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Heuristics for Multi-Attributed Vehicle Routing Problems: A Survey and Synthesis

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Abstract. The attributes of Vehicle Routing Problems are additional characteristics or constraints that aim to better take into account the specificities of real applications. The variants thus formed are supported by a well-developed literature, including a large variety of heuristics. This article first reviews the main classes of attributes, providing a survey of heuristics and meta-heuristics for Multi-Attribute Vehicle Routing Problems (MAVRP). It then takes a closer look at the concepts of 64 remarkable meta-heuristics, selected objectively for their outstanding performance on 15 classic MAVRP with different attributes. This cross-analysis leads to the identification of “winning strategies” in designing effective heuristics for MAVRP. This is an important step in the development of general and efficient solution methods for dealing with the large range of vehicle routing variants.

Keywords: Vehicle routing, multi-attribute problems, heuristics, meta-heuristics, survey, analysis, algorithmic design principles.

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1 Introduction

Vehicle routing problems have been the subject of intensive research for more than 50 years, due to their great scientific interest as difficult combinatorial optimization problems and their importance in many application fields, including transportation, logistics, communications, manufacturing, military and relief systems, and so on. The “traditional” Capacitated Vehicle Routing Problem (CVRP) involves designing least cost delivery routes to service a geographically-dispersed customer set, while respecting vehicle-capacity constraints. This NP-hard optimization problem combines the characteristics of a Bin Packing Problem aiming to assign loads to capacitated vehicles, and a Travelling Salesman Problem (TSP) that aims to find the best route for each vehicle, i.e., the least costly sequence of visits for the customers assigned to it.

The extremely broad range of actual applications where routing issues are found leads to the definition of many VRP variants with additional characteristics and constraints aiming to capture a higher level of system detail or decision choices, including but not limited to richer system structures (e.g., several depots, vehicle fleets, and commodities), customer requirements (e.g., multi-period visits and within-period time windows), vehicle operation rules (e.g., load placement, route restrictions on total distance or time, and driver work rules), and decision context (e.g., traffic congestion and planning over extended time horizons). Combined with the traditional CVRP, these *problem attributes* make up a vast research, development, and literature domain. The dimensions of most problem instances of interest hinders the applicability of exact methods, while the few software systems currently presented as general heuristic solvers are increasingly challenged as the number and variety of attributes grows. Hence, thousands thousands of heuristics, meta-heuristics, and solution concepts specialized for specific *Multi-Attribute Vehicle Routing Problems* (MAVRPs) have been proposed in the literature.

This literature, vast and difficult to classify, has been historically articulated around several streams of research dedicated to a number of major attributes. Such diverse research lines would be justified if the nature of the various problem settings would call for radically different solution approaches. Yet, MAVRPs naturally share many common features, and most heuristic strategies developed for specific problems can be applied to a broader range of VRP variants. The identification of such fundamental design elements for MAVRP metaheuristics is of primary interest to progress toward more generalist and efficient VRP algorithms, thus providing the means to quickly address various application cases without extensive problem-tailored algorithmic developments.

To respond to these challenges, we introduce a unifying synthesis and analysis of MAVRP solution methods, providing the means to identify the main concepts of successful heuristics and metaheuristics. The analysis is based on two main ideas. On the one hand, we analyse from a general perspective detached from the particular characteristics of the attributes. On the other hand, we adopt a synthetic approach to deal with the abundance of contributions. Thus, in particular, the scope of the analysis has been limited to MAVRPs with complete and exact data, demands on nodes (no arc-routing settings), and a single objective.

We identified, classified, and analysed fifteen (15) MAVRPs, which have been the object of a consistent body of well-acknowledged research resulting in a considerable

number of heuristic methods and a number of common benchmark sets of test instances. We then selected a limited number, three to five, of the most efficient heuristic methods for each of these MAVRP variants. The resulting sixty-four (64) methods were then analysed in detail, resulting in the identification of broad concepts and main algorithmic-design principles, an objective synthesis of “winning strategies” for MAVRP heuristics, and perspectives on major research challenges.

The article unfolds in three main parts. Section 2 recalls the “traditional” CVRP and reviews the fundamental elements of heuristics developed to address it. Most of these elements are also found in the next sections when analysing heuristics for multi-attribute problems. Section 3 introduces an attribute-classification system, and presents the selected MAVRPs and the corresponding subset of selected high-performance heuristics, thus providing the necessary material for our unifying analysis of state-of-the-art MAVRP heuristics in Section 4. Section 5 concludes with a discussion of a number of challenges for the field and possible research perspectives.

2 Heuristics for the CVRP

The CVRP was introduced in the seminal article by Dantzig and Ramser (1959) under the name “Truck Dispatching Problem”. It was only several years later, following the publication of the article by Christofides (1976), that the current name of the problem became widespread. We initiate the section by recalling the CVRP setting and formulation. We then review the main categories of heuristic solution methods: *constructive heuristics*, *local improvement heuristics*, and *metaheuristics*, *hybrid methods*, and *parallel and cooperative metaheuristics*.

2.1 Problem statement

Like numerous previous articles, we define the CVRP as follows. Let $G = (\mathcal{V}, \mathcal{E})$ be a complete undirected graph with $|\mathcal{V}| = n + 1$ nodes. The node $v_0 \in \mathcal{V}$ represents a depot, where a fleet of m identical vehicles with capacity Q is based, and where the product to be distributed is stored. The other nodes $v_i \in \mathcal{V} \setminus \{v_0\}$, for $i \in \{1, \dots, n\}$, represent the customers, characterized by demands for non-negative amounts of product q_i . Edges $(i, j) \in \mathcal{E}$ represent the possibility of travelling directly from a node (customer or depot) $v_i \in \mathcal{V}$ to a different node $v_j \in \mathcal{V}$ for a transportation cost of c_{ij} . The objective of the CVRP is to find a set of m or less routes such that, all customers are visited, the demand of each customer is delivered by a single vehicle, the total cumulated demand of the customers serviced by any route is smaller than Q , and the total transportation cost is minimized. Applications considering an unlimited fleet can be modelled by setting $m = n$ as at most one vehicle per customer is needed.

Many different formulations of this problem can be found in the literature. Introduced by Fisher and Jaikumar (1981), the integer linear programming formulation (1) - (8) has the advantage of presenting explicitly the combined assignment and sequencing characteristics of the CVRP. It is based on two families of binary variables, y_{ik} , designating the assignment of customer i to vehicle k by the value 1 (and 0, otherwise; $y_{0k} = 1$

signals vehicle k operates), and x_{ijk} , taking the value 1 when vehicle k visits node v_j immediately after node v_i ($i \neq j$).

$$\text{Min. } \sum_{i=0}^n \sum_{j=0}^n \sum_{k=1}^m c_{ij} x_{ijk} \quad (1)$$

$$\text{Subject to } \sum_{k=1}^m y_{ik} = 1 \quad i = 1, \dots, n \quad (2)$$

$$\sum_{k=1}^m y_{0k} \leq m \quad (3)$$

$$\sum_{i=1}^n q_i y_{ik} \leq Q \quad k = 1, \dots, m \quad (4)$$

$$\sum_{j=0}^n x_{ijk} = \sum_{j=0}^n x_{jik} = y_{ik} \quad i = 0, \dots, n; k = 1, \dots, m \quad (5)$$

$$\sum_{v_i \in S} \sum_{v_j \in S} x_{ijk} \leq |S| - 1 \quad k = 1, \dots, m; S \in V \setminus \{0\}; |S| \geq 2 \quad (6)$$

$$y_{ik} \in \{0, 1\} \quad i = 0, \dots, n; k = 1, \dots, m \quad (7)$$

$$x_{ijk} \in \{0, 1\} \quad i = 0, \dots, n; j = 0, \dots, n; k = 1, \dots, m \quad (8)$$

Constraints (2) - (4) present the structure of a Bin Packing problem with m bins. These constraints enforce, respectively, the assignment of each customer to a single vehicle, the maximum number of vehicles operating out of the depot, and the vehicle capacity. Constraints (5) - (6) are then related to the structure of the routes, guaranteeing the selection of an adequate number of arcs entering into and exiting from each node (depot and customers), and eliminating sub-tours (i.e., routes that don't pass through the depot). The number of constraints of this latter type grows exponentially with the number of customers. The CVRP includes the TSP as a special case when $m = 1$ and $Q = +\infty$, and is thus NP-hard.

An additional constraint on the maximum length of each route, Relation (9), is often found in the literature. A service duration τ_i is associated to each customer, the sum of the customer service times and the travel time of the route being then limited to T .

$$\sum_{i=0}^n \sum_{j=0}^n (c_{ij} + \tau_i) x_{ijk} \leq T \quad k = 1, \dots, m \quad (9)$$

The CVRP has been the subject of intensive research since the 1960s. Numerous exact methods, heuristics, and metaheuristics have been presented in the literature, as illustrated by various surveys (see Baldacci et al. 2007, Cordeau et al. 2007, Gendreau et al. 2008, Eksioglu et al. 2009, Potvin 2009 and Laporte 2009, for the most recent) and books (Golden and Assad 1988, Toth and Vigo 2002b, Golden et al. 2008). Today, the best exact methods for VRP are still limited to relatively small problems. For example, the largest instances solved by Fukasawa et al. (2006) and Baldacci et al. (2008b) have

a maximum of 135 customers. Because of this, the development of heuristics and meta-heuristics constitutes a very active domain in the literature. The main families of such methods for the CVRP are reviewed in the following.

2.2 Constructive heuristics

Mainly proposed between the 1960s and the 1980s, a large number of heuristics attempted to produce solutions constructively. One key characteristic of these heuristics is that they operate in a *greedy* manner, producing a set of definitive decisions (e.g., customer insertion or the merging of two routes) that cannot be reversed afterwards.

The *savings method* of Clarke and Wright (1964) is the best-known example of a constructive heuristic. Starting from an initial solution s_0 in which each customer is served by a different route, the heuristic searches for and merges two route extremities i and j , maximizing the distance saved $s_{ij} = c_{i0} + c_{0j} - c_{ij}$, under the condition that the merged route is feasible. The original method has been revised and improved several times, notably by Gaskell (1967) and Yellow (1970) who parametrized the original equation to give more, or less, importance to the distance to the depot ($s_{ij} = c_{i0} + c_{0j} - \lambda c_{ij}$ with $\lambda \geq 0$), thus correcting a flaw in the original method, which produced routes with a high “circular” tendency.

Another heuristic, called “sweep” (Gillett and Miller 1974), is remarkable in its simplicity. The approach explores the customers circularly, in increasing polar angle around the depot. Each customer is successively inserted in this order at the end of the current route. If this insertion is infeasible because of the route constraints, then a new route is initiated. At the end of this construction phase, Gillett and Miller (1974) proposed to apply a λ -opt improvement heuristic (see Section 2.3) to post-optimize each route separately.

Other heuristics perform the assignment and sequencing in two separate phases. The approach called “route-first cluster-second” (Newton and Thomas 1974, Bodin and Berman 1979, Beasley 1983) first constructs a giant circuit that visits all customers, like a TSP solution. This giant tour is then cut into several routes from the depot. The segmentation problem can be solved exactly as a shortest path problem in an acyclic graph.

Proposed by Fisher and Jaikumar (1981), the “cluster-first route-second” approach, first creates customer clusters, and then optimizes the order of visits for each cluster as a TSP subproblem. The creation of the clusters is performed by solving a Generalized Assignment Problem (GAP) for the customers, around m locations chosen to represent zones with a high customer density. A linear estimate of the route costs is used as the objective function of the GAP. This approach is strongly linked to the visual solution approach of human planners. In addition, the priority given to the assignment allows capacity constraints to be better dealt with for highly constrained problems presenting few feasible solutions. This specificity is significant in the CVRP literature where most constructive heuristics manage the capacity constraints as a by-product of a policy exclusively dedicated to the geometrical creation of routes.

The heuristics presented in this section are generally capable of producing solutions that are within 10% or 15% of the optimum in a very short time. A detailed review of

these methods can be found in Laporte and Semet (2002). Today, constructive methods are still used to produce initial solutions for a wide range of heuristics, and have been adapted to many MAVRPs. Furthermore, certain metaheuristics (e.g., GRASP or Ant Colony Optimization) rely on iteratively calling on constructive heuristics, biased by information gathered during the global search, to create new solutions.

2.3 Local-improvement heuristics

Sequence-based combinatorial optimization problems lend themselves well to the application of local search (LS) improvement heuristics (Aarts and Lenstra 2003). Based on an initial solution s , a local search heuristic explores a *neighbourhood* $\mathcal{N}(s)$, generally defined by perturbations (*moves*) on s , in order to find an improving solution s' that replaces s for a new iteration of the heuristic. The local search stops at a solution \bar{s} when no improving solution can be found in $\mathcal{N}(\bar{s})$. This solution is a *local optimum* of the problem and the neighbourhood used. The set of solutions – or states characterizing solutions – linked by neighbourhood relationships is usually called *search space*, while the succession of states reached in the course of the method constitutes a *search trajectory* in the graph thus formed. Many neighbourhoods have been defined in the VRP literature. For the sake of brevity, we will only describe those which are still frequently used and named as such in the current literature.

A first category, coming directly from the TSP literature, relies on arc exchanges to optimize separately the routes. In the terminology of Lin (1965), a neighbourhood of the type λ -*opt* contains the set of solutions obtained by deleting and reinserting λ arcs. The neighbourhood size is $|\mathcal{N}^{\lambda-opt}| = O(n^\lambda)$. The most commonly used neighbourhoods in the literature include *2-opt* and *3-opt*, as well as *Or-exchange* (Or 1976). The latter neighbourhood involves relocating sequences of visits of bounded length, and constitutes a subset of *3-opt* of size $O(n^2)$. Examples of the *2-opt* and *Or-exchange* moves are illustrated in Figure 1. Noteworthy is also the *GENI* insertion operator (Gendreau et al. 1992), which effectively evaluates combined customer insertions in a route with restricted *3-opt* or *4-opt* optimization.

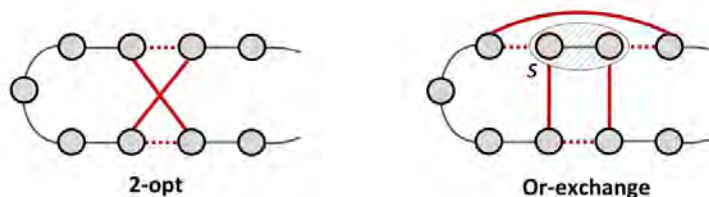


Figure 1: *2-opt* and *Or-exchange* illustration. The deleted/inserted arcs are indicated with dotted/bold lines.

Other CVRP local-search neighbourhoods allow several routes to be improved simultaneously, generally by exchanging arcs or moving visits between the sequences. Among the most commonly used neighbourhoods of this type, the *insert* neighbourhood (also

called *shift* neighbourhood in Osman 1993) consists of moving a visit from one route to another, while a *swap* (also called *1-interchange*) exchanges 2 visits between their respective routes. The *2-opt** neighbourhood (Potvin and Rousseau 1995) is based on the deletion and reinsertion of two arc pairs from two different routes. This neighbourhood, which can also be assimilated to an exchange of “route ends”, is sometimes called *crossover* neighbourhood, and is illustrated in Figure 2. The three previously mentioned neighbourhoods contain $O(n^2)$ solutions.

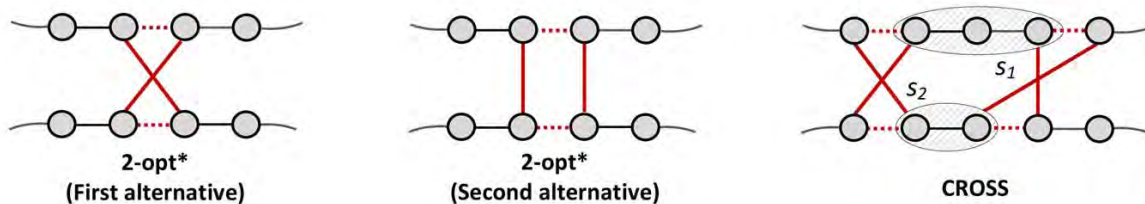


Figure 2: 2-opt* and CROSS-exchange illustration. The deleted/inserted arcs are indicated with dotted/bold lines.

Finally, the CROSS-exchange neighbourhood (Taillard et al. 1997) exchanges two (one being potentially empty) customer sequences s_1 and s_2 , thus generalizing the three neighbourhoods introduced previously, *insert*, *swap*, and *2-opt**. The customer sequences s_1 and s_2 can eventually be reversed in this process to produce a slightly larger neighbourhood, called *I-CROSS* in Bräysy (2003). The CROSS and I-CROSS neighbourhoods are of size $O(n^4)$ and would be costly to evaluate exhaustively. In practice, the size of the exchanged sequences is often limited by a value L_{max} , so that the size of the neighbourhood becomes $O(L_{max}^2 n^2)$. CROSS and I-CROSS are themselves special cases of λ -interchanges moves (Osman 1993), which involve exchanging any (potentially non-consecutive) set of less than λ customers between two routes.

Even the evaluation of quadratically sized neighbourhoods can be impracticable for certain large problems. Thus, further *neighbourhood pruning* procedures are frequent in the literature. A common method, called *granular search*, requires computing for each node v_i a list $\Gamma(v_i)$ of spatially related neighbours, and only considering moves that involve v_i and $v_j \in \Gamma(v_i)$ (Gendreau et al. 1992, Johnson and McGeoch 1997, Toth and Vigo 2003). Another type of limitation, introduced early in the TSP literature by Christofides and Eilon (1972), Lin and Kernighan (1973), and generalized for the CVRP by Irnich et al. (2006) under the name of *sequential search*, is based on the observation that all profitable λ -opt can be broken down into a list of arc exchanges $(\phi_1, \dots, \phi_\lambda)$ with gains (g_1, \dots, g_λ) , such that all subsets of $k \leq \lambda$ first arc exchanges have a positive partial gain $\sum_{i=1}^k g_i$. This observation allows to rapidly eliminate a lot of unpromising neighbour solutions.

The computational efficiency of local-improvement heuristics can also be improved by means of memory structures to store move evaluations (Zachariadis and Kiranoudis 2010a). When additional attributes of the VRP (e.g., time windows) are taken into account, it is also possible to accelerate the cost and feasibility evaluations by means of

judiciously-defined global variables on subsequences of visits (Savelsbergh 1985, Baker and Schaffer 1986, Savelsbergh 1992, Kindervater and Savelsbergh 1997, Taillard et al. 1997, Irnich 2008, Vidal et al. 2011c).

Larger neighbourhoods, with an exponential number of solutions, have also been widely studied and used in the literature. In particular, the procedure of Lin and Kernighan (1973) is a remarkably effective method for optimizing a TSP sequence. Like the *ejection chains* strategy developed by Glover (1992, 1996) and extensively applied to the CVRP by Rego (2001), this procedure attempts to find a cycle that alternates existing and non-existing arcs in the current solution, so that the solution obtained by replacing existing arcs in a cycle with the non-existing arcs is feasible and improving. Such a method can be viewed as an incomplete investigation of a λ -opt neighbourhood with large λ values. Closely related to the previous concepts, the cyclical transfers of Thompson and Psaraftis (1993) explore a large neighbourhood obtained by moving k customers within b routes. The search for an improving neighbour solution is formulated as a negative-cost cycle detection problem in an auxiliary graph. Although NP-hard, this latter subproblem can be solved effectively by means of heuristics.

Other ruin-and-recreate neighbourhoods (Shaw 1998) operate deletions and reinsertions of customer visits within customer sequences. Methods of this kind vary in the nature of their destruction and reconstruction operators, and may exploit heuristic methods, constraint programming, or integer programming for reconstruction. Finally, generalizing the work of Sarvanov and Doroshko (1981) for the TSP, De Franceschi et al. (2006) and Toth (2008) propose neighbourhoods based on fixing some customers and re-assigning unfixed customers between fixed ones, which are explored by solving an integer-programming model. Other, less often used, large neighbourhoods and exploration techniques are reviewed in Ahuja et al. (2002) and Pisinger and Ropke (2010). Additional literature reviews on local-search methods for the VRP can be found in Van Breedam (1995), Thompson and Psaraftis (1993), Kindervater and Savelsbergh (1997), Laporte and Semet (2002), Bräysy and Gendreau (2005a) and Funke et al. (2005). Local search constitutes an essential building block of metaheuristics for the CVRP, described in the next section.

2.4 Metaheuristics

The term “metaheuristic” was first coined by Glover (1986) to designate a broad class of heuristic methods that continue the search beyond the first encountered local optimum. A somewhat crude but telling definition characterizes metaheuristics as *heuristics guiding other heuristics*.

Metaheuristics constitute a core research domain in combinatorial optimization as illustrated by many literature reviews (e.g., Osman and Laporte 1996, Blum and Roli 2003, Gendreau and Potvin 2005) and books (e.g., Corne et al. 1999, Glover and Kochenberger 2003, Dréo et al. 2003, Gendreau and Potvin 2010). The CVRP is a testing ground particularly appreciated for such methods, as illustrated by the reviews of Gendreau et al. (2002), Cordeau et al. (2005), Gendreau et al. (2008), Laporte (2009) and Potvin (2009). We distinguish between so-called neighbourhood-centred methods, which generally proceed by iteratively exploring the neighbourhoods of a single incumbent solu-

tion, population-based strategies evolving a set of solutions by generating one of several “new” solutions out of combinations of existing ones, and approaches that either combine elements of different metaheuristics, the so-called hybrids, or harness the exploration capabilities of several solution methods exploiting their interaction, the parallel and cooperative search methods.

2.4.1 Neighbourhood-centred search

Simulated Annealing (SA) (Kirkpatrick et al. 1983, Černý 1985) overcomes the limitation of local-improvement heuristics, the rapid attraction to a local optimum, by accepting solution-deterioration moves with a probability governed by a statistical process, the so-called *temperature* parameter. The higher the temperature, the more likely it is to accept a deteriorating move. Temperature evolves dynamically during the search relatively to a cooling scheme, first favouring a vast exploration and frequent degradations, then gradually accepting fewer and fewer degradations to intensify the search for good-quality solutions. For the CVRP, efficient deterministic “Record-to-Record” (R-to-R) variants (Dueck 1993, Li et al. 2005) accept any neighbor solution which is *not much worse* than the incumbent solution, and prevent degradations that are too significant relatively to the best-found solution s^* , subject to re-starting the search from s^* .

Tabu search (Glover 1986, 1989, 1990, Glover and Laguna 1998) associates a search trajectory centred on the choice the best neighbour of the incumbent solution, with learning capabilities, generally represented as short-, medium- and long-term memories on solution elements, which replace or significantly complement the randomization used in other metaheuristics. The method is thus escaping from local optima by accepting moves that degrade the solution, providing their are the “best” in the explored neighbourhood. This decision process is enhanced by two mechanisms, the first aiming to avoid cycling and relying on short-term memories to reject solutions that contain recently examined *tabu* elements, the second accepting solutions that fulfil some *aspiration criteria* such as “the best solution in value or containing a given solution element”. Of central importance are the medium- and long-term memories used to manage significant trajectory-inflecting procedures known as *intensification*, e.g., focusing the search around elite solutions while promoting high-quality elements, and *diversification*, e.g., moving the search to an under-explored area of the search space, promoting infrequent elements, and so on. The challenge of balancing diversification and intensification is still a key research question in the literature.

Tabu search led to very effective CVRP metaheuristics, including TABUROUTE (Gendreau et al. 1994), Adaptive Memory (AM) variants (Taillard 1993, Rochat and Taillard 1995, Tarantilis 2005), and the Unified Tabu Search (UTS) (Cordeau et al. 1997, 2001). In TABUROUTE and UTS, diversification and intensification occur through penalties (incentives, respectively) on frequently (rarely) encountered solution elements, while AM approaches regularly redirect the search to a region around a new solution built out of promising fragments from the memory.

Concepts from tabu search have inspired other metaheuristics. Long-term memories for penalizing frequent solution elements can also be viewed as a basis of *Guided Local Search* (Voudouris and Tsang 1999), applied by Kilby et al. (1999), Tarantilis et al.

(2007), Kytöjoki et al. (2007), and Zachariadis and Kiranoudis (2010a) to the CVRP. In this case, modifying the search space by means of penalties is a primary tool for escaping from local optima. Similarly, aspiration criteria take a preponderant role in the *Attribute Based Hill Climber (ABHC)* method (Whittle and Smith 2004, Derigs and Kaiser 2007).

Variable neighbourhood Search (VNS) (Mladenović and Hansen 1997, Hansen et al. 2010) exploits the fact that a local optimum is defined for a given neighbourhood. Thus, changing the nature of the neighbourhood during the search, or at least some of its parameters, provide the means for further solution improvements. The order of neighbourhood evaluations and the solution acceptance criteria can be either deterministic or probabilistic. For the CVRP, additional solution perturbation mechanisms and long-term memories inspired from tabu search are sometimes employed (Kytöjoki et al. 2007, Fleszar et al. 2009, Chen et al. 2010). Metaheuristic hybrids (see Section 2.4.3) based on VNS are thus frequent.

In the same spirit, the *Adaptive Large neighbourhood Search (ALNS)* by Pisinger and Ropke (2007) exploits the benefits of varied neighbourhoods based on ruin-and-recreate moves (Shaw 1998). The frequency of use of these neighbourhoods is adapted throughout the search relatively to their past performance. Finally, the *Iterated Local Search (ILS)* (see Lourenço et al. 2010, for a recent review) applies successively a local-improvement phase, which ends up in a local optima, and a perturbation phase to escape from the local optima. Scaling appropriately the strength of the perturbation operator is a crucial point of the method. Prins (2009a) provide a simple and efficient application of ILS to the CVRP, where several solutions are iteratively produced from the same incumbent solution by means of improvement and perturbation mechanisms, the best one being selected for the next iteration.

2.4.2 Population-based methods

Population-based methods are often inspired from natural mechanisms. *Genetic Algorithms (GA)* and *Evolutionary Algorithms (EA)* were introduced during the late 1950s, and developed in their current form in Holland (1975). These algorithms interpret genetic laws and natural selection to evolve a population of individuals assimilated to solutions, through elitist selection, crossover, and mutation operators. With EA, it is also common to simultaneously make the search strategies (e.g., operator parameters) evolve with the solutions. Traditional GA and EA have a tendency to progress too slowly, however, and have thus been enhanced with various mechanisms, such as local search, which is also sometimes called an “education operator”. The algorithms thus obtained are sometimes called “genetic local searches” (Mühlenbein et al. 1988) or “memetic algorithms” (Moscato 1989, Moscato and Cotta 2010).

Some of these enhanced genetic methods performed remarkably well on classical CVRP benchmark instances (Prins 2004, Alba and Dorronsoro 2006, Marinakis et al. 2006, Nagata et al. 2010, Vidal et al. 2011a). We refer to Potvin (2009) for a thorough coverage of the field. It is noteworthy that many successful genetic algorithms for the CVRP use a *giant-tour* solution representation *without trip delimiters* (Prins 2004), along with clustering procedures to optimally *Split* a tour into a set of routes. This feature, directly inspired by route-first cluster-second constructive procedures (Section 2.2), en-

ables to rely on very simple genetic crossover operators working on permutations. On the other hand, an adequate management of population diversity also appears to be of critical importance for the considered problems (Prins 2004, Sörensen and Sevaux 2006, Vidal et al. 2011a).

The *Path Relinking (PR)* and *Scatter Search (SS)* metaheuristics (Glover 1977, Resende et al. 2010) are other population methods based on solutions recombinations. These methods promote strategic recombination over randomization, and differ essentially from the GA-type of methods in the manner in which solutions are crossed and in the size of the solution pool, which is generally smaller. Recombinations in PR involve an initial solution s^{DEP} and a guiding solution s^{GD} , both selected from an elite solution population. Characteristics of s^{GD} are progressively inserted in s^{DEP} in order to create a trajectory connecting these two solutions, potentially containing new improving solutions. On the other hand, the recombinations operators used in SS can involve more than two solutions. Path relinking was applied to the CVRP by Ho and Gendreau (2006).

Ant Colony Optimization (ACO) approaches (Dorigo and Stützle 2004) were inspired by the social behaviour of ants foraging for food and are for now the swarm-type of method most used in optimization. ACO was applied to the CVRP by Bullnheimer et al. (1999), Bell and Mc Mullen (2004), Doerner et al. (2004), Reimann et al. (2004) and Yu et al. (2009), among others. The individual behaviour of ants is embodied by constructive heuristics, exploiting informations on the search history (i.e., pheromones). Other swarm-inspired methods were proposed for the CVRP by Marinakis and Marinaki (2011) (*bee colonies*) and Marinakis and Marinaki (2010) (*particle swarms*). All these methods exploiting some form of learning, as are *neural networks* (Ghaziri 1996, Vakhutinsky and Golden 1994, Créput and Koukam 2008), and *artificial immune systems* (Masutti and De Castro 2008), to name but a few. These algorithms are often combined with local-improvement procedures, thus complicating the task to estimate the proper impact of collective intelligence paradigms on the search performance.

2.4.3 Hybrid metaheuristics

Hybrid metaheuristics blend concepts from various solution methodologies, metaheuristic classes most often, to take advantage of their respective strengths. The blending may take the form of a juxtaposition of methods (e.g., two algorithms called on consecutively) or an indissociable inclusion of elements from one method into a fully-functional different metaheuristic (e.g., tabu search-inspired memories in VNS). Hybrids may exclusively combine metaheuristic concepts, or also involve algorithmic ideas and modules from mathematical programming, constraint programming, tree-search procedures, and so on.

Although much effort has been recently put into properly defining the scope of hybrid metaheuristics (Raidl et al. 2010, Blum et al. 2011), the term remains very general and covers very different strategies. One can indeed argue that metaheuristics, described as *heuristics guiding other heuristics*, are hybrid in nature. This shows the shortcomings of a too-encompassing definition or, even, of trying to find a precise definition. Within the scope of this paper, we identify hybridisation as a strong concept in metaheuristic design, rather than a well-defined class of methods, aiming to take advantage of the synergy among different solution-method ideas to explore a broad variety of solution

strategies, often yielding superior results.

A large variety of hybrid methods has thus been proposed for the CVRP. Several approaches involve combined neighbourhood-centred search concepts, such as SA+tabu (Osman 1993), GRASP+ILS (Prins 2009a), ILS+VND (Chen et al. 2010), tabu+ILS (Cordeau and Maischberger 2011), among others. Hybridization schemes of this kind are frequent in recent local search-based methods, which are frequently enriched with restart procedures (a main characteristic of GRASP), probabilistic acceptance of deteriorating moves (a main characteristic of SA), variable neighbourhoods (VNS), or long-term memories and penalties on solutions attributes (GLS).

Population- and neighbourhood-search hybrids are also widespread. The wide majority of population-based approaches for the CVRP actually integrates some kind of combined local-search components, and can be characterized as hybrid. Furthermore, two of the three most efficient current CVRP metaheuristics (Nagata and Bräysy 2009, Vidal et al. 2011a) combine GA and LS. Other advanced hybridization schemes involve combined GA+tabu (Perboli et al. 2008), or combined population-based concepts such as GA+PSO (Marinakis and Marinaki 2010) and PR+PSO (Marinakis et al. 2010).

Finally, a number of metaheuristics for the CVRP integrated integer or constraint programming components to recombine promising elements of solutions into complete solutions (Rochat and Taillard 1995, Tarantilis 2005, Alvarenga et al. 2007, Groër and Golden 2011), or to explore large neighbourhoods based on ruin-and-recreate (Shaw 1998, De Franceschi et al. 2006, Salari et al. 2010). One actually observes a trend towards proposing *mathheuristics* for vehicle routing problems, combining metaheuristic and mathematical programming components, and explicitly using the model formulation in defining elements of the method (Doerner and Schmid 2010).

2.4.4 Parallel and cooperative metaheuristics

Parallel metaheuristics (Toulouse et al. 1996, Alba 2005, Crainic and Toulouse 2010) are concerned with the efficient exploitation of simultaneous work (often on several processors) to solve a given problem instance, and have proved of great interest for routing problems (Crainic 2008).

Several types of parallelism may be distinguished according to how parallelism is obtained, how communications among the tasks running in parallel are defined, as well as how the global search is conducted. In the most straightforward classification, *low-level* parallelism involves decomposing parts of the algorithm into independent tasks, thus providing the means to exploit parallel resources without changing the general behaviour of the method. To be efficient, such a strategy must target the computationally expensive “bottleneck” procedures, which most frequently are the evaluation of moves in neighbourhood-centred methods, and crossover, selection, and evaluation in population-based ones. To our knowledge, although many papers are concerned with the development of such strategies for metaheuristics in general, few studies on low-level procedures have been directly focused on the CVRP. A notable exception is the recent work of Schulz (2011), considering the efficient solving of CVRPs on Graphic Processing Units (GPU). In this case, the change in hardware has direct implications on the resolution methodology.

In contrast, metaheuristics based on *high-level* parallelism either partition the set of decisions, leading to problem decompositions, or conduct multiple concurrent searches on one or several search spaces. The simplest method of the latter kind, noted as *parallel independent multi-search*, involves to gather the best final solution of a set of methods not linked by any communication or information exchange. This parallel implementation of the multi-start strategy can offer very interesting performances for the CVRP. Yet, to fully profit from parallelism, more advanced *cooperation schemes* integrate mechanisms to share information during the course of the methods and, in the most advanced settings, to create new information out of the exchanged data. Thus, the nature of the information shared, the frequency of the communications, and the scope (utilisation) of the received information are the main characteristics of cooperation strategies.

For the CVRP, as for most combinatorial optimization cases, the most efficient parallel metaheuristics are built on asynchronous communications, triggered individually by the cooperating algorithms, and often taking the form of exchanges of solutions or elements of solutions. Most multi-search strategies are based on either *adaptive* (Rochat and Taillard 1995, Badeau et al. 1997) or *central memory* (Rego 2001, Jin et al. 2010, Cordeau and Maischberger 2011, Groër and Golden 2011) principles. The former gathers promising solution fragments and constructs new solutions out of such fragments. Tabu searches improve these new solutions, and return the best found solutions to the memory. In central memory-based cooperation, participating solution methods, which may be metaheuristics, exact algorithms, or any other method, exchange solutions and, possibly, various other data, through a common data repository (the “central memory”). Thus, all information is always available on request to any of the cooperating processes and, moreover, can be used to generate new relevant information, e.g., new solutions, performance measures on solution components, promising areas of the search space, and so on. Currently, tabu search threads (in a hybrid setting for Cordeau and Maischberger 2011) cooperate in most central-memory methods proposed for the CVRP, while Groër and Golden (2011) also added integer programming solvers.

Other parallel strategies arose in the field of evolutionary computation. According to fine-grained parallel ideas, individuals are arrayed according to some geometrical form (2-dimensional toroidal grid in Alba and Dorronsoro 2006) and interact only with the (four, in this case) individuals directly connected to. This sets up a diffusion mechanism of good individual characteristics throughout the population. GA cooperation is generally built according to a coarse-grained strategy, where populations evolve separately and cooperate through migrations of elite solutions (e.g., Dorronsoro et al. 2007).

Doerner et al. (2006) performed extensive sensitivity analyses on several cooperative ACO metaheuristics that communicate synchronously through exchanges of solutions, ants, or pheromones. Experiments reproduce the results obtained for the parallel strategies for other metaheuristics, and show that parallel methods tend to outperform sequential ones, that sharing populations of elite solutions is more relevant than solely broadcasting the global best solution, and that episodic re-initializations of pheromone matrices contribute towards a better search. Furthermore, exploiting the spatial decomposition of Reimann et al. (2004) in a parallel context leads to increased speed-ups.

2.5 Relative performance of CVRP heuristics

In the last 30 years of research on the CVRP, two main sets of instances have been widely used by the literature to measure the relative performance of heuristics. The 14 benchmark instances of Christofides et al. (1979) involve from 50 to 199 customers, which are spatially randomly distributed for the first 10 instances, and otherwise clustered. The 20 large-scale instances (pr01-pr20) of Golden et al. (1998), range from 200 to 483 customers and present geometric symmetries.

Nowadays, many metaheuristics reach systematically the best known solution (BKS) on almost all instances from Christofides et al. (1979). A comparison of state-of-the-art metaheuristics based on this benchmark tends to be less statistically significant, as only slight differences on 3 or 4 instances are now reported. To state the best performing methods, we therefore rely on the larger scale instances of Golden et al. (1998), for which the results of well-performing approaches remain significantly different. Table 1 provides a comparative analysis of the performance of the best heuristics on this latter set (as recommended by the authors, the results of Groër and Golden 2011, are given for 8 processors). We restricted the comparison to methods providing detailed results on all 20 instances. The solutions quality is measured as the average “Gap” to the current BKS for each instance, gathered in all the considered articles, while the computation time is indicated in column “T”. For a fair comparison, we estimate for each method a scaled computation time “T[#]”, as it would be on a Pentium IV 3.0 GHz, using the factors of Dongarra (2009). Algorithm performances are also presented graphically in Figure 3 relatively to the dual objective of solution quality and scaled time.

Table 1: Best performing metaheuristics for CVRP on Golden et al. (1998) instances

Ref.	Approach	Nb Runs	Gap	T(min)	CPU	T [#] (min)
VCGLR11-s : Vidal et al. (2011a) slow	Hybrid GA	Avg 10	+0.15 %	113.1	Opt 2.4 GHz	92.72
VCGLR11-f : Vidal et al. (2011a) fast	Hybrid GA	Avg 10	+0.26 %	34.79	Opt 2.4 GHz	28.53
NB09 : Nagata and Bräysy (2009)	Hybrid GA	Avg 10	+0.27 %	35.60	Opt 2.4 GHz	29.19
GG11 : Groër and Golden (2011)	Para. R-to-R	Best 5	+0.29 %	40×5.0	8×Xe 2.3 GHz	112.4
MB07-s : Mester and Bräysy (2007) slow	EA+ELS	Single	+0.32 %	24.35	P IV 2.8 GHz	22.4
ZK10 : Zachariadis and K. (2010a)	GLS+Tabu	Avg 10	+0.42 %	40.5	T5500 1.6 GHz	26.7
MM11 : Marinakis and Marinaki (2011)	Bees mating	Best 50	+0.55 %	50×3.96	P-M 1.86 GHz	117
JCL10 : Jin et al. (2010)	Coop Tabu	Avg 10	+0.59 %	8×41.9	8×Xe 3.0 GHz	341
P09 : Prins (2009a)	GRASP+ELS	Single	+0.62 %	7.27	P-IV 2.8 GHz	6.09
T05 : Tarantilis (2005)	Ad.M.+Tabu	Single	+0.92 %	45.47	P-II 400 Mhz	2.02
RDH04 : Reimann et al. (2004)	ACO	Avg 10	+0.92 %	49.33	P-III 900 Mhz	7.05
CM11 : Cordeau and Maischberger (2011)	Iter. Tabu	Avg 10	+0.93 %	31.28	Xe 2.93 GHz	31.07
MM10 : Marinakis and Marinaki (2010)	GA+PSO	Avg 50	+0.98 %	4.20	P-M 1.86 GHz	2.48
DK07 : Derigs and Kaiser (2007)	ABHC	Single	+1.01 %	113.34	Cel 2.4 GHz	105.62
MB07-f : Mester and Bräysy (2007) fast	EA+ELS	Single	+1.22 %	0.22	P-IV 2.8 GHz	0.20
PR07 : Pisinger and Ropke (2007)	ALNS	Avg 10	+1.34 %	10.77	P-IV 3.0 GHz	10.77
LGW05 : Li et al. (2005)	R-to-R	Single	+1.38 %	1.13	Ath 1.0 GHz	0.33
MMP06 : Marinakis et al. (2006)	Hybrid GA	Single	+1.55 %	3.44	P-III 667 GHz	0.23
P04 : Prins (2004)	Hybrid GA	Single	+1.65 %	66.60	P-III 1.0 GHz	10.59

Current state-of-the-art methods exploit neighbourhood-centred searches such as local-improvement heuristics, record-to-record, or tabu search. The best performances are

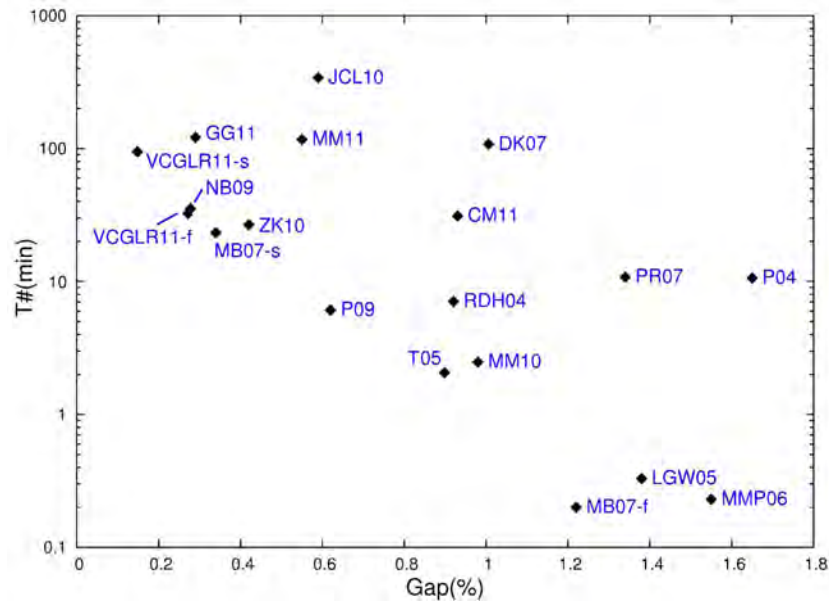


Figure 3: State-of-the-art CVRP methods: solution quality and scaled computation time

achieved, however, by hybrid methods combining neighbourhood-based search with collective intelligence and population concepts (MB07, NB09, VCGLR11), and by parallel cooperative methods based on tabu search and solution-recombination procedures (T05, JCL10, GG11). Eleven (11) algorithms in Table 1 produce solutions that are extremely close or identical to the BKS, with deviations of less than 1% on instances with up to 483 customers. Six methods (VCGLR11-s, VCGLR11-f, MB-07-s, P09, T05, MB07-f) constitute a dominating set with regards to the bi-objective of quality and computational efficiency. A special mention should also be made to (GG11, ZK10, JCL10, P09, CM11, LGW05, P04) which stand out by their simplicity. Finally, some larger instances including thousands of customers have been tackled by LGW05, MB07, as well as Kytöjoki et al. (2007), thus filling the need for quick methods for large real-world CVRPs.

In light of the methods presented in this section, the “traditional” CVRP is remarkably well addressed by metaheuristics. The problem still remains, however, an interesting testing ground for many methodological studies aimed towards more efficient local search methods, new metaheuristic concepts, hybrid and cooperative methods.

The literature shows that more emphasis has been put in recent years on VRPs with additional attributes, for which applications are still challenging. In fact, although many of the methods presented in this section can be rapidly adapted to these VRP extensions, very few general-purpose methods are able to handle the wide range of MAVRPs, and particular method targeting individual cases were generally proposed. The objective of the following sections is to present and classify the main MAVRP attributes, and analyse the concepts of successful heuristics, as a first step on the path toward the development of agile methods able to handle a greater variety of VRP variants.

3 MAVRP Classification & State-of-the-Art Heuristics

Most VRP attributes directly derive from the requirements of real applications. They are the subject of a vast amount of studies, grouping several thousands of articles. Several classification systems for VRP attributes have been proposed. Bodin (1975) and Bodin and Golden (1981) inventoried about a dozen attributes of VRP related to service times. With the same outlook, Ronen (1988) proposed a similar taxonomy centred on practical problem characteristics, and discussed the challenges of the solution methods. Desrochers et al. (1990) introduced a more complete classification system, as well as a four-field notation inspired by Graham et al. (1979). This notation served as a support for the creation of a management system for algorithms and models, based on inference mechanisms for selecting or producing appropriate solution methods (Desrochers et al. 1999). Eksioglu et al. (2009) provided the most complete taxonomy of the MAVRP literature, integrating most of the most common variants as well as several general observations on the nature of the articles. This taxonomy was accompanied by bibliometric data, illustrating the growth in the number of articles, the main authors, subjects, and journals. In addition to the previously listed taxonomies, other papers proposed thematic literature reviews of routing problems (Assad 1988, Desrosiers et al. 1995, Bräysy et al. 2008a,b, Andersson et al. 2010). The annotated bibliographies by Laporte and Osman (1995) and Gendreau et al. (2008) are also noteworthy, providing pointers towards hundreds of articles dealing with MAVRPs and other related problems.

Unlike scheduling, however, where the classification system of Graham et al. (1979) is still used and updated, none of the previously-listed classification systems has been used on a large scale in the vehicle routing literature. This is probably connected to the large variety of attributes, which makes all exhaustive taxonomies extremely difficult. In addition, although the previous classification systems permitted the organization of the different attributes and contributions, few hints were given as to what heuristic concepts to privilege for the different types of attributes.

3.1 Three main classes of attributes

To analyse the concepts of MAVRP heuristics, we distinguish three main classes of attributes, relative to their impact on three aspects of the problem that must imperatively be dealt with by solvers: the *Assignment of customers and routes to resources* (**ASSIGN**), the *Sequence choices* (**SEQ**), and the *Evaluation of fixed sequences* (**EVAL**). This simple classification is intimately connected with the resolution methodologies, as dealing with these three problem aspects leads to a complete solution method. Please note that an attribute may impact several aspects of the problem, and thus possibly appear in several categories.

ASSIGN attributes impact the assignment of limited resources, e.g., vehicles, vehicle types, depots, and service periods over a planning horizon, to customer services and routes. Most common ASSIGN attributes include *multiple depots*, *heterogeneous fleets*, *periodic*, *split deliveries*, *site dependencies*, *inventory*, *location* and *profits collec-*

tion. Furthermore, two main sub-families of ASSIGN attributes may be distinguished. Some variants of the VRP, e.g., multiple depot or heterogeneous VRP, are concerned with the assignment of *resources to routes*. In these settings, an entire route can potentially be re-assigned. In other cases, such as the VRP with multiple periods (PVRP) or inventory routing, the assignment issues are performed on *resources to customers*, and re-assigning a full route is likely to be infeasible due to the independent assignment constraints.

SEQ attributes impact directly the nature and structure of the routes. In a *backhaul* setting, for example, the route is a compound of two sequences of linehaul and backhaul services, respectively. In presence of *multiple trips* or *intermediate facilities*, the routes pass several times at depots, while in the *generalized* VRP, groups of customers are defined, and only one visit per group is performed. In *truck-and-trailer* problems, the routes involve sections with and without trailer. Finally, some other SEQ attributes are related to specificities of the graph, when performing routing on a *tree*, a *grid*, or a *shoreline*.

Finally, EVAL attributes impact a large variety of evaluations and constraint checks that must be performed once the route contents and orders are chosen, including the optimization of the remaining variables, such as service times for problems with time characteristics, idle-time and break placement, speed choices, or the explicit consideration of product placement in trucks. The literature is extremely rich on attributes of this kind, some of the most common being *time windows*, *time-dependent route durations or costs*, *loading constraints*, *open routes*, and *working regulations*. Again, two sub-categories of attributes can be identified. The wide majority of EVAL attributes are inherent to *separate* routes, and thus the evaluations of routes can still be performed independently in the related VRP variants. However, there are also some *linking* EVAL attributes, like *synchronization*, *transshipment* and *consistency*, which link together the fixed-route evaluations, and result in very challenging problems.

Separating attributes among the three previously described categories allows to emphasize relationships between problems, and also estimate the solution method adjustments necessary to deal with them. For example, an EVAL attribute may be managed by an existing algorithm completed with appropriate sequence evaluation methods, while maintaining the resource assignment and sequence creation procedures. In a similar manner, an ASSIGN attribute may be tackled with new assignment procedures without impacting the evaluation of the routes.

Table 2 gathers attributes frequently encountered in the literature. Fifteen of these attributes, marked in boldface, were selected to serve as support to our study on MAVRP heuristics, relatively to two main criteria: 1) the resulting VRP variant is the subject of a significant literature, including exact and heuristic methods, and is possibly mentioned in specialized literature reviews; and 2) benchmark instances are available for comparisons between methods. The first criterion illustrates the importance of the variant in the domain, while the second guarantees that some remarkably efficient algorithms can be objectively selected. The 15 resulting variants are now briefly reviewed in Sections (3.2 - 3.4). In each case, we describe the respective MAVRP, report the size of instances currently solvable with exact methods, and indicate the selected subset of well performing heuristics.

Table 2: Frequently encountered attributes in the literature

ASSIGN		SEQ	EVAL	
<i>Resources to Routes</i>	<i>Resources to Cust.</i>		<i>Separate</i>	<i>Linking</i>
Multiple Depots	Periodic	Backhauls	Open	Synchronization
Heterogeneous Fleet	Split Deliveries	1→1 Pick & Deliv	Time windows	Transshipment
Location	Profits collection	Multiple Trips	Time dependent	Consistency
	Site Dependencies	Intermed Facilities	Working Regulations	
	Inventory	Multiple Echelons	Special loading constr	
	Consistency	Truck and Trailer	Soft & Multiple TW	
		Generalized	Duration Constraints	
		Graph specificities:	Other timing features	
		– Tree	Cumulative costs	
		– Grid	Simult Pick & Deliv	
		– Shoreline	Lunch Breaks	
			Load-dependent Costs	
			Hazardous Materials	

3.2 Heuristics for VRP variants with ASSIGN attributes

Multiple depots. The multi-depot VRP (MDVRP) deals with a number of depots $d > 1$. Each vehicle is assigned to a single depot, which is generally both the origin and the destination of the vehicle’s route. Some variants, called “non-fixed” problems, relax this latter requirement. Furthermore, in the classical MDVRP, no limit on supply at depots is considered. Recent elements of literature review can be found in Ombuki-Berman and Hanshar (2009) and Vidal et al. (2011a). The best exact method (Baldacci and Mingozzi 2009) can solve problem instances up to 75 customers, as well as a few instances with up to 199 customers. When considering metaheuristics, high-quality solutions were produced by the parallel UTS of Cordeau and Maischberger (2011), the ALNS of Pisinger and Ropke (2007), the fuzzy logic-guided hybrid GA of Lau et al. (2010), and the Hybrid Genetic Search with Advanced Diversity Control (HGSADC) of Vidal et al. (2011a).

Heterogeneous fleet. Customers are assigned to vehicle types with different characteristics: capacity, maximum route times, fixed costs, and variable costs in terms of the distance. When the number of vehicles is not constrained, the problem is usually referred to as the *Fleet Size and Mix* problem (FSM), otherwise the more difficult version is called Heterogeneous VRP (HVRP) (see Baldacci et al. 2008a, for a review). The exact algorithm of Baldacci and Mingozzi (2009) solves most problem instances with 75 customers or less, as well as some instances with 100 customers. State-of-the-art metaheuristics are of various kinds: based on column generation (Choi and Tcha 2007), tabu search (Brandão 2011), hybrid GA (Prins 2009b), or ILS and VNS hybridizations (Penna et al. 2011). Specific to the FSM, the tabu search of Brandão (2009), the VNS of Imran et al. (2009), and the hybrid GA of Liu et al. (2009) also produce solutions of good quality.

Periodic. A time dimension is introduced in the Periodic VRP (PVRP) as route planning is to be performed over a horizon of several periods. Each customer requires a total number of services according to some acceptable combinations of visit periods called *patterns*. The assignment of customer visits is thus subject to compatibility constraints with the patterns. The PVRP is reviewed in Francis et al. (2008). Exact methods (Bal-

dacci et al. 2011a) were able to solve some instances with up to 100 customers and 6 time periods. Several efficient neighbourhood-centred searches have been designed, such as UTS (Cordeau et al. 1997, 2001) and its parallel extension (Cordeau and Maischberger 2011), the VNS of Hemmelmayr et al. (2009), and the hybrid record-to-record and integer programming matheuristic of Gulczynski et al. (2011). The population approach of Alegre et al. (2007), dedicated to large temporal horizons, focuses on assignment optimization, while using constructive methods to create routes. Up to date, the best results were produced by HGSADC (Vidal et al. 2011a).

Split deliveries. Customer demands can be satisfied by several vehicles, each moving a partial load. This variant is called VRP with split deliveries (SDVRP), as reviewed in Chen et al. (2007), Gulczynski et al. (2008) and Archetti and Speranza (2011). Instances with up to 50 customers (Belenguer et al. 2000, Lee et al. 2006) can be exactly solved. When time windows on customer services are added, larger instances involving 100 customers can be solved by branch-and-cut-and-price (Desaulniers 2010, Archetti et al. 2011). Two well-performing heuristics (Chen et al. 2007, Archetti et al. 2008) involve integer programming optimization with either record-to-record or tabu search. Derigs et al. (2009) compared several neighbourhood-centred metaheuristics, the *Attribute Based Hill Climber* (ABHC) yielding the best solution quality. Finally, a scatter search was proposed by Mota et al. (2007), while Boudia et al. (2007) introduced an efficient hybrid GA with giant-tour representation.

Prize collection. For several customers, service is optional but is rewarded with a prize. Hence, customers must be implicitly distributed among two subsets, following whether their service is omitted or performed. Several objectives were dealt with in the literature, notably the optimization of a weighted sum of route lengths and prizes (Dell’Amico et al. 1995), or the maximization of the prizes under a route length constraint, usually called the *team orienteering problem* (see the reviews of Feillet et al. 2005 and Vansteenwegen et al. 2010). Exact methods can solve instances with up to 100 customers (Boussier et al. 2006). Most efficient metaheuristics for this problem are population-based. Ke et al. (2008) proposed a hybrid ACO method with a local search. Souffriau et al. (2010) introduced a path relinking method, in which elements of the solution set undergo an *aging* process. Bouly et al. (2009) introduced a hybrid GA based on giant-tour solution representation, which is hybridized later on with PSO in Dang et al. (2011). Finally, Archetti et al. (2006) proposed a hybrid tabu search and VNS.

3.3 Heuristics for VRP variants with SEQ attributes

Backhauls. Customers are separated into two groups: delivery customers (i.e., *linehaul customers*) and pickup customers (i.e., *backhaul customers*). All routes mixing both groups of customers must serve all linehaul customers before the first backhaul customer, thus leading to particular route structures. Toth and Vigo (2002a) and Parragh et al. (2008a) conducted literature reviews dedicated to the VRPB. Instances of maximum 100 customers were solved exactly by Toth and Vigo (1997) and Mingozi et al. (1999). The best metaheuristics include the ALNS of Ropke and Pisinger (2006); the tabu search of Brandão (2006), which, as Zachariadis and Kiranoudis (2011), uses long-term memories in order to direct the search toward inadequately exploited characteristics; and finally

the ACO of Gajpal and Abad (2009), which concurrently evolves two ant families to work on both assignment and sequences.

Pickups & Deliveries. Each service is characterized by a pair of locations designating the pickup and delivery spots. All pickups must be made before the deliveries. This type of problem is dealt with in numerous literature reviews as a *one-to-one Pickup and Delivery Problem (PDP)* (Desaulniers et al. 2002, Berbeglia et al. 2007, Cordeau et al. 2008, Berbeglia et al. 2010) or simply as *VRP with Pickup and Deliveries (VRPPD)* (Parragh et al. 2008a,b). This problem is often coupled with time-window constraints. Ropke et al. (2007) solved exactly instances involving up to 96 requests. Efficient neighbourhood-based metaheuristics have been proposed, including the ALNS of Ropke et al. (2007) and the two-phase method of Bent and Van Hentenryck (2006), which combines simulated annealing (SA) to reduce the number of routes with large neighbourhood search (LNS) to optimize the distance. These methods were recently outperformed by the memetic algorithm of Nagata and Kobayashi (2011), which exploits a well-designed crossover focused on transmitting parent characteristics without introducing too many new arcs in the offspring. For problem variants arising from the domain of transportation on demand (dial-a-ride problem), UTS (Cordeau and Laporte 2003) and the VNS of Parragh et al. (2010) produce solutions of good quality.

Multiple trips. During its tour, a vehicle can pass several times by the depot to load or unload. By doing so, the global constraints on the routes, such as the maximum duration and time windows, continue to be considered. Some time-window constrained instances of VRP with multiple trips (VRPM) with up to 50 customers have been solved optimally by Azi et al. (2010b) and Hernandez et al. (2011). Regarding metaheuristics, no consensus has been found, up to date, on the benchmark instances and the objective used for methods comparisons, thus complicating our task to select well-performing methods. Alonso et al. (2008) recently generalized UTS to a complex periodic VRP with multiple routes and compatibility constraints between vehicles and customers. Olivera and Viera (2007) proposed an adaptive memory-based search. In the same spirit, Battarra et al. (2009) relies on simple alternating heuristics for creation of tours and aggregation within a guidance scheme based on the detection of critical time intervals, when many routes are active. The GA of Salhi and Petch (2007) is based on a route representation in the form of circular sectors and continual injection of new genetic material. Finally, Azi et al. (2010a) apply ALNS with problem-tailored ruin-and-recreate operators.

3.4 Heuristics for VRP variants with EVAL attributes

Time windows. The VRP with time windows (VRPTW) is certainly the most extensively studied VRP variant to date. Time windows are associated to customer visits and depot, each arc being characterized by a route duration. Waiting time is allowed upon an early arrival to a customer. Late arrival is forbidden in presence of *hard* time windows, or sanctioned with penalties in *soft* time-window settings. Recent literature reviews can be found in Bräysy and Gendreau (2005b,a) and Gendreau and Tarantilis (2010). Most efficient exact methods (Kallehauge et al. 2006, Jepsen et al. 2008, Baldacci et al. 2011b) can solve most instances with up to 100 customers, and a few instances with up to 1000 customers. However, exact resolution is highly dependent upon the charac-

teristics of the instance and the width of time windows. Actual state-of-the-art VRPTW metaheuristics are of various kinds. The guided EA of Repoussis et al. (2009) combines evolution, ruin-and-recreate mutations, and guided local search. Prescott-Gagnon et al. (2009) proposed a LNS combined with branch-and-price for solution reconstruction. The HGA proposed by Nagata et al. (2010) uses a particularly effective crossover operator. This latter method, as well as the path relinking of Hashimoto and Yagiura (2008) and HGSADC of Vidal et al. (2011b), apply temporal-constraint relaxations in the course of the search to benefit from infeasible solutions in the search space.

Time-dependent. In practical settings, when facing network congestion especially, travel times on an arc depend on the departure date, leading to a *Time-Dependent VRP* (TDVRP). This problem is frequently combined with time-window constraints, and a *First-In, First-Out (FIFO)* property for the transportation durations is frequently assumed, meaning that a vehicle starting earlier arrives at its destination earlier. Specialized literature reviews were conducted by Malandraki and Daskin (1992), Ichoua et al. (2003) and Fleischmann et al. (2004). Among the particularly efficient heuristics, the adaptive memory search of Ichoua et al. (2003) manages a population of good-quality routes, which are recombined and improved by tabu search. The ILS of Hashimoto et al. (2008) draws its strength from a temporary relaxation of the problem combined with efficient neighbourhood evaluation procedures. Balseiro et al. (2011) proposed a cooperative ACO, hybridized with local searches and ejection chains, which rely on two ant colonies to perform respectively fleet-size and distance minimization.

Other time-related attributes. Many other temporal features on routes have been introduced in the literature, such as speed choices, waiting-time constraints, and multiple time windows, time-dependent service costs, or the minimization of the average time to reach customers, also called *cumulative VRP* (CCVRP). All these variants require to determine the service time to customers for the routes produced during the search in order to evaluate their cost and feasibility. The resulting sub-problems, called *optimal start time problems* or *timing problems*, are reviewed in Hashimoto et al. (2010), and in Vidal et al. (2011c) within a unifying framework that goes beyond vehicle routing. A set of ILS heuristics allowed to address effectively VRP problems with general or convex piecewise linear service costs as a function of service times (Ibaraki et al. 2005, 2008), and with flexible travel time (Hashimoto et al. 2006). These three heuristics are based on remarkably efficient move evaluations for the problems considered. For the CCVRP, Ngueveu et al. (2010) and Ribeiro and Laporte (2012) successfully extended the concepts of, respectively, the hybrid GA with giant tour representation of (Prins 2004), and the ALNS of Pisinger and Ropke (2007),

Working and driving hours regulations. Regulations related to long-distance transportation impose complex rules for driving time and driver breaks. Combining the VRP with break scheduling leads to difficult route feasibility checks. The recent literature on this subject is mainly oriented on the laws in the United States and the European Union. When considering a fixed sequence of visits, the break scheduling sub-problem can be solved exactly in $O(n^2)$ for the laws of the United States (Goel and Kok 2011). For the European Union laws, the complexity of the resulting problem has not yet been determined (Goel 2010). Since routes are costly to evaluate, simple neighbourhood-centred approaches are usually preferred. Both Goel and Kok (2011)

and Prescott-Gagnon et al. (2010) rely on LNS, the latter method relying on integer programming for visit reinsertions. Rancourt et al. (2010) designed a tabu search to address the United States regulations with multiple time windows.

Special loading constraints. Less-than-truckload routing activities are the source of a large range of constraints related to the loading of packages and pallets in 2D or 3D space (*2L-CVRP* or *3L-CVRP*), and their effective loading and unloading. Other lines of research consider the explicit placement of different products in different compartments, and the transportation of hazardous material, which involves additional constraints related to product incompatibility and spacing (see Iori and Martello 2010 for a survey). Tricoire et al. (2009) developed an exact method for the pallet-loading problem, that allows to deal with up to 36 customers. The most effective heuristics for the VRP with 2D and 3D packing include the ACO of Fuellerer et al. (2009, 2010), the tabu search of Zachariadis et al. (2009), and the GRASP \times ELS of Duhamel et al. (2011) that solves a relaxation of the 2L-CVRP as a project scheduling problem with resource constraints, and yields the actual best solution quality. For the pallet-loading problem, the best performances has been reached with the VNS of Tricoire et al. (2009).

Open. Related to the invoicing practices of road transportation suppliers, the last return to the depot is not counted towards the transportation costs in the *Open VRP* (*OVRP*). This variant has been reviewed by Li et al. (2007). Currently, the exact method of Letchford et al. (2006) can solve problems with up to 100 customers. The OVRP is very similar to the “traditional” CVRP from the point of view of a heuristic approach, and a lot of effective methods are adaptations of metaheuristics originally intended for the CVRP. Zachariadis and Kiranoudis (2010b) proposed a tabu search, involving memories to store route evaluations, while Fleszar et al. (2009) proposed an effective VNS relying on some infeasible solutions. A good performance was also achieved by the hybrid EA of Repoussis et al. (2010).

4 A Synthesis of “Winning” MAVRP Strategies

In the previous section, we identified a set of 64 metaheuristics for 15 different MAVRPs, which perform remarkably well on classical literature benchmarks. As illustrated in Table 3, these methods are of various natures and neighbourhood- and population-based methods tend to be equally represented.

Table 3: Main metaheuristic concepts used in the 64 winning methods

Neighbourhood-centred	Freq.	Population-based	Freq.
Tabu Search	14	Genetic or Evolutionary Algorithm	18
Variable Neighbourhood Search	6	Ant Colony Optimization	4
Iterated Local Search	5	Scatter Search	2
Simulated Annealing & Record-to-record	4	Path Relinking	2

To progress towards a better understanding of the winning strategies of MAVRP metaheuristics, we detail the analysis relatively to the 19 main metaheuristic features

presented in Table 4.

Table 4: Fundamental metaheuristic features

Search space	1) presence of infeasible solutions
	2) use of indirect representations of solutions
Neighbourhoods	3) presence of multiple neighbourhoods
	4) use of polynomially enumerable neighbourhoods
	5) use of pruning procedures
	6) use of large neighbourhoods
	7) use of solution recombinations
Trajectory	8) presence of random components
	9) continuous aspect of trajectories
	10) discontinuous aspect
	11) mixed aspect
Control and memories	12) use of populations
	13) diversity management
	14) parameter adaptation
	15) advanced guidance mechanisms
Hybrid strategies	16) use of hybridization
	17) matheuristics with integer programming
Parallelism	18) use of parallelism or cooperation concepts
Problem decompositions	19) use of problem decompositions

Tables 5 and 6 provide a summary of our analysis, each line being associated to a method, and each of the 19 columns (3 - 21) corresponding to a feature that is potentially present. An X sign where line i meets column j indicates that method i relies on concept j . The rest of this section details how these features are used in the 64 state-of-the-art metaheuristics under consideration.

Search Space. Metaheuristics are generally described relatively to the concept of search space, that is, a set of solutions, or more generally a set of states describing solutions, in which the method evolves. Basing the search-space definition on solutions is appropriate for the CVRP. For many MAVRPs, however, defining a complete solution goes beyond route description, as additional decisions related to attributes must be specified. Many metaheuristics are then designed to explore a search space made of indirect representations of solutions, containing, for example, only the route information, on which an efficient *decoder* algorithm can be applied to extract one or several complete solutions. This widely applied methodology is in itself a structural problem decomposition.

In the heuristics surveyed, 13/64 methods rely explicitly on indirect solution representations and decoders. A well-known example is the representation of Prins (2004) as a *giant tour without trip delimiters*, used in many of the selected GAs (Boudia et al. 2007, Prins 2009b, Liu et al. 2009, Nguveu et al. 2010), and in the GRASP+ELS of Duhamel et al. (2011). In this case, the optimal insertion of depot visits in the tour can be solved in a quadratic number of route evaluations with a shortest path-based *Split*

Table 5: Successful metaheuristics for CVRP and MAVRPs with ASSIGN attributes

		SP.		NEIGHBOUR.					TRAJEC.				CONTROL								
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	
		SOL. REPRESENT. RELAXATION		MULT NEIGHB. ENUMERATIVE PRUNING LARGE RECOMBIN.					RANDOMNESS CONTINUOUS DISCONTINUOUS MIXED				POPULATIONS DIV. MANAG. PARAM. ADAPT. GUIDANCE					HYBRIDIZATION MATHEUR.		PARALLELISM DECOMPOSITION	
CAPACITATED VRP																					
Mester and Bräysy (2007)	Guided EA + ELS			X	X	X	X		X	X				X	X		X				
Nagata and Bräysy (2009)	HGA	X		X	X	X		X		X			X				X				
Zachariadis and K. (2010a)	Tabu			X	X	X			X	X					X						
Groër and Golden (2011)	Para. R-to-R			X	X	X			X	X	X	X	X		X		X	X	X	X	
Vidal et al. (2011a)	HGA + Div.Man.	X	X	X	X	X		X		X			X	X	X		X				
MULTIPLE DEPOTS																					
Pisinger and Ropke (2007)	ALNS	X		X			X		X	X				X	X		X				
Lau et al. (2010)	Genetic							X	X	X			X	X	X						
Vidal et al. (2011a)	HGA + Div.Man.	X	X	X	X	X		X	X	X			X	X	X		X				
Cordeau and M. (2011)	Parallel Tabu	X		X	X		X	X	X	X				X	X				X		
HETEROGENEOUS FLEET																					
Liu et al. (2009)	HGA		X	X	X			X	X	X			X	X			X				
Imran et al. (2009)	VNS			X	X				X	X											
Prins (2009b)	HGA		X	X	X			X	X	X			X	X			X				
Brandão (2009, 2011)	Tabu	X		X	X	X			X	X					X		X				
Penna et al. (2011)	ILS + VNS			X	X				X	X							X				
PERIODIC																					
Alegre et al. (2007)	Scatter Search		X	X	X			X	X	X			X	X						X	
Hemmelmayr et al. (2009)	VNS	X		X	X				X	X				X							
Gulczynski et al. (2011)	Rec-to-Rec + IP			X	X		X		X	X							X	X			
Vidal et al. (2011a)	HGA + Div.Man.	X	X	X	X	X		X	X	X			X	X	X		X				
Cordeau and M. (2011)	Parallel Tabu	X		X	X		X	X	X	X				X	X				X		
SPLIT DELIVERIES																					
Chen et al. (2007)	Rec-to-Rec			X	X	X	X			X				X			X	X			
Boudia et al. (2007)	HGA		X	X	X			X	X	X			X	X			X				
Mota et al. (2007)	Scatter Search			X	X			X		X			X								
Derigs et al. (2009)	ABHC			X	X					X					X						
Archetti et al. (2008)	Tabu + IP			X	X				X	X					X		X	X			
PRIZE COLLECTING																					
Archetti et al. (2006)	Tabu + VNS	X		X	X				X	X	X	X		X			X				
Ke et al. (2008)	ACO				X				X	X					X		X				
Souffriau et al. (2010)	Path Relinking			X	X			X	X	X			X		X						
Bouly et al. (2009)	HGA + LNS		X	X	X		X	X	X	X			X	X			X				
Dang et al. (2011)	PSO + HGA		X	X	X		X	X	X	X			X	X	X		X				

Table 6: Successful metaheuristics for MAVRPs with SEQ and EVAL attributes

		SP.		NEIGHBOUR.					TRAJEC.				CONTROL							
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
		SOL. REPRESENT. RELAXATION		MULT NEIGHB. ENUMERATIVE PRUNING LARGE RECOMBIN.					RANDOMNESS CONTINUOUS DISCONTINUOUS MIXED				POPULATIONS DIV. MANAG. PARAM. ADAPT. GUIDANCE HYBRIDIZATION				MATHEUR. PARALLELISM		DECOMPOSITION	
BACKHAULS																				
Brandão (2006)	Tabu	X		X	X				X	X				X	X					
Ropke and Pisinger (2006)	ALNS	X		X			X		X	X				X	X	X				
Gajpal and Abad (2009)	ACO			X	X	X			X		X				X	X				
Zachariadis and K. (2011)	Attrib. driven LS			X	X	X				X					X					
PICK-UP AND DELIVERIES																				
Bent and V.H. (2006)	SA + LNS			X	X	X	X		X	X							X	X		
Ropke et al. (2007)	ALNS	X		X			X		X	X				X	X	X				
Cordeau and Laporte (2003)	Tabu	X			X	X				X				X	X					
Parragh et al. (2010)	VNS	X		X	X		X		X	X				X						
Nagata and Kobayashi (2011)	HGA			X	X	X	X	X	X		X		X		X	X				
MULTIPLE TRIPS																				
Alonso et al. (2008)	Tabu	X			X					X				X	X					
Olivera and Viera (2007)	Adapt. M. + Tabu	X		X	X	X		X	X	X	X	X	X		X	X	X			
Salhi and Petch (2007)	HGA	X	X	X	X		X		X	X	X		X	X						
Battarra et al. (2009)	Adapt. Guid.				X				X	X					X					X
Azi et al. (2010b)	ALNS			X			X		X	X				X						
TIME WINDOWS																				
Hashimoto et Y. (2008)	Path Relinking	X		X	X	X	X	X	X		X	X	X	X	X	X				
Repoussis et al. (2009)	Guided EA			X	X	X	X	X	X	X	X	X	X	X	X	X	X			
P.-Gagnon et al. (2009)	LNS + Col. Gen.			X			X		X	X							X	X		
Nagata et al. (2010)	HGA	X		X	X	X		X	X	X			X				X			
Vidal et al. (2011b)	HGA + Div.Man.	X	X	X	X	X		X	X	X			X	X	X		X			X
TIME DEPENDENT																				
Ichoua et al. (2003)	Adapt. M. + Tabu			X	X			X	X	X	X	X	X		X	X	X	X	X	X
Hashimoto et al. (2008)	ILS	X		X	X	X			X	X										
Balseiro et al. (2011)	ACO	X		X	X	X			X		X				X	X	X	X		
OTHER TEMPORAL FEATURES																				
Ibaraki et al. (2005, 2008)	ILS	X		X	X	X	X		X	X										
Ngueveu et al. (2010)	HGA		X	X	X		X		X		X		X	X			X			
Ribeiro and Laporte (2012)	ALNS	X		X			X		X	X				X	X	X				
WORKING HOURS REGULATIONS																				
Goel and Kok (2011)	LNS			X	X		X		X	X										
P.-Gagnon et al. (2010)	LNS + Col. Gen.			X			X		X	X							X	X		
Rancourt et al. (2010)	Tabu	X		X	X					X				X						
LOADING CONSTRAINTS																				
Fuellerer et al. (2009, 2010)	ACO			X	X				X		X			X	X	X	X	X		X
Zachariadis et al. (2009)	Guided Tabu			X	X	X			X	X				X						
Duhamel et al. (2011)	GRASP + ELS	X	X	X	X				X	X	X	X								
Tricoire et al. (2009)	VNS			X	X				X	X										
OPEN VRP																				
Fleszar et al. (2009)	VNS	X		X	X				X	X										
Repoussis et al. (2010)	Guided EA			X	X	X	X	X	X	X	X	X	X	X	X	X	X			
Zachariadis and K. (2010b)	Tabu			X	X	X			X	X					X					

procedure. In Alegre et al. (2007), solutions are characterized exclusively by decisions on assignments to time periods. The role of the decoder is assumed by a VRP algorithm (a quick constructive method in this case) that creates the routes for each period separately. Finally, Salhi and Petch (2007) rely on an abstract solution representation in the form of circular sectors. Decoding is performed by means of a cluster-first route-second heuristic that relies on the sectors for the clustering phase. One should finally note that the use of multiple structurally different search spaces, relying on different (representations, decoder) pairs, may efficiently reduce the risks of getting trapped in a local optimum.

Another main characteristic of the search space comes from the potential use of infeasible solutions. Since the early literature on tabu search with the *strategic oscillation* concept (Glover 1986, 2011), studies report that a controlled exploitation of infeasible solutions may enhance the search, by allowing it to transition more easily between structurally different feasible solutions. Furthermore, the use of infeasible solutions may contribute toward improving the *robustness* of the method, which is less dependent upon the availability of a feasible initial solution (finding a feasible solution is often in itself a NP-hard problem).

About half of the selected MAVRP heuristics (28/64) rely on penalized infeasible solutions in the search space, which violate either the route constraints (load, duration, or time windows), the fleet size limit, or do not service all customers. Moreover, iteratively decrementing the fleet size limit while relaxing route constraints provides the means to address the “fleet size minimization” objective without relying on complex route elimination procedures. Elements of sensitivity analyses on the role of infeasible solutions in the context of PVRP and MDVRP can be found in Vidal et al. (2011a). In the methods surveyed, relaxations of route constraints are usually privileged over fleet-size relaxations, as it can be difficult to progress from a solution with too many routes to a feasible solution.

Neighbourhoods. With the exception of some methods that use exclusively large neighbourhoods, and the GA of Lau et al. (2010) that appears to rely exclusively on crossover and random mutation, all mentioned MAVRP heuristics are based on at least one type of *enumerable* neighbourhood using the arc exchanges described in Section 2.3. The size of these enumerable neighbourhoods is usually $O(n^2)$ in practice. Exponentially large neighbourhoods are also frequently used (23/64). Besides ruin-and-recreate neighbourhoods or perturbation mechanisms that are well represented in recent methods, cyclic transfers or ejection chains are also used (Ibaraki et al. 2005), as well as variants of the Sarvanov-Doroshko IP refinement heuristic (Chen et al. 2007, Gulczynski et al. 2011). Finally, 25/64 methods combine solutions, or fragments of solutions, into new solutions, thus transmitting good sequence elements as the evolutionary, genetic, scatter search and path relinking algorithms do. Not only population-based searches use this mechanisms. Consider for example the adaptive memory approaches of Olivera and Viera (2007) and Ichoua et al. (2003), which operate recombinations of solution fragments, and the set covering based approach of Groër and Golden (2011) involving recombinations of routes issued from multiple solutions.

Almost all the methods surveyed (60/64) exploit multiple neighbourhoods, either successively, or in a compound way. The successive exploration of multiple neighbourhoods makes the basis of the VNS methodology, and is recognized as an important success factor for metaheuristics in general, especially on complex problems with multiple con-

straints and characteristics such as MAVRPs. The methods of Archetti et al. (2006) and Parragh et al. (2010) push very far the concept of neighbourhood variation by exploiting structurally-different, enumerative and large, neighbourhoods. Other VNS for MAVRPs may gradually increase the size of the enumerative neighbourhoods by varying the number of arcs to be exchanged (Hemmelmayr et al. 2009, Tricoire et al. 2009), but, strictly speaking, do not involve structural neighbourhood differences.

Searching efficiently these neighbourhoods is critical for performance, as it generally makes for the biggest part of the computation effort. Therefore, many techniques aim at pruning the neighbourhoods, or at enumerating them more efficiently. Move restrictions based on customer neighbourhood lists (granular search) are frequently used (Ibaraki et al. 2005, Mester and Bräysy 2007, Olivera and Viera 2007, Hashimoto and Yagiura 2008, Vidal et al. 2011a), as well as neighbourhood limitation strategies based on recently modified solution features (Nagata and Bräysy 2008, 2009, Nagata et al. 2010, Nagata and Kobayashi 2011). In presence of EVAL attributes, re-optimization information developed on subsequence of successive customers can increase the efficiency of neighbour evaluations (Kindervater and Savelsbergh 1997, Cordeau and Laporte 2003, Nagata et al. 2010, Vidal et al. 2011b), and approximate evaluations of neighbours can be performed to reduce complexity (Ichoua et al. 2003). Among the selected methods, 25/64 algorithms use at least one of the listed techniques.

Memories of previous computations, aimed at reducing computational redundancy without changing the method behaviour, are also frequently used. Although such procedures may be viewed as a matter of algorithmic engineering, and thus not necessarily mentioned, they are critical to reach a good performance, especially on problems for which route evaluations are costly such as the 2L- or 3L-CVRP, or the VRP with break scheduling. Most common memories of this kind are dedicated to manage move informations (Cordeau and Laporte 2003, Alegre et al. 2007, Zachariadis and Kiranoudis 2010a, Vidal et al. 2011a) and route evaluations (Tricoire et al. 2009, Zachariadis et al. 2009, Duhamel et al. 2011). Addressing all the attributes of the problem with well-designed neighbourhood-based searches is, and should remain, a primary concern when addressing complex MAVRPs.

Search trajectories. The inclusion of random components in the various algorithm choices, mentioned explicitly in 57/64 methods, is a dominant characteristic of search trajectories. Randomisation is a prerequisite of asymptotic convergence properties of metaheuristics such as SA or GA. In practice, however, it is mostly used as a simple and efficient way to avoid cyclic behaviour and increase the diversity of solutions. Only a few current methods for MAVRPs are deterministic. For example, although tabu search has been first built on deterministic arguments (Glover 1986), recent applications involve random diversification operations, or tabu lists whose size vary probabilistically. Adding random noise to the objective function, as in Pisinger and Ropke (2007), is another way to exploit randomization to diversify the search.

The amount of change from one solution to the next is also characteristic of the methods. In neighbourhood-centred methods, successive solutions tend to be in close proximity, sharing many common elements. This kind of trajectory can be qualified as continuous, unlike the trajectories of most population-based metaheuristics with crossovers, which are discontinuous, and display a “jumpy” behaviour between successive solutions.

Finally, mixed trajectories, combining continuous search and jumps, aim to profit from both kinds of exploration.

We identified 40/64 methods that use mostly a continuous trajectory, and 31/64 methods that often use discontinuous trajectories. Seven algorithms use mixed trajectories, with large continuous search phases as well as regular jumps. These are neighbourhood-centred metaheuristics that include mechanisms to change abruptly the search region by “jumping” to an elite solution (e.g., Archetti et al. 2006, Groër and Golden 2011), during GRASP restarts (Duhamel et al. 2011), or when complete solutions are reconstituted from fragments or separate routes (Ichoua et al. 2003, Olivera and Viera 2007). Note that, ruin-and-recreate LNS and perturbation moves were included among the *continuous* class. Our main motivation is that the effective amount of arcs that are actually changed from one LNS iteration to the next can remain rather small, and such moves are generally operated in a spirit of single solution improvement.

Memories and control. The judicious acquisition, management, and exploitation of knowledge on the problem and on the past search history is a complex task that belongs to the core of metaheuristics. Glover (1986) described three types of memories in the case of tabu search: short-term memories (e.g., tabu lists), which allow the search to be influenced locally in order to evade local optima, and medium- and long-term memories (e.g., memories on solutions elements), which are used to direct the overall exploration of the search space. These kind of memories have since been developed into various forms, and exploited for many means in other metaheuristics, including those surveyed for MAVRPs.

In particular, 26/64 of the selected metaheuristics bring into play populations as memories to manage promising or good-quality solutions, solution representations, routes, or solutions fragments. This is naturally the case for GA-based methods, path relinking, and scatter search, as well as metaheuristics relying on adaptive (Ichoua et al. 2003, Olivera and Viera 2007) or central memory cooperation (Cordeau and Maischberger 2011, Groër and Golden 2011). The populations of solution elements are used as the support for recombination procedures, including through set covering formulations, yielding new incumbent solutions.

Usually, a mix of diverse and high-quality elements is stored, thus aiming to find a balance between exploring new solution elements and focusing on champion features. Maintaining both diversity and elitism simultaneously in a population is a difficult task, as the aggressive local-improvement procedures, used in most efficient metaheuristics, tend to strongly drive the population towards a few local optima, resulting in premature convergence. Population-diversity management has thus been shown to be a key success factor in achieving good performance for MAVRPs (Prins 2004, Vidal et al. 2011a). It is especially critical in addressing *rich* VRPs combining several attributes, as finding new high-quality solutions on such intricate problems seems to require a good diversity of solution elements.

Among the above-mentioned methods, 15/26 operate diversity management procedures, relying usually on a distance metric between individuals for both measuring diversity and driving the population management. For MAVRPs, this metric is usually based on solution differences in the objective space (Prins 2004, Liu et al. 2009, Ngueveu et al. 2010) or similarities in the route sequences (Prins 2009b, Vidal et al. 2011a), or are de-

signed specifically for the attributes considered (those of the ASSIGN category especially, e.g., Alegre et al. 2007, Vidal et al. 2011a). Diversity can then be controlled by different means. Lau et al. (2010) rely on fuzzy logic to adapt search parameters relatively to population diversity and quality measures. Prins (2004), as well as several other recent genetic algorithms with population management (Sörensen and Sevaux 2006), impose distance constraints for acceptance in the population. Souffriau et al. (2010) implement ageing concepts to discard too “old” solutions from the pool. Finally, HGSADC (Vidal et al. 2011a) does not consider diversity as a constraint, but as an integral part of the objective that competes with solution quality during individual evaluations. Empirical studies show that the latter mechanism leads to a higher solution diversity and quality.

Population management parameters are not the only ones to be adapted throughout the search. Parameter adaptation tends to be widespread in the methods analysed (26/64) to drive the infeasibility penalties (Cordeau et al. 1997, Vidal et al. 2011a), mutation or crossover rates (Repoussis et al. 2009, Lau et al. 2010), or other algorithm strategies such as the frequency of use of operators and neighbourhoods (Ropke and Pisinger 2006, Pisinger and Ropke 2007, Azi et al. 2010b). Evolving search parameters directly within the genetic material of individuals is a common practice in EAs, while general metaheuristics adaptation is a main focus of hyper-heuristics (Burke et al. 2010).

More advanced forms of *guidance*, aiming to explicitly collect, analyse, and exploit knowledge on the past search to orient the future trajectories, are used in 29/64 methods. In MAVRP metaheuristics, the information is usually built as statistics on solution features, arcs, sets of arcs, routes, or problem specific attributes. The search context, e.g., the value of the incumbent solution and, eventually, the evolution of the value of the best solution (overall or for the current phase of the search), the value of particular counters resulting from the search history, and so on, is also part of the knowledge which is built.

This body of information, once collected and analysed, serves as support for *guidance actions*. The purpose of such actions is generally to either intensify the search, by focusing on promising solution features, or diversify it towards under-explored areas of the search space. Various methods are used in the methods surveyed to undertake such intensification and diversification actions, such as, penalties or incentives on solution attributes (see Cordeau and Laporte 2003, Battarra et al. 2009, Derigs et al. 2009, Repoussis et al. 2009, 2010, Zachariadis et al. 2009, among others), “jumps” toward elite solutions or new solutions recombined from elite elements (Ichoua et al. 2003, Brandão 2006, Oliveira and Viera 2007), target solutions in path relinking (Hashimoto et al. 2008, Souffriau et al. 2010), neighbourhood choices governed by pheromone matrices (Ke et al. 2008, Fuellerer et al. 2009, 2010, Balseiro et al. 2011), or history-based ruin-and-recreate operators (Ropke and Pisinger 2006, Pisinger and Ropke 2007, Ribeiro and Laporte 2012). Guidance actions may be undertaken continuously, as part of the fundamental search pattern of the metaheuristic (e.g., path relinking or TABUROUTE and UTS incorporating dynamically adjusted penalties on solution stagnation or infeasibility elements), or discreetly through a purposeful move.

Balancing intensification and diversification is particularly important for MAVRPs, where many problem features may be exploited in order to drive more efficiently the search. It is thus well-known that *statistically frequent* features of high-quality solutions

are more likely to appear in the global optimum, thus explaining partly the recent success for MAVRP of population-based metaheuristics (Jones 1995), which favour the apparition and transmission of good solution elements, called *building blocks* in Holland (1975). Similarly, concepts of identification and combination of statistically promising solution attributes appeared with tabu search under the name of *vocabulary building* (Glover and Laguna 1998). Problem knowledge can thus be exploited in many ways in MAVRPs to intensify the search around relevant solution elements. Much of this same information can also be used for diversification, as it does, and should not play second violin. Indeed, as MAVRP search spaces, although sometimes metaphorically described as globally convex “big valleys” (Boese 1995, Kubiak 2007), remain nonetheless rugged and some near-optimal solutions may be substantially different from the global optimum, diversification procedures play a critical role in search efficiency.

Finally, among the papers surveyed, many sensitivity analyses on parameters seek a good balance between intensification and diversification, though modifications of diversity management, tabu lists, temperature controls in SA, pheromone matrices, thresholds choices in R-to-R, neighbourhood choices in LNS or VNS, and so on. However, due to the balance that must be established, such parameters are subject to correlations, and advanced calibration methods, meta-calibration (De Landgraaf et al. 2007) or other statistical methods (Nannen and Eiben 2007) that address all parameters together may be necessary.

Hybridization. The metaheuristics surveyed rely to a large extent (39/64) on hybridization. By decreasing order of appearance, we report genetic algorithms and ACO methods combined with local search, sometimes using large neighbourhoods; tabu search methods combined with diversification operators based on solution recombinations (Ichoua et al. 2003, Olivera and Viera 2007); and hybrid neighbourhood-centred methods combining SA and LNS (Gajpal and Abad 2009), tabu search and VNS (Archetti et al. 2006), or ILS with VNS (Penna et al. 2011). Eight hybrid metaheuristics involve mathematical programming components. These components are used to handle attributes of the problem, such as loading constraints (Fuellerer et al. 2009) or split deliveries (Chen et al. 2007, Archetti et al. 2008). In other cases, exact methods are used to search large neighbourhoods (Bent and Van Hentenryck 2006, Prescott-Gagnon et al. 2010, Gulczynski et al. 2011), or recombine solution elements (Groër and Golden 2011).

Parallelism and cooperation. With the exception of multi-start methods that can be considered as a straightforward form of parallelism, 5/64 efficient methods relying on advanced parallelism and cooperation mechanisms were identified. Most of these methods involve neighbourhood-based heuristics, tabu search in particular, that communicate through an *adaptive memory* of elements of solutions (Ichoua et al. 2003) or through a *central memory* of complete solutions (Cordeau and Maischberger 2011, Groër and Golden 2011). Integer programming solvers are used in (Groër and Golden 2011) to recreate solutions from the routes present in memory. In Balseiro et al. (2011), cooperation is based on pheromone exchanges between two ant colonies that optimize simultaneously travel times and fleet size.

It should finally be mentioned that more advanced cooperative metaheuristics are emerging for *rich* MAVRPs. In particular, Le Bouthillier and Crainic (2005) introduced an advanced cooperative method for the VRPTW based on central memory. The method

was complemented in Le Bouthillier et al. (2005) with advanced guidance features. It served then as a building block of the *Integrative Cooperative Search (ICS)* framework of Crainic et al. (2009), which relies on a structural problem decomposition among several such central memories. Each central memory involves several *partial solvers* that cooperate to produce *partial solutions* of the sub-problems, while integrators take on the role of reconstituting complete solutions from partial solutions picked in the partial memories. A global search coordinator is in charge of guiding the overall search as well as modifying the parameters and procedures.

Problem Decompositions. MAVRPs lend themselves well to various decomposition approaches, centred on assignments or geometry (Ostertag 2008, Bent and Van Hentenryck 2010), temporal aspects (Bent and Van Hentenryck 2010), or on solution attribute subsets (Crainic et al. 2009). Among the methods analysed, Ichoua et al. (2003), Vidal et al. (2011b) and Fuellerer et al. (2009, 2010) separate temporarily the routes of an elite solution using geometrical arguments, the different customer sets corresponding to sub-problems that are solved separately. Such decompositions thus makes it possible to improve the assignments of an elite solution in a view of intensification. Structural problem decompositions, involving successive or simultaneous solutions of sub-problems presenting less attributes, are also used. Alegre et al. (2007) apply to the PVRP a scatter search to optimize the assignment to periods, while a simple CVRP heuristic is repeatedly used for route creation. Finally, for the VRP with multiple trips, the algorithm of Battarra et al. (2009) solves successively sub-problems that aim, respectively, to generate simple routes and aggregate them together. Decomposition become essential to handle rich MAVRPs but, in this context, the sequential approaches that independently solve problem characteristics consecutively are not sufficient to attain high-quality solutions. A clever management of the successive decompositions, sub-problem resolutions, and full solution reconstructions becomes thus essential.

5 Conclusions and Perspectives

This unifying survey and synthesis responds to the considerable challenge related to the abundance of VRP variants and to the relatively few general classifications and analyses of these problems and solution methods. The survey underlines that, while few general and efficient metaheuristics were proposed in the literature for this important class of problems, MAVRPs naturally share many common features, and most heuristic strategies developed for specific problems can be applied to a broader range of VRP variants. Hence, we conducted this analysis from a general perspective detached from the particular characteristics of the VRP attributes, and adopted a synthetic approach providing the means to cope with the abundance of contributions. We analysed in detail sixty-four successful metaheuristics for fifteen well-studied MAVRPs, identifying the main concepts and algorithmic-design principles, and highlighting the winning strategies of many efficient metaheuristics for a wide variety of variants.

When considering state-of-the-art methods, we observed recurrent notions such as *mix*, *variability*, *hybridisation*, *cooperation*, *diversity*, *multiplicity*, as well as *balance*, *equilibrium*, *trade-off*. It appears that most successful metaheuristics are not determined

by a single factor but are the result of a good balance between several elements of methodology: the use of different search spaces, variable neighbourhoods, mixed continuous and discontinuous search, short-, medium- and long-term memories, trade-off between diversification and intensification, cooperation and collective intelligence, hybridisation, and so on. In brief, *in unity and diversity lies strength*. The performance of those methods indicates that each element plays an important role. On the one hand, long-term memories, jumps, recombinations and, generally, advanced guidance mechanisms providing diversification and, when relevant, population-diversity management methods have the potential to make the search progress in the general “big rugged valley” of MAVRPs. On the other hand, short and medium-term memories and well-designed solution-improvement methods provide the aggressive search capabilities to complete the refinement of solutions.

We also observed that a clever implementation of algorithms is a necessary condition to yield competitive and scalable methods. Neighbourhood pruning procedures (granularity, sequential searches) or memories on already evaluated routes, route segments, and moves, are necessary in many cases. Furthermore, one may notice that many algorithms rely on randomization and dedicate most of their computing time to evaluating various potential choices, without taking much advantage of history and already performed computations that may in many cases be profitably used. More intelligent guidance schemes have thus the potential to lead to performance improvements.

The research avenues for developing efficient MAVRP heuristics are numerous. We conclude the paper by summing up some open research questions. In the previous sections, we identified a number of search-space, neighbourhoods, and trajectory choices leading to successful MAVRP metaheuristics. One may then ask to what extent these choices should depend upon the variant of the problem, and whether it is possible to identify desirable search spaces and neighbourhoods for some broad MAVRP classes. Of a similar nature are studies related to the definition of population-diversity metrics (e.g., what type of distance for MAVRPs) and management methods, and whether it should be dependent upon the particular problem setting. Designing adequate and general neighbourhood pruning procedures for MAVRPs is another important issue of similar nature, which may also be stated in terms of making current mechanisms, e.g., granular and sequential search, efficiently applicable to a large variety of attributes and problem settings. Such algorithmic developments and proof-of-concept studies make up a very challenging research area.

The integration of diversification and the appropriate balance between intensification and diversification are critical factors for efficient MAVRP metaheuristics. This area is closely related to the development of advanced mechanisms to extract knowledge out of the explored search-space areas and to globally guide the metaheuristics. Links to the fields of hyper-heuristics and landscape analysis should also be more thoroughly explored.

As this survey illustrates, a number of metaheuristic families, tabu search, adaptive large neighbourhood search, and hybrid genetic algorithms, in particular, are widely acknowledged for their performance on a variety of MAVRPs. Given how differently these metaheuristics define and explore the search space, they are very likely to lead to extremely effective hybrid algorithms and parallel cooperative methods. This is an extremely rich and promising research field, particularly given the trend toward problem settings including a continuously increasing number of attributes and solution methods

capable of addressing these attributes simultaneously.

To conclude, more general-purpose solvers, capable of handling a wide range of MAVRPs, are necessary to efficiently address practical routing applications in a timely manner. Many research questions have been answered by personalizing algorithms for each particular variant and by case-by-case improvements. However, solving generically (e.g., using a single solver and parameter set) a wide range of MAVRPs requires a better understanding of the problem foundations and the methods. This unifying survey and synthesis is a step toward reaching these goals.

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References

- Aarts, E.H.L., J.K. Lenstra, eds. 2003. *Local search in combinatorial optimization*. Princeton Univ Pr.
- Ahuja, Ravindra K., Özlem Ergun, James B. Orlin, Abraham P. Punnen. 2002. A survey of very large-scale neighborhood search techniques. *Discrete Applied Mathematics* **123**(1-3) 75–102.
- Alba, E. 2005. *Parallel metaheuristics: a new class of algorithms*. Wiley-Interscience.
- Alba, E., B. Dorronsoro. 2006. Computing nine new best-so-far solutions for Capacitated VRP with a cellular Genetic Algorithm. *Information Processing Letters* **98**(6) 225–230.
- Alegre, J., M. Laguna, J. Pacheco. 2007. Optimizing the periodic pick-up of raw materials for a manufacturer of auto parts. *European Journal of Operational Research* **179**(3) 736–746.
- Alonso, F., M. J. Alvarez, J. E. Beasley. 2008. A tabu search algorithm for the periodic vehicle routing problem with multiple vehicle trips and accessibility restrictions. *Journal of the Operational Research Society* **59**(7) 963–976.
- Alvarenga, G.B., G.R. Mateus, G. De Tomi. 2007. A genetic and set partitioning two-phase approach for the vehicle routing problem with time windows. *Computers & Operations Research* **34**(6) 1561–1584.

- Andersson, H., A. Hoff, M. Christiansen, G. Hasle, A. Lokketangen. 2010. Industrial aspects and literature survey: Combined inventory management and routing. *Computers & Operations Research* **37**(9) 1515–1536.
- Archetti, C., M. Bouchard, G. Desaulniers. 2011. Enhanced Branch and Price and Cut for Vehicle Routing with Split Deliveries and Time Windows. *Transportation Science* **45**(3) 285–298.
- Archetti, C., A. Hertz, M. G. Speranza. 2006. Metaheuristics for the team orienteering problem. *Journal of Heuristics* **13**(1) 49–76.
- Archetti, C., M. G. Speranza. 2011. Vehicle routing problems with split deliveries. *International Transactions in Operational Research, Forthcoming* .
- Archetti, C., M.G. Speranza, M.W.P. Savelsbergh. 2008. An optimization-based heuristic for the split delivery vehicle routing problem. *Transportation Science* **42**(1) 22–31.
- Assad, A.A. 1988. Modeling and Implementation Issues in Vehicle Routing. B.L. Golden, A.A. Assad, eds., *Vehicle Routing: Methods and Studies*. North-Holland, Amsterdam, 7–45.
- Azi, N., M. Gendreau, J.-Y. Potvin. 2010a. An adaptive large neighborhood search for a vehicle routing problem with multiple trips. *CIRRELT Working Paper 8* .
- Azi, N., M. Gendreau, J.-Y. Potvin. 2010b. An exact algorithm for a vehicle routing problem with time windows and multiple use of vehicles. *European Journal of Operational Research* **202**(3) 756–763.
- Badeau, P., F. Guertin, M. Gendreau, J.-Y. Potvin, E.D. Taillard. 1997. A parallel tabu search heuristic for the vehicle routing problem with time windows. *Transportation Research Part C: Emerging Technologies* **5**(2) 109–122.
- Baker, E.K., J.R. Schaffer. 1986. Solution improvement heuristics for the vehicle routing and scheduling problem with time window constraints. *American Journal of Mathematical and Management Sciences* **6**(3) 261–300.
- Baldacci, R., E. Bartolini, A. Mingozzi, A. Valletta. 2011a. An Exact Algorithm for the Period Routing Problem. *Operations Research* **59**(1) 228–241.
- Baldacci, R., M. Battarra, D. Vigo. 2008a. Routing a heterogeneous fleet of vehicles. B. Golden, S. Raghavan, E. Wasil, eds., *The Vehicle Routing Problem: Latest Advances and New Challenges*. Springer, 3–27.
- Baldacci, R., N. Christofides, A. Mingozzi. 2008b. An exact algorithm for the vehicle routing problem based on the set partitioning formulation with additional cuts. *Mathematical Programming* **115**(2) 351–385.
- Baldacci, R., A. Mingozzi. 2009. A unified exact method for solving different classes of vehicle routing problems. *Mathematical Programming* **120**(2) 347–380.
- Baldacci, R., A. Mingozzi, R. Roberti. 2011b. New route relaxation and pricing strategies for the vehicle routing problem. *Operations Research - Forthcoming* .
- Baldacci, R., P. Toth, D. Vigo. 2007. Recent advances in vehicle routing exact algorithms. *4OR* **5**(4) 269–298.
- Balseiro, S.R., I. Loiseau, J. Ramonet. 2011. An Ant Colony algorithm hybridized with insertion heuristics for the Time Dependent Vehicle Routing Problem with Time Windows. *Computers & Operations Research* **38**(6) 954–966.
- Battarra, M., M. Monaci, D. Vigo. 2009. An adaptive guidance approach for the heuristic solution of a minimum multiple trip vehicle routing problem. *Computers & Operations Research* **36**(11) 3041–3050.

- Beasley, J.E. 1983. Route first - cluster second methods for vehicle routing. *Omega* **11**(4) 403–408.
- Belenguer, J. M., M. C. Martinez, E. Mota. 2000. A Lower Bound for the Split Delivery Vehicle Routing Problem. *Operations Research* **48**(5) 801–810.
- Bell, J., P. Mc Mullen. 2004. Ant colony optimization techniques for the vehicle routing problem. *Advanced Engineering Informatics* **18**(1) 41–48.
- Bent, R., P. Van Hentenryck. 2006. A two-stage hybrid algorithm for pickup and delivery vehicle routing problems with time windows. *Computers & Operations Research* **33**(4) 875–893.
- Bent, R., P. Van Hentenryck. 2010. Spatial, temporal, and hybrid decompositions for large-scale vehicle routing with time windows. *Proceedings of the 16th international conference on Principles and practice of constraint programming, CP'10, St. Andrews, Scotland*. Springer-Verlag, 99–113.
- Berbeglia, G., J.-F. Cordeau, I. Gribkovskaia, G. Laporte. 2007. Static pickup and delivery problems: a classification scheme and survey. *Top* **15**(1) 1–31.
- Berbeglia, G., J.-F. Cordeau, G. Laporte. 2010. Dynamic pickup and delivery problems. *European Journal of Operational Research* **202**(1) 8–15.
- Blum, C., J. Puchinger, G. Raidl, A. Roli. 2011. Hybrid metaheuristics in combinatorial optimization: A survey. *Applied Soft Computing* **11**(6) 4135–4151.
- Blum, C., A. Roli. 2003. Metaheuristics in Combinatorial Optimization : Overview and Conceptual Comparison. *ACM Computing Surveys (CSUR)* **35**(3) 268–308.
- Bodin, L., B. Golden. 1981. Classification in vehicle routing and scheduling. *Networks* **11**(2) 97–108.
- Bodin, L. D. 1975. A taxonomic structure for vehicle routing and scheduling problems. *Computers & Urban Society* **1**(1) 11–29.
- Bodin, L.D., L. Berman. 1979. Routing and scheduling of school buses by computer. *Transportation Science* **13**(2) 113.
- Boese, K.D. 1995. Cost versus distance in the traveling salesman problem. *Unpublished preliminary report* .
- Boudia, M., C. Prins, M. Reghioui. 2007. An effective memetic algorithm with population management for the split delivery vehicle routing problem. *Lecture Notes in Computer Science* **4771** 16–30.
- Bouly, H., D.-C. Dang, A. Moukrim. 2009. A memetic algorithm for the team orienteering problem. *4OR* **8**(1) 49–70.
- Boussier, S., D. Feillet, M. Gendreau. 2006. An exact algorithm for team orienteering problems. *4OR* **5**(3) 211–230.
- Brandão, J. 2006. A new tabu search algorithm for the vehicle routing problem with backhauls. *European Journal of Operational Research* **173**(2) 540–555.
- Brandão, J. 2009. A deterministic tabu search algorithm for the fleet size and mix vehicle routing problem. *European Journal of Operational Research* **195**(3) 716–728.
- Brandão, J. 2011. A tabu search algorithm for the heterogeneous fixed fleet vehicle routing problem. *Computers & Operations Research* **38**(1) 140–151.
- Bräysy, O. 2003. A Reactive Variable Neighborhood Search for the Vehicle-Routing Problem with Time Windows. *INFORMS Journal on Computing* **15**(4) 347–368.

- Bräysy, O., M. Gendreau. 2005a. Vehicle routing problem with time windows, Part I : Route construction and local search algorithms. *Transportation Science* **39**(1) 104–118.
- Bräysy, O., M. Gendreau. 2005b. Vehicle routing problem with time windows, Part II : Meta-heuristics. *Transportation Science* **39**(1) 119–139.
- Bräysy, O., M. Gendreau, G. Hasle, A. Lokketangen. 2008a. A Survey of Heuristics for the Vehicle Routing Problem Part I: Basic Problems and Supply Side Extensions. Tech. rep., SINTEF, Norway.
- Bräysy, O., M. Gendreau, G. Hasle, A. Lokketangen. 2008b. A Survey of Heuristics for the Vehicle Routing Problem Part II: Demand Side Extensions. Tech. rep., SINTEF, Norway.
- Bullnheimer, B., R.F. Hartl, C. Strauss. 1999. An improved ant system algorithm for the vehicle routing problem. *Annals of Operations Research* **89** 319–328.
- Burke, E.K., M. Hyde, G. Kendall, G. Ochoa, E. Özcan, J.R. Woodward. 2010. A classification of hyper-heuristic approaches. M. Gendreau, J.-Y. Potvin, eds., *Handbook of Metaheuristics*. Springer, 449–468.
- Chen, P., H.-K. Huang, X.-Y. Dong. 2010. Iterated variable neighborhood descent algorithm for the capacitated vehicle routing problem. *Expert Systems with Applications* **37**(2) 1620–1627.
- Chen, S., B. Golden, E. Wasil. 2007. The split delivery vehicle routing problem: Applications, algorithms, test problems, and computational results. *Networks* **49**(4) 318–329.
- Choi, E., D. Tcha. 2007. A column generation approach to the heterogeneous fleet vehicle routing problem. *Computers & Operations Research* **34**(7) 2080–2095.
- Christofides, N. 1976. The Vehicle Routing Problem. *RAIRO Operations Research* **10** 55–70.
- Christofides, N., Mingozzi A., P. Toth. 1979. The Vehicle Routing Problem. N. Christofides, Mingozzi A., P. Toth, C. Sandi, eds., *Combinatorial Optimization*. John Wiley, New York, 315–338.
- Christofides, N., S. Eilon. 1972. Algorithms for Large-Scale Travelling Salesman Problems. *Operational Research Quarterly* **23**(4) 511.
- Clarke, G., J. W. Wright. 1964. Scheduling of Vehicles from a Central Depot to a Number of Delivery Points. *Operations Research* **12**(4) 568–581.
- Cordeau, J.-F., M. Gendreau, A. Hertz, G. Laporte, J.S. Sormany. 2005. New heuristics for the vehicle routing problem. A. Langevin, D. Riopel, eds., *Logistics systems: design and optimization*. Springer, 279–297.
- Cordeau, J.-F., M. Gendreau, G. Laporte. 1997. A tabu search heuristic for periodic and multi-depot vehicle routing problems. *Networks* **30**(2) 105–119.
- Cordeau, J.-F., G. Laporte. 2003. A tabu search heuristic for the static multi-vehicle dial-a-ride problem. *Transportation Research Part B* **37**(6) 579–594.
- Cordeau, J.-F., G. Laporte, A. Mercier. 2001. A unified tabu search heuristic for vehicle routing problems with time windows. *Journal of the Operational Research Society* **52**(8) 928–936.
- Cordeau, J.-F., G. Laporte, M.W.F Savelsbergh, D. Vigo. 2007. Vehicle Routing. Barnhart, C., Laporte, G., eds., *Transportation*. Handbooks in Operations Research and Management Science, North-Holland, Amsterdam, 367–428.
- Cordeau, J.-F., M. Maischberger. 2011. A Parallel Iterated Tabu Search Heuristic for Vehicle Routing Problems. Tech. rep., CIRRELT.

- Cordeau, J.F., G. Laporte, S. Ropke. 2008. Recent models and algorithms for one-to-one pickup and delivery problems. B. Golden, S. Raghavan, E. Wasil, eds., *The Vehicle Routing Problem: Latest Advances and New Challenges*. Springer, 327–357.
- Corne, D., M. Dorigo, F. Glover. 1999. *New ideas in optimisation*. McGraw-Hill, Maidenhead, UK, England.
- Crainic, T.G. 2008. Parallel Solution Methods for Vehicle Routing Problems. Golden, B.L., Raghavan, S., Wasil, E.A., eds., *The Vehicle Routing Problem: Latest Advances and New Challenges*. Springer, New York, 171–198.
- Crainic, T.G, G. C. Crisan, M. Gendreau, N. Lahrichi, W. Rei. 2009. Multi-thread cooperative optimization for rich combinatorial problems. *Proceedings of the 23rd IEEE International Parallel & Distributed Processing Symposium, IPDPS 2009*.
- Crainic, T.G, M. Toulouse. 2010. Parallel meta-heuristics. M. Gendreau, J.-Y. Potvin, eds., *Handbook of Metaheuristics*. Springer, 497–541.
- Créput, J.-C., A. Koukam. 2008. Self-organization in evolution for the solving of distributed terrestrial transportation problems. B. Prasad, ed., *Soft Computing Applications in Industry, Studies in Fuzziness and Soft Computing*, vol. 226. Springer Berlin / Heidelberg, 189–205.
- Dang, D.-C., R. Guibadj, A. Moukrim. 2011. A PSO-Based Memetic Algorithm for the Team Orienteering Problem. *EvoApplications, LNCS 6625*. Turin, Italy, 471–480.
- Dantzig, G.B., J.H. Ramser. 1959. The truck dispatching problem. *Management science* **6**(1) 80–91.
- De Franceschi, R., M. Fischetti, P. Toth. 2006. A new ILP-based refinement heuristic for vehicle routing problems. *Mathematical Programming* **105**(2) 471–499.
- De Landgraaf, W.A., AE Eiben, V Nannen. 2007. Parameter calibration using meta-algorithms. *IEEE Congress on Evolutionary Computation, CEC 2007*. IEEE, 71–78.
- Dell’Amico, M., F. Maffioli, P. Värbrand. 1995. On prize-collecting tours and the asymmetric travelling salesman problem. *International Transactions in Operational Research* **2**(3) 297–308.
- Derigs, U., R. Kaiser. 2007. Applying the attribute based hill climber heuristic to the vehicle routing problem. *European Journal of Operational Research* **177**(2) 719–732.
- Derigs, U., B. Li, U. Vogel. 2009. Local search-based metaheuristics for the split delivery vehicle routing problem. *Journal of the Operational Research Society* 1356–1364.
- Desaulniers, G. 2010. Branch-and-price-and-cut for the split delivery vehicle routing problem with time windows. *Operations research* **58**(1) 179–192.
- Desaulniers, G., J. Desrosiers, A. Erdmann, M.M. Solomon, F. Soumis. 2002. VRP with pickup and delivery. P. Toth, D. Vigo, eds., *The vehicle routing problem*. SIAM Monographs on Discrete Mathematics and Applications, SIAM, Philadelphia, PA, 225–242.
- Desrochers, J.K., J.K. Lenstra, M.W.P. Savelsbergh. 1990. A classification scheme for vehicle routing and scheduling problems. *European Journal of Operational Research* **46**(3) 322–332.
- Desrochers, M, C.V. Jones, J.K. Lenstra, M.W.P. Savelsbergh, L. Stougie. 1999. Towards a model and algorithm management system for vehicle routing and scheduling problems. *Decision support systems* **25**(2) 109–133.

- Desrosiers, J., Y. Dumas, M.M Solomon, F. Soumis. 1995. Time Constrained Routing and Scheduling. M. Ball, Magnanti, T.L., Monma, C.L., Nemhauser, G.L., eds., *Network Routing, Handbooks in Operations Research and Management Science*, vol. 8. North-Holland, Amsterdam, 35–139.
- Doerner, K., V. Schmid. 2010. Survey: Matheuristics for rich vehicle routing problems. M. Blesa, C. Blum, G. Raidl, A. Roli, M. Sampels, eds., *Hybrid Metaheuristics, Lecture Notes in Computer Science*, vol. 6373. Springer Berlin / Heidelberg, 206–221.
- Doerner, K.F., R.F. Hartl, S. Benkner, M. Lucka. 2006. Parallel cooperative savings based ant colony optimization : multiple search and decomposition approaches. *Parallel processing letters* **16**(3) 351.
- Doerner, K.F., R.F. Hartl, G. Kiechle, M. Lucka, M. Reimann. 2004. Parallel ant systems for the capacitated vehicle routing problem. J. Gottlieb, G. Raidl, eds., *Evolutionary Computation in Combinatorial Optimization, LNCS, 3004*. Springer, 72–83.
- Dongarra, J. 2009. Performance of various computers using standard linear equations software. Tech. rep., University of Tennessee.
- Dorigo, M., T. Stützle. 2004. *Ant colony optimization*. Cambridge, MIT Press.
- Dorronsoro, B., D. Arias, F. Luna, A.J. Nebro, E. Alba. 2007. A grid-based hybrid cellular genetic algorithm for very large scale instances of the CVRP. W. Smari, ed., *High Performance Computing & Simulation Conference (HPCS 2007)*. Prague, Czech Republic, 759–765.
- Dréo, J., A. Pétrowski, P. Siarry, É. Taillard. 2003. *Métaheuristiques pour l'optimisation difficile*. Eyrolles.
- Dueck, G. 1993. New optimization heuristics: The great deluge algorithm and the record-to-record travel. *Journal of Computational Physics* **104**(1) 86–92.
- Duhamel, C., P. Lacomme, A. Quilliot, H. Toussaint. 2011. A multi-start evolutionary local search for the two-dimensional loading capacitated vehicle routing problem. *Computers & Operations Research* **38**(3) 617–640.
- Eksioglu, B., A.V. Vural, A. Reisman. 2009. The vehicle routing problem: A taxonomic review. *Computers & Industrial Engineering* **57**(4) 1472–1483.
- Feillet, D., P. Dejax, M. Gendreau. 2005. Traveling Salesman Problems with Profits. *Transportation Science* **39**(2) 188–205.
- Fisher, M.L., R. Jaikumar. 1981. A generalized assignment heuristic for vehicle routing. *Networks* **11**(2) 109–124.
- Fleischmann, B., M. Gietz, S. Gnutzmann. 2004. Time-Varying Travel Times in Vehicle Routing. *Transportation Science* **38**(2) 160–173.
- Fleszar, K., I. Osman, K. Hindi. 2009. A variable neighbourhood search algorithm for the open vehicle routing problem. *European Journal of Operational Research* **195**(3) 803–809.
- Francis, P. M., K. R. Smilowitz, M. Tzur. 2008. The period vehicle routing problem and its extensions. B. L. Golden, S. Raghavan, E. A. Wasil, eds., *The Vehicle Routing Problem: Latest Advances and New Challenges*. Springer, 73–102.
- Fuellerer, G., K.F. Doerner, R.F. Hartl, M. Iori. 2009. Ant colony optimization for the two-dimensional loading vehicle routing problem. *Computers & Operations Research* **36**(3) 655–673.

- Fuellerer, G., K.F. Doerner, R.F. Hartl, M. Iori. 2010. Metaheuristics for vehicle routing problems with three-dimensional loading constraints. *European Journal of Operational Research* **201**(3) 751–759.
- Fukasawa, R., H. Longo, J. Lysgaard, M.P. Aragão, M. Reis, E. Uchoa, R.F. Werneck. 2006. Robust branch-and-cut-and-price for the capacitated vehicle routing problem. *Mathematical programming* **106**(3) 491–511.
- Funke, B., T. Grünert, S. Irnich. 2005. Local search for vehicle routing and scheduling problems: Review and conceptual integration. *Journal of Heuristics* **11**(4) 267–306.
- Gajpal, Y., P. Abad. 2009. Multi-ant colony system (MACS) for a vehicle routing problem with backhauls. *European Journal of Operational Research* **196**(1) 102–117.
- Gaskell, T.J. 1967. Bases for vehicle fleet scheduling. *Operational Research Quarterly* **18**(3) 281–295.
- Gendreau, M., A. Hertz, G. Laporte. 1992. New insertion and postoptimization procedures for the traveling salesman problem. *Oper. Res.* **40**(6) 1086–1094.
- Gendreau, M, A Hertz, G Laporte. 1994. A tabu search heuristic for the vehicle routing problem. *Management Science* **40**(10) 1276–1290.
- Gendreau, M., G. Laporte, J.-Y. Potvin. 2002. Metaheuristics for the capacitated vrp. P. Toth, D. Vigo, eds., *The vehicle routing problem*. SIAM Monographs on Discrete Mathematics and Applications, SIAM, Philadelphia, PA, 129–154.
- Gendreau, M., J.-Y. Potvin. 2005. Metaheuristics in Combinatorial Optimization. *Annals of Operations Research* **140**(1) 189–213.
- Gendreau, M., J.-Y. Potvin, eds. 2010. *Handbook of Metaheuristics*. Springer.
- Gendreau, M., J.Y. Potvin, O. Bräysy, G. Hasle, A. Løkketangen. 2008. Metaheuristics for the vehicle routing problem and its extensions: A categorized bibliography. B. Golden, S. Raghavan, E. Wasil, eds., *The Vehicle Routing Problem: Latest Advances and New Challenges*. Springer, New York, 143–169.
- Gendreau, M., C.D. Tarantilis. 2010. Solving large-scale vehicle routing problems with time windows: The state-of-the-art. *CIRRELT Working Paper 04* .
- Ghaziri, H. 1996. Supervision in the self-organizing feature map: Application to the vehicle routing problem. I.H. Osman, J.P. Kelly, eds., *Meta-heuristics: Theory & applications*. Kluwer, Boston, 651–660.
- Gillett, B.E., L.R. Miller. 1974. A heuristic algorithm for the vehicle-dispatch problem. *Operations Research* **22**(2) 340–349.
- Glover, F. 1977. Heuristics for integer programming using surrogate constraints. *Decision Sciences* **8** 156–166.
- Glover, F. 1986. Future paths for integer programming and links to artificial intelligence. *Computers & Operations Research* **13**(5) 533–549.
- Glover, F. 1989. Tabu Search – Part I. *ORSA Journal on Computing* **1**(3) 190–206.
- Glover, F. 1990. Tabu Search – Part II. *ORSA Journal on Computing* **2**(1) 4–32.
- Glover, F. 1992. New ejection chain and alternating path methods for traveling salesman problems. O. Balci, R. Sharda, S. Zenios, eds., *Computer Science and Operations Research: New Developments in Their Interfaces*. Pergamon Press, 449–509.
- Glover, F. 1996. Ejection chains, reference structures and alternating path methods for traveling salesman problems. *Discrete Applied Mathematics* **65**(1-3) 223–253.

- Glover, F. 2011. The case for strategic oscillation. *Annals of Operations Research* **183**(1) 163–173.
- Glover, F., M. Laguna. 1998. *Tabu search*. Kluwer Academic Publishers, Boston, MA.
- Glover, F.W., G.A. Kochenberger, eds. 2003. *Handbook of Metaheuristics*. Springer.
- Goel, A. 2010. Truck driver scheduling in the European Union. *Transportation Science* **44**(4) 429–441.
- Goel, A., L. Kok. 2011. Truck driver scheduling in the united states. *Transportation Science, Articles in Advance* .
- Golden, B.L., A.A. Assad. 1988. *Vehicle routing: methods and studies*, vol. 16. Elsevier.
- Golden, B.L., S. Raghavan, E.A. Wasil, eds. 2008. *The Vehicle Routing Problem: Latest Advances and New Challenges*. Springer.
- Golden, B.L., E.A. Wasil, J.P. Kelly, I.M. Chao. 1998. The impact of metaheuristics on solving the vehicle routing problem: algorithms, problem sets, and computational results. T.G. Crainic, G. Laporte, eds., *Fleet management and logistics*. Kluwer Academic Publishers, Boston, Ma, 33–56.
- Graham, R.L., E.L. Lawler, J.K. Lenstra, A.H.G Rinnooy Kan. 1979. Optimization and approximation in deterministic sequencing and scheduling: a survey. *Annals of Discrete Mathematics* **5** 287–326.
- Groër, C., B. Golden. 2011. A Parallel Algorithm for the Vehicle Routing Problem. *INFORMS Journal on Computing* **23**(2) 315–330.
- Gulczynski, D., B. Golden, E. Wasil. 2011. The period vehicle routing problem: New heuristics and real-world variants. *Transportation Research Part E: Logistics and Transportation Review* **47**(5) 648–668.
- Gulczynski, D.J., B. Golden, E. Wasil. 2008. Recent Developments in Modeling and Solving the Split Delivery Vehicle Routing Problem. *2008 Tutorials in Operations Research: State-of-the-Art Decision-Making Tools in the Information-Intensive Age* 170–180.
- Hansen, P., N. Mladenović, J.A. Moreno Pérez. 2010. Variable neighbourhood search: methods and applications. *Annals of Operations Research* **175**(1) 367–407.
- Hashimoto, H., T. Ibaraki, S. Imahori, M. Yagiura. 2006. The vehicle routing problem with flexible time windows and traveling times. *Discrete Applied Mathematics* **154**(16) 2271–2290.
- Hashimoto, H., M. Yagiura. 2008. A Path Relinking Approach with an Adaptive Mechanism to Control Parameters for the Vehicle Routing Problem with Time Windows. *EvoCOP'08 Proceedings of the 8th European conference on Evolutionary computation in combinatorial optimization, LNCS 4972*, vol. 4972. Springer, 254–265.
- Hashimoto, H., M. Yagiura, T. Ibaraki. 2008. An iterated local search algorithm for the time-dependent vehicle routing problem with time windows. *Discrete Optimization* **5**(2) 434–456.
- Hashimoto, H., M. Yagiura, S. Imahori, T. Ibaraki. 2010. Recent progress of local search in handling the time window constraints of the vehicle routing problem. *4OR* **8**(3) 221–238.
- Hemmelmayr, V.C., K.F. Doerner, R.F. Hartl. 2009. A variable neighborhood search heuristic for periodic routing problems. *European Journal of Operational Research* **195**(3) 791–802.
- Hernandez, F., D. Feillet, R. Giroudeau, O. Naud. 2011. A new exact algorithm to solve the Multi-trip vehicle routing problem with time windows and limited duration. Tech. rep., Hal Archives Ouvertes.

- Ho, S.C., M. Gendreau. 2006. Path relinking for the vehicle routing problem. *J. Heuristics* **12**(1-2) 55–72.
- Holland, J.H. 1975. *Adaptation in natural and artificial systems. an introductory analysis with applications to biology, control and artificial intelligence*. Ann Arbor: The University of Michigan Press.
- Ibaraki, T., S. Imahori, M. Kubo, T. Masuda, T. Uno, M. Yagiura. 2005. Effective Local Search Algorithms for Routing and Scheduling Problems with General Time-Window Constraints. *Transportation Science* **39**(2) 206–232.
- Ibaraki, T., S. Imahori, K. Nonobe, K. Sobue, T. Uno, M. Yagiura. 2008. An iterated local search algorithm for the vehicle routing problem with convex time penalty functions. *Discrete Applied Mathematics* **156**(11) 2050–2069.
- Ichoua, S., M. Gendreau, J.Y. Potvin. 2003. Vehicle dispatching with time-dependent travel times. *European Journal of Operational Research* **144**(2) 379–396.
- Imran, A., S. Salhi, N.A. Wassan. 2009. A variable neighborhood-based heuristic for the heterogeneous fleet vehicle routing problem. *European Journal of Operational Research* **197**(2) 509–518.
- Iori, M., S. Martello. 2010. Routing problems with loading constraints. *Top* **18**(1) 4–27.
- Irnich, S. 2008. Resource extension functions: properties, inversion, and generalization to segments. *OR Spectrum* **30** 113–148.
- Irnich, S., B. Funke, T. Grünert. 2006. Sequential search and its application to vehicle-routing problems. *Computers & Operations Research* **33**(8) 2405–2429.
- Jepsen, M., B. Petersen, S. Spoorendonk, D. Pisinger. 2008. Subset-Row Inequalities Applied to the Vehicle-Routing Problem with Time Windows. *Operations Research* **56**(2) 497–511.
- Jin, J., T.G. Crainic, A. Lokketangen. 2010. A parallel multi-neighborhood cooperative tabu search for capacitated vehicle routing problem. Tech. Rep. 54, CIRRELT.
- Johnson, D.S., L.A. McGeoch. 1997. The traveling salesman problem: A case study in local optimization. E.H.L. Aarts, J.K. Lenstra, eds., *Local search in combinatorial optimization*. Princeton Univ Pr, 215–310.
- Jones, T. 1995. Fitness distance correlation as a measure of problem difficulty for genetic algorithms. *Proceedings of the 6th international conference on genetic algorithms*. Morgan Kaufmann, San Francisco, CA, USA, 184–192.
- Kallehauge, B., J. Larsen, O. Madsen. 2006. Lagrangian duality applied to the vehicle routing problem with time windows. *Computers & Operations Research* **33**(5) 1464–1487.
- Ke, L., C. Archetti, Z. Feng. 2008. Ants can solve the team orienteering problem. *Computers & Industrial Engineering* **54**(3) 648–665.
- Kilby, P., P. Prosser, P. Shaw. 1999. Guided local search for the vehicle routing problem with time windows. S. Voss, S. Martello, I.H. Osman, C. Roucairol, eds., *Meta-Heuristics: Advances and Trends in Local Search Paradigms for Optimization*. Kluwer Academic Publishers, 321–361.
- Kindervater, G.A.P., M.W.P. Savelsbergh. 1997. Vehicle routing: Handling edge exchanges. E. Aarts, J.K. Lenstra, eds., *Local Search in Combinatorial Optimization*. Wiley, 337–360.
- Kirkpatrick, S., C.D. Gelatt, M.P. Vecchi. 1983. Optimization by simulated annealing: Quantitative studies. *Science* **220**(4598) 671–680.

- Kubiak, M. 2007. Distance measures and fitness-distance analysis for the capacitated vehicle routing problem. K.F. Doerner, M. Gendreau, P. Greistorfer, W.J. Gutjahr, R.F. Hartl, M. Reimann, eds., *Metaheuristics: Progress in Complex Systems Optimization*. Number 39 in Operations Research/Computer Science Interfaces Series, Springer, New York, 345–364.
- Kytöjoki, J., T. Nuortio, O. Bräysy, M. Gendreau. 2007. An efficient variable neighborhood search heuristic for very large scale vehicle routing problems. *Computers & Operations Research* **34**(9) 2743–2757.
- Laporte, G. 2009. Fifty Years of Vehicle Routing. *Transportation Science* **43**(4) 408–416.
- Laporte, G., I.H. Osman. 1995. Routing problems: A bibliography. *Annals of Operations Research* **61**(1) 227–262.
- Laporte, G., F. Semet. 2002. Classical Heuristics for the Vehicle Routing Problem. Toth, P., Vigo, D., eds., *The Vehicle Routing Problem*. SIAM Monographs on Discrete Mathematics and Applications, SIAM, Philadelphia, PA, 109–128.
- Lau, H.C.W., T.M. Chan, W.T. Tsui, W.K. Pang. 2010. Application of Genetic Algorithms to Solve the Multidepot Vehicle Routing Problem. *IEEE Transactions on Automation Science and Engineering* **7**(2) 383–392.
- Le Bouthillier, A., T.G Crainic. 2005. A Cooperative Parallel Meta-Heuristic for the Vehicle Routing Problem with Time Windows. *Computers & Operations Research* **32**(7) 1685–1708.
- Le Bouthillier, A., T.G Crainic, P. Kropf. 2005. A Guided Cooperative Search for the Vehicle Routing Problem with Time Windows. *IEEE Intelligent Systems* **20**(4) 36–42.
- Lee, C., M. Epelman, C. White III, Y.A. Bozer. 2006. A shortest path approach to the multiple-vehicle routing problem with split pick-ups. *Transportation Research Part B: Methodological* **40**(4) 265–284.
- Letchford, A.N., J. Lysgaard, R.W. Eglese. 2006. A branch-and-cut algorithm for the capacitated open vehicle routing problem. *Journal of the Operational Research Society* **58**(12) 1642–1651.
- Li, F., B. Golden, E. Wasil. 2005. Very large-scale vehicle routing: new test problems, algorithms, and results. *Computers & Operations Research* **32**(5) 1165–1179.
- Li, F., B. Golden, E. Wasil. 2007. The open vehicle routing problem: Algorithms, large-scale test problems, and computational results. *Computers & Operations Research* **34**(10) 2918–2930.
- Lin, S. 1965. Computer solutions of the traveling salesman problem. *Bell System Technical Journal* **44**(10) 2245–2269.
- Lin, S., B.W. Kernighan. 1973. An effective heuristic algorithm for the traveling-salesman problem. *Operations research* **21**(2) 498–516.
- Liu, S., W. Huang, H. Ma. 2009. An effective genetic algorithm for the fleet size and mix vehicle routing problems. *Transportation Research Part E: Logistics and Transportation Review* **45**(3) 434–445.
- Lourenço, H.R., O.C. Martin, T. Stützle. 2010. Iterated Local Search: Framework and Applications. M. Gendreau, J.-Y. Potvin, eds., *Handbook of Metaheuristics, International Series in Operations Research & Management Science*, vol. 146. Springer, Boston, MA, 363–397.
- Malandraki, C., M.S. Daskin. 1992. Time dependent vehicle routing problems: Formulations, properties and heuristic algorithms. *Transportation Science* **26**(3) 185–200.

- Marinakis, Y., M. Marinaki. 2010. A Hybrid Genetic - Particle Swarm Optimization Algorithm for the Vehicle Routing Problem. *Expert Systems with Applications* **37**(2) 1446–1455.
- Marinakis, Y., M. Marinaki. 2011. Bumble bees mating optimization algorithm for the vehicle routing problem. B.K. Panigrahi, Y. Shi, M.-H. Lim, L.M. Hiot, Y.S. Ong, eds., *Handbook of Swarm Intelligence, Adaptation, Learning, and Optimization*, vol. 8. Springer Berlin Heidelberg, 347–369.
- Marinakis, Y., M. Marinaki, G. Dounias. 2010. A hybrid particle swarm optimization algorithm for the vehicle routing problem. *Engineering Applications of Artificial Intelligence* **23**(4) 463–472.
- Marinakis, Y., A. Migdalas, P.M. Pardalos. 2006. A new bilevel formulation for the vehicle routing problem and a solution method using a genetic algorithm. *Journal of Global Optimization* **38**(4) 555–580.
- Masutti, T., L. De Castro. 2008. A neuro-immune algorithm to solve the capacitated vehicle routing problem. Peter Bentley, Doheon Lee, Sungwon Jung, eds., *Artificial Immune Systems, Lecture Notes in Computer Science*, vol. 5132. Springer Berlin / Heidelberg, 210–219.
- Mester, D., O. Bräysy. 2007. Active-guided evolution strategies for large-scale capacitated vehicle routing problems. *Computers & Operations Research* **34**(10) 2964–2975.
- Mingozzi, A., S. Giorgi, R. Baldacci. 1999. An exact method for the vehicle routing problem with backhauls. *Transportation Science* **33**(3) 315–329.
- Mladenović, N., P. Hansen. 1997. Variable neighborhood search. *Computers & Operations Research* **24**(11) 1097–1100.
- Moscato, P. 1989. On Evolution, Search, Optimization, Genetic Algorithms and Martial Arts: Towards Memetic Algorithms. Tech. rep., California Institute of Technology, Pasadena, Calif.
- Moscato, P., C. Cotta. 2010. A modern introduction to memetic algorithms. M. Gendreau, J.-Y. Potvin, eds., *Handbook of Metaheuristics*. Springer, 141–183.
- Mota, E., V. Campos, Á. Corberán. 2007. A new metaheuristic for the vehicle routing problem with split demands. *Lecture Notes in Computer Science* **4446** 121–129.
- Mühlenbein, H., M. Gorges-Schleuter, O. Krämer. 1988. Evolution algorithms in combinatorial optimization. *Parallel Computing* **7**(1) 65–85.
- Nagata, Y., O. Bräysy. 2008. Efficient local search limitation strategies for vehicle routing problems. *EvoCOP'08 Proceedings of the 8th European conference on Evolutionary computation in combinatorial optimization*. Springer, 48–60.
- Nagata, Y., O. Bräysy. 2009. Edge Assembly based Memetic Algorithm for the Capacitated Vehicle Routing Problem. *Networks* **54**(4) 205–215.
- Nagata, Y., O. Bräysy, W. Dullaert. 2010. A penalty-based edge assembly memetic algorithm for the vehicle routing problem with time windows. *Computers & Operations Research* **37**(4) 724–737.
- Nagata, Y., S. Kobayashi. 2011. A memetic algorithm for the pickup and delivery problem with time windows using selective route exchange crossover. *Parallel Problem Solving from Nature, PPSN XI, LNCS* **6238** 536–545.
- Nannen, V, AE Eiben. 2007. Efficient relevance estimation and value calibration of evolutionary algorithm parameters. *Evolutionary Computation, 2007. CEC 2007. IEEE Congress on*, vol. pages. IEEE, 103–110.

- Newton, R.M., W.H. Thomas. 1974. Bus routing in a multi-school system. *Computers & Operations Research* **1**(2) 213–222.
- Ngueveu, S.U., C. Prins, R. Wolfler Calvo. 2010. An effective memetic algorithm for the cumulative capacitated vehicle routing problem. *Computers & Operations Research* **37**(11) 1877–1885.
- Olivera, A., O. Viera. 2007. Adaptive memory programming for the vehicle routing problem with multiple trips. *Computers & Operations Research* **34**(1) 28–47.
- Ombuki-Berman, B., T. Hanshar. 2009. Using genetic algorithms for multi-depot vehicle routing. F. B. Pereira, J. Tavares, eds., *Bio-inspired Algorithms for the Vehicle Routing Problem*. Springer, 77–99.
- Or, I. 1976. Traveling salesman-type combinatorial problems and their relation to the logistics of regional blood banking. Ph.D. thesis, Northwestern University, Evanston, IL.
- Osman, I.H. 1993. Metastrategy simulated annealing and tabu search algorithms for the vehicle routing problem. *Annals of Operations Research* **41**(1-4) 421–451.
- Osman, I.H., G. Laporte. 1996. Metaheuristics: A bibliography. *Annals of Operations Research* **63**(5) 511–623.
- Ostertag, A. 2008. Decomposition strategies for large scale multi depot vehicle routing problems. Ph.D. thesis, Universität Wien.
- Parragh, S.N., K.F. Doerner, R.F. Hartl. 2008a. A survey on pickup and delivery problems Part I: Transportation between customers and depot. *Journal für Betriebswirtschaft* **58**(1) 21–51.
- Parragh, S.N., K.F. Doerner, R.F. Hartl. 2008b. A survey on pickup and delivery problems Part II: Transportation between pickup and delivery locations. *Journal für Betriebswirtschaft* **58**(2) 81–117.
- Parragh, S.N., K.F. Doerner, R.F. Hartl. 2010. Variable neighborhood search for the dial-a-ride problem. *Computers & Operations Research* **37**(6) 1129–1138.
- Penna, P.H.V., A. Subramanian, L.S. Ochi. 2011. An Iterated Local Search heuristic for the Heterogeneous Fleet Vehicle Routing Problem. *Journal of Heuristics, forthcoming*.
- Perboli, G., F. Pezzella, R. Tadei. 2008. EVE-OPT: a hybrid algorithm for the capacitated vehicle routing problem. *Mathematical Methods of Operations Research* **68**(2) 361–382.
- Pisinger, D., S. Ropke. 2007. A general heuristic for vehicle routing problems. *Computers & Operations Research* **34**(8) 2403–2435.
- Pisinger, D., S. Ropke. 2010. Large neighborhood search. M. Gendreau, J.-Y. Potvin, eds., *Handbook of Metaheuristics*. Springer, 399–419.
- Potvin, J.-Y. 2009. State-of-the Art Review : Evolutionary Algorithms for Vehicle Routing. *INFORMS J. Comput* **21**(4) 518–548.
- Potvin, J.-Y., J.-M. Rousseau. 1995. An Exchange Heuristic for Routing Problems with Time Windows. *The Journal of the Operational Research Society* **46**(12) 1433–1446.
- Prescott-Gagnon, E., G. Desaulniers, M. Drexl, L.-M. Rousseau. 2010. European Driver Rules in Vehicle Routing with Time Windows. *Transportation Science* **44**(4) 455–473.
- Prescott-Gagnon, E., G. Desaulniers, L.M. Rousseau. 2009. A branch-and-price-based large neighborhood search algorithm for the vehicle routing problem with time windows. *Networks* **54**(4) 190–204.

- Prins, C. 2004. A Simple and Effective Evolutionary Algorithm for the Vehicle Routing Problem. *Computers & Operations Research* **31**(12) 1985–2002.
- Prins, C. 2009a. A GRASP - evolutionary local search hybrid for the vehicle routing problem. F.B. Pereira, J. Tavares, eds., *Bio-Inspired Algorithms for the Vehicle Routing Problem*. Springer-Verlag Berlin Heidelberg, 35–53.
- Prins, C. 2009b. Two memetic algorithms for heterogeneous fleet vehicle routing problems. *Engineering Applications of Artificial Intelligence* **22**(6) 916–928.
- Raidl, R., J. Puchinger, C. Blum. 2010. Metaheuristic hybrids. M. Gendreau, J.-Y. Potvin, eds., *Handbook of Metaheuristics*. Springer, 469–496.
- Rancourt, M.È., J.-F. Cordeau, G. Laporte. 2010. Long-Haul Vehicle Routing and Scheduling with Working Hour Rules. Tech. rep., CIRRELT.
- Rego, C. 2001. Node-ejection chains for the vehicle routing problem: Sequential and parallel algorithms. *Parallel Computing* **27**(3) 201–222.
- Reimann, M., K.F. Doerner, R.F. Hartl. 2004. D-Ants: Savings based ants divide and conquer the vehicle routing problem. *Comput. Oper. Res* **31** 563–591.
- Repoussis, P.P., C.D. Tarantilis, O. Bräysy, G. Ioannou. 2010. A hybrid evolution strategy for the open vehicle routing problem. *Computers & Operations Research* **37**(3) 443–455.
- Repoussis, P.P., C.D. Tarantilis, G. Ioannou. 2009. Arc-Guided Evolutionary Algorithm for the Vehicle Routing Problem With Time Windows. *IEEE Transactions on Evolutionary Computation* **13**(3) 624–647.
- Resende, M.G.C., C.C. Ribeiro, F. Glover, R. Marti. 2010. Scatter search and path-relinking: Fundamentals, advances, and applications. M. Gendreau, J.-Y. Potvin, eds., *Handbook of Metaheuristics*. International Series in Operations Research & Management Science, Springer US, Boston, MA, 87–107.
- Ribeiro, G.M., G. Laporte. 2012. An Adaptive Large Neighborhood Search Heuristic for the Cumulative Capacitated Vehicle Routing Problem. *Computers & Operations Research* **39**(3) 728–735.
- Rochat, Y., E.D. Taillard. 1995. Probabilistic Diversification and Intensification in Local Search for Vehicle Routing. *Journal of Heuristics* **1**(1) 147–167.
- Ronen, D. 1988. Perspectives on practical aspects of truck routing and scheduling. *European Journal of Operational Research* **35**(2) 137–145.
- Ropke, S., J.F. Cordeau, G. Laporte. 2007. Models and branch-and-cut algorithms for pickup and delivery problems with time windows. *Networks* **49**(4) 258–272.
- Ropke, S., D. Pisinger. 2006. A unified heuristic for a large class of Vehicle Routing Problems with Backhauls. *European Journal of Operational Research* **171**(3) 750–775.
- Salari, M., P. Toth, A. Tramontani. 2010. An ILP improvement procedure for the Open Vehicle Routing Problem. *Computers & Operations Research* **37**(12) 2106–2120.
- Salhi, S., R. J. Petch. 2007. A GA Based Heuristic for the Vehicle Routing Problem with Multiple Trips. *Journal of Mathematical Modelling and Algorithms* **6**(4) 591–613.
- Sarvanov, V.I., N.N. Doroshko. 1981. The approximate solution of the travelling salesman problem by a local algorithm with scanning neighborhoods of factorial cardinality in cubic time (in Russian). *Software: Algorithms and Programs 31, Mathematical Institute of the Belorussian Academy of Sciences, Minsk* 11–13.
- Savelsbergh, M.W.P. 1985. Local Search in Routing Problems with Time Windows. *Annals of Operations Research* **4**(1) 285–305.

- Savelsbergh, M.W.P. 1992. The Vehicle Routing Problem with Time Windows: Minimizing Route Duration. *ORSA Journal on Computing* **4**(2) 146–154.
- Schulz, C. 2011. Efficient Local Search on the GPU Investigations on the Vehicle Routing Problem. *SINTEF Report 2011-05-24* .
- Shaw, P. 1998. Using constraint programming and local search methods to solve vehicle routing problems. M. Maher, J.-F. Puget, eds., *Principles and Practice of Constraint Programming - CP98, Lecture Notes in Computer Science*, vol. 1520. Springer Berlin / Heidelberg, 417–431.
- Sörensen, K, M Sevaux. 2006. MAPM: memetic algorithms with population management. *Computers & Operations Research* **33**(5) 1214–1225.
- Souffriau, W., P. Vansteenwegen, G. Vanden Berghe, D. Van Oudheusden. 2010. A Path Relinking approach for the Team Orienteering Problem. *Computers & Operations Research* **37**(11) 1853–1859.
- Taillard, E. 1993. Parallel iterative search methods for vehicle routing problems. *Networks* **23**(8) 661–673.
- Taillard, E., P. Badeau, M. Gendreau, F. Guertin, J.-Y. Potvin. 1997. A tabu search heuristic for the vehicle routing problem with soft time windows. *Transportation Science* **31**(2) 170–186.
- Tarantilis, C.D. 2005. Solving the vehicle routing problem with adaptive memory programming methodology. *Computers & Operations Research* **32**(9) 2309–2327.
- Tarantilis, C.D., E.E. Zachariadis, C.T. Kiranoudis. 2007. A guided tabu search for the heterogeneous vehicle routing problem. *Journal of the Operational Research Society* **59**(12) 1659–1673.
- Thompson, P.M., H.N. Psaraftis. 1993. Cyclic transfer algorithms for multivehicle routing and scheduling problems. *Operations research* **41**(5) 935–946.
- Toth, P. 2008. An integer linear programming local search for capacitated vehicle routing problems. Bruce Golden, S. Raghavan, Edward Wasil, eds., *The vehicle routing problem: latest advances and new challenges*. Operations Research/Computer Science Interfaces, Springer US, Boston, MA, 275–295.
- Toth, P., D. Vigo. 1997. An exact algorithm for the vehicle routing problem with backhauls. *Transportation Science* **31**(4) 372–385.
- Toth, P., D. Vigo. 2002a. VRP with backhauls. P. Toth, D. Vigo, eds., *The vehicle routing problem*. SIAM Monographs on Discrete Mathematics and Applications, SIAM, Philadelphia, PA, 195–224.
- Toth, P., D. Vigo, eds. 2002b. *The Vehicle Routing Problem*. SIAM Monographs on Discrete Mathematics and Applications, SIAM, Philadelphia, PA.
- Toth, P., D. Vigo. 2003. The Granular Tabu Search and Its Application to the Vehicle-Routing Problem. *INFORMS Journal on Computing* **15**(4) 333–346.
- Toulouse, M., T.G. Crainic, M. Gendreau. 1996. Communication Issues in Designing Cooperative Multi Thread Parallel Searches. I.H. Osman, J.P. Kelly, eds., *Meta-Heuristics: Theory & Applications*. Kluwer Academic Publishers, Norwell, MA, 501–522.
- Tricoire, F., K.F. Doerner, R.F. Hartl, M. Iori. 2009. Heuristic and exact algorithms for the multi-pile vehicle routing problem. *OR Spectrum* 1–29.

- Vakhutinsky, A.I., B.L. Golden. 1994. Solving vehicle routing problems using elastic nets. *Proceedings of 1994 IEEE International Conference on Neural Networks (ICNN'94)* **7**(2) 4535–4540.
- Van Breedam, A. 1995. Improvement heuristics for the vehicle routing problem based on simulated annealing. *European Journal of Operational Research* **86**(3) 480–490.
- Vansteenwegen, P., W. Souffriau, D.V. Oudheusden. 2010. The orienteering problem: A survey. *European Journal of Operational Research* **209**(1) 1–10.
- Černý, V. 1985. Thermodynamical approach to the traveling salesman problem: An efficient simulation algorithm. *Journal of optimization theory and applications* **45**(1) 41–51.
- Vidal, T., T.G. Crainic, M. Gendreau, N. Lahrichi, W. Rei. 2011a. A Hybrid Genetic Algorithm for Multi-Depot and Periodic Vehicle Routing Problems. *Operations Research, Forthcoming* .
- Vidal, T., T.G. Crainic, M. Gendreau, C. Prins. 2011b. A hybrid genetic algorithm with adaptive diversity management for a large class of vehicle routing problems with time-windows. Tech. Rep. 61, CIRRELT.
- Vidal, T., T.G. Crainic, M. Gendreau, C. Prins. 2011c. A unifying view on timing problems and algorithms. Tech. Rep. 43, CIRRELT.
- Voudouris, C., E. Tsang. 1999. Guided local search and its application to the traveling salesman problem. *European journal of operational research* **113**(2) 469–499.
- Whittley, I.M., G.D. Smith. 2004. The attribute based hill climber. *Journal of Mathematical Modelling and Algorithms* **3**(2) 167–178.
- Yellow, P.C. 1970. A Computational Modification to the Savings Method of Vehicle Scheduling. *Operational Research Quarterly* **21**(2) 281–283.
- Yu, B., Z. Yang, B. Yao. 2009. An improved ant colony optimization for vehicle routing problem. *European Journal of Operational Research* **196**(1) 171–176.
- Zachariadis, E.E., C.T. Kiranoudis. 2010a. A strategy for reducing the computational complexity of local search-based methods for the vehicle routing problem. *Computers & Operations Research* **37**(12) 2089–2105.
- Zachariadis, E.E., C.T. Kiranoudis. 2010b. An open vehicle routing problem metaheuristic for examining wide solution neighborhoods. *Computers & Operations Research* **37**(4) 712–723.
- Zachariadis, E.E., C.T. Kiranoudis. 2011. An effective local search approach for the Vehicle Routing Problem with Backhauls. *Expert Systems with Applications, in Press* .
- Zachariadis, E.E., C.D. Tarantilis, C.T. Kiranoudis. 2009. A Guided Tabu Search for the Vehicle Routing Problem with two-dimensional loading constraints. *European Journal of Operational Research* **195**(3) 729–743.