CIRRELT

Centre interuniversitaire de recherche sur les réseaux d'entreprise, la logistique et le transport

Interuniversity Research Centre on Enterprise Networks, Logistics and Transportation

Modeling and Solving a Logging Camp Location Problem

Sanjay Dominik Jena
Jean-François Cordeau
Bernard Gendron

September 2011

CIRRELT-2011-55

Bureaux de Montréal :
Université de Montréal
C.P. 6128, succ. Centre-ville
Montréal (Québec)
Canada H3C 3J7
Téléphone : 514 343-7575
Télécopie : 514 343-7121

Bureaux de Québec :
Université Laval
2325, de la Terrasse, bureau 2642
Québec (Québec)
Canada G1V 0A6
Téléphone : 418 656-2073
Télécopie : 418 656-2624

www.cirrelt.ca

Modeling and Solving a Logging Camp Location Problem

Sanjay Dominik Jena[1,2,*], Jean-François Cordeau[1,3], Bernard Gendron[1,2]

[1] Interuniversity Research Centre on Enterprise Networks, Logistics and Transportation (CIRRELT)

[2] Department of Computer Science and Operations Research, Université de Montréal, P.O. Box 6128, Station Centre-Ville, Montréal, Canada H3C 3J7

[3] Department of Logistics and Operations Management, HEC Montréal, 3000 Côte-Sainte-Catherine, Montréal, Canada H3T 2A7

Abstract. Harvesting plans for Canadian logging companies tend to cover wider territories than before. Long transportation distances for the workers involved in logging activities have thus become a significant issue. Often, cities or villages to accommodate the workers are far away. A common practice is thus to construct camps close to the logging regions, containing the complete infrastructure to host the workers. The problem studied in this paper consists in finding the optimal number, location and size of logging camps. We investigate the relevance and advantages of constructing additional camps as well as expanding and relocating existing ones since the harvest areas change over time. We model this problem as an extension of the Capacitated Facility Location Problem including the representation of economies of scale on several levels of the cost structure, the possibility of temporarily closing parts of the facilities as well as particular capacity constraints that involve integer rounding on the left hand side. Results for real-world data and for a large set of randomly generated instances are presented.

Keywords. Logging camps, forest industry, capacitated facility location problem, mixed integer programming.

Acknowledgements. The authors would like to thank Mathieu Blouin and Jean Favreau from FPInnovations for their valuable support throughout this study and for providing the data used in the experiments. The authors are also grateful to MITACS, the Natural Sciences and Engineering Research Council of Canada (NSERC) and FPInnovations for their financial support.

---

* Corresponding author: Sanjay.Jena@cirrelt.ca

Dépôt légal – Bibliothèque et Archives nationales du Québec
Bibliothèque et Archives Canada, 2011

1 Introduction

1.1 Context and Scope

Context. Log harvest planning in the forestry sector has changed throughout the last decades. Both silviculture and harvesting in Canada have become more sophisticated and now pose complex planning problems to get the most from the available regions and harvest cycles. Based on a wide variety of considerations, a long-term plan is designed to determine the volume and regions for wood logging. These decisions are commonly divided into smaller time periods, as logging activities and road construction within a single logging region typically take several months.

Due to political and environmental issues, as well as the size of the country, harvesting plans tend to cover wider territories than they used to. Often, sparse logging is necessary to certify the forestry operations. Several questions arise such as the location and capacity for administrative services, sorting yards and central log processing stations. Similarly, the location where the workers involved in forestry activities are accommodated gains in importance. If villages or cities are close, workers can be hosted at their homes or at motels. However, logging regions in Canada are often widely distributed and located far from such hosting options. In that case, accommodating the workers in the closest village or city is rarely an attractive option, as the commuting time and transportation costs are too high. Transportation times would consume a significant portion of the potential productive time. Furthermore, an additional salary is commonly paid when the transportation times exceed a certain threshold.

A common solution to this problem is the construction of logging camps in which the workers are accommodated. Logging camps are typically located close to the logging regions so that the transportation costs for the workers are reasonable. When allocating each work crew to a camp, the accommodation costs are given as a cost per day per worker. In order to host all workers, the construction of new accommodations may be necessary. The larger a camp, the smaller the daily cost per person. Hence, a small number of large camps results in smaller accommodation costs than a large number of small camps. However, the fewer camps are available, the higher the transportation costs tend to be, because their location is less flexible. The construction of a new camp or the relocation of an existing one may pay off in the long term as the traveling costs to the logging regions may be much lower.

Scope. This work investigates the possibility of constructing and relocating camps for the accommodation of workers, considering the harvest planning for the next five years. The problem, motivated by the needs of a Canadian logging company, consists in finding the number of camps that have to be constructed or relocated, their size in terms of capacity and their location such that the total costs for accommodation and transportation are minimal. The interesting question is whether such an investment in camp construction and relocation pays off, considering the operational logging and road construction planning for the next five years. It is important to note that the actual work crew assignment between accommodations and work regions is not relevant in practice, but is only used to determine the minimum capacity level necessary to host all workers. For the operational work crew assignment, other planning tools will be used. It is assumed that all information about work crews, logging regions and distances are known at the beginning of the planning and are not subject to uncertainty.

1.2 Contributions and Organization of the Paper

Contributions. Due to the complexity of the problem, manual planning approaches usually do not yield optimal solutions. The main objective of this paper is to propose a formulation for the problem that can be solved by a general-purpose solver for instances of reasonable size. The impact of different instance and model properties on the difficulty of the problem is studied. The presence of economies of scales on several levels of the cost structure as well as partial facility closing are part of the main concerns. Further aspects include particular capacity constraints that involve integer rounding on the left hand side. It is shown how such capacity constraints can be useful in other applications, but increase the integrality gap of the problem. We derive valid inequalities to effectively reduce this integrality gap.

Organization. This paper is organized as follows. Section 2 describes the relevant problem details. As the problem can be modeled as a facility location problem, the literature review in Section 3 focuses on relevant extensions in that domain. The mathematical formulation in Section 4 gradually extends the Capacitated Facility Location Problem to model the problem being addressed. This includes the particular capacity constraints, valid inequalties and additional features such as the relocation and partial closing of camps. Section 5 summarizes the results of the computational experiments performed. Finally, Section 6 concludes the work.

2 Problem Description

Based on an existing strategic plan, the logging company provides a harvesting plan for the next five years. Each year is divided into two seasons: winter and summer, each with a certain number of available working days. Depending on the geographical location, some regions will be logged more in winter whereas other regions will be logged more in summer. Each region is defined by its estimated log volume (measured in m^3) that is subject to harvesting (it may be part of the strategic decision that not the entire region will be harvested) within each season and the length of the road (measured in km) that has to be constructed in that region in order to access the logging areas and transport the log.

2.1 Work Crews, Demands and Hosting Capacities

There are two types of work crews: logging and road construction. Crews of the same type contain the same number of members. The members of a crew always stay together during work and are hosted at the same accommodation. For each logging region and season, a logging and road construction demand is given. Based on given productivity rates for the work crews one can compute the average number of crews necessary to cover the demand at each region for each season.

Example: Logging crews work 100 days within a given season and cut $180m^3$ per day, i.e., $18{,}000m^3$ within the season. A certain region holds a total demand of $27{,}000m^3$ for the season. Throughout 50 days, two logging crews will be working (i.e., $2 \cdot 50 \cdot 180m^3 = 18{,}000m^3$). The other 50 days, a single logging crew will be working (i.e., $1 \cdot 50 \cdot 180m^3 = 9{,}000m^3$). This results in an average allocation of $27{,}000/18{,}000 = 1.5$ logging crews in that season.

As the operational assignment of logging crews is not our final concern, we can assume that the crews of each working type are flexible with respect to the days they work within each season. That is, if a crew works only a few days in a season, we may assume that the exact days do not matter. In our example, it does not matter in which of the 100 days we use two crews and in which we use only one crew. In practice, a work crew may work a number of days in one region and then in another region in the same season. To determine the minimum capacity necessary to host all work crews allocated to a certain accommodation, consider the following example.

The workers from two regions are hosted at the same camp. One region has an average demand of 1.5 logging crews and 0.7 road construction crew. The other region has an average demand of 1.25 logging crews and 0.5 road construction crew. Figure 1 (a) illustrates this scenario for the logging crews. In total, we have a demand of $1.5 + 1.25 = 2.75$ logging crews and $0.7 + 0.5 = 1.2$ road construction crews. Hence, for 75% of the time during the season there will be $\lceil 2.75 \rceil = 3$ logging crews and 25% of the time there will be $\lfloor 2.75 \rfloor = 2$ logging crews, which is illustrated in Figure 1 (b). In the same way, for 20% of the season there will be $\lceil 1.2 \rceil = 2$ road construction crews and for the other 80% there will be only $\lfloor 1.2 \rfloor = 1$ road construction crew. Assuming that a logging crew has six workers and a road construction crew has three workers, we will need accommodation for $\lceil 2.75 \rceil \cdot 6 + \lceil 1.2 \rceil \cdot 3 = 18 + 6 = 24$ workers. To determine the minimum capacity of an accommodation, we can add the average numbers of crews allocated to this accommodation and round up the sum to the next highest integer (for each crew type).

Transportation. Workers are usually transported by pick-ups, using a given road network. Costs are composed of the travel and working time of the workers as well as the vehicle costs, i.e., renting and gas. An additional salary has to be paid if a certain transportation time (usually one hour per day) is exceeded. This makes large travel distances very costly. Workers of the same crew are transported in one or more vehicles. Workers of different crews do not share the same vehicle.

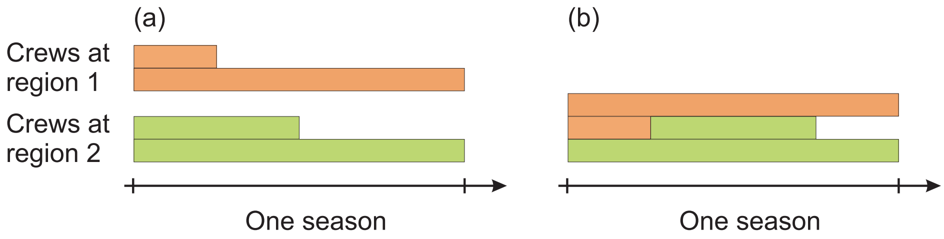

Figure 1: Example of logging demands hosted at the same accommodation.

Supervisors. In addition to the work crews, there are fixed numbers of logging and road construction supervisors. Supervisors have to be considered for the accommodation capacities and their individual transportation costs. Although it is not clearly predictable how many days a supervisor will be at which region, one may assume that their presence in a region is proportional to the demand for work crews at that region. Hosting regions for supervisors are often limited to accommodations with administrative units.

2.2 Camps and Trailers

Certain accommodations for the workers may already exist. These accommodations can either be hosting options in villages or cities (e.g., apartments, hotels or the employees' own homes) in reasonable distance of the logging regions, or camps that are usually located in the forest close to the logging regions. Accommodations vary in their capacity and their hosting costs. Camps are composed of trailers. A trailer contains the infrastructure to host a certain number of workers. In practice, trailers of different capacities are available. However, for the purpose of this study, we may assume that the trailer with a capacity for twelve persons is the most common one and hence all trailers have the same capacity. In addition to the trailers that host workers, a camp contains a number of additional trailers that provide complementary, but necessary infrastructure, such as a kitchen and leisure facilities. The number of additional trailers directly depends on the total hosting capacity of the camp, i.e., the number of hosting trailers. In the following, we will measure the capacity of a camp by the number of hosting trailers. Hence, the construction costs for a number of hosting trailers already include the costs for the necessary number of additional trailers.

Trailers can be either open or closed. Only open trailers are available for use. Trailers that are not in use have to be closed, involving one-time closing costs. Once a trailer is closed, it cannot be used in subsequent seasons until it is reopened, involving one-time reopening costs. Closing or reopening operations can be performed before each season. Costs for such operations usually involve economies of scale in the number of hosting trailers, since common resources are shared. The use of hosting trailers to accommodate workers involves two types of daily costs: fixed costs for each open trailer (including the cost for the trailer itself, its equipment, the cook, etc.) and variable costs (food, etc.) for each worker. The fixed costs are paid for each open trailer per day. Costs for closed trailers are so small that they do not have to be considered. Variable costs are paid for each worker hosted at the camp. If a trailer is open, its fixed costs have to be paid throughout the entire season, independent of its use. All costs may follow the principle of economies of scale, i.e., the larger the quantity, the lower the price-per-worker/trailer. New camps can only be constructed at certain places from a given set of potential locations. It is very common that several logging regions are served by workers from the same accommodation. Though it is rare, one logging region may also be served by workers from different accommodations.

2.3 Capacity Expansion and Camp Relocation

At certain points during the planning it may be interesting to increase the capacity of existing camps. Such capacity expansion is performed by adding new trailers. It is assumed that the cost of adding n trailers is the same as the construction of a new camp with n trailers. Trailers may also be permanently shut down. For the sake of simplicity, it is assumed that this is done by closing these trailers.

Logging regions are not equally harvested every year. That is, a camp may be close to logging regions with demands in certain years, but far away from logging regions that will be harvested afterwards. Instead of constructing a new camp, which involves high costs, camps can be moved from one location to another.

The relocation of camps can only be performed once a year, before the summer season. The distance between the origin and destination for a relocation has very little impact on the total relocation costs. We may thus assume that the total cost for relocating a camp depends only on the camp size (i.e., the number of trailers it includes). All trailers have to be closed before relocation. After the relocation, all trailers that are supposed to be in use have to be reopened again. In theory, camps from two distinct locations can also be joined to further reduce the costs per unit. Trailers from the same camp could also be relocated to distinct locations. In practice, these features are observed rather rarely. For the sake of simplicity, it is hence assumed that camps can only be relocated as a whole and that two different camps cannot be merged at the same location.

2.4 Objective

Given that all logging and road construction demands must be covered, we must ensure that sufficient accommodations are available to host the workers. We want to minimize the total costs, which are composed of two parts:

- All costs involved in providing the necessary accommodations: camp construction, camp relocation, maintenance for open trailers, closing and reopening of trailers and hosting costs for workers.
- The transportation costs between the accommodations and the logging regions. This includes the costs for using the vehicles and an additional salary for long transportation times.

A solution to the problem consists of the following information, given for each of the seasons in each of the years of the planning horizon:

- For each camp construction: the location and camp size.
- For each camp relocation: the origin, destination and size of the relocated camp.
- For each camp: the number of trailers that will be closed or reopened.

An insight into the suggested assignment of work crew demands to the accommodations may also be interesting for decision-makers. The assignment is necessary to determine the minimum level of camp capcities. However, it is not explicitly part of the problem solution.

Throughout this work, we will refer to this problem as the *Camp Size and Location Problem (CSLP).*

3 Literature Review

The forestry sector has been an extensive user of Operations Research (OR) methods for strategic, tactical and operational planning. Optimization is mainly used for supply chain design (D'Amours et al., 2008), harvesting (Bredström et al., 2010) and transportation planning (Carlsson et al., 2009). Strong interest is shown by both the public and private sector, typically in countries where logs represent a large portion of the net exports, such as Canada, Chile, New Zealand and the Scandinavian countries. Several recent surveys provide broad overviews of optimization in the forestry sector (see, e.g., D'Amours et al., 2008; Rönnqvist, 2003; Weintraub and Romero, 2006).

Rönnqvist (2003) compares different planning levels in terms of planning horizon, allowable solution time and required solution quality. These characteristics strongly vary among the different applications. Board cutting is individually decided for each tree and has to be optimally solved within less than a second. Harvesting plans typically cover an entire year. Near optimal solutions are desired within an hour of computation time. Forest management plans have to be evaluated quickly in order to allow for manual comparisons. However, the planning includes a strategic outlook for more than 100 years. To the best of our knowledge, the problem of locating logging camps has not yet been addressed in the OR literature. Its solution requirements are similar to those of road planning: one aims at near-optimal solutions, planning includes decisions for five years and one can allow computation times of several hours. Mathematical programming appears to be an appropriate tool, since it provides high quality solutions and it allows to model particular industrial constraints.

Several known problems present features similar to those found in the CSLP. Such problems typically belong to the family of Facility Location Problems. The CSLP can be formulated as an extension of the well studied Capacitated Facility Location Problem (CFLP), which aims at finding the optimal locations to construct an unknown number of facilities with capacity constraints. All customer demands have to be covered and the total costs, usually composed by costs for facility construction, production and transportation, are minimized. In the last decades, practical needs led to many extensions of the CFLP such as multiple periods, multiple commodities, multiple capacity levels and multiple stages. As demands are likely to change over time, many models focused on the dynamic (i.e., multi-period) case of the problem in order to address dynamic aspects such as capacity reduction, expansion and relocation.

The diversity, importance and maturity of facility location problems has been confirmed by many recent literature surveys (Hamacher and Nickel, 1998; Klose and Drexl, 2005; Melo et al., 2009; Revelle and Eiselt, 2005; Revelle et al., 2008). Melo et al. (2009) focus on the context of supply chains. Smith et al. (2009) review the development of location analysis from its early beginning and highlights today's most important applications. Many of the extensions proposed for the CFLP can be found in the proposed CSLP. Camps are translated to facilities and hosting demands to customers. The relevant literature regarding these features will now be reviewed.

Dynamic Facility Location Problems. The CSLP contains strong dynamic aspects, since logging regions tend to be harvested within a few seasons. Hence, a customer may have high demands in some time periods and no demand at all in the other periods. Early works in the domain of dynamic facility location were initiated by Ballou (1968) and Wesolowsky (1973). Recent works include Albareda-Sambola et al. (2009), Canel et al. (2001), Dias (2006), Melo et al. (2005), Peeters and Antunes (2001), Shulman (1991) and Troncoso and Garrido (2005). Many more references can be found in the previously cited reviews as well as in the one of Owen and Daskin (1998), which focuses on approaches that are based on either dynamic or stochastic facility location problems.

In addition to the optimal timing and sizes for facility construction, further dynamic features have been found beneficial to adapt to changing demand and market conditions. Capacity expansion has been incorporated by Melo et al. (2005), Peeters and Antunes (2001) and Troncoso and Garrido (2005). Capacity reduction or facility shut-down is addressed by Canel et al. (2001), Dias (2006), Melo et al. (2005) and Peeters and Antunes (2001). In an early work, Wesolowsky and Truscott (1975) considered a simple case of relocation of facilities. Melo et al. (2005) provide an extensive modeling framework for dynamic multi-commodity facility location problems. Their model focuses on the relocation of existing facilities and gradual capacity transfer from existing facilities to new ones while considering generic multi-level supply chain network structures.

Multiple Commodities. In some applications, customers have demands for several distinct commodities. The models must then distinguish between the different commodities to satisfy the demand for each of them as well as to control their capacity at the facilities. In the context of the CSLP, the different work crew types (i.e., logging crews and road construction crews) and supervisors can be modeled as different commodities.

In the multi-commodity facility location literature, models commonly assume that the customers have an individual demand for each commodity. However, on the facility side, the capacity constraints can be formulated in two different ways:

1. Each facility holds an individual capacity for each of the commodities.
2. Each facility holds a global capacity for the sum of all commodities.

The first option is the more common one in the literature (Canel et al., 2001; Geoffrion and Graves, 1974; Lee, 1991; Warszawski, 1973). In the CSLP, we rather consider the second case. While customers have a demand distinguished between the different commodities, the total capacity at the camps applies to the sum of all workers, whether they are logging or road construction workers. This idea of a common capacity for all commodities is also followed in the modeling framework of Melo et al. (2005).

Multiple Capacity Levels. The presence of production capacities automatically raises the question of the dimension of such capacities. While some applications allow for several facilities at the same place, most consider only one facility per location. Facilities may have fixed capacities or may choose among different capacity levels. Often, facility construction and unit production costs follow the principle of economies of scale, i.e., the larger the facility, the cheaper the price per unit in terms of facility construction and commodity production. One finds this feature in the CSLP, where camps are composed of trailers. The more hosting trailers exist, the larger the capacity and the better common resources (such as supplementary infrastructure) are shared. The choice of different capacity levels allows to represent such economies of scale.

Early works considering different capacity levels are Lee (1991), Shulman (1991) and Sridharan (1991). The choice of the capacity level is modeled as an additional variable index, having only one variable of a certain capacity level active for each facility. The cost part in the objective function thus corresponds to a piecewise linear function. In the literature, this has been the most common way to represent such cost functions (Paquet et al., 2004; Troncoso and Garrido, 2005).

Holmberg (1994) and Holmberg and Ling (1997) introduce an incremental approach to model staircase functions, where all variables up to the chosen capacity level are active. Similar approaches have since been adapted to more complex problems (Correia and Captivo, 2003; Gouveia and Saldanha da Gama, 2006).

Conclusions. Many of the features found in the CSLP have already been addressed in isolation in the facility location literature. However, very few models consider modular capacity levels in a dynamic context (Melo et al., 2005; Peeters and Antunes, 2001; Shulman, 1991; Troncoso and Garrido, 2005). These works do not address dynamic features such as facility closing/reopening or relocation. The closest related works are those of Melo et al. (2005) and Troncoso and Garrido (2005). The latter authors represent economies of scale for facility construction, but not for operational costs. Capacity relocation is also not considered. Melo et al. (2005) focus on capacity relocation, but consider modular capacity decisions only for relocation.

While many models consider closing an entire facility or reducing its capacity, none of the reviewed works present the possibility of partially or entirely deactivating a facility for a certain time period, as it is possible with trailers in logging camps. In addition, the capacity constraints found in the CSLP have not yet been addressed in the context of facility location problems.

4 Mathematical Formulation

The CSLP can be modeled as an extension of the CFLP. Some of the additional features have been considered in variations of that classical problem. However, to the best of our knowledge, no extension of the CFLP considered all features at the same time. In particular, two of them have not been mentioned in the related literature:

1. Round-up (integer) capacity constraints for the camps.
2. Partial closing and reopening of trailers throughout the planning periods.

In the following, the CFLP will be gradually extended in order to explore the impact of the additional features on the model size and solution difficulty. In a first step, a formulation for the Dynamic Modular (i.e., multiple capacity levels) Multi-Commodity Facility Location Problem with Multi-Source Assignment is studied. Then, the dynamic features are added, namely the relocation of camps and the closing and reopening of trailers. These latter decisions are transformed into modular ones to represent economies of scale.

4.1 Extending the CFLP

The classical CFLP, as presented by Sridharan (1995), is extended. To be more precise, the following features are added:

- Multiple periods. We study the problem in a dynamic context, i.e., over multiple time periods with independent demands.
- Multiple commodities. We assume the existence of different commodities, one for each work crew type. Each customer may have independent demands for each of these commodities.
- Multiple capacity levels. We assume that a facility may have different capacities, i.e., different numbers of hosting trailers. These capacities are modular and can represent cost structures involving economies of scale.

Due to its additional characteristics, we refer to this problem as the *Dynamic Modular Multi-Commodity Facility Location Problem (DMCFLP).*

4.1.1 Input Data and Decision Variables

Input Data. Consider the following input data:

- I - set of potential camp locations (facilities).
- J - set of logging/road construction regions (customers).
- K - set of possible camp sizes (with respect to the number of hosting trailers), $K = \{1, 2, .., \overline{K}\}$.
- P - set of existing work crew types (commodities).
- T - set of seasons (time periods), $T = \{1, 2, 3, .., |T|\}$.
- d_{jpt} - demand (in number of crews) for commodity $p \in P$ in region $j \in J$ and period $t \in T$.
- u_{ik} - total capacity of a camp of size $k \in K$ at location $i \in I$.
- c^C_{ik} - construction cost of a camp of size $k \in K$ at location $i \in I$.
- c^V_{ijkpt} - variable operational costs (including transportation and hosting costs) for the entire time period $t \in T$ for one crew of working type $p \in P$ accommodated at a camp of size $k \in K$ at location $i \in I$ and working at region $j \in J$. The total cost is typically not linear with respect to the Euclidean distance between the work region and the accommodation.

Decision Variables. The decision variables are:

- $x_{ijkpt} \in \mathbb{R}^+$ - total demand (in number of crews) of crew type $p \in P$ assigned from a camp of size $k \in K$ at location $i \in I$ to region $j \in J$ at time period $t \in T$.
- $y_{ik} \in \{0, 1\}$ - 1, if a camp of size $k \in K$ is constructed at location $i \in I$ at the beginning of the horizon, 0 otherwise.

4.1.2 Mathematical Model

The model is given by:

$$\min \sum_{i\in I}\sum_{k\in K} c^C_{ik} y_{ik} + \sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\sum_{p\in P}\sum_{t\in T} c^V_{ijkpt} x_{ijkpt} \tag{1}$$

$$s.t. \quad \sum_{i\in I}\sum_{k\in K} x_{ijkpt} = d_{jpt} \quad ;\forall j \in J \quad ;\forall p \in P \quad ;\forall t \in T \tag{2}$$

$$\sum_{p\in P}\sum_{j\in J} x_{ijkpt} \le u_{ik} y_{ik} \quad ;\forall i \in I \quad ;\forall k \in K \quad ;\forall t \in T \tag{3}$$

$$\sum_{k\in K} y_{ik} \le 1 \quad ;\forall i \in I \tag{4}$$

$$x_{ijkpt} \le d_{jpt} y_{ik} \quad ;\forall i \in I \quad ;\forall j \in J \quad ;\forall k \in K \quad ;\forall p \in P \quad ;\forall t \in T \tag{5}$$

$$x_{ijkpt} \in \mathbb{R}^+ \quad ;\forall i \in I \quad ;\forall j \in J \quad ;\forall k \in K \quad ;\forall p \in P \quad ;\forall t \in T \tag{6}$$

$$y_{ik} \in \{0, 1\} \quad ;\forall i \in I \quad ;\forall k \in K \tag{7}$$

The objective function (1) minimizes the camp construction cost and the operational costs. The set of constraints (2) guarantees that all customer demands are satisfied. Note that demands are likely to be fractional, as illustrated in Figure 1. Constraints (3) require that the hosting demands assigned to each camp do not exceed the camp capacities. Constraints (4) ensure that only one capacity level is selected for each facility. The set of valid inequalities (5), also referred to as *Strong Inequalities (SI)* (Gendron and Crainic, 1994), provide a stronger upper bound for the demand assignment variables. Computational experiments show that CPLEX solves the problem more effectively when adding only the violated SIs (using CPLEX user cuts) than when adding all SIs *a priori* or not adding them at all.

Non-movable accommodations. In addition to logging camps, we may model accommodations such as motels and apartments to host workers. We do so by representing them as a restricted case of a camp, with two types of information: hosting costs and total capacity. Such accommodations possess a single capacity level and cannot be relocated.

4.2 Round-Up Capacity Constraints

As explained above, the CSLP involves particular capacity constraints where the sum of all demands assigned to a certain accommodation is rounded up to the next integer value. Adding, for example, demands of 1.5 crews and 1.25 crews, one only needs a total capacity for three crews (if all crews are hosted at the same camp) instead of four (compare Figure 1).

Let N_p be the number of workers in a crew of type p. We introduce additional integer variables z_{ikpt} for the integer rounding, indicating the total number of crews of type p assigned to a size k camp at location $i \in I$ at period $t \in T$. The existing capacity constraints (3) are replaced by two new constraints (8) and (9), which we will refer to as the *round-up capacity constraints (RUC)*. Instead of using the continuous sum of the facility/customer assignment variables (x variables), the capacity constraints (9) take into account the next highest integer value, bounded by the z variables in constraints (8):

$$\sum_{j \in J} x_{ijkpt} \leq z_{ikpt} \quad ;\forall i \in I \;\; ;\forall k \in K \;\; ;\forall p \in P \;\; ;\forall t \in T \tag{8}$$

$$\sum_{p \in P} N_p z_{ikpt} \leq u_{ik} y_{ik} \quad ;\forall i \in I \;\; ; k \in K \;\; ;\forall t \in T \tag{9}$$

$$z_{ikpt} \in \mathbb{Z}^+ \quad ;\forall i \in I \;\; ;\forall k \in K \;\; ;\forall p \in P \;\; ;\forall t \in T \tag{10}$$

This type of capacity constraints is likely to appear in other applications. In the context of facility location problems, scenarios can be modeled where a facility may not be able to produce any arbitrary amount of a product, but only modular sized packages of products.

4.2.1 Strengthening the Formulation

Experiments have shown that the average integrality gap increases significantly (see Section 5.2 for details) when using round-up capacity constraints (8)-(10) instead of the usual constraints (3). Consider the following *aggregrated demand* inequalities which are known to be redundant for the linear relaxation of the model:

$$\sum_{i \in I} \sum_{k \in K} u_{ik} y_{ik} \geq \sum_{p \in P} \sum_{j \in J} d_{jpt} N_p \quad ;\forall t \in T$$

We will now strengthen these inequalities, based on the fact that z is integer. Substituting (2) in (8) shows that one can always round up the sum of all demands from different regions for the same product. We replace the right hand side (RHS) of the previous inequality by D_t, where:

$$D_t = \sum_{p \in P} \left\lceil \sum_{j \in J} d_{jpt} \right\rceil N_p \quad ;\forall t \in T$$

We now express the resulting inequality in terms of the number of trailers instead of the number of crews. Assuming that each trailer hosts exactly M workers, i.e., $u_{ik} = Mk$, we have:

$$\sum_{i \in I} \sum_{k \in K} k y_{ik} \geq \frac{D_t}{M} \quad ;\forall t \in T$$

These inequalities state the minimum number of open trailers necessary to satisfy all customer demands. We know that the RHS, the minimum number of open trailers, is always integer. We can thus replace the RHS by S_t, where:

$$S_t = \left\lceil \frac{D_t}{M} \right\rceil \quad ;\forall t \in T$$

In a final step, we aim at reducing the coefficients of the y variables on the left hand side. Suppose that $\overline{K} > S_t$. It is then sufficient that only one $y_{ik'}$ with $k' \geq S_t$ is active in order to satisfy the entire customer demand in the integer solution. That is, we may set the coefficient of a variable $y_{ik'}$ to S_t whenever $k' \geq S_t$:

$$\sum_{i \in I} \sum_{k \in K} \min\{k, S_t\}\, y_{ik} \geq S_t \quad ; \forall t \in T \tag{11}$$

In the following, we will refer to these constraints as the *strengthened aggregated demand (SAD)* inequalities.

4.3 Partial Camp Closing, Relocation and Modular Costs

In this section, the previous model will be extended with the following features that may appear in a dynamic context:

1. Construction of new camps/trailers at any time period.
2. Closing and reopening of trailers at any time period.
3. Relocation of camps at any time period.
4. Modular costs for trailer closing/reopening and camp relocation.

This problem corresponds to the CSLP. A network flow structure, illustrated in Figure 2, is added on top of the previously introduced model to manage the first three features. For each time period, two nodes for open trailers and two nodes for closed trailers are used. Arcs between these nodes represent certain operations to modify the number of open and closed trailers at each location and to relocate them to other locations. The flow on these arcs indicates the number of trailers involved in the corresponding operation. New trailers can be constructed at the beginning of any season (s arcs). Open trailers can be closed (v^{OC} arcs) and closed trailers can be reopened (v^{CO} arcs). The arcs v^{OO} represent trailers that were open at the beginning of the season and remain open during the current season. The arcs v^{CC} indicate closed trailers that are not relocated to another region. These trailers can still be reopened for the current season. Finally, l^O and l^C indicate the number of trailers that are open and closed, respectively, at each location throughout the entire season.

Relocation is allowed only for closed trailers. One could model relocation by the use of direct arcs between all location pairs. However, this would result in very large models. Hence, relocation is modeled by the use of a central node, here referred to as a *hub node (H)*. The flow of relocated trailers is first passed to the hub node (w^O arcs) and then further distributed to another location (w^I arcs).

4.3.1 Input Data and Decision Variables

Additional Input Data. In addition to the previously introduced input data, additional parameters are considered. These data may already consider economies of scale with respect to k, the number of trailers involved in the operation: c_k^{TO} and c_k^{TC} are the costs to reopen and close k trailers of the same camp, respectively. The maintenance costs for a camp with k open trailers during season t is given by c_{kt}^{M}. Finally, c_k^R represents the costs for relocating a camp with k closed trailers.

Additional Decision Variables. To incorporate the new features, some variables have to be extended and new variables have to be added to the model. A new index k for variable y_{ikt} indicates the number of open hosting trailers during period t. A separate binary variable s_{iqt} indicates the construction of q new trailers at location i before period t. In addition, arc flow variables for the network are added to manage the closing and reopening of trailers: l_{it}^O, l_{it}^C, v_{it}^{OO}, v_{it}^{OC}, v_{it}^{CO} and v_{it}^{CC}.

Finally, binary variables are needed to incorporate modular costs: v_{ikt}^{BCO} and v_{ikt}^{BOC} indicate whether k trailers are reopened or closed, respectively, at location i before time period t. Variables w_{ikt}^{BO} and w_{ikt}^{BI} indicate whether a size k camp is relocated from or to, respectively, location i before period t.

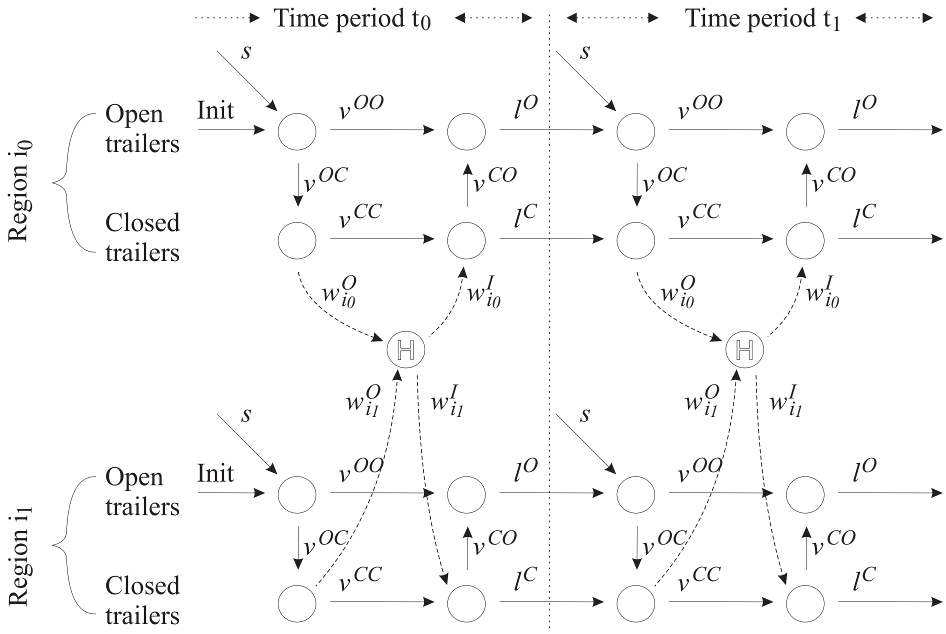

Figure 2: Network model to manage open and closed trailers at each location.

4.3.2 Mathematical Model

Objective Function. The objective function minimizes all costs: maintenance for open trailers, operational hosting and transportation, trailer construction, camp relocation and trailer reopening and closing:

$$\begin{aligned}\min \sum_{i\in I}\sum_{k\in K}\sum_{t\in T} c^M_{kt} y_{ikt} + \sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\sum_{p\in P}\sum_{t\in T} c^V_{ijkpt} x_{ijkpt} \\ + \sum_{i\in I}\sum_{q\in K}\sum_{t\in T} c^C_{in} s_{iqt} + \sum_{i\in I}\sum_{k\in K}\sum_{t\in T} c^R_{ik} w^{BO}_{ikt} \\ + \sum_{i\in I}\sum_{k\in K}\sum_{t\in T} c^{TO}_{k} v^{BCO}_{ikt} + \sum_{i\in I}\sum_{k\in K}\sum_{t\in T} c^{TC}_{k} v^{BOC}_{ikt}\end{aligned}$$

Demand and Capacity Constraints. The constraints representing the part of the facility location problem are identical to the ones in the previously presented model. However, the y variables now represent the number of open trailers at each location and time period:

$$\sum_{i\in I}\sum_{k\in K} x_{ijkpt} = d_{jpt} \quad ;\forall j\in J \ ;\forall p\in P \ ;\forall t\in T \tag{12}$$

$$\sum_{j\in J} x_{ijkpt} \le z_{ikpt} \quad ;\forall i\in I \ ;\forall k\in K \ ;\forall p\in P \ ;\forall t\in T \tag{13}$$

$$\sum_{p\in P} N_p z_{ikpt} \le u_{ik} y_{ikt} \quad ;\forall i\in I \ ;\forall k\in K \ ;\forall t\in T \tag{14}$$

$$\sum_{k\in K} y_{ikt} \le 1 \quad ;\forall i\in I \ ;\forall t\in T \tag{15}$$

$$x_{ijkpt} \le d_{jpt} y_{ikt} \quad ;\forall i\in I \ ;\forall j\in J \ ;\forall k\in K \ ;\forall p\in P \ ;\forall t\in T \tag{16}$$

Flow Conservation and Consistency Constraints. The network is modeled by the following constraints. Constraints (17), (18), (19) and (20) represent the first nodes for open and closed trailers and the second nodes for open and closed trailers, respectively. Note that the variables l^O_{it} and l^C_{it} do not exist for $t = 0$, i.e., in constraints (17) and (18), we have $l^O_{i(t=0)} = 0$ and $l^C_{i(t=0)} = 0$. If a region $i \in I$ already possesses a camp at the beginning of the planning horizon, then a constant $\Gamma_{it} > 0$ (with $t = 1$) indicates the number of hosting trailers of that camp. Clearly, $\Gamma_{it} = 0$ for all $t > 1$. Constraints (21) guarantee that the number of existing trailers at a camp never exceeds the maximum camp size, while (22) link the y variables to the number of open trailers:

$$\Gamma_{it} + l^O_{i(t-1)} + \sum_{q \in K} ns_{iqt} = v^{OO}_{it} + v^{OC}_{it} \quad ; \forall i \in I \quad ; \forall t \in T \tag{17}$$

$$l^C_{i(t-1)} + v^{OC}_{it} = v^{CC}_{it} + w^O_{it} \quad ; \forall i \in I \quad ; \forall t \in T \tag{18}$$

$$v^{OO}_{it} + v^{CO}_{it} = l^O_{it} \quad ; \forall i \in I \quad ; \forall t \in T \tag{19}$$

$$v^{CC}_{it} + w^I_{it} = v^{CO}_{it} + l^C_{it} \quad ; \forall i \in I \quad ; \forall t \in T \tag{20}$$

$$l^O_{it} + l^C_{it} \leq \overline{K} \quad ; \forall i \in I \quad ; \forall t \in T \tag{21}$$

$$\sum_{k \in K} k y_{ikt} = l^O_{it} \quad ; \forall i \in I \quad ; \forall t \in T \tag{22}$$

Relocation Consistency Constraints. Equalities (24) enforce that if a camp of size k is removed from a location, then a camp of the same size must be placed at another region. It ensures that trailers of different camps will not be mixed if they are relocated at the same time period. Constraints (23) ensure that camps are only relocated as a whole. Together, constraints (23) and (24) also ensure that camps are not merged at a region. Although redundant, constraints $\sum_{k \in K} w^{BO}_{ikt} \leq 1$ and $\sum_{k \in K} w^{BI}_{ikt} \leq 1$ are explicitly added to the model, since they help CPLEX generate further cuts.

$$v^{CC}_{it} + v^{OO}_{it} \leq \overline{K} \left(1 - \sum_{k \in K} w^{BO}_{ikt} \right) \quad ; \forall i \in I \quad ; \forall t \in T \tag{23}$$

$$\sum_{i \in I} w^{BO}_{ikt} = \sum_{i \in I} w^{BI}_{ikt} \quad ; \forall k \in K \quad ; \forall t \in T \tag{24}$$

Linking Constraints for Modular Costs. Linking constraints as suggested by Melo et al. (2005) are used to link the continuous arc flow variables to the binary variables for modular decisions:

$$\sum_{k \in K} k v^{BCO}_{ikt} = v^{CO}_{it} \quad ; \forall i \in I \quad ; \forall t \in T \tag{25}$$

$$\sum_{k \in K} k v^{BOC}_{ikt} = v^{OC}_{it} \quad ; \forall i \in I \quad ; \forall t \in T \tag{26}$$

$$\sum_{k \in K} k w^{BO}_{ikt} = w^O_{it} \quad ; \forall i \in I \quad ; \forall t \in T \tag{27}$$

$$\sum_{k \in K} k w^{BI}_{ikt} = w^I_{it} \quad ; \forall i \in I \quad ; \forall t \in T \tag{28}$$

Variable Domains. Once the y variables are fixed, the remaining subproblem defined by the network flow structure can be stated as a *Mininum Cost Network Flow Problem.* All l^O arcs are then fixed according to the y values due to the equality constraints (22). Thus, the remaining network matrix has the *unimodularity property.* We could thus state all arc variables as continuous without losing their integrality property in the solution. However, we keep integrality on the arc variables since experiments showed that it slightly facilitates the solution by CPLEX.

$$x_{ijkpt} \in \mathbb{R}^+ \quad ;\forall i \in I \quad ;\forall j \in J \quad ;\forall k \in K \quad ;\forall p \in P \quad ;\forall t \in T \tag{29}$$
$$z_{ikpt} \in \mathbb{Z}^+ \quad ;\forall i \in I \quad ;\forall k \in K \quad ;\forall p \in P \quad ;\forall t \in T \tag{30}$$
$$y_{ikt} \in \{0,1\} \quad ;\forall i \in I \quad ;\forall k \in K \quad ;\forall t \in T \tag{31}$$
$$s_{iqt} \in \{0,1\} \quad ;\forall i \in I \quad ;\forall q \in K \quad ;\forall t \in T \tag{32}$$
$$l_{it}^{O}, l_{it}^{C}, v_{it}^{CC}, v_{it}^{CO}, v_{it}^{OO}, v_{it}^{OC}, w_{it}^{O}, w_{it}^{I} \in \mathbb{Z}^+ \quad ;\forall i \in I \quad ;\forall t \in T \tag{33}$$
$$v_{ikt}^{BCO}, v_{ikt}^{BOC}, w_{ikt}^{BO}, w_{ikt}^{BI} \in \{0,1\} \quad ;\forall i \in I \quad ;\forall k \in K \quad ;\forall t \in T \tag{34}$$

4.3.3 Linear Costs for Relocation and Trailer Reopening/Closing

We can state the CSLP without modular costs for the relocation of camps and trailer reopening/closing. We refer to this problem variant as the *CSLP with linear costs (CSLPlc)*. In the formulation introduced below, we do not use hub nodes for relocation, but rather represent direct relocation between each pair of regions. To be specific, we use the variables $w_{i_1 i_2 t}^{B} \in \{0,1\}$ and $w_{i_1 i_2 t} \in \mathbb{Z}^+$ to indicate whether a camp is relocated from i_1 to i_2 at time period t and the size of that camp, respectively. Binary variables and linking constraints for modular costs are excluded. Let c^R, c^{TO} and c^{OC} be the costs to relocate, reopen and close one trailer, respectively. The modified model is given by:

$$\begin{aligned}
\min & \sum_{i\in I}\sum_{k\in K}\sum_{t\in T} c_{kt}^{M} y_{ikt} + \sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\sum_{p\in P}\sum_{t\in T} c_{ijkpt}^{V} x_{ijkpt} \\
& + \sum_{i\in I}\sum_{q\in K}\sum_{t\in T} c_{in}^{C} s_{iqt} + \sum_{i_1\in I}\sum_{i_2\in I}\sum_{k\in K}\sum_{t\in T} c^{R} w_{i_1 i_2 t} \\
& + \sum_{i\in I}\sum_{t\in T} c^{TO} v_{it}^{CO} + \sum_{i\in I}\sum_{t\in T} c^{TC} v_{it}^{OC} \\
s.t. \; & (12)-(16), (17), (19), (21), (22) \\
& (29)-(33)
\end{aligned}$$

$$l_{i(t-1)}^{C} + v_{it}^{OC} = v_{it}^{CC} + \sum_{i_2\in I} w_{i i_2 t} \quad ;\forall i \in I \quad ;\forall t \in T \tag{35}$$
$$v_{it}^{CC} + \sum_{i_1\in I} w_{i_1 i t} = v_{it}^{CO} + l_{it}^{C} \quad ;\forall i \in I \quad ;\forall t \in T \tag{36}$$
$$v_{it}^{CC} + v_{it}^{OO} \le \overline{K}\left(1 - \sum_{i_2\in I\setminus\{i\}} w_{i i_2 t}^{B}\right) \quad ;\forall i \in I \quad ;\forall t \in T \tag{37}$$
$$w_{i_1 i_2 t} \le \overline{K} \cdot w_{i_1 i_2 t}^{B} \quad ;\forall i_1 \in I \quad ;\forall i_2 \in I\setminus\{i_1\} \quad ;\forall t \in T \tag{38}$$
$$\sum_{i_2\in I} w_{i i_2 t}^{B} \le 1 \quad ;\forall i \in I \quad ;\forall t \in T \tag{39}$$
$$\sum_{i_1\in I} w_{i_1 i t}^{B} \le 1 \quad ;\forall i \in I \quad ;\forall t \in T \tag{40}$$
$$w_{i_1 i_2 t} \in \mathbb{Z}^+, w_{i_1 i_2 t}^{B} \in \{0,1\} \quad ;\forall i_1 \in I \quad ;\forall i_2 \in I \quad ;\forall t \in T \tag{41}$$

Constraints (35), (36), (37) and (38) replace (18), (20), (23) and (24), respectively. The binary relocation variables are necessary to enable the modeling of constraints (39) and (40), which ensure that camps are only relocated as a whole and not merged after relocation. In this formulation, these constraints are essential to ensure feasibility. In the CSLP, relocation can also be modeled by using variables for each pair of regions instead of central hub nodes. However, experiments showed that this significantly increases the model size and therefore also the difficulty of solving the problem.

5 Computational Experiments

5.1 Instance Generation and Experimentation Environment

In order to test the robustness of the model, instances have been generated with different parameters. Certain data have been adapated from a real-world (RW) instance, based on data provided by a Canadian logging company (see Section 5.3). Key parameters are found to be the ones that may change the difficulty of the problem, namely:

- **Problem dimension**. Instances have been generated with the following dimensions *(#facility locations/#customers)*: *(10/20)*, *(10/50)*, *(50/50)* and *(50/100)*.
- **Distances and transportation costs**. For each of the problem sizes, three different networks have been randomly generated on squares of the following sizes: $300km \times 300km$, $380km \times 380km$ and $450km \times 450km$. Transportation costs have been computed as explained in Section 2.1.
- **Number of commodities**. Demands are generated either only for logging and road construction (i.e., two commodities) or additionally for the corresponding supervisors (i.e., four commodities).
- **Concavity of the cost curves**. Two extreme cases are considered: construction and operational costs are either linear or concave. In addition, the cost curves given in the RW instance with linear construction costs and concave operational costs are considered.
- **Demand distribution**. The demand for each region within each season is randomly generated so that the total demand in each season throughout all regions is similar. For each region, the demand is either uniformly distributed over all seasons or randomly distributed over up to four seasons.
- **Cost distribution**. The ratio between camp construction/relocation and transportation costs is generated for different ratios. The transportation costs were set to 20%, 100% and 200% of the original transportation costs indicated in the RW instance.
- **Initial demand coverage**. Instances are generated with different numbers of initially existing camps. The total capacity of such camps covers either 0%, 50% or 100% of the total demand.

All generated instances contain ten time periods. Camp relocation costs and the costs to close or reopen trailers have been adapted from the RW instance. The maximum camp size $\overline{K}$ has been chosen so that a single camp with $\overline{K}$ trailers is capable to host the entire worker demand. The combination of all different configurations explained above resulted in 1296 instances, 432 for each of the three transportation networks. Experiments are either based on all instances or on a certain subset. Instance set *ISall* contains all 1296 instances, whereas instance set *ISsel* represents the more difficult instances of reasonable size. It includes 216 instances: all instances of size (10/20) and (10/50), excluding those with only two commodities or linear cost curves.

The code has been written in C/C++ using the Callable Library of IBM ILOG CPLEX 12.2.0.2 and has been compiled and executed on openSUSE 11.3. Each problem instance has been run on a single AMD Opteron 250 processor (2.4 GHz), limited to 4GB of RAM. If not stated otherwise, CPLEX computation times have been limited to 60 minutes.

5.2 Computational Results

The following variants of the problem have been considered to investigate the impact of the different problem features on the difficulty of solving the problem:

- The DMCFLP as described in Section 4.1. Both versions without and with round-up capacity constraints (see Section 4.2) and SAD inequalities are considered.
- The CSLPlc (i.e., the CSLP with linear costs for relocation and trailer reopening/closing) as described in Section 4.3.3.
- The CSLP, as described in Section 4.3, with all modular costs.

Table 1 shows the average sizes of the models for all three problem variants: the number of binary variables (*#V bin*), the total number of variables (*#V tot*), the total number of constraints (*#C tot*) and the number of strong inequalities (*#SIs tot*). The number of binary variables increases by a factor of around five when allowing camp relocation and closing/reopening of trailers. Interestingly, the CSLPlc holds almost the same number of binary variables. However, in the CSLPlc the binary variables represent camp relocation decisions for all pairs of regions, whereas in the CSLP the binary variables are numerous due to the modular capacity levels. The latter is expected to make the problem more difficult to solve as the capacity level directly impacts the costs. Note that the number of capacity levels (*#cLev*) depends on the total demand and therefore also on the number of customers, which therefore impacts on the number of binary variables as well.

		DMCFLP			CSLPlc			CSLP			
Inst Size	# cLev	#V bin (k)	#V tot (k)	#C tot (k)	#V bin (k)	#V tot (k)	#C tot (k)	#V bin (k)	#V tot (k)	#C tot (k)	#SIs tot (k)
10/20	5.6	<1	37	3	2	29	4	3	30	4	28
10/50	12.3	1	165	6	3	136	8	8	140	7	137
50/50	12.3	7	1,067	33	37	722	54	37	699	32	690
50/100	23.5	14	4,936	62	49	2,537	77	71	2,537	56	2,529

Table 1: Average model sizes of the DMCFLP, CSLPlc and CSLP.

The SIs, given by the inequalities (16) and (5) for the DMCFLP and the CSLP, respectively, are very effective to strengthen the model. For the *ISsel* instances, the integrality gap of the DMCFLP with SAD inequalities decreased from 20.3% to 2.2% on average by adding the SIs without increasing the linear relaxation solution time. However, the number of SIs becomes very large when increasing the number of customers, as can be seen in Table 1. It was observed that the solver often did not find feasible solutions for such large models. On the other hand, adding the SIs dynamically to the model (as CPLEX user cuts) often slowed down the solution of the root node. In the following experiments, SIs have been added as user cuts in the case of the DMCFLP. For the CSLPlc and CSLP, all SIs have been added to the model *a priori*, since the improved linear relaxation bound proved to have a significant impact on the difficulty of the problem. Further experiments on the *ISsel* instances indicate that CPLEX performs best when the parameter *MIPEmphasis* is set to *feasibility*.

5.2.1 Impact of the round-up capacity constraints and inequalities

Table 2 shows the average values of the integrality gap, its standard deviation and the average time to solve the linear relaxation of the DMCFLP model (with all SIs) without and with RUC constraints. Results for the latter one are reported without and with SAD inequalities. The average values have been computed over all instances for which an optimal integer and linear relaxation solutions were available for all three problem versions (indicated by *# Inst*; note that, for instance size (50/50) and (50/100), very few optimal integer and linear relaxation solutions were found). The results indicate that the RUC constraints significantly increase the integrality gap of the model. However, the use of the SAD inequalities effectively decreases both the integrality gap and the solution time for the linear relaxation. The same trend is observed for the standard deviation of the integrality gap.

Table 3 summarizes the average optimality gaps after one hour of computation time. For each instance size, the total number of instances in the set (*# Inst all*) and the number of instances for which both versions found feasible integer solutions (*# Inst sel*) are reported. The optimality gaps (*gap %*) are either average values over all instances (*all*) or only those for which feasible solutions have been found for both problems (*sel*). The column *# ns* indicates the number of instances where either no feasible integer solution has been found or the solver ran out of memory. Both problems are solved fairly well for small instances. For larger instances, the results indicate that using RUC adds to the difficulty of solving the problem. Most of the instances of size (50/100) could not be solved at all within the given computation time, mostly due to memory limitations. In both problem versions, SIs are added as CPLEX user cuts. Experiments showed that adding all SIs *a priori* to the model worsens the results for both problem versions.

		w/o RUC			w/ RUC w/o SAD			w/ RUC w/ SAD		
Inst Size	# Inst	gap %	Std Dev	time (sec)	gap %	Std Dev	time (sec)	gap %	Std Dev	time (sec)
10/20	304	3.6	3.1	28	8.3	5.7	27	1.3	1.3	12
10/50	238	2.1	2.2	575	4.5	3.4	497	1.5	1.9	570
50/50	3	1.0	1.5	1,888	3.1	4.4	2,145	1.8	2.5	1,834
50/100	0	-	-	-	-	-	-	-	-	-
Total Avg	545	2.8	2.6	244	6.0	4.3	261	1.4	1.5	231

Table 2: Comparing the integrality gap and the linear relaxation solution time for the DMCFLP (*ISall*) with and without RUC constraints and SAD inequalities.

			DMCFLP w/o RUC			DMCFLP w/ RUC w/ SAD		
Inst Size	# Inst all	# Inst sel	gap % (sel)	gap % (all)	# ns	gap % (sel)	gap % (all)	# ns
10/20	324	324	0.0	0.0	0	0.0	0.0	0
10/50	324	311	4.5	5.1	2	6.4	6.3	11
50/50	324	185	12.4	22.2	23	16.1	16.1	136
50/100	324	18	14.1	55.1	220	13.2	14.9	303
Total	1296	838	4.8	13.4	245	6.2	6.3	450

Table 3: Comparing the solution quality for the DMCFLP (*ISall*) without and with RUC constraints and SAD inequalities after one hour of computation time.

5.2.2 Solving the problem and solution properties

The impact of different problem features and instance properties are now explored. In the following, we investigate the difficulty of solving the DMCFLP, the CSLPlc and the CSLP. We show relations between the optimal solutions of the DMCFLP and the CSLP by comparing the number of constructed and relocated trailers. This leads to the idea of using DMCFLP solutions as starting solutions for the CSLP. The impact of certain properties such as the demand distribution over time, the initial camp capacity and the dimension of transportation costs is evaluated.

Solving the DMCFLP, CSLPlc and CSLP. Table 4 summarizes the average optimality gaps (*gap %*), solution time and number of instances where no feasible integer solution has been found (*# ns*) for the *ISsel* instance set. It can be observed that most of the DMCFLP instances can be well solved within one hour of computation time. On the contrary, the CSLPlc appears much more difficult to solve. Although the models hold a similar number of variables and constraints, the implementation of modular costs in the model of the CSLP significantly complicates the solution of the problem.

		DMCFLP			CSLPlc			CSLP		
Inst Size	# Inst	gap %	# ns	time (sec)	gap %	# ns	time (sec)	gap %	# ns	time (sec)
10/20	108	0.0	0	506	2.9	0	2,387	7.4	24	2,320
10/50	108	15.4	5	3,073	2.2	85	2,353	3.2	90	2,795
Total	216	7.5	5	1,790	2.8	85	2,382	6.7	114	2,404

Table 4: Comparing the solution quality for the different problem variants (*ISsel*) after one hour of computation time.

DMCFLP warm start solutions for the CSLP. The above results indicate that DMCFLP solutions of fair quality can easily be obtained. For the CSLP, we may have trouble to find any feasible integer solution at all. However, a feasible solution for the DMCFLP is also feasible for the CSLP. We may thus use these solutions as warm start solutions for the CSLP. Table 5 shows the average optimality gaps of optimal

DMCFLP solutions in the CSLP as well as the results for the CSLP when DMCFLP solutions are used as warm start solutions. To obtain a feasible CSLP solution, the y variable values of the optimal DMCFLP solution are fixed. CPLEX then heuristically finds feasible values for the missing variables (parameter *effortLevel* has been set to 3). The average optimality gap of such solutions (all *ISsel* instances have been considered, except five instances of size (10/50) where no optimal DMCFLP solution has been found) is around 15%. Separating the instances by certain characteristics gives us a better insight into the impact of those properties. One would assume that DMCFLP solutions perform better for instances where the demand is uniformly distributed over time, since the relocation of camps seems less probable. However, the results do not show any clear evidence of a better performance.

On the other hand, the capacity of existing camps seems to have more impact on the DMCFLP solution quality in the CSLP. The less camps initially exist, the better the DMCFLP solution quality. This is because in both versions camps have to be constructed. This is summarized in Table 6, which reports the average number of constructed and relocated trailers according to the demand distribution and the number of initially existing camps (only solutions with a proven optimality gap smaller than or equal to 10% have been considered). Instances with demand uniformly distributed over all time periods tend to have less constructions and relocations than instances in which demand is irregularly distributed over time. In addition, the less camp capacity is initially available, the smaller the chance that existing camps are relocated instead of constructing new ones. Thus, new optimal placed camps in a DMCFLP solution are more likely to be a good choice for the CSLP as well.

The last two columns (*CSLPheur*) in Table 5 indicate the results after one hour of computation time for the CSLP, when the best DMCFLP solution obtained after one hour of computation time is used as a warm start solution. Compared with the single execution of the CSLP (see Table 4), CPLEX now finds feasible solutions for most of the instances while maintaining a similar average optimality gap.

Inst	all	Demand distribution		Initial demand coverage			CSLPheur	
Size		Uniform	Clustered	0%	50%	100%	gap	# ns
10/20	12.8	11.6	13.9	9.2	12.9	16.1	6.6	0
10/50	17.2	18.8	15.7	12.1	15.8	23.9	14.2	22
Total	14.9	15.0	14.8	10.6	14.4	19.8	10.7	22

Table 5: The average optimality gaps of optimal DMCFLP solutions (*ISsel*) in the CSLP as well as the CSLP optimality gaps when DMCFLP solutions are used as warm start solutions.

Inst	Demand distribution		Initial demand coverage		
Size	Uniform	Clustered	0%	50%	100%
# Constructions	4.9	6.7	7.8	4.6	2.3
# Relocations	0.8	1.1	0.0	1.2	1.8

Table 6: The average number of constructed and relocated trailers within near optimal CSLP solutions.

The impact of the cost ratio. The ratio between transportation costs and the costs to construct or relocate camps has also been found to have a strong impact on the difficulty of solving the problem. A total of 264 additional instances of the sizes (10/20) and (10/50) have been generated with eleven different transportation costs, set between 1% and 3000% of the original transportation costs given in the RW instance. We refer to this percentage as $TC\%$. All instances contain sufficient camp capacities to cover 50% of the average demand per season. Denote R the estimated ratio between transportation and construction costs. To compute R, we first compute, for each existing camp, the total transportation costs necessary to satisfy all customers demands by that camp (i.e., we ignore all other camps). We then scale the average value of these total transportation costs down to one trailer. R is set to this value divided by the cost to construct a camp with one trailer. For $TC\% \in [1, 3000]$, the average of the cost ratios R grows linearly from 0.04 to 106.41.

Figure 3 (a) and (b) illustrate the difficulty of solving the generated instances for the CSLP subject to their $TC\%$ ratios (in one hour of computation time). For each of the $TC\%$ cost ratios, the number of instances where no feasible solution has been found (see Figure 3 (a)) and the average optimality gap of the final solutions (see Figure 3 (b)) are reported. The results indicate that the problem gets more difficult to solve when $TC\% = 100$ ($R = 3.55$). With $TC\%$ values greater than 1500 ($R = 53.20$), it seems that the solution of the problem gets slightly easier again. Figure 3 (c) shows the average number of constructed and relocated trailers within the final solutions (again, only solutions with a proven optimality gap smaller than or equal to 10% have been considered). The results indicate that the number of constructed trailers grows faster than the number of relocations when the transportation costs increase.

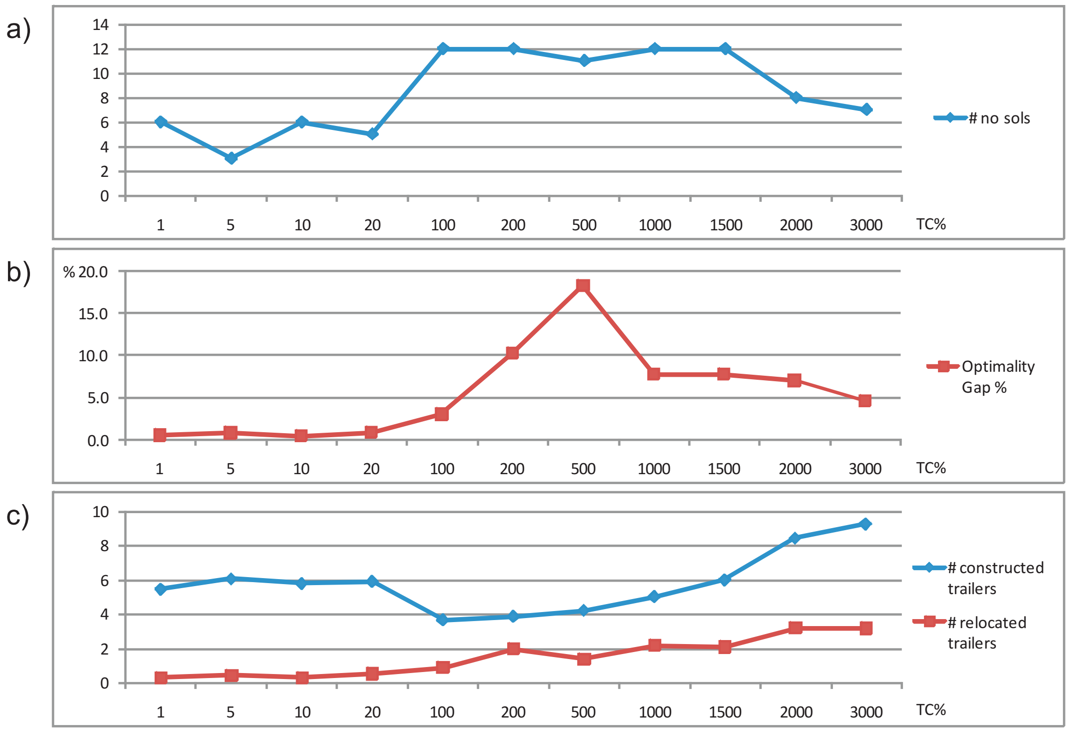

Figure 3: The impact of the transportation cost ratio on (a) the number of CSLP instances where no solutions have been found, (b) the average optimality gaps and (c) the average number of constructed and relocated trailers in near optimal solutions.

Yearly camp relocation. All previous experiments have assumed that camp relocation is allowed after each season. In the case of the Canadian logging company that provided the real-world instance, relocation is possible only once a year. We investigate the difficulty of solving this slightly simplified problem, considering all instances (*ISall*). We use the *CSLPheur* approach, i.e., we first solve the DMCFLP with a time limit of one hour and then use the best solution as a starting solution for the CSLP, also limited to one hour of computation time. The results, summarized in Table 7, show that instances of reasonable size (i.e., 10/20 and 10/50) can be fairly well solved. Most of the larger instances exceed either the given memory limit of 4GB or CPLEX capabilities to solve the problem in the given time limit.

5.3 Case Study

Real World Instance. Data from a Canadian logging company have been provided by FPInnovations. This real-world (RW) instance contains 29 logging regions. Figure 4 illustrates the logging regions, the road network and the existing accommodations. The harvest areas are commonly located at the extremities of the road network tree. Independent logging and road construction demands are given for five years, each year being divided into two periods, summer and winter. These demands are not necessarily clustered within subsequent sequences. Demands require up to eight logging and four road construction crews, each with six and three workers, respectively. Demands for three logging supervisors and one road construction supervisor

Inst Size	# Inst	gap %	# ns	# opt	time (sec)
10/20	324	4.3	0	134	3992
10/50	324	14.6	17	24	5664
50/50	324	24.2	134	12	7173
50/100	324	19.7	295	31	7447
Total Avg	1296	11.5	446	201	4984

Table 7: Results (ISall) with CSLPheur when camp relocation is allowed only once a year.

are estimated in proportion to the regions' work crew demands. All 29 locations are available for potential camp construction or relocation. The complete demand is easily covered by five existing accommodations: one non-movable (marked as 5) and four camps (marked as 1 to 4, with 2, 3, 4 and 4 trailers, respectively). Based on a detailed road network that considers different road conditions, travel times and costs have been computed. These costs take into account gasoline, vehicle renting and additional salary due to long travel times.

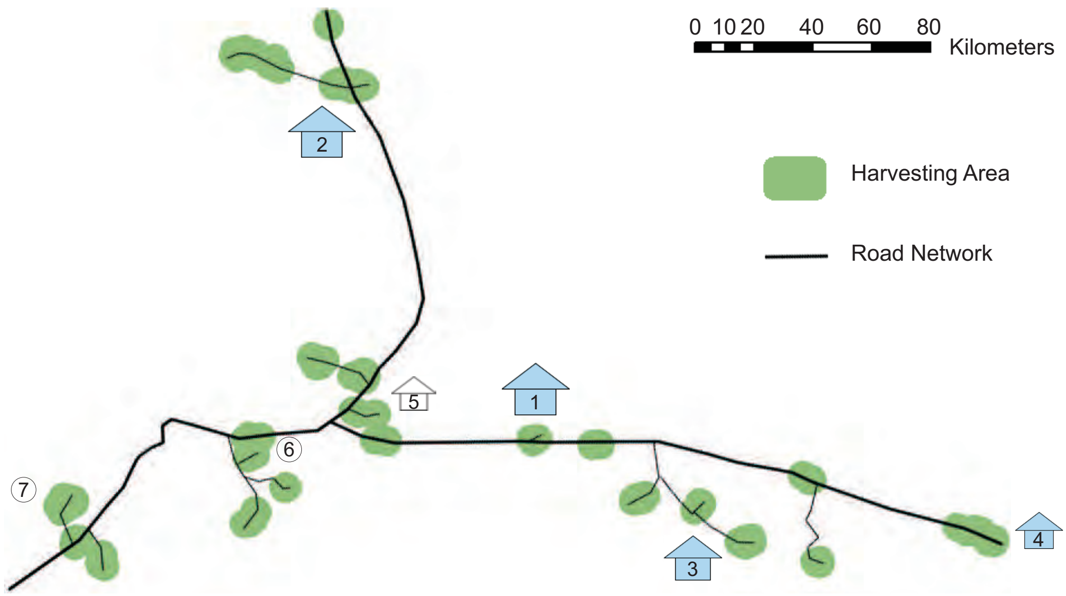

Figure 4: Simplified illustration of the logging regions and the road network for the real-world instance.

Solution analysis for different scenarios. We explore three different scenarios for this instance:

1. Only the existing accommodations can be used to host the workers.
2. In addition to existing camps, new camps can be constructed at the beginning of any season. Camps cannot be relocated to another region.
3. In addition to existing camps, new camps can be constructed at the beginning of any season and camps can be relocated to another region once a year.

Table 8 shows how costs are distributed in the optimal solution of each scenario. The existing accommodations possess sufficient capacity to host all required workers. Scenario 2 suggests the construction of a new camp with two trailers in the region which is labeled as location 7 in Figure 4, after the fifth season. However, the small savings involved do not seem to justify such an investment. Scenario 3 suggests the relocation of a camp with four trailers from the region labeled as location 1 to the one labeled as location 6, also after the fifth season. The additional camp relocation costs are outweighed by the savings in the transportation costs, which reduced by more than 40%. This results in a very beneficial solution, reducing the total costs by 8.6%. One recognizes that constructing a new camp gives us the possibility to add capacity close to remote logging regions, because former camps close to other logging regions will still be available. In the case of a relocation, however, we rather need to consider placing a camp close to all logging regions. This explains the more centralized location for the camp relocation in Scenario 3.

Costs ($)	Scenario 1	Scenario 2	Scenario 3
Construction	0	606,000	0
Relocation	0	0	302,470
Hosting	1,879,905	1,536,549	1,476,083
Transportation	2,261,809	1,468,012	1,353,561
Trailer Change	252,521	247,992	242,751
Maintenance	2,983,112	3,490,962	3,365,490
Total	7,377,347	7,349,515	6,740,355

Table 8: Cost distribution in the optimal solutions for the three different scenarios of the real-world instance.

Clearly, the reduction of the transportation costs is directly linked to the traveled time and distance. As can be seen in Table 9, the average distance traveled by logging and road construction crews is reduced significantly (23% and 16%, respectively, in Scenario 3) when a camp is constructed or relocated. Though the relocation of an existing camp is economically more beneficial, it involves slightly higher travel distances than the construction of a new camp, since fewer camps are available. Interestingly, for the supervisors, we find the opposite trend. Their travel distances increased on average by 22%. Supervisor demands are much lower than work crew demands, as there are only four supervisors, but up to 62 workers at the same time. In addition, supervisors do not earn additional salary for long distances. This explains why the actual usage of available camp capacities, i.e., already open trailers, are economically more important than the distance traveled. Finally, one can observe that we open slightly more trailers in Scenario 3. Maintenance costs increase, but lower transportation costs may be involved as such trailers are closer to certain logging regions.

	Scenario 1	Scenario 2	Scenario 3
Trailers open	49.4%	47.9%	54.8%
Average travel distance (km):			
Logging crews	114	83	88
Road construction crews	129	102	109
Supervisors	153	195	187

Table 9: Usage of existing trailers and travel distances for the three different scenarios of the real-world instance.

Variation of instance properties. Based on the RW instance, 71 additional instances have been generated by varying the previously explained properties: number of commodities, cost curve, initial demand coverage and cost ratio. Table 10 summarizes the results without and with supervisors (i.e., two and four commodities, respectively) demands (*w/ sv*) for the *CSLPheur* approach. All instances could be reasonably solved in the given time limit. As expected, solving an instance with demands for supervisors is more difficult than solving the same problem without supervisor demands.

w/ sv	gap %	# ns	# opt	time (sec)
No	9.8	0	12	3169
Yes	15.8	0	4	4815
All	12.8	0	16	3992

Table 10: Results for the 72 RW instance variations with CSLPheur when camp relocation is allowed only once a year.

6 Conclusions and Future Research

A mixed-integer model for the location of logging camps has been presented. This model extends the classical Capacitated Facility Location Problem by several features. Next to the well known features of multiple periods, multiple commodities and multiple capacity levels, further extensions include the partial and temporary closing of facilities, particular capacity constraints that include integer rounding and the integration of economies of scale on several levels of the cost structure. In addition, the model allows the extensions and relocation of existing facilities. Such integer rounding capacity constraints can be useful in other applications. As they increase the integrality gap and therefore the difficulty to solve the problem, new valid inequalities are derived to effectively reduce this integrality gap.

Instances based on a large variety of different properties have been generated. Experiments on these instances illustrated the impact of the different problem features on the difficulty to solve the problem. It is shown that general purpose solvers such as CPLEX are capable of solving most of the instances up to a realistic size in reasonable time, when using optimal solutions of a simplified problem as warm start solution for the entire problem. The optimal solution of a problem instance based on real world data from a Canadian logging company has been analyzed. A saving potential of more than 8% of the total costs is found by relocating an existing camp.

Though most of the smaller and medium sized instances can be solved in reasonable time, some of the instances remain unsolved. The models for larger instances typically exceed the memory limitations of current standard computers, such as the ones used in the experiments. In order to solve these instances, more sophisticated solution techniques will be necessary, such as mathematical decomposition. Interesting extensions of the model for future research include the possibility of partial relocation of camps as well as the use of trailers of different sizes.

Acknowledgements

We would like to thank Mathieu Blouin and Jean Favreau from FPInnovations for their valuable support throughout this study and for providing the data used in the experiments. The authors are also grateful to MITACS, the Natural Sciences and Engineering Research Council of Canada (NSERC) and FPInnovations for their financial support.

References

Albareda-Sambola, M., E. Fernandez, Y. Hinojosa, and J. Puerto (2009). The multi-period incremental service facility location problem. *Computers & Operations Research 36*(5), 1356–1375.

Ballou, R. H. (1968). Dynamic warehouse location analysis. *Journal of Marketing Research 5*.

Bredström, D., P. Jönsson, and M. Rönnqvist (2010). Annual planning of harvesting resources in the forest industry. *International Transactions in Operational Research 17*(2), 155–177.

Canel, C., B. M. Khumawala, J. Law, and A. Loh (2001). An algorithm for the capacitated, multi-commodity multi-period facility location problem. *Computers & Operations Research 28*(5), 411–427.

Carlsson, D., S. D'Amours, A. Martel, and M. Rönnqvist (2009). Supply Chain Planning Models in the Pulp and Paper Industry. *INFOR 47*(3), 167–183.

Correia, I. and M. Captivo (2003). A Lagrangean heuristic for a modular capacitated location problem. *Annals of Operations Research 122*, 141–161.

D'Amours, S., M. Rönnqvist, and A. Weintraub (2008). Using operational research for supply chain planning in the forest products industry. *INFOR 46*(4), 265–281.

Dias, J. (2006). Capacitated dynamic location problems with opening, closure and reopening of facilities. *IMA Journal of Management Mathematics 17*(4), 317–348.

Gendron, B. and T. Crainic (1994). Relaxations for multicommodity capacitated network design problems. Technical report, Publication CRT-945, Centre de recherche sur les transports, Université de Montréal.

Geoffrion, a. M. and G. W. Graves (1974). Multicommodity distribution system design by benders decomposition. *Management Science 20*(5), 822–844.

Gouveia, L. and F. Saldanha da Gama (2006). On the capacitated concentrator location problem: a reformulation by discretization. *Computers & Operations Research 33*(5), 1242–1258.

Hamacher, H. and S. Nickel (1998). Classification of location models. *Location Science 6*(1-4), 229–242.

Holmberg, K. (1994). Solving the staircase cost facility location problem with decomposition and piecewise linearization. *European Journal of Operational Research 75*(1), 41–61.

Holmberg, K. and J. Ling (1997). A Lagrangean heuristic for the facility location problem with staircase costs. *European Journal of Operational Research 97*(1), 63–74.

Klose, A. and A. Drexl (2005). Facility location models for distribution system design. *European Journal of Operational Research 162*(1), 4–29.

Lee, C. Y. (1991). An optimal algorithm for the multiproduct capacitated facility location problem with a choice of facility type. *Computers & Operations Research 18*(2), 167–182.

Melo, M., S. Nickel, and F. Saldanha da Gama (2005). Dynamic multi-commodity capacitated facility location: a mathematical modeling framework for strategic supply chain planning. *Computers & Operations Research 33*(1), 181–208.

Melo, M., S. Nickel, and F. Saldanha da Gama (2009). Facility location and supply chain management - a review. *European Journal of Operational Research 196*(2), 401–412.

Owen, S. H. and M. S. Daskin (1998). Strategic facility location: a review. *European Journal of Operational Research 111*(3), 423–447.

Paquet, M., A. Martel, and G. Desaulniers (2004). Including technology selection decisions in manufacturing network design models. *International Journal of Computer Integrated Manufacturing 17*(2), 117–125.

Peeters, D. and A. P. Antunes (2001). On solving complex multi-period location models using simulated annealing. *European Journal of Operational Research 130*(1), 190–201.

Revelle, C. and H. Eiselt (2005). Location analysis: a synthesis and survey. *European Journal of Operational Research 165*(1), 1–19.

Revelle, C., H. Eiselt, and M. S. Daskin (2008). A bibliography for some fundamental problem categories in discrete location science. *European Journal of Operational Research 184*(3), 817–848.

Rönnqvist, M. (2003). Optimization in forestry. *Mathematical programming 284*, 267–284.

Shulman, A. (1991). An algorithm for solving dynamic capacitated plant location problems with discrete expansion sizes. *Operations Research 39*(3), 423–436.

Smith, H. K., G. Laporte, and P. R. Harper (2009). Locational analysis: highlights of growth to maturity. *Journal of the Operational Research Society 60*, S140–S148.

Sridharan, R. (1991). A lagrangian heuristic for the capacitated plant location problem with side constraints. *Journal of the Operational Research Society 42*(7), 579–585.

Sridharan, R. (1995). The capacitated plant location problem. *European Journal of Operational Research 87*(2), 203–213.

Troncoso, J. and R. Garrido (2005). Forestry production and logistics planning: an analysis using mixed-integer programming. *Forest Policy and Economics 7*(4), 625–633.

Warszawski, A. (1973). Multi-dimensional location problems. *Operational Research Quarterly (1970-1977) 24*(2), 165–179.

Weintraub, A. and C. Romero (2006). Operations research models and the management of agricultural and forestry resources: a review and comparison. *Interfaces 36*(5), 446–457.

Wesolowsky, G. O. (1973). Dynamic facility location. *Management Science 19*(11), 1241–1248.

Wesolowsky, G. O. and W. G. Truscott (1975). The multiperiod location-allocation problem with relocation of facilities. *Management Science 22*(1), 57–65.