otherwise;

- *Customer assignment*: $w_{sz} = 1$, if customer $z, z \in \mathcal{Z}$, is assigned to satellite $s, s \in \mathcal{S}$, 0, otherwise;

- *Vehicle selection*: $u_i = 1$, if vehicle $i, i \in \mathcal{T} \cup \mathcal{V}$, is used, and 0, otherwise;

- *Flows*: $f_{ps}^t \geq 0$, flow from platform $p, p \in \mathcal{P}$, to satellite $s, s \in \mathcal{S}$, on urban vehicle $t, t \in \mathcal{T}$.

We refer to this formulation as 2E-LRP *three-index* formulation because routing variables, r_{ij}^t and x_{ig}^v, are defined using three indices. Figure 2 illustrates the parameters and variables of this formulation, the latter being indicated in lower-case bold letters.

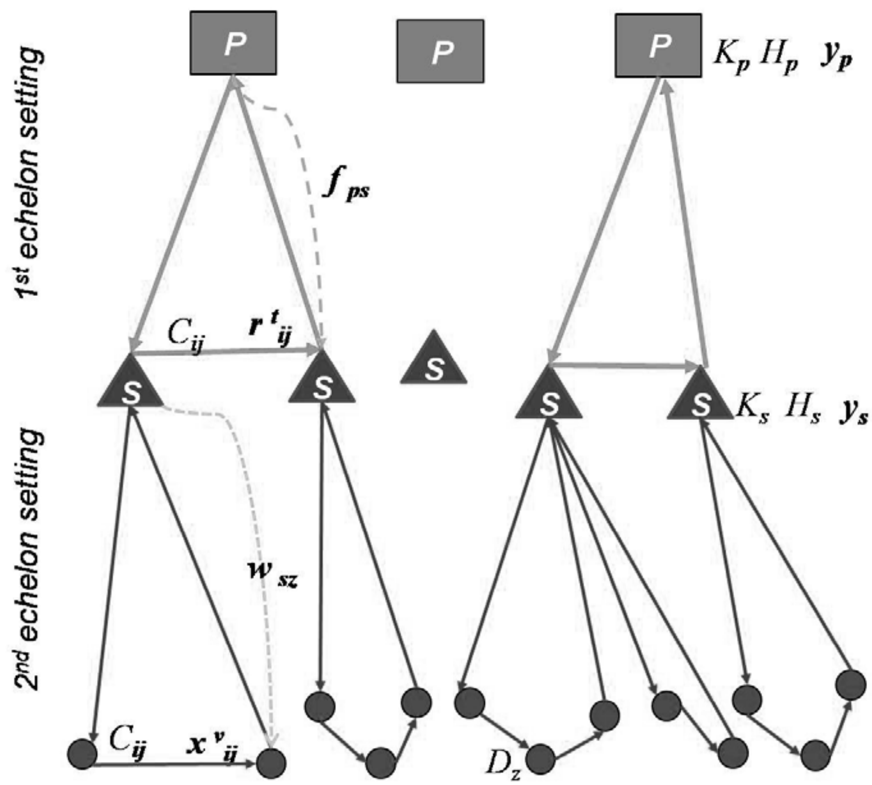

Figure 2: Variables and parameters of the *2E-LRP* three-index formulation

The problem can be formulated as follows:

$$\text{Minimize} \quad \sum_{p\in\mathcal{P}} H_p\ y_p + \sum_{s\in\mathcal{S}} H_s\ y_s + \sum_{t\in\mathcal{T}} h^t\ u_t + \sum_{v\in\mathcal{V}} h^v\ u_v + \\ + \sum_{v\in\mathcal{V}} \sum_{i\in\mathcal{S}\cup\mathcal{Z}} \sum_{j\in\mathcal{S}\cup\mathcal{Z}} C_{ij}\ x_{ij}^v + \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{P}\cup\mathcal{S}} \sum_{j\in\mathcal{P}\cup\mathcal{S}} C_{ij}\ r_{ij}^t \tag{1}$$

$$\text{Subject to} \quad \sum_{v\in\mathcal{V}} \sum_{j\in\mathcal{S}\cup\mathcal{Z}} x_{zj}^v = 1 \qquad \forall z \in \mathcal{Z} \tag{2}$$

$$\sum_{l\in\mathcal{S}\cup\mathcal{Z}} x_{lj}^v - \sum_{l\in\mathcal{S}\cup\mathcal{Z}} x_{jl}^v = 0 \qquad \forall j \in \mathcal{Z}\cup\mathcal{S}, \forall v \in \mathcal{V} \tag{3}$$

$$L_i - L_j + (|S| + |Z|) \sum_{v\in\mathcal{V}} x_{ij}^v \le (|S| + |Z| - 1) \qquad \forall i,j \in \mathcal{Z}\cup\mathcal{S}, i \ne j \tag{4}$$

$$\sum_{l\in\mathcal{S}\cup\mathcal{Z}} \sum_{j\in\mathcal{S}} x_{lj}^v \le 1 \qquad \forall v \in \mathcal{V} \tag{5}$$

$$\sum_{t\in\mathcal{T}} \sum_{j\in\mathcal{P}\cup\mathcal{S}} r_{lj}^t = y_l \qquad \forall l \in \mathcal{S} \tag{6}$$

$$\sum_{l\in\mathcal{P}\cup\mathcal{S}} r_{lh}^t - \sum_{l\in\mathcal{P}\cup\mathcal{S}} r_{hl}^t = 0 \qquad \forall h \in \mathcal{P}\cup\mathcal{S}, \forall t \in \mathcal{T} \tag{7}$$

$$L_i - L_j + (|P| + |S|) \sum_{t\in\mathcal{T}} r_{ij}^t \le (|P| + |S| - 1) \qquad \forall i,j \in \mathcal{S}\cup\mathcal{P}, i \ne j \tag{8}$$

$$\sum_{l\in\mathcal{P}\cup\mathcal{S}} \sum_{j\in\mathcal{P}} r_{lj}^t \le 1 \qquad \forall t \in \mathcal{T} \tag{9}$$

$$\sum_{h\in\mathcal{S}\cup\mathcal{Z}} x_{zh}^v + \sum_{h\in\mathcal{S}\cup\mathcal{Z}} x_{sh}^v - w_{zs} \le 1 \qquad \forall z \in \mathcal{Z}, \forall v \in \mathcal{V}, \forall s \in \mathcal{S} \tag{10}$$

$$\sum_{s\in\mathcal{S}} w_{zs} = 1 \qquad \forall z \in \mathcal{Z} \tag{11}$$

$$\sum_{p\in\mathcal{P}} \sum_{t\in\mathcal{T}} f_{ps}^t - \sum_{z\in\mathcal{Z}} D_z\ w_{zs} = 0 \qquad \forall s \in \mathcal{S} \tag{12}$$

$$\sum_{s\in\mathcal{S}} f_{ps} - K_p\ y_p \le 0 \qquad \forall p \in \mathcal{P} \tag{13}$$

$$\sum_{p\in\mathcal{P}} \sum_{t\in\mathcal{T}} f_{ps}^t - K_s\ y_s \le 0 \qquad \forall s \in \mathcal{S} \tag{14}$$

$$k^t \sum_{h\in\mathcal{S}\cup\mathcal{P}} r_{sh}^t - f_{ps}^t \ge 0 \qquad \forall t \in \mathcal{T}, \forall s \in \mathcal{S}, \forall p \in \mathcal{P} \tag{15}$$

$$k^v \sum_{h\in\mathcal{S}\cup\mathcal{P}} r_{ph}^t - f_{ps}^t \ge 0 \qquad \forall t \in \mathcal{T}, \forall s \in \mathcal{S}, \forall p \in \mathcal{P} \tag{16}$$

$$\sum_{i \in \mathcal{Z}} D_z \sum_{j \in \mathcal{S} \cup \mathcal{Z}} x^v_{zj} \leq k^v\ u_v \qquad \forall v \in \mathcal{V} \tag{17}$$

$$\sum_{p \in \mathcal{P}} \sum_{s \in \mathcal{S}} f^t_{ps} \leq k_t\ u^t \qquad \forall t \in \mathcal{T} \tag{18}$$

$$\begin{aligned}
r^t_{ij} &= \{0,1\} && \forall i,j \in \mathcal{P} \cup \mathcal{S}, t \in \mathcal{T} \\
x^v_{ij} &= \{0,1\} && \forall i,j \in \mathcal{S} \cup \mathcal{Z}, v \in \mathcal{V} \\
w_{zs} &= \{0,1\} && \forall z \in \mathcal{Z}, s \in \mathcal{S} \\
y_p &= \{0,1\} && \forall p \in \mathcal{P} \\
y_s &= \{0,1\} && \forall s \in \mathcal{S} \\
u_t &= \{0,1\} && \forall t \in \mathcal{T} \\
u_v &= \{0,1\} && \forall v \in \mathcal{V} \\
f^t_{ps} &\geq 0 && \forall p \in \mathcal{P}, s \in \mathcal{S}, t \in \mathcal{T}
\end{aligned} \tag{19}$$

The objective function (1) is the sum of six cost components: opening cost for platforms and satellites, utilization costs for vehicles, transportation cost on the two echelons. The model constraints control the routing at the first and second echelons, the flow conservation, the capacities of facilities and vehicles, and the consistency (linking) relations among variables.

Constraints (2) impose that each customer z, $z \in \mathcal{Z}$, is served by exactly one city freighter v, $v \in \mathcal{V}$. Constraints (3) impose that for each city freighter v, $v \in \mathcal{V}$, the number of arcs entering a node i, $i \in \mathcal{Z} \cup \mathcal{S}$, is equal to the number of arcs leaving the node. Constraints (4) are subtour elimination constraints, where L_i and L_j are continuous non-negative variables, imposing the inclusion of at least a satellite in each route performed by a city freighter. Constraints (5) impose that each city freighter v, $v \in \mathcal{V}$, has to be assigned unambiguously to one satellite s, $s \in \mathcal{S}$, i.e. each vehicle can perform one route only. Constraints (6) to (9) are the routing constraints imposing on the first echelon the same conditions that constraints (2) to (5) impose on second echelon.

Constraints (12) are the flow conservation constraints at satellites..

Constraints (13) and (14) impose that the flow leaving a platform p, $p \in \mathcal{P}$ or entering a satellite s, $s \in \mathcal{S}$, has to be less than the capacity of the respective facility, if it is open. Constraints (17) require that the demand satisfied by city freighter v, $v \in \mathcal{V}$, has to be less than the vehicle capacity, if used. Similarly, constraints (18) impose that the amount of flow transferred by an urban vehicle t, $t \in \mathcal{T}$, has to be less than its capacity, if used.

Constraints (10) link allocation and routing variables. Actually, according to constraints (2), each customer is assigned to exactly one city freighter v, $v \in \mathcal{V}$. This, together with constraints (3) to (5), imply that exactly one satellite is on the route of v. Then, for any customer z^*, assigned to a route v^* containing a satellite s^*, the

$\sum_{h\in\mathcal{S}\cup\mathcal{Z}} x^{v^*}_{z^*h} = 1$ and $\sum_{h\in\mathcal{S}\cup\mathcal{Z}} x^{v^*}_{s^*h} = 1$ and, consequently, $w_{z^*s^*} = 1$. When the customer is not on a route starting from satellite s^*, constraints (10) are satisfied for both $w_{zs} = 0$ and $w_{zs} = 1$, but since each customer has to be assigned to just one satellite, it will be assigned to the one satisfying its demand. Constraints (11), imposing that each customer z has to be assigned to a satellite s, are thus redundant, but allow to slightly improve the bounds of linear relaxations.

Constraints (15) and (16) guarantee that the amount of flow on a vehicle t, $t \in \mathcal{T}$, from a platform p, $p \in \mathcal{P}$, to a satellite s, $s \in \mathcal{S}$, is positive if and only if both satellite and platform are visited by the same vehicle. Constraints (19) determine feasible range for variables in terms of integrality and non-negativity.

The three index model yields a very flexible formulation that can address location-routing problems with both symmetric and asymmetric cost matrices. It can be extended to take into account other features, such as multi-commodity flows, introduction of time windows for customers, maximum route length constraints, heterogeneous fleets, etc. On the other side it is hard to solve, since it requires the definition of a large number of variables and constraints.

4.3 Two-index formulation: *2i-2E-LRP*

In the two-index 2E-LRP formulation, routing decisions are defined with two-index variables, relative to assignment and sequencing. The formulation is inspired by the MDVRP model of Dondo and Cerdá (2007).

We define the additional sets of variables

- *Customer-to-freighter assignment*: $a_{zv} = 1$, if customer $z, z \in \mathcal{Z}$, is assigned to city freighter $v, v \in \mathcal{V}$, 0, otherwise;
- *Freighter-to-satellite assignment*: $b_{sv} = 1$, if city freighter $v, v \in \mathcal{V}$, is assigned to satellite $s, s \in \mathcal{S}$, 0, otherwise;
- *Customer sequencing*: $x_{ij} = 1$, if node i precedes node j, $i, j \in \mathcal{Z} \mid i < j$, and 0, otherwise;
  Note that a single variable is defined for each couple of nodes, based of relative ordering of nodes, i.e. x_{ij} exists if $ord(i) < ord(j)$, where $ord(i)$ indicates the relative position of element i in the customer set $\mathcal{Z}$); In this way number of sequencing variable is cut by half;
- *Satellite-to-vehicle assignment*: $m_{st} = 1$, if satellite $s, s \in \mathcal{S}$, is assigned to urban vehicle $t, t \in \mathcal{T}$, and 0, otherwise;

- *Vehicle-to-platform assignment*: $n_{pt} = 1$, if urban vehicle $t, t \in \mathcal{T}$, is assigned to platform $p, p \in \mathcal{P}$, and 0, otherwise;

- *Satellite sequencing*: $r_{ij} = 1$, if satellite i is visited before satellite j, $i, j \in \mathcal{S} \mid i < j$, and 0, otherwise;
  The discussion above relative to the number of variables x_{ij} is valid also for r_{ij} variables;

- *Assignment*: $w_{ij} = 1$, if node i is assigned to node j, and 0, otherwise; For the first echelon, $i \in \mathcal{S}, j \in \mathcal{P}$; for the second, $i \in \mathcal{S}, j \in \mathcal{P}$.

Figure 3 illustrates the variables of this formulation.

We also explicitly define a number of non-negative transportation costs

- $C^1(s)$, routing cost (first echelon) from an open platform up to satellite $s, s \in \mathcal{S}$;
- $C^2(z)$, routing cost (second echelon) from an open satellite up to customer $z, z \in \mathcal{Z}$;
- $C^V(v)$, total routing cost for city freighter $v, v \in \mathcal{V}$;
- $C^T(t)$, total routing cost for urban vehicle $t, t \in \mathcal{T}$.

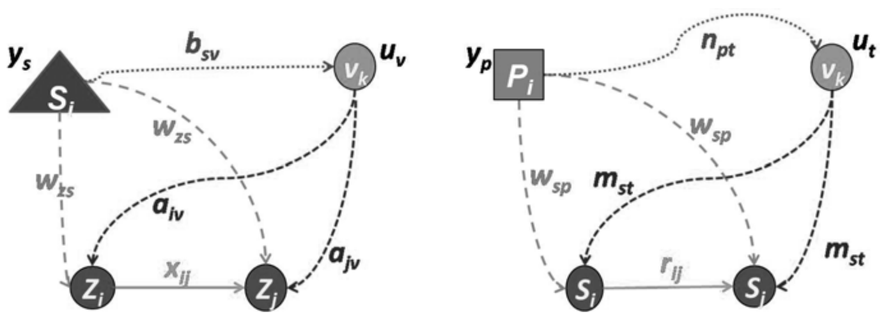

Figure 3: Variables of the *2E-LRP* two-index formulation

The two-index formulation for the 2E-LRP then takes the following form:

$$\text{Minimize} \quad \sum_{p \in \mathcal{P}} H_p \, y_p + \sum_{s \in \mathcal{S}} H_s \, y_s + h^t \sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} n_{pt} + \\ + h^v \sum_{s \in \mathcal{S}} \sum_{v \in \mathcal{V}} b_{sv} + \sum_{v \in \mathcal{V}} C^V(v) + \sum_{t \in \mathcal{T}} C^T(t) \tag{20}$$

$$\text{Subject to} \quad \sum_{v \in \mathcal{V}} a_{zv} = 1 \qquad \forall z \in \mathcal{Z} \tag{21}$$

$$\sum_{s \in \mathcal{S}} b_{sv} \leq 1 \qquad \forall v \in \mathcal{V} \tag{22}$$

$$C^2(i) \geq C_{si}\left(b_{jv} + a_{iv} - 1\right) \qquad \forall i \in \mathcal{Z}, s \in \mathcal{S}, v \in \mathcal{V} \tag{23}$$

$$C^2(j) \geq C^2(i) + C_{ji} - M\left(1 - x_{ij}\right) - M\left(2 - a_{jv} + a_{iv}\right) \qquad \forall i, j \in \mathcal{Z}_{i<j}, v \in \mathcal{V} \tag{24}$$

$$C^2(i) \geq C^2(j) + C_{ij} - M\left(x_{ij}\right) - M\left(2 - a_{jv} + a_{iv}\right) \qquad \forall i, j \in \mathcal{Z}_{j<i}, v \in \mathcal{V} \tag{25}$$

$$C^V v \geq C^2(i) + C_{is} - M\left(2 - b_{sv} - a_{iv}\right) \qquad \forall i \in \mathcal{Z}, s \in \mathcal{S}, v \in \mathcal{V} \tag{26}$$

$$\sum_{t \in \mathcal{T}} m_{st} = y_s \qquad \forall s \in \mathcal{S} \tag{27}$$

$$\sum_{p \in \mathcal{P}} n_{pt} \leq 1 \qquad \forall t \in \mathcal{T} \tag{28}$$

$$C^1(i) \geq C_{ji}\left(n_{jt} + a_{it} - 1\right) \qquad \forall i \in \mathcal{S}, j \in \mathcal{P}, t \in \mathcal{T} \tag{29}$$

$$C^1(j) \geq C^1(i) + C_{ji} - M\left(1 - r_{ij}\right) - M\left(2 - a_{jt} + a_{it}\right) \qquad \forall i, j \in \mathcal{S}_{i<j}, t \in \mathcal{T} \tag{30}$$

$$C^1(i) \geq C^1(j) + C_{ij} - M\left(r_{ij}\right) - M\left(2 - a_{jt} + a_{it}\right) \qquad \forall i, j \in \mathcal{S}_{j<i}, t \in \mathcal{T} \tag{31}$$

$$C^T t \geq C^1(i) + c_{ip} - M\left(2 - b_{pt} - a_{it}\right) \qquad \forall i \in \mathcal{S}, p \in \mathcal{P}, t \in \mathcal{T} \tag{32}$$

$$\sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} f_{ps}^t - \sum_{z \in \mathcal{Z}} D_z\ w_{zs} = 0 \qquad \forall s \in \mathcal{S} \tag{33}$$

$$\sum_{z \in \mathcal{Z}} D_z\ w_{zs} \leq K_s\ y_s \qquad \forall s \in \mathcal{S} \tag{34}$$

$$\sum_{s \in \mathcal{S}} f_{ps}^t - K_p\ y_p \leq 0 \qquad \forall p \in \mathcal{P} \tag{35}$$

$$\sum_{z \in \mathcal{Z}} D_z\ a_{zv} \leq k^v \sum_{s \in \mathcal{S}} b_{sv} \qquad \forall v \in \mathcal{V} \tag{36}$$

$$\sum_{p \in \mathcal{P}} \sum_{s \in \mathcal{S}} f_{ps}^t - k^t\ n_{pt} \leq 0 \qquad \forall t \in \mathcal{T} \tag{37}$$

$$a_{zv} + b_{sv} - w_{zs} \leq 1 \qquad \forall i \in \mathcal{Z}, s \in \mathcal{S}, v \in \mathcal{V} \tag{38}$$

$$k^t\ n_{pt} - f_{ps}^t \geq 0 \qquad \forall t \in \mathcal{T}, \forall s \in \mathcal{S}, \forall p \in \mathcal{P} \tag{39}$$

$$k^t\ m_{st} - f_{ps}^t \geq 0 \qquad \forall t \in \mathcal{T}, \forall s \in \mathcal{S}, \forall p \in \mathcal{P} \tag{40}$$

$$\begin{aligned}
a_{zv} &= \{0,1\} \qquad \forall z \in \mathcal{Z}, v \in \mathcal{V} & b_{sv} &= \{0,1\} \qquad \forall s \in \mathcal{S}, v \in \mathcal{V} \\
w_{zs} &= \{0,1\} \qquad \forall z \in \mathcal{Z}, s \in \mathcal{S} & x_{ij} &= \{0,1\} \qquad \forall i, j \in \mathcal{Z}_{(i<j)} \\
m_{st} &= \{0,1\} \qquad \forall s \in \mathcal{S}, t \in \mathcal{T} & n_{pt} &= \{0,1\} \qquad \forall p \in \mathcal{P}, t \in \mathcal{T} \\
r_{ij} &= \{0,1\} \qquad \forall i, j \in \mathcal{S}_{(i<j)} & y_s &= \{0,1\} \qquad \forall s \in \mathcal{S} \\
y_p &= \{0,1\} \qquad \forall p \in \mathcal{P}
\end{aligned} \tag{41}$$

The objective function (20) minimizes the total cost. The first group of constraints enforce routing restrictions, first for the second level, (21) to (26), and then for the first, (27) to (32). We detail for the former. Constraints (21) and (22) assign customers and satellites, respectively, to a single city freighter. Constraints (23) define the least cost for a city freighter to reach a customer. Constraints (24) and (25) define the relationships between the traveling costs up to customers i and j being serviced by same tour, with respect to the order of visit. Recall that C_{ij} is the travel cost from node i to adjacent node j. When both nodes are on the same tour performed by vehicle v, i.e., $y_{iv} = y_{jv} = 1$, and node i is visited before j, i.e., ($x_{ij} = 1$), then constraints (24) states that the routing cost from the satellite to customer j, $C^2(j)$, must be greater than $C^2(i)$, by at least C_{ij}. The reverse statement holds when j is visited before i, i.e., ($x_{ij} = 0$). Constraints (26) determine the total city-freighter routing cost.

Equations (33) are flow conservation constraints at satellites. Constraints (34), (35), (36), and (37) enforce capacity restrictions for satellites, platforms, city freighters, and urban vehicles, respectively.

Constraints (38) are consistency constraints among assignment variables. Constraints (39) and (40) are linking constraints between flow and assignment variables. Finally, Constraints (41) define the integrality or non-negativity of decision variables.

This formulation is suitable for symmetric problems. Similarly to the three index formulation, it requires the definition of a large set of constraints and variables. The two models present opposite features, however, the two-index formulation requiring less variables but a larger number of constraints. To illustrate this difference, consider that for an instance with 2 platforms, 8 satellites, 20 customers, 5 urban vehicles, and 8 city freighters, we obtain 7 063 variables and 2 242 constraints for the three-index formulation, and 783 variables and 7 552 constraints for two-index formulation.

4.4 Single-index formulation: *1i-2E-LRP*

The single-index formulation is path based. It generalizes the single-echelon location-routing formulation present in literature, where a variable is defined for all feasible routes. Two different sets of routes need to be defined for the 2E-LRP, one for each echelon. Let $\mathcal{R}^1$ and $\mathcal{R}^2$ represent the two sets. Let also $\mathcal{R}^1_p$ be the subset of $\mathcal{R}^1$ composed of routes starting from platform p. Define the following route-selection and flow decision variables:

- $x_i = 1$, if second-echelon route $i, i \in \mathcal{R}^2$, is selected, and 0, otherwise;
- $r_i = 1$, if first-echelon route $i, i \in \mathcal{R}^1$, is selected, and 0, otherwise;
- $f(i) \geq 0$, the flow on first echelon route $i, i \in \mathcal{R}^1$, from a platform to a satellite.

A cost C_i is associated to each route $i, i \in \mathcal{R}^1 \cup \mathcal{R}^2$. Two incidence matrices, A and B, are defined to specify the definition of paths (routes) in terms of nodes (satellites and customers), where

- $\alpha_{is} = 1$, if satellite $s, s \in \mathcal{S}$, is covered by first-echelon route $i, i \in \mathcal{R}^1$, and 0, otherwise;
- $\beta_{iz} = 1$, if customer $z, z \in \mathcal{Z}$, is covered by second-echelon route $i, i \in \mathcal{R}^2$, and 0, otherwise.

Similarly, two incidence matrices, E and F, specify the feasible platform-to-satellite and satellite-to-customer assignments, where

- $\epsilon_{ps} = 1$, if satellite $s, s \in \mathcal{S}$, may be services by platform $p, p \in \mathcal{P}$, and 0, otherwise;
- $\varphi_{sz} = 1$, if customer $z, z \in \mathcal{Z}$, may be services by satellite $s, s \in \mathcal{S}$, and 0, otherwise.

The 2E-LRP can then be formulated as a single-index model:

$$\text{Minimize} \quad \sum_{p \in \mathcal{P}} H_p\ y_p + \sum_{s \in \mathcal{S}} H_s\ y_s + h^t \sum_{i \in \mathcal{R}^1} r_i + h^v \sum_{i \in \mathcal{R}^2} x_i + \sum_{i \in \mathcal{R}^1} C_i\ r_i + \sum_{i \in \mathcal{R}^2} C_i\ x_i \tag{42}$$

$$\text{Subject to} \quad \sum_{s \in \mathcal{S}} \varphi_{sz}\ y_s = 1 \qquad \forall z \in \mathcal{Z} \tag{43}$$

$$\sum_{i \in \mathcal{R}^2} \beta_{iz}\ x_i = \sum_{j \in \mathcal{S}} \varphi_{jz}\ y_j \qquad \forall z \in \mathcal{Z} \tag{44}$$

$$\sum_{p \in \mathcal{P}} \epsilon_{ps}\ y_p = y_s \qquad \forall j \in \mathcal{S} \tag{45}$$

$$\sum_{i \in \mathcal{R}^1} \alpha_{is}\ r_i = \sum_{p \in \mathcal{P}} \epsilon_{ps}\ y_p \qquad \forall s \in \mathcal{S} \tag{46}$$

$$\sum_{i \in \mathcal{R}^1} f_i\ \alpha_{is} - \sum_{z \in \mathcal{Z}} D_z\ \varphi_{sz}\ y_s = 0 \qquad \forall p \in \mathcal{P} \tag{47}$$

$$\sum_{z \in \mathcal{Z}} D_z\ \varphi_{sz}\ y_s \leq K_s\ y_s \qquad \forall s \in \mathcal{S} \tag{48}$$

$$\sum_{i \in \mathcal{R}^1_p} f_i \leq K_p\ y_p \qquad \forall p \in \mathcal{P} \tag{49}$$

$$k^g\ r_i - f_i \geq 0 \qquad \forall i \in \mathcal{R}^1 \tag{50}$$

$$
\begin{aligned}
r_i &= \{0,1\} && \forall i \in \mathcal{R}^1 \\
x_i &= \{0,1\} && \forall i \in \mathcal{R}^2 \\
y_p &= \{0,1\} && \forall p \in \mathcal{P} \\
y_s &= \{0,1\} && \forall s \in \mathcal{S} \\
f_i &\geq 0 && \forall i \in \mathcal{R}^1 \qquad (51)
\end{aligned}
$$

The objective function (42) minimizes the total cost. Constraints (43) impose that each customer is served by just one satellite, while Constraints (44) specify that if a customer is served by a satellite, then it has to be served on just one route passing through that satellite. Constraints (45) and (46) impose to the first echelon the routing conditions just described for the second. Equations (47) are flow-balance constraints at satellites. Constraints (48) enforce the satellite capacity restrictions. Constraints (49) and (50) are linking constraints for platforms and routes, respectively, enforcing their capacity restrictions. Constraints (50) enforce the binary and non-negativity definition of decision variables.

This model, referred to in literature also as a set partitioning formulation, is very compact and flexible. As extensively shown in the VRP and scheduling literature, it can account for a large variety of quite complex routing requirements. On the other hand, this formulation requires a huge number of variables to represent all feasible first and second echelon routes, which can be performed for each possible platform-satellite configuration. The number of these configurations increases very rapidly with the instance size. For example, a small instance with two platform locations, three satellite locations, and eight customers has 21 possible platform-satellite configurations, whereas an instance with three platforms, five satellites, and fifteen customers has 217 possible platform-satellite configurations. Therefore, solving a one-index formulation requires either a heavy route-generation phase, possible for very small instances only or, calling on column generation techniques.

5 Computational Experiments

We present the results of computational tests on small and medium instances of the three and two-index formulations proposed for the 2E-LRP. Results were obtained by the XPRESS-MP solver and are compared in terms of computation time, bounds, and quality of solutions. The instance generation is introduced first, followed by the result analysis..

The single-index formulation does not appear in this comparison, because the instances that might be directly addressed are too small and developing a column generation-based solver goes way beyond the scope of the present paper.. We can, however, provide an estimation of the computational effort required to solve the single-index model. Recall that this formulation is based on the determination, explicit or not, of all possible routes for each platform-satellite configuration. ¿From this point of view, solving the 2E-LRP is equivalent to solving a two-echelon multi-depot vehicle routing problem, *2E-MDVRP*, for each configuration, and keeping the best solution, minimizing both location and routing costs. No method has yet been proposed for the 2E-MDVRP. A solution approach is however available for the two-echelon capacitated vehicle routing problem, *2E-CVRP*, with one platform and two or more satellites in Perboli et al. (2010b). We can, based on the results reported in that paper, estimate a lower bound for the computation time of the 2E-LRP, which reaches several hours for the small instances described above. This further explains why such results are not part of the paper and confirms the need for column generation-based solution methods.

5.1 Instance generator

No instances were available to test and compare performances of the proposed 2E-LRP models. We therefore developed a random instance generator, coded in C++, and used it to generate several instance sets, which may be obtained from the authors on request.

The aim of the instance generator is to reproduce a schematic representation of a multi-level urban area. Customers and facilities are located within concentric circle rings of increasing ray, illustrated in Figure 4.

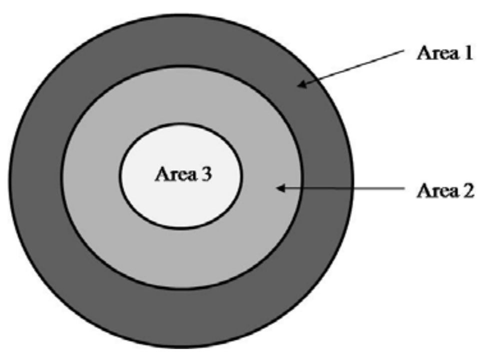

Figure 4: Schematic representation of a multi-level urban area

The instance generator is based on a number of parameters

- Urban area size represented by the ray values corresponding to the three areas, with the condition that $ray_1 \geq ray_2 \geq ray_3$;
- Instance size specifying the number of customers, $|\mathcal{Z}|$, and of facilities $|\mathcal{S}|$ and $|\mathcal{P}|$ to locate;

- Customer and facility spatial distribution. The positions of customers, satellites, and platforms were fixed randomly, generating the (X, Y) coordinates in the appropriate urban area according to the following criteria:
  - *Customers* were randomly located within Area 3;
  - *Satellites* were located within Area 2 or Area 3 according to a parameter α: α % of total number of satellites within Area 2 and $(1-\alpha)$% within Area 3;
  - *Platforms* were located within Area 1;
- Euclidean distances were computed among nodes;
- Customer demands, D_z, were randomly generated in the range $[D_{min}, D_{max}]$;
- Facility capacities, K_p and K_s, were randomly generated in the range $[K_{min}, K_{max}]$ and subdivided into a predetermined number of classes of equal length;
- Facility location costs were randomly generated in the range $[H_{min}, H_{max}]$ and subdivided into the same number of classes of equal length as for facility capacity. Moreover, facility costs were related to capacity values. Thus, for each randomly chosen capacity value, a random location cost is chosen within the correspondent cost class, as illustrated in Figure 5.
- Single values for vehicle capacities, k^t and k^v, and costs, h^t and h^v.

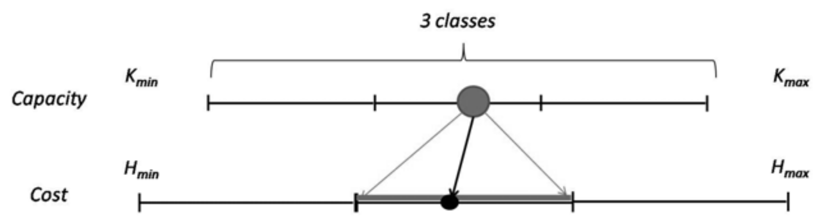

Figure 5: Capacity and location cost values

Three sets of instances were generated differing in the satellite distribution. Set *I1* has all satellites in Area 2, Set *I2* has them equally distributed in Areas 2 and 3, while in Set *I3*, satellites are only in Area 3. The distribution of satellites within the specified areas in the three instance sets is graphically represented in Figure 6.

Different combinations of the number of customers, satellites, and platforms were used to generate instances in each set, and the corresponding ranges appear in the first three lines of Table 1. The rest of the table displays the values used for the other parameters when generating instances.

In the following, instances will be indicated with a notation composed of the name of the instance set it belongs to, and the cardinalities of the customer and potential facility sites. For example *I1/2-8-20* stands for an instance of set I1 with 2 platforms, 8 satellites and 20 customers.

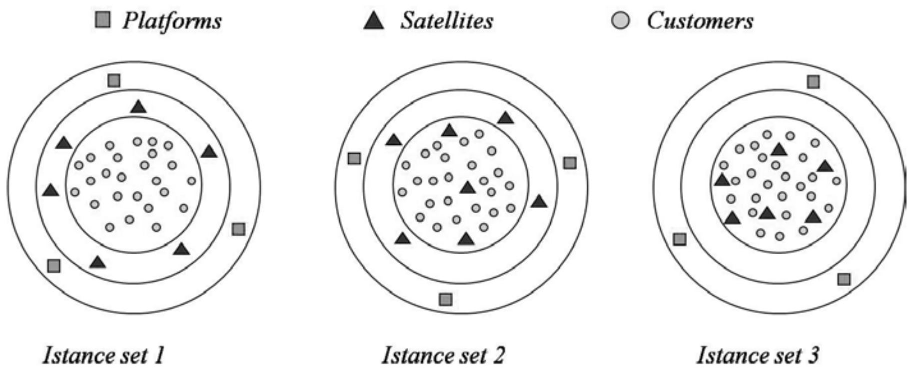

Figure 6: Satellite distribution in the three sets of instances

Parameter	Range or Value
Customer cardinality	$\{8, 10, 15, 20, 25\}$
Satellite cardinality	$\{3, 5, 8, 10\}$
Platform cardinality	$\{2, 3\}$
Customer demands	$D_z \in [1, 100]$
Satellite capacity	Three classes with ranges $U_s \in [300, 600]$
Satellite costs	Three classes with ranges $H_s \in [50, 90]$
Platform capacities	Three classes with ranges $U_p \in [900, 1600]$
Platform costs	Three classes with ranges $H_p \in [100, 160]$
Urban vehicle capacity and cost	$k_t = 800$ and $h_t = 50$
City freighter capacity and cost	$k_v = 200$ and $h_v = 25$

Table 1: Instance parameter values

5.2 Computational results

The three and two-index models were solved for the instances in the three sets defined above by XPRESS-MP. Experiments were run on an Intel(R) Pentium(R) 4 (2.40 GHz, RAM 4.00 GB) computer.

Tables 2, 3, and 4 display the results obtained on small and medium instances. For each instance and model, the tables display the values of the linear relaxation (*LR*), best lower bound (*BL*), best found solution (*BS*), percentage gap between *BS* and related *BL* (*Gap*%), and computation time in CPU seconds (*Time*). The evaluation of the gap for a generic instance *I*, is computed as GAP (%) = 100 (BS(I) - BL(I))/BS(I) . The execution stops when an overload occurs. The best solution for each instance is highlighted in bold and the optimal solutions are marked by an asterisk.

Instance	3i-2E-LRP					2i-2E-LRP				
	LR	BL	BS	Gap(%)	Time	LR	BB	BS	Gap(%)	Time
I1/2-3-8	415.46	678.28	**678.27***	**0.00**	61	224.03	678.28	**678.27***	**0.00**	8
I1/2-3-10	552.09	877.65	**877.64***	**0.00**	4198	272.42	877.65	**877.64***	**0.00**	671
I1/2-4-10	444.24	838.22	**838.21***	**0.00**	3152	215.69	838.22	**838.21***	**0.00**	363
I1/2-4-15	646.12	999.44	**1155.71**	13.52	21223	411.26	897.00	1274.40	29.61	19678
I1/2-8-20	628.91	821.72	**1104.13**	25.58	24043	520.49	642.67	1057.42	39.22	27131
I1/2-8-25	823.94	1068.98	**1445.43**	26.04	21043	646.05	850.05	1514.24	43.86	29913
I1/2-10-15	415.11	642.37	**760.00**	15.48	19275	332.50	437.38	771.54	43.31	22123
I1/2-10-20	701.57	929.49	**1051.87**	11.63	27336	505.15	719.15	1098.23	34.52	31879
I1/2-10-25	745.20	956.89	**1331.46**	28.13	31087	590.63	773.45	1307.90	40.86	35471
I1/3-5-10	399.11	848.17	**848.16***	**0.00**	4745	226.89	848.17	**848.16***	**0.00**	92
I1/3-5-15	623.11	830.96	**1003.12**	17.16	22153	375.46	830.96	**1003.12**	17.16	21187
I1/3-8-10	338.11	512.91	**512.90***	**0.00**	3300	260.00	447.86	**512.91**	12.68	15200
I1/3-8-15	577.42	719.92	**842.13**	14.51	23215	410.89	603.25	875.34	31.08	28768
I1/3-8-20	603.84	715.29	**898.21**	20.37	24000	441.49	627.55	949.96	33.94	29980
I1/3-8-25	763.77	877.46	**1212.42**	27.63	26467	557.38	798.00	1288.02	38.04	31732
I1/3-10-15	427.71	564.15	**711.12**	20.67	21533	328.19	437.50	754.54	42.02	27533
I1/3-10-20	591.30	790.11	**1030.02**	23.29	19325	436.71	595.55	1099.52	45.84	29815
I1/3-10-25	847.81	1437.24	1696.74	15.29	36000	524.18	871.76	**1527.42**	42.93	39138

Table 2: Results of 3- and 2-index formulations on instance set I1

We observe, examining the three tables, that the solver is able to determine the optimal solutions for both formulations in the case of small instances. Both optima are reached within the specified time limit, but the two-index formulation requires much lower computation times.

The situation changes when medium instances, with more than five satellites, are considered. The three-index formulation performs better than the two-index one, in terms of both quality of solution and comparison with the related best bound. This is basically due to the fact that the linear relaxation of the three-index formulation provides higher lower-bound values. Moreover, the improvement in the best bound determined during the solution process is much higher for three-index formulation compared to the two-index model, which stays very near to the linear relaxation. This is basically due to the *big M* parameter used in subtour elimination constraints. A good estimation of M in

Instance	3i-2E-LRP					2i-2E-LRP				
	LR	BL	BS	Gap$^{(\%)}$	Time	LR	BB	BS	Gap$^{(\%)}$	Time
I2/2-3-8	501.19	648.19	**648.18***	**0.00**	179	283.44	648.19	**648.19**	**0.00**	13
I2/2-3-10	693.57	972.55	**972.55***	**0.00**	3416	443.28	972.55	**972.55**	**0.00**	541
I2/2-4-10	723.89	931.82	**931.816***	**0.00**	8221	428.53	931.82	**931.82**	**0.00**	3134
I2/2-4-15	805.11	936.09	1078.07	13.17	21604	387.63	874.65	**987.96**	11.47	18345
I2/2-8-20	676.00	765.22	**945.79**	19.09	28132	468.75	697.22	1133.04	38.46	30254
I2/2-8-25	766.33	899.13	**1280.14**	29.76	29123	553.38	777.22	1331.88	41.64	33398
I2/2-10-15	559.54	741.16	**835.09**	11.25	22737	465.66	579.36	910.37	36.36	26458
I2/2-10-20	666.63	805.76	**1104.33**	27.04	26734	478.57	678.32	1265.49	46.40	31123
I2/2-10-25	845.75	1009.21	**1435.67**	29.70	27452	624.21	861.35	1505.50	42.79	37139
I2/3-5-10	574.64	755.84	**793.21**	4.71	19803	422.18	793.21	**793.21***	**0.00**	3186
I2/3-5-15	680.10	785.13	**957.25**	17.98	19245	470.40	813.76	**957.25**	14.99	17132
I2/3-8-10	449.23	633.22	**654.20**	3.21	22546	344.73	617.28	**654.20**	5.64	25854
I2/3-8-15	501.70	690.37	**822.78**	16.09	24004	376.63	516.78	825.52	37.40	28452
I2/3-8-20	695.41	846.78	**1058.34**	19.99	26887	509.24	708.75	1085.03	34.68	30475
I2/3-8-25	830.34	1063.13	**1462.03**	27.28	27698	558.91	864.50	1596.02	45.83	37484
I2/3-10-15	617.68	872.14	**1016.48**	14.20	25316	442.69	656.09	1186.14	44.69	30211
I2/3-10-20	619.15	800.27	**1064.02**	24.79	36342	444.32	632.20	1076.06	41.25	39876
I2/3-10-25	841.57	989.04	**1463.65**	32.43	39941	623.62	855.16	1594.71	46.38	45124

Table 3: Results of 3- and 2-index formulations on instance set I2

Instance	3i-2E-LRP					2i-2E-LRP				
	LR	BL	BS	Gap$^{(\%)}$	Time	LR	BB	BS	Gap$^{(\%)}$	Time
I3/2-3-8	712.05	976.20	**976.19***	**0.00**	352	502.04	976.20	**976.19***	**0.00**	35
I3/2-3-10	597.36	807.60	**807.60***	**0.00**	3103	368.19	807.60	**807.60***	**0.00**	57
I3/2-4-10	689.27	984.85	**984.85***	**0.00**	5106	475.38	984.85	**984.85***	**0.00**	1858
I3/2-4-15	849.39	1077.12	**1405.15**	23.34	19345	496.68	1114.28	1431.67	22.17	17456
I3/2-8-20	883.99	999.13	**1424.26**	29.85	27321	620.40	893.75	1537.08	41.85	31477
I3/2-8-25	772.87	967.84	**1375.75**	29.65	28653	528.38	795.52	1418.50	43.92	33223
I3/2-10-15	600.63	677.68	**906.15**	25.21	18765	397.60	609.83	1021.49	40.30	25567
I3/2-10-20	704.11	765.09	**1255.21**	39.05	27335	502.42	716.14	1328.19	46.08	36245
I3/2-10-25	830.79	937.23	**1364.25**	31.30	30000	574.30	870.00	1640.28	46.96	39987
I3/3-5-10	445.86	983.60	**983.59***	**0.00**	15995	243.62	983.60	**983.59***	**0.00**	2369
I3/3-5-15	560.14	843.10	**1045.86**	19.39	21225	297.48	933.19	**1045.86**	10.77	22733
I3/3-8-10	478.27	625.35	**736.49**	15.09	18436	305.87	493.23	741.36	33.47	21298
I3/3-8-15	548.07	701.03	**953.55**	26.48	25598	419.05	599.82	1024.88	41.47	29984
I3/3-8-20	648.58	822.39	**1084.05**	24.14	23987	472.31	665.49	1280.12	48.01	31135
I3/3-8-25	803.14	876.74	**1199.91**	26.93	28766	538.13	825.00	1412.43	41.59	35567
I3/3-10-15	505.44	679.13	**929.60**	26.94	21435	340.67	532.50	986.65	46.03	27745
I3/3-10-20	658.60	784.99	**1101.76**	28.75	27645	480.86	671.16	1322.60	49.26	40567
I3/3-10-25	798.53	1030.87	**1419.73**	27.39	36000	544.86	847.50	1690.61	49.87	42113

Table 4: Results of 3- and 2-index formulations on instance set I3

function of the maximum length of the routes slightly improves the performance of the solver.

	Average Gap (%) Values			
	3i-2E-LRP	**2i-2E-LRP**	**3i-2E-LRP***	**2i-2E-LRP***
Set 1	16.37	27.50	14.50	15.72
Set 2	17.22	27.48	15.98	17.93
Set 3	20.75	31.21	20.13	25.02

Table 5: Average GAP values (%)s on the three set of instances

Finally, Table 5 displays the average percentage gap values for each formulation and instance set. The first two columns, *3i-2E-LRP* and *3i-2E-LRP*, report the average of the already shown gaps of Tables 2, 3, and 4. The third and the fourth columns, *3i-2E-LRP** and *3i-2E-LRP**, report the average gap values computed by using the higher best bound obtained by the two models. ¿From the data in this table, we can observe that the performance of the two-index model significantly improves when compared to the best available lower bound, whereas it is almost equivalent for the three-index formulation. This emphasizes the need for research to enhance the bounds.

6 Conclusions

Designing a two-echelon freight distribution system is an important logistic problem, and we addressed this issue as a two-echelon location-routing problem. The 2E-LRP represents a new class of location-routing problems, not yet addressed in the literature, where one desires to simultaneously determine the location of facilities on two adjacent echelons of the distribution system, while explicitly accounting for the routing of vehicles on both echelons.

We explored modeling issues for the general 2E-LRP class, before focusing on the basic, static, deterministic, single-commodity version. We proposed three mixed integer programming formulations for this basic 2E-LRP differentiated by the level of routing detail explicitly appearing in the model definition. We then compared the formulations on their flexibility and resulting problem dimensions. We also computationally compared them on a large set of problem instances newly generated (and freely available from the authors), by means of a commercial solver. The results are very encouraging in terms of quality of solution and computation time, and justify continuing working on this topic to develop exact (branch-and-cut-and-price with column generation based on the third formulation) and meta-heuristic solution methods proper to the 2E-LRP.

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