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Abstract. Practical vehicle routing settings bring forth a variety of problem attributes to traduce customers, vehicles, drivers, network specificities and needs. The resulting multi-attribute vehicle routing problems (MAVRP) have been the subject of an abundant literature in recent years. Yet, because of the multiplicity of possible attribute combinations, existing algorithms are frequently unable to treat emerging applications, and the design of a novel problem-tailored method is a long-winded process, often inadequate for the industry. This paper contributes towards addressing this challenge with a new general-purpose metaheuristic for vehicle routing problems. The proposed Unified Hybrid Genetic Search (UHGS) uses efficient generic local search, genetic operators, *Split* algorithm, and advanced diversity management methods, which are independent of the problem variant. These latter procedures, instead, rely on adaptive assignment, sequencing, and route evaluation components to make the interface with problem specific knowledge. Extensive computational experiments on 26 different MAVRPs and 39 benchmarks demonstrate the remarkable performance of the method, which outperforms the current state-of-the-art problem-tailored methods, thus revealing that generality does not necessarily alters efficiency for the considered settings.

Keywords: Vehicle routing, general-purpose solver, component-based design.

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1 Introduction

General-purpose solvers for combinatorial optimization are algorithms that can be re-used to address a large class of problems without requiring an extensive user involvement and expertise. Such tools are necessary for the timely application of current optimization techniques in industry. However, high generality is often paid in terms of computational efficiency, and current general-purpose solvers may not be sufficient to efficiently address some sub-classes of optimization problems without extensive customization work.

When considering the Vehicle Routing Problem (VRP) and its variants (i.e., the problem of designing least cost vehicle routes to service geographically dispersed customers), the challenges related to generic resolution take all their sense. Practical VRP models, indeed, exhibit many structurally different constraints and characteristics, called *attributes* in Vidal et al. (2012b), to model the specificities of practical applications and better integrate routing optimization in the decision chain. This variety of attributes leads to a wide collection of Multi-Attribute Vehicle Routing Problems (MAVRP), some of which, especially the *rich* ones combining many attributes, being very challenging.

At present, the stand-alone application of general-purpose integer programming or constraint programming solvers does not allow to efficiently address large-scale instances of many MAVRPs. Thus, some research has been focused on producing unified vehicle routing solvers, usually local search-based metaheuristics (Cordeau et al. 1997, Ropke and Pisinger 2006a,b, Kritzinger et al. 2012, Subramanian 2012), which have the potential to address a variety of variants. Current methods of this kind generally address a *rich* VRP formulation including several MAVRPs as special cases, or require additional development to be adapted. These methods are limited in the classes, the properties and the number of attributes they manage. When dealing with subproblems, they may also suffer from superfluous computations due to reminiscent attributes from the rich formulation. Building a general-purpose MAVRP solver which is both efficient in terms of solution quality and computational effort thus remains a major research challenge.

To address this challenge, we propose a MAVRP heuristic design based on polymorphic components, and rely on it to propose a Unified local search and Hybrid Genetic Search (UHGS) for MAVRPs. The unified local search addresses a large subset of vehicle routing variants while still allowing for state-of-the-art move evaluations. It relies on a library of problem-specific *route evaluation operators*, automatically selected and combined relatively to the attributes of the problem, to perform data preprocessing on partial routes and achieve efficient route and move evaluations. This local search constitutes a main building block of the proposed unified hybrid genetic local search. UHGS exploits a giant-tour solution representation with a unified *Split* procedure (Prins 2004) to optimally position the depot visits, relies on route constraint relaxations and a local search-based *education* of individuals, and considers the diversity contribution as a proper component of fitness to enhance exploration.

The contributions of this work are the following: 1) A component-based heuristic design is proposed to address the wide family of vehicle routing problems. 2) A unified local search is built on these principles, whose computational complexity matches the state-of-the-art approaches on a wide range of MAVRPs despite its high generality. 3) A unified solution representation, *Split* algorithm, and genetic operators are introduced. 4) A new general-purpose Hybrid Genetic Search for MAVRPs is built from these components, which addresses a large set of variants in a single implementation and with a single parameter set. Extensive computational experiments demonstrate the remarkable performance of the proposed approach on the classical VRP as well as on multi-attribute variants with multiple periods, multiple depots, vehicle-site dependencies, soft, multiple, and general time windows, backhauls, cumulative or load-dependent costs, simultaneous pickup and delivery, fleet mix, time dependency, service site choice, driving and working hour regulations, and many of their combinations. For all the 26 different VRP variants and 39 sets of benchmark instances considered, UHGS matches or outperforms state-of-the-art problem-tailored procedures in the literature, thus showing that generality does not alter efficiency for these settings.

This paper is structured as follows. Section 2 states the problem and reviews the main classes of general-purpose MAVRPs solvers as well as the proposed general method design. Section 3 details the unified local search and its route evaluation operators. Section 4 describes the UHGS which is built on top of the generic local search. Finally, computational experiments on a wide range of problems are reported in Section 5 and Section 6 concludes.

2 Problem Statement and General Methodology

Vehicle routing problems have been studied for more than 50 years, serving as support for a vast literature, including numerous surveys (see Gendreau et al. 2008, Andersson et al. 2010, Vidal et al. 2012b among others), books (Toth and Vigo 2002, Golden et al. 2008), and overall more than a thousand dedicated journal articles (Eksioglu et al. 2009). Emphasis on the topic is still growing today, because of its major economic impact, the large difficulty of many settings, and above all because of the considerable variety of attributes combinations encountered in practice.

2.1 Vehicle routing problems, notations and attributes

The classical Capacitated Vehicle Routing Problem (CVRP) can be stated as follows. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a complete undirected graph with $|\mathcal{V}| = n+1$ vertices, vertex $v_0 \in \mathcal{V}$ representing a depot, where a fleet of m identical vehicles with capacity Q is based, the other vertices $v_i \in \mathcal{V}\backslash\{v_0\}$ for $i \in \{1, \dots, n\}$ representing customers characterized by a demand for q_i units of product. Edges $(i, j) \in \mathcal{E}$ illustrate the possibility to travel from a customer v_i to a customer v_j for a cost d_{ij} (assimilated to the distance). The CVRP requires designing up to m cycles (vehicle routes) starting and ending at a depot v_0 in order to service each customer once.

Relatively to the requirements of practical applications, many VRP variants with *attributes* have emerged, aiming at better accounting for customer requirements (e.g., time-dependent service costs, time windows, multiple planning periods), network and vehicle characteristics (multiple depots, congestion, heterogeneous fleet, vehicle-site dependencies), driver's needs (working hour regulations, lunch breaks), or at better integrating the decisions in a longer-term tactical or strategic planning (inventory or location routing). The large variety of practical settings characteristics and VRP attributes is supported by a vast literature, including hundreds of methods specialized in various attribute combinations. For the purpose of conciseness, a detailed presentation of all VRP variants addressed in this paper is outside the scope of this paper. Detailed surveys are provided in Gendreau et al. (2008), Golden et al. (2008), Andersson et al. (2010) and Vidal et al. (2012b).

Following Vidal et al. (2012b), three main categories of attributes are discerned in this paper. ASSIGN attributes are problem specificities which requires additional decisions on the assignment of customers to some globally constrained ASSIGN Attribute Resources (AARs), for example depots, days or vehicle types. SEQ attributes are characteristics of the problem that explicitly impact the nature of the routes, such as backhaul trips, multiple trips, or multi-echelon structures. Finally, EVAL attributes affect the way routes are evaluated. This latter class of attributes encompasses advanced route costs or feasibility evaluations, as well as the eventual optimization of additional decisions on routes (e.g., service dates, waiting time, packing of objects in the truck) when the sequence of visits if fixed. Each family of attribute thus impacts the resolution methodologies in a very different way.

2.2 Unified solution approaches for MAVRPs.

General-purpose solvers for combinatorial optimization, integer programming or constraint programming solvers are frequently insufficient for generating fast and high quality MAVRP solutions. Since high generality is often paid in terms of performance, designing a general-purpose solver dedicated to the wide family of VRP variants appears as an alternative to better exploit the problem structure and achieve higher efficiency. In the following, we distinguish three main approaches for achieving generality: **rich solvers**, **modeling and solution frameworks**, and **component libraries**.

Rich solvers are designed to address a rich VRP which generalizes several MAVRPs associated to subsets of attributes. Several well-known heuristics are included in this category, including the Unified Tabu Search (UTS: Cordeau et al. 1997, 2001, Cordeau and Laporte 2001, 2003, Cordeau et al. 2004), the Adaptive Large Neighborhood Search algorithm (ALNS: Ropke and Pisinger 2006a,b, Pisinger and Ropke 2007), the Iterated Local Searches of Ibaraki et al. (2005, 2008) and Hashimoto et al. (2006, 2008) (ILS), and Subramanian (2012) (ILS-SP), the latter being hybridized with integer programming components, and finally the exact integer programming approach of Baldacci and Mingozzi (2009), Baldacci et al. (2011a,b) (IPSP), based on a set partitioning formulation.

Finally, Hybrid Genetic Algorithms (HGA) with a *giant-tour* solution representation and local search (Prins 2004) have proven their ability in solving many MAVRPs (Labadi et al. 2008, Prins 2009, Ngueveu et al. 2010, Vidal et al. 2012a). In addition, a large class of mixed node and arc routing problem variants has been addressed in Prins and Bouchenoua (2005). Still, no unifying implementation of this class of methods has been proposed up to date. In particular, different MAVRPs lead to different hard-coded implementations of solution representation, crossover, *Split* and local search procedures. Generalizing these procedures to address a larger range of variants is an important challenge which is addressed in this paper.

Table 1 displays the variants addressed by each algorithm, and indicates for each method the largest subset of MAVRPs that was addressed in a single implementation. In the aforementioned methods, the rich VRP formulation may be a periodic VRP with time windows (UTS case), a rich pick-up and delivery problem with time windows (case of ALNS), a VRP with general time windows, time-dependent and flexible travel-times (ILS case), or a heterogeneous pickup-and-delivery problem with time windows (ILS-SP case). However, relying on a rich model results in two main limitations. Firstly, rich problems become more intricate and difficult to solve as the number of compound attributes grows. Secondly, all features of the rich general model are still accounted for when dealing with the sub-problems, leading to wasted computations induced by deactivated attributes and sometimes higher complexity for some algorithm components.

Table 1: Attributes addressed by some well known rich VRP solvers in the literature

Type	**Attribute**	**Acronym**	**UTS**	**ALNS**	**ILS**	**ILS-SP**	**IPSP**	**UHGS**
ASSIGN	Multiple depots	MDVRP	X	X		X	X	X
	Multiple periods	PVRP	X				X	X
	Heterogeneous fleet	HVRP				X	X	X
	Site-dependent	SDVRP	X	X			X	X
	Split deliveries	VRPSD						
SEQ	Multiple trips	MTVRP						
	Pickup & deliveries	VRPPD	X	X			X	
	Backhauls	VRPB		X				X
EVAL	Open	OVRP		X		X		X
	Cumulative	CCVRP						X
	Load-dependent costs	LDVRP						X
	Simultaneous P.&D.	VRPSDP		X		X		X
	Vehicle Fleet Mix	VFMP				X	X	X
	Duration constraints	DurVRP	X		X			X
	Hard TW	VRPTW	X	X	X		X	X
	Soft TW	VRPSTW			X			X
	Multiple TW	VRPMTW			X			X
	General TW	VRPGTW			X			X
	Time-dep. travel time	TDVRP			X			X
	Flexible travel time	VRPFTT			X			
	Lunch breaks	VRPLB						X
	Work hours reg	VRTDSP						X
	Service site choice	GVRP[2]						X

[2] Problem known as "Generalized Vehicle Routing Problem"

Modeling and solution frameworks seek to capture the general properties of the attributes to transform them into machine readable components. The framework of Desaulniers et al. (1998), especially, formulates some classes of attributes by means of resources (e.g., load, distance, time...) which are extended to successive customer visits by means of *resource extension functions (REFs)*, and subject to interval constraints. This framework was applied to formulate various crew scheduling and routing problem variants and solve them efficiently by column generation (Desaulniers et al. 2005).

It is also known that the performance of many heuristics for MAVRPs is directly linked to the capability of evaluating new routes produced during the search. Hence, a large body of literature has been focused on reducing the complexity of route evaluations in presence of difficult EVAL attributes (Savelsbergh 1985, 1992, Garcia 1996, Kindervater and Savelsbergh 1997, Campbell and Savelsbergh 2004). These approaches share the common characteristic that they develop meaningful information on subsequences of successive visits (partial routes) to speed up evaluations of new routes. Using these methodologies, time windows, simultaneous pickups and deliveries, or load-dependent costs attributes can be efficiently managed in the course of local searches, leading to notable gains in computational complexity.

Merging together these two avenues of research, Irnich (2008b) considered both forward and backward extension of resources, and the management of generalized resources extension functions on visits subsequences to perform efficient route evaluations. This extended REF methodology, combined with *sequential search* concepts, led to a unified solution approach (Irnich 2008a). Yet, strong properties on REFs inversion and generalization to segments are required for the framework to apply.

Finally, Puranen (2011) introduced a domain model able to express VRP variants and transform them into a routing metamodel workable by an optimization method. The routing metamodel is based on the concepts of *actors*, *activities*, *resources*, and *capabilities.* It exploits both the concept of resource extension functions, and a generalization called mapping-ordering constraints. The methodology covers the complete resolution process flow, from the domain model, to the routing metamodel and its resolution. However, few computational experiments were presented up to date to demonstrate the capabilities of the approach.

2.3 Proposed component-based framework

As underlined in this review, some unifying methodologies and algorithms have been proposed for multi-attribute VRPs. However, these approaches were limited in the classes, the properties and the number of attributes they manage. Vehicle routing modeling and solutions frameworks especially (Desaulniers et al. 1998, Irnich 2008a,b, Puranen 2011) provide a remarkable formalism for many attributes, but in counterpart require strong properties to be efficiently applied, such as the existence of REFs which are invertible and generalizable to segments.

In this paper, we investigate a different approach towards general-purpose MAVRP resolution based on object-programming concepts and component-based design. Indeed, even a general-purpose solver must ultimately account for the specific attributes, objective, and constraints of the problem. Yet, to achieve higher generality, these problem-specificities can be relegated to restricted components of the algorithm (classes in object-oriented programming) defined relatively to a subset of functionalities. These components should be *polymorphic* (Meyer 1997), meaning that they can be implemented differently relatively to the problem specificities, but their functionalities can always be used in the same manner, independently of the problem, and without requiring the knowledge of what is inside the component. The implementation of components requires a library of basic attribute-dependent operators, out of which the algorithm can automatically select the necessary operators relatively to the problem specification. Furthermore, components can be designed to still leave the possibility to rely on attribute-specific strategies within the operators, opening the way for example to efficient incremental solution evaluation procedures.

It is worth noticing that some related designs have been also used in the combinatorial optimization literature to build general-purpose solvers and software libraries (see Fink and Voss 2003, Cahon et al. 2004, among others), as well as in the design of some hyper-heuristics (Burke et al. 2010) and cooperative methods (Crainic and Toulouse 2010). However, component-based heuristic approaches are rare in the VRP literature (Du and Wu 2001, Groër et al. 2010). Especially, polymorphism has been efficiently used to generate adaptable resolution strategies, i.e. configurable metaheuristics or local search strategies, but the variability of problems still remains a challenge. In addition, although hyper-heuristics and cooperative methods achieve more robust solving by making several basic methods adapt or cooperate, they are still depending upon the availability of multiple elementary problem-tailored methods.

Restricting the scope of the proposed approach to the vehicle routing class enables to better account for problem particularities. Indeed, MAVRPs and several other combinatorial optimization problems present a particular structure combining decisions on assignment (and partitioning), sequencing, and fixed sequence optimization and evaluations (Section 2.1). Three categories of attributes can thus be differentiated relatively to their impact on heuristic resolution: 1) ASSIGN attributes, which change the choice and validation of assignments to general resources (depots, days, vehicle types), 2) SEQ attributes, which change the nature of the network and the sequences, and 3) EVAL attributes, which impact the solution evaluations. To account for these attributes, polymorphic *assignment*, *sequence choice*, and *route evaluation* components can be designed to provide the following functionalities:

- **Assignment:** Select and check the feasibility of customer and route re-assignments to different ASSIGN attribute resources (day, depot, vehicle type...) ;
- **Sequence choice:** Generate neighbor solutions with different sequencing alternatives with regards to SEQ attributes ;
- **Route evaluations:** Evaluate a fixed route and optimize side decisions related to EVAL attributes (timing or loading sub-problems).

As demonstrated in the next Sections, these components can serve as support to build a wide range of general-purpose neighborhood-based or population-based metaheuristics for MAVRPs. Section 3 first describes a unified local search with efficient route evaluation operators designed to address MAVRPs with EVAL attributes, and Section 4 follows with a description of the proposed Unified Hybrid Genetic Search (UHGS) for MAVRPs.

3 Unified Local Search for Vehicle Routing Problems

Designing a general-purpose high-performance local search for MAVRPs is a notable research challenge in itself. Hence, the methodology developed to this end is presented before going further towards the full UHGS framework. The focus is oriented towards a subset of attributes, called *EVAL* attributes (Vidal et al. 2012b), and which main characteristic is that they impact heuristic resolution during the separate evaluation of routes. Examples of such attributes include most loading constraints (simultaneous pickups and deliveries, loading problems), timing aspects (time windows, flexible or time-dependent travel-times, breaks) and complex route cost functions, among others. In the proposed approach, these problem specificities are enclosed in the *route evaluation* components, which are polymorphic problem-dependent elements of the methodology, and provides the basic functionalities for route, move evaluations and feasibility statements. Since high performance is sought, these components were designed to store information on sub-sequences and avoid redundant computations.

3.1 Route evaluation components

The route evaluation components exploit the fact that any local-search move issued from a bounded number of edge exchanges and node relocations can be assimilated to a recombination of a bounded number of sequence of visits from an incumbent solution (Kindervater and Savelsbergh 1997, Vidal et al. 2011b). As illustrated in Figure 1, an inter-route RELOCATE move of a sequence of visits $[\sigma_r(u), \dots, \sigma_r(v)]$ next to a visit $\sigma_{r'}(w)$ yields two recombined routes $\rho = [\sigma_r(1), \dots, \sigma_r(u-1)] \oplus [\sigma_r(v+1), \dots, \sigma_r(|r|)]$ and $\rho' = [\sigma_{r'}(1), \dots, \sigma_{r'}(w)] \oplus [\sigma_r(u), \dots, \sigma_r(v)] \oplus [\sigma_{r'}(w+1), \dots, \sigma_{r'}(|r'|)]$, $\oplus$ denoting the concatenation operator.

We thus introduce in Table 2 five functionalities of route evaluation components. The first three functionalities, called INIT(σ), FORW(σ), and BACK(σ) provide respectively the means to initialize and build the re-optimization information on sequences by forward and backward concatenation of single visits. In a local search, they should be called in a pre-processing phase to build the information on sub-sequences. The evaluation of new sequences made of a concatenation of several sub-sequences is then performed by using an evaluator which takes advantage from the previously developed information of sub-sequences. Two such evaluators,

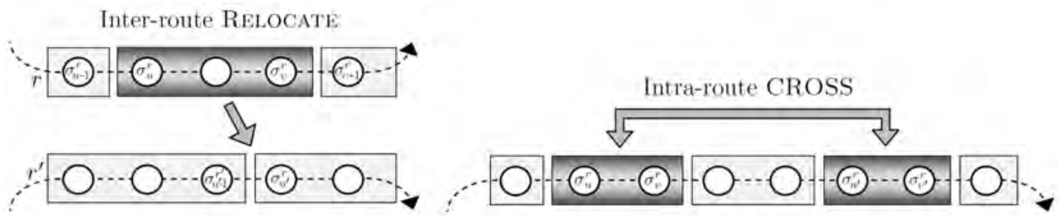

Figure 1: Moves assimilated to recombinations of sequences

EVAL2(σ_1, σ_2) or EVALN($\sigma_1, \ldots, \sigma_n$), are presented. The former considers the concatenation of two segments, while the latter allows for any number of segments. The reasons for designing two different functionalities relate to the fact that all attributes do not allow for an efficient implementation of EVALN, and thus in some well-defined settings the algorithm must exclusively rely on EVAL2 and other construction functionalities to perform route evaluations (Section 3.3).

Table 2: Route evaluation component functionalities

Functionalities for data construction:	
INIT(σ)	Initialize the data $\mathcal{D}(v_0)$ for a sub-sequence containing a single visit.
FORW(σ)	Compute the data of $\mathcal{D}(\sigma \oplus v_i)$ from the data of sub-sequence σ and vertex v_i.
BACK(σ)	Compute the data of $\mathcal{D}(v_i \oplus \sigma)$ from the data of vertex v_i and sub-sequence σ.
Functionalities for route evaluations:	
EVAL2(σ_1, σ_2)	Evaluate the cost and feasibility of the concatenated sequence $\sigma_1 \oplus \sigma_2$.
EVALN($\sigma_1, \ldots, \sigma_n$)	Evaluate the cost and feasibility of the concatenated sequence $\sigma_1 \oplus \cdots \oplus \sigma_n$.

The route evaluation component provides the basis structure to obtain state-of-the-art local search procedures for all $EVAL$ attributes. Its relies on a library of route evaluation operators, specific to each attributes, which are selected automatically by the method relatively to the problem specification. How to implement the route evaluation operators for different attributes is discussed in Section 3.2. A unified local search implementation based on these operators is then presented in Section 3.3.

3.2 Route evaluation operators for several attributes

Route evaluations operators are specific to each attribute, but always respect the five functionality scheme described in Section 3.1.

Three cases of attributes arise. For some VRP attributes, some type of information on subsequences, including the cost characterization among others, is efficiently computable by induction on the concatenation operation, such that a single equation can serve as the basis for all functionalities. Such situation correspond in the framework of Irnich (2008b) to the case of REFs that are invertible and generalizable to segments. Among the MAVRPs that can be managed in this way, we detail here the cases of the VRP with capacity, distance constraints, backhauls, cumulative costs, with hard (eventually multiple) time windows, with simultaneous deliveries and pickups, and with lunch breaks.

For some other VRP attributes, such as soft time windows, time-dependent travel times or fuel optimization, the structure of the re-optimization information is more complex and

Forw(σ) or Back(σ) functions may become more computationally expensive than quick concatenation evaluations. In addition, EvalN functions may not be available in all cases.

Finally, a more advanced role may be given to the route evaluation operator for some MAVRPs. These operators can indeed assume the optimization of additional decisions on visit locations within groups of customers (case of the generalized VRP), explicitly determine the break times placement for drivers (case of VRP with truck driver schedule regulations), or position the objects in the truck (case of VRP with loading constraints). Bi-directional shortest path procedures, tree search methods, or integer programming components are then potentially employed in the implementation. Several important implementations of route evaluation operators are now described.

Capacity and distance. The classical CVRP is perhaps the simplest setting on which information preprocessing is frequently used. Indeed, it is natural to manage for each sub-sequence σ its partial load $Q(\sigma)$ and partial distance $D(\sigma)$ to speed-up the load constraint checks and distance computations. Equations (1) and (2) enable to compute these quantities by induction on the concatenation operator, and provide the means to implement both Forw, Back and EvalN functionalities in $O(1)$ time. It also is worth noting that other globally constrained resources accumulated on arcs or vertices on the routes can be managed in the same way (see Irnich 2008b).

$$Q(\sigma_1 \oplus \sigma_2) = Q(\sigma_1) + Q(\sigma_2) \tag{1}$$

$$D(\sigma_1 \oplus \sigma_2) = D(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + D(\sigma_2) \tag{2}$$

Cumulative costs. The Cumulative VRP (CCVRP) is based on a different objective seeking the minimization of the sum of arrival times to customers. Evaluating the cost of a route subject to some modifications requires more advanced methods than for the classical CVRP, since arrival times to many customers in the route are impacted. Still evaluations remain manageable in amortized $O(1)$ operations for several families of classical local search neighborhoods (Ngueveu et al. 2010). Vidal et al. (2011b) and Silva et al. (2012) show that three types of information on subsequences are sufficient to efficiently evaluate route costs: the duration $D(\sigma)$ to perform the sequence of visits σ, the cumulative cost $C(\sigma)$ when starting at time 0, thus representing the cost of the sequence, and the delay cost $W(\sigma)$ for each unit of time delay in the starting date. For a sequence σ_0 containing a single vertex, the information can be initialized by setting $D(\sigma_0) = 0$ as no travel time is performed, $C(\sigma_0) = 0$, and $W(\sigma_0) = 1$ when the vertex is a customer, otherwise $W(\sigma_0) = 0$. Equations (3-5) then enable to compute this information by induction on the concatenation operation, thus allowing to efficiently implement all route evaluation functions.

$$D(\sigma_1 \oplus \sigma_2) = D(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + D(\sigma_2) \tag{3}$$

$$C(\sigma_1 \oplus \sigma_2) = C(\sigma_1) + W(\sigma_2)(D(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)}) + C(\sigma_2) \tag{4}$$

$$W(\sigma_1 \oplus \sigma_2) = W(\sigma_1) + W(\sigma_2) \tag{5}$$

Load-dependent costs. The fuel consumption f_{ij} of a vehicle is estimated in Xiao et al. (2012) to grow linearly with the load q_{ij} on a segment, and thus $f_{ij} = (f_1 q_{ij} + f_2)d_{ij}$, where f_1 represents the fuel cost per mile and unit of load, and f_2 stands for the base cost per mile. We propose an efficient evaluation of fuel consumption on a route which involves the computation of cumulated demand $Q(\sigma)$, distance $D(\sigma)$, and the load-factor $F(\sigma)$ (load-times-distance) on sequences. The fuel consumption $C(\sigma)$ can be derived from this information since $C(\sigma) = f_1 F(\sigma) + f_2 D(\sigma)$. For a sequence σ_0 containing a single vertex v_i, $Q(\sigma_0) = q_i$, $D(\sigma_0) = 0$, and $F(\sigma_0) = 0$. Furthermore, Equations (6-8) enable to compute these values by induction on larger subsequences, leading to route evaluations in $O(1)$ time.

$$D(\sigma_1 \oplus \sigma_2) = D(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + D(\sigma_2) \tag{6}$$

$$Q(\sigma_1 \oplus \sigma_2) = Q(\sigma_1) + Q(\sigma_2) \tag{7}$$

$$F(\sigma_1 \oplus \sigma_2) = F(\sigma_1) + Q(\sigma_2)(D(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)}) + F(\sigma_2) \tag{8}$$

Backhauls. In the VRP with Backhauls (VRPB), to each customer is either associated a delivery quantity $q_i \neq 0$ of a product, or a pickup quantity $p_i \neq 0$ of a different product. The capacity of the truck is limited to Q product units. Furthermore, a structural route constraint is imposed, pick-up customers being necessarily serviced at the end of the route, after at least one delivery customer. This structural constraint can be modeled directly in the distance matrix by setting $c_{ij} = +\infty$ if vertex v_i corresponds to a pickup customer and v_j is a delivery customer, and by setting the distance from the depot $c_{0j} = +\infty$ for any pickup customer v_j. Evaluating the routes then requires checking the load constraints and summing up the distances. Three types of information are developed on sequences σ to that extent: the partial distance $D(\sigma)$, the total delivery quantity $Q^{\mathrm{D}}(\sigma)$, and the total pickup quantity $Q^{\mathrm{P}}(\sigma)$. Since the two types of products are never jointly in the truck because of structural route constraints, checking load feasibility on a sequence involves simply to check whether $Q^{\mathrm{D}}(\sigma) \leq Q$ and $Q^{\mathrm{P}}(\sigma) \leq Q$. Hence, both $Q^{\mathrm{D}}(\sigma)$ and $Q^{\mathrm{P}}(\sigma)$ can be independently evaluated as previously described in Equation (1) to implement the route evaluation functionalities.

Simultaneous deliveries and pickups. The VRP with simultaneous deliveries and pickups (VRPSDP) also involves two different products to be respectively delivered and picked-up. In contrast with the VRPB, no structural constraint is imposed on the routes, and a vertex can require both a delivery and a pick-up. As the truck can now contain both types of products in the meantime, the load feasibility must be ensured at each vertex of the trip. To that extent, three kinds of data are managed on subsequences: $Q^{\mathrm{D}}(\sigma)$ and $Q^{\mathrm{P}}(\sigma)$ the sum of deliveries and, respectively, pick-ups on the sequence σ, and $Q^{\mathrm{MAX}}(\sigma)$ the maximum load in the truck while processing the sequence σ when starting with an initial load of $Q^{\mathrm{D}}(\sigma)$. These values can be computed by induction on the concatenation operator using Equations (9-11), leading to efficient constant time implementations of the route evaluation functions.

$$Q^{\mathrm{P}}(\sigma_1 \oplus \sigma_2) = Q^{\mathrm{P}}(\sigma_1) + Q^{\mathrm{P}}(\sigma_2) \tag{9}$$

$$Q^{\mathrm{D}}(\sigma_1 \oplus \sigma_2) = Q^{\mathrm{D}}(\sigma_1) + Q^{\mathrm{D}}(\sigma_2) \tag{10}$$

$$Q^{\mathrm{MAX}}(\sigma_1 \oplus \sigma_2) = \max\{Q^{\mathrm{MAX}}(\sigma_1) + Q^{\mathrm{D}}(\sigma_2), Q^{\mathrm{MAX}}(\sigma_2) + Q^{\mathrm{P}}(\sigma_1)\} \tag{11}$$

Another variant of VRPSDP has been addressed in Kindervater and Savelsbergh (1997). In this latter setting, however, a single commodity is considered and products picked-up at a location may be used to service further customers in the route, leading to different equations.

Time windows and duration constraints. The VRP with hard time windows (VRPTW) imposes interval constraints $[e_i, l_i]$ on arrival dates to each customer v_i, as well as service durations s_i (by default $s_0 = 0$). Waiting time is allowed on the route. The VRPTW is the first variant on which information on sub-sequences was managed and exploited (Savelsbergh 1985, 1992, Garcia 1996, Kindervater and Savelsbergh 1997). Previous authors proposed to characterize any sub-sequence with four types of information: a feasibility statement $F(\sigma)$, the sum of travel and service times $T(\sigma)$, the earliest possible completion time for the sequence of visits $E(\sigma)$, and the latest feasible starting date $L(\sigma)$. For a sequence $\sigma_0 = (v_i)$ containing a single vertex, $T(\sigma_0) = s_i$, $E(\sigma_0) = e_i + s_i$, $L(\sigma_0) = l_i$ and $F(\sigma_0) = true$. Equations (12-15) enable then to compute by induction the information for a concatenation of sequences.

$$T(\sigma_1 \oplus \sigma_2) = T(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + T(\sigma_2) \tag{12}$$

$$E(\sigma_1 \oplus \sigma_2) = \max\{E(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + T(\sigma_2), E(\sigma_2)\} \tag{13}$$

$$L(\sigma_1 \oplus \sigma_2) = \min\{L(\sigma_1), L(\sigma_2) - d_{\sigma_1(|\sigma_1|)\sigma_2(1)} - T(\sigma_1)\} \tag{14}$$

$$F(\sigma_1 \oplus \sigma_2) \equiv F(\sigma_1) \ \wedge \ F(\sigma_2) \ \wedge \ (E(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} \leq L(\sigma_2)) \tag{15}$$

When the departure date of the vehicle is not fixed, starting dates have an influence on the total waiting time on the route. The minimum duration for the route, still, can be obtained from the previous information as $DUR(\sigma) = \max\{E(\sigma_i) - L(\sigma_i), T(\sigma_i)\}$. Route evaluations are thus manageable in $O(1)$ time.

Lunch breaks and depot choices. Lunch breaks appear in several practical applications (Sahoo et al. 2005, Bostel et al. 2008), but have been only the focus of moderate attention in the literature. Let the VRPTW with lunch break (VRPTWLB) be defined as a VRPTW variant such that for any non-empty route a single break of duration s_{LB} must be taken between $[e_{\text{LB}}, l_{\text{LB}}]$ at one dedicated location v^{LB} chosen in a set of potential locations $\mathcal{V}^{\text{LB}}$. Let also the variant with flexible break (VRPTWFB) represent the case where the location of the break is unconstrained. As shown in the following, lunch placement choices can be addressed by adequately designing the route evaluation operators, having thus only a minor impact on the general algorithm behavior.

Consider the case of the VRPTWFB. Any sub-sequence σ can be characterized by two sets of information: a data set $T(\sigma)$, $E(\sigma)$, $L(\sigma)$, $F(\sigma)$ which characterizes the time windows as in Equations (12-15) when no break has been taken in the sub-sequence, and another data set $E'(\sigma)$, $L'(\sigma)$, $F'(\sigma)$ which characterizes the case where a break is taken *somewhere* between the first and the last visit of σ. By definition $T'(\sigma) = T(\sigma) + s_{\text{LB}}$ for any σ. Initially, for a sequence $\sigma_0 = (v_i)$ containing a single vertex, $T(\sigma_0) = s_i$, $E(\sigma_0) = e_i + s_i$, $L(\sigma_0) = l_i$ and $F(\sigma_0) = true$. Furthermore, breaks are exclusively taken inside the sequence, and thus a sequence made of a single visit shall not include a break, such that $E'(\sigma_0) = +\infty$, $L'(\sigma_0) = 0$ and $F'(\sigma_0) = false$. Computing $T(\sigma_1 \oplus \sigma_2)$, $E(\sigma_1 \oplus \sigma_2)$, $L(\sigma_1 \oplus \sigma_2)$, $F(\sigma_1 \oplus \sigma_2)$ can be done as previously with Equations (12-15). Computing their counterparts with breaks by induction comes to select a best case out of three: the break is either taken during σ_1 (case 1), between σ_1 and σ_2 (case 2), or during σ_2 (case 3). These computations are displayed in Equations (17-27).

$$E'(\sigma_1 \oplus \sigma_2) = \min(\{E'_{\text{case i}} | F'_{\text{case i}} = true\} \cup +\infty) \tag{16}$$

$$L'(\sigma_1 \oplus \sigma_2) = \max(\{L'_{\text{case i}} | F'_{\text{case i}} = true\} \cup -\infty) \tag{17}$$

$$F'(\sigma_1 \oplus \sigma_2) = F'_{\text{case 1}} \vee F'_{\text{case 2}} \vee F'_{\text{case 3}} \tag{18}$$

$$E'_{\text{case }1} = \max\{E'(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + T(\sigma_2), E(\sigma_2)\} \tag{19}$$
$$E'_{\text{case }2} = \max\{E(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + s_{\text{LB}} + T(\sigma_2), e_{\text{LB}} + s_{\text{LB}} + T(\sigma_2), E(\sigma_2)\} \tag{20}$$
$$E'_{\text{case }3} = \max\{E(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + T'(\sigma_2), E'(\sigma_2)\} \tag{21}$$
$$L'_{\text{case }1} = \min\{L'(\sigma_1), L(\sigma_2) - d_{\sigma_1(|\sigma_1|)\sigma_2(1)} - T'(\sigma_1)\} \tag{22}$$
$$L'_{\text{case }2} = \min\{L(\sigma_1), l_{\text{LB}} - T(\sigma_1), L(\sigma_2) - d_{\sigma_1(|\sigma_1|)\sigma_2(1)} - s_{\text{LB}} - T(\sigma_1)\} \tag{23}$$
$$L'_{\text{case }3} = \min\{L(\sigma_1), L'(\sigma_2) - d_{\sigma_1(|\sigma_1|)\sigma_2(1)} - T(\sigma_1)\} \tag{24}$$
$$F'_{\text{case }1} = F'(\sigma_1) \wedge F(\sigma_2) \wedge (E'(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} \leq L(\sigma_2)) \tag{25}$$
$$F'_{\text{case }2} = F(\sigma_1) \wedge F(\sigma_2) \wedge (E(\sigma_1) \leq l_{\text{LB}}) \wedge (E(\sigma_1) + s_{\text{LB}} + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} \leq L(\sigma_2)) \tag{26}$$
$$F'_{\text{case }3} = F(\sigma_1) \wedge F'(\sigma_2) \wedge (E(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} \leq L'(\sigma_2)) \tag{27}$$

It is worth mentioning that a similar methodology can also be used to adjust dynamically, within the evaluation of the routes, the break location choices in the VRPTWLB case, and even similarly the choice and placement of depot visits in a multi-depot setting. Integrating these decisions in the evaluation operators enables to combine the placement or assignment features within local search moves, and considerably reduce the combinations of choices to be worked out in the remaining parts of the method.

Soft and general time windows. For all previously-mentioned attributes, constant-size characteristic data was available for the segments, as well as general concatenation equations (segment REFs in the terminology of Irnich 2008b). However, several MAVRPs fall outside of this scope. This is the case for the VRP with soft time windows (VRPSTW), which allows penalized late arrival to customer, and more generally for the generalization of the VRPTW where the service cost $c_i(t_i)$ of each customer v_i is a piecewise linear function of the service date. For this latter variant, the placement of departure times and waiting times, and thus the determination of a good schedule for a fixed route makes for a non-trivial *timing problem with separable activity costs* (Vidal et al. 2011b) and no judicious $O(1)$ size data structure is known to characterize the sub-sequences and their exact cost when concatenated together.

In this case, the route evaluation information can be developed as a set of piecewise functions (Hendel and Sourd 2006, Ibaraki et al. 2005, 2008). Each sub-sequence is characterized by a function $F(\sigma)(t)$ representing the minimum cost to service the sequence σ while arriving at the last customer before time t, and $B(\sigma)(t)$ stating the minimum cost of servicing σ after time t.

For a sequence $\sigma_0 = (v_i)$ with a single vertex, $F_{\sigma_0}(t) = \min_{x \leq t} c_i(x)$ and $B_{\sigma_0}(t) = \min_{x \geq t} c_i(x)$. The construction operator Forw relies on forward dynamic programming (Equation 28) to build explicitly the information for the concatenation of a sequence σ with a vertex v_i. In reverse, the functionality Back is based on backward dynamic programming (Equation 29). Equation (30) provides the cost $Z^*(\sigma_1 \oplus \sigma_2)$ of the concatenated sequence $\sigma_1 \oplus \sigma_2$ when $F(\sigma_1)(t)$ and $B(\sigma_2)(t)$ are available, thus leading to an efficient Eval2 functionality.

$$F(\sigma \oplus v_i)(t) = \min_{0 \leq x \leq t}\{c_i(x) + F(\sigma)(x - s_{\sigma(|\sigma|)} - d_{\sigma(|\sigma|),i})\} \tag{28}$$

$$B(v_i \oplus \sigma)(t) = \min_{x \geq t}\{c_i(t) + B(\sigma)(x + s_i + d_{i,\sigma(1)})\} \tag{29}$$

$$Z^*(\sigma_1 \oplus \sigma_2) = \min_{x \geq 0}\{F(\sigma_1)(x) + B(\sigma_2)(x + s_{\sigma_1(|\sigma_1|)} + d_{\sigma_1(|\sigma_1|)\sigma_2(1)})\} \tag{30}$$

In our implementation, the data structures $F(\sigma)(t)$ and $B(\sigma)(t)$ are managed as linked lists of function pieces characterized by interval, origin value and slope. The data construction functionalities $\text{FORW}(\sigma)$, $\text{BACK}(\sigma)$ and EVAL2 work in $O(\Sigma_i \xi(c_i))$ time, $\xi(c_i)$ representing the number of pieces of a piecewise cost function c_i. However the EVALN functionality is not efficiently manageable. In the particular case where all functions $c_i(t)$ are convex, more advanced implementations based either on heaps (Hendel and Sourd 2006), or on search trees (Ibaraki et al. 2008), achieve a complexity of $O(\log \Sigma_i \xi(c_i))$ for both EVAL2 and EVALN.

Other time features. The literature contains various other EVAL attributes related to time, such as duration constraints, multiple time windows, time-dependent trip durations, flexible travel times, minimum and maximum intervals of time between pairs of services, among others. We refer to Vidal et al. (2011b) for a comprehensive review and analysis of state-of-the-art algorithms for the underlying *timing* sub-problems for route evaluations, and their incremental resolution during local searches. These approaches were used to implement UHGS route evaluation operators for the related problems with time characteristics.

Service site choices. In the Generalized Vehicle Routing Problem (GVRP), each request v_i is associated to a set of λ_i alternative locations $L_i = \{l_{i1}, \dots, l_{i|\lambda_i|}\}$. Exactly one location of each set must be serviced. As illustrated in Baldacci et al. (2009), the GVRP is relevant for several practical applications and directly generalizes other variants of vehicle routing.

Recent efficient heuristics for this problem (Moccia et al. 2012) include local-search procedures on the order of services which do not consider explicitly in the neighborhoods the locations to be serviced. In this scope, evaluating a sequence of services comes to find the associated shortest sequence of visits to locations, leading to a shortest path problem.

Again, efficient route evaluations require to store for each sequence of services adequate data to speed-up the shortest path computation. In this case, the information to be stored for a sequence σ is the shortest path $S(\sigma)[i,j]$ between the i^{th} location of $\sigma(1)$ and the j^{th} location of $\sigma(|\sigma|)$, where $i \in \{1, \dots, \lambda_{\sigma(1)}\}$ and $j \in \{1, \dots, \lambda_{\sigma(|\sigma|)}\}$. For a sequence $\sigma_0 = (v_i)$ containing a single service, $S(\sigma_0)[x,x] = 0$ for any $x \in \{1, \dots, \lambda_i\}$ and $S(\sigma_0)[x,y] = +\infty$ if $x \neq y$. Equation (31) enables then to develop this information on larger subsequences by induction on the concatenation operator.

$$\begin{aligned} S(\sigma_1 \oplus \sigma_2)[i,j] = \min_{1 \leq x \leq \lambda_{\sigma_1(|\sigma_1|)}, 1 \leq y \leq \lambda_{\sigma_2(1)}} & S(\sigma_1)[i,x] + d_{xy} + S(\sigma_2)[y,j] \\ & \forall i \in \{1, \dots, \lambda_{\sigma_1(1)}\}, \forall j \in \{1, \dots, \lambda_{\sigma_2(|\sigma_2|)}\} \end{aligned} \tag{31}$$

Equation (31) provides the means to implement efficiently in $O(\lambda^2)$ operations all route evaluation functionalities, λ standing for the maximum number of locations associated to a service. This complexity is notably better than the complexity of computing each shortest path from scratch, which would be $O(n_r \lambda^2)$ operations for a route containing n_r services.

Hours of service regulations. Governments worldwide impose complex regulations on truck drivers schedules to limit the amount of work and driving within intervals of time and impose a minimum frequency and duration for break and rest periods. Because of their large impact on driving times, these regulations should be accounted for when optimizing the routes, leading to combined vehicle routing and truck driver scheduling problems (VRTDSP). However, even checking the existence of a feasible placement of breaks for a fixed sequence of visits makes for a highly complex problem which is known to be solvable in a quadratic time for United States hours of service regulations (Goel 2011), while in other cases, with European Union, Canadian, or Australian rules, no polynomial algorithm is known up to this date.

Despite this high complexity, most efficient methods for the VRTDSP integrate break scheduling feasibility checks directly in the local search (Prescott-Gagnon et al. 2010, Goel and Vidal 2012), and thus during each route evaluation. In UHGS, these break scheduling procedures are implemented in the route evaluation operators. A set of schedule alternatives is maintained for each subsequence of consecutive visits. The schedule information is extended to larger subsequences by appending new driving and break activities at the end of the schedules, and selecting only a relevant subset by means of dominance relationships. The current implementation is exclusively based on forward operators, and thus EVAL2(σ_1,σ_2) is implemented by iteratively completing the schedule of σ_1 with services of σ_2.

Summary. As reviewed in this section, efficient route evaluations operators relatively to different VRP attributes may require to develop radically different information on sequences, and using more or less complex evaluation procedures. Still, all previously-mentioned approaches respect the same five functionalities, based on the forward or backward propagation of labels (or generally of any information to characterize the sequences), and the evaluation of the concatenation of two or more sequences using the information developed on sequences. As shown in the following, this library of route evaluation operators provides the means to implement a general-purpose state-of-the-art local search for many MAVRPs.

3.3 Unified local search procedure

The five functionalities of the route evaluation component can be used to build any local search procedure with moves involving a bounded number of edge exchanges and node relocations, since all these moves can be evaluated as a recombination of partial subsequences from the incumbent solution. This general process is illustrated in Algorithm 1. To efficiently evaluate the moves, the unified local search manages information on subsequences of consecutive visits (and reverse subsequences in presence of moves that impact the route orientation), using the INIT, FORW and BACK route construction functionalities. This information is built during a pre-processing phase at the beginning of the local search, and is then updated whenever any route is modified. Moves are then evaluated by means of the EVAL2 and EVALN functions.

Algorithm 1 Unified local search based on route evaluation operators

```
Detect the good combination of evaluation operators relatively to the problem attributes
Build re-optimization data on subsequences using the INIT, FORW and BACK operators.
while some improving moves exist in the neighborhood N do
   for each move μ_i in N do
      for each route r_j^μ produced by the move do
         Determine the k sub-sequences [σ_1, ..., σ_k] that are concatenated to produce r_j^μ
         if k = 2, then NEWCOST(r) = EVAL2(σ_1,σ_2)
         else if k > 2, then NEWCOST(r) = EVALN(σ_1,...,σ_k)
      if ACCEPTCRITERIA(μ_i) then perform the move μ and update the re-optimization
      data on for each route r_j^μ using the INIT, FORW and BACK operators.
```

In the specific implementation of this paper, the neighbor solutions issued from moves are explored in random order, using the acceptance criterion of Vidal et al. (2011a) and terminating whenever no improving move can be found in the whole neighborhood. As in Vidal et al. (2011a), the classical 2-OPT*, and 2-OPT neighborhoods are used, as well as the inter-route and intra-route CROSS and I-CROSS neighborhoods, restricted to subsequences of length smaller than $L_{max} = 2$ and including relocate moves as special cases. Only moves involving *neighbor vertices* is terms of distance and time characteristics (Vidal et al. 2011a) are attempted, leading to a neighborhood size of $O(L^2_{max}\Gamma n)$ instead of $O(L^2_{max}n^2)$ where Γ stands for the number of neighbor vertices per vertex.

It should be noted that all *inter-route* moves such as CROSS, I-CROSS and 2-OPT*, require either EVAL2(σ_1, σ_2) or EVALN$(\sigma_1, \sigma_L, \sigma_2)$ where σ_L is a sequence of size bounded by L_{max}. When no efficient EVALN is available, in presence of attributes such as soft and general time windows for example, this first family of *inter-route* moves can still be implemented efficiently as EVALN$(\sigma_1, \sigma_L, \sigma_2)$ can be replaced by less than L_{max} successive calls to FORW to yield the information on $\sigma' = \sigma_1 \oplus \sigma_L$, with a final call to EVAL2(σ', σ_2). *Intra-route* CROSS and I-CROSS and 2-OPT moves require calling EVALN on a set of 3 to 5 subsequences. If no efficient EVALN is available, the same reasoning for replacement can still be used, but in this case the number of necessary calls to FORW becomes linear in the route size since the size of intermediate subsequences is not bounded. However, since intra-route moves are usually in minority, this increased number of operations did not impact the method speed.

The good combination of data structures and route evaluation operators is automatically determined relatively to the problem attributes according to the component-based framework of Section 2.3, and thus the route evaluation operators allow to use advanced move evaluation techniques which were until now considered as problem-specific in a unified framework for MAVRPs. The resulting unified local search is efficient and applicable to many VRP variants. It can be derived into any generic neighborhood-based metaheuristic such as tabu search, iterated local search, or variable neighborhood search. Relatively to the recent advances with regards to genetic algorithms and diversity management for vehicle routing, we opted to combine this procedure with the approach of Vidal et al. (2012a) to obtain a Unified Hybrid Genetic Search (UHGS). Such integration requires addressing several additional challenges, related to the design of a generic solution representation, genetic operators and population management methods. The next section explains how to address them.

4 Unified Hybrid Genetic Search

The proposed UHGS is an extension of the Hybrid Genetic Search with Advanced Diversity Control of Vidal et al. (2012a), and aims to address MAVRPs in a unified manner by means of component-based design. The method stands out from previous works since all its elements (solution representation, genetic operators, local searches) are fully generic and detached from the attributes of the problem, relying on the subset of polymorphic *assignment* and *route evaluation* components to make the interface with problem-specific knowledge (Section 2.3). Please note that in the scope of this work, only single echelon problems with a route structure as a single sequence have been addressed, thus allowing to rely on a unique *sequencing component* based on standard VRP neighborhoods (Section 3.3). This Section briefly recalls the general behavior of UHGS, then details in turn each element of the unified method.

4.1 General behavior

UHGS combines four main optimization methodologies: 1) an hybridization of genetic algorithms with local search procedures; 2) the use of penalized infeasible solutions, managed into two distinct sub-populations during the search; 3) a solution representation *without trips delimiters* (Prins 2004) with an optimal *Split* procedure for delimiters computation; 4) an advanced population management method with a *diversity and cost objective* for solution evaluation.

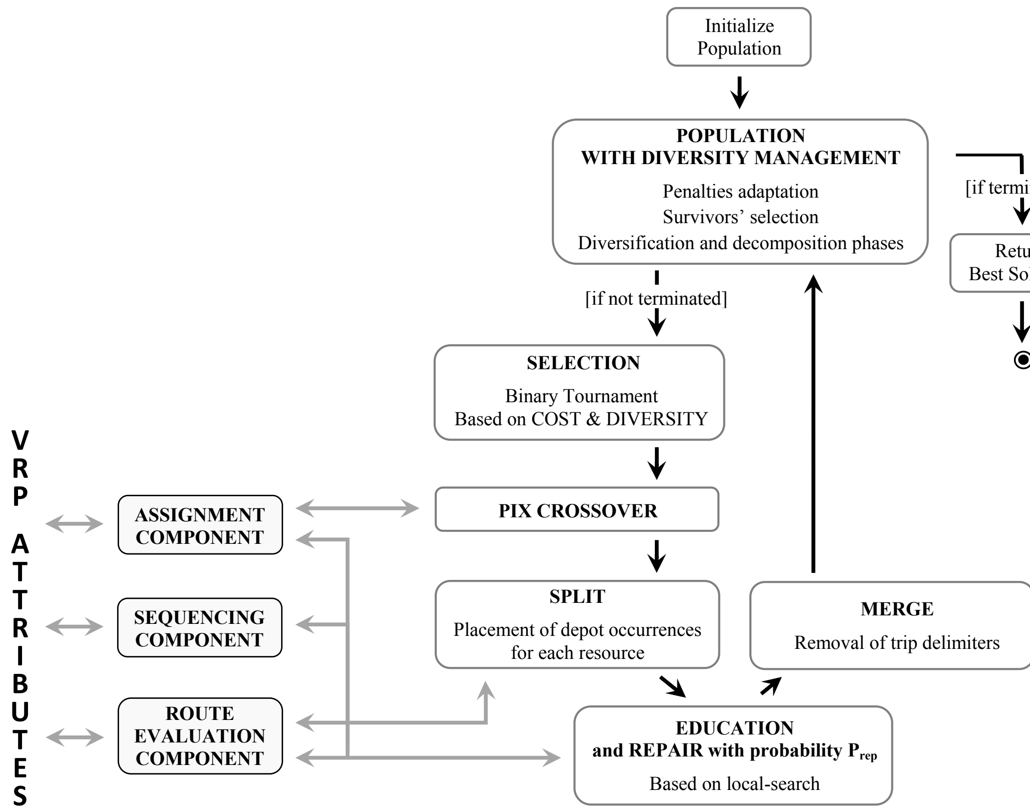

Figure 2: UHGS behavior and relationships with problem attributes

As illustrated in Algorithm 2, UHGS iteratively selects two individuals in the merge of the sub-populations to serve as input of a crossover operator, yielding a single offspring. After going through the *Split* procedure to compute trip delimiters, the offspring is *Educated* by means of a local search, *Repaired* with probability P_{rep} when infeasible, and transfered to the suitable sub-population. Each sub-population is managed separately to trigger a *Survivor Selection* procedure when reaching a maximum size. *Diversification procedures* and *decomposition phases* are regularly used to further enhance the diversity and intensify the search around some elite solution characteristics. The algorithm terminates when It_{max} successive iterations (individual generations) have been performed without improving the best solution, or when a time limit T_{max} is reached.

4.2 Unified solution representation and Split

MAVRPs generally involve two levels of decisions relative to the assignment of customer services to some ASSIGN Attribute Resources (AARs), and the optimization of routes for each AAR. In accordance with this problem structure, solutions are represented in the course of UHGS as a collection of giant tours without explicit mention of visits to the depot (Prins 2004). As illustrated in Figure 3, each giant tour corresponds to a different combination of AAR, for example a (vehicle type/day) couple in a heterogeneous periodic VRP. Problems without ASSIGN attribute lead to only one AAR, and thus to a solution representation as a single giant tour.

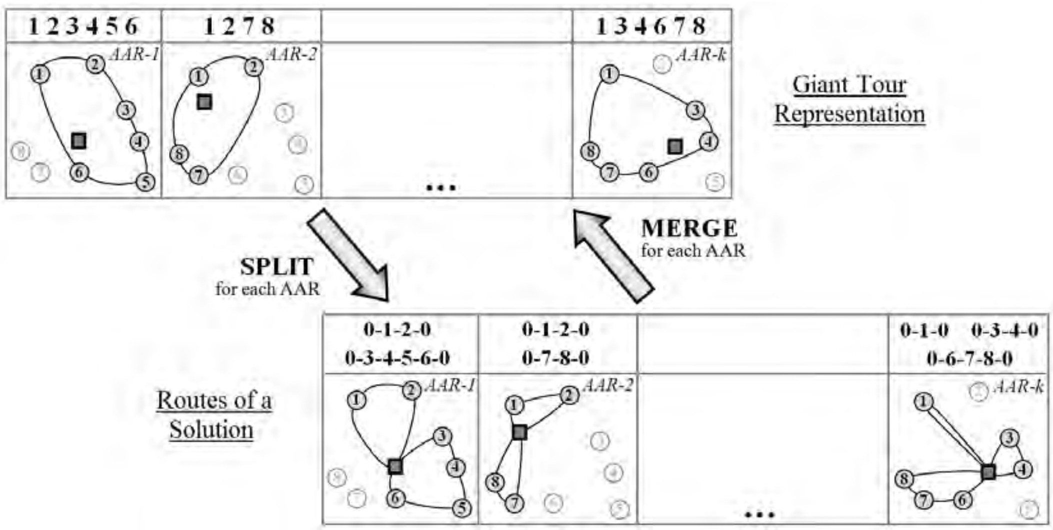

Figure 3: Solution representation as a giant tour per AAR, illustration of the Split procedure and its reverse Merge operation

Not considering trip delimiters in the solution representation allows for simpler crossover procedures such as the ones introduced in Lacomme et al. (2005), Bostel et al. (2008) and Vidal et al. (2012a). On the other hand, a *Split* algorithm must be applied on each giant tour to insert depot visits, to support for solution evaluations and local-search procedures.

A fully generic *Split* procedure for MAVRPs based on route evaluation components is introduced in Algorithm 2. For any giant tour $\tau = (\tau_1, \ldots, \tau_\nu)$ containing ν customers, the splitting problem is assimilated to a shortest path problem on a directed acyclic auxiliary graph $\mathcal{G}' = (\mathcal{V}, \mathcal{A})$, where $\mathcal{V}$ includes $\nu + 1$ nodes notated 0 to ν. Any arc $a_{ij} \in \mathcal{A}$ with $i < j$ represents the route originating from the depot, visiting customers σ_{i+1} to σ_j, and returning. The cost of each arc is set to the cost of the associated route.

All arc costs can be computed by calling $\mathcal{O}(\nu^2)$ times the functionalities FORW and EVAL2 of the route evaluations component (Lines 1-5 of Algorithm 2). Setting a maximum value $\bar{r}$ on the number of customers in a route enables to reduce this number of calls to $\mathcal{O}(\nu\bar{r})$. Once this pre-processing is achieved, the shortest path is solved by means of m iterations of the Bellman-Ford algorithm (see Cormen et al. 2001) in presence of a fleet size limit to m. If no limit on the fleet size is imposed, a shortest path based on the topological order of indexes is used. The final complexity of the proposed unified *Split* algorithm is $\mathcal{O}([m + \xi(\text{FORW}) + \xi(\text{EVAL2})]\nu\bar{r})$

Algorithm 2 Generic Split

for each node $i \in \{1, \ldots, \nu\}$ **do**
  $SeqData(\sigma) = \textsc{Init}(\{v_0\})$ //*Initialize with depot vertex*
  for each node $j \in \{i+1, \ldots, \min(i+\bar{r}, \nu)\}$ **do**
    $SeqData(\sigma) = \textsc{Forw}(\sigma,\{\tau_j\})$ //*Append a new customer to the route end*
    $\phi(a_{ij}) = \textsc{Eval2}(\sigma,\{v_0\})$ //*Evaluate the route*
Solve the shortest path problem on $\mathcal{G}' = (\mathcal{V}, \mathcal{A})$ with cost $\phi(a_{ij})$ for each arc a_{ij}
Return the set of routes associated to the set of arcs of the shortest path

where $\xi(\textsc{Forw})$ and $\xi(\textsc{Eval2})$ represent respectively the complexity of the Forw and Eval2 operators. This algorithm is applicable to all VRP variants mentioned in Section 3.2.

4.3 Individuals Evaluation

Any individual p in UHGS is evaluated relatively to its *feasibility*, *cost*, and *contribution to the population diversity*. Define the *penalized cost* $\phi_{\mathcal{P}}^{\text{COST}}(p)$ of p as the sum on all routes of total distance and penalized excesses relatively to load and other constraint violation of N_{ATT} EVAL attributes. For any route σ with distance $\varphi^{\text{D}}(\sigma)$, load excess $\varphi^{\text{Q}}(\sigma)$, and excesses $\varphi^{\text{E}_i}(\sigma)$ for $i \in \{1, \ldots, N_{\text{ATT}}\}$ relatively to EVAL attributes, the penalized cost $\phi(r)$ is given by Equation (32), where ω^{Q} and ω^{E_i} for $i \in \{1, \ldots, N_{\text{ATT}}\}$ represent the associated penalty coefficients. The set of excesses $\varphi^{\text{E}_i}(\sigma)$ depends upon the EVAL attributes of the problem, and can include the excess of pickup load (variants of VRPB), excess in duration (variants of DurVRP), time-window relaxations in the sense of Nagata et al. (2010) or service lateness (VRTDSP, TDVRP). Penalty coefficients are adapted during the search relatively to the proportion of feasible individuals as in Vidal et al. (2011a, 2012a).

$$\phi(r) = \varphi^{\text{D}}(r) + \omega^{\text{Q}}\varphi^{\text{Q}}(r) + \sum_{i=1}^{N_{\text{ATT}}} \omega^{\text{E}_i}\varphi^{\text{E}_i}(\sigma) \tag{32}$$

Define the *diversity contribution* $\phi_{\mathcal{P}}^{\text{DIV}}(p)$ of an individual p as its average distance with the μ^{CLOSE} most similar individuals in the sub-population. The Hamming distance on assignment decisions is used in presence of ASSIGN attributes, while in the other case the broken pairs distance (Prins 2009) is automatically used to measure the proportion of common edges.

Equation (33) finally states the *biased fitness* $f_{\mathcal{P}}(p)$ of an individual p in the sub-population $\mathcal{P}$ as a weighted sum of its penalized cost rank $f_{\mathcal{P}}^{\text{COST}}(p)$ and its rank $f_{\mathcal{P}}^{\text{DIV}}(p)$ relatively to diversity contribution. This trade-off between diversity and cost is balanced by parameter μ^{ELITE} and was shown to play an essential role in the performance of the method.

$$f_{\mathcal{P}}(p) = f_{\mathcal{P}}^{\text{COST}}(p) + \left(1 - \frac{\mu^{\text{ELITE}}}{|\mathcal{P}|}\right) f_{\mathcal{P}}^{\text{DIV}}(p) \tag{33}$$

4.4 Selection and Crossover

During the course of UHGS, two parent individuals are iteratively selected by binary tournament in the merge of the feasible and infeasible sub-populations to serve as input to the crossover and produce a single offspring. The Assignment and Insertion Crossover (AIX) is then used for problems involving at least one ASSIGN attribute, otherwise the simple Ordered Crossover (OX) is applied (see Prins 2004).

AIX is a direct generalization of the PIX crossover of Vidal et al. (2012a). This crossover first decides for each ASSIGN Attribute Resource (AAR) whether the genetic material of p_1, p_2 or both parents is transmitted. To that extent, two random numbers are first picked between 0 and n_{AAR} according to a uniform distribution. Let n_1 and n_2 be the smallest and the largest of these numbers. For n_1 random AARs, the genetic material will be inherited from p_1 exclusively. For $(n_2 - n_1)$ other random AARs, the material will be inherited from p_2 exclusively. Finally, for the remaining $(n_{\mathrm{AAR}} - n_2)$ AARs the material will be jointly inherited from p_1 and p_2. Such method leaves the possibility of unequal inheritance of genetic material from parents, even in presence of a large number of AARs, and thus provides the means to perform both small solution refinement and complex structural recombinations.

The selected material from p_1 is then fully transmitted, resulting to a partial assignment of customers to AARs which is by definition a subset of a feasible assignment. The method then proceeds by inheriting in turn each selected delivery from p_2 and appending it to the end of the giant tour corresponding to its AAR. At each insertion, the polymorphic *assignment* component (Section 2.3) is called to check whether this inheritance of a customer still allows for completion into a feasible assignment relatively to ASSIGN attributes. If this latter property in not fulfilled, then the delivery is not transmitted into the offspring.

Finally, as the previous constraints can lead to an incomplete offspring, missing visits to customers are inserted in turn in a random order to the best location relatively to the penalized route cost. Good insertion procedures require the knowledge of the depot occurrences, and thus this final step of AIX shall be completed after using the unified *Split* algorithm (Section 4.2).

4.5 Education and Repair

Any offspring issued from the crossover undergoes the *Split* procedure, and is then improved by means of an *Education* operator based on two local searches. Firstly, the local search procedure of Section 3.3 is applied independently for each AAR to perform Route Improvements (RI). Secondly, an Assignment Improvement (AI) procedure is designed to optimize on assignment decisions. Finally, RI is applied a last time.

The AI procedure is the generalization of the *pattern improvement* procedure of Vidal et al. (2012a). AI tentatively removes all services to a customer, and chooses the best combination of insertion locations in AARs for reinsertion. AI relies on the *assignment* component to list the tentative combinations of resource assignments, and evaluates each insertion position by means of the *route evaluation* component. Any insertion is thus assimilated to a call to $\textsc{EvalN}(\sigma_1, \sigma_0, \sigma_2)$ where σ_0 contains a single vertex. Customer re-assignments are exhaustively tried in a random order, the best re-assignment position being systematically chosen. AI stops when no improving re-assignment can be found.

The solutions issued from *Education* are directly accepted in the adequate sub-population relatively to their feasibility. Furthermore, any infeasible solution with penalty (see Section 4.3) is *Repaired* with probability P_{rep}. The *Repair* operator temporarily increases the penalty coefficients by a factor of 10 and calls *Education* to redirect the search towards feasible solutions.

4.6 Population management

Sub-populations are independently managed to always contain between μ^{MIN} and $\mu^{\text{MIN}} + \mu^{\text{GEN}}$ individuals, by triggering a *survivor selection* phase each time a sub-population reaches a maximum size of $\mu^{\text{MIN}} + \mu^{\text{GEN}}$. *Survivor selection* consists in iteratively removing μ^{GEN} times the worst individual with regards to the biased fitness function of Section 4.3, privileging first the removal of *clones* individuals having null distance to at least another individual. To regularly introduce new genetic material, these population management mechanisms are completed by diversification phases (Vidal et al. 2012a, 2011a) which take place after each It_{div} successive iterations without improvement of the best solution, and consists in retaining the best $\mu/3$ individuals and replacing the others by new individuals. This advanced population management procedure coupled with the diversity-based evaluation of individuals plays a main role in the success of the overall algorithm.

4.7 Decomposition Phases

Finally, a decomposition phase is triggered after each It_{dec} iterations. In a decomposition phase, an elite solution selected in the 25% best feasible individuals is used to define subproblems, by fixing the assignments to AAR resources and by separating routes in several subsets as in Vidal et al. (2011a). UHGS is then independently applied on each subproblem, including the partial solution issued from the elite individual in the initial population, and three different best solutions are reconstituted out of the best solutions of the subproblems. This decomposition phase introduces a strong intensification around elite characteristics, and contributes in finding high quality solutions for problems of large size. For this reason, it is only activated for problem instances involving more than 150 customers.

5 Computational Experiments

Extensive computational experiments have been conducted on a wide range of MAVRPs to assess the performance of the general-purpose UHGS relatively to the best problem-tailored algorithms for each setting. Both "academic" problems and rich VRPs have been addressed in these studies. A single parameter setting, the same as in Vidal et al. (2011a, 2012a), was used for all the experiments in order to examine the applicability of the method without extensive problem-tailored parameter customization. The termination criteria has been set to ($It_{max} = 5000$; $T_{max} = 30min$) to compare with other authors in similar time. Problems requiring fleet minimization are solved by iteratively decrementing the fleet size limit and running UHGS until no feasible solution can be found. The algorithm was implemented in C++ and run on Opteron 250 2.4GHz and Opteron 275 2.2GHz processors.

Tables 4 and 5 compare the results of UHGS with the best current methods in the literature for each problem class taken separately. Columns (1-4) indicate the variant considered, the origin of the benchmark instances, the number n of customers in these instances and the objective function ("C" standing for distance, "D" for duration, i.e., the time elapsed between departure and return, "T" for travel time, "F" for fleet size, "TW" for time-window violations). Hierarchical objectives are presented by decreasing order of priority, and separated with the sign "/". In the last column is reported for each state-of-the-art method the gap of an average or single run with respect to the current Best Known Solutions (BKS), the gap of the best solution produced by the method, the average run time to achieve these results (for parallel

Table 3: List of acronyms for benchmarks and methods

List of acronyms for benchmarks					
B11	Bektas et al. (2011)	G84	Golden (1984)	LS99	Liu and Shen (1999)
CGL97	Cordeau et al. (1997)	G09	Goel (2009)	MG06	Montané and Galvão (2006)
CL01	Cordeau and Laporte (2001)	GH99	Gehring and Homberger (1999)	SD88	Solomon and Desrosiers (1988)
CMT79	Christofides et al. (1979)	GJ89	Goetschalckx and J.-B. (1989)	SN99	Salhi and Nagy (1999)
F94	Fisher (1994)	GWKC98	Golden et al. (1998)		
List of acronyms for state-of-the-art algorithms					
B10	Belhaiza (2010)	KTDHS12	Kritzinger et al. (2012)	RT10	Repoussis and Tarantilis (2010)
BDHMG08	Bräysy et al. (2008)	MB07	Mester and Bräysy (2007)	RTBI10	Repoussis et al. (2010)
BER11	Bektas et al. (2011)	MCR12	Moccia et al. (2012)	RTI09a	Repoussis et al. (2009a)
BLR11	Balseiro et al. (2011)	NB09	Nagata and Bräysy (2009)	RTI09b	Repoussis et al. (2009b)
BPDRT09	Bräysy et al. (2009)	NBD10	Nagata et al. (2010)	S12	Subramanian (2012)
CM12	Cordeau and M. (2012)	NPW10	Ngueveu et al. (2010)	SDBOF10	Subramanian et al. (2010)
F10	Figliozzi (2010)	P09	Prins (2009)	SPUO12	Subramanian et al. (2012)
FEL07	Fu et al. (2007)	PBDH08	Polacek et al. (2008)	XZKX12	Xiao et al. (2012)
GA09	Gajpal and Abad (2009)	PDDR10	Prescott-Gagnon et al. (2010)	ZTK10	Zachariadis et al. (2010)
GG11	Groër and Golden (2011)	PR07	Pisinger and Ropke (2007)	ZK10	Zachariadis and Kiranoudis (2010)
HDH09	Hemmelmayr et al. (2009)	PR08	Pirkwieser and Raidl (2008)	ZK11	Zachariadis and Kiranoudis (2011)
ISW09	Imran et al. (2009)	RL12	Ribeiro and Laporte (2012)	ZK12	Zachariadis and Kiranoudis (2012)

methods, the computation time on a single processor is reported in italics), and the processor used. For each benchmark and problem class, the algorithm yielding the best result quality is indicated in boldface. Table 4 also includes the results from previous HGSADC applications on the PVRP, MDVRP, CVRP, and VRTDSP with European Union regulations (Vidal et al. 2011a, 2012a, Goel and Vidal 2012) since UHGS works in the same way when instantiated on these problems. Detailed results on all benchmarks are given in Appendix.

As reported in Tables 4 and 5, UHGS produces high-quality solutions for all problems and benchmark sets, from pure academic problems such as the CVRP to a large variety of VRP variants and rich settings. In addition to its high generality and its potential to address many MAVRPs, UHGS matches or outperforms the wide majority of problem-tailored approaches on each separate benchmark and problem class. The average standard deviation of solutions, measured separately for each single objective problem, ranges between 0.002% (GVRP) and 0.66% (MDPVRPTW), thus showing that the meta-heuristic produces high-quality solutions in a consistent manner. The average run time remains in most cases smaller than 10 minutes for average-sized problems (100 to 200 customers), being thus adequate for daily or weekly planning. Overall 954/1008 BKS have been either retrieved or improved during these experiments, and 550/1008 BKS have been strictly improved.

6 Conclusions and Perspectives

A new Unified Local Search and a Hybrid Genetic Search relying on a component-based design have been introduced to address a large variety of difficult VRP variants. The methods rely on automatically-configured assignment, sequencing and route evaluation components to address problem specificities. These components serve as the basis to implement generic local-search improvement, *Split*, and genetic operators. Furthermore, the use of individual fitness measures based on both diversity and quality, and advanced population management schemes provides the means for a thorough and efficient search.

The remarkable performance of UHGS has been demonstrated on a wide range of problems. On 26 VRP variants and 39 sets of benchmark instances, UHGS matches or outperforms the current state-of-the-art problem-tailored algorithms. Overall, 954/1008 best known solutions

Table 4: Performance analysis on several VRP variants with various objectives

Variant	**Bench.**	n	**Obj.**	**State-of-the-art methods**				
				Author	Avg.%	Best%	T(min)	CPU
CVRP	CMT79	[50,199]	C	GG11	—	+0.03%	*2.38*	8×Xe 2.3G
				MB07	+0.03%	—	2.80	P-IV 2.8G
				UHGS*	**+0.02%**	**+0.00%**	11.90	Opt 2.4G
CVRP	GWKC 98	[200,483]	C	GG11	—	+0.29%	*5.00*	8×Xe 2.3G
				NB09	+0.27%	+0.16%	21.51	Opt 2.4G
				UHGS*	**+0.15%**	**+0.02%**	71.41	Opt 2.4G
VRPB	GJ89	[25,200]	C	ZK12	+0.38%	+0.00%	1.09	T5500 1.67G
				GA09	+0.09%	+0.00%	1.13	Xe 2.4G
				UHGS	**+0.01%**	**+0.00%**	0.99	Opt 2.4G
CCVRP	CMT79	[50,199]	C	NPW10	+0.74%	+0.28%	5.20	Core2 2G
				RL12	+0.37%	+0.07%	2.69	Core2 2G
				UHGS	**+0.01%**	**-0.01%**	1.42	Opt 2.2G
CCVRP	GWKC 98	[200,483]	C	NPW10	+2.03%	+1.38%	94.13	Core2 2G
				RL12	+0.34%	+0.07%	21.11	Core2 2G
				UHGS	**-0.14%**	**-0.23%**	17.16	Opt 2.2G
VRPSDP	SN99	[50,199]	C	SDBOF10	+0.16%	+0.00%	*0.37*	256×Xe 2.67G
				ZTK10	—	+0.11%	—	T5500 1.66G
				UHGS	**+0.01%**	**+0.00%**	2.79	Opt 2.4G
VRPSDP	MG06	[100,400]	C	SDBOF10	+0.30%	+0.17%	*3.11*	256×Xe 2.67G
				UHGS	+0.20%	+0.07%	12.00	Opt 2.4G
				S12	**+0.08%**	**+0.00%**	7.23	I7 2.93G
VFMP-F	G84	[20,100]	C	ISW09	—	+0.07%	8.34	P-M 1.7G
				SPUO12	+0.12%	+0.01%	0.15	I7 2.93G
				UHGS	**+0.04%**	**+0.01%**	1.13	Opt 2.4G
VFMP-V	G84	[20,100]	C	ISW09	—	+0.02%	8.85	P-M 1.7G
				SPUO12	+0.17%	+0.00%	0.06	I7 2.93G
				UHGS	**+0.03%**	**+0.00%**	0.85	Opt 2.4G
VFMP-FV	G84	[20,100]	C	P09	—	+0.02%	0.39	P4M 1.8G
				UHGS	+0.01%	+0.00%	0.99	Opt 2.4G
				SPUO12	**+0.01%**	**+0.00%**	0.13	I7 2.93G
LDVRP	CMT79	[50,199]	C	XZKX12	+0.48%	+0.00%	1.3	NC 1.6G
				UHGS	**-0.28%**	**-0.33%**	2.34	Opt 2.2G
LDVRP	GWKC 98	[200,483]	C	XZKX12	+0.66%	+0.00%	3.3	NC 1.6G
				UHGS	**-1.38%**	**-1.52%**	23.81	Opt 2.2G
PVRP	CGL97	[50,417]	C	HDH09	+1.69%	+0.28%	3.09	P-IV 3.2G
				UHGS*	+0.43%	+0.02%	6.78	Opt 2.4G
				CM12	**+0.24%**	**+0.06%**	*3.55*	64×Xe 3G
MDVRP	CGL97	[50,288]	C	CM12	+0.09%	+0.03%	*3.28*	64×Xe 3G
				S12	+0.07%	+0.02%	11.81	I7 2.93G
				UHGS*	**+0.08%**	**+0.00%**	5.17	Opt 2.4G
GVRP	B11	[16,262]	C	BER11	+0.06%	—	0.01	Opt 2.4G
				MCR12	+0.11%	—	0.34	Duo 1.83G
				UHGS	**+0.00%**	**-0.01%**	1.53	Opt 2.4G

Table 5: Performance analysis on several VRP variants with various objectives (continued)

Variant	Bench.	n	Obj.	State-of-the-art methods Author	Avg.%	Best%	T(min)	CPU
OVRP	CMT79 &F94	[50,199]	F/C	RTBI10	0%/+0.32%	—	9.54	P-IV 2.8G
				S12	—/+0.16%	0%/+0.00%	2.39	I7 2.93G
				UHGS	**0%/+0.11%**	**0%/+0.00%**	1.97	Opt 2.4G
OVRP	GWKC 98	[200,480]	F/C	ZK10	0%/+0.39%	0%/+0.21%	14.79	T5500 1.66G
				S12	0%/+0.13%	0%/+0.00%	64.07	I7 2.93G
				UHGS	**0%/-0.11%**	**0%/-0.19%**	16.82	Opt 2.4G
VRPTW	SD88	100	F/C[4]	RTI09	0%/+0.11%	0%/+0.04%	17.9	Opt 2.3G
				UHGS*	0%/+0.04%	0%/+0.01%	2.68	Xe 2.93G
				NBD10	**0%/+0.02%**	**0%/+0.00%**	5.0	Opt 2.4G
VRPTW	HG99	[200,1000]	F/C[4]	RTI09b	—	+0.16%/+3.36%	270	Opt 2.3G
				NBD10	+0.20%/+0.42%	+0.10%/+0.27%	21.7	Opt 2.4G
				UHGS*	**+0.18%/+0.11%**	**+0.08%/-0.10%**	141	Xe 2.93G
OVRPTW	SD88	100	F/C[4]	RTI09a	+0.89%/+0.42%	0%/+0.24%	10.0	P-IV 3.0G
				KTDHS12	0%/+0.79%	0%/+0.18%	10.0	Xe 2.67G
				UHGS	**+0.09%/-0.10%**	**0%/-0.10%**	5.27	Opt 2.2G
TDVRPTW	SD88	100	F/C	KTDHS12	+2.25%	0%	10.0	Xe 2.67G
				UHGS	**-3.31%**	**-3.68%**	21.94	Opt 2.2G
VFMPTW	LS99	100	D	BDHMG08	—	+0.59%	10.15	Ath 2.6G
				RT10	+0.22%	—	16.67	P-IV 3.4G
				UHGS	**-0.15%**	**-0.24%**	4.58	Opt 2.2G
VFMPTW	LS99	100	C	BDHMG08	—	+0.25%	3.55	Ath 2.6G
				BPDRT09	—	+0.17%	0.06	Duo 2.4G
				UHGS	-0.38%	-0.49%	4.82	Opt 2.2G
PVRPTW	CL01	[48,288]	C	PR08	—	+1.75%	—	Opt 2.2G
				CM12	+1.10%	+0.76%	*11.3*	64×Xe 3G
				UHGS*	**+0.63%**	**+0.22%**	32.7	Xe 2.93G
MDVRPTW	CL01	[48,288]	C	PBDH08	—	+1.37%	147	P-IV 3.6G
				CM12	+0.36%	+0.15%	*6.57*	64×Xe 3G
				UHGS*	**+0.19%**	**+0.03%**	6.49	Xe 2.93G
SDVRPTW	CL01	[48,288]	C	B10	+2.23%	—	2.94	Qd 2.67G
				CM12	+0.62%	+0.36%	*5.60*	64×Xe 3G
				UHGS*	**+0.36%**	**+0.10%**	5.48	Xe 2.93G
VRPSTW (type 1, α=100)	SD88	100	F/TW/C[5]	F10	0%	—	9.69	P-M 1.6G
				UHGS	**-3.05%**	**-4.42%**	18.62	Opt 2.2G
VRPSTW (type 1, α=1)	SD88	100	C+TW	KTDHS12	+0.62%	+0.00%	10.0	Xe 2.67G
				UHGS	**-0.13%**	**-0.18%**	5.82	Opt 2.2G
VRPSTW (type 2, α=100)	SD88	100	F/TW/C[5]	FEL07	0%	—	5.98	P-II 600M
				UHGS	**-13.91%**	**-13.91%**	41.16	Opt 2.2G
VRPSTW (type 2, α=1)	SD88	100	C+TW	**UHGS**	**+0.26%**	**0%**	29.96	Opt 2.2G
MDPVRPTW	New	[48,288]	C	**UHGS**	**+0.77%**	**0%**	16.89	Opt 2.2G
VRTDSP (E.U. rules)	G09	100	F/C	PDDR10	0%/0%	0%/0%	88	Opt 2.3G
				UHGS*	**-0.56%/-0.54%**	**-0.85%/-0.70%**	228	Xe 2.93G

∗ These results have been originally presented in Vidal et al. (2011a, 2012a) and Goel and Vidal (2012).
[4] The gaps to BKS in terms of fleet size and distance are averaged on groups of instances C1-100,R1-100,...,RC2-1000.
[5] For the sake of brevity, only the fleet size is reported in this Table.

have been either retrieved or improved. Hence, it appears that the proposed heuristic design is particularly efficient for dealing with MAVRPs, and also that generality does not necessarily play against performance for the considered classes of VRP variants.

This general-purpose VRP solver opens the way to experimentations and sensitivity analyses of local search and meta-heuristic components on a wide range of structurally different problems. Finally, perspectives of research involve the generalization of the method towards a wider variety of ASSIGN and SEQ attributes, multi-objective and stochastic settings.

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Appendix: Detailed experimental results

Tables 6 to 35 present the detailed results of UHGS and other state-of-the-art methods for a variety of vehicle routing variants and benchmarks. The first group of columns displays the instance identifier, number of customers n, vehicle fleet limit m when applicable, and also the number of customer clusters c in the GVRP case, the number of vehicle types w in fleet size and mix settings, and the number of periods t and depots d in the MDPVRP case. The next group of columns presents the results of actual state-of-the-art methods for each problem, as well as the results of UHGS. When available, both average and best results on several runs (number of runs specified in the headings of the Table) are provided.

We indicate in boldface the best average result among algorithms for each instance as well as the previous Best Known Solution (BKS) in the last column. New best known solutions are underlined. Finally, average measures over sets of instances are presented in the last lines: the computation time for each method, the average percentage of error (Gap) relative to the previous BKS, and the processor used. Some specific details for each problem and benchmark are listed in the following.

VRP with Backhauls (VRPB). "Double" precision values have been used for distance computations. Comparison is made with methods that rely on the same assumption. A fleet size value m is specified in the instances. As in the previous works, we consider a fixed fleet size without allowing less or more than m vehicles. To that extent, the distance matrix has been modified by setting $d_{00} = +\infty$. Other variants of the VRPB, such as the vehicle routing with mixed backhauls, the VRPB with time windows or with multiple depots, can be addressed by UHGS. For the sake of conciseness, results on these variants are not reported in this paper.

Cumulative VRP (CCVRP). Following the guidelines of Ngueveu et al. (2010), the duration constraint is not considered and the fleet size limit is fixed to the minimum feasible value. In the original paper of NPC10, only the best solution on the benchmark instances of Christofides et al. (1979) were reported. This algorithm was then provided to Ribeiro and Laporte (2012) who ran more extensive experiments on all instances. We rely on these latter values for our experimental comparison.

VRP with Simultaneous Deliveries and Pickups (VRPSDP). "Double" precision values have been used for distances and demands. We only compare to recent methods that rely on the same convention.

Load Dependent VRP (LDVRP). As in previous work, maximum trip duration constraints are taken into account (sum of driving length plus service time) and the fleet size is unconstrained.

Generalized VRP (GVRP). Distances were rounded to the closest integer to compare with the recent works of Bektas et al. (2011) and Moccia et al. (2012) using the same assumptions.

Open VRP (OVRP). The hierarchical objective of fleet minimization and then distance is used. As almost all recent methods succeeded in reaching the same best known fleet size for each problem, this fleet size "$\mathrm{m}_{\mathrm{BKS}}$" is presented in a single column. The same convention as

Repoussis et al. (2010) is used: the route duration from the benchmark instances of Christofides et al. (1979) is multiplied by a factor of 0.9, whereas for the benchmark instances of Golden et al. (1998) the duration constraint is not considered.

Vehicle Fleet Mix Problem with Time Windows (VFMPTW). The benchmark instances of Liu and Shen (1999) are considered with three different fleet cost settings (type A,B,C instances). As in the former paper and several following works, we addressed the minimization of fixed fleet cost plus the *trip duration*, i.e., the time elapsed between departure from the depot and return minus the service time (Tables 25-27). Is should be noted that the departure time is not constrained to be time 0, and several best known solutions require a delayed departure from the depot. Additional experiments have also been conducted to address the more standard objective of fixed fleet cost plus total distance (Tables 28-30).

VRP with Soft Time Windows (VRPSTW). A classification and notation for soft time windows settings is introduced in Fu et al. (2007). Type 1 and type 2 soft time windows have been addressed in this paper. Several criteria have been considered by previous authors to assess on the quality of solutions, these criteria include the fleet size, the number of customers serviced outside of their time windows, the amount of lateness and earliness, and the route distance. To optimize on these objectives, we implemented a general formulation of service costs $c_i(t_i)$ to customer as a function of the service date t_i, given in Equation (34). In these equations, γ represents a fixed penalty for servicing a customer outside of its time window, and α and β are respectively the penalties for one unit of tardiness or earliness.

$$c_i(t_i) = \begin{cases} \gamma + \beta(e_i - t_i) & \text{if } t_i < e_i \\ 0 & \text{if } e_i \le t_i \le l_i \\ \gamma + \alpha(t_i - l_i) & \text{if } l_i < t_i \end{cases} \tag{34}$$

Two settings of type 1 soft time windows have been addressed. To address the hierarchical objective of minimizing first the fleet size, then the number of customers serviced outside of their time windows, then lateness, and finally distance, the parameters values have been to $\beta = +\infty$, $\gamma = 100000$ and $\alpha = 100$ and the fleet size has been minimized by iteratively reducing the fleet limit (Table 31). Another objective, involving the minimization of distance plus lateness with $\alpha = 1$ has been also addressed (Table 32). In this case, the other parameters have been set to $\beta = +\infty$ and $\gamma = 0$.

Furthermore, two settings of type 2 soft time windows have been addressed. The hierarchical objective has been addressed by setting $\beta = 100$, $\gamma = 100000$ and $\alpha = 100$ (Table 33). We also provide results for the objective seeking to minimize the sum of distance, earliness and lateness by setting $\beta = 1$, $\gamma = 0$ and $\alpha = 1$ (Table 34).

Time-Dependent VRP with Time Windows (TDVRPTW). The same setting as Kritzinger et al. (2012) has been considered, involving the minimization of the total time dependent travel time. The same convention as the authors is used, and thus all routes are constrained to start at time 0, waiting time being allowed only at a customer location upon an early arrival. The benchmark instances of Solomon and Desrosiers (1988) are used, with the three different travel time scenarios of Ichoua et al. (2003). Furthermore, the planning horizon has been divided into three equal periods of time, and the category matrix of Balseiro et al.

(2011) has been used. As a result, not all benchmark instances may be feasible, and methods do not find feasible solutions on the exact same subset of instances. In Tables (22-24), the computation of overall gaps to BKS was based on the subset of instances for which a feasible solution has been found by all methods.

Multi-Depot Periodic VRP with Time Windows (MDPVRPTW). The set of MD-PVRP instances from Vidal et al. (2012a), originally issued from the combination of multi-depot and periodic instances of Cordeau et al. (1997, 2001), has been completed with the time windows values from Cordeau et al. (2001). Experiments have been conducted to set the fleet size value close to the minimum fleet size. The corresponding values of m are reported, along with the results, in Table 35.

Table 6: Results on the VRPB, instances of Goetschalckx (1989)

Inst	n	m	GA09		ZK11		UHGS			BKS
			Avg 8	Best X	Avg 10	Best 10	Avg 10	Best 10	T(min)	
A1	25	8	**229.89**	**229.89**	**229.89**	**229.89**	**229.89**	**229.89**	0.11	229.89
A2	25	5	**180.12**	**180.12**	**180.12**	**180.12**	**180.12**	**180.12**	0.12	180.12
A3	25	4	**163.41**	**163.41**	**163.41**	**163.41**	**163.41**	**163.41**	0.13	163.41
A4	25	3	**155.80**	**155.80**	**155.80**	**155.80**	**155.80**	**155.80**	0.16	155.80
B1	30	7	**239.08**	**239.08**	**239.08**	**239.08**	**239.08**	**239.08**	0.14	239.08
B2	30	5	**198.05**	**198.05**	**198.05**	**198.05**	**198.05**	**198.05**	0.15	198.05
B3	30	3	**169.37**	**169.37**	**169.37**	**169.37**	**169.37**	**169.37**	0.18	169.37
C1	40	7	**250.56**	**250.56**	**250.56**	**250.56**	**250.56**	**250.56**	0.22	250.56
C2	40	5	**215.02**	**215.02**	**215.02**	**215.02**	**215.02**	**215.02**	0.24	215.02
C3	40	5	**199.35**	**199.35**	**199.35**	**199.35**	**199.35**	**199.35**	0.23	199.35
C4	40	4	**195.37**	**195.37**	**195.37**	**195.37**	**195.37**	**195.37**	0.24	195.37
D1	38	12	**322.53**	**322.53**	**322.53**	**322.53**	**322.53**	**322.53**	0.18	322.53
D2	38	11	**316.71**	**316.71**	**316.71**	**316.71**	**316.71**	**316.71**	0.17	316.71
D3	38	7	**239.48**	**239.48**	**239.48**	**239.48**	**239.48**	**239.48**	0.19	239.48
D4	38	5	**205.83**	**205.83**	**205.83**	**205.83**	**205.83**	**205.83**	0.24	205.83
E1	45	7	**238.88**	**238.88**	**238.88**	**238.88**	**238.88**	**238.88**	0.27	238.88
E2	45	4	**212.26**	**212.26**	**212.26**	**212.26**	**212.26**	**212.26**	0.32	212.26
E3	45	4	**206.66**	**206.66**	**206.66**	**206.66**	**206.66**	**206.66**	0.36	206.66
F1	60	6	**263.17**	**263.17**	263.27	**263.17**	**263.17**	**263.17**	0.38	263.17
F2	60	7	**265.21**	**265.21**	265.66	**265.21**	**265.21**	**265.21**	0.39	265.21
F3	60	5	241.48	**241.12**	**241.12**	**241.12**	**241.12**	**241.12**	0.49	241.12
F4	60	4	**233.86**	**233.86**	234.60	**233.86**	**233.86**	**233.86**	0.54	233.86
G2	57	6	**245.44**	**245.44**	**245.44**	**245.44**	**245.44**	**245.44**	0.38	245.44
G3	57	5	**229.51**	**229.51**	229.97	**229.51**	**229.51**	**229.51**	0.43	229.51
G4	57	6	**232.52**	**232.52**	**232.52**	**232.52**	**232.52**	**232.52**	0.45	232.52
G5	57	5	**221.73**	**221.73**	**222.87**	**221.73**	**221.73**	**221.73**	0.46	221.73
G6	57	4	**213.46**	**213.46**	214.38	**213.46**	**213.46**	**213.46**	0.54	213.46
H1	68	6	269.00	**268.93**	270.06	**268.93**	**268.93**	**268.93**	0.62	268.93
H2	68	5	**253.37**	**253.37**	253.91	**253.37**	**253.37**	**253.37**	0.57	253.37
H3	68	4	**247.45**	**247.45**	**247.45**	**247.45**	**247.45**	**247.45**	0.64	247.45
H4	68	5	**250.22**	**250.22**	251.09	**250.22**	**250.22**	**250.22**	0.59	250.22
H5	68	4	**246.12**	**246.12**	**246.12**	**246.12**	**246.12**	**246.12**	0.62	246.12
H6	68	5	**249.14**	**249.14**	250.06	**249.14**	**249.14**	**249.14**	0.59	249.14
I1	90	10	350.40	**350.25**	351.08	**350.25**	350.37	**350.25**	0.89	350.25
I2	90	7	310.32	**309.94**	309.98	**309.94**	**309.94**	**309.94**	0.85	309.94
I3	90	5	294.84	**294.51**	294.79	**294.51**	**294.51**	**294.51**	0.99	294.51
I4	90	6	296.13	**295.99**	297.91	**295.99**	**295.99**	**295.99**	0.92	295.99
I5	90	7	301.83	**301.24**	303.49	**301.24**	**301.24**	**301.24**	0.82	301.24
J1	94	10	335.12	**335.01**	335.78	**335.01**	**335.01**	**335.01**	0.83	335.01
J2	94	8	**310.42**	**310.42**	312.51	**310.42**	**310.42**	**310.42**	0.84	310.42
J3	94	6	279.34	**279.22**	280.43	**279.22**	**279.22**	**279.22**	0.93	279.22
J4	94	7	296.58	**296.53**	298.32	**296.53**	**296.53**	**296.53**	1.12	296.53
K1	113	10	396.14	**394.07**	397.38	**394.07**	394.35	**394.07**	1.33	394.07
K2	113	8	362.56	**362.13**	365.46	**362.13**	**362.13**	**362.13**	1.40	362.13
K3	113	9	366.71	**365.69**	369.44	**365.69**	**365.69**	**365.69**	1.30	365.69
K4	113	7	350.32	**348.95**	349.72	**348.95**	**348.95**	**348.95**	1.29	348.95
L1	150	10	420.06	**417.90**	421.68	**417.90**	418.16	**417.90**	3.94	417.90
L2	150	8	401.36	**401.23**	405.20	**401.23**	**401.23**	**401.23**	2.96	401.23
L3	150	9	404.32	**402.68**	405.76	**402.68**	**402.68**	**402.68**	2.73	402.68
L4	150	7	384.83	**384.64**	388.14	**384.64**	**384.64**	**384.64**	2.35	384.64
L5	150	8	390.33	**387.56**	390.46	387.57	**387.56**	**387.56**	2.72	387.56
M1	125	11	399.12	**398.59**	400.50	**398.59**	398.66	**398.59**	1.58	398.59
M2	125	10	398.16	**396.92**	401.91	**396.92**	396.93	**396.92**	2.39	396.92
M3	125	9	377.81	**375.70**	378.07	**375.70**	375.93	**375.70**	2.70	375.70
M4	125	7	348.46	**348.14**	352.03	**348.14**	348.20	**348.14**	1.72	348.14
N1	150	11	408.17	**408.10**	411.72	**408.10**	**408.10**	**408.10**	2.72	408.10
N2	150	10	408.25	**408.07**	412.31	**408.07**	408.13	**408.07**	2.55	408.07
N3	150	9	394.70	**394.34**	398.76	**394.34**	394.94	**394.34**	2.46	394.34
N4	150	10	394.87	**394.79**	396.16	**394.79**	395.13	**394.79**	2.37	394.79
N5	150	7	374.12	**373.48**	376.90	**373.48**	373.55	**373.48**	3.12	373.48
N6	150	8	374.79	**373.76**	379.78	**373.76**	373.76	**373.76**	3.42	373.76
Time			1.13 min		1.09 min		0.99 min			
Gap			+0.09%	+0.00%	+0.38%	+0.00%	+0.01%	+0.00%		
CPU			T5500 1.67G		Xe 2.4G		Opt 2.4G			

Table 7: Results on the CCVRP, instances of Christofides et al. (1979)

Inst	**n**	**m**	**NPC10**		**RL12**		**UHGS**			**BKS**
			Avg 5	Best 5	Avg 5	Best 5	Avg 10	Best 10	T(min)	
p01	50	5	**2230.35**	**2230.35**	2235.27	**2230.35**	**2230.35**	**2230.35**	0.44	2230.35
p02	75	10	2443.07	2421.90	2401.72	**2391.63**	2394.00	**2391.63**	0.70	2391.63
p03	100	8	4073.12	4073.12	4063.98	**4045.42**	**4045.42**	**4045.42**	0.93	4045.42
p04	150	12	5020.75	**4987.52**	4994.93	**4987.52**	**4987.52**	**4987.52**	1.84	4987.52
p05	199	17	5842.00	5810.20	5857.76	5838.32	5809.94	**5806.02**	4.02	5810.12
p11	120	7	7395.83	7317.98	7341.28	7315.87	**7314.55**	**7314.55**	1.35	7315.87
p12	100	10	3559.23	**3558.92**	3566.06	**3558.92**	**3558.93**	**3558.93**	0.64	3558.93
Time			5.20 min		2.69 min		1.42 min			
Gap			+0.74%	+0.28%	+0.37%	+0.07%	+0.01%	-0.01%		
CPU			Core2 2G		Core2 2G		Opt 2.2G			

Table 8: Results on the CCVRP, instances of Golden et al. (1998)

Inst	**n**	**m**	**NPC10**[1]		**RL12**		**UHGS**			**BKS**
			Avg 5	Best 5	Avg 5	Best 5	Avg 10	Best 10	T(min)	
pr01	240	9	54878.25	54815.17	54853.76	54786.92	54742.20	**54739.85**	7.40	54786.92
pr02	320	10	100918.54	100836.90	100934.34	100662.53	100562.52	**100560.16**	11.00	100662.53
pr03	400	10	171400.35	171277.26	172231.14	171613.59	170964.42	**170923.53**	23.95	171277.26
pr04	480	10	262830.96	262584.23	265207.46	263433.03	262044.19	**261993.33**	26.14	262584.23
pr05	200	5	114237.00	114163.64	114846.27	114494.66	**114163.63**	**114163.63**	6.95	114163.64
pr06	280	7	140456.96	140430.09	140929.71	140804.64	**140430.08**	**140430.08**	12.65	140430.09
pr07	360	8	186702.15	183282.64	181610.82	180481.56	178976.20	**178880.44**	26.66	180481.56
pr08	440	10	194510.99	194312.60	195174.85	194988.74	193683.21	**193659.14**	26.07	194273.58
pr09	255	14	4740.42	4730.70	4728.05	4725.58	4724.01	**4722.06**	6.97	4725.58
pr10	323	16	6747.10	6732.36	6717.76	6713.92	6720.04	**6713.26**	10.30	6713.92
pr11	399	18	9259.66	9243.05	9216.60	**9214.07**	9222.92	9219.42	14.61	9214.07
pr12	483	19	12649.21	12629.37	12543.04	12526.17	12516.98	**12500.52**	28.66	12526.17
pr13	252	26	3660.93	3653.07	3638.50	3628.30	3632.63	**3627.45**	7.16	3628.30
pr14	320	29	6045.20	5770.02	5257.95	5216.80	5206.53	**5187.56**	16.25	5216.80
pr15	396	33	7140.11	7077.48	7023.12	7010.41	7015.51	**7005.47**	18.70	7010.41
pr16	480	37	9339.45	9300.74	9268.30	9250.98	9247.68	**9239.10**	27.28	9250.98
pr17	240	22	3103.99	3089.99	3068.29	3065.46	3061.28	**3060.14**	6.06	3065.46
pr18	300	27	4582.44	4528.16	4244.60	4221.14	4211.80	**4199.43**	11.64	4221.14
pr19	360	33	5589.12	5570.35	5531.78	5523.38	5502.59	**5496.39**	17.06	5523.38
pr20	420	38	7473.69	7413.58	7240.86	7223.08	7188.59	**7184.19**	21.82	7223.08
Time			94.13 min		21.11 min		17.16 min			
Gap			+2.03%	+1.38%	+0.34%	+0.07%	-0.14%	-0.23%		
CPU			Core2 2G		Core2 2G		Opt 2.2G			

[1] *Solutions reported in Ribeiro and Laporte (2012) using the same code as NPC10*

Table 9: Results on the VRPSDP, instances of Salhi and Nagy (1999)

Inst	n	ZTK10	SDBOF10		S12		UHGS			BKS
		—	Avg 50	Best 50	Avg 50	Best 50	Avg 10	Best 10	T(min)	
CMT1X	50	469.80	**466.77**	**466.77**	**466.77**	**466.77**	**466.77**	**466.77**	0.72	466.77
CMT1Y	50	469.80	**466.77**	**466.77**	**466.77**	**466.77**	**466.77**	**466.77**	0.71	466.77
CMT2X	75	**684.21**	684.49	**684.21**	684.78	**684.21**	684.43	**684.21**	1.32	684.21
CMT2Y	75	**684.21**	684.43	**684.21**	684.59	**684.21**	684.36	**684.21**	1.35	684.21
CMT3X	100	**721.27**	**721.27**	**721.27**	721.46	**721.27**	**721.27**	**721.27**	1.69	721.27
CMT3Y	100	**721.27**	**721.27**	**721.27**	721.50	**721.27**	**721.27**	**721.27**	1.79	721.27
CMT12X	100	**662.22**	**662.22**	**662.22**	663.44	**662.22**	**662.22**	**662.22**	1.74	662.22
CMT12Y	100	**662.22**	662.25	**662.22**	663.12	**662.22**	**662.22**	**662.22**	1.69	662.22
CMT11X	120	**833.92**	842.78	**833.92**	848.65	846.23	**833.92**	**833.92**	2.75	833.92
CMT11Y	120	**833.92**	842.78	**833.92**	848.74	846.23	**833.92**	**833.92**	2.66	833.92
CMT4X	150	**852.46**	**852.46**	**852.46**	853.02	**852.46**	852.65	**852.46**	4.16	852.46
CMT4Y	150	**852.46**	**852.46**	**852.46**	852.73	**852.46**	852.72	**852.46**	3.95	852.46
CMT5X	199	1030.55	1029.66	**1029.25**	1029.52	**1029.25**	1029.60	**1029.25**	7.99	1029.25
CMT5Y	199	1030.55	1029.71	**1029.25**	**1029.25**	**1029.25**	1029.79	**1029.25**	6.60	1029.25
Time		—	256×0.36 min		—		2.79 min			
Gap		+0.11%	+0.16%	+0.00%	+0.30%	+0.21%	+0.01%	+0.00%		
CPU		T5500 1.66G	Xe 2.67G		I7 2.93G		Opt 2.4G			

Table 10: Results on the VRPSDP, instances of Montané and Galvão (2006)

Inst	n	ZTK10	SDBOF10		S12		UHGS			BKS
		—	Avg 50	Best 50	Avg 50	Best 50	Avg 10	Best 10	T(min)	
r101	100	**1009.95**	1010.54	**1009.95**	1010.08	**1009.95**	1011.60	**1009.95**	1.23	1009.95
r201	100	**666.20**	**666.20**	**666.20**	**666.20**	**666.20**	**666.20**	**666.20**	2.39	666.20
c101	100	1220.99	1220.64	**1220.18**	1220.43	**1220.18**	1220.99	1220.99	1.10	1220.18
c201	100	**662.07**	**662.07**	**662.07**	**662.07**	**662.07**	**662.07**	**662.07**	1.63	662.07
rc101	100	**1059.32**	**1059.32**	**1059.32**	**1059.32**	**1059.32**	**1059.32**	**1059.32**	1.21	1059.32
rc201	100	**672.92**	**672.92**	**672.92**	**672.92**	**672.92**	**672.92**	**672.92**	1.99	672.92
R1_2_1	200	3376.30	3369.93	3360.02	3355.04	**3353.80**	3364.40	3355.37	6.77	3353.80
R2_2_1	200	**1665.58**	**1665.58**	**1665.58**	**1665.58**	**1665.58**	**1665.58**	**1665.58**	8.95	1665.58
C1_2_1	200	3643.82	3635.87	3629.89	3636.53	**3628.51**	3639.00	3637.42	7.25	3628.51
C2_2_1	200	**1726.59**	**1726.59**	**1726.59**	**1726.59**	**1726.59**	**1726.59**	**1726.59**	5.52	1726.59
RC1_2_1	200	3323.56	3317.51	3306.00	3306.73	**3303.70**	3315.35	3304.39	8.10	3303.70
RC2_2_1	200	**1560.00**	**1560.00**	**1560.00**	**1560.00**	**1560.00**	**1560.00**	**1560.00**	7.35	1560.00
R1_4_1	400	9691.60	9647.24	9618.97	9539.56	**9519.45**	9594.08	9547.85	24.53	9519.45
R2_4_1	400	3572.38	3557.43	3551.38	3549.49	**3546.49**	3555.10	**3546.49**	24.57	3546.49
C1_4_1	400	11179.36	11118.98	11099.54	11075.60	**11047.19**	11113.63	11077.26	29.67	11047.19
C2_4_1	400	3549.27	3558.92	3546.10	3543.65	**3539.50**	3541.44	**3539.50**	29.76	3539.50
RC1_4_1	400	9645.27	9564.86	9536.77	9478.12	**9447.53**	9509.39	9469.44	29.84	9447.53
RC2_4_1	400	3423.62	3404.62	**3403.70**	**3403.70**	**3403.70**	3404.08	**3403.70**	24.10	3403.70
Time		—	256×3.11 min		7.23 min		12.00 min			
Gap		+0.47%	+0.30%	+0.17%	+0.08%	+0.00%	+0.20%	+0.07%		
CPU		T5500 1.66G	Xe 2.67G		I7 2.93G		Opt 2.4G			

Table 11: Results on the VFMP-F, only fixed vehicle costs, instances of Golden (1984)

Inst	n	w	ISW09	P09	SPUO12		UHGS			BKS
			Best 5-7	Best 5	Avg 10	Best 10	Avg 10	Best 10	T(min)	
F3	20	5	**961.03**	**961.03**	**961.03**	**961.03**	**961.03**	**961.03**	0.20	961.03
F4	20	3	**6437.33**	**6437.33**	**6437.33**	**6437.33**	**6437.33**	**6437.33**	0.23	6437.33
F5	20	5	**1007.05**	**1007.05**	1008.76	**1007.05**	**1007.05**	**1007.05**	0.23	1007.05
F6	20	3	**6516.47**	**6516.47**	**6516.47**	**6516.47**	**6516.47**	**6516.47**	0.23	6516.47
F13	50	6	**2406.36**	**2406.36**	2411.31	**2406.36**	2406.57	**2406.36**	1.02	2406.36
F14	50	3	**9119.03**	**9119.03**	**9119.03**	**9119.03**	**9119.03**	**9119.03**	0.88	9119.03
F15	50	3	**2586.37**	**2586.37**	**2586.37**	**2586.37**	**2586.37**	**2586.37**	0.73	2586.37
F16	50	3	**2720.43**	2729.08	2724.55	**2720.43**	**2720.43**	**2720.43**	0.66	2720.43
F17	75	4	1741.95	1746.09	1744.23	**1734.53**	1735.37	**1734.53**	1.75	1734.53
F18	75	6	**2369.65**	**2369.65**	2373.79	**2369.65**	2374.16	**2369.65**	1.73	2369.65
F19	100	3	8665.05	8665.12	8662.54	**8661.81**	8663.97	8662.86	3.70	8661.81
F20	100	3	4044.68	4044.78	4038.63	4032.81	4037.77	4034.42	2.26	4029.74
Time			8.34 min	0.71 min	0.15 min		1.13 min			
Gap			+0.07%	+0.12%	+0.13%	+0.01%	+0.04%	+0.01%		
CPU			PM 1.7G	PM 1.8G	I7 2.93G		Opt 2.4G			

Table 12: Results on the VFMP-V, only variable vehicle costs, instances of Golden (1984)

Inst	n	w	ISW09	P09	SPUO12		UHGS			BKS
			Best 5-7	Best 5	Avg 10	Best 10	Avg 10	Best 10	T(min)	
V3	20	5	NC	NC	**623.22**	**623.22**	**623.22**	**623.22**	0.17	623.22
V4	20	3	NC	NC	387.34	**387.18**	**387.18**	**387.18**	0.19	387.18
V5	20	5	NC	NC	**742.87**	**742.87**	**742.87**	**742.87**	0.20	742.87
V6	20	3	NC	NC	**415.03**	**415.03**	**415.03**	**415.03**	0.22	415.03
V13	50	6	**1491.86**	**1491.86**	1492.01	**1491.86**	**1491.86**	**1491.86**	0.72	1491.86
V14	50	3	**603.21**	**603.21**	605.00	**603.21**	**603.21**	**603.21**	0.56	603.21
V15	50	3	**999.82**	**999.82**	1001.03	**999.82**	**999.82**	**999.82**	0.61	999.82
V16	50	3	**1131.00**	**1131.00**	1131.85	**1131.00**	**1131.00**	**1131.00**	0.57	1131.00
V17	75	4	**1038.60**	**1038.60**	1042.48	**1038.60**	**1038.60**	**1038.60**	1.14	1038.60
V18	75	6	**1800.80**	**1800.80**	1802.89	**1800.80**	1801.40	1801.40	1.34	1800.80
V19	100	3	**1105.44**	**1105.44**	1106.71	**1105.44**	1106.93	**1105.44**	1.71	1105.44
V20	100	3	1533.24	1535.12	1534.23	**1530.43**	1531.82	**1530.43**	2.80	1530.43
Time			8.85 min	0.41 min	0.06 min		0.85 min			
Gap			+0.02%	+0.03%	+0.17%	+0.00%	+0.03%	+0.00%		
CPU			PM 1.7G	PM 1.8G	I7 2.93G		Opt 2.4G			

Table 13: Results on the VFMP-FV, fixed and variable vehicle costs, instances of Golden (1984)

Inst	**n**	**w**	**ISW09**	**P09**	**SPUO12**		**UHGS**			**BKS**
			Best 5-7	Best 5	Avg 10	Best 10	Avg 10	Best 10	T(min)	
FV3	20	5	**1144.22**	**1144.22**	**1144.22**	**1144.22**	**1144.22**	**1144.22**	0.17	1144.22
FV4	20	3	**6437.33**	**6437.33**	**6437.33**	**6437.33**	**6437.33**	**6437.33**	0.23	6437.33
FV5	20	5	**1322.26**	**1322.26**	**1322.26**	**1322.26**	**1322.26**	**1322.26**	0.17	1322.26
FV6	20	3	**6516.47**	**6516.47**	**6516.47**	**6516.47**	**6516.47**	**6516.47**	0.23	6516.47
FV13	50	6	**2964.65**	**2964.65**	**2964.65**	**2964.65**	**2964.65**	**2964.65**	0.51	2964.65
FV14	50	3	**9126.90**	**9126.90**	**9126.90**	**9126.90**	**9126.90**	**9126.90**	0.79	9126.90
FV15	50	3	**2634.96**	2635.21	**2634.96**	**2634.96**	2635.06	**2634.96**	0.71	2634.96
FV16	50	3	3169.10	3169.14	**3168.92**	**3168.92**	**3168.92**	**3168.92**	0.80	3168.92
FV17	75	4	2008.14	**2004.48**	2007.12	**2004.48**	2007.04	**2004.48**	1.33	2004.48
FV18	75	6	3157.20	3153.16	3148.91	**3147.99**	3148.99	3148.99	1.28	3147.99
FV19	100	3	8665.88	8664.67	8662.89	**8661.81**	8663.04	**8661.81**	3.91	8661.81
FV20	100	3	4154.87	4154.49	4153.12	**4153.02**	**4153.02**	**4153.02**	1.73	4153.02
Time			8.42 min	0.39 min	0.13 min		0.99 min			
Gap			+0.05%	+0.02%	+0.01%	+0.00%	+0.01%	+0.00%		
CPU			PM 1.7G	PM 1.8G	I7 2.93G		Opt 2.4G			

Table 14: Results on the LDVRP, instances of Christofides et al. (1979)

Inst	**n**	**XZKX12**		**UHGS**			**BKS**
		Avg 10	Best 10	Avg 10	Best 10	T(min)	
p01	50	751.43	751.11	<u>**746.39**</u>	<u>**746.39**</u>	0.50	751.11
p02	75	1188.62	1179.53	<u>**1172.62**</u>	<u>**1172.62**</u>	0.99	1179.53
p03	100	1153.56	**1147.83**	**1147.83**	**1147.83**	1.49	1147.83
p04	150	1461.69	1452.88	<u>**1446.64**</u>	<u>**1446.64**</u>	4.49	1452.88
p05	199	1865.30	1844.87	1840.54	<u>**1834.31**</u>	6.26	1844.87
p11	120	1516.42	1513.48	<u>**1511.99**</u>	<u>**1511.99**</u>	1.74	1513.48
p12	100	1175.59	**1174.02**	**1174.02**	**1174.02**	0.92	1174.02
Time		1.3 min		2.34 min			
Gap		+0.48%	+0.00%	-0.28%	-0.33%		
CPU		—		Opt 2.2G			

Table 15: Results for the LDVRP, instances of Golden et al. (1998)

Inst	**n**	**XZKX12**		**UHGS**			**BKS**
		Avg 10	Best 10	Avg 10	Best 10	T(min)	
pr01	240	7714.29	7683.952	7661.10	**7660.64**	12.24	7683.95
pr02	320	11195.02	11172.71	11178.93	**11148.74**	25.71	11172.71
pr03	400	14566.73	14497.64	14525.36	**14480.67**	25.90	14497.64
pr04	480	18605.37	18327.03	18225.56	**18206.84**	30.43	18327.03
pr05	200	8576.91	8561.53	8457.61	**8457.60**	5.71	8561.53
pr06	280	11121.04	11102.22	11056.72	**11056.47**	12.51	11102.22
pr07	360	13477.07	13422.16	13408.06	**13392.93**	27.56	13422.16
pr08	440	16098.60	15928.26	15538.15	**15491.34**	29.76	15928.26
pr09	255	858.34	850.80	835.55	**834.73**	23.17	850.80
pr10	323	1090.85	1083.00	1062.66	**1061.36**	27.86	1083.00
pr11	399	1360.20	1352.32	1319.47	**1316.59**	30.34	1352.32
pr12	483	1661.07	1630.81	1599.59	**1596.68**	30.02	1630.81
pr13	252	1269.37	1261.93	1235.32	**1232.99**	13.84	1261.93
pr14	320	1604.83	1595.48	1564.18	**1562.73**	22.87	1595.48
pr15	396	1987.76	1970.43	1934.13	**1930.84**	28.80	1970.43
pr16	480	2408.72	2391.12	2340.21	**2337.60**	30.00	2391.12
pr17	240	1033.88	1027.21	1018.17	**1018.02**	11.22	1027.21
pr18	300	1469.97	1462.31	1440.00	**1435.34**	20.15	1462.31
pr19	360	2014.26	2007.62	1967.85	**1966.77**	28.32	2007.62
pr20	420	2699.29	2687.85	2626.61	**2621.48**	30.18	2687.85
Time		3.3 min		23.81 min			
Gap		+0.66%	+0.00%	-1.38%	-1.52%		
CPU		—		Opt 2.2G			

Table 16: Results on the GVRP, instances of Bektas et al. (2011)

Inst	**n**	**c**	**m**	**BER11**	**MCL11**	**UHGS**			**BKS**
				Single	Single	Avg 10	Best 10	T(min)	
A-n32-k5-C16-V2	32	16	2	**519.00**	**519.00**	**519.00**	**519.00**	0.64	519.00
A-n33-k5-C17-V3	33	17	3	**451.00**	**451.00**	**451.00**	**451.00**	0.69	451.00
A-n33-k6-C17-V3	33	17	3	**465.00**	**465.00**	**465.00**	**465.00**	0.69	465.00
A-n34-k5-C17-V3	34	17	3	**489.00**	**489.00**	**489.00**	**489.00**	0.73	489.00
A-n36-k5-C18-V2	36	18	2	**505.00**	**505.00**	**505.00**	**505.00**	0.83	505.00
A-n37-k5-C19-V3	37	19	3	**432.00**	**432.00**	**432.00**	**432.00**	0.76	432.00
A-n37-k6-C19-V3	37	19	3	**584.00**	**584.00**	**584.00**	**584.00**	0.83	584.00
A-n38-k5-C19-V3	38	19	3	**476.00**	**476.00**	**476.00**	**476.00**	0.89	476.00
A-n39-k5-C20-V3	39	20	3	**557.00**	**557.00**	**557.00**	**557.00**	0.99	557.00
A-n39-k6-C20-V3	39	20	3	**544.00**	**544.00**	**544.00**	**544.00**	1.05	544.00
A-n44-k6-C22-V3	44	22	3	**608.00**	**608.00**	**608.00**	**608.00**	1.36	608.00
A-n45-k6-C23-V4	45	23	4	**613.00**	**613.00**	**613.00**	**613.00**	1.10	613.00
A-n45-k7-C23-V4	45	23	4	**674.00**	**674.00**	**674.00**	**674.00**	1.21	674.00
A-n46-k7-C23-V4	46	23	4	**593.00**	**593.00**	**593.00**	**593.00**	1.00	593.00
A-n48-k7-C24-V4	48	24	4	**667.00**	**667.00**	**667.00**	**667.00**	1.26	667.00
A-n53-k7-C27-V4	53	27	4	**603.00**	**603.00**	**603.00**	**603.00**	1.33	603.00
A-n54-k7-C27-V4	54	27	4	**690.00**	**690.00**	**690.00**	**690.00**	1.39	690.00
A-n55-k9-C28-V5	55	28	5	**699.00**	**699.00**	**699.00**	**699.00**	1.32	699.00
A-n60-k9-C30-V5	60	30	5	**769.00**	**769.00**	**769.00**	**769.00**	1.46	769.00
A-n61-k9-C31-V5	61	31	5	**638.00**	**638.00**	**638.00**	**638.00**	1.59	638.00
A-n62-k8-C31-V4	62	31	4	**740.00**	**740.00**	**740.00**	**740.00**	1.89	740.00
A-n63-k10-C32-V5	63	32	5	**801.00**	**801.00**	**801.00**	**801.00**	1.63	801.00
A-n63-k9-C32-V5	63	32	5	**912.00**	**912.00**	**912.00**	**912.00**	1.71	912.00
A-n64-k9-C32-V5	64	32	5	**763.00**	**763.00**	**763.00**	**763.00**	1.84	763.00
A-n65-k9-C33-V5	65	33	5	**682.00**	**682.00**	**682.00**	**682.00**	1.62	682.00
A-n69-k9-C35-V5	69	35	5	**680.00**	**680.00**	**680.00**	**680.00**	1.83	680.00
A-n80-k10-C40-V5	80	40	5	**997.00**	**997.00**	**997.00**	**997.00**	2.65	997.00
B-n31-k5-C16-V3	31	16	3	**441.00**	**441.00**	**441.00**	**441.00**	0.63	441.00
B-n34-k5-C17-V3	34	17	3	**472.00**	**472.00**	**472.00**	**472.00**	0.70	472.00
B-n35-k5-C18-V3	35	18	3	**626.00**	**626.00**	**626.00**	**626.00**	0.65	626.00
B-n38-k6-C19-V3	38	19	3	**451.00**	**451.00**	**451.00**	**451.00**	0.86	451.00
B-n39-k5-C20-V3	39	20	3	**357.00**	**357.00**	**357.00**	**357.00**	0.79	357.00
B-n41-k6-C21-V3	41	21	3	**481.00**	**481.00**	**481.00**	**481.00**	1.08	481.00
B-n43-k6-C22-V3	43	22	3	**483.00**	**483.00**	**483.00**	**483.00**	1.15	483.00
B-n44-k7-C22-V4	44	22	4	**540.00**	**540.00**	**540.00**	**540.00**	1.13	540.00
B-n45-k5-C23-V3	45	23	3	**497.00**	**497.00**	**497.00**	**497.00**	1.28	497.00
B-n45-k6-C23-V4	45	23	4	**478.00**	**478.00**	**478.00**	**478.00**	1.23	478.00
B-n50-k7-C25-V4	50	25	4	**449.00**	**449.00**	**449.00**	**449.00**	1.08	449.00
B-n50-k8-C25-V5	50	25	5	**916.00**	**916.00**	**916.00**	**916.00**	1.36	916.00
B-n51-k7-C26-V4	51	26	4	**651.00**	**651.00**	**651.00**	**651.00**	1.44	651.00
B-n52-k7-C26-V4	52	26	4	**450.00**	**450.00**	**450.00**	**450.00**	1.18	450.00
B-n56-k7-C28-V4	56	28	4	**486.00**	492.00	**486.00**	**486.00**	1.35	486.00
B-n57-k7-C29-V4	57	29	4	**751.00**	**751.00**	**751.00**	**751.00**	1.43	751.00
B-n57-k9-C29-V5	57	29	5	**942.00**	**942.00**	**942.00**	**942.00**	1.60	942.00
B-n63-k10-C32-V5	63	32	5	**816.00**	**816.00**	**816.00**	**816.00**	1.74	816.00
B-n64-k9-C32-V5	64	32	5	**509.00**	**509.00**	**509.00**	**509.00**	1.37	509.00
B-n66-k9-C33-V5	66	33	5	**808.00**	**808.00**	**808.00**	**808.00**	1.88	808.00
B-n67-k10-C34-V5	67	34	5	**673.00**	**673.00**	**673.00**	**673.00**	1.91	673.00
B-n68-k9-C34-V5	68	34	5	**704.00**	**704.00**	**704.00**	**704.00**	1.87	704.00
B-n78-k10-C39-V5	78	39	5	**803.00**	804.00	**803.00**	**803.00**	2.38	803.00
G-n262-k25-C131-V12	262	131	12	3249.00	3319.00	3241.80	<u>**3229.00**</u>	22.98	3249.00
M-n101-k10-C51-V5	101	51	5	**542.00**	**542.00**	**542.00**	**542.00**	2.85	542.00
M-n121-k7-C61-V4	121	61	4	**719.00**	720.00	**719.00**	**719.00**	5.45	719.00

Table 17: Results on the GVRP, instances of Bektas et al. (2011) (continued)

Inst	**n**	**c**	**m**	**BER11**	**MCL11**	**UHGS**			**BKS**
				Single	Single	Avg 10	Best 10	T(min)	
M-n151-k12-C76-V6	151	76	6	**659.00**	**659.00**	**659.00**	**659.00**	4.94	659.00
M-n200-k16-C100-V8	200	100	8	**791.00**	805.00	**786.00**	**786.00**	11.47	791.00
P-n101-k4-C51-V2	101	51	2	**455.00**	**455.00**	**455.00**	**455.00**	4.83	455.00
P-n16-k8-C8-V5	16	8	5	**239.00**	**239.00**	**239.00**	**239.00**	0.09	239.00
P-n19-k2-C10-V2	19	10	2	**147.00**	**147.00**	**147.00**	**147.00**	0.15	147.00
P-n20-k2-C10-V2	20	10	2	**154.00**	**154.00**	**154.00**	**154.00**	0.15	154.00
P-n21-k2-C11-V2	21	11	2	**160.00**	162.00	**160.00**	**160.00**	0.19	160.00
P-n22-k2-C11-V2	22	11	2	**162.00**	163.00	**162.00**	**162.00**	0.24	162.00
P-n22-k8-C11-V5	22	11	5	**314.00**	**314.00**	**314.00**	**314.00**	0.18	314.00
P-n23-k8-C12-V5	23	12	5	**312.00**	**312.00**	**312.00**	**312.00**	0.23	312.00
P-n40-k5-C20-V3	40	20	3	**294.00**	**294.00**	**294.00**	**294.00**	1.00	294.00
P-n45-k5-C23-V3	45	23	3	**337.00**	**337.00**	**337.00**	**337.00**	1.13	337.00
P-n50-k10-C25-V5	50	25	5	**410.00**	**410.00**	**410.00**	**410.00**	1.08	410.00
P-n50-k7-C25-V4	50	25	4	**353.00**	**353.00**	**353.00**	**353.00**	1.23	353.00
P-n50-k8-C25-V4	50	25	4	**392.00**	421.00	**392.00**	**392.00**	1.39	392.00
P-n51-k10-C26-V6	51	26	6	**427.00**	**427.00**	**427.00**	**427.00**	1.04	427.00
P-n55-k10-C28-V5	55	28	5	**415.00**	**415.00**	**415.00**	**415.00**	1.22	415.00
P-n55-k15-C28-V8	55	28	8	**555.00**	565.00	**555.00**	**555.00**	1.07	555.00
P-n55-k7-C28-V4	55	28	4	**361.00**	**361.00**	**361.00**	**361.00**	1.41	361.00
P-n55-k8-C28-V4	55	28	4	**361.00**	**361.00**	**361.00**	**361.00**	1.36	361.00
P-n60-k10-C30-V5	60	30	5	**443.00**	**443.00**	**443.00**	**443.00**	1.52	443.00
P-n60-k15-C30-V8	60	30	8	**565.00**	**565.00**	**565.00**	**565.00**	1.24	565.00
P-n65-k10-C33-V5	65	33	5	**487.00**	**487.00**	**487.00**	**487.00**	1.55	487.00
P-n70-k10-C35-V5	70	35	5	**485.00**	**485.00**	**485.00**	**485.00**	1.74	485.00
P-n76-k4-C38-V2	76	38	2	**383.00**	**383.00**	**383.00**	**383.00**	2.80	383.00
P-n76-k5-C38-V3	76	38	3	**405.00**	**405.00**	**405.00**	**405.00**	2.44	405.00
A-n32-k5-C11-V2	32	11	2	**386.00**	**386.00**	**386.00**	**386.00**	0.33	386.00
A-n33-k5-C11-V2	33	11	2	318.00	**315.00**	**315.00**	**315.00**	0.30	315.00
A-n33-k6-C11-V2	33	11	2	**370.00**	**370.00**	**370.00**	**370.00**	0.28	370.00
A-n34-k5-C12-V2	34	12	2	**419.00**	**419.00**	**419.00**	**419.00**	0.39	419.00
A-n36-k5-C12-V2	36	12	2	**396.00**	**396.00**	**396.00**	**396.00**	0.39	396.00
A-n37-k5-C13-V2	37	13	2	**347.00**	**347.00**	**347.00**	**347.00**	0.42	347.00
A-n37-k6-C13-V2	37	13	2	**431.00**	**431.00**	**431.00**	**431.00**	0.47	431.00
A-n38-k5-C13-V2	38	13	2	**367.00**	**367.00**	**367.00**	**367.00**	0.46	367.00
A-n39-k5-C13-V2	39	13	2	**364.00**	**364.00**	**364.00**	**364.00**	0.43	364.00
A-n39-k6-C13-V2	39	13	2	**403.00**	**403.00**	**403.00**	**403.00**	0.53	403.00
A-n44-k6-C15-V2	44	15	2	**503.00**	**503.00**	**503.00**	**503.00**	0.74	503.00
A-n45-k6-C15-V3	45	15	3	**474.00**	**474.00**	**474.00**	**474.00**	0.65	474.00
A-n45-k7-C15-V3	45	15	3	**475.00**	**475.00**	**475.00**	**475.00**	0.62	475.00
A-n46-k7-C16-V3	46	16	3	**462.00**	**462.00**	**462.00**	**462.00**	0.87	462.00
A-n48-k7-C16-V3	48	16	3	**451.00**	**451.00**	**451.00**	**451.00**	0.90	451.00
A-n53-k7-C18-V3	53	18	3	**440.00**	**440.00**	**440.00**	**440.00**	1.10	440.00
A-n54-k7-C18-V3	54	18	3	**482.00**	**482.00**	**482.00**	**482.00**	1.04	482.00
A-n55-k9-C19-V3	55	19	3	**473.00**	**473.00**	**473.00**	**473.00**	1.10	473.00
A-n60-k9-C20-V3	60	20	3	**595.00**	**595.00**	**595.00**	**595.00**	1.33	595.00
A-n61-k9-C21-V4	61	21	4	**473.00**	**473.00**	**473.00**	**473.00**	1.18	473.00
A-n62-k8-C21-V3	62	21	3	**596.00**	**596.00**	**596.00**	**596.00**	1.61	596.00
A-n63-k10-C21-V4	63	21	4	**593.00**	**593.00**	**593.00**	**593.00**	1.33	593.00
A-n63-k9-C21-V3	63	21	3	**642.00**	643.00	**642.00**	**642.00**	1.51	642.00
A-n64-k9-C22-V3	64	22	3	**536.00**	**536.00**	**536.00**	**536.00**	1.64	536.00
A-n65-k9-C22-V3	65	22	3	**500.00**	**500.00**	**500.00**	**500.00**	1.53	500.00
A-n69-k9-C23-V3	69	23	3	**520.00**	**520.00**	**520.00**	**520.00**	1.60	520.00
A-n80-k10-C27-V4	80	27	4	**710.00**	**710.00**	**710.00**	**710.00**	2.14	710.00
B-n31-k5-C11-V2	31	11	2	**356.00**	**356.00**	**356.00**	**356.00**	0.35	356.00

Table 18: Results on the GVRP, instances of Bektas et al. (2011) (continued)

Inst	n	c	m	BER11	MCL11	UHGS			BKS
				Single	Single	Avg 10	Best 10	T(min)	
B-n34-k5-C12-V2	34	12	2	**369.00**	**369.00**	**369.00**	**369.00**	0.35	369.00
B-n35-k5-C12-V2	35	12	2	**501.00**	**501.00**	**501.00**	**501.00**	0.36	501.00
B-n38-k6-C13-V2	38	13	2	**370.00**	**370.00**	**370.00**	**370.00**	0.53	370.00
B-n39-k5-C13-V2	39	13	2	**280.00**	**280.00**	**280.00**	**280.00**	0.42	280.00
B-n41-k6-C14-V2	41	14	2	**407.00**	**407.00**	**407.00**	**407.00**	0.57	407.00
B-n43-k6-C15-V2	43	15	2	**343.00**	**343.00**	**343.00**	**343.00**	0.73	343.00
B-n44-k7-C15-V3	44	15	3	**395.00**	**395.00**	**395.00**	**395.00**	0.54	395.00
B-n45-k5-C15-V2	45	15	2	422.00	**410.00**	**410.00**	**410.00**	0.73	410.00
B-n45-k6-C15-V2	45	15	2	**336.00**	**336.00**	**336.00**	**336.00**	0.76	336.00
B-n50-k7-C17-V3	50	17	3	**393.00**	**393.00**	**393.00**	**393.00**	1.00	393.00
B-n50-k8-C17-V3	50	17	3	**598.00**	**598.00**	**598.00**	**598.00**	0.98	598.00
B-n51-k7-C17-V3	51	17	3	**511.00**	**511.00**	**511.00**	**511.00**	0.72	511.00
B-n52-k7-C18-V3	52	18	3	**359.00**	**359.00**	**359.00**	**359.00**	0.99	359.00
B-n56-k7-C19-V3	56	19	3	**356.00**	**356.00**	**356.00**	**356.00**	1.10	356.00
B-n57-k7-C19-V3	57	19	3	**558.00**	**558.00**	**558.00**	**558.00**	1.26	558.00
B-n57-k9-C19-V3	57	19	3	**681.00**	**681.00**	**681.00**	**681.00**	1.18	681.00
B-n63-k10-C21-V3	63	21	3	**599.00**	**599.00**	**599.00**	**599.00**	1.61	599.00
B-n64-k9-C22-V4	64	22	4	**452.00**	**452.00**	**452.00**	**452.00**	1.34	452.00
B-n66-k9-C22-V3	66	22	3	**609.00**	**609.00**	**609.00**	**609.00**	1.43	609.00
B-n67-k10-C23-V4	67	23	4	**558.00**	**558.00**	**558.00**	**558.00**	1.37	558.00
B-n68-k9-C23-V3	68	23	3	**523.00**	**523.00**	**523.00**	**523.00**	1.71	523.00
B-n78-k10-C26-V4	78	26	4	**606.00**	**606.00**	**606.00**	**606.00**	1.83	606.00
G-n262-k25-C88-V9	262	88	9	2476.00	2463.00	2469.00	**<u>2460.00</u>**	13.80	2463.00
M-n101-k10-C34-V4	101	34	4	**458.00**	**458.00**	**458.00**	**458.00**	2.62	458.00
M-n121-k7-C41-V3	121	41	3	**527.00**	**527.00**	**527.00**	**527.00**	4.18	527.00
M-n151-k12-C51-V4	151	51	4	**483.00**	**483.00**	**483.00**	**483.00**	5.19	483.00
M-n200-k16-C67-V6	200	67	6	**605.00**	**605.00**	**605.00**	**605.00**	6.06	605.00
P-n101-k4-C34-V2	101	34	2	374.00	**370.00**	**370.00**	**370.00**	3.26	370.00
P-n16-k8-C6-V4	16	6	4	**170.00**	**170.00**	**170.00**	**170.00**	0.05	170.00
P-n19-k2-C7-V1	19	7	1	**111.00**	**111.00**	**111.00**	**111.00**	0.07	111.00
P-n20-k2-C7-V1	20	7	1	**117.00**	**117.00**	**117.00**	**117.00**	0.07	117.00
P-n21-k2-C7-V1	21	7	1	**117.00**	**117.00**	**117.00**	**117.00**	0.07	117.00
P-n22-k2-C8-V1	22	8	1	**111.00**	**111.00**	**111.00**	**111.00**	0.10	111.00
P-n22-k8-C8-V4	22	8	4	**249.00**	**249.00**	**249.00**	**249.00**	0.12	249.00
P-n23-k8-C8-V3	23	8	3	**174.00**	**174.00**	**174.00**	**174.00**	0.13	174.00
P-n40-k5-C14-V2	40	14	2	**213.00**	**213.00**	**213.00**	**213.00**	0.55	213.00
P-n45-k5-C15-V2	45	15	2	**238.00**	**238.00**	**238.00**	**238.00**	0.71	238.00
P-n50-k10-C17-V4	50	17	4	**292.00**	**292.00**	**292.00**	**292.00**	0.69	292.00
P-n50-k7-C17-V3	50	17	3	**261.00**	**261.00**	**261.00**	**261.00**	0.84	261.00
P-n50-k8-C17-V3	50	17	3	**262.00**	**262.00**	**262.00**	**262.00**	0.84	262.00
P-n51-k10-C17-V4	51	17	4	**309.00**	**309.00**	**309.00**	**309.00**	0.73	309.00
P-n55-k10-C19-V4	55	19	4	**301.00**	**301.00**	**301.00**	**301.00**	1.01	301.00
P-n55-k15-C19-V6	55	19	6	**378.00**	**378.00**	**378.00**	**378.00**	0.76	378.00
P-n55-k7-C19-V3	55	19	3	**271.00**	**271.00**	**271.00**	**271.00**	1.15	271.00
P-n55-k8-C19-V3	55	19	3	**274.00**	**274.00**	**274.00**	**274.00**	1.15	274.00
P-n60-k10-C20-V4	60	20	4	**325.00**	**325.00**	**325.00**	**325.00**	1.31	325.00
P-n60-k15-C20-V5	60	20	5	**382.00**	**382.00**	**382.00**	**382.00**	1.10	382.00
P-n65-k10-C22-V4	65	22	4	**372.00**	**372.00**	**372.00**	**372.00**	1.25	372.00
P-n70-k10-C24-V4	70	24	4	**385.00**	**385.00**	**385.00**	**385.00**	1.55	385.00
P-n76-k4-C26-V2	76	26	2	320.00	**309.00**	**309.00**	**309.00**	2.02	309.00
P-n76-k5-C26-V2	76	26	2	**309.00**	**309.00**	**309.00**	**309.00**	2.31	309.00
Time				0.01 min	0.34 min		1.53 min		
Gap				+0.06%	+0.11%	+0.00%	-0.01%		
CPU				Opt 2.4G	Duo 1.83G		Opt 2.4G		

Table 19: Results on the OVRP, instances of Christofides et al. (1979) and Fisher (1994)

Inst	n	m_{BKS}	ZK10		RTBI10	S12		UHGS			BKS
			Avg 10	Best 10	Single	Avg 50	Best 50	Avg 10	Best 10	T(min)	
p01	50	5	**416.06**	**416.06**	**416.06**	**416.06**	**416.06**	**416.06**	**416.06**	0.41	416.06
p02	75	10	568.38	**567.14**	**567.14**	**567.14**	**567.14**	568.15	**567.14**	0.51	567.14
p03	100	8	639.98	**639.74**	**639.74**	639.81	**639.74**	**639.74**	**639.74**	0.85	639.74
p04	150	12	733.93	**733.13**	**733.13**	**733.13**	**733.13**	**733.13**	**733.13**	1.73	733.13
p05	199	16	895.62	893.39	894.11	895.55	**883.50**	890.15	884.08	4.13	883.50
p06	50	6	—	—	**412.96**	**412.96**	**412.96**	**412.96**	**412.96**	0.55	412.96
p07	75	10	—	—	584.15	*582.07**	**583.19**	584.59	**583.19**	0.77	583.19
p08	100	9	—	—	**644.63**	644.95	**644.63**	644.79	**644.63**	1.79	644.63
p09	150	13	—	—	764.56	759.38	757.91	760.75	**757.07**	5.18	757.84
p10	199	17	—	—	888.46	877.68	**874.71**	875.49	**874.71**	6.10	874.71
p11	120	7	682.34	**682.12**	**682.12**	**682.12**	**682.12**	**682.12**	**682.12**	1.51	682.12
p12	100	10	**534.24**	**534.24**	**534.24**	**534.24**	**534.24**	**534.24**	**534.24**	0.61	534.24
p13	120	11	—	—	910.26	904.02	**899.16**	900.22	**899.16**	3.39	899.16
p14	100	11	—	—	**591.87**	**591.87**	**591.87**	**591.87**	**591.87**	1.70	591.87
f11	71	4	**177.00**	**177.00**	**177.00**	177.21	**177.00**	**177.00**	**177.00**	0.65	177.00
f12	134	7	770.57	**769.55**	**769.55**	770.00	**769.55**	769.68	**769.55**	1.71	769.55
Time			—		9.54 min	2.39 min		1.97 min			
Gap			—	—	+0.32%	*+0.16%**	+0.00%	+0.11%	+0.00%		
CPU			T5500 1.66G		P-IV 2.8G	I7 2.93G		Opt 2.4G			

∗ The minimum fleet size was not attained by S12 on all runs.

Table 20: Results on the OVRP, instances of Golden et al. (1998)

Inst	n	m^*	ZK10		RTBI10	S12		UHGS			BKS
			Avg 10	Best 10	Single	Avg 50	Best 50	Avg 10	Best 10	T(min)	
pr01	240	9	4562.88	4557.38	4583.70	4551.74	4544.46	4546.35	**4543.00**	8.20	4544.46
pr02	320	10	7264.32	7253.20	7271.24	7229.56	7215.48	7218.41	**7213.56**	14.94	7215.48
pr03	400	9	9824.44	9793.72	9821.09	9784.52	9773.83	9763.39	**9750.63**	22.66	9773.83
pr04	480	10	12430.06	12415.36	12428.20	12393.40	12389.43	12387.82	**12380.66**	26.41	12389.43
pr05	200	5	**6018.52**	**6018.52**	**6018.52**	**6018.52**	**6018.52**	**6018.52**	**6018.52**	5.71	6018.52
pr06	280	7	7735.10	7731.00	7733.77	7728.77	7721.16	7704.91	**7704.59**	11.23	7721.16
pr07	360	8	9243.69	9193.15	9254.15	9205.01	9180.93	9132.27	**9127.70**	19.01	9180.93
pr08	440	10	10363.28	10347.70	10363.40	10342.10	10326.57	10316.60	**10289.70**	26.43	10326.57
Time			14.79 min		17.53 min	64.07 min		16.82 min			
Gap			+0.39%	+0.21%	+0.47%	+0.13%	+0.00%	-0.11%	-0.19%		
CPU			T5500 1.66G		P-IV 2.8G	I7 2.93G		Opt 2.4G			

Table 21: Results on the OVRPTW, instances of Solomon and Desrosiers (1988)

Inst	**n**	**RTI09**		KTDHS12		**UHGS**			**BKS**
		Avg 10	Best 10	Avg 10	Best 10	Avg 10	Best 10	T(min)	
R101	100	**19.00/1192.85**	**19/1192.85**	**19.00/1192.95**	**19/1192.85**	**19.00/1192.85**	**19/1192.85**	2.86	19/1192.85
R102	100	17.00/1081.65	**17/1079.39**	**17.00/1079.39**	**17/1079.39**	**17.00/1079.39**	**17/1079.39**	2.81	17/1079.39
R103	100	13.00/1017.28	**13/1016.78**	13.00/1016.83	**13/1016.78**	**13.00/1016.78**	**13/1016.78**	3.57	13/1016.78
R104	100	9.53/844.32	9/869.63	9.00/837.28	9/834.44	9.00/833.52	**9/832.50**	4.86	9/834.44
R105	100	14.00/1055.98	**14/1055.04**	14.00/1055.34	**14/1055.04**	**14.00/1055.04**	**14/1055.04**	3.92	14/1055.04
R106	100	12.00/1001.04	12/1000.95	12.00/1003.15	12/1001.41	12.00/1000.93	**12/1000.36**	4.88	12/1000.95
R107	100	10.00/915.82	10/912.99	10.00/918.47	**10/910.75**	10.00/914.75	10/912.08	6.14	10/910.75
R108	100	9.00/760.30	9/760.30	9.00/765.63	9/760.30	9.00/760.32	**9/759.86**	4.80	9/760.30
R109	100	11.00/934.77	11/934.53	11.00/937.86	**11/934.15**	11.00/934.52	**11/934.15**	4.36	11/934.15
R110	100	10.00/851.01	**10/846.49**	10.00/881.91	10/874.64	10.00/885.18	**10/873.75**	5.08	10/846.49
R111	100	10.00/902.45	**10/895.21**	10.00/904.25	10/895.56	**10.00/895.21**	**10/895.21**	5.34	10/895.21
R112	100	9.47/814.33	9/811.73	9.00/815.43	9/805.17	9.00/811.76	**9/801.43**	5.81	9/805.17
C101	100	**10.00/556.18**	**10/556.18**	**10.00/556.18**	**10/556.18**	**10.00/556.18**	**10/556.18**	0.67	10/556.18
C102	100	**10.00/556.18**	**10/556.18**	**10.00/556.18**	**10/556.18**	**10.00/556.18**	**10/556.18**	1.00	10/556.18
C103	100	**10.00/556.18**	**10/556.18**	**10.00/556.18**	**10/556.18**	**10.00/556.18**	**10/556.18**	1.10	10/556.18
C104	100	**10.00/555.41**	**10/555.41**	**10.00/555.41**	**10/555.41**	**10.00/555.41**	**10/555.41**	1.06	10/555.41
C105	100	**10.00/556.18**	**10/556.18**	**10.00/556.18**	**10/556.18**	**10.00/556.18**	**10/556.18**	0.83	10/556.18
C106	100	**10.00/556.18**	**10/556.18**	**10.00/556.18**	**10/556.18**	**10.00/556.18**	**10/556.18**	0.85	10/556.18
C107	100	**10.00/556.18**	**10/556.18**	**10.00/556.18**	**10/556.18**	**10.00/556.18**	**10/556.18**	0.84	10/556.18
C108	100	**10.00/555.80**	**10/555.80**	**10.00/555.80**	**10/555.80**	**10.00/555.80**	**10/555.80**	0.95	10/555.80
C109	100	**10.00/555.80**	**10/555.80**	**10.00/555.80**	**10/555.80**	**10.00/555.80**	**10/555.80**	0.97	10/555.80
RC101	100	14.00/1228.76	**14/1227.37**	**14.00/1227.37**	**14/1227.37**	**14.00/1227.37**	**14/1227.37**	3.37	14/1227.37
RC102	100	12.00/1205.65	12/1203.05	12.00/1197.16	**12/1185.43**	12.50/1125.04	12/1195.20	4.12	12/1185.43
RC103	100	11.00/927.62	11/923.15	11.00/919.32	**11/918.65**	**11.00/918.65**	**11/918.65**	3.92	11/918.65
RC104	100	10.00/789.21	**10/787.02**	10.00/790.58	**10/787.02**	**10.00/787.02**	**10/787.02**	4.51	10/787.02
RC105	100	13.00/1220.96	13/1195.20	13.00/1201.73	13/1195.20	13.10/1186.34	**13/1185.43**	4.45	13/1195.20
RC106	100	11.00/1113.20	11/1095.65	11.00/1073.78	**11/1071.83**	11.00/1074.52	**11/1071.83**	4.24	11/1071.83
RC107	100	11.00/862.44	11/861.28	11.00/862.88	11/861.28	11.00/860.62	**11/860.62**	4.03	11/861.28
RC108	100	10.00/832.05	**10/831.09**	10.00/837.38	10/833.03	10.00/832.27	**10/831.09**	4.31	10/831.09
R201	100	**4.00/1182.43**	**4/1182.43**	4.00/1187.99	**4/1182.43**	**4.00/1182.43**	**4/1182.43**	5.80	4/1182.43
R202	100	3.00/1150.07	**3/1149.59**	3.00/1152.70	3/1151.14	**3.00/1149.59**	**3/1149.59**	9.73	3/1149.59
R203	100	3.00/894.34	**3/889.12**	3.00/900.51	3/894.40	3.00/890.02	**3/889.12**	11.72	3/889.12
R204	100	2.00/803.54	2/801.46	2.00/821.67	2/803.50	2.00/798.13	**2/797.83**	7.67	2/801.46
R205	100	3.00/950.21	**3/943.33**	3.00/966.18	3/952.83	**3.00/943.33**	**3/943.33**	11.21	3/943.33
R206	100	3.00/870.96	3/869.27	3.00/883.18	3/874.78	**3.00/865.32**	**3/865.32**	13.01	3/869.27
R207	100	2.77/865.93	2/857.08	2.00/880.56	2/857.08	**2.00/854.40**	**2/854.40**	6.65	2/857.08
R208	100	2.00/700.67	2/700.53	2.00/710.74	2/700.63	2.00/694.67	**2/694.24**	8.00	2/700.53
R209	100	3.00/853.86	**3/851.69**	3.00/864.91	**3/851.69**	3.00/852.42	**3/851.69**	11.57	3/851.69
R210	100	3.00/894.38	3/892.45	3.00/908.77	3/901.87	3.00/891.23	**3/890.02**	11.00	3/892.45
R211	100	2.00/887.41	2/886.90	2.00/894.55	2/874.49	2.00/852.15	**2/846.92**	8.14	2/874.49
C201	100	**3.00/548.51**	**3/548.51**	**3.00/548.51**	**3/548.51**	**3.00/548.51**	**3/548.51**	1.33	3/548.51
C202	100	**3.00/548.51**	**3/548.51**	**3.00/548.51**	**3/548.51**	**3.00/548.51**	**3/548.51**	2.09	3/548.51
C203	100	**3.00/548.13**	**3/548.13**	**3.00/548.13**	**3/548.13**	**3.00/548.13**	**3/548.13**	2.73	3/548.13
C204	100	**3.00/547.55**	**3/547.55**	3.00/549.02	**3/547.55**	**3.00/547.55**	**3/547.55**	3.21	3/547.55
C205	100	**3.00/545.83**	**3/545.83**	**3.00/545.83**	**3/545.83**	**3.00/545.83**	**3/545.83**	1.76	3/545.83
C206	100	**3.00/545.45**	**3/545.45**	**3.00/545.45**	**3/545.45**	**3.00/545.45**	**3/545.45**	1.87	3/545.45
C207	100	**3.00/545.24**	**3/545.24**	**3.00/545.24**	**3/545.24**	**3.00/545.24**	**3/545.24**	1.92	3/545.24
C208	100	**3.00/545.28**	**3/545.28**	**3.00/545.28**	**3/545.28**	**3.00/545.28**	**3/545.28**	2.16	3/545.28
RC201	100	4.00/1309.06	**4/1303.73**	4.00/1321.87	4/1304.50	**4.00/1303.73**	**4/1303.73**	6.50	4/1303.73
RC202	100	3.00/1329.52	3/1321.43	3.00/1335.13	3/1292.35	3.00/1289.86	**3/1289.04**	10.92	3/1292.35
RC203	100	3.00/995.02	3/993.29	3.00/1004.88	3/993.22	3.00/987.28	**3/977.56**	10.81	3/993.22
RC204	100	3.00/719.92	**3/718.97**	3.00/736.97	3/722.20	**3.00/718.97**	**3/718.97**	9.52	3/718.97
RC205	100	4.00/1190.67	**4/1189.84**	4.00/1193.05	**4/1189.84**	**4.00/1189.84**	**4/1189.84**	7.92	4/1189.84
RC206	100	3.00/1092.09	3/1091.79	3.00/1102.53	3/1092.66	**3.00/1087.97**	**3/1087.97**	10.72	3/1091.79
RC207	100	3.00/1005.32	**3/998.70**	3.00/1015.46	3/1006.06	3.00/999.29	**3/998.70**	9.43	3/998.70
RC208	100	3.00/772.76	3/769.40	3.00/786.41	3/778.32	3.00/769.12	**3/768.75**	11.94	3/769.40
Time		10.0 min		10.0 min		5.27 min			
Gap		+0.89%/+0.42%	0%/+0.24%	0%/+0.79%	0%/+0.18%	+0.09%/-0.10%	0%/-0.10%		
CPU		P-IV 3G		Xe 2.67G		Opt 2.2G			

Table 22: Results on the TDVRPTW. Scenario 1 for time-dependent travel times. Instances of Solomon and Desrosiers (1988).

Inst	n	m	KTDHS12		UHGS				BKS
			Avg 10	Best 10	Avg 10	Best 10	Feas	T(min)	
R101	100	19	NF	NF	NF	NF	0	NF	NF
R102	100	17	1257.35	1251.90	**1244.65**	**1244.65**	10	7.14	1251.90
R103	100	13	1063.09	1044.74	**1027.69**	**1027.69**	10	7.59	1044.74
R104	100	9	884.21	871.04	858.18	**856.83**	10	15.94	871.04
R105	100	14	1204.71	1198.83	1192.17	**1192.06**	10	10.89	1198.83
R106	100	12	1091.37	1077.66	1073.34	**1071.23**	10	10.81	1077.66
R107	100	10	947.49	940.68	938.23	**936.36**	10	16.77	940.68
R108	100	9	838.48	820.88	812.33	**806.94**	10	14.01	820.88
R109	100	11	982.32	964.29	**954.98**	**954.98**	10	14.23	964.29
R110	100	10	935.46	915.37	906.91	**905.12**	10	17.83	915.37
R111	100	10	939.81	922.89	911.94	**909.01**	10	13.44	922.89
R112	100	9	860.37	833.53	822.85	**819.19**	10	18.57	833.53
C101	100	10	843.46	843.46	**829.83**	**829.83**	10	5.72	843.46
C102	100	10	822.42	813.57	**792.69**	**792.69**	10	6.52	813.57
C103	100	10	795.80	791.12	**770.69**	**770.69**	10	6.86	791.12
C104	100	10	751.89	742.65	**733.59**	**733.59**	10	5.89	742.65
C105	100	10	837.63	837.63	**799.88**	**799.88**	10	6.89	837.63
C106	100	10	829.92	829.92	**813.90**	**813.90**	10	6.89	829.92
C107	100	10	830.22	830.22	**798.79**	**798.79**	10	6.88	830.22
C108	100	10	798.23	798.23	**769.03**	**769.03**	10	7.01	798.23
C109	100	10	783.78	782.05	**753.44**	**753.44**	10	6.56	782.05
RC101	100	14	1534.30	1528.60	1523.94	**1516.19**	10	13.50	1528.60
RC102	100	12	1332.03	1312.46	1308.27	**1301.60**	10	14.07	1312.46
RC103	100	11	1093.73	1088.96	1071.40	**1068.81**	10	15.32	1088.96
RC104	100	10	992.50	983.82	952.18	**951.62**	10	11.29	983.82
RC105	100	13	1375.86	1331.29	1334.13	**1330.73**	10	15.63	1331.29
RC106	100	11	1225.86	**1197.62**	**1197.62**	**1197.62**	10	11.86	1197.62
RC107	100	11	1049.10	1043.66	**1025.97**	**1025.97**	10	9.58	1043.66
RC108	100	10	1014.88	999.83	946.87	**946.70**	10	17.57	999.83
R201	100	4	1149.48	1132.33	**1127.55**	**1127.55**	10	23.01	1132.33
R202	100	3	1096.50	1079.97	1069.16	**1067.42**	10	30.00	1079.97
R203	100	3	867.75	850.40	825.77	**825.51**	10	29.37	850.40
R204	100	2	768.38	737.02	711.80	**706.95**	10	30.06	737.02
R205	100	3	943.16	928.32	893.12	**893.10**	10	30.02	928.32
R206	100	3	857.91	836.60	812.95	**812.07**	10	30.04	836.60
R207	100	2	863.58	838.53	778.06	**777.10**	10	30.01	838.53
R208	100	2	696.29	676.85	642.31	**642.07**	10	30.05	676.85
R209	100	3	801.74	787.51	760.81	**757.84**	10	30.00	787.51
R210	100	3	889.18	878.33	840.17	**835.98**	10	30.04	878.33
R211	100	2	861.16	834.05	763.44	**757.13**	10	30.01	834.05
C201	100	3	**687.85**	**687.85**	**687.85**	**687.85**	10	15.08	687.85
C202	100	3	674.53	669.94	**669.71**	**669.71**	10	20.74	669.94
C203	100	3	658.68	650.26	**649.34**	**649.34**	10	21.21	650.26
C204	100	3	655.08	636.09	**627.28**	**627.28**	10	29.25	636.09
C205	100	3	**657.31**	**657.31**	**657.31**	**657.31**	10	17.94	657.31
C206	100	3	652.91	**652.74**	**652.74**	**652.74**	10	19.68	652.74
C207	100	3	**637.56**	**637.56**	**637.56**	**637.56**	10	18.86	637.56
C208	100	3	632.14	**632.07**	**632.07**	**632.07**	10	18.26	632.07
RC201	100	4	1388.04	1380.04	**1353.87**	**1353.87**	10	27.45	1380.04
RC202	100	3	1280.62	1261.00	1228.58	**1218.29**	10	30.01	1261.00
RC203	100	3	959.91	949.37	923.44	**921.53**	10	30.01	949.37
RC204	100	3	764.69	747.93	715.30	**713.42**	10	29.96	747.93
RC205	100	4	1211.44	1190.98	1148.82	**1148.80**	10	28.32	1190.98
RC206	100	3	1124.87	1094.28	1081.40	**1074.54**	10	30.03	1094.28
RC207	100	3	1000.90	965.80	884.82	**881.25**	10	30.00	965.80
RC208	100	3	769.45	753.06	710.09	**705.01**	10	30.00	753.06
Time			10.00 min		18.81 min				
Gap			+1.42%	+0.00%	-2.23%	-2.40%			
CPU			Xe 2.67G		Opt 2.2G				

Table 23: Results on the TDVRPTW. Scenario 2 for time-dependent travel times. Instances of Solomon and Desrosiers (1988).

Inst	**n**	**m**	**KTDHS12**		**UHGS**				**BKS**
			Avg 10	Best 10	Avg 10	Best 10	Feas	T(min)	
R101	100	19	NF	NF	NF	NF	0	NF	NF
R102	100	17	NF	NF	NF	NF	0	NF	NF
R103	100	13	902.75	887.75	868.71	**867.91**	10	12.05	887.75
R104	100	9	788.45	770.74	741.35	**739.67**	10	20.76	770.74
R105	100	14	1128.87	1115.13	1108.06	**1106.53**	10	14.33	1115.13
R106	100	12	969.51	956.49	927.84	**924.24**	10	19.68	956.49
R107	100	10	841.29	828.09	788.52	**785.88**	10	18.87	828.09
R108	100	9	753.85	744.12	697.70	**694.34**	10	18.18	744.12
R109	100	11	828.75	820.63	802.40	**800.74**	10	19.13	820.63
R110	100	10	818.41	805.87	776.47	**774.00**	10	25.78	805.87
R111	100	10	805.12	780.30	758.02	**755.96**	10	19.46	780.30
R112	100	9	739.13	726.37	696.51	**693.77**	10	18.59	726.37
C101	100	10	938.79	938.79	**909.64**	**909.64**	10	7.39	938.79
C102	100	10	903.91	882.57	**831.26**	**831.26**	10	8.03	882.57
C103	100	10	808.15	782.62	**756.16**	**756.16**	10	9.32	782.62
C104	100	10	729.25	707.24	**690.88**	**690.88**	10	10.02	707.24
C105	100	10	890.03	890.03	**868.48**	**868.48**	10	7.50	890.03
C106	100	10	985.14	950.51	**941.54**	**941.54**	10	12.89	950.51
C107	100	10	876.98	875.48	**842.85**	**842.85**	10	8.31	875.48
C108	100	10	782.11	781.82	**765.20**	**765.20**	10	7.55	781.82
C109	100	10	753.26	748.27	**735.40**	**735.40**	10	8.95	748.27
RC101	100	14	1347.83	**1323.17**	1374.09	1368.70	7	19.16	1323.17
RC102	100	12	1136.54	1099.21	1093.30	**1085.49**	10	21.43	1099.21
RC103	100	11	946.09	934.14	902.78	**902.53**	10	16.45	934.14
RC104	100	10	868.43	853.10	811.31	**810.52**	10	19.39	853.10
RC105	100	13	1171.57	1157.13	**1123.10**	**1123.10**	10	10.26	1157.13
RC106	100	11	1081.85	1061.79	1040.33	**1033.77**	10	22.75	1061.79
RC107	100	11	920.69	899.41	869.49	**866.80**	10	18.31	899.41
RC108	100	10	872.04	850.54	806.32	**805.52**	10	16.28	850.54
R201	100	4	1085.02	1062.61	**1053.15**	**1053.15**	10	29.16	1062.61
R202	100	3	1006.56	987.93	969.72	**964.92**	10	30.00	987.93
R203	100	3	804.56	769.34	742.06	**741.43**	10	30.00	769.34
R204	100	2	680.34	666.19	607.90	**601.30**	10	30.01	666.19
R205	100	3	884.45	850.71	810.82	**807.44**	10	30.02	850.71
R206	100	3	797.33	783.52	718.53	**717.75**	10	30.01	783.52
R207	100	2	759.04	725.98	691.82	**675.67**	10	30.01	725.98
R208	100	2	616.52	588.26	548.61	**543.85**	10	30.01	588.26
R209	100	3	712.84	699.02	625.19	**622.91**	10	30.02	699.02
R210	100	3	778.63	764.40	722.49	**716.22**	10	30.00	764.40
R211	100	2	758.16	713.55	656.20	**644.33**	10	30.01	713.55
C201	100	3	790.34	**788.10**	NF	NF	0	NF	788.10
C202	100	3	764.97	757.12	**751.74**	**751.74**	10	28.77	757.12
C203	100	3	733.05	694.39	**690.35**	**690.35**	10	25.11	694.39
C204	100	3	700.42	670.69	**628.67**	**628.27**	10	29.82	670.69
C205	100	3	728.56	**727.15**	**727.15**	**727.15**	10	19.61	727.15
C206	100	3	676.52	**665.33**	**665.33**	**665.33**	10	23.64	665.33
C207	100	3	675.34	674.67	**674.36**	**674.36**	10	21.44	674.67
C208	100	3	640.02	636.29	**636.00**	**636.00**	10	21.73	636.29
RC201	100	4	1344.22	1312.33	1293.48	**1292.56**	10	29.32	1312.33
RC202	100	3	1192.80	1168.10	1102.68	**1098.28**	10	30.04	1168.10
RC203	100	3	887.28	857.83	824.70	**818.77**	10	30.00	857.83
RC204	100	3	660.05	645.97	611.44	**607.98**	10	30.00	645.97
RC205	100	4	1131.87	1111.67	1055.54	**1048.86**	10	30.03	1111.67
RC206	100	3	1062.80	1032.97	995.23	**989.51**	10	30.10	1032.97
RC207	100	3	872.36	835.43	743.73	**743.12**	10	30.00	835.43
RC208	100	3	670.33	641.81	591.67	**586.27**	10	30.00	641.81
Time			10.00 min		21.69 min				
Gap			+2.26%	+0.00%	-3.63%	-3.94%			
CPU			Xe 2.67G		Opt 2.2G				

Table 24: Results on the TDVRPTW. Scenario 3 for time-dependent travel times. Instances of Solomon and Desrosiers (1988).

Inst	**n**	**m**	**KTDHS12**		**UHGS**				**BKS**
			Avg 10	Best 10	Avg 10	Best 10	Feas	T(min)	
R101	100	19	NF	NF	NF	NF	0	NF	NF
R102	100	17	NF	NF	NF	NF	0	NF	NF
R103	100	13	NF	NF	NF	NF	0	NF	NF
R104	100	9	853.79	832.46	817.95	**813.06**	10	26.57	832.46
R105	100	14	NF	NF	NF	NF	0	NF	NF
R106	100	12	1084.34	1066.01	1024.24	**1019.23**	10	23.47	1066.01
R107	100	10	941.97	919.95	862.84	**859.71**	10	21.12	919.95
R108	100	9	827.52	812.56	761.10	**760.26**	10	23.22	812.56
R109	100	11	859.67	837.42	815.10	**812.99**	10	23.31	837.42
R110	100	10	876.13	849.11	805.66	**798.49**	10	28.55	849.11
R111	100	10	857.17	827.99	790.81	**785.37**	10	23.20	827.99
R112	100	9	838.76	793.71	752.65	**747.92**	10	28.03	793.71
C101	100	10	NF	NF	NF	NF	0	NF	NF
C102	100	10	1329.45	1311.99	**1269.54**	**1269.54**	10	10.92	1311.99
C103	100	10	1072.50	1041.54	**981.06**	**981.06**	10	15.12	1041.54
C104	100	10	916.64	894.51	836.17	**836.14**	10	17.31	894.51
C105	100	10	NF	NF	NF	NF	0	NF	NF
C106	100	10	NF	NF	NF	NF	0	NF	NF
C107	100	10	1372.59	1343.68	**1279.38**	**1279.38**	10	13.70	1343.68
C108	100	10	1080.69	1022.15	**999.19**	**999.19**	10	15.13	1022.15
C109	100	10	951.30	928.89	905.70	**904.29**	10	13.46	928.89
RC101	100	14	NF	NF	NF	NF	0	NF	NF
RC102	100	12	1310.70	**1289.28**	1318.06	1304.26	10	28.81	1289.28
RC103	100	11	1090.99	1065.00	1028.33	**1024.48**	10	22.71	1065.00
RC104	100	10	982.88	958.57	902.89	**896.50**	10	22.25	958.57
RC105	100	13	NF	NF	NF	NF	0	NF	NF
RC106	100	11	NF	1161.21	1131.44	**1118.98**	10	27.03	1161.21
RC107	100	11	1020.87	983.72	942.39	**938.91**	10	25.63	983.72
RC108	100	10	1014.18	967.10	902.39	**897.99**	10	24.88	967.10
R201	100	4	1362.68	1349.52	1320.70	**1315.38**	10	30.00	1349.52
R202	100	3	1294.18	1224.77	1144.71	**1141.05**	10	30.01	1224.77
R203	100	3	935.58	910.99	869.19	**867.38**	10	30.01	910.99
R204	100	2	753.51	722.38	640.90	**632.29**	10	30.01	722.38
R205	100	3	1024.49	1002.42	954.34	**944.11**	10	30.00	1002.42
R206	100	3	883.12	865.91	814.67	**805.69**	10	30.00	865.91
R207	100	2	892.48	832.61	776.67	**760.18**	10	30.01	832.61
R208	100	2	647.42	623.78	580.91	**571.58**	10	30.01	623.78
R209	100	3	677.58	634.49	591.28	**583.67**	10	30.00	634.49
R210	100	3	839.39	808.99	766.77	**765.89**	10	30.06	808.99
R211	100	2	818.26	765.52	704.36	**687.64**	10	30.01	765.52
C201	100	3	1259.63	**1258.26**	NF	NF	0	NF	1258.26
C202	100	3	1012.87	**1000.70**	NF	NF	0	NF	1000.70
C203	100	3	860.24	**835.76**	944.84	938.18	10	30.01	835.76
C204	100	3	906.17	850.95	798.71	**789.25**	10	30.02	850.95
C205	100	3	916.38	907.52	NF	NF	0	NF	907.52
C206	100	3	941.36	**919.21**	**919.21**	**919.21**	10	27.66	919.21
C207	100	3	1001.65	981.36	973.93	**945.40**	10	29.80	981.36
C208	100	3	812.24	803.73	**798.62**	**798.62**	10	28.75	803.73
RC201	100	4	NF	1745.53	1710.68	**1709.25**	10	30.00	1745.53
RC202	100	3	1452.26	1391.35	1344.76	**1338.53**	10	30.01	1391.35
RC203	100	3	1028.38	995.73	973.43	**965.94**	10	30.00	995.73
RC204	100	3	686.81	665.00	635.57	**630.70**	10	30.00	665.00
RC205	100	4	1328.71	1306.59	1271.21	**1262.93**	10	30.00	1306.59
RC206	100	3	1281.50	1214.31	1166.72	**1159.44**	10	30.01	1214.31
RC207	100	3	842.13	821.26	729.82	**720.06**	10	30.00	821.26
RC208	100	3	650.86	637.50	562.23	**549.13**	10	30.01	637.50
Time			10.00 min		26.15 min				
Gap			+3.33%	+0.00%	-4.32%	-5.02%			
CPU			Xe 2.67G		Opt 2.2G				

Table 25: Results on the VFMPTW, minimization of duration, type A fleet, instances of Liu and Shen (1999)

Inst	**n**	**w**	**BDHMG08**	**RT10**	**UHGS**			**BKS**
			Best 3	Single	Avg 10	Best 10	T(min)	
R101	100	5	4631.31	**4536.40**	4617.95	4608.62	6.03	4536.40
R102	100	5	4401.30	**4348.92**	4376.11	4369.74	6.38	4348.92
R103	100	5	4182.16	**4119.04**	4149.67	4145.68	4.65	4119.04
R104	100	5	3981.28	3986.35	3965.21	**3961.39**	5.23	3981.28
R105	100	5	4236.84	4229.67	4215.84	**4209.84**	5.11	4229.67
R106	100	5	4118.48	4130.82	4112.20	**4109.08**	6.32	4118.48
R107	100	5	4035.96	4031.16	4012.58	**4007.87**	5.45	4031.16
R108	100	5	3970.26	3962.20	3936.47	**3934.48**	5.12	3962.20
R109	100	5	4060.17	4052.21	4037.40	**4020.75**	5.06	4052.21
R110	100	5	3995.18	3999.09	3971.53	**3965.88**	5.30	3995.18
R111	100	5	4017.81	4016.19	3992.07	**3985.68**	6.05	4016.19
R112	100	5	3947.30	3954.65	3923.21	**3918.88**	6.77	3947.30
C101	100	3	**7226.51**	**7226.51**	**7226.51**	**7226.51**	3.19	7226.51
C102	100	3	**7119.35**	7137.79	**7119.35**	**7119.35**	2.82	7119.35
C103	100	3	7107.01	7141.03	7104.46	**7102.86**	2.46	7107.01
C104	100	3	**7081.50**	7086.70	7081.51	7081.51	2.22	7081.50
C105	100	3	7199.36	**7169.08**	7196.06	7196.06	3.40	7169.08
C106	100	3	7180.03	**7157.13**	7177.41	7176.68	3.67	7157.13
C107	100	3	7149.17	**7135.38**	7144.73	7144.49	3.19	7135.38
C108	100	3	7115.81	7113.57	**7111.23**	**7111.23**	2.78	7113.57
C109	100	3	7094.65	7092.49	**7091.66**	**7091.66**	2.39	7092.49
RC101	100	4	5253.97	5237.19	5225.17	**5217.90**	5.03	5237.19
RC102	100	4	5059.58	5053.62	5044.63	**5018.47**	5.71	5053.48
RC103	100	4	4868.94	4885.58	4830.08	**4822.21**	6.03	4868.94
RC104	100	4	4762.85	4761.28	4741.69	**4737.00**	4.10	4761.28
RC105	100	4	5119.80	5110.86	5110.51	**5097.35**	5.61	5110.86
RC106	100	4	4960.78	4966.27	4947.46	**4935.91**	6.60	4960.78
RC107	100	4	4828.17	4819.91	4791.19	**4783.08**	5.32	4819.91
RC108	100	4	4734.15	4749.44	4710.71	**4708.85**	5.17	4734.15
R201	100	4	3922.00	**3753.42**	3791.54	3782.88	7.66	3753.42
R202	100	4	3610.38	3551.12	3540.39	**3540.03**	13.37	3551.12
R203	100	4	3350.18	3336.60	3314.09	**3311.35**	9.07	3334.08
R204	100	4	3390.14	3103.84	3076.13	**3075.95**	8.87	3103.84
R205	100	4	3465.81	3367.90	3334.35	**3334.27**	9.25	3367.90
R206	100	4	3268.36	3264.70	3246.09	**3242.40**	9.01	3264.70
R207	100	4	3231.26	3158.69	3145.79	**3145.08**	9.40	3158.69
R208	100	4	3063.10	3056.45	3020.52	**3017.12**	8.07	3056.45
R209	100	4	3192.95	3194.74	3186.18	**3183.36**	9.49	3191.63
R210	100	4	3375.38	3325.28	3288.82	**3287.66**	10.21	3325.28
R211	100	4	3042.48	3053.08	3021.67	**3019.93**	9.08	3042.48
C201	100	4	5891.45	**5820.78**	5878.54	5878.54	5.17	5820.78
C202	100	4	5850.26	5783.76	**5776.88**	**5776.88**	5.15	5779.59
C203	100	4	5741.90	**5736.94**	5741.82	5741.12	5.72	5736.94
C204	100	4	5691.51	5718.49	**5680.46**	**5680.46**	4.31	5691.51
C205	100	4	5786.71	**5747.67**	5782.53	5781.15	6.56	5747.67
C206	100	4	5795.15	**5738.09**	5767.70	5767.70	4.74	5738.09
C207	100	4	5743.52	**5721.16**	5731.54	5731.44	5.14	5721.16
C208	100	4	5884.20	5732.95	**5725.03**	**5725.03**	4.52	5732.95
RC201	100	6	4740.21	**4701.88**	4740.49	4737.59	5.28	4701.88
RC202	100	6	4522.36	4509.11	**4487.48**	**4487.48**	4.48	4509.11
RC203	100	6	4312.52	4313.42	4305.63	**4305.49**	5.88	4312.52
RC204	100	6	4141.04	4157.32	4140.16	**4137.93**	6.68	4141.04
RC205	100	6	4652.57	**4585.20**	4625.21	4615.04	6.40	4585.20
RC206	100	6	4431.64	4427.73	4408.63	**4405.16**	5.14	4416.95
RC207	100	6	4310.11	4313.07	4295.07	**4290.14**	6.52	4310.11
RC208	100	6	4091.92	4103.31	4076.12	**4075.04**	5.74	4091.92
Time			13.14 min	16.67 min		5.86 min		
Gap			+0.72%	+0.08%	-0.13%	-0.21%		
CPU			Ath 2.6G	P-IV 3.4G		Opt 2.2G		

Table 26: Results on the VFMPTW, minimization of duration, type B fleet, instances of Liu and Shen (1999)

Inst	**n**	**w**	**BDHMG08**	**RT10**	**UHGS**			**BKS**
			Best 3	Single	Avg 10	Best 10	T(min)	
R101	100	5	2486.76	**2421.19**	2487.11	2486.77	3.89	2421.19
R102	100	5	2227.48	**2209.50**	2223.80	2222.15	4.31	2209.50
R103	100	5	1938.93	1953.50	1931.17	<u>**1930.21**</u>	4.18	1938.93
R104	100	5	1714.73	1713.36	1694.06	<u>**1688.12**</u>	4.34	1713.36
R105	100	5	2027.98	2030.83	<u>**2017.56**</u>	<u>**2017.56**</u>	3.83	2027.98
R106	100	5	1919.03	1919.02	1916.36	<u>**1913.84**</u>	5.10	1919.02
R107	100	5	1789.58	1780.52	1775.34	<u>**1774.50**</u>	4.27	1780.52
R108	100	5	**1649.24**	1665.78	1657.01	1654.68	5.83	1649.24
R109	100	5	1828.63	1840.54	1818.15	<u>**1818.15**</u>	5.09	1828.63
R110	100	5	1774.46	1788.18	1765.50	<u>**1761.53**</u>	5.77	1774.46
R111	100	5	1769.71	1772.51	1757.34	<u>**1751.10**</u>	5.57	1769.71
R112	100	5	1669.78	1667.00	1664.36	<u>**1663.09**</u>	6.33	1667.00
C101	100	3	**2417.52**	**2417.52**	**2417.52**	**2417.52**	2.06	2417.52
C102	100	3	2350.55	**2350.54**	2350.55	2350.55	2.98	2350.54
C103	100	3	2353.64	2347.99	2345.31	<u>**2345.31**</u>	4.02	2347.99
C104	100	3	2328.62	**2325.78**	2327.84	2327.84	2.43	2325.78
C105	100	3	**2373.53**	2375.04	**2373.53**	**2373.53**	3.35	2373.53
C106	100	3	2404.56	**2381.14**	2386.03	2386.03	3.17	2381.14
C107	100	3	2370.01	2357.67	2364.21	2364.21	3.10	2357.52
C108	100	3	**2346.38**	**2346.38**	**2346.38**	**2346.38**	3.28	2346.38
C109	100	3	2339.89	**2336.29**	**2336.29**	**2336.29**	2.60	2336.29
RC101	100	4	2462.60	2464.66	2461.29	<u>**2456.10**</u>	4.69	2462.60
RC102	100	4	2263.45	2272.68	2261.83	<u>**2259.25**</u>	4.24	2263.45
RC103	100	4	2035.62	2041.24	2028.38	<u>**2025.30**</u>	4.74	2035.62
RC104	100	4	1905.06	1916.85	1901.04	<u>**1901.04**</u>	4.37	1905.06
RC105	100	4	**2308.59**	2325.99	2329.30	2329.14	4.76	2308.59
RC106	100	4	2149.56	2160.45	2152.58	<u>**2146.00**</u>	3.68	2149.56
RC107	100	4	2000.77	2003.26	1990.20	<u>**1989.34**</u>	4.09	2000.77
RC108	100	4	1910.83	1908.72	1900.80	<u>**1898.96**</u>	3.26	1906.69
R201	100	4	2002.53	**1953.42**	1975.28	1973.43	6.39	1953.42
R202	100	4	1790.38	1751.12	1747.39	<u>**1740.03**</u>	8.05	1751.12
R203	100	4	1541.19	1536.60	1513.38	<u>**1511.35**</u>	6.44	1535.08
R204	100	4	1284.33	1303.84	1276.31	<u>**1275.95**</u>	7.58	1284.33
R205	100	4	1563.62	1560.07	<u>**1534.27**</u>	<u>**1534.27**</u>	6.45	1560.07
R206	100	4	1464.53	1464.70	1443.43	<u>**1441.35**</u>	5.92	1464.53
R207	100	4	1380.41	1358.69	1345.42	<u>**1345.08**</u>	6.97	1358.69
R208	100	4	1244.74	1256.45	1219.25	<u>**1217.12**</u>	6.00	1244.74
R209	100	4	1431.37	1394.74	1382.44	<u>**1380.79**</u>	7.75	1394.74
R210	100	4	1516.66	1525.28	1486.85	<u>**1485.65**</u>	7.72	1516.66
R211	100	4	1255.06	1253.08	1220.46	<u>**1219.93**</u>	7.36	1253.08
C201	100	4	1820.64	**1816.14**	1820.64	1820.64	2.90	1816.14
C202	100	4	1795.40	**1768.51**	1775.21	**1768.51**	5.22	1768.51
C203	100	4	**1733.63**	1734.82	**1733.63**	**1733.63**	3.29	1733.63
C204	100	4	1708.69	1716.18	<u>**1680.46**</u>	<u>**1680.46**</u>	3.30	1708.69
C205	100	4	1782.74	**1747.68**	1778.30	1778.30	5.48	1747.68
C206	100	4	1772.87	**1756.01**	1767.70	1767.70	3.84	1756.01
C207	100	4	1729.49	**1729.39**	1729.49	1729.49	3.48	1729.39
C208	100	4	1724.20	**1723.20**	1724.20	1724.20	3.40	1723.20
RC201	100	6	2343.79	**2230.54**	2331.33	2329.59	4.34	2230.54
RC202	100	6	2091.53	2022.15	2059.81	2057.66	6.69	2002.62
RC203	100	6	1852.74	1841.26	1825.14	<u>**1824.54**</u>	5.33	1841.26
RC204	100	6	1565.31	1575.18	1557.77	<u>**1555.75**</u>	5.50	1565.31
RC205	100	6	2195.75	**2166.62**	2179.31	2174.74	5.43	2166.62
RC206	100	6	1923.56	1893.13	<u>**1883.08**</u>	<u>**1883.08**</u>	4.33	1887.23
RC207	100	6	1745.85	1743.23	1719.07	<u>**1714.14**</u>	5.65	1743.23
RC208	100	6	1488.19	1526.78	<u>**1483.20**</u>	<u>**1483.20**</u>	4.80	1488.19
Time			9.12 min	16.67 min	4.80 min			
Gap			+0.59%	+0.23%	-0.16%	-0.25%		
CPU			Ath 2.6G	P-IV 3.4G	Opt 2.2G			

Table 27: Results on the VFMPTW, minimization of duration, type C fleet, instances of Liu and Shen (1999)

Inst	n	w	BDHMG08	RT10	UHGS			BKS
			Best 3	Single	Avg 10	Best 10	T(min)	
R101	100	5	2199.78	**2134.90**	2199.79	2199.79	3.38	2134.90
R102	100	5	1925.55	**1913.37**	1926.50	1925.56	5.06	1913.37
R103	100	5	**1609.94**	1631.47	1616.42	1615.38	3.62	1609.94
R104	100	5	1370.84	1377.81	1365.32	<u>**1363.26**</u>	4.58	1370.84
R105	100	5	**1722.05**	1729.57	**1722.05**	**1722.05**	3.60	1722.05
R106	100	5	1602.87	1607.96	1603.06	<u>**1599.04**</u>	4.77	1602.87
R107	100	5	1456.02	1452.52	1447.86	<u>**1442.97**</u>	3.72	1452.52
R108	100	5	1336.28	1330.28	1321.96	<u>**1321.68**</u>	5.44	1330.28
R109	100	5	1507.77	1519.37	1508.36	<u>**1506.59**</u>	5.02	1507.77
R110	100	5	1446.41	1457.43	1446.96	<u>**1443.92**</u>	5.73	1446.41
R111	100	5	1447.88	1443.34	1427.82	<u>**1423.47**</u>	6.99	1443.34
R112	100	5	1335.41	1339.44	1329.24	<u>**1329.07**</u>	4.77	1335.41
C101	100	3	**1628.31**	1628.94	1628.94	1628.94	1.83	1628.31
C102	100	3	**1610.96**	**1610.96**	**1610.96**	**1610.96**	2.43	1610.96
C103	100	3	1619.68	**1607.14**	**1607.14**	**1607.14**	2.77	1607.14
C104	100	3	1613.96	**1598.50**	1599.90	1599.90	2.70	1598.50
C105	100	3	**1628.38**	1628.94	1628.94	1628.94	1.88	1628.38
C106	100	3	**1628.94**	**1628.94**	**1628.94**	**1628.94**	1.88	1628.94
C107	100	3	**1628.38**	1628.94	1628.94	1628.94	1.96	1628.38
C108	100	3	**1622.89**	**1622.89**	**1622.89**	**1622.89**	3.00	1622.89
C109	100	3	**1614.99**	**1614.99**	1615.93	1615.93	3.62	1614.99
RC101	100	4	2084.48	2089.37	2084.16	<u>**2082.95**</u>	4.91	2084.48
RC102	100	4	1895.92	1906.68	1898.52	<u>**1895.05**</u>	4.28	1895.92
RC103	100	4	1660.62	1666.24	1661.76	<u>**1650.30**</u>	3.98	1660.62
RC104	100	4	1537.09	1540.13	<u>**1526.04**</u>	<u>**1526.04**</u>	3.57	1537.09
RC105	100	4	1957.52	**1953.99**	1962.82	1957.14	4.71	1953.99
RC106	100	4	1776.08	1787.69	1775.84	<u>**1774.94**</u>	3.80	1776.08
RC107	100	4	1614.04	1622.90	1611.28	<u>**1607.11**</u>	3.83	1614.04
RC108	100	4	1535.14	1531.69	1524.10	<u>**1523.96**</u>	3.38	1531.69
R201	100	4	1729.92	1728.42	<u>**1716.02**</u>	<u>**1716.02**</u>	4.54	1728.42
R202	100	4	1537.35	1527.92	1524.96	<u>**1515.03**</u>	8.84	1527.92
R203	100	4	1308.70	1311.60	1287.36	<u>**1286.35**</u>	6.24	1308.70
R204	100	4	1062.46	1085.71	1051.19	<u>**1050.95**</u>	7.62	1062.46
R205	100	4	1311.84	1335.07	1309.29	<u>**1309.27**</u>	6.44	1311.84
R206	100	4	1251.51	1239.70	1216.87	<u>**1216.35**</u>	5.34	1239.70
R207	100	4	1149.23	1139.61	<u>**1120.08**</u>	<u>**1120.08**</u>	7.23	1139.61
R208	100	4	1009.26	1022.11	992.66	<u>**992.12**</u>	6.01	1009.26
R209	100	4	1178.45	1171.41	1156.97	<u>**1155.79**</u>	7.50	1171.41
R210	100	4	1289.35	1281.08	1259.42	<u>**1257.89**</u>	6.54	1281.08
R211	100	4	1013.84	1028.08	995.54	<u>**994.93**</u>	6.59	1013.84
C201	100	4	**1269.41**	**1269.41**	**1269.41**	**1269.41**	2.86	1269.41
C202	100	4	1242.66	1244.54	<u>**1239.54**</u>	<u>**1239.54**</u>	3.85	1242.66
C203	100	4	**1193.63**	1203.42	**1193.63**	**1193.63**	3.03	1193.63
C204	100	4	**1176.52**	1188.18	**1176.52**	**1176.52**	3.90	1176.52
C205	100	4	1245.62	1239.60	<u>**1238.30**</u>	<u>**1238.30**</u>	4.36	1239.60
C206	100	4	1245.05	**1229.23**	1238.30	1238.30	4.87	1229.23
C207	100	4	1215.42	1213.07	<u>**1209.49**</u>	<u>**1209.49**</u>	3.00	1213.07
C208	100	4	**1204.20**	1205.18	**1204.20**	**1204.20**	3.03	1204.20
RC201	100	6	2004.53	**1915.42**	1996.79	1996.79	3.67	1915.42
RC202	100	6	1766.52	**1677.62**	1733.23	1732.66	6.53	1677.62
RC203	100	6	1517.98	1504.35	1496.48	<u>**1496.11**</u>	6.15	1504.35
RC204	100	6	1238.66	1241.45	<u>**1220.75**</u>	<u>**1220.75**</u>	5.45	1238.66
RC205	100	6	1854.22	**1822.07**	1844.74	1844.74	5.07	1822.07
RC206	100	6	1590.22	1586.61	1557.19	<u>**1553.65**</u>	4.45	1586.61
RC207	100	6	1396.16	1406.26	1382.17	<u>**1377.52**</u>	6.06	1396.16
RC208	100	6	1145.84	1175.23	1141.47	<u>**1140.10**</u>	6.32	1145.84
Time			8.18 min	16.67 min		4.58 min		
Gap			+0.45%	+0.35%	-0.17%	-0.25%		
CPU			Ath 2.6G	P-IV 3.4G		Opt 2.2G		

Table 28: Results on the VFMPTW, minimization of distance, type A fleet, instances of Liu and Shen (1999)

Inst	n	w	BDHMG08	BPDRT09	UHGS			BKS
			Best 3	Best 5	Avg 10	Best 10	T(min)	
R101	100	5	4349.80	4342.72	4322.04	**4314.36**	4.61	4342.72
R102	100	5	4196.46	4189.21	4175.05	**4166.28**	6.03	4182.47
R103	100	5	4052.85	4051.62	4034.88	**4027.36**	5.35	4051.62
R104	100	5	3973.48	3972.65	3938.92	**3936.40**	4.81	3972.65
R105	100	5	4161.72	4152.50	4130.71	**4122.50**	6.49	4152.50
R106	100	5	4095.20	4085.30	4058.95	**4048.59**	5.57	4085.07
R107	100	5	4006.61	3996.74	3979.18	**3970.51**	5.56	3996.74
R108	100	5	3961.38	3949.50	3932.46	**3928.12**	4.68	3949.50
R109	100	5	4048.29	4035.89	4020.93	**4015.71**	4.80	4035.89
R110	100	5	3997.88	3991.63	3966.47	**3961.68**	6.49	3991.63
R111	100	5	4011.63	4009.61	3973.49	**3964.99**	5.28	4008.88
R112	100	5	3962.73	3954.19	3926.32	**3918.88**	4.92	3954.19
C101	100	3	7098.04	7097.93	**7093.45**	**7093.45**	2.96	7097.13
C102	100	3	7086.11	7085.47	**7080.17**	**7080.17**	2.14	7085.47
C103	100	3	7080.35	7080.41	**7079.21**	**7079.21**	2.09	7080.35
C104	100	3	7076.90	**7075.06**	**7075.06**	**7075.06**	2.19	7075.06
C105	100	3	7096.19	7096.22	**7093.45**	**7093.45**	3.33	7095.13
C106	100	3	7086.91	7088.35	**7083.87**	**7083.87**	2.28	7086.91
C107	100	3	7084.92	7090.91	**7084.61**	**7084.61**	2.23	7084.92
C108	100	3	7082.49	7081.18	**7079.66**	**7079.66**	2.19	7081.18
C109	100	3	7078.13	7077.68	**7077.30**	**7077.30**	2.04	7077.68
RC101	100	4	5180.74	5168.23	5154.95	**5150.86**	5.21	5168.23
RC102	100	4	5029.59	5025.22	5000.28	**4987.24**	4.81	5025.22
RC103	100	4	4895.57	4888.53	4821.61	**4804.61**	7.08	4888.53
RC104	100	4	4760.56	4747.38	4724.10	**4717.63**	5.30	4747.38
RC105	100	4	5060.37	5068.54	5035.76	**5035.35**	5.57	5060.37
RC106	100	4	4997.86	4972.11	4944.74	**4936.74**	5.63	4972.11
RC107	100	4	4865.76	4861.04	4795.35	**4788.69**	5.08	4861.04
RC108	100	4	4765.37	4753.12	4709.09	**4708.85**	4.78	4753.12
R201	100	4	3484.95	3530.24	**3446.78**	**3446.78**	6.51	3484.95
R202	100	4	3335.74	3335.61	**3308.16**	**3308.16**	7.68	3335.61
R203	100	4	3173.95	3164.03	**3141.09**	**3141.09**	5.65	3162.84
R204	100	4	3065.15	3029.83	3018.83	**3018.14**	6.96	3029.83
R205	100	4	3277.69	3261.19	3220.56	**3218.97**	6.40	3252.43
R206	100	4	3173.30	3165.85	3150.61	**3146.34**	10.30	3165.85
R207	100	4	3136.47	3102.79	3080.64	**3077.58**	8.70	3100.64
R208	100	4	3050.00	3009.13	2999.35	**2997.24**	5.37	3009.13
R209	100	4	3155.73	3155.60	3123.30	**3122.42**	6.37	3141.17
R210	100	4	3219.23	3206.09	3178.57	**3174.85**	6.93	3206.09
R211	100	4	3055.04	3026.02	3021.67	**3019.93**	9.10	3026.02
C201	100	4	5701.45	5700.87	**5695.02**	**5695.02**	3.71	5695.02
C202	100	4	5689.70	5689.70	**5685.24**	**5685.24**	3.78	5687.07
C203	100	4	5685.82	**5681.55**	**5681.55**	**5681.55**	4.21	5681.55
C204	100	4	5690.30	5677.69	**5677.66**	**5677.66**	4.27	5677.66
C205	100	4	5691.70	5691.70	**5691.36**	**5691.36**	3.98	5691.70
C206	100	4	5691.70	5691.70	**5689.32**	**5689.32**	3.82	5691.70
C207	100	4	5689.82	5692.36	**5687.35**	**5687.35**	4.24	5689.82
C208	100	4	5686.50	5689.59	**5686.50**	**5686.50**	3.86	5686.50
RC201	100	6	4407.68	4404.07	4378.21	**4374.09**	5.92	4398.21
RC202	100	6	4277.67	4266.96	4244.65	**4244.63**	4.63	4266.96
RC203	100	6	4204.85	4189.94	4171.47	**4170.17**	7.73	4185.70
RC204	100	6	4109.86	4098.34	**4087.11**	**4087.11**	5.79	4098.34
RC205	100	6	4329.96	4304.52	4295.41	**4291.93**	5.46	4304.52
RC206	100	6	4272.08	4272.82	4253.57	**4251.88**	5.12	4272.08
RC207	100	6	4232.81	4219.52	4186.43	**4185.98**	4.82	4213.66
RC208	100	6	4095.71	4093.83	4076.27	**4075.04**	4.08	4082.58
Time			4.13 min	—	5.09 min			
Gap			+0.25%	+0.06%	-0.41%	-0.48%		
CPU			Ath 2.6G	Duo 2.4G	Opt 2.2G			

Table 29: Results on the VFMPTW, minimization of distance, type B fleet, instances of Liu and Shen (1999)

Inst	**n**	**w**	**BDHMG08**	**BPDRT09**	**UHGS**			**BKS**
			Best 3	Best 5	Avg 10	Best 10	T(min)	
R101	100	5	**2226.94**	—	2229.24	2228.67	5.05	2226.94
R102	100	5	**2071.90**	—	2073.91	2073.63	3.59	2071.90
R103	100	5	1857.22	—	1855.83	**1853.66**	4.57	1857.22
R104	100	5	1707.31	—	1686.09	**1683.33**	5.37	1707.31
R105	100	5	1995.07	—	**1988.86**	**1988.86**	3.30	1995.07
R106	100	5	1903.95	—	1889.58	**1888.31**	4.64	1903.95
R107	100	5	1766.18	—	1754.75	**1753.35**	4.15	1766.18
R108	100	5	1666.89	—	1651.75	**1647.88**	4.54	1666.89
R109	100	5	1833.54	—	1819.14	**1818.15**	3.66	1833.54
R110	100	5	1781.15	—	1765.34	**1758.64**	5.11	1781.15
R111	100	5	1768.74	—	1747.08	**1740.86**	5.32	1768.74
R112	100	5	1675.76	—	1664.41	**1661.85**	5.36	1675.76
C101	100	3	2340.98	—	**2340.15**	**2340.15**	3.12	2340.98
C102	100	3	2326.53	—	**2325.70**	**2325.70**	2.61	2326.53
C103	100	3	2325.61	—	**2324.60**	**2324.60**	3.03	2325.61
C104	100	3	**2318.04**	—	**2318.04**	**2318.04**	2.44	2318.04
C105	100	3	2344.64	—	**2340.15**	**2340.15**	3.00	2344.64
C106	100	3	2345.85	—	**2340.15**	**2340.15**	3.46	2345.85
C107	100	3	2345.60	—	**2340.15**	**2340.15**	3.20	2345.60
C108	100	3	2340.17	—	**2338.58**	**2338.58**	3.18	2340.17
C109	100	3	**2328.55**	—	**2328.55**	**2328.55**	2.72	2328.55
RC101	100	4	2417.16	—	2416.23	**2412.71**	4.16	2417.16
RC102	100	4	2234.47	—	2213.93	**2213.92**	5.86	2234.47
RC103	100	4	2025.74	—	2016.28	**2016.28**	3.50	2025.74
RC104	100	4	1912.65	—	1908.66	**1897.04**	4.07	1912.65
RC105	100	4	2296.16	—	2293.21	**2287.51**	4.54	2296.16
RC106	100	4	2157.84	—	2141.32	**2140.86**	3.95	2157.84
RC107	100	4	2008.02	—	1990.13	**1989.34**	2.99	2008.02
RC108	100	4	1920.91	—	1900.72	**1898.96**	3.91	1920.91
R201	100	4	1687.44	—	1721.54	**1646.78**	5.96	1687.44
R202	100	4	1527.74	—	1509.06	**1508.16**	9.15	1527.74
R203	100	4	1379.15	—	**1341.09**	**1341.09**	3.75	1379.15
R204	100	4	1243.56	—	1218.47	**1218.14**	5.11	1243.56
R205	100	4	1471.97	—	1419.52	**1418.97**	7.28	1471.97
R206	100	4	1400.84	—	1350.32	**1346.34**	7.23	1400.84
R207	100	4	1333.53	—	1279.63	**1277.58**	6.28	1333.53
R208	100	4	1225.37	—	1198.41	**1197.24**	4.33	1225.37
R209	100	4	1370.30	—	1323.60	**1322.42**	6.15	1370.30
R210	100	4	1418.54	—	1376.24	**1374.31**	6.97	1418.54
R211	100	4	1263.72	—	1219.99	**1219.93**	6.79	1263.72
C201	100	4	1700.87	—	**1695.02**	**1695.02**	2.97	1700.87
C202	100	4	1687.84	—	**1685.24**	**1685.24**	2.83	1687.84
C203	100	4	1696.25	—	**1681.55**	**1681.55**	3.36	1696.25
C204	100	4	1705.94	—	**1677.66**	**1677.66**	3.47	1705.94
C205	100	4	1711.00	—	**1691.36**	**1691.36**	3.21	1711.00
C206	100	4	1691.70	—	**1689.32**	**1689.32**	2.99	1691.70
C207	100	4	1704.88	—	**1687.35**	**1687.35**	3.44	1704.88
C208	100	4	1689.59	—	**1686.50**	**1686.50**	3.24	1689.59
RC201	100	6	1965.31	—	1942.19	**1938.36**	5.94	1965.31
RC202	100	6	**1771.87**	—	1773.04	1772.81	5.92	1771.87
RC203	100	6	1619.55	—	1606.56	**1604.04**	5.99	1619.55
RC204	100	6	1501.10	—	**1490.25**	**1490.25**	4.28	1501.10
RC205	100	6	1853.58	—	1835.74	**1832.53**	5.11	1853.58
RC206	100	6	1761.49	—	**1725.44**	**1725.44**	5.79	1761.49
RC207	100	6	1666.03	—	1651.09	**1646.37**	5.65	1666.03
RC208	100	6	1494.11	—	**1483.20**	**1483.20**	4.39	1494.11
Time			3.45 min	—		4.50 min		
Gap			+0.00%	—	-0.94%	-1.10%		
CPU			Ath 2.6G	Duo 2.4G		Opt 2.2G		

Table 30: Results on the VFMPTW, minimization of distance, type C fleet, instances of Liu and Shen (1999)

Inst	**n**	**w**	**BDHMG08**	**BPDRT09**	**UHGS**			**BKS**
			Best 3	Best 5	Avg 10	Best 10	T(min)	
R101	100	5	**1951.20**	1951.89	**1951.20**	**1951.20**	4.60	1951.20
R102	100	5	**1770.40**	1778.29	1785.35	1785.35	2.92	1770.40
R103	100	5	1558.17	1555.26	1552.64	**1552.34**	3.55	1555.26
R104	100	5	1367.82	1372.08	1356.70	**1355.15**	5.37	1361.46
R105	100	5	1696.67	1698.26	**1694.56**	**1694.56**	3.25	1696.67
R106	100	5	1589.25	1590.11	1590.25	**1583.17**	4.12	1589.25
R107	100	5	1435.21	1439.81	1433.44	**1428.08**	5.36	1435.21
R108	100	5	1334.75	1334.68	1315.47	**1314.88**	5.32	1334.68
R109	100	5	1515.22	1514.13	1507.97	**1506.59**	4.68	1507.10
R110	100	5	1457.42	1461.85	1448.83	**1443.92**	5.06	1457.42
R111	100	5	1439.43	1439.14	1423.43	**1420.15**	5.58	1435.93
R112	100	5	1358.17	1343.26	1329.70	**1327.58**	4.97	1337.68
C101	100	3	**1628.94**	**1628.94**	**1628.94**	**1628.94**	2.12	1628.94
C102	100	3	**1597.66**	**1597.66**	**1597.66**	**1597.66**	2.35	1597.66
C103	100	3	**1596.56**	**1596.56**	**1596.56**	**1596.56**	2.88	1596.56
C104	100	3	1594.06	1590.86	**1590.76**	**1590.76**	2.32	1590.86
C105	100	3	**1628.94**	**1628.94**	**1628.94**	**1628.94**	1.96	1628.94
C106	100	3	**1628.94**	**1628.94**	**1628.94**	**1628.94**	2.05	1628.94
C107	100	3	**1628.94**	**1628.94**	**1628.94**	**1628.94**	2.17	1628.94
C108	100	3	**1622.75**	**1622.75**	**1622.75**	**1622.75**	3.20	1622.75
C109	100	3	**1614.99**	**1614.99**	1615.93	1615.93	3.88	1614.99
RC101	100	4	2048.44	2053.55	2047.33	**2043.48**	4.39	2048.44
RC102	100	4	1860.48	1872.49	1849.38	**1847.92**	4.10	1860.48
RC103	100	4	1660.81	1663.08	1654.30	**1646.35**	4.19	1660.81
RC104	100	4	1536.24	1540.61	1523.42	**1522.04**	5.64	1536.24
RC105	100	4	1913.09	1929.89	1925.66	**1913.06**	4.01	1913.09
RC106	100	4	1772.05	1776.52	1770.95	1770.95	3.77	1761.63
RC107	100	4	1615.74	1633.29	1609.88	**1607.11**	4.08	1615.74
RC108	100	4	1527.35	1527.87	1524.10	**1523.96**	3.35	1527.35
R201	100	4	1441.46	1466.13	1462.03	1443.41	4.90	1439.76
R202	100	4	1298.10	1296.78	1297.43	**1283.16**	7.91	1288.70
R203	100	4	1145.38	1127.28	**1116.09**	**1116.09**	3.95	1127.28
R204	100	4	1019.77	1000.89	993.16	**993.14**	6.63	1000.89
R205	100	4	1222.03	1240.74	1196.73	**1193.97**	7.33	1222.03
R206	100	4	1138.26	1141.13	1123.21	**1121.34**	6.00	1138.26
R207	100	4	1086.42	1067.97	1055.39	**1052.58**	6.97	1067.97
R208	100	4	976.11	979.50	971.36	**969.90**	5.78	976.11
R209	100	4	1140.96	1140.38	1098.89	**1097.42**	5.94	1123.19
R210	100	4	1161.87	1170.29	1153.34	**1149.85**	6.80	1161.87
R211	100	4	1015.84	1008.54	**994.93**	**994.93**	6.51	1008.54
C201	100	4	**1194.33**	**1194.33**	**1194.33**	**1194.33**	4.82	1194.33
C202	100	4	1189.35	**1185.24**	**1185.24**	**1185.24**	2.57	1185.24
C203	100	4	**1176.25**	**1176.25**	**1176.25**	**1176.25**	3.60	1176.25
C204	100	4	1176.55	1176.55	**1175.37**	**1175.37**	4.32	1176.55
C205	100	4	**1190.36**	**1190.36**	**1190.36**	**1190.36**	4.46	1190.36
C206	100	4	**1188.62**	**1188.62**	**1188.62**	**1188.62**	4.01	1188.62
C207	100	4	**1184.88**	1187.71	**1184.88**	**1184.88**	3.66	1184.88
C208	100	4	1187.86	**1186.50**	**1186.50**	**1186.50**	2.99	1186.50
RC201	100	6	1632.41	1630.53	1623.78	**1623.36**	6.81	1630.53
RC202	100	6	1459.84	1461.44	1450.70	**1447.27**	5.29	1459.84
RC203	100	6	1295.07	1292.92	**1274.04**	**1274.04**	4.49	1292.92
RC204	100	6	1171.26	1162.91	1159.37	**1159.00**	6.01	1162.91
RC205	100	6	1525.28	1532.67	1517.09	**1512.53**	5.24	1525.28
RC206	100	6	1425.15	1420.89	1400.62	**1395.18**	3.92	1420.89
RC207	100	6	1332.40	1328.29	1319.56	**1314.44**	6.24	1328.29
RC208	100	6	1155.02	1152.92	**1140.10**	**1140.10**	6.81	1152.92
Time			3.08 min	—		4.56 min		
Gap			+0.26%	+0.27%	-0.36%	-0.51%		
CPU			Ath 2.6G	Duo 2.4G		Opt 2.2G		

Table 31: Results on the type 1 VRPSTW (only lateness) with $\alpha = 100$, hierarchical objective involving first the minimization of the Fleet Size "Fleet", then the number of customers serviced outside of their time windows "TW", then the overall lateness "L", and finally distance "Dist". Instances of Solomon and Desrosiers (1988)

Inst	n	F10				UHGS								
		Single				Avg 10				Best 10				
		Fleet	TW	L	Dist	Fleet	TW	L	Dist	Fleet	TW	L	Dist	T(min)
R101	100	12	56	—	1128.70	11.00	27.70	1644.70	1329.70	11	27	1720.97	1331.85	11.78
R102	100	11	46	—	1058.70	10.00	22.20	1069.17	1226.84	10	21	1206.72	1252.78	18.64
R103	100	10	34	—	1027.40	9.80	7.50	258.62	1171.84	9	5	90.68	1208.63	11.05
R104	100	9	18	—	947.30	9.00	0.50	17.15	1009.30	9	0	0.00	1007.31	7.79
R105	100	11	42	—	1073.50	10.20	19.50	1106.53	1239.15	10	9	680.04	1381.88	13.74
R106	100	10	33	—	1047.40	9.90	8.10	433.32	1236.89	9	6	358.86	1259.11	12.59
R107	100	10	24	—	987.60	9.00	7.00	354.81	1029.30	9	7	343.45	1042.96	11.24
R108	100	9	14	—	947.20	9.00	0.00	0.00	963.50	9	0	0.00	960.88	8.10
R109	100	10	28	—	1001.40	10.00	5.00	208.62	1194.64	10	5	200.72	1183.42	8.80
R110	100	9	29	—	1013.40	9.20	9.50	461.42	1043.08	9	0	0.00	1118.84	12.57
R111	100	10	26	—	983.30	9.00	7.80	385.46	1038.01	9	6	443.34	1047.09	12.72
R112	100	9	17	—	940.90	9.00	0.00	0.00	988.59	9	0	0.00	982.14	9.96
R201	100	3	56	—	984.00	3.00	2.00	197.62	1502.80	3	2	174.47	1497.06	27.05
R202	100	3	40	—	943.50	2.00	21.30	4703.07	991.60	2	20	4757.71	991.27	30.61
R203	100	2	30	—	901.80	2.00	6.10	1124.60	991.09	2	6	1076.65	995.76	30.04
R204	100	2	19	—	836.30	2.00	0.00	0.00	830.79	2	0	0.00	825.52	26.74
R205	100	3	36	—	911.90	2.00	14.40	2998.08	983.30	2	14	2767.13	973.84	26.49
R206	100	2	25	—	956.90	2.00	3.50	652.23	994.39	2	3	465.02	993.13	29.92
R207	100	2	18	—	876.60	2.00	0.00	0.00	893.85	2	0	0.00	893.33	29.54
R208	100	2	11	—	833.40	2.00	0.00	0.00	727.17	2	0	0.00	726.82	14.16
R209	100	2	26	—	950.50	2.00	7.30	1814.72	990.71	2	7	1386.37	994.41	29.28
R210	100	2	29	—	963.80	2.00	6.00	1339.34	995.19	2	6	1264.94	997.75	29.75
R211	100	2	14	—	906.80	2.00	0.00	0.00	900.56	2	0	0.00	892.72	27.23
C101	100	—	—	—	—	10.00	0.00	0.00	828.94	10	0	0.00	828.94	2.32
C102	100	—	—	—	—	10.00	0.00	0.00	828.94	10	0	0.00	828.94	2.28
C103	100	—	—	—	—	10.00	0.00	0.00	828.06	10	0	0.00	828.06	2.12
C104	100	—	—	—	—	10.00	0.00	0.00	824.78	10	0	0.00	824.78	1.96
C105	100	—	—	—	—	10.00	0.00	0.00	828.94	10	0	0.00	828.94	2.73
C106	100	—	—	—	—	10.00	0.00	0.00	828.94	10	0	0.00	828.94	2.56
C107	100	—	—	—	—	10.00	0.00	0.00	828.94	10	0	0.00	828.94	2.48
C108	100	—	—	—	—	10.00	0.00	0.00	828.94	10	0	0.00	828.94	2.19
C109	100	—	—	—	—	10.00	0.00	0.00	828.94	10	0	0.00	828.94	2.05
C201	100	—	—	—	—	3.00	0.00	0.00	591.56	3	0	0.00	591.56	5.82
C202	100	—	—	—	—	3.00	0.00	0.00	591.56	3	0	0.00	591.56	8.35
C203	100	—	—	—	—	3.00	0.00	0.00	591.17	3	0	0.00	591.17	9.79
C204	100	—	—	—	—	3.00	0.00	0.00	590.60	3	0	0.00	590.60	8.08
C205	100	—	—	—	—	3.00	0.00	0.00	588.88	3	0	0.00	588.88	6.40
C206	100	—	—	—	—	3.00	0.00	0.00	588.49	3	0	0.00	588.49	6.53
C207	100	—	—	—	—	3.00	0.00	0.00	588.29	3	0	0.00	588.29	6.54
C208	100	—	—	—	—	3.00	0.00	0.00	588.32	3	0	0.00	588.32	6.51
RC101	100	11	44	—	1255.30	11.00	12.10	789.34	1486.11	11	12	808.44	1486.99	7.01
RC102	100	10	32	—	1230.10	10.30	10.80	476.57	1399.34	10	4	41.41	1539.29	10.51
RC103	100	10	25	—	1154.60	10.00	1.20	18.41	1320.22	10	1	4.18	1333.71	8.03
RC104	100	10	12	—	1083.90	10.00	0.00	0.00	1135.48	10	0	0.00	1135.48	5.57
RC105	100	11	38	—	1219.70	10.90	7.20	345.49	1516.88	10	6	278.42	1538.72	6.89
RC106	100	10	27	—	1150.30	10.00	7.60	308.59	1310.32	10	7	344.38	1307.76	8.85
RC107	100	10	28	—	1123.00	10.00	1.50	65.46	1300.57	10	1	64.05	1325.19	9.70
RC108	100	10	10	—	1071.60	10.00	0.00	0.00	1139.82	10	0	0.00	1139.82	6.97
RC201	100	3	48	—	1147.40	3.00	3.00	482.47	1663.69	3	3	482.47	1663.69	32.65
RC202	100	3	35	—	1073.50	3.00	0.00	0.00	1377.25	3	0	0.00	1365.65	26.22
RC203	100	3	29	—	906.30	2.10	13.80	3013.28	929.95	2	0	0.00	1064.14	30.49
RC204	100	2	14	—	850.70	2.00	2.00	261.17	915.94	2	2	257.81	916.79	29.27
RC205	100	3	40	—	1158.40	3.00	1.00	87.83	1623.09	3	1	9.38	1605.20	32.78
RC206	100	3	40	—	978.40	3.00	0.00	0.00	1149.37	3	0	0.00	1146.32	24.48
RC207	100	3	33	—	986.40	3.00	0.00	0.00	1061.49	3	0	0.00	1061.14	24.51
RC208	100	2	21	—	885.50	2.00	6.40	946.83	910.34	2	6	949.50	917.64	22.51
Time		9.69 min				18.62 min								
CPU		P-M 1.6G				Opt 2.2G								

Table 32: Results on the type 1 VRPSTW (only lateness) with $\alpha = 1$. Minimization of the sum of distance and lateness under a fleet size limit. Instances of Solomon and Desrosiers (1988).

Inst	n	m	KTDHS12		UHGS			BKS
			Avg 10	Best 10	Avg 10	Best 10	T(min)	
R101	100	19	1562.98	**1562.58**	1562.89	**1562.58**	1.93	1562.58
R102	100	17	1379.62	**1379.11**	1379.21	**1379.11**	1.90	1379.11
R103	100	13	1160.64	1159.54	1159.51	**1159.28**	3.34	1159.54
R104	100	9	1009.02	1003.73	**999.77**	**999.77**	3.93	1003.73
R105	100	14	1348.89	**1347.75**	**1347.75**	**1347.75**	2.17	1347.75
R106	100	12	1237.29	**1236.58**	**1236.58**	**1236.58**	2.91	1236.58
R107	100	10	1089.84	1084.96	**1083.62**	**1083.62**	4.55	1084.96
R108	100	9	951.24	949.94	947.04	**946.60**	4.20	949.94
R109	100	11	1176.40	**1173.21**	**1173.21**	**1173.21**	3.54	1173.21
R110	100	10	1114.66	**1106.66**	1111.57	1107.26	3.55	1106.66
R111	100	10	1086.36	1080.25	1076.41	**1074.84**	4.97	1080.25
R112	100	9	981.82	972.11	975.78	**971.31**	5.13	972.11
C101	100	10	**828.94**	**828.94**	**828.94**	**828.94**	2.10	828.94
C102	100	10	**828.94**	**828.94**	**828.94**	**828.94**	2.15	828.94
C103	100	10	828.07	828.07	**828.06**	**828.06**	2.02	828.07
C104	100	10	**824.78**	**824.78**	**824.78**	**824.78**	1.96	824.78
C105	100	10	**828.94**	**828.94**	**828.94**	**828.94**	2.12	828.94
C106	100	10	**828.94**	**828.94**	**828.94**	**828.94**	1.91	828.94
C107	100	10	**828.94**	**828.94**	**828.94**	**828.94**	1.89	828.94
C108	100	10	**828.94**	**828.94**	**828.94**	**828.94**	1.94	828.94
C109	100	10	**828.94**	**828.94**	**828.94**	**828.94**	1.90	828.94
RC101	100	14	1591.59	**1590.22**	**1590.22**	**1590.22**	2.57	1590.22
RC102	100	12	1429.90	**1428.21**	**1428.21**	**1428.21**	2.83	1428.21
RC103	100	11	1242.33	**1239.54**	1239.73	**1239.54**	3.09	1239.54
RC104	100	10	1128.74	**1126.31**	**1126.31**	**1126.31**	3.25	1126.31
RC105	100	13	1451.38	**1450.84**	**1450.84**	**1450.84**	2.59	1450.84
RC106	100	11	1350.17	**1349.30**	1349.72	**1349.30**	3.55	1349.30
RC107	100	11	1208.96	**1208.81**	1208.98	**1208.81**	3.37	1208.81
RC108	100	10	1119.61	**1118.00**	1118.31	**1118.00**	3.72	1118.00
R201	100	4	1237.17	**1237.11**	**1237.11**	**1237.11**	4.13	1237.11
R202	100	3	1169.23	**1165.32**	**1165.32**	**1165.32**	6.19	1165.32
R203	100	3	942.96	937.35	934.01	**933.52**	10.55	937.35
R204	100	2	840.79	832.38	824.73	**824.02**	24.72	832.38
R205	100	3	1006.79	**994.43**	**994.43**	**994.43**	6.53	994.43
R206	100	3	920.13	912.81	**906.14**	**906.14**	7.39	912.81
R207	100	2	1044.87	908.70	888.44	**887.28**	24.63	908.70
R208	100	2	735.26	728.92	727.08	**726.82**	13.72	728.92
R209	100	3	917.21	909.30	**909.16**	**909.16**	6.98	909.30
R210	100	3	958.58	948.80	941.95	**938.34**	8.57	948.80
R211	100	2	923.85	901.18	892.50	**885.71**	19.66	901.18
C201	100	3	**591.56**	**591.56**	**591.56**	**591.56**	4.62	591.56
C202	100	3	**591.56**	**591.56**	**591.56**	**591.56**	6.90	591.56
C203	100	3	**591.17**	**591.17**	**591.17**	**591.17**	7.67	591.17
C204	100	3	**590.60**	**590.60**	**590.60**	**590.60**	6.84	590.60
C205	100	3	**588.88**	**588.88**	**588.88**	**588.88**	5.24	588.88
C206	100	3	**588.49**	**588.49**	**588.49**	**588.49**	5.84	588.49
C207	100	3	**588.29**	**588.29**	**588.29**	**588.29**	5.25	588.29
C208	100	3	**588.32**	**588.32**	**588.32**	**588.32**	5.89	588.32
RC201	100	4	1380.47	**1380.33**	**1380.33**	**1380.33**	4.34	1380.33
RC202	100	3	1322.17	**1317.28**	**1317.28**	**1317.28**	9.59	1317.28
RC203	100	3	1057.10	1046.05	1045.00	**1040.77**	10.73	1046.05
RC204	100	3	809.09	797.41	**797.04**	**797.04**	6.71	797.41
RC205	100	4	1305.97	1299.61	**1298.00**	**1297.65**	6.34	1299.61
RC206	100	3	1135.90	**1135.26**	**1135.26**	**1135.26**	6.56	1135.26
RC207	100	3	1073.58	1061.14	1058.16	**1056.88**	7.43	1061.14
RC208	100	3	834.82	829.00	827.90	**827.67**	7.98	829.00
Time			10.00 min		5.82 min			
Gap			+0.62%	+0.00%	-0.13%	-0.18%		
CPU			Xe 2.67G		Opt 2.2G			

Table 33: Results on the type 2 VRPSTW (earliness and lateness) with $\alpha = 100$, hierarchical objective involving first the minimization of the Fleet Size "Fleet", then the number of customers serviced outside of their time windows "TW", then the overall earliness plus lateness "E+L", and finally distance "Dist". Instances of Solomon and Desrosiers (1988)

Inst	n	FEL07				UHGS								
		Best X				Avg 10				Best 10				
		Fleet	TW	E+L	Dist	Fleet	TW	E+L	Dist	Fleet	TW	E+L	Dist	T(min)
R101	100	14	44	—	1872.94	9.00	57.30	2998.08	1025.65	9	56	2742.45	1018.58	61.19
R102	100	13	29	—	1732.54	9.00	38.70	1825.32	1018.77	9	37	1934.47	1012.28	60.94
R103	100	12	9	—	1542.79	9.00	17.30	676.55	1020.72	9	16	621.25	1022.96	60.65
R104	100	10	0	—	1107.18	9.00	1.60	44.39	1014.39	9	1	19.05	1013.65	60.44
R105	100	—	—	—	—	9.00	39.10	2008.68	1030.78	9	37	2050.22	1037.70	61.04
R106	100	—	—	—	—	9.00	24.30	1107.09	1035.12	9	24	972.07	1021.48	60.73
R107	100	—	—	—	—	9.00	7.90	341.56	1031.17	9	7	294.55	1033.97	60.58
R108	100	10	0	—	968.34	9.00	0.00	0.00	980.60	9	0	0.00	970.15	60.37
R109	100	11	4	—	1379.87	9.00	22.40	961.34	1023.20	9	21	833.92	1028.18	60.66
R110	100	—	—	—	—	9.00	12.60	572.97	1025.82	9	11	552.12	1034.03	60.49
R111	100	—	—	—	—	9.00	8.10	334.60	1015.26	9	7	248.93	1013.65	60.65
R112	100	—	—	—	—	9.00	1.10	30.57	1018.88	9	1	1.25	1025.86	60.30
C101	100	10	0	—	828.94	10.00	0.00	0.00	828.94	10	0	0.00	828.94	30.55
C102	100	10	0	—	828.94	10.00	0.00	0.00	828.94	10	0	0.00	828.94	30.25
C103	100	10	0	—	918.08	10.00	0.00	0.00	828.06	10	0	0.00	828.06	30.15
C104	100	10	0	—	899.00	10.00	0.00	0.00	824.78	10	0	0.00	824.78	30.32
C105	100	10	0	—	828.94	10.00	0.00	0.00	828.94	10	0	0.00	828.94	30.43
C106	100	10	0	—	828.94	10.00	0.00	0.00	828.94	10	0	0.00	828.94	30.78
C107	100	10	0	—	828.94	10.00	0.00	0.00	828.94	10	0	0.00	828.94	30.55
C108	100	10	0	—	828.94	10.00	0.00	0.00	828.94	10	0	0.00	828.94	30.18
C109	100	10	0	—	828.94	10.00	0.00	0.00	828.94	10	0	0.00	828.94	30.15
RC101	100	13	26	—	1851.22	9.00	46.70	2631.68	1120.65	9	44	2787.38	1122.73	39.23
RC102	100	13	1	—	1772.42	9.00	31.30	1573.76	1123.74	9	29	1508.22	1119.92	38.13
RC103	100	11	0	—	1416.81	9.00	17.30	698.94	1125.40	9	16	640.61	1131.62	37.56
RC104	100	10	0	—	1262.55	9.00	5.80	155.00	1117.95	9	5	161.57	1126.69	33.99
RC105	100	12	1	—	1531.57	9.00	34.60	1648.93	1130.18	9	33	1580.23	1127.29	39.71
RC106	100	11	0	—	1224.72	9.00	28.63	1161.00	1123.24	9	28	1207.60	1111.51	38.08
RC107	100	—	—	—	—	9.00	18.67	720.80	1112.72	9	18	601.38	1106.99	35.73
RC108	100	—	—	—	—	9.00	11.20	352.37	1107.38	9	10	284.23	1123.11	35.30
R201	100	—	—	—	—	2.00	41.20	9159.21	985.32	2	40	8583.79	988.84	31.01
R202	100	—	—	—	—	2.00	25.90	4631.88	986.18	2	24	5062.88	986.21	31.00
R203	100	—	—	—	—	2.00	10.80	1924.46	979.73	2	10	1514.13	988.64	30.89
R204	100	—	—	—	—	2.00	0.00	0.00	873.07	2	0	0.00	851.66	30.96
R205	100	—	—	—	—	2.00	20.00	3979.80	985.48	2	19	3289.15	979.97	30.89
R206	100	—	—	—	—	2.00	9.00	1699.39	983.37	2	7	1624.34	988.52	30.89
R207	100	—	—	—	—	2.00	1.50	135.63	973.57	2	1	26.44	933.74	30.91
R208	100	—	—	—	—	2.00	0.00	0.00	741.26	2	0	0.00	730.54	30.69
R209	100	—	—	—	—	2.00	12.70	2383.53	980.17	2	10	1910.46	963.47	30.85
R210	100	—	—	—	—	2.00	12.30	2352.23	981.78	2	11	2015.68	983.07	30.88
R211	100	—	—	—	—	2.00	0.70	40.99	968.68	2	0	0.00	931.99	30.90
C201	100	—	—	—	—	3.00	0.00	0.00	591.56	3	0	0.00	591.56	30.19
C202	100	—	—	—	—	3.00	0.00	0.00	591.56	3	0	0.00	591.56	30.33
C203	100	—	—	—	—	3.00	0.00	0.00	591.17	3	0	0.00	591.17	30.38
C204	100	—	—	—	—	3.00	0.00	0.00	590.93	3	0	0.00	590.60	30.37
C205	100	—	—	—	—	3.00	0.00	0.00	588.88	3	0	0.00	588.88	30.17
C206	100	—	—	—	—	3.00	0.00	0.00	588.49	3	0	0.00	588.49	30.22
C207	100	—	—	—	—	3.00	0.00	0.00	588.29	3	0	0.00	588.29	30.18
C208	100	—	—	—	—	3.00	0.00	0.00	588.32	3	0	0.00	588.32	30.24
RC201	100	—	—	—	—	2.00	52.70	12622.22	908.21	2	50	12420.98	912.51	31.01
RC202	100	—	—	—	—	2.00	33.80	7893.30	911.08	2	33	7335.92	907.59	30.98
RC203	100	—	—	—	—	2.00	16.30	3181.25	907.44	2	15	2418.65	910.63	30.89
RC204	100	—	—	—	—	2.00	3.40	601.79	902.05	2	2	460.90	894.01	30.72
RC205	100	—	—	—	—	2.00	39.30	8993.74	909.28	2	37	8706.52	911.66	30.96
RC206	100	—	—	—	—	2.00	35.30	8265.45	906.10	2	33	7424.35	913.25	30.82
RC207	100	—	—	—	—	2.00	25.00	5291.51	908.94	2	24	4805.29	908.80	30.85
RC208	100	—	—	—	—	2.00	12.70	1828.13	911.30	2	11	1694.30	912.64	30.74
Time		5.98 min				41.16 min								
CPU		P-II 0.6G				Opt 2.2G								

Table 34: Results on the type 2 VRPSTW (earliness and lateness) with $\alpha = 1$. Minimization of the sum of distance, earliness and lateness under a fleet size limit. Instances of Solomon and Desrosiers (1988)

Inst	**n**	**m**	**UHGS**		
			Avg 10	Best 10	T(min)
R101	19	100	1546.91	1546.91	24.13
R102	17	100	1377.38	1377.38	26.93
R103	13	100	1158.83	1158.31	30.14
R104	9	100	1004.57	1000.33	30.10
R105	14	100	1342.57	1342.57	30.03
R106	12	100	1223.09	1223.09	30.12
R107	10	100	1080.90	1079.12	30.43
R108	9	100	948.23	945.64	30.08
R109	11	100	1164.68	1164.68	30.11
R110	10	100	1108.30	1104.59	30.16
R111	10	100	1065.76	1065.76	30.08
R112	9	100	991.50	969.91	30.10
C101	10	100	828.94	828.94	30.10
C102	10	100	828.94	828.94	30.08
C103	10	100	828.06	828.06	30.20
C104	10	100	824.78	824.78	29.34
C105	10	100	828.94	828.94	30.12
C106	10	100	828.94	828.94	30.07
C107	10	100	828.94	828.94	30.07
C108	10	100	828.94	828.94	30.33
C109	10	100	828.94	828.94	30.09
RC101	14	100	1584.20	1584.20	29.67
RC102	12	100	1409.36	1409.36	30.07
RC103	11	100	1231.67	1231.67	30.16
RC104	10	100	1123.25	1121.84	30.12
RC105	13	100	1433.37	1433.37	30.18
RC106	11	100	1334.89	1334.89	30.39
RC107	11	100	1203.06	1203.06	30.45
RC108	10	100	1115.44	1115.44	30.12
R201	4	100	1235.14	1235.14	30.28
R202	3	100	1159.76	1159.76	30.13
R203	3	100	937.04	934.10	30.15
R204	2	100	837.21	820.90	30.18
R205	3	100	996.24	994.43	30.12
R206	3	100	910.99	906.54	30.11
R207	2	100	937.79	906.81	30.19
R208	2	100	735.31	730.52	30.13
R209	3	100	911.61	909.16	30.11
R210	3	100	948.91	938.77	30.14
R211	2	100	921.81	912.39	30.17
C201	3	100	591.56	591.56	30.09
C202	3	100	591.56	591.56	30.11
C203	3	100	591.17	591.17	30.11
C204	3	100	590.60	590.60	30.09
C205	3	100	588.88	588.88	30.12
C206	3	100	588.49	588.49	30.13
C207	3	100	588.29	588.29	30.11
C208	3	100	588.32	588.32	30.11
RC201	4	100	1380.33	1380.33	30.12
RC202	3	100	1312.05	1312.05	30.10
RC203	3	100	1047.43	1044.74	30.13
RC204	3	100	796.91	796.68	30.13
RC205	4	100	1300.98	1297.86	30.11
RC206	3	100	1135.44	1135.26	30.12
RC207	3	100	1061.92	1056.88	30.12
RC208	3	100	832.30	827.67	30.13
Time			29.96 min		
Gap			+0.26%	+0.00%	
CPU			Opt 2.2G		

Table 35: Results on the new MDPVRPTW instances.

Inst	**n**	**m**	**t**	**d**	**UHGS**		
					Avg 10	Best 10	T(min)
pr01	48	1	4	4	2483.81	2482.78	0.87
pr02	96	2	4	4	4474.72	4468.60	2.93
pr03	144	3	4	4	5758.43	5735.59	6.93
pr04	192	4	4	4	6708.98	6680.76	18.96
pr05	240	4	4	4	7275.26	7202.79	29.02
pr06	288	5	4	4	8263.29	8207.18	29.68
pr07	72	1	6	6	5497.20	5496.76	2.06
pr08	144	2	6	6	7791.98	7716.08	11.21
pr09	216	3	6	6	10579.81	10504.77	29.49
pr10	288	4	6	6	13612.91	13343.55	30.00
pr11	48	1	4	4	2043.74	2043.74	0.85
pr12	96	2	4	4	3851.07	3825.34	3.47
pr13	144	3	4	4	4781.02	4755.10	9.48
pr14	192	4	4	4	5535.82	5471.17	21.36
pr15	240	4	4	4	5871.13	5830.71	30.00
pr16	288	5	4	4	6913.56	6832.53	30.01
pr17	72	1	6	6	4787.41	4782.74	2.47
pr18	144	2	6	6	6465.43	6402.79	21.78
pr19	216	3	6	6	8940.97	8785.80	30.01
pr20	288	3	6	6	10930.86	10662.62	30.00
pr01b	48	1	4	4	2423.29	2423.29	0.64
pr02b	96	2	4	4	4521.26	4486.88	2.84
pr03b	144	3	4	4	5664.54	5649.74	7.20
pr04b	192	4	4	4	6724.52	6694.32	16.69
pr05b	240	4	4	4	7345.82	7284.81	28.18
pr06b	288	5	4	4	8591.64	8551.01	30.00
pr07b	72	1	6	6	5255.06	5255.06	1.67
pr08b	144	2	6	6	7468.25	7444.70	10.10
pr09b	216	3	6	6	10930.40	10797.34	29.67
pr10b	288	4	6	6	12673.68	12494.65	30.00
pr11b	48	1	4	4	2100.23	2100.23	0.80
pr12b	96	2	4	4	3782.07	3748.45	2.85
pr13b	144	3	4	4	4891.31	4883.31	9.40
pr14b	192	4	4	4	5474.16	5442.94	21.13
pr15b	240	4	4	4	5902.56	5809.17	29.71
pr16b	288	5	4	4	6992.78	6941.25	29.90
pr17b	72	1	6	6	4794.89	4794.89	1.86
pr18b	144	2	6	6	6332.54	6288.46	22.25
pr19b	216	3	6	6	9003.87	8825.36	30.00
pr20b	288	3	6	6	10998.39	10774.34	30.00
Time						16.09 min	
Gap					+0.77%	+0.00%	
CPU						Opt 2.2G	