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August 2011

CIRRELT-2011-50

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Performance Evaluation of a Reconfigurable Production Line

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Abstract. This paper presents an analytical method for evaluating the performances of a two-machine-one-buffer reconfigurable production line. The reconfiguration helps to increase the machine performances. The graceful degradation encountered in reconfigurable machines is seen as a very nice middle ground between the expensive fault-tolerance of redundancy and the low cost of non-robust systems. In the considered case, each machine is composed of essential and non essential equipments. The failure of any essential equipment induces the shutdown of the entire machine, while the failure of the non essential equipment implies the continuity of the machine service with a reduced level of functionality. To assess the accuracy of the proposed method, simulation and numerical experiments have been conducted. The proposed model can then be used as the building block for performance evaluation of longer production lines using either the decomposition or the aggregation techniques.

Keywords. Reconfiguration, manufacturing production lines, productivity.

Acknowledgements. The authors would like to thank the Natural Sciences and Engineering Research Council of Canada (NSERC) for financial support.

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1. Introduction

A reconfigurable system is defined a as a system that may allow service continuity under failure, on the basis of a reduced level of performance. Such a system is considered in [1] as a set of equipments partitioned into a subset of essential equipments and a subset of non-essential equipments. Essential equipment is one whose failure causes the entire system shut-down. In contrast, at the failure of non-essential equipment, the service is allowed to continue but implies degradation in production. This continuity of the service may be accomplished elegantly, rather than just by throwing money at the problem with brute-force redundancy. Redundant units involve higher deployment costs, provide functionality that is only useful in the case of failure, and cannot help if the failure is systematic. While it may be that brute-force redundancy is the only way to satisfy stringent availability requirements for essential functions, not every function is essential. In fact, much of the increasing level of computing power, automation and flexibility in manufacturing systems provides extra functionality rather than basic essential functions. Thus, there is a room in many manufacturing systems to implement reconfiguration and graceful degradation of functionality, in order to enhance the dependability for non-essential (but highly desirable) functions. A gracefully degrading system can be viewed as a system in which faults are masked and only manifest themselves in a reduced level of system functionality.

In the domain of real-time control of manufacturing systems, automatic reconfiguration mechanisms are being increasingly used for the design of robust reconfigurable systems. This can be achieved by automatically installing, at the occurrence of a failure, a new control strategy to obtain maximal functionality using remaining system resources, resulting in a system that still functions, albeit with lower overall utility. Reconfiguration has been identified as a key mechanism for an automatic graceful degradation facility in several works (e.g., [2-4] and references therein). In [2], the key insight of such reconfiguration is to re-synthesize the supervisory control, in response to a failure, in such a way as to allow automated manufacturing system operational safety. In [3], the authors propose the concept of intercellular transfers at failure to allow the reconfiguration, and to improve overall performance of cellular manufacturing systems. In [4], the authors describe an architecture-based approach to gracefully degrading systems based upon product family architectures combined with automatic reconfiguration. Examples of reconfiguration mechanisms can be found in [2] and [5].

Figure 1 represents the alternance between nominal and degraded operating of a reconfigurable machine.

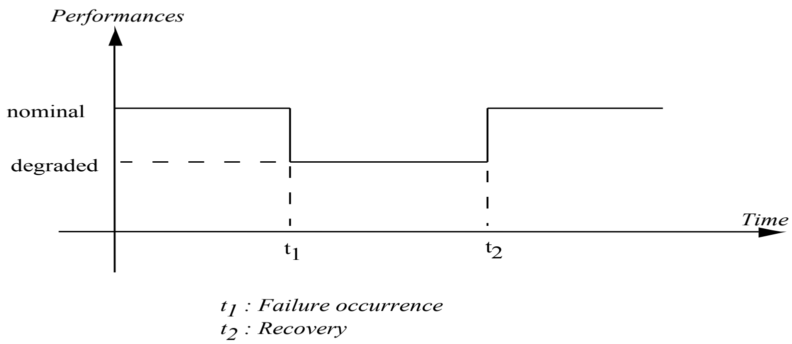

Figure 1. Nominal/degraded operating alternance of a reconfigurable machine [1]

1.1 Background

There are three research areas related to the research presented in this paper. The first is the literature of flow models of unreliable production lines, most often presented for binary state machines: one up (good) state and one failure state. The second concerns multi-state system (MSS) reliability models, which are frequently used in evaluating performance measures of series and parallel systems (in the reliability block diagram sense) without any intermediate buffer. Finally, our model can be seen as an outgrowth of a small body of literature that discusses approaches for evaluating the availability and the throughput of unreliable production lines with multi-state machines.

First, there is a substantial literature on flow models of unreliable production lines. The objective of the majority of these models is to evaluate the throughput for lines composed of several machines and intermediate buffers: see for example [11, 12] for a survey. The throughput is in general difficult to evaluate exactly by analytical Markov models. As a result, most of the methods used to analyze long lines are based either on analytical approximation methods or simulation. Simulation models are applicable to a wide class of systems, but are more expensive computationally [13]. Analytical approximation methods are generally based on the Markov model developed for a line with two machines and one buffer [14] and either aggregation approach [15-17] or decomposition approach [18-22]. Other papers developing aggregation methods are [23-26]. All these papers consider binary-state machines.

Second, machines with different operating modes are usually studied, for buffer-less systems, at the basis of MSS reliability theory [27]. Different reliability measures can be considered for MSS evaluation and design. Among them, the throughput is a common performance measure of MSS, but it is usually evaluated without considering any intermediate buffer.

Finally, our model can be viewed as an outgrowth of the small body of literature devoted to the throughput evaluation of unreliable production lines with multi-state machines. In [28], the authors develop models for two multi-state machine systems with intermediate finite buffers. These models consider that each machine has three states. In the first state, the machine is operating and producing good parts. In the second state, the machine is operating and producing bad parts, but the operator does not know this yet (quality failures). In the third state, the machine is not operating (operational failures). Others papers that study quality–quantity interactions are [29, 30]. To approximate general processing, failure, and repair time distributions by using phase-type distributions, more detailed models of production systems where each stage is modelled by using more than two states have been used [31-34]. Finally, the objective of the models in [35] is to analyze how production system design, quality, and productivity are inter-related in production systems. They calculate total throughput, effective throughput (i.e., the throughput of good parts), and yield. Unlike [35], we consider in this paper that the machines are reconfigurable, i.e., they can continue their service with a reduced level of functionality.

1.2 Objectives

The objective of this paper is to evaluate analytically the throughput of a production line composed of two reconfigurable machines separated by a finite-capacity buffer. This means that each machine is composed of essential and non essential equipments. The failure of any essential equipment induces the shutdown of the entire machine. The failure of the non essential equipment implies the continuity of the machine service with a reduced level of functionality. Parts are moved from machine 1 to machine 2 by some kind of transfer mechanism. The parts are stocked in the buffer when machine 2 is down or busy.

1.3 Outline

The remainder of this paper is organized as follows. In Section 2, the single reconfigurable machine model is presented, while the two-machines-one-buffer model is detailed in Section 3. In this section, the assumptions and notations of the model, all the details of both the internal and boundary equations and the solution methodology are described. In Section 4, the validation results of the proposed model are shown. Finally, conclusions are given in Section 5.

2. Single reconfigurable machine model

A reconfigurable machine is defined by a set of equipments partitioned into a subset of essential equipments and a subset of non essential equipments. The failure of essential equipment causes the entire machine shut down. In contrast, at the failure of non-essential equipment, the service is allowed to continue with a reduced level of functionality.

The following assumptions are considered for the single reconfigurable machine model:

1. Essential equipment is either up (good), or failed with a constant failure rate;
2. Equipment can fail only when the machine is in one of its operating states;
3. Only one repair crew is assigned to repair the failing equipments. If the system is in a state where both non essential equipment and essential equipment are failed, the essential equipment will be repaired first;
4. Any repaired equipment is as good as new, and its repair rate is constant. Repair times are independent and identically distributed [6];
5. Failure times are independent and identically distributed;
6. The system is completely observable, i.e. there is perfect information to determine instantaneously the failure of any essential or non essential equipment;

7. The system is considered as a perfect fault-coverage wherein all failures can be repaired and are therefore covered. The perfect fault-coverage model does not have any absorbing state and thus leads to the steady state availability analysis [7].

The single machine can be modeled by a continuous-time Markov process. We denote by N_i the state of the machine when it is operating normally. The state D_i refers to the degraded operating state, i.e., the state reached at the failure of non essential equipment. Let F_i^N be the state of the system when a failure of essential equipment occurs while the machine is operating normally. We denote by F_i^D the state of the machine when a failure of essential equipment occurs while the system is in degraded mode. The state transition diagram of the system is given in figure 2.

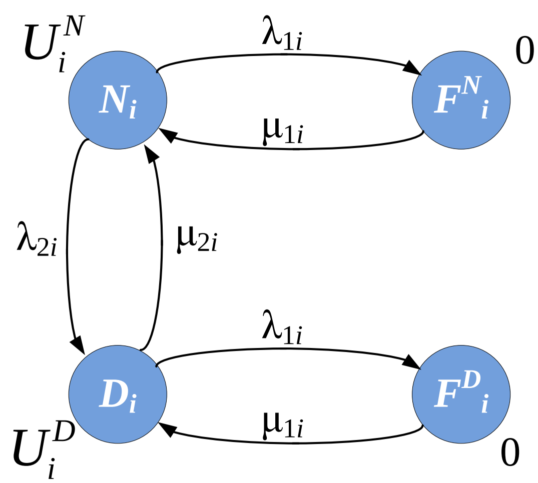

Figure 2. State transition diagram of a single reconfigurable machine

The reconfiguration helps to increase the machine performances. The graceful degradation encountered in reconfigurable machines is seen as a very nice middle ground between the expensive fault-tolerance of redundancy and the low cost of non-robust systems. In the next section, we consider a production line containing two reconfigurable machines separated by a finite buffer. The throughput evaluation of such a line is very important because it is at the basis of the development of any decomposition or aggregation method for longer lines. This evaluation is however quite challenging because of the introduction of the degraded mode at each machine level.

3. Two-machine-one-buffer model

The performance evaluation of a two-machine-one-buffer production line is based on the following steps. Considering the Markov Chain that represents the two-machines-one-buffer line where each machine can operate in a degraded operating mode, the Chapman-Kolmogorov (CK) equations are defined for both internal and boundary states. These equations and the condition that all probabilities sum up to one lead to a system of equations that can be solved simultaneously to compute state probabilities and density functions which allow us to determine the availability, the production rate and the average buffer level. In the next subsections, we will describe each of these steps in more details.

3.1 Additional assumptions and notations

The throughput evaluation of production line containing two reconfigurable machines separated by a finite buffer is very important, because it is at the basis of the development of any decomposition or aggregation method for longer lines. This evaluation is however challenging because of the introduction of the degraded mode at each machine level. The considered tandem production line is shown in Figure 3, where the machines are denoted by M_1 and M_2 and the intermediate buffer is denoted by B_1. Parts flow from outside the system to machine M_1, then to buffer B_1, then to machine M_2 after which they leave the system. It is assumed that the flow of processed parts resembles a continuous fluid. The buffer capacity separating the two machines M_1 and M_2 is denoted by h_1. A machine is *starved* if its upstream buffer is empty. It is called *blocked* if its downstream buffer is full. Indeed, the production rate of the tandem production line may be improved by the buffer, as it may prevent blocking and/or starvation of machines. As it is usually the case for production systems, we assume that the failures are operation-dependent, *i.e.* a machine can fail only while it is processing parts (it is said to be working). Thus, if a machine is operational (*i.e.* not down) but starved or blocked, it cannot fail.

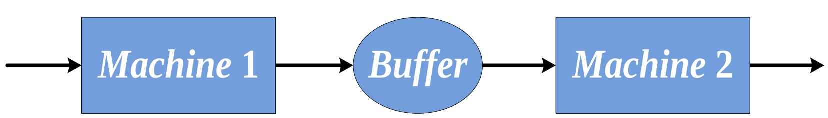

Figure 3. Two-machine-one-buffer reconfigurable production line

Each machine M_j (with j = 1, 2) can experience three states: nominal, degraded, and failed. We assume that all times to failure and times to repair are exponentially distributed. Let $MTBF_{1j}$ denote the *Mean Time Between Failures* of machine M_j from the nominal state; then $\lambda_{1j} = \frac{1}{MTBF_{1j}}$ is its corresponding failure rate. Similarly, $MTTR_{1j}$ and $\mu_{1j} = \frac{1}{MTTR_{1j}}$ are the *Mean Time To Repair* and the repair rate. On the other hand, $MTBF_{2j}$ is the *Mean Time To Degrade* of machine M_j, and $\lambda_{2j} = \frac{1}{MTBF_{2j}}$ is its corresponding degradation rate, while $MTTR_{2j}$ denote the *Mean Time To Recover* machine M_j to the nominal operating mode with a corresponding repair rate $\mu_{2j} = \frac{1}{MTTR_{2j}}$.

We assume that the processing times of each machine are deterministic, *i.e.* a fixed amount of time is required to perform the operation, in both nominal operating and degraded operating modes. Thus, a machine M_j has constant nominal cycle time θ_j^N and nominal production rate $U_j^N = \frac{1}{\theta_j^N}$. The nominal rate U_j^N represents the maximum rate at which the machine M_j can operate while being in its nominal operating state and not slowed down by an upstream or a downstream machine [9]. Similarly, a machine M_j has a constant degraded cycle time θ_j^D and a degraded production rate $U_j^D = \frac{1}{\theta_j^D}$. The degraded rate U_j^D represents the maximum rate at which the machine M_j can operate while being in its degraded operating state and not slowed down by an upstream or a downstream machine. It is assumed that the considered line is homogenous, which means that all machines have the same nominal processing time and the same degraded processing time. That is, $U_1^N = U_2^N = U^N$ and $U_1^D = U_2^D = U^D$. The following additional assumptions are also used:

- The first machine is never starved, *i.e.* there is always available part at the input;
- The last machine is never blocked, *i.e.* finished parts leave machine M_2 immediately or there is always available space for part storage at the output of the line;

- The degraded rate for a given machine is less than the nominal rates of all the machines of the line;
- There exists a stationary regime where steady-state behaviour is reached.

3.2 Internal states equations

There are 16 internal states corresponding to the stochastic composition [8] of the stochastic automata of machines M_1 and M_2 given by Figure 2. Thus, there are 16 internal states equations which are, in steady-state:

$$\left(U_2^N - U_1^N\right)\frac{\partial f\left(x, N_1, N_2\right)}{\partial x} - \left(\lambda_{11} + \lambda_{12} + \lambda_{21} + \lambda_{22}\right) f\left(x, N_1, N_2\right) + \mu_{11} f\left(x, F_1^N, N_2\right) + \mu_{12} f\left(x, N_1, F_2^N\right) + \mu_{21} f\left(x, D_1, N_2\right) + \mu_{22} f\left(x, N_1, D_2\right) = 0 \tag{1}$$

$$-U_1^N \frac{\partial f\left(x, N_1, F_2^N\right)}{\partial x} - \left(\lambda_{11} + \lambda_{21} + \mu_{12}\right) f\left(x, N_1, F_2^N\right) + \mu_{11} f\left(x, F_1^N, F_2^N\right) + \mu_{21} f\left(x, D_1, F_2^N\right) + \lambda_{12} f\left(x, N_1, N_2\right) = 0 \tag{2}$$

$$\left(U_2^D - U_1^N\right)\frac{\partial f\left(x, N_1, D_2\right)}{\partial x} - \left(\lambda_{11} + \lambda_{12} + \lambda_{21} + \lambda_{22}\right) f\left(x, N_1, D_2\right) + \mu_{11} f\left(x, F_1^N, D_2\right) + \mu_{12} f\left(x, N_1, F_2^D\right) + \mu_{21} f\left(x, D_1, D_2\right) + \mu_{22} f\left(x, N_1, N_2\right) = 0 \tag{3}$$

$$-U_1^N \frac{\partial f\left(x, N_1, F_2^D\right)}{\partial x} - \left(\lambda_{11} + \lambda_{21} + \mu_{12}\right) f\left(x, N_1, F_2^D\right) + \mu_{11} f\left(x, F_1^N, F_2^D\right) + \mu_{21} f\left(x, D_1, F_2^D\right) + \lambda_{12} f\left(x, N_1, D_2\right) = 0 \tag{4}$$

$$-\left(\mu_{11} + \mu_{12}\right) f\left(x, F_1^N, F_2^N\right) + \lambda_{11} f\left(x, N_1, F_2^N\right) + \lambda_{12} f\left(x, F_1^N, N_2\right) = 0 \tag{5}$$

$$U_2^N \frac{\partial f\left(x, F_1^N, N_2\right)}{\partial x} - \left(\mu_{11} + \lambda_{12} + \lambda_{22}\right) f\left(x, F_1^N, N_2\right) + \lambda_{11} f\left(x, N_1, N_2\right) + \mu_{12} f\left(x, F_1^N, F_2^N\right)$$
$$+ \mu_{22} f\left(x, F_1^N, D_2\right) = 0 \tag{6}$$

$$U_2^D \frac{\partial f\left(x, F_1^N, D_2\right)}{\partial x} - \left(\mu_{11} + \lambda_{12} + \mu_{22}\right) f\left(x, F_1^N, D_2\right) + \lambda_{11} f\left(x, N_1, D_2\right) + \mu_{12} f\left(x, F_1^N, F_2^D\right)$$
$$+ \lambda_{22} f\left(x, F_1^N, N_2\right) = 0 \tag{7}$$

$$-\left(\mu_{11} + \mu_{12}\right) f\left(x, F_1^N, F_2^D\right) + \lambda_{11} f\left(x, N_1, F_2^D\right) + \lambda_{12} f\left(x, F_1^N, D_2\right) = 0 \tag{8}$$

$$-U_1^D \frac{\partial f\left(x, D_1, F_2^N\right)}{\partial x} - \left(\lambda_{11} + \mu_{21} + \mu_{12}\right) f\left(x, D_1, F_2^N\right) + \mu_{11} f\left(x, F_1^D, F_2^N\right) + \lambda_{21} f\left(x, N_1, F_2^N\right)$$
$$+ \lambda_{12} f\left(x, D_1, N_2\right) = 0 \tag{9}$$

$$\left(U_2^N - U_1^D\right) \frac{\partial f\left(x, D_1, N_2\right)}{\partial x} - \left(\lambda_{11} + \lambda_{12} + \mu_{21} + \lambda_{22}\right) f\left(x, D_1, N_2\right) + \mu_{11} f\left(x, F_1^D, N_2\right)$$
$$+ \mu_{12} f\left(x, D_1, F_2^N\right) + \lambda_{21} f\left(x, N_1, N_2\right) + \mu_{22} f\left(x, D_1, D_2\right) = 0 \tag{10}$$

$$\left(U_2^D - U_1^D\right) \frac{\partial f\left(x, D_1, D_2\right)}{\partial x} - \left(\lambda_{11} + \lambda_{12} + \mu_{21} + \mu_{22}\right) f\left(x, D_1, D_2\right) + \mu_{11} f\left(x, F_1^D, D_2\right)$$
$$+ \mu_{12} f\left(x, D_1, F_2^D\right) + \lambda_{21} f\left(x, N_1, D_2\right) + \lambda_{22} f\left(x, D_1, N_2\right) = 0 \tag{11}$$

$$-U_1^D \frac{\partial f\left(x, D_1, F_2^D\right)}{\partial x} - \left(\lambda_{11} + \mu_{21} + \mu_{12}\right) f\left(x, D_1, F_2^D\right) + \mu_{11} f\left(x, F_1^D, F_2^D\right) + \lambda_{21} f\left(x, N_1, F_2^D\right)$$
$$+ \lambda_{12} f\left(x, D_1, D_2\right) = 0 \tag{12}$$

$$-\left(\mu_{11} + \mu_{12}\right) f\left(x, F_1^D, F_2^N\right) + \lambda_{11} f\left(x, D_1, F_2^N\right) + \lambda_{12} f\left(x, F_1^D, N_2\right) = 0 \tag{13}$$

$$U_2^N \frac{\partial f\left(x,F_1^D,N_2\right)}{\partial x} - \left(\mu_{11}+\lambda_{12}+\lambda_{22}\right) f\left(x,F_1^D,N_2\right) + \lambda_{11} f\left(x,D_1,N_2\right) + \mu_{12} f\left(x,F_1^D,F_2^N\right) + \mu_{22} f\left(x,F_1^D,D_2\right) = 0 \quad (14)$$

$$U_2^D \frac{\partial f\left(x,F_1^D,D_2\right)}{\partial x} - \left(\mu_{11}+\lambda_{12}+\mu_{22}\right) f\left(x,F_1^D,D_2\right) + \lambda_{11} f\left(x,D_1,D_2\right) + \mu_{12} f\left(x,F_1^D,F_2^D\right) + \lambda_{22} f\left(x,F_1^D,N_2\right) = 0 \quad (15)$$

$$-\left(\mu_{11}+\mu_{12}\right) f\left(x,F_1^D,F_2^D\right) + \lambda_{11} f\left(x,D_1,F_2^D\right) + \lambda_{12} f\left(x,F_1^D,D_2\right) = 0 \quad (16)$$

Figure 4 shows the internal states transition diagram.

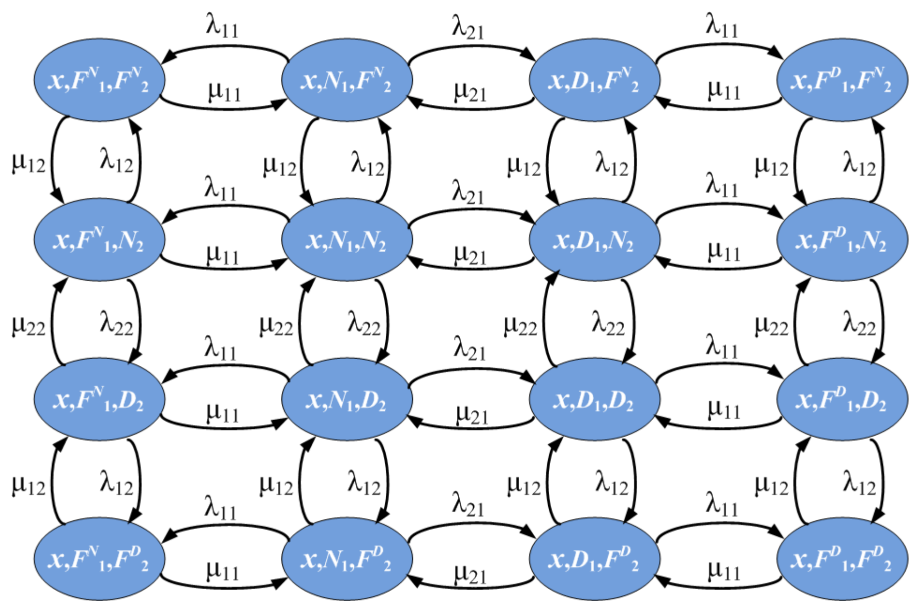

Figure 4. Internal states transition diagram

3.3 Solution of the internal states equations

We assume the form $f(x,\alpha_1,\alpha_2) = Ce^{\rho x}G_1(\alpha_1)G_2(\alpha_2)$, which was successfully adopted in [9], where:

$$G_i(\alpha_i) = \begin{cases} 1 \;\; if \;\; \alpha_i = N_i \;\; or \;\; \alpha_i = D_i \\ G_i(F_i^N) \;\; if \;\; \alpha_i = F_i^N \\ G_i(F_i^D) \;\; if \;\; \alpha_i = F_i^D \end{cases} \qquad i = 1,2 \quad (17)$$

In order to define the steady-states probabilities of the system, we need to solve the internal states equations. We then need to find the values of C, ρ and $G_i(\alpha_i)$ with $i=1,2$.

Substituting the above form in equations (1) – (16) and after some manipulations [10], we obtain:

$$\left(U_1^N - U_2^N\right)K\left[G_1\left(F_1^N\right)+G_2\left(F_2^N\right)\right]+\left[U_1^N\left(\mu_{22}-\lambda_{22}\right)+U_2^N\left(\lambda_{21}-\mu_{21}\right)\right]=0, \tag{18}$$

which is an 3rd degree polynomial in K (where K is independent from both M_1 and M_2.). Let us denote K_m as the m^{th} root of the polynomial (with $m=1,...,3$). Once the roots are defined, by solving equation (18), we can determine the values of $G_i(\alpha_i)$ with $i=1,2$ and ρ:

$$G_{1m}\left(F_1^N\right)=G_{1m}\left(F_1^D\right)=\frac{\lambda_{11}}{\mu_{11}+K_m} \tag{19}$$

$$G_{2m}\left(F_2^N\right)=G_{2m}\left(F_2^D\right)=\frac{\lambda_{12}}{\mu_{12}-K_m} \tag{20}$$

$$\rho_m=\frac{-K_m}{U_2^N}\left[\left(\frac{\lambda_{11}}{\mu_{11}+K_m}\right)+\left(\frac{\lambda_{12}}{\mu_{12}-K_m}\right)\right]+\frac{\left(\mu_{22}-\lambda_{22}\right)}{U_2^N} \tag{21}$$

We can then write:

$$f\left(x,\alpha_1,\alpha_2\right)=\sum_{m=1}^{3}C_m e^{\rho_m x}G_{1m}(\alpha_1)G_{2m}(\alpha_2), \tag{22}$$

where the remaining unknowns are the C_m (with $m=1,...,3$) which are determined from the solutions of the boundary states equations.

3.4 Boundary states equations

There are also 16 possible states of the form $\left(h_1,\alpha_1,\alpha_2\right)$ (i.e. high boundary states) and 16 possible states of the form $\left(0,\alpha_1,\alpha_2\right)$ (i.e. low boundary states). However, not all of these states are reachable.

Tables 1 and 2 indicate, with justification, if a given boundary state is reachable or not. In these tables, the state of a machine M_i that is starved while being in its nominal-operating mode is denoted by $\underline{N_i}$ and the state of a machine M_i that is blocked while being in its nominal-operating mode is denoted by $\overline{N_i}$. Similarly, the state of a machine M_i that is starved while being in its degraded-operating mode is denoted by $\underline{D_i}$ and the state of a machine M_i that is blocked while being in its degraded-operating mode is denoted by $\overline{D_i}$.

Table 1. *Combinations of states when the buffer is full*

Reference	State	Justification
H1	(h_1, N_1, N_2)	Reachable: Since M_1 and M_2 are both operational in their nominal operating, this state is reachable only if $U_1^N = U_2^N$.
H2	(h_1, N_1, D_2)	Reachable: if M_1 is operational in its nominal operating mode and M_2 is operational in its degraded operating mode, this state is reachable since $U_1^N > U_2^D$.
H3	$(h_1, \overline{N_1}, F_2^N)$	Reachable: Since M_1 is operational and M_2 is down, the buffer level will increase until it reaches its capacity. Then, M_1 becomes blocked.
H4	$(h_1, \overline{N_1}, F_2^D)$	Reachable: Similar to H3.
H5	(h_1, D_1, N_2)	Unreachable: Since M_1 is operational in its degraded operating mode and M_2 is operational in its nominal operating mode, then the buffer level will necessarily decrease since $U_2^N > U_1^D$.
H6	(h_1, D_1, D_2)	Reachable: Similar to H1.
H7	$(h_1, \overline{D_1}, F_2^N)$	Unreachable: See Remark 1.
H8	$(h_1, \overline{D_1}, F_2^D)$	Reachable: Similar to H3.
H9	(h_1, F_1^N, N_2)	Unreachable: Since M_1 is down and M_2 is operational, the buffer level will necessarily decrease.
H10	(h_1, F_1^N, D_2)	Unreachable: Similar to H9.
H11	(h_1, F_1^N, F_2^N)	Unreachable: See Remark 2.
H12	(h_1, F_1^N, F_2^D)	Unreachable: Similar to H11.
H13	(h_1, F_1^D, N_2)	Unreachable: Similar to H9.
H14	(h_1, F_1^D, D_2)	Unreachable: Similar to H9.
H15	(h_1, F_1^D, F_2^N)	Unreachable: Similar to H11.
H16	(h_1, F_1^D, F_2^D)	Unreachable: Similar to H11.

Remark 1.

State $\left(h_1, \overline{D_1}, F_2^N\right)$ is unreachable because the only way to reach it is either to transit:

- from state $\left(h_1, D_1, N_2\right)$ which is unreachable; or
- from state $\left(h_1, \overline{N_1}, F_2^N\right)$; but since M_1 is blocked, it cannot fail and such transition is impossible.

Remark 2.

State $\left(h_1, F_1^N, F_2^N\right)$ is unreachable because the only way to reach it is either to transit:

- from states $\left(h_1, F_1^N, N_2\right)$ or $\left(h_1, F_1^N, D_2\right)$ which are unreachable; or
- from states $\left(h_1, \overline{N_1}, F_2^N\right)$ or $\left(h_1, \overline{D_1}, F_2^N\right)$; but since M_1 is blocked, it cannot fail and such transitions are impossible.

Remark 3.

State $\left(0, F_1^N, \underline{D_2}\right)$ is unreachable because the only way to reach it is either to transit:

- from state $\left(0, N_1, D_2\right)$ which is unreachable; or
- from state $\left(0, F_1^N, \underline{N_2}\right)$; but since M_2 is starved, it cannot fail and such transition is impossible.

Remark 4.

State $\left(0, F_1^N, F_2^N\right)$ is unreachable because the only way to reach it is either to transit:

- from states $\left(0, N_1, F_2^N\right)$ or $\left(0, D_1, F_2^N\right)$ which are unreachable; or
- from states $\left(0, F_1^N, \underline{N_2}\right)$ or $\left(0, F_1^N, \underline{D_2}\right)$; but since M_2 is starved, it cannot fail and such transitions are impossible.

Table 2. *Combinations of states when the buffer is empty*

Reference	State	Justification
L1	$\left(0, N_1, N_2\right)$	Reachable: Since M_1 and M_2 are both operational in their nominal operating mode, this state is reachable only if $U_1^N = U_2^N$.
L2	$\left(0, N_1, D_2\right)$	Unreachable: Since M_1 is operational in its nominal operating mode and M_2 is operational in its degraded operating mode, then the buffer level will necessarily increase since $U_1^N > U_2^D$.
L3	$\left(0, N_1, F_2^N\right)$	Unreachable: Since M_1 is operational and M_2 is down, the buffer level will necessarily increase.
L4	$\left(0, N_1, F_2^D\right)$	Unreachable: Similar to L3.
L5	$\left(0, D_1, N_2\right)$	Reachable: Since M_1 is operational in its degraded operating mode and M_2 is operational in its nominal operating mode, this state is reachable since $U_1^D < U_2^N$.
L6	$\left(0, D_1, D_2\right)$	Reachable: Similar to L1.
L7	$\left(0, D_1, F_2^N\right)$	Unreachable: Similar to L3.
L8	$\left(0, D_1, F_2^D\right)$	Unreachable: Similar to L3.
L9	$\left(0, F_1^N, \underline{N_2}\right)$	Reachable: Since M_1 is down and M_2 is operational, the buffer level will necessarily decrease until it becomes empty. Then, M_2 becomes starved.
L10	$\left(0, F_1^N, \underline{D_2}\right)$	Unreachable: See Remark 3.
L11	$\left(0, F_1^N, F_2^N\right)$	Unreachable: See Remark 4.
L12	$\left(0, F_1^N, F_2^D\right)$	Unreachable: Similar to L11.
L13	$\left(0, F_1^D, \underline{N_2}\right)$	Reachable: Similar to L9.
L14	$\left(0, F_1^D, \underline{D_2}\right)$	Reachable: Similar to L9.
L15	$\left(0, F_1^D, F_2^N\right)$	Unreachable: Similar to L11.
L16	$\left(0, F_1^D, F_2^D\right)$	Unreachable: Similar to L11.

As the reachable boundary states are now identified, we have to find the remaining feasible transitions for the Markov Chain that represents the two-machines-one-buffer production line. For this, we consider each boundary state and we determine all feasible transitions from and to this state.

High boundary states

- From state (h_1, N_1, N_2), the transitions that are physically possible correspond to $\lambda_{11}, \lambda_{12}, \lambda_{21}$ and λ_{22}. At the occurrence of λ_{11} (failure of M_1), the buffer level decreases and the system reaches state (x, F_1^N, N_2). At the occurrence of λ_{21} (degradation of M_1), the buffer level decreases and the system reaches state (x, D_1, N_2). At the occurrence of λ_{12} (failure of M_2) or λ_{22} (degradation of M_2), the buffer remains full but the system reaches states $(h_1, \overline{N_1}, F_2^N)$ and (h_1, N_1, D_2), respectively.

- From states $(h_1, \overline{N_1}, F_2^N)$ and $(h_1, \overline{D_1}, F_2^N)$, the only feasible transitions are μ_{12} and μ_{21} corresponding to the repair of M_2 and the recovery of M_1, respectively. The system then reaches states (h_1, N_1, N_2) and (h_1, D_1, N_2), respectively.

- From state (h_1, N_1, D_2), the feasible transitions correspond to $\lambda'_{11}, \lambda_{12}, \lambda'_{21}$ and μ_{22}. At the occurrence of μ_{22} (recovery of M_2), the buffer remains full but the system reaches state (h_1, N_1, N_2). At the occurrence of λ'_{11} (failure of M_1), the buffer level decreases and the system reaches state (x, F_1^N, D_2). At the occurrence of λ'_{21} (degradation of M_1), the buffer remains full but the system reaches state (h_1, D_1, D_2). At the occurrence of λ_{12} (failure of M_2), the buffer remains full but the system reaches state $(h_1, \overline{N_1}, F_2^D)$.

- From state (h_1, D_1, D_2), the feasible transitions correspond to $\lambda_{11}, \lambda_{12}, \mu_{21}$ and μ_{22}. At the occurrence of μ_{22}, the buffer level decreases and the system reaches (x, D_1, N_2). At the occurrence of λ_{11}, the buffer level also decreases and the system reaches state (x, F_1^D, D_2).

At the occurrence of μ_{21}, the buffer remains full but the system reaches state $\left(h_1, N_1, D_2\right)$. At the occurrence of λ_{12}, the buffer remains full but the system reaches state $\left(h_1, \overline{D_1}, F_2^D\right)$.

- From state $\left(h_1, \overline{N_1}, F_2^D\right)$, the only feasible transition is μ_{12} corresponding to the repair of M_2. The system then reaches state $\left(h_1, N_1, D_2\right)$.

- From state $\left(h_1, \overline{D_1}, F_2^D\right)$, the only feasible transitions are μ_{12} and μ_{21} corresponding to the repair of M_2 and the recovery of M_1, respectively. The system then reaches states $\left(h_1, D_1, D_2\right)$ and $\left(h_1, \overline{N_1}, F_2^D\right)$, respectively.

Figure 5 illustrates the partial graph obtained for the high boundary states described above.

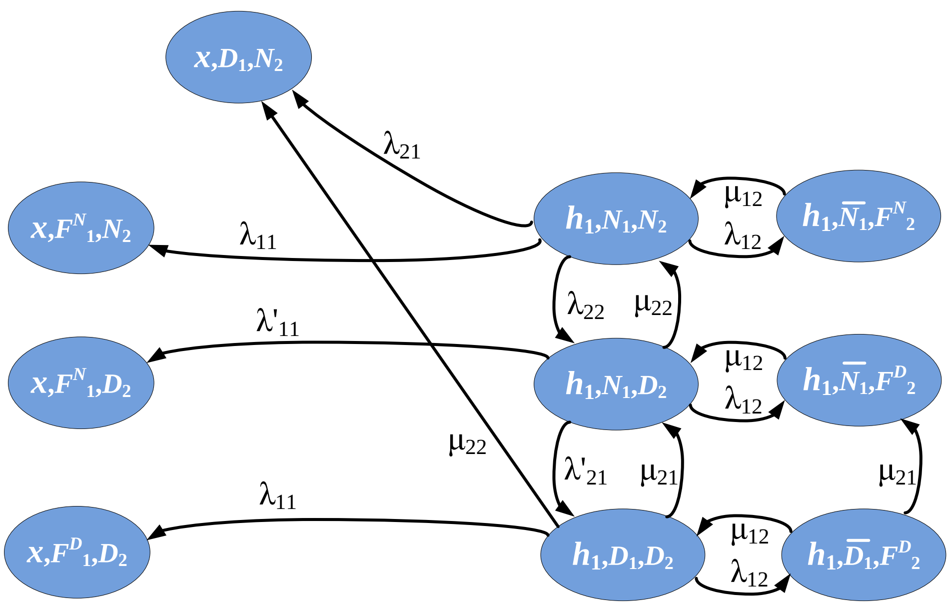

Figure 5. Partial graph for high boundary states

Low boundary states

- From state $\left(0, N_1, N_2\right)$, the feasible transitions correspond to $\lambda_{11}, \lambda_{12}, \lambda_{21}$ and λ_{22}. At the occurrence of λ_{11} (failure of M_1), the buffer remains empty but the system reaches state $\left(0, F_1^N, \underline{N_2}\right)$. At the occurrence of λ_{12} (failure of M_2), the buffer level increases and the system reaches state $\left(x, N_1, F_2^N\right)$. At the occurrence of λ_{21} (degradation of M_1), the buffer remains empty but the system reaches state $\left(0, D_1, N_2\right)$. At the occurrence of λ_{22} (degradation of M_2), the buffer level increases and the system reaches state $\left(x, N_1, D_2\right)$.

- From states $\left(0, F_1^N, \underline{N_2}\right)$ and $\left(0, F_1^D, \underline{N_2}\right)$, the only feasible transitions correspond to the repair of M_1. The system then reaches states $\left(0, N_1, N_2\right)$ and $\left(0, D_1, N_2\right)$, respectively.

- From state $\left(0, F_1^D, \underline{D_2}\right)$, the only feasible transitions are μ_{11} and μ_{22} corresponding to the repair of M_1 and the recovery of M_2, respectively. The system then reaches states $\left(0, D_1, D_2\right)$ and $\left(0, F_1^D, \underline{N_2}\right)$, respectively.

- From state $\left(0, D_1, D_2\right)$, the feasible transitions correspond to $\lambda_{11}, \lambda_{12}, \mu_{21}$ and μ_{22}. At the occurrence of λ_{11}, the buffer remains empty but the system reaches state $\left(0, F_1^D, \underline{D_2}\right)$. At the occurrence of λ_{12}, the buffer level increases and the system reaches state $\left(x, D_1, F_2^D\right)$. At the occurrence of μ_{22}, the buffer remains empty but the system reaches state $\left(0, D_1, N_2\right)$. At the occurrence of μ_{21}, the buffer level increases and the system reaches state $\left(x, N_1, D_2\right)$.

- From state $\left(0, D_1, N_2\right)$, the feasible transitions correspond to $\lambda_{11}, \lambda'_{12}, \mu_{21}$ and λ'_{22}. At the occurrence of λ_{11}, the buffer remains empty but the system reaches state $\left(0, F_1^D, \underline{N_2}\right)$. At the occurrence of λ'_{22}, the buffer remains empty but the system reaches state $\left(0, D_1, D_2\right)$. At the occurrence of μ_{21}, the buffer also remains empty but the system reaches state $\left(0, N_1, N_2\right)$. At the occurrence of λ'_{12}, the buffer level increases and the system reaches state $\left(x, D_1, F_2^N\right)$.

Figure 6 illustrates the partial graph obtained for the low boundary states described above.

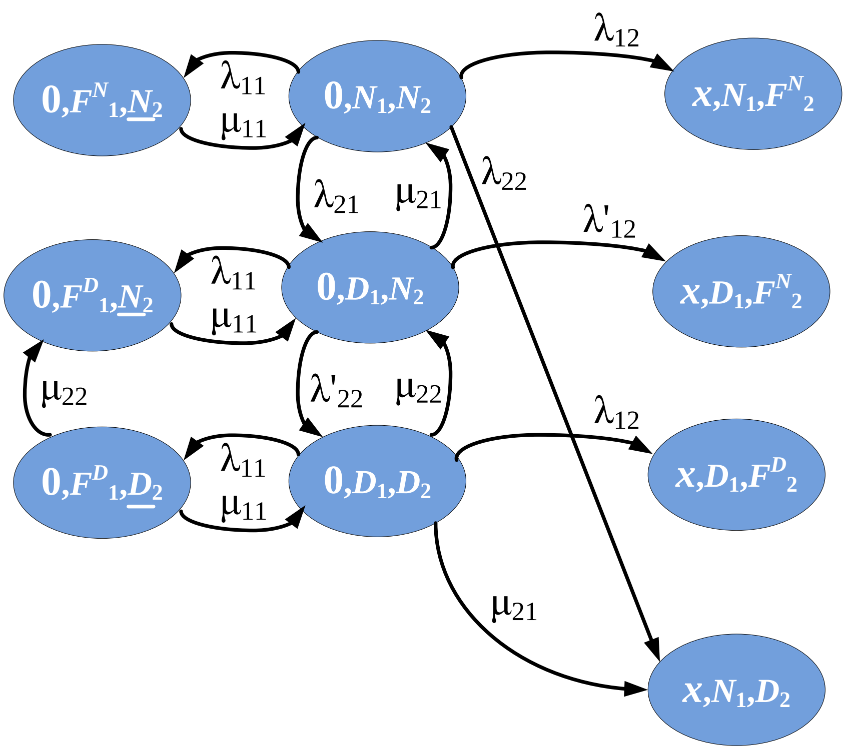

Figure 6. Partial graph for low boundary states

Finally, by merging figures 4-6, we obtain the Markov Chain for the two-machines-one-buffer production line as sketched in Figure 7.

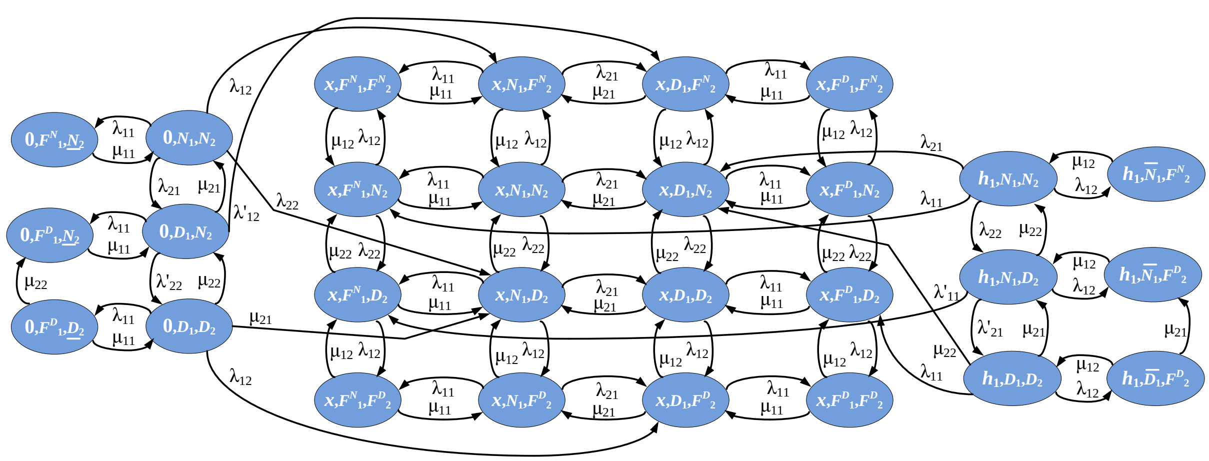

Figure 7. The Markov Chain of the two-machines-one-buffer line

As one may notice, on Figures 4, 5 and 6, some transition rates are reduced. This is due to the assumption that the failures are operation-dependent. That is, we obtain:

The transitions from state $\left(h_1, N_1, D_2\right)$ to states $\left(h_1, D_1, D_2\right)$ and $\left(x, F_1^N, D_2\right)$ leads to the reduction of the transition rates because of the slowdown of M_1 by M_2 since $U_1^N > U_2^D$. Thus, we obtain the following transition rates: $\lambda'_{21} = \lambda_{21} \times \frac{U_2^D}{U_1^N}$ and $\lambda'_{11} = \lambda_{11} \times \frac{U_2^D}{U_1^N}$.

Similarly, the transitions from state $\left(0, D_1, N_2\right)$ to states $\left(0, D_1, D_2\right)$ and $\left(x, D_1, F_2^N\right)$ leads to the reduction of the transition rates because of the slowdown of M_2 by M_1 since $U_1^D < U_2^N$. Thus, we obtain the following transition rates: $\lambda'_{22} = \lambda_{22} \times \frac{U_1^D}{U_2^N}$ and $\lambda'_{12} = \lambda_{12} \times \frac{U_1^D}{U_2^N}$.

3.5 Solution of the boundary states equations

Once the reachable boundary states have been identified, we get 6 equations with 6 unknowns for the case of the high boundary states and 6 equations with 6 unknowns for the case of the low boundary states.

These two systems of equations can be rewritten in a matrix form (see Appendix A). By solving these systems of equations, we can then express all the states probabilities of both internal and boundary states as functions of the only remaining unknowns C_m (with $m = 1, ..., 3$). Using the normalization equation below, we can determine the values of these unknowns and thus, evaluate the state probabilities of the two-machines-one-buffer production line.

$$\sum_{\alpha_1 = N_1, D_1, F_1^N, F_1^D} \sum_{\alpha_2 = N_2, D_2, F_2^N, F_2^D} \left[\int_0^{h_1} f\left(x, \alpha_1, \alpha_2\right) dx + p\left(0, \alpha_1, \alpha_2\right) + p\left(h_1, \alpha_1, \alpha_2\right) \right] = 1 \tag{23}$$

3.6 Performance measures of the system

Once the states probabilities of the system have been determined, we can evaluate the performance measures of the system, namely the availability and the production rate of each machine as well as the average buffer level.

3.6.1 Average buffer level

The average buffer level can be determined from the following expression:

$$\bar{x} = \sum_{\alpha_1 = N_1, D_1, F_1^N, F_1^D} \sum_{\alpha_2 = N_2, D_2, F_2^N, F_2^D} \left[\int_0^{h_1} x \times f\left(x, \alpha_1, \alpha_2\right) dx + p\left(h_1, \alpha_1, \alpha_2\right) \right] \tag{24}$$

3.6.2 Availability and production rate of the machines

The availability of machine M_1 can be determined from the following expression:

$$\begin{aligned} E_1 = & \sum_{\alpha_2 = N_2, D_2, F_2^N, F_2^D} \int_0^{h_1} p\left(x, N_1, \alpha_2\right) dx + p\left(0, N_1, N_2\right) + \sum_{\alpha_2 = N_2, D_2, F_2^N, F_2^D} \int_0^{h_1} p\left(x, D_1, \alpha_2\right) dx \\ & + p\left(0, D_1, N_2\right) + p\left(0, D_1, D_2\right) + p\left(h_1, N_1, N_2\right) + p\left(h_1, D_1, D_2\right) + \frac{U^D}{U^N} p\left(h_1, N_1, D_2\right) \end{aligned} \tag{25}$$

And the production rate of machine M_1 is given by:

$$\begin{aligned} PR_1 = & U^N \times \left[\sum_{\alpha_2 = N_2, D_2, F_2^N, F_2^D} \int_0^{h_1} p\left(x, N_1, \alpha_2\right) dx + p\left(0, N_1, N_2\right) + p\left(h_1, N_1, N_2\right) \right] + \\ & U^D \times \left[\sum_{\alpha_2 = N_2, D_2, F_2^N, F_2^D} \int_0^{h_1} p\left(x, D_1, \alpha_2\right) dx + p\left(0, D_1, N_2\right) + p\left(0, D_1, D_2\right) + p\left(h_1, N_1, D_2\right) + p\left(h_1, D_1, D_2\right) \right] \end{aligned} \tag{26}$$

Similarly, the availability of machine M_2 can be determined by the following expression:

$$\begin{aligned} E_2 = & \sum_{\alpha_1 = N_1, D_1, F_1^N, F_1^D} \int_0^{h_1} p\left(x, \alpha_1, N_2\right) dx + p\left(h_1, N_1, N_2\right) + \sum_{\alpha_1 = N_1, D_1, F_1^N, F_1^D} \int_0^{h_1} p\left(x, \alpha_1, D_2\right) dx \\ & + p\left(0, N_1, N_2\right) + \frac{U^D}{U^N} p\left(0, D_1, N_2\right) + p\left(0, D_1, D_2\right) + p\left(h_1, D_1, D_2\right) + p\left(h_1, N_1, D_2\right) \end{aligned} \tag{27}$$

And the production rate of machine M_2 is given by:

$$\begin{aligned} PR_2 = & U^N \times \left[\sum_{\alpha_1 = N_1, D_1, F_1^N, F_1^D} \int_0^{h_1} p\left(x, \alpha_1, N_2\right) dx + p\left(0, N_1, N_2\right) + p\left(h_1, N_1, N_2\right) \right] + \\ & U^D \times \left[\sum_{\alpha_1 = N_1, D_1, F_1^N, F_1^D} \int_0^{h_1} p\left(x, \alpha_1, D_2\right) dx + p\left(0, D_1, N_2\right) + p\left(0, D_1, D_2\right) + p\left(h_1, N_1, D_2\right) + p\left(h_1, D_1, D_2\right) \right] \end{aligned} \tag{28}$$

4. Validation of the proposed method

In order to assess the accuracy of the proposed method, simulation experiments were conducted. For this purpose, an object-oriented discrete events simulation tool has been developed. Many examples were considered where all failure and repair rates are assumed to have an exponential distribution and for each example. Five levels of degradation were taken into account. Each example was run for 20 replications of 500 time-units warm-up period and 4,000 time-units simulation period.

The obtained results are compiled in the Table 3 below. For each of the evaluated three performance measures, a percentage error is calculated according to the following expression:

$$Error\% = \frac{Analytical - Simulation}{Simulation} \times 100 \qquad (29)$$

Table 3. Numerical Results and comparison

Example	$\frac{U^D}{U^N}$	Average Buffer Level			System Availability			System Production Rate		
		Analytical	Simulation	Error%	Analytical	Simulation	Error%	Analytical	Simulation	Error%
Example 1	0.9	41.73	42.09	-0.85	0.9214	0.942	-2.18	1.05	1.036	1.35
	0.85	41.69	41.95	-0.61	0.9215	0.9281	-0.71	1.023	1.013	0.98
	0.75	41.6	41.13	1.14	0.9229	0.9491	-2.76	0.9686	0.9926	-2.41
	0.6	41.44	40.71	1.79	0.9218	0.9299	-0.87	0.886	0.8956	-1.07
	0.5	41.31	42.05	-1.75	0.9214	0.9224	-0.1	0.8306	0.8438	-1.56
Example 2	0.9	31.5	31.94	-1.38	0.722	0.7331	-1.52	0.8231	0.815	0.99
	0.85	31.49	31.63	-0.45	0.7219	0.7353	-1.83	0.8014	0.7987	0.33
	0.75	31.48	31.74	-0.82	0.7218	0.7231	-0.19	0.7582	0.7566	0.2
	0.6	31.45	31.17	0.88	0.7215	0.7164	0.71	0.6933	0.7112	-2.53
	0.5	31.43	30.96	1.49	0.7212	0.7008	2.91	0.65	0.6701	-2.99

Example 3	0.9	18.807	18.16	3.52	0.9167	0.9279	-1.2	1.2235	1.2382	-1.18
	0.85	18.5594	19.02	-2.47	0.9181	0.9227	-0.49	1.1966	1.1757	1.77
	0.75	18.0042	18.6482	-3.42	0.9203	0.9134	0.75	1.1413	1.1719	-2.61
	0.6	16.9764	17.45	-2.75	0.9205	0.9186	0.2	1.0541	1.0413	1.22
	0.5	16.1092	16.55	-2.71	0.9166	0.9253	-0.94	0.9925	1.0125	-1.97
Example 4	0.9	18.4582	19.25	-4.15	0.7431	0.7361	0.95	0.984	0.9633	2.14
	0.85	18.3999	18.97	-3	0.7436	0.7234	2.79	0.9602	0.9578	0.25
	0.75	18.2759	19.03	-3.99	0.7443	0.7432	0.14	0.9122	0.9074	0.52
	0.6	18.0681	18.4669	-2.16	0.7447	0.7312	1.84	0.8386	0.8185	2.45
	0.5	17.9119	18.1391	-1.25	0.7445	0.7408	0.49	0.7884	0.7877	0.08
Example 5	0.9	15.8757	16.4269	-3.35	0.9694	0.9792	-1	0.9489	0.984	-3.56
	0.85	15.8371	15.3257	3.33	0.969	0.9671	0.19	0.9238	0.9546	-3.22
	0.75	15.7517	16.4786	-3.89	0.9677	0.976	-0.85	0.8736	0.8983	-2.74
	0.6	15.595	16.33	-4.5	0.9637	0.9791	-1.57	0.7982	0.8006	-0.29
	0.5	15.4621	15.9976	-3.36	0.9591	0.9747	-1.6	0.7483	0.7646	-2.13
Example 6	0.9	39.9214	40.057	-0.33	0.9635	0.9585	0.52	0.9428	0.9802	-3.81
	0.85	39.9207	38.453	3.81	0.9635	0.962	0.15	0.918	0.9379	-2.12
	0.75	39.919	39.54	0.95	0.9635	0.9587	0.5	0.8683	0.8977	-3.27
	0.6	39.916	39.17	1.9	0.9634	0.9517	1.22	0.7939	0.8197	-3.14
	0.5	39.9133	40.028	-0.28	0.9634	0.9634	0	0.7442	0.7486	-0.58
Example 7	0.9	42.5958	43.11	-1.19	0.9215	0.9056	1.75	0.9631	0.9555	0.79
	0.85	42.5681	43.47	-2.07	0.9216	0.9315	-1.06	0.9379	0.9319	0.64
	0.75	42.5074	44.2467	-3.93	0.9218	0.928	-0.66	0.8875	0.9249	-4.04
	0.6	42.3984	43.494	-2.51	0.9218	0.9213	0.05	0.8115	0.8458	-4.05
	0.5	42.308	43.0099	-1.63	0.9216	0.9257	-0.44	0.7606	0.7899	-3.7

Example 8	0.9	32.101	32.8029	2.14	0.7223	0.7271	-0.67	0.7548	0.7659	-1.46
	0.85	32.0968	32.9164	-2.49	0.7223	0.7408	-2.5	0.735	0.7445	-1.28
	0.75	32.0876	31.3723	2.28	0.7222	0.728	-0.81	0.6953	0.6893	0.86
	0.6	32.0713	32.2195	-0.46	0.722	0.7241	-0.27	0.6358	0.6417	-0.92
	0.5	32.0582	32.833	-2.36	0.7218	0.7152	0.91	0.5961	0.593	0.52
Example 9	0.9	48.2591	49.9602	-3.4	0.9158	0.9164	-0.06	0.9574	0.99	-3.29
	0.85	48.1685	47.7112	0.95	0.9156	0.9016	1.55	0.9323	0.9699	-3.87
	0.75	47.9753	46.9087	2.27	0.915	0.9177	-0.29	0.882	0.9174	-3.85
	0.6	47.6491	46.7764	1.86	0.9131	0.9113	0.19	0.8059	0.8403	-4.09
	0.5	47.3997	46.6922	1.51	0.911	0.9235	-1.35	0.755	0.7781	-2.96
Example 10	0.9	21.3732	22.298	-4.14	0.9301	0.9291	0.1	0.9742	0.9823	-0.82
	0.85	20.9701	21.69	-3.31	0.9288	0.9249	0.42	0.9493	0.9676	-1.89
	0.75	20.0987	20.54	-2.14	0.9241	0.9227	0.15	0.8976	0.8981	-0.05
	0.6	18.6031	18.5783	0.13	0.9095	0.9153	-0.63	0.8159	0.8342	-2.19
	0.5	17.4582	17.2612	1.14	0.8928	0.9081	-1.68	0.7592	0.7817	-2.87
Example 11	0.9	64.7611	65.3446	-0.89	0.9507	0.9604	-1	3.1972	3.3005	-3.13
	0.85	64.7101	65.4141	-1.07	0.9506	0.9716	-2.16	3.1131	3.2107	-3.03
	0.75	64.5979	63.96	0.99	0.9501	0.9678	-1.82	2.9447	3.0178	-2.42
	0.6	64.3954	62.1088	3.68	0.9488	0.9518	-0.31	2.6914	2.5901	3.91
	0.5	64.228	64.5483	-0.49	0.9472	0.9662	-1.96	2.522	2.4928	1.17

The obtained results have shown that the average absolute value of the percentage error for the production rate estimation of the system is less than 4.09 %, while this error is less than 2.91 % for the system availability.

5. Conclusion

In this paper, we proposed an analytical method for the performance evaluation of a tandem production line where the machines can operate in a degraded operating mode. The main characteristic of the proposed method is its ability to evaluate performance measures of the line, namely the availability, the production rate, and the average buffer level. Indeed, this is more realistic since machines can be in other states than either nominal or down states. It can be in a degraded operating state which results in continuity of service but at a lower rate. Simulation experiments and numerical results have shown the accuracy of the proposed method. The proposed method can be used as the building block in the throughput evaluation of longer production lines with reconfigurable machines using either the decomposition or the aggregation techniques.

References

[1]. Nourelfath, M., Ait-Kadi, D., Soro, W. I., *Availability modeling and optimization of reconfigurable manufacturing systems*, Journal of Quality in Maintenance Engineering, 9 (3), 284-302, 2003.

[2]. Nourelfath, M., Extension de la théorie de la supervision à la surveillance et à la commande des systèmes à évènements discrets : Application à la sécurité opérationnelle des systèmes de production, Ph.D. Thesis, Institut National des Sciences Appliquées, Lyon, France, 1997.

[3]. Dhouib, K., Ait-Kadi, D., Bouti, A. *Contribution to availability assessment of flexible manufacturing cells*, Second International Conference and Exhibition on CIM, 2, Singapore, 347-354, 1993.

[4]. Nace, W., Koopman, P., *A product family approach to graceful degradation*, Proceedings of DIPES 2000, International IFIP WG Workshop on Distributed and Parallel Embedded Systems, Paderborn University, Paderborn, 2000.

[5]. Nourelfath, M., Niel, E., *Modular supervisory control of an experimental automated manufacturing system*, Control Engineering Practice (IFAC), 12, 205-216, 2004.

[6]. Sherwin, D. G, Availability and productiveness of systems of redundant stages of like items in term of item availability, International Journal of Production Research, 22 (1), 109-120, 1984.

[7]. Moustafa, M. S., *Availability of k-out-of-n:g systems with m failure modes*, Microelectronics Reliability. 36 (3), 385-388, 1996.

[8]. Hopcroft, J. E., Ullman, J. D., *Introduction to automata theory, languages, and computation*, Addison-Wesley series in computer science, 1979.

[9]. Gershwin, S. B., *Manufacturing systems engineering*, Prentice Hall, 1994.

[10]. Belmansour A. T., Nourelfath, M., An aggregation method for performance evaluation of a tandem homogenous production line with machines having multiple failure modes, Reliability Engineering and System Safety, 95, 1193-1201, 2010.

[11]. Dallery, Y., Gershwin, S. B., *Manufacturing flow line system: A review of models and analytical results*, Queuing systems theory and applications, 12 (1-2): 3-94, 1992.

[12]. Papadopoulos, H.T., Heavey, C., Queuing theory in manufacturing systems analysis and design: A classification of models for production and transfer lines, European Journal of Operational Research, 92 (1): 1-27, 1996.

[13]. Sörensen, K., Janssens, G. K., *A Petri net model of a continuous flow transfer line with unreliable machines*, European Journal of Operational Research, 152 (1): 248-262, 2004.

[14]. Dubois, D., Forestier, J. P., Production et en-cours moyens d'un ensemble de deux machines séparées par une zone de stockage, RAIRO Automatique, 16 (2): 105-132, 1982.

[15]. De Koster, M. B. M., *Estimation of line efficiency by aggregation*, International Journal of Production Research, 25 (4): 615-626, 1987.

[16]. Terracol, C., David, R. Performances d'une ligne composée de machines et de stocks intermédiaires. R.A.I.R.O APII 1987; 21 (3) : 239-262.

[17]. Terracol, D., David, R., *An aggregation method for performance evaluation of transfer lines with unreliable machines and finite buffers*, Proceedings of the IEEE International Conference on Robotics and Automation, 1333-1338, 1987.

[18]. Gershwin, S. B., An efficient decomposition method for the approximate evaluation of tandem queues with finite storage space and blocking, Operations Research, 35 (2): 291-305, 1987.

[19]. Dallery, Y., David, R., Xie, X., *Approximate analysis of transfer lines with unreliable machines and finite buffers*, IEEE Transactions on Automatic Control, 34 (9): 943-953, 1989.

[20]. Tolio T., Matta A., Gershwin S. B., *Analysis of two-machine lines with multiple failure modes*, IIE Transactions, 34 (1): 51-62, 2002.

[21]. Levantesi, R., Matta, A., Tolio, T., Performance evaluation of continuous production lines with machines having different processing times and multiple failure modes, Performance Evaluation, 51 (2-4): 247-268, 2003.

[22]. Li, J., Overlapping decomposition: A system-theoretic method for modeling and analysis of complex manufacturing systems, IEEE Transactions on Automation Science and Engineering, 2 (1): 40-53, 2005.

[23]. Ancelin, B., Semery, A., *Calcul de la productivité d'une ligne intégrée de fabrication*, R.A.I.R.O APII, 21 : 209-238, 1987.

[24]. De Koster, M. B. M., *Capacity oriented analysis and design of production systems*, Lecture Notes in Economics and Mathematical Systems no. 323, Springer-Verlag, London, 1989.

[25]. Glassey, C. R., Hong, Y., Analysis of behaviour of an unreliable n-stage transfer line with (n − 1) inter-stage storage buffers, International Journal of Production Research, 31 (3): 519–530, 1993.

[26]. Chahdi El Ouazzani, R., Modélisation et analyse des performances des systèmes de production utilisant des stocks tampons à capacités finies, Ph.D. Thesis, Université Laval, Quebec, Canada, 2007.

[27]. Lisnianski, A., Levitin, G., *Multi-State System Reliability, Assessment, Optimization and Applications*. World Scientific Publishing Co. Pte. Ltd., Singapore, 2003.

[28]. Kim, J., Gershwin, S. B., *Integrated quality and quantity modelling of a production line*, OR Spectrum, 27(2–3):287–314, 2005.

[29]. Tempelmeier, H., Bürger, M., Performance evaluation of unbalanced flow lines with general distributed processing times, failures and imperfect production, IIE *Transactions*, 33(4):293–302, 2001.

[30]. Poffe, A., Gershwin, S. B., Integrating quality and quantity modelling in a production line, Working paper, 2005.

[31]. Altiok, T., *Production lines with phase-type operation and repair times and finite buffers*, International Journal of Production Research, 23:489–498, 1985.

[32]. Özdogru, U., Altıok, T., *Analysis of two-valve fluid flow systems with general repair times*, In: Analysis and Modeling of Manufacturing Systems. Kluwer Publications, Dordrecht, 255–288, 2003.

[33]. Dallery, Y., On modeling failure and repair times in stochastic models of manufacturing systems using generalized exponential distributions, Queuing Systems Theory and Applications, 15(1–4):199–209, 1994.

[34]. Tan, B., Effects of variability on the due-time performance of a continuous materials flow production system in series, International Journal of Production Economics, 54:87–100, 1998.

[35]. Kim, J., Gershwin, S. B., Modeling and analysis of long flow lines with quality and operational failures, IIE Transactions, 40:284–296, 2008.

Appendix A. Solution of the boundary states probabilities

A.1 Solution of the high boundary states probabilities

$$\begin{bmatrix} -\mu_{12} & 0 & 0 & \lambda_{12} & 0 & 0 \\ 0 & -\mu_{12} & \mu_{21} & 0 & 0 & \lambda_{12} \\ 0 & 0 & -(\mu_{12}+\mu_{21}) & 0 & \lambda_{12} & 0 \\ \mu_{12} & 0 & 0 & -(\lambda_{11}+\lambda_{12}+\lambda_{21}+\lambda_{22}) & 0 & \mu_{22} \\ 0 & 0 & \mu_{12} & 0 & -(\lambda_{11}+\lambda_{12}+\mu_{21}+\mu_{22}) & \lambda'_{21} \\ 0 & \mu_{12} & 0 & \lambda_{22} & \mu_{21} & -(\lambda'_{11}+\lambda'_{21}+\lambda_{12}+\mu_{22}) \end{bmatrix} \times \begin{bmatrix} p\left(h_1,\overline{N_1},F_2^N\right) \\ p\left(h_1,\overline{N_1},F_2^D\right) \\ p\left(h_1,\overline{D_1},F_2^D\right) \\ p\left(h_1,N_1,N_2\right) \\ p\left(h_1,D_1,D_2\right) \\ p\left(h_1,N_1,D_2\right) \end{bmatrix}$$

$$= \begin{bmatrix} -\left[\dfrac{U_1^D\mu_{21}}{(\mu_{12}+\mu_{21})}+U_1^N\right]\sum_{m=1}^{3} C_m G_{2m}(F_2^D) \\ -U_1^N\sum_{m=1}^{3} C_m G_{2m}(F_2^D) \\ -U_1^D\sum_{m=1}^{3} C_m G_{2m}(F_2^D) \\ 0 \\ 0 \\ \left(U_1^N - U_2^D\right)\sum_{m=1}^{3} C_m \end{bmatrix}$$

A.2 Solution of the low boundary states probabilities

$$\begin{bmatrix} -\mu_{11} & \lambda_{11} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\mu_{11} & \lambda_{11} & \mu_{22} & 0 \\ 0 & 0 & 0 & 0 & -\left(\mu_{11}+\mu_{22}\right) & \lambda_{11} \\ 0 & 0 & 0 & \lambda'_{22} & \mu_{11} & -\left(\lambda_{11}+\lambda_{12}+\mu_{21}+\mu_{22}\right) \\ \mu_{11} & -\left(\lambda_{11}+\lambda_{12}+\lambda_{21}+\lambda_{22}\right) & 0 & 0 & 0 & \mu_{21} \\ 0 & \lambda_{21} & \mu_{11} & -\left(\lambda_{11}+\lambda'_{12}+\mu_{21}+\lambda'_{22}\right) & 0 & \mu_{22} \end{bmatrix} \times \begin{bmatrix} p\left(0,F_1^N,\underline{N_2}\right) \\ p\left(0,N_1,N_2\right) \\ p\left(0,F_1^D,\underline{N_2}\right) \\ p\left(0,D_1,N_2\right) \\ p\left(0,F_1^D,\underline{D_2}\right) \\ p\left(0,D_1,D_2\right) \end{bmatrix}$$

$$= \begin{bmatrix} \left[\dfrac{U_2^D\mu_{22}}{\left(\mu_{11}+\mu_{22}\right)}+U_2^N\right]\displaystyle\sum_{m=1}^{3} C_m G_{1m}(F_1^N) \\ -U_2^N \displaystyle\sum_{m=1}^{3} C_m G_{1m}(F_1^D) \\ -U_2^D \displaystyle\sum_{m=1}^{3} C_m G_{1m}(F_1^D) \\ 0 \\ 0 \\ \left(U_1^D - U_2^N\right)\displaystyle\sum_{m=1}^{3} C_m \end{bmatrix}$$