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Multi-Zone Multi-Trip Vehicle Routing Problem with Time Windows

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Abstract. We introduce a new vehicle routing problem class in which customers are divided into a number of customer zones defined through geographical or timing characteristics. The customers of each of these zones must be serviced within time windows by dedicated routes originating at associated supply points characterized by hard time windows and very limited waiting facilities, if any. A key feature of the problem is that a vehicle can be used to cover routes in different zones at different times. The objective is to minimize the total transportation cost to ensure that the customers are serviced on time and that vehicles arrive at the next customer zone just in time for the next assignment. The problem is addressed by a decomposition-based heuristic. Lower-bound procedures and benchmark problem instances are introduced, highlighting the satisfactory performance of the heuristic. Finally, a wide range of sensitivity analyses on several key parameters reveal interesting facets of the behavior of this new problem class.

Keywords: Vehicle routing, multi-trip, time windows, synchronization, City Logistics.

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1 Introduction

The Vehicle Routing Problem (VRP) involves the design of a set of minimum cost delivery or collection routes from one or several depots to a number of geographically scattered customers subject to some side constraints such as vehicle capacity, time windows, route length, etc. Depending upon these side constraints, a large variety of vehicle routing problems have been classified in the literature. We present a new VRP variant where customers are divided into a number of customer zones and a vehicle is allowed to perform multiple tours for different zones. This problem can be stated as the *Multi-Zone Multi-Trip Vehicle Routing Problem with Time Windows* (*MZMT-VRPTW*). The problem has the following four distinctive features.

1. Multi-Zone. We assume that a large city is divided into a number of customer zones on the basis of some common attributes such as geographical proximity, delivery times, type of product, type of vehicle used for delivery, etc. Thus, each customer zone has a set of customers associated with it. Each customer zone also has one supply point, which works as a distribution center to satisfy the demand of the associated customers. Customer zones are not necessarily disjoint, i.e., they may overlap.

2. Time-Constrained. Each supply point has a fixed opening time and the loading of a vehicle starts exactly at the opening time of a supply point. This forces vehicles to be readily available at supply points upon their opening time. We also assume that there is a limited *allowable waiting time* available at any supply point. Thus vehicles cannot arrive much in advance at supply points. However, vehicles can wait at designated waiting stations before moving to supply points to arrive just before their opening time.

3. Multiple-Tour. A vehicle can perform more than one tour for different customer zones in a given planning period. However, the fixed opening time of supply points restricts vehicles to perform at most one tour in each customer zone.

4. Time Windows. A time window is imposed on the start of service at each customer.

A vehicle route starts from the main depot, performs multiple tours to service different customer zones and finally returns to the main depot. The problem involves the design of a set of minimum cost vehicle routes to ensure that vehicles deliver the goods on time and that they arrive at supply points on time for their next assignment. The objective is to minimize the total transportation cost. This cost cost includes two terms: 1) a fixed cost for operating a vehicle, 2) a variable cost in terms of the total distance travelled by all the vehicles. The main challenge in this problem is to effectively synchronize the successive portions of vehicle routes in different customer zones, since vehicles must meet the time windows at both customer locations and supply points.

In this paper, we propose a heuristic solution approach for the MZMT-VRPTW and we study the properties of solutions for various parameter values. The contributions of the paper are the following: 1) A formal definition of a new variant of vehicle routing problem, the MZMT-VRPTW, and a general model formulation; 2) A heuristic solution procedure for the MZMT-VRPTW; 3) A set of benchmark instances for the problem; 4) An extensive sensitivity analysis for a wide range of values of key problem parameters.

The remainder of this paper is organized as follows. In Section 2, we describe the problem and review the related literature. The problem is formulated in Section 3 and a solution procedure is proposed in Section 4. In Section 5, we present lower bound calculations. In Section 6, we introduce benchmark instance sets for the MZMT-VRPTW and describe how these problem sets are generated. Computational results are reported in Section 7, followed by conclusions in Section 8.

2 Problem description

In this section we describe the problem in detail and present the literature review. We first define some necessary notation.

Let S be the set of supply points. Since we assume that there is a unique supply point associated with each zone and that each supply point supplies a single zone, S also refers to the set of customer zones, i.e., index $s \in S$ is used to refer both to customer zone s and its associated supply point, depending on the context. Let $t(s)$ be the opening time for supply point s. To simplify the exposition, we assume that supply points (and customer zones) are indexed in non-decreasing order of opening time, i.e., $s < s'$ implies that $t(s) \leq t(s')$. Let D_s denote the set of customers associated with customer zone s and $D = \cup_{s \in S} D_s$. A non-negative demand q_d is associated with each customer $d \in D$. A fleet of n_ν identical vehicles of capacity Q are based at main depot g. When a vehicle is moved from the main depot g to a supply point $s \in S$, no travel cost is incurred, but a fixed cost F is charged.

At supply point s, loading of vehicles starts exactly at opening time $t(s)$. Thus, all the vehicles required to serve customers in D_s must be available at supply point s at time $t(s)$. A vehicle takes $\delta(s)$ time units for loading all the freight that it will deliver in zone s. Vehicles load the designated freight at supply point s at time $t(s)$ and leave this point at time $t(s)+\delta(s)$ to perform the delivery route for servicing a subset of customers from D_s. A time window $[a(d), b(d)]$ is imposed on the start of service at customer d. A vehicle takes $\delta(d)$ time units to unload the freight at customer d and τ_{ij} time units to travel from point i to point j, where $i, j \in S \cup D$. After performing its route in customer zone s, a vehicle can move to another customer zone s' for the next tour. Eeach supply point has a limited allowable time η for a vehicle to wait. The fixed opening time and limited allowable waiting time restrict vehicles to arrive at supply point s' between time instants $t(s') - \eta$ and $t(s')$. However, vehicles are allowed to wait (longer) at designated waiting stations $w \in W$ before moving to a supply point s'. Finally, vehicles return back to the main depot g.

Our interest in the MZMT-VRPTW arises from the two-tier city logistics distribution system described by Crainic et al. (2009). In this system, two types of vehicles, urban trucks and city freighters, are used. The two tiers of the system operate from two different facilities called City Distribution Centers (CDC) and satellites. In the first tier, freight is moved from the CDC to satellites by urban trucks. Each urban truck travels to a subset of satellites and returns back to the CDC. At satellites, freight is transferred from the urban truck to city freighters. In the second tier of the system, goods are moved from each satellite to their final destination by city freighters. Each city freighter performs a route to serve designated customers. After performing a route at a satellite, city freighters travel to another satellite to perform another route. When a city freighter has completed the successive routes that were assigned to it for the day, the vehicle returns to the depot where it is based, thus completing its *workday*.

The MZMT-VRPTW corresponds to the second tier of the two-tier city logistics system, which involves the design of optimal delivery routes for city freighters. The correspondance between the second tier of the two-tier city logistics system and the MZMT-VRPTW is further explained below:

- Multi-Zone. In the second tier of the system, the set of customer demands that have to be satisfied through a particular satellite at a given time is already determined by the distribution strategy of urban trucks. The combination (satellite, time) is called in this paper a *supply point*. This leads to the main assumption of our model on dividing a large city into a number of customer zones each with a unique supply point.
- Time-Constrained. Satellites are assumed to be cross-docking points where goods are transferred from urban trucks to city freighters. Upon the arrival of an urban truck, goods are moved directly to the appropriate city freighters without using any intermediate facility. Therefore, in our model, we assume that the loading of vehicles starts exactly at the opening instant of supply points gioven by the arrival time of the urban truck. Traffic regulations and space availability do not allow city freighters to wait at supply points for a long period, if at all, and we thus assume a (very) limited *allowable waiting time* at supply points. This leads to the introduction of *waiting stations*, which are dedicated locations (e.g., parkings) where vehicles can wait for longer periods.

Another similar situation in which customers are divided into a number of customer zones arises in the School Bus Routing Problem (SBRP). In the SBRP, a fleet of buses is used to service several schools. Each school has a set of bus stops associated with it and for each of these stops there is a known group of students. Each school has a fixed opening time and there are strict time windows for the delivery of students to the schools. School buses pickup students from their bus stop and take them to their school. After performing its route for any given school zone (including the delivery of students to the school itself), a bus typically moves to another school zone to pick up its students. The problem is to optimize the level of service in such a way that all students are picked up brought to their respective school, while satisfying all the time windows.

In this paper, we present a new variant of the VRP. While numerous variations of the original VRP have been proposed since the original paper of Dantzig and Ramser (1959), none of these corresponds exactly to our problem. Some variants, however, share some common aspects with our work. The Multi-Trip Vehicle Routing and Scheduling Problem (MTVRSP) and the VRP with Inter-Depot Routes (VRPIDR) are two such variants in which each vehicle is allowed to perform more than one route during its planning period. In the MTVRSP, a single depot is used to replenish vehicles before they perform subsequent trips. This problem has been addressed by Taillard et al. (1996), Brandao and Mercer (1997, 1998), Petch and Salhi (2003), Salhi and Petch (2007) and Olivera and Viera (2007). Azi et al. (2007) propose an exact algorithm for a single-vehicle MTVRSP with time windows, while Azi et al. (2010) tackle the multi-vehicle case. In the VRPIDR, vehicles are allowed to replenish at intermediate points and can thus perform more than one trip in a given planning period. Angelelli and Speranza (2002) address a periodic version of this problem, while Crevier et al. (2007) study a multi-depot one.

As described earlier, the School Bus Routing Problem (SBRP) is the only problem that closely resembles the problem that we address. While there is a large number of articles on the SBRP, there is no standard definition of the problem. In fact, most of the papers on the SBRP focus on real-world applications and thus each of them deals with very specific assumptions and constraints (see the surveys by Desrosiers et al. (1981), Braca et al. (1997) and Park and Kim (2009)). According to Desrosiers et al. (1981), the SBRP can be decomposed into a five subproblems: 1) Data preparation, 2) Bus stop selection, 3) Bus route generation, 4) School bell time adjustment and 5) Route scheduling. Most of the studies consider either a single or a subset of these subproblems. The fifth subproblem, i.e., the route scheduling one, is close to our problem. Given exact starting and finishing times for routes, this subproblem consists in determining subsets of routes that can be executed by the same bus. Different heuristic approaches have been proposed for solving this subproblem, such as the ones presented by Newton and Thomas (1974), Bodin (1975), Bodin and Berman (1979), Desrosiers et al. (1981, 1986), Braca et al. (1997), Li and Fu (2002), and Spada et al. (2005). In the school bell-time adjustment subproblem, the starting and ending times of a school are considered to be decision variables that can be set to minimize the number of buses. The issue of bell-timing adjustment is considered by Desrosiers et al. (1981, 1986), Bodin et al. (1983) and Fugenschuh (2009). As stated earlier, the structure of our problem is similar to the multi-school bus route scheduling problem, however, the characteristics, objectives and issues are completely different. For example, one of the objectives considered by a number of studies on the SBRP is to minimize the total travel time spent by students on the bus, while in our case the total time spent by freight in the vehicle is not a issue. For more information on the SBRP, we refer readers to the recent survey by Park and Kim (2009).

3 Problem formulation

In this section we provide a formulation for the MZMT-VRPTW. This formulation is inspired from the one found in Crainic et al. (2009). The problem is defined on a space-time network (V, A), where the set of nodes V represents physical locations and the arcs in A stand for the possible movements between these nodes that are feasible with respect to the associated time windows. Set V is made up of the main depot g and the sets of supply points, customers, and waiting stations, i.e., $V = g \cup S \cup D \cup W$.

Several subsets of arcs make up set A:

$$A = \cup_{s \in S}[A_s^{SD} \cup A_s^{DS} \cup A_s^{DD}] \cup A^{DW} \cup A^{DG} \cup A^{GS} \cup A^{WS}.$$

- Arcs in $A_s^{SD} = \{(s, d) | d \in D_s, t(s) + \delta(s) + \tau_{sd} \leq b(d)\}, s \in S$, go from a supply point s to each customer $d \in D_s$, such that the vehicle can arrive before due date $b(d)$ to serve node d.
- Arcs in $A_s^{DS} = \{(d, s') | s' \in S, d \in D_s, t(s') > t(s), a(d) + \delta(d) + \tau_{ds'} \leq t(s')\}, s \in S$, link customers $d \in D_s$ to supply points s' $(s' > s)$ that can be reached before their opening time $t(s')$.
- We can also define the backward-star of a supply node $s \in S$ with respect to customers as the set $A_s^{S^-} = \{(d, s) | d \in D_{s'}, s' \in S, s' < s, a(d) + \delta(d) + \tau_{ds} \leq t(s)\}$, i.e., the sets of links from some customers to s such that a vehicle can travel from these customers to s before its opening time $t(s)$.
- An arc exists between each pair of customers $(d, j), d, j \in D_s$, for which the movement is feasible with respect to the respective time-window constraints. Given the time window $[a(d), b(d)]$ and the service time $\delta(d)$ of customer $d \in D_s$, one considers only the arcs to customer j such that $a(d) + \delta(d) + \tau_{dj} \leq b(j)$. Set A_s^{DD} contains these arcs.
- Define the back-star of node $d \in D_s, s \in S$, with respect to customers as the set of arcs $A_d^{D^-} = \{(i, d) | i \in D_s, a(i) + \delta(i) + \tau_{id} \leq b(d)\}$, such that the vehicle can arrive before latest starting time $b(d)$ to serve node d.

- Arcs in $A^{DW} = \{(d,w)|d \in \cup_{s\in S} D_s, w \in W\}$ go from customers in D to waiting stations in W.
- Arcs in $A^{DG} = \{(d,g)|d \in \cup_{s\in S} D_s \}$ go from customers to the main depot g.
- Arcs in $A^{GS} = \{(g,s),|s \in S\}$ go from the main depot g to supply points in S.
- Arcs in $A^{WS} = \{(w,s),|s \in S, w \in W\}$ go from waiting stations in W to supply points in S.

Let F stand for the fixed cost for operating a vehicle and c_{ij}, $(i,j) \in \bar{A} = A \setminus (A^{GS} \cup A^{DG})$ represent the unit transportation cost between two nodes $i, j \in V$. We define the following decision variables:

- x_{ij}^k, a binary variable that takes value 1 if arc (i,j) is used by vehicle k and value 0 otherwise;
- ω_i^k, a continuous variable specifying the start of service at demand node $i \in D$ when serviced by vehicle k. For supply points and waiting stations, ω_i^k represents the arrival time of vehicle k.

The MZMT-VRPTW can then be formulated as the following three-index based vehicle flow model:

$$(MZMT - VRPTW) \qquad \text{Minimize} \sum_{k=1}^{n_\nu} \left[\sum_{(i,j)\in\bar{A}} c_{ij} x_{ij}^k + F \sum_{(i,j)\in A^{GS}} x_{gs}^k \right] \tag{1}$$

$$\text{Subject to} \qquad \sum_{(s,d)\in A_s^{SD}} x_{sd}^k \leq 1 \quad s \in S, \quad k = 1, ..., n_\nu, \tag{2}$$

$$\sum_{k=1}^{n_\nu} \sum_{(d,j)\in A_s^{DD}} x_{dj}^k + \sum_{k=1}^{n_\nu} \sum_{(d,s')\in A_s^{DS}} x_{ds'}^k + \sum_{k=1}^{n_\nu} \sum_{(d,g)\in A^{DG}} x_{dg}^k + \sum_{k=1}^{n_\nu} \sum_{(d,w)\in A^{DW}} x_{dw}^k = 1 \quad d \in D_s, \quad s \in S \tag{3}$$

$$\sum_{k=1}^{n_\nu} \left[x_{sd}^k + \sum_{(i,d)\in A_d^{D-}} x_{id}^k \right] = 1 \quad d \in D_s, \quad s \in S, \tag{4}$$

$$\sum_{(g,s)\in A^{GS}} x_{gs}^k + \sum_{(w,s)\in A^{WS}} x_{ws}^k + \sum_{(d,s)\in A_s^{S-}} x_{ds}^k = \sum_{(s,d)\in A_s^{SD}} x_{sd}^k \qquad s \in S, \quad k = 1, ..., n_\nu, \tag{5}$$

$$\sum_{k=1}^{n_\nu} \sum_{(d,g)\in A^{DG}} x_{dg}^k = \sum_{k=1}^{n_\nu} \sum_{(g,s)\in A^{GS}} x_{gs}^k, \tag{6}$$

$$\sum_{(d,w)\in A^{DW}} x^k_{dw} = \sum_{(w,s)\in A^{WS}} x^k_{ws} \quad w \in W, \quad k = 1, ..., n_\nu, \tag{7}$$

$$\sum_{d\in D_s} q_d x^k_{sd} + \sum_{(i,j)\in D_s} q_j x^k_{ij} \leq Q \quad s \in S \quad k = 1, ..., n_\nu, \tag{8}$$

$$\begin{aligned} t(s) + \delta(s) + \tau_{sd} - \omega^k_d \leq & (1 - x^k_{sd})(t(s) + \delta(s) + \tau_{sd}) \\ & (s,d) \in A^{SD}_s, \quad s \in S, \quad k = 1, ..., n_\nu, \end{aligned} \tag{9}$$

$$\begin{aligned} \omega^k_i + \delta(i) + \tau_{ij} - \omega^k_j \leq & \; (1 - x^k_{ij}))(b(i) + \delta(i) + \tau_{ij}) \\ & (i,j) \in A^{DD}_s, \quad s \in S, \quad k = 1, ..., n_\nu, \end{aligned} \tag{10}$$

$$\begin{aligned} & a(d)\left[x^k_{sd} + \sum_{(i,d)\in A^{D-}_s} x^k_{id}\right] \leq \omega^k_d \\ & \leq b(d)\left[\sum_{(d,i)\in A^{DD}_s} x^k_{di} + \sum_{(d,s')\in A^{DS}_s} x^k_{ds'} + \sum_{(d,g)\in A^{DG}} x^k_{dg} + \sum_{(d,w)\in A^{DW}} x^k_{dw}\right] \\ & d \in D_s, \quad s \in S, \quad k = 1, ..., n_\nu, \end{aligned} \tag{11}$$

$$(t(s) - \eta) \sum_{(s,d)\in A^{SD}_s} x^k_{sd} \leq \omega^k_s \leq t(s) \sum_{(s,d)\in A^{SD}_s} x^k_{sd} \quad s \in S, \quad k = 1, ..., n_\nu, \tag{12}$$

$$\begin{aligned} \omega^k_d + \delta(d) + \tau_{ds} - \omega^k_s \leq & (1 - x^k_{ds})(b(d) + \delta(d) + \tau_{ds}) \\ & (d,s) \in A^{S^-}_s, \quad s \in S, \quad k = 1, ..., n_\nu, \end{aligned} \tag{13}$$

$$\begin{aligned} \omega^k_w + \tau_{ws} - \omega^k_s \leq & (1 - x^k_{ws})(max_{d\in D}(b(d) + \tau_{dw}) + \tau_{ws}) \\ & (w,s) \in A^{GS}, \quad s \in S, \quad k = 1, ..., n_\nu, \end{aligned} \tag{14}$$

$$\begin{aligned} \omega^k_d + \delta(d) + \tau_{dw} - \omega^k_w \leq & (1 - x^k_{dw})(b(d) + \delta(d) + \tau_{dw}) \\ & (w,s) \in A^{GS}, \quad s \in S, \quad k = 1, ..., n_\nu, \end{aligned} \tag{15}$$

$$x^k_{ij} \in \{0, 1\}, \quad (i,j) \in A, \quad k = 1, ..., n_\nu. \tag{16}$$

The objective function (1) minimizes the total transportation cost, including the fixed costs incurred for using vehicles. Constraint (2) ensures that a vehicle leaving a supply point visits a customer, while constraint (3) forces the single assignment of customers to routes. Constraint (3) also ensures that a vehicle leaving a customer d goes either to another customer from the same set D_s, a supply point $s'(s' > s)$, a waiting station $w \in W$ or the main depot g. These two sets of constraints together with (4) also enforce the flow conservation at customer nodes. The conservation of flow at supply point is completed by constraint (5). Constraints (6) and (7) represent the conservation of flow at main depot and waiting stations respectively. Constraint (8) enforces the restrictions on the vehicle capacity, each time a city freighter leaves a supply point s to deliver customer demands from set D_s.

Constraints (9), (10) and (11) enforce schedule feasibility with respect to the service time consideration for movements between customers, but no restrictions are imposed on the actual arrival time of a vehicle. Constraints (12), (13), (14) and (15) impose the synchronization between vehicle arrival and opening time of supply points. Constraint (12) ensure that if a vehicle k is used to serve customers associated with supply point s then it should arrive at supply point s between time interval $t(s)-\eta$ and $t(s)$. Constraint (13) enforce the schedule feasibility for movements between a customer node $d \in D_{s'}(s' < s)$ and a supply point s. Constraint (14) enforce the schedule feasibility for movements between a waiting station w and a supply point s. Constraint (15) enforce the schedule feasibility for movements between a customer node $d \in \cup_{s \in S} D$ and a waiting station w. Finally, constraint (16) impose binary values on the flow variables.

4 Solution Method

Because the MZMT-VRPTW is an extension of the VRP and since only small instances of the VRP can be solved exactly, it is clear that one cannot expect to solve large instances of the MZMT-VRPTW with the formulation presented in Section 3. We have therefore opted for the development of a heuristic solution procedure. In the MZMT-VRPTW, customers are divided into a number of customer zones. Taking advantage of this special structure of the problem, we use a decomposition-based solution method for the MZMT-VRPTW. The general idea behind our approach, inspired by Crainic et al. (2009), is to decompose the problem by customer zone, to solve the resulting small VRPTW in each customer zone, and finally to determine the flow of vehicles among different customer zones to serve the routes associated with these customer zones. The method proceeds in two phases:

1. Routing, in which we solve an independent VRPTW for each customer zone s. This independent VRPTW consists of supply point s and the set of customers D_s associated with zone s.
2. Circulation, where we solve the problem of moving vehicles among supply points to determine the flow of vehicles at minimum cost. The flow of vehicles originates from the main depot, visits subset of supply points, serves the routes associated with these supply points, and finally returns back to the main depot.

4.1 Routing

We solve an independent VRPTW for each customer zone s. This individual VRPTW is defined on a complete graph $G_s = (V_s, A_s)$, where $V_s = \{s\} \cup D_s$ is the vertex set and

$A_s = \{(i,j) : i,j \in V_s, i \neq j\}$ is the arc set. Vertex s represents the supply point where vehicles start delivery tours, while the other vertices of V_s represent the customers to be served. With each customer $i \in D_s$ are associated a nonnegative demand q_i, a service time $\delta(i)$ and a time window $[a(i), b(i)]$. At supply point s, a maximum of n_ν vehicles (city freighters) are available to serve customers.

The VRPTW then consists in designing a maximum of n_ν routes on G_s such that: (i) every route starts and ends at supply point s; (ii) every customer belongs to exactly one route; (iii) the total load of each vehicle does not exceed the vehicle capacity Q; (iv) customers are visited within their respective time window; (v) the total travel time of all vehicles is minimized. In case of early arrival at a customer location, the vehicle serving this customer must wait until the beginning of the timi window before starting service.

In a standard VRPTW, each vehicle returns back to the depot after serving its customers. However, in the final solution of MZMT-VRPTW, a vehicle moves to some other customer zone s' (with $s' > s$) after performing a route in customer zone s. Therefore the cost of arcs $(i, s), i \in D_s$, going back to supply point s affect the solution quality. We consider two different alternatives to address this issue : 1) set the cost of all of these arcs to zero, 2) set the cost of arc cost $(i, s), i \in D_s$, to the cost of moving to nearest supply point from node i. In section 7, we perform experiments with these two alternatives to see which one produces the most effective solutions.

To solve the VRPTW for graph G_s, we use the Unified Tabu Search designed by Cordeau et al. (2001), a method which has proved to be highly effective for tackling a wide range of classical vehicle routing problems. Since we have an individual VRPTW problem for each customer zone, a total of $|S|$ individual VRPTWs must be solved. The solutions of these VRPTWs provide routes for the vehicles. These routes are assigned to be serviced by one of the vehicles in the circulation phase.

4.2 Circulation

The output of the routing phase determines the number of vehicles required to serve each customer zone and the actual vehicle routes. In the circulation phase, we determine the flow of vehicles among customer zones to serve the predefined routes generated in the routing phase.

The workday of a city freighter can be described as follows: The vehicle starts from the main depot and moves to some customer zone, say s, where it is assigned to serve one of the routes generated in the routing phase for this zone. After performing this route, the vehicle can move to another customer zone, say $s'(s' > s)$. The vehicle is allowed to move to supply point s' only if it is possible to reach it before its opening time $t(s')$. At the same time, a vehicle is not allowed to reach s' before $t(s') - \eta$. If the direct movement of a vehicle would make it arrive at s' before $t(s')-\eta$, then it must go to some waiting station to wait before moving to s' or finishing its workday. The vehicle leaves the waiting station in such a way that it arrives at supply point s' between $t(s')-\eta$ and $t(s')$. In this way, a vehicle moves through a subset of customer zones to perform exactly one predefined route in each zone and finally returns back to the main depot.

Let N_s represent the number of vehicles required to service customer zone s. Let $\Delta_\phi(s)$ and $k_\phi(s)$ represent the total time and total distance for route $\phi = 1, ..., N_s$, from supply point s to the last customer visited on route ϕ, which is denoted $d(s_\phi)$. Then $t(s)+\Delta_\phi(s)$ is the time at which route ϕ is completed and the vehicle is ready to proceed to the next customer zone or to the main depot. Let $w(s_\phi, s') \in G$ represent the nearest waiting station between demand node $d(s_\phi)$ and supply point s'. A minimum cost network flow problem may then be defined to yield a circulation plan for the vehicles.

The set V is made up of the node sets s for supply points and g for main depot plus the set of route nodes s_ϕ standing for the routes $\phi = 1, ..., N_s$ associated with each node s. The arcs of the network are:

- Arcs $(s, s_\phi) \in A_s^{SD}$ go from supply-node s to each route node s_ϕ, $\phi = 1, ..., N_s$, $s \in S$.
- Arcs in $A_s^{DS} = \{s_\phi, s'\}$ link each route-node s_ϕ to supply-nodes s', such that $t(s') - \eta \leq t(s) + \Delta_\phi(s) + \delta_{d(s_\phi),s'} \leq t(s')$ or $t(s) + \Delta_\phi(s) + \delta_{d(s_\phi),g(s_\phi,s')} + \delta_{g(s_\phi,s'),s'} \leq t(s')$. These two conditions represent the schedule feasibility at supply point s' either directly or while using waiting station.
  The set $A_s^{S-} = \{s'_\phi, s\}$ represents the backstar of node s with respect to route-nodes s'_ϕ such that $t(s)-\eta \leq t(s')+\Delta_\phi(s')+\delta_{d(s'_\phi),s} \leq t(s)$ or $t(s')+\Delta_\phi(s')+\delta_{d(s'_\phi),g(s'_\phi,s)} + \delta_{g(s'_\phi,s),s} \leq t(s)$.
- Arcs in $A^{GS} = (g, s)$ go from main depot g to supply point s.
- Arcs in $A^{DG} = (s_\phi, g)$ go from route node s_ϕ to main depot g.

Define the decision variables y_{ij} to stand for the number of city freighters that move between nodes $i, j \in V$. Let F stand for the fixed cost for operating a vehicle and c_{ij}, $(i, j) \in \bar{A} = A \setminus (A^{GS} \cup A^{DG})$ represent the unit transportation cost between two nodes $i, j \in V$. The minimum cost network formulation for the circulation problem then becomes

$$\text{Minimize} \sum_{(i,j)\in\bar{A}} c_{ij} y_{ij} + F \sum_{(g,s)\in A^{GS}} y_{gs} \tag{17}$$

$$\text{Subject to} \sum_{(s,s_\phi)\in A_s^{SD}} y_{ss_\phi} = N_s, \quad s \in S, \tag{18}$$

$$y_{ss_\phi} = 1, \quad (s, s_\phi) \in A_s^{SD}, \quad s \in S, \tag{19}$$

$$y_{gs} + \sum_{(i,j)\in A_s^{S-}} y_{ij} = \sum_{(s,s_\phi)\in A_s^{SD}} y_{ss_\phi}, \quad s \in S, \tag{20}$$

$$y_{ss_\phi} = y_{s_\phi g} + \sum_{(s_\phi,s')\in A_s^{DS}} y_{s_\phi s'}, \quad s \in S, \tag{21}$$

$$\sum_{(g,s)\in A^{GS}} y_{gs} = \sum_{(s_\phi,g)\in A^{DG}} y_{s_\phi g}, \tag{22}$$

$$y_{ij} \geq 0, (i,j) \in A. \tag{23}$$

The objective function (17) minimizes the total transportation cost. Constraint (18) fixes the number of city freighters that must arrive at each supply point. Constraint (19) restricts single vehicle per route condition at each route node. Constraints (20) and (21) enforce the flow conservation conditions at supply points and route nodes respectively. Conservation of flow at main depot g is enforced by constraint (22). Finally, condition (23) imposes binary values on the flow variables.

4.3 Improvement Scheme

The routing and circulation phases determine the complete routes of city freighters, as well as the number of vehicles used to serve each zone. Since in this complete solution city freighters do not return to the supply point of the zone in which they just performed a route, it is possible to improve the solution by reassigning customers between the routes servicing each zone. To maintain the consistency of the overall delivery plan, *we only consider customer reassignments between routes that service the same zone.* We may thus perform this improvement step separately for each customer zone. This is done using again the Unified Tabu Search of Cordeau et al. (2001), but the method is applied in a slightly different way than in the routing phase: 1) The solution obtained after the routing and circulation phases is used as an initial feasible solution; 2) The number of vehicles routes is known and kept constant; 3) Each route has a known (and fixed) endpoint, which is either another supply point or the main depot; 5) Each route has a maximum duration, which depends upon the opening time of its endpoint.

5 Lower bounds

To assess the quality of the solutions produced by the heuristic described in the previous section, we would like, ideally, to compare these solutions with optimal ones. Unfortunately, the MZMT-VRPTW integer programming model presented in section 3 is not tractable within reasonable CPU times even for small problem instances. This stems from the fact that this model relies on a three-index vehicle flow formulation and therefore requires a large number of decisions variables. We must therefore settle down to comparisons between heuristic solution values and lower bounds, which should be obtained by relaxing some of the constraints of the original problem.

Going back to the three-index formulation, we note that we must resort to it to ensure the schedule feasibility at waiting stations and supply points. If we allow unlimited waiting time at supply points, then vehicles can move directly from a demand node to a supply point without using a waiting station. In this case, we can remove the time variables ω_i^k for supply points and waiting stations from the formulation and then formulate the relaxed problem as a two-index vehicle flow model. We formulate this model for the relaxed MZMT-VRPTW on a space-time network (V, A). The vertex set V is made up of the main depot g, the set of supply points S, and customer demands $d \in D_s, s \in S$. The arc set is made up of sets $A_s^{SD}, A_s^{DS}, A_s^{DD}, A^{DG}, A^{GS}$ from section 3, i.e., feasible movements corresponds to the arcs of $A = \cup_{s \in S}[A_s^{SD} \cup A_s^{DS} \cup A_s^{DD}] \cup A^{DG} \cup A^{GS}$.

As in Section 3, let F stand for the fixed cost for operating a vehicle and c_{ij}, $(i, j) \in \bar{A} = A \setminus (A^{GS} \cup A^{DG})$, represent the transportation cost between nodes $i, j \in V$. We define the following decision variables:

- x_{ij}, a binary variable which takes value 1 if arc (i, j) is used by a vehicle and 0 otherwise;
- $\omega(i)$, a continuous variable specifying the start time of service for customer $i \in D$;
- u_i, a continuous variable representing the load of the vehicle that visits customer i, as it departs.

The relaxed MZMT-VRPTW can then be written as the following multi-commodity network flow model with time window and capacity constraints:

$$(LB1) \qquad \text{Minimize} \sum_{(i,j)\in(\bar{A})} c_{ij}x_{ij} + F \sum_{(g,s)\in A^{GS}} x_{gs} \tag{24}$$

$$\text{S.t.} \quad \sum_{(d,j)\in A_s^{DD}} x_{dj} + \sum_{(d,s')\in A_s^{DS}} x_{ds'} + \sum_{(d,g)\in A^{DG}} x_{dg} = 1, \quad d \in D_s, \quad s \in S, \tag{25}$$

$$x_{sd} + \sum_{(i,d)\in A_s^{D-}} x_{id} = 1, \quad d \in D_s, \quad s \in S, \tag{26}$$

$$\sum_{(g,s)\in A^{GS}} x_{gs} + \sum_{(d,s)\in A_s^{S-}} x_{ds} = \sum_{(s,d)\in A_s^{SD}} x_{sd}, \quad s \in S, \tag{27}$$

$$\sum_{(d,g)\in A^{DG}} x_{dg} = \sum_{(g,s)\in A^{GS}} x_{gs}, \tag{28}$$

$$u_i + q_j - u_j \leq (1 - x_{ij})Q, \quad (i,j) \in A_s^{DD}, \quad s \in S, \tag{29}$$

$$q_d \leq U(d) \leq Q, \quad d \in D, \tag{30}$$

$$\begin{aligned} t(s) + \delta(s) + \tau_{sd} - \omega(d) \leq & (1 - x_{sd})(t(s) + \delta(s) + \tau_{sd} - a(d)) \\ & (s,d) \in A_s^{SD}, \quad s \in S, \end{aligned} \tag{31}$$

$$\begin{aligned} \omega(i) + \delta(i) + \tau ij - \omega(j) \leq & (1 - x_{ij})(b(i) + \delta(i) + \tau ij - a(j)) \\ & (i,j) \in A_s^{DD}, \quad s \in S, \end{aligned} \tag{32}$$

$$a(d) \leq \omega(d) \leq b(d), \quad d \in D, \tag{33}$$

$$\begin{aligned} \omega(d) + \delta(d) + \tau ds - t(s) \leq & (1 - x_{sd})(b(d) + \delta(d) + \tau ds - t(s)) \\ & (d,s) \in A_s^{S^-}, \quad s \in S, \end{aligned} \tag{34}$$

$$x_{ij} \in \{0,1\}, \quad (i,j) \in A. \tag{35}$$

The objective function (24) minimizes the total transportation-related cost, as well as the number of vehicles used. Constraints (25) and (26) enforce the flow conservation at customer nodes. These constraints also guarantee that each customer node will be visited exactly once. Constraints (27) and (28) represent the conservation of flow at supply points and main depot respectively. Constraints (29) and (30) enforce the restrictions on the vehicle capacity. These constraints also ensure the connectivity requirements for customer nodes.

Constraints (31), (32), (33) and (34) enforce schedule feasibility with respect to the time consideration at customer nodes and supply points. Constraint (31) enforces the schedule feasibility for movements between supply point s and customers $d \in D_s$. Constraints (32) and (33) enforce the schedule feasibility for movements between customers. Constraint (34) enforces the schedule feasibility for movements between customer-demand $d \in D_{s'}(s' < s)$ and supply point s. Finally, constraint (35) ensures that the flow variables take binary values.

We call the above formulation the LB1 model. This model can solve larger problem instances compared to the MZMT-VRPTW model. Still, LB1 cannot solve the really large problem instances because of time-window and capacity constraints. Therefore, we now introduce the LB2 model to solve even larger problem instances. The LB2 model is a relaxation obtained from LB1 by dropping constraints on time windows and vehicle capacity. Let $n(D_s)$ represents the minimum number of vehicles required to serve customer set D_s, then LB2 can be formulated as follows:

$$(LB2) \qquad \text{Minimize} \sum_{(i,j) \in (\bar{A})} c_{ij} x_{ij} + F \sum_{(g,s) \in A^{GS}} x_{gs} \tag{36}$$

$$\text{Subject to} \quad \sum_{(s,d) \in A_s^{SD}} x_{sd} \geq n(D_s) \quad s \in S, \tag{37}$$

$$\sum_{(d,j)\in A_s^{DD}} x_{dj} + \sum_{(d,s')\in A_s^{DS}} x_{ds'} + \sum_{(d,g)\in A^{DG}} x_{dg} = 1 \quad d \in D_s, \quad s \in S, \tag{38}$$

$$x_{sd} + \sum_{(i,d)\in A_s^{D-}} x_{id} = 1 \quad d \in D_s, \quad s \in S, \tag{39}$$

$$\sum_{(g,s)\in A^{GS}} x_{gs} + \sum_{(d,s)\in A_s^{S-}} x_{ds} = \sum_{(s,d)\in A_s^{SD}} x_{sd} \quad s \in S, \tag{40}$$

$$\sum_{(d,g)\in A^{DG}} x_{dg} = \sum_{(g,s)\in A^{GS}} x_{gs}, \tag{41}$$

$$x_{ij} \in \{0,1\} \quad (i,j) \in A. \tag{42}$$

Constraint (37) enforces the restriction on minimum number of vehicles required to serve customer zone s. Other constraints are the same as in model LB1 described earlier. To compute the value of $n(D_s)$, we use a lower-bound calculation for the Bin Packing Problem (BPP).

6 Development of Sets of Problem Instances

In order to perform computational experiments on the methods we present, we generated two sets of problem instances, a large and small one. The total number of customers in the former set varies from 400 to 3,600, while in the latter it varies from 25 to 90. Large problem instances correspond to real-world applications, but the lower-bound formulation $LB1$ cannot be solved for them. This motivated the generation of the smaller instances in order to be able to use the lower bound $LB1$ in the evaluation of the performance of the proposed heuristic.

We generated six sets of 10 large instances each, for a total of 60 large problem instances. The six sets are called A1, A2, B1, B2, C1, and C2. The number of customer zones for these sets is 4, 8, 16, 32, 36, and 72, respectively. Instances in set A are the smallest in size with 400 customers, instances in set B are of medium size with 1600 customers, and instances in set C are the largest ones with 3600 customers. Supply points, waiting stations, and customers were uniformly distributed in a square, with the X and Y coordinates in the interval [0,100], [0, 200], and [0, 300] for sets A, B, and C, respectively. We included one waiting station for every group of 100 customers. A summary of these parameters is given in Table 1.

Other parameters were generated as follows. The opening time of the first supply point was set to zero and generated randomly in the [0, 14,400] range (in minutes) for the others. The opening times were first ordered and then assigned to to the supply points

Problem set	Number of customer zones / supply points	Number of customers	Number of waiting stations	X,Y customer coordinates
A1	4	400	4	[0, 100]
A2	8	400	4	[0, 100]
B1	16	1600	16	[0,200]
B2	32	1600	16	[0, 200]
C1	36	3600	36	[0, 300]
C2	72	3600	36	[0, 300]

Table 1: Parameters for large instances

to enforce their non-decreasing order. Waiting and vehicle-loading times at supply points were set at 100 and 30, respectively, for all supply points.

A customer is assigned to a customer zone based on its proximity to the corresponding supply point. A customer is randomly assigned to one of the four nearest supply points, the assignment probability being inversely proportional to the distance between the customer and the supply point. More precisely, the weights for assigning customers to their first, second, third, and fourth nearest supply points were set to 50%, 25%, 15%, and 10%, respectively. The ready time of a customer was generated randomly in the range of [0, 300], a time window duration was then randomly generated in the interval [150, 450], and finally, the due date for delivery was set by adding the time window duration to the ready time. These ready times and due dates may, however, not be feasible, because customers are serviced by different supply points with different opening times. To ensure feasibility, a number of values were added to the ready times and due dates: the opening time of the supply point to which the customer is assigned, the loading time at the supply point, and the smallest integer higher than the value of the distance between the customer and the supply point. The demand of each customer was randomly generated in the interval [5, 25]. Since we fix the vehicle capacity to 100 and the average customer demand is 15, a vehicle should handle on average 6-7 customers (per zone). The service time of customers was set to 20 for all customers.

The capacity of all vehicles was set to 100. The main depot was located in the middle of the region served, with X and y coordinates equal to [50,50] for set A, [100,100] for set B, and [150,150] for set C. The location of main depot does not affect the solution since the distance from the main depot to other nodes is not included in the objective function. We assume that the required number of vehicles is readily available at supply points. The fixed cost for operating a vehicle was set to 500 for all instance sets.

The six sets of small problem instances, D1 to D6 with 10 instances in each set, were generated following the same main ideas. We seek to evaluate the performance gap of our heuristic solution relative to the lower bound solution. In addition, we would like to

Problem set	Number of customer zones	Number of customers / zone	Total umber of customers	X,Y customer coordinates
D1	5	5	25	[0, 100]
D2	5	7	35	[0, 100]
D3	5	9	45	[0,100]
D4	5	6	30	[0, 100]
D5	10	6	60	[0, 100]
D6	15	6	90	[0, 100]

Table 2: Parameters for small instances

analyze the correlation between this gap and the problem size/complexity. Unlike other VRP variants, however, the total number of customers may not necessarily reflect the difficulty of addressing a particular problem instance, which is principally affected by two parameters: i) the number of customers per customer zone $|D_s|$, and ii) the number of supply points $|S|$. Therefore, the small problem sets were divided into two groups. The number of supply points $|S|$ was kept the same (at 5) for the instances of the first group but the number of customers per customer zone $|D_s|$ varied, being set to 5, 7 and 9 for D1, D2, and D3, respectively. The reverse policy was applied to the second group of instance sets, the number of customers per zone being kept the same at 6, but the number of supply points varied, being set respectively to 5, 10, and 15 for sets D4, D5, and D6. Small instances have no waiting stations but allow waiting at supply points. A summary of the parameters for these six problem sets is given in Table 2.

The other parameters were generated similarly to the large-instance case, except for the assignment of customers to customer zones. Here, we considered again the four nearest supply points for possible assignment, but if these supply points were already assigned with the required number of customers, we consider all other supply points for possible assignment.

7 Experiments

This section presents the computational results of the proposed heuristic procedure. First, we present the numerical results on the large problem instances generated in Section 6. We then compare our heuristic solution with lower bound values to evaluate the performance of the heuristic under consideration. We complete the presentation with the results of sensitivity analyses with different parameters, for a better understanding of the behavior of the model and heuristic.

The proposed heuristic was coded in C and implemented on AMD Opteron 2.3 GHz workstation with 16 GB of RAM. The circulation phase and lower bound formulations were solved with CPLEX 10.1. All computations were performed in double precision arithmetic and the final results are rounded to their nearest integer value. All travel times were set equal to the corresponding distances. Similarly, the unit transportation cost between two nodes was set equal to the distance between those nodes. The abbreviations used in reporting the results are defined in Table 3.

Name	Definition
NR	Total number of routes obtained in the routing phase by solving individual VRPTW for all customer zones;
TL	Total distance traveled by all vehicles;
NV	Total number of vehicles used in the final solution;
TC	Total transportation cost, including the fixed cost for operating vehicles and the variable cost for the total distances traveled by all vehicles;
DM	Number of times vehicles move directly from one customer zone to another customer zone without using waiting stations;
MWS	Number of times waiting stations are used by vehicles before moving to the next customer zone;
CPU	Total CPU time taken by the algorithm under consideration.

Table 3: Performance measure definitions

7.1 Numerical Results

We investigate first the effect of distances associated to arcs going back to the depot in the routing phase, which make up an important factor in determining the solution quality. We consider two alternatives: 1) zero cost for all the arcs, and 2) cost of going to the nearest supply point s' that can be reached feasibly from node i i.e., $a(i)+\delta(i)+\tau_{is} <= t(s)$. The average results of the 60 large problem instances for these two alternatives are reported in Table 4. The total cost for first alternative is 59,358, while the total cost of the second alternative is 59,338, which indicates that there is no significant difference between the alternatives, even though using zero costs for all arcs displays a slight advantage. It seems that considering the nearest supply point makes the solution slightly biased towards choosing this supply point, which might not be the most appropriate choice. Giving equal importance to the arcs going back to the depot thus seems to make the solution unbiased. In the remainder of the paper, therefore, we consider zero distances for all the arcs going back to the supply point in the routing phase of our solution method.

Scenario	NR	TL	NV	TC	DM	MWS
1	288	36016	47	59358	25	217
2	289	35863	47	59338	24	218

Table 4: Impact of return-to-depot-arc distances in the routing phase

We now turn to the results of the heuristic on the large instances, average results being reported in Table 5 (detailed results are reported in Tables 16 and 17 in the Appendix). The total cost reported in these tables is the sum of the fixed cost of operating the displayed number of vehicles and the variable cost corresponding to the distance traveled by the same vehicles. The CPU times indicate the proposed heuristic is efficient even for large instance dimensions.

Comparing reults for sets A1 and A2 shows that the average total cost of set A2 is less than for set A1. The same observation holds also for the average number of vehicles used and the average total distance traveled by vehicles. Remember that both sets have 400 customers, but set A1 has 4 customer zones while set A2 has 8 customer zones. This result provides an important insight on correlation between the total cost and the number of customer zones. The total transportation cost can thus be reduced by increasing the number of supply points, i.e., customer zones. Increasing the number of customer zones increases the utilization of the vehicles, which finally reduces the total transportation cost. This argument is supported by results from the other two problem sets.

Problem Set	NR	TL	NV	TC	DM	MWS	CPU
A1	61	6625	24	18575	1	36	290
A2	63	6161	19	15411	2	42	145
B1	245	30353	51	55653	14	180	1289
B2	251	28146	39	47396	20	193	607
C1	549	74126	87	117426	39	423	2990
C2	566	69770	64	101570	68	434	1351
Average	289	35863	47	59338	24	218	1112

Table 5: Average results for the large instance sets

Detailed results show that, in total, 2817 vehicles are used in the 60 problem instances, servicing a total of 17,337 routes. Hence, on average, a vehicle services 6.15 routes, which shows a reasonably high *vehicle utilization*. On the other hand, detailed results also show that a waiting station is used by vehicles on 13,037 occasions, while they move directly from one customer zone to another customer zone on 1449 occasions only. This low, 9.98 %, factor of direct moves to another customer zone without using a waiting station is mainly due to the small waiting time δ allowed at supply points. We investigate the effect of allowable waiting time at supply points on the transportation cost in the sensitivity analysis subsection (Section 7.3).

7.2 Comparisons with lower bounds

The performance of the heuristic solution is evaluated by comparing the heuristic solution (HS) with the lower bound solution LB1 and LB2 (Section 5). We used small instances sets when LB1 was involved, and large ones otherwise (involving LB2).

The comparative results of HS and LB1 for problem sets D1-D6 (averages over 10 problem instances for each set) are reported in Table 6 (detailed results are reported in Tables 18 and 19 in the Appendix). Other than the total length traveled by all vehicles TL, number of vehicles used NV, and total transportation cost TC, the tables also display the *% Gap* between HS and LB1. Notice that the HS solutions were obtained with an allowable waiting time δ set to infinity, in which case, the MZMT-VRPTW reduces to the relaxed problem LB1. Hence, the percentage gaps reported in Table 6 stand for the optimal gaps between heuristic and optimal solution. The optimality gap of HS with respect to the total cost is on average 9.5 % away from the LB1 bound. This is mainly due to the large gap in the number of vehicles (of the order of 20% on average). Indeed, small-size instances are highly sensitive to the number of vehicles used. On the other hand, no trend was observed in the relationships between the gaps and the instance size.

Problem	TL			NV			TC		
Set	HS	LB1	% Gap	HS	LB1	% Gap	HS	LB1	% Gap
D1	977	988	0.1	1.8	1.3	50.0	1877	1638	16.8
D2	1227	1123	9.3	2.2	2.2	0.0	2327	2223	4.7
D3	1367	1331	3.2	3	2.5	23.3	2867	2581	11.4
D4	1075	1091	0.2	2.2	1.8	30.0	2175	1991	9.0
D5	1718	1648	4.4	3.1	2.6	18.3	3268	2948	10.5
D6	2187	2022	8.5	3.3	3.3	0.8	3837	3672	4.5
Average	1425	1367	4.3	2.6	2.3	20.4	2725	2509	9.5

Table 6: Average comparative results between HS and LB1 for small instances

Table 7 display comparative results on the performance of LB2 with respect to that of LB1. Experiments were performed on the small instances, the gap columns displaying the percentage "away" of LB2 solutions from LB1 solution. Detailed results of LB1 and LB2 bounding procedures are reported in Tables 18 - 21 in the Appendix. Table 7 shows that, on average, LB2 solutions are 13.4 % away from the optimum. It is intuitive that LB2 will provide a looser lower bound since all critical constraints, such as time windows and vehicle capacity, have been relaxed. The percentage gap in terms of the number of vehicles used is 7.5%, which is low comparatively to the gap in terms of total cost. The lower gap on the number of vehicles follows from the constraint (37), which enforces the restriction on the minimum number of vehicles at customer zones. On the other hand, the gap is high regarding the total distances traveled (16.6 %). This performance could be improved by adding sub-tour elimination constraints to the LB2 formulation, but this would make it significantly less tractable. Again, no significant correlation between gap and instance size may be observed.

Finally, average comparative results for HS, with the allowable waiting time at supply points δ set to ∞, and LB2 are reported in Tables 8 and 9, for small and large instance sets, respectively. The gaps are 26.1 % and 27.2%, respectively (with no significant correlation between gap and instance size or number of supply points).

Problem Set	TL LB1	TL LB2	TL % Gap	NV LB1	NV LB2	NV % Gap	TC LB1	TC LB2	TC % Gap
D1	988	813	15.9	1.3	1.0	15.0	1638	1313	18.5
D2	1123	919	18.1	2.2	2.1	3.3	2223	1969	11.3
D3	1331	1100	16.9	2.5	2.4	3.3	2581	2300	11.1
D4	1091	868	19.1	1.8	1.7	5.0	1991	1718	14.1
D5	1648	1387	15.6	2.6	2.4	5.8	2948	2587	12.1
D6	2022	1736	14.1	3.3	2.9	12.5	3672	3186	13.2
Average	1367	1137	16.6	2.3	2.1	7.5	2509	2179	13.4

Table 7: Average comparative results between LB1 and LB2 for small instances

Problem Set	TL HS	TL LB2	TL % Gap	NV HS	NV LB2	NV % Gap	TC HS	TC LB2	TC % Gap
D1	976	813	20.5	1.8	1.0	80.0	1876	1313	43.4
D2	1211	919	32.8	2.2	2.1	5.0	2311	1969	17.6
D3	1320	1100	20.6	3.0	2.4	30.0	2820	2300	23.5
D4	1070	868	23.2	2.2	1.7	40.0	2170	1718	27.7
D5	1734	1387	25.1	2.9	2.4	21.7	3184	2587	23.2
D6	2198	1736	27.0	3.3	2.9	18.3	3848	3186	21.1
Average	1418	1137	24.9	2.6	2.1	32.5	2701	2179	26.1

Table 8: Average comparative results between HS and LB2 for small instances

The gaps are large. Yet, the previous discussion underlined the observation that LB2 is rather loose. This is supported by the figures in these two tables, where the comparative behavior of HS and LB2 is the same for both small and large instances. Recalling the very good behavior of LS relative to LB1 on mall instances, we infer that, on the one hand, the large gaps in these comparisons follow mainly from the relative quality of the bound and, on the other hand, the actual behavior of the proposed heuristic should be similar on large instances to the satisfying one on the small ones.

7.3 Sensitivity Analysis

The goal of this part of the experimentation is to better understand the nature of the MZMT-VRPTW variant of the routing family of problem setting by exploring the behavior of the solution relative to variations in the value of a number of key parameters. We focus, in particular, on the waiting time allowed at supply points, the number of waiting stations, the proportion of supply points providing waiting facilities, the amplitude of the customer time windows. the location of supply points relative to customers, and the importance of the vehicle fixed costs.

Problem	TL			NV			TC		
Set	HS	LB2	% Gap	HS	LB2	% Gap	HS	LB2	% Gap
A1	5818	3903	51.1	24	22	10.9	17768	14903	20.5
A2	5171	4179	24.7	19	14	36.3	14421	11029	31.0
B1	27373	21610	27.7	51	38	34.9	52673	40510	29.9
B2	24593	20538	20.0	39	28	35.8	43843	34738	26.2
C1	68261	54550	25.1	87	65	33.0	111561	87200	28.0
C2	62114	47498	30.7	64	52	22.5	93964	73498	27.8
Average	32222	25380	29.9	47	37	28.9	55705	43646	27.2

Table 9: Average comparative results between HS and LB2 for large instances

7.3.1 Allowable waiting time at supply points

The time vehicles may wait at supply points constitutes one of the main complicating factors for the problem setting we address. In actual practice, the waiting-time capabilities of supply points is determined, and limited, by many factors, including the location, organization, and environment (e.g., type of urban neighborhood and activity zone) of the supply point, traffic and parking regulations, and so on.

The experiment consisted in solving the large instances with various values for the allowable waiting time δ parameter. We selected two extreme values, 0 and ∞, and three intermediate one, namely 25, 50, and 100. The 0 extreme value corresponds to the case when vehicles cannot wait at supply points, being forced to arrive just in time for the opening time of the supply points. On the other hand, the infinite value indicates no restrictions on waiting time at supply points.

Average results for the large instances are reported in Table 10. The figures in the table show that all performance measures (except the number of routes, which depends only on the customers assigned to each supply point and are thus invariant), total transportation cost, number of vehicles used, and distance traveled, decrease when more waiting time is allowed at supply points. More waiting capabilities at supply points thus increases the probability of moving directly to the next customer zone, without using the waiting stations and thus avoiding traveling additional distances. Examining the performance measures for the extreme values reinforces this observation. No waiting possibilities at supply points results in the highest utilization of waiting stations and highest costs, 7.19% higher in total cost compared to the $\delta = \infty$ case, when no waiting station is in use.

The experimental results thus confirms that the allowable waiting time at supply points is one of the critical elements of the problem setting and the model. Recall that the relaxed problem LB1 represents the model for infinite allowable waiting time. One may thus infer that part of the gap of HS relatively to LB1 finally, notice that the results of this analysis provide an economic estimation of the value of waiting facilities associated to the supply points.

δ	NR	TL	NV	TC	DM	MWS
0	289	36112	47	59712	0	242
25	289	36073	47	59565	10	232
50	289	35978	47	59453	15	227
100	289	35863	47	59338	24	218
∞	289	32222	47	55705	242	0

Table 10: Average results for the large instances for different values of δ

7.3.2 Number of waiting stations

The number of waiting stations, where vehicles may wait before moving to the customer zone to reach it on time, is the second main synchronization characteristic of the problem studied, directly influencing the performance of the corresponding system. We consider four cases of limited parking availability by defining 1, 2, 4, and 50 waiting stations for each 100 customers. The latest figure is used to model the situation of an "infinite" number of waiting stations, corresponding to the case of unlimited availability of parking space in the city (street parking, for example). One may thus wait just outside the supply point, which is equivalent to going there directly and waiting for the opening time. Infinite number of waiting stations is thus equivalent to infinite allowable waiting time at supply points.

More waiting stations, if suitably distributed among customer locations, should result in more direct routes toward the next supply point and, thus, in lower distances traveled. This intuition is validated by the results of the four cases are reported in Table 11, where Column W/C identifies the case through the number of waiting stations for every 100 customers included in the large instances. These results indicate that the total cost decreases with the number of waiting stations, while the number of vehicles used remains the same. Thus, the decrease in total cost is due only to the decrease in the distance traveled by vehicles.

W/C	NR	TL	NV	TC	DM	MWS	% TC Saving
1	289	35863	47	59338	24	218	0
2	289	34208	47	57683	22	220	2.79
4	289	33260	47	56743	20	222	4.37
50	289	32285	47	55769	18	224	6.02

Table 11: Average results, large instances, for different values of number of waiting stations

The last column of Table 11 displays the transportation cost savings computed for Cases 2 to 4 with respect to the base Case 1. These figures provide upper bounds, within the parameter of the large instances, on the additional cost for opening and operating additional waiting stations.

7.3.3 Proportion of supply points with waiting facilities

It is actually possible in some applications (e.g., City Logistics), that some supply points be provided with waiting facilities (e.g., when located within parking lots) for vehicles to wait for their net service. We therefore performed an analysis letting the proportion of supply points with facilities allowing vehicle waiting (allowable waiting time = 100) vary from 0 to 100% by increments of 25. Results are reported in Table 12. Observations are similar to those of the sensitivity analysis of the allowable waiting time δ (Table 10). Indeed, similar to the impact of allowable waiting time at supply points, when the proportion of supply points allowing waiting of vehicles increases, the total travel distance reduces because more direct trips to the next customer zone become possible, while the utilization of waiting stations decreases, which reduces the total distances traveled by the vehicles.

Proportion	NR	TL	NV	TC	DM	MWS
0 %	289	36112	47	59712	0	242
25 %	289	36063	47	59638	6	236
50 %	289	35994	47	59535	12	230
75 %	289	35923	47	59423	18	224
100 %	289	35863	47	59338	24	218

Table 12: Impact of the proportion of supply points with allowable waiting time (average results, large instances)

7.3.4 Customer time windows

Table 13 displays the average results over the large instances of the analysis of the impact of the width of the customer service time window on global system performance. We compared the results of the base case to those of applying the heuristic to the same instances but with customer due dates yielding time windows twice as large as before.

Problem sets	NR	TL	NV	TC	DM	MWS
Initial	289	35863	47	59338	24	218
Wide	289	34839	47	58314	19	223

Table 13: Impact of customer time window width (average results, large instances)

Notice that the number of routes was the same for all previous analyses, since the routing phase of the heuristic solved the same problem. This is no longer true in the present case, as modifying the width of the time windows changes the routing-problem setting. Examining the detailed results, one observes minor differences in the number of routes. More significantly, broadening the customer time windows modifies the customer-to-route assignments lowering the total transportation cost, from 59,338 to 58,314 and the total distance traveled by vehicles from 35,863 to 34,839.

7.3.5 Location of supply points

A supply point can be located either inside the customer zones or at their periphery, on the outer boundary of all customer zones. The former organization characterizes the original instance sets, identified as "Within the zone" in Table 14. We aim with this analysis to measure the impact, if any, of this location, by changing the location of supply points to the nearest outer boundary of the customer zones, identified as "On the border" in the same table.

Consider the example of a supply point s. Assume the coordinates of s are [50, 10] in the original problem set. The nearest outer boundary for s is the line $y = 0$. Therefore, we locate the supply point s on the line $y = 0$, and its new coordinates become [50, 0]. Changing the coordinates of a supply point will generally modify the travel times to customer and, consequently, some previous customer time windows might no longer be feasible. We therefore modify the customer ready time and due date by replacing in the original computation (see Section refsection6) the initial distance between the customer and the supply point by the maximum value between this distance and the distance between the customer and the new location of the supply point. Table 14 reports the results of this analysis averaged over the large instances.

Supply point location	NR	TL	NV	TC	DM	MWS
Within the zone	290	36575	48	60417	21	221
On the border	287	47606	48	71489	21	218

Table 14: Average results on large instances with different supply-point locations

As expected, moving the supply points outside the customer zones increases the distance traveled by the vehicles and, thus, the total transportation cost. It also decreases the utilization of waiting stations. This increase must be put into the perspective of each particular application, however. Thus, for City Logistics, a location of the supply points within the customer zone implies longer distances traveled by first-tier vehicles and, thus, generally, higher impacts on congestion and the environment. While the full City-Logistics analysis is beyond the scope of this paper, the present results emphasize the interest of the MZMT-VRPTW problem setting and the proposed methodology.

7.3.6 Vehicle fixed costs

One of the objectives of the MZMT-VRPTW is to minimize the total number of vehicles used. This number is directly related to the fixed cost F associated with the utilization of the vehicles. We therefore, vary the fixed cost F, setting it to 0, 50, 200, 500, and 1000. The average results over large instances are reported in Table 15.

Fixed Cost	NR	TL	NV	DM	MWS
0	289	17590	289	0	0
50	289	28410	90	11	188
200	289	34986	49	22	218
500	289	35863	47	24	218
1000	289	35864	47	24	218

Table 15: Effect of fixed cost on solution quality (average results on large instances)

The fixed cost set to zero represents the case where the only objective is to minimize the total distances traveled by all the vehicles. The results clearly show the corresponding measure as the minimum distance required for the instances. No waiting stations are necessary in this case. This performance is achieved, however, at the cost of using the minimum number of vehicles, i.e., one vehicle for each of the routes performed. Increasing the fixed costs increases the distance traveled and decreases the utilization of waiting stations and the number of vehicles operated. Variations may be dramatic, witness the passage from 0 to 50, which emphasizes the major role of fixed vehicle costs within this problem setting. Actually, one has already reached the good-results region when the fixed costs are set at 200. Imposing higher values, e.g. 1000, increasing represents the case when travel costs are not important, the main objective being the minimization of the number of vehicles used (the last row of the table indeed displays the lowest number of vehicles, for the highest total distance traveled and level of utilization of the waiting stations).

8 Conclusions

We have studied the MZMT-VRPTW, a new vehicle routing problem class in which customers are divided into a number of zones along geographic or temporal lines defined by the customer time windows and the tight time windows of the supply points of each zone where demand becomes available. Vehicles in MZMT-VRPTW may perform multiple tours for different zones, aiming to reach the next zone just-in-time for the beginning of its time window. The global objective is the minimization of the total cost computed as the sum of the fixed costs of selecting vehicles and the variable costs of moving them around to service customers.

We have developed and analyzed a decomposition-based heuristic approach, which addresses the problem in two phases. Small VRPTW for each customer zone are solved in the first phase, while the flow of vehicles among different customer zones is determined in the second phase. The decomposition heuristic was tested on randomly generated problem instances. We have also performed sensitivity analyses on several parameters of the MZMT-VRPTW, which revealed interesting facets of the behavior of this new problem class. We also provided two lower bounds that, even though not very tight, indicate the heuristic performs reasonably well. The MZMT-VRPTW is a new problem and no previous results were available, but we hope our results have opened the field for investigation and provide a benchmark for future research.

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Appendix

The Appendix displays the detailed results for the heuristic on the large instances, Tables 16 and 17, and lower bound computations for the small instance sets in Tables 18 and 19 for LB1, and Tables 20 and 21 for LB2. The definition of the column identifiers is:

NR	Total number of routes obtained in the routing phase by solving individual VRPTW for all customer zones;
TL	Total distance traveled by all vehicles;
NV	Total number of vehicles used in the final solution;
TC	Total transportation cost, including the fixed cost for operating vehicles and the variable cost for the total distances traveled by all vehicles;
DM	Number of times vehicles move directly from one customer zone to another customer zone without using waiting stations;
MWS	Number of times waiting stations are used by vehicles before moving to the next customer zone;
CPU	Total CPU time taken by the algorithm under consideration.

Inst.	NR	TL	NV	TC	DM	MWS	CPU
A1-1	63	8267	19	17767	0	44	280.96
A1-2	59	6783	24	18783	0	35	320.84
A1-3	60	6614	19	16114	0	41	283.14
A1-4	61	5436	31	20936	0	30	275.76
A1-5	63	6850	18	15850	11	34	282.08
A1-6	60	7456	20	17456	0	40	283.43
A1-7	61	7169	24	19169	0	37	328.08
A1-8	62	6790	20	16790	2	40	289.32
A1-9	63	5570	32	21570	0	31	273.8
A1-10	58	5316	32	21316	0	26	277.98
A2-1	63	5880	21	16380	1	41	162.09
A2-2	62	6433	24	18433	2	36	156.94
A2-3	64	5654	18	14654	4	42	140.14
A2-4	62	6556	13	13056	0	49	143.32
A2-5	63	7256	14	14256	3	46	156.91
A2-6	61	5480	24	17480	7	30	142.34
A2-7	65	6455	15	13955	1	49	136.06
A2-8	64	5975	18	14975	0	46	135.96
A2-9	61	6430	16	14430	0	45	133.02
A2-10	63	5490	22	16490	6	35	147.49

Table 16: Results for the A large instance set

Inst.	NR	TL	NV	TC	DM	MWS	CPU
B1-1	246	26507	89	71007	4	153	1290.51
B1-2	246	31419	44	53419	22	180	1278.53
B1-3	247	30675	41	51175	20	186	1155.21
B1-4	243	33331	42	54331	25	176	1415.79
B1-5	243	31572	45	54072	12	186	1338.3
B1-6	248	29593	50	54593	16	182	1378.36
B1-7	241	30322	46	53322	15	180	1315.15
B1-8	246	31423	48	55423	5	193	1121.7
B1-9	240	28708	53	55208	19	168	1214.03
B1-10	248	29979	48	53979	5	195	1377.85
B2-1	246	28889	32	44889	21	193	545.2
B2-2	247	26927	47	50427	22	178	570.26
B2-3	253	28941	40	48941	25	188	681.78
B2-4	253	31894	28	45894	19	206	590.88
B2-5	253	26023	41	46523	17	195	602.21
B2-6	251	28441	36	46441	16	199	573.18
B2-7	249	27894	34	44894	16	199	594.27
B2-8	252	30549	28	44549	23	201	635.97
B2-9	256	28301	37	46801	21	198	619.19
B2-10	250	23606	62	54606	16	172	660.81
C1-1	547	72967	86	115967	67	394	2814.88
C1-2	546	67176	92	113176	43	411	2750.84
C1-3	551	74773	80	114773	62	409	2937.05
C1-4	548	72810	83	114310	30	435	2908.79
C1-5	548	71245	100	121245	25	423	3094.12
C1-6	552	71824	87	115324	39	426	2905.78
C1-7	548	77443	86	120443	31	431	3069.55
C1-8	551	76473	78	115473	40	433	2911.42
C1-9	550	81560	89	126060	34	427	3428.86
C1-10	545	74993	85	117493	18	442	3076.06
C2-1	562	66732	69	101232	70	423	1347.69
C2-2	571	71789	53	98289	85	433	1326.31
C2-3	562	73680	65	106180	71	426	1344.62
C2-4	559	60387	74	97387	35	450	1347.63
C2-5	569	74590	53	101090	74	442	1347.21
C2-6	565	64347	85	106847	54	426	1462.27
C2-7	562	67471	62	98471	76	424	1348.63
C2-8	570	72448	55	99948	73	442	1308.74
C2-9	568	73801	57	102301	79	432	1307.41
C2-10	567	72450	63	103950	67	437	1371.44
Average sets A,B,C	288.95	35863	46.95	59338	24.15	217.85	1111.97

Table 17: Results for large instance sets B and C

Inst.	NR	TL	NV	TC	CPU
D1-1	6	845	2	1845	1.82
D1-2	5	1052	1	1552	0.03
D1-3	5	1324	1	1824	0.27
D1-4	5	1223	1	1723	0.06
D1-5	5	876	1	1376	0.19
D1-6	5	827	1	1327	0.03
D1-7	5	1029	2	2029	0.06
D1-8	5	858	2	1858	0.25
D1-9	5	935	1	1435	0.04
D1-10	5	909	1	1409	0.17
D2-1	7	1139	2	2139	0.27
D2-2	7	970	3	2470	7.68
D2-3	9	1188	2	2188	9.37
D2-4	7	1082	3	2582	2.19
D2-5	9	1249	2	2249	0.43
D2-6	7	1177	2	2177	25.95
D2-7	9	1173	2	2173	0.08
D2-8	6	1078	2	2078	29.04
D2-9	5	1119	2	2119	254.76
D2-10	8	1054	2	2054	0.25

Table 18: LB1 results for the small instances sets D1, D2

Inst.	NR	TL	NV	TC	CPU
D3-1	10	1165	3	2665	243.53
D3-2	10	1315	2	2315	107.61
D3-3	10	1111	4	3111	143.68
D3-4	10	1383	2	2383	2.58
D3-5	9	1235	2	2235	80.86
D3-6	10	1538	2	2538	534.16
D3-7	9	1435	2	2435	21.50
D3-8	10	1459	2	2459	51.52
D3-9	10	1149	4	3149	104.06
D3-10	10	1523	2	2523	103.04
D4-1	6	1092	2	2092	0.28
D4-2	5	788	2	1788	2.36
D4-3	5	1436	1	1936	2.44
D4-4	7	1012	2	2012	0.33
D4-5	7	976	2	1976	3.17
D4-6	5	1285	1	1785	60.65
D4-7	6	1121	2	2121	68.72
D4-8	6	1006	2	2006	0.24
D4-9	7	1109	2	2109	0.34
D4-10	7	1085	2	2085	10.43
D5-1	13	1696	3	3196	6.35
D5-2	12	1785	2	2785	592.90
D5-3	12	1854	3	3354	48.52
D5-4	12	1624	2	2624	2.91
D5-5	13	1742	2	2742	2649.32
D5-6	13	1536	4	3536	734.38
D5-7	13	1483	3	2983	113.63
D5-8	11	1420	2	2420	97.27
D5-9	12	1785	2	2785	2756.95
D5-10	13	1559	3	3059	345.02
D6-1	18	2254	3	3754	4020.97
D6-2	19	2057	3	3557	640.39
D6-3	18	1809	3	3309	1867.42
D6-4	16	1901	4	3901	2977.89
D6-5	21	2186	4	4186	21272.70
D6-6	23	2031	4	4031	1870.13
D6-7	18	1722	3	3222	7413.11
D6-8	18	2139	3	3639	5275.44
D6-9	19	1846	3	3346	882.86
D6-10	17	2279	3	3779	8871.51
Average all sets	10	1367	2	2509	1071.90

Table 19: LB1 results for the small instances sets D3 - D5

Inst.	NR	TL	NV	TC	CPU
D1-1	5	830	1	1330	0.01
D1-2	5	949	1	1449	0.00
D1-3	5	755	1	1255	0.01
D1-4	5	789	1	1289	0.00
D1-5	5	747	1	1247	0.00
D1-6	5	770	1	1270	0.01
D1-7	5	954	1	1454	0.01
D1-8	5	799	1	1299	0.00
D1-9	5	822	1	1322	0.01
D1-10	5	712	1	1212	0.00
D2-1	7	945	2	1945	0.01
D2-2	6	777	3	2277	0.01
D2-3	9	1032	2	2032	0.01
D2-4	7	1021	2	2021	0.01
D2-5	7	1024	2	2024	0.01
D2-6	7	837	2	1837	0.00
D2-7	9	966	2	1966	0.01
D2-8	6	793	2	1793	0.01
D2-9	5	846	2	1846	0.01
D2-10	8	946	2	1946	0.01

Table 20: LB2 results for the small instances D1, D2

Inst.	NR	TL	NV	TC	CPU
D3-1	10	1210	2	2210	0.01
D3-2	10	1024	2	2024	0.01
D3-3	10	886	4	2886	0.01
D3-4	10	1164	2	2164	0.01
D3-5	9	1075	2	2075	0.01
D3-6	10	1172	2	2172	0.01
D3-7	9	1230	2	2230	0.01
D3-8	9	957	2	1957	0.01
D3-9	10	982	4	2982	0.01
D3-10	10	1299	2	2299	0.01
D4-1	6	928	2	1928	0.01
D4-2	5	769	1	1269	0.01
D4-3	5	976	1	1476	0.00
D4-4	7	884	2	1884	0.01
D4-5	6	794	2	1794	0.00
D4-6	5	807	1	1307	0.01
D4-7	6	838	2	1838	0.01
D4-8	6	886	2	1886	0.01
D4-9	7	917	2	1917	0.01
D4-10	6	884	2	1884	0.01
D5-1	13	1481	3	2981	0.02
D5-2	11	1447	2	2447	0.02
D5-3	11	1534	3	3034	0.02
D5-4	12	1339	2	2339	0.02
D5-5	12	1389	2	2389	0.02
D5-6	12	1298	3	2798	0.02
D5-7	13	1431	2	2431	0.02
D5-8	11	1234	2	2234	0.02
D5-9	12	1405	2	2405	0.02
D5-10	13	1313	3	2813	0.02
D6-1	17	1900	3	3400	0.04
D6-2	19	1987	2	2987	0.04
D6-3	17	1657	2	2657	0.04
D6-4	16	1555	4	3555	0.04
D6-5	18	1723	4	3723	0.04
D6-6	21	1839	3	3339	0.05
D6-7	17	1392	3	2892	0.04
D6-8	17	1673	3	3173	0.04
D6-9	18	1612	3	3112	0.04
D6-10	16	2023	2	3023	0.04
Average all sets	9.55	1137	2	2179	0.01

Table 21: LB2 results for the small instances D3 - D5