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Scheduled Service Network Design with Synchronization and Transshipment Constraints for Intermodal Container Transportation Networks

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Abstract. In this paper we address the problem of scheduled service network design for container freight distribution along rivers, canals, and coastlines. We propose a new concise continuous-time mixed-integer linear programming model that accurately evaluates the time of occurrence of transportation events and the number of containers transshipped between vehicles. Given the transportation network, the fleet of available vehicles, the demand and the supply of containers, the sailing time of vehicles, and the structure of costs, the objective of the model is to build a minimum cost service network design and container distribution plan that defines services, their departure and arrival times, as well as vehicle and container routing. The model is solved with a commercial solver and is tested on data instances inspired from real-world problems encountered by EU carrier companies. The results of the computational study show that in scheduled service networks direct routes happen more often when either the fleet capacity is tight or the handling costs and the lead time interval increase. The increase of the same parameters leads to the decrease of the number of containers transshipped between vehicles.

Keywords. Transportation, multicommodity scheduled service network design with asset-management consideration, continuous-time model, container distribution, container transshipment, vehicles synchronization.

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1 Introduction

Following the growth in world trade, freight containerized transportation has become the backbone of international commerce. The importance of containerization for global trade is reflected by the growth of the container fleet. For the last twenty years, the global container fleet has gone up more than fourfold and reached 29 million TEUs. The efficiency of the container deployment has also grown. If in 1990 each container was loaded or unloaded approximately 14 times during the year, then in 2010, this figure reached up to about 19 port moves per container (United United Nations 2011). This has become possible due to the improvements in the port handling machinery, the optimization of terminal yard operations, and the continuous development of coordination and planning tools for intermodal transportation systems.

Much attention was recently paid to the modeling of maritime shipping networks. Most of the existing studies (Christiansen et al., 2007, 2004; Crainic, 2003) address maritime container and bulk freight transportation. Although, the land leg of the international maritime container route has been receiving increasing attention, relatively little effort was directed at the development of models for container and bulk freight distribution along rivers, canals and coastlines (Andersen et al., 2009a; Crainic and Kim, 2007).

We address the problem of scheduled service network design for container freight distribution along rivers, canals, and coastlines that is a major concern for terminals, freight forwarders and other consolidation-based carrier companies. These companies, that from now on we address as service providers, aim to meet the consignee (and shipper) demand with minimum possible costs subject to the lead time restrictions. To achieve this aim, such companies provide intermodal transportation services between river, canal, coastline terminals, and the port area. Intermodal transportation implies that two or more modes of transport are used to move the same loading unit (i.e., container) between its origin and destination in an integrated manner, without loading or unloading cargo (Unspecified, 1997). Then, the major planning issues faced by a service provider are the selection and the scheduling of services, the coordination of services at terminals, the routing of container flows, and the transshipment of containers between vehicles at the terminals. These aspects of service network design problems encountered in intermodal transportation are reviewed in Crainic and Kim (2007).

The increasing requirements for efficient utilization of available resources demand new time-dependent service network design model formulations. Among them, the modeling of vehicle-management aspects integrated to service network planning deserves a particular attention. Andersen et al. (2009a,b); Pedersen and Crainic (2007) focus on the transportation service design. They study the influence of introducing asset-management constraints on the generated output and performance of service network design problem formulations. Christiansen (1999); Andersen et al. (2009a) address the accurate estimation of product transshipments between vehicles (services), that eventually appeal to the problem of vehicle moves (services) synchronization at transshipment points. However, the integration of these real-world transportation planning aspects into time-dependent service network design problems leads to problem formulations of a very large size.

One of the key issues in mathematical modeling of time-dependent service network design problems is the treatment of time. All existing formulations can be classified into two categories: discrete-time models and continuous-time models. In the discrete-time approach, the time horizon is uniformly divided into a number of time intervals. Events such as vehicle moves, container flow moves, inventory repositioning, loading and unloading of containers are determined at the boundaries of these time intervals. To achieve an accurate approximation of the original problem, an appropriate time discretization that suits the interests of the decision maker is needed. To capture the dynamic environment of intermodal transportation systems, smaller time intervals have to be chosen. For example, the travel time of the fastest transportation mode or the time of unloading a vehicle can be considered as the size of the time intervals. As a dedicated set of state variables is used for each time interval, the total number of the state variables is proportional to the number of considered time intervals. Time discretization with considerable small time intervals leads to very large and complex combinatorial problems.

In contrast with the discrete-time approach, in the continuous-time models, events can take place at any point in time. To capture these events, additional variables corresponding to their starting and ending times, are introduced. Continuous-time models are usually of smaller sizes than the discrete-time models and require less computational effort for their solution. However, due to the continuous treatment of time, it becomes more challenging to model the scheduling process. Additional constraints have to be introduced to coordinate the timing and the sequence of occurrence of the events. In the intermodal transportation environment, this relates to the transfer of containers between vehicles, and the synchronization of vehicle arrivals to the same terminal.

This paper proposes a new multicommodity scheduled service network design model that accurately evaluates the time of occurrence of transportation events and the number of containers transshipped between vehicles. We use a continuous-time approach to represent the transportation environment with varying duration of events, such as vehicle travel times and the time required for loading and unloading a vehicle. We target tighter problem formulations as compared to corresponding discrete-time problem formulations. The model can be used for the tactical planing level of scheduled service network designs of intermodal transportation systems, where accurate estimation of time is an important decision factor. Given the network, the fleet of available vehicles, the demand and the supply of containers, the sailing time of vehicles, and the structure of costs, the objective is to build a minimum cost service network design plan that defines services, their departure and arrival times, vehicle and container routing.

The contribution of this paper is a new continuous-time mixed-integer linear programming model for multicommodity scheduled service network design and container distribution problem with asset-management consideration, where we allow the transportation of a single container by several vehicles, that are different in speed and capacity. In such cases, the transshipment of containers has to be synchronized with vehicle moves. For that, we develop a set of constraints that defines the occurrence of container transshipment events and synchronizes the departure and the arrival times of the vehicles involved in the transshipment. We solve our model with an existing solver and test it on data instances inspired from real-world problems encountered by EU intermodal terminals and short sea shipping carrier companies.

The paper is organized as follows. The literature review on the designated topics is provided in Section 2. The problem statement is given in Section 3. In Section 4 we discuss the general settings of the multicommodity scheduled service network design and container distribution problem with asset-management consideration and its main components and characteristics. Section 5 is dedicated to the development of the mathematical model for this problem. In this section, we provide a simple four-node example to give evidence of the model's adequate decision support. This example illustrates the effect of synchronization and transshipment constraints on scheduled service design and container distribution plan. In Section 6 we evaluate the performance of the proposed model with respect to the generated scheduled service network design, container distribution plan, and solution times. Our conclusions and future research directions are presented in Section 7.

2 Literature review

Numerous studies have addressed modeling of service network design problems in the past decades. In these studies, authors developed decision support systems that aim to ensure an optimal allocation and utilization of resources to achieve the economic and customer service goals of the transportation service provider. The proposed models usually take the form of network design formulations, a class of mixed-integer network optimization problems for which no efficient, exact solution method exists, except for special cases. Most of the models use discrete-time formulations to represent vehicle scheduling, container or bulk product routing and empty container repositioning.

Crainic et al. (1993) proposed a general modeling framework for the dynamic allocation of empty containers that includes the specifics of container distribution along rivers, canals or coastlines. They developed discretized time-space formulations for single and multicommodity capacitated service network design problems that minimize the total operational costs of the company. Several types of containers and the possibility of their substitution were taken into account, though container handling and transfer were not considered.

Christiansen (1999) used a continuous-time approach to model a combined inventory management and ship routing problem. A fleet of ships transports a single commodity between the production and consumption harbors. The model is based on the assumption that the number of possible arrivals and departures to a node is known in advance. The lack of such information dramatically increases the size of the model. The author proposed several problem adjustments which allow the decomposition of the problem and its solution by Dantzig-Wolfe decomposition.

Al-Khayyal and Hwang (2007) addressed the fleet scheduling and the multi-commodity pickup and delivery problem. Their continuous-time model defines the minimum cost routing of heterogeneous ships that have to distribute various liquid bulk products across a set of supply and demand harbors with specified product availabilities and needs. The interaction between multiple ships arriving at the same destination made the formulation bilinearly constrained. Authors used linearization schemes to develop an equivalent mixed-integer linear programming reformulation of the problem. They implemented it in a commercial solver for mixed-integer linear programming.

Agarwal and Ergun (2008) considered the problem of service network design faced by liner shipping companies. They proposed a mixed-integer linear program for the simultaneous solution of the ship and cargo routing problems. This discrete-time formulation seeks for an efficient and profitable service route design given the set of feasible cycles and the number of vehicles maintaining the weekly frequencies on a cycle. The transshipment costs are ignored at the stage of liner network design, since the authors claimed this would lead to a tremendous size of the graph representing the given network. To solve the problem, the authors proposed algorithms that exploit the separability of the problem: a greedy heuristic, a column generation-based algorithm, and a two-phase Benders decomposition-based algorithm.

Andersen et al. (2009a) considered a cargo scheduling and routing problem, where cargo has to be carried by internal rail services and shipped by external ferry services. For a given demand of commodities and a set of internal and external services, the proposed discretized time-space model formulation defines the intermodal service network design and the departure times of services, such that the throughput time of commodities in the system is minimized. The transfer of commodities is modeled through transfer arcs that connect a pair of nodes: an actual intermodal terminal and an artificial commodity cargo-transfer node. This dramatically increases the size of the problem. The loading and unloading times are assumed to be constant. The authors implemented the problem formulation in a standard solver.

Wang and Meng (2012) proposed a cost minimization model for a ship fleet deployment problem arising in the liner shipping industry. Their model captures the duration and the costs of the container transshipment, loading and discharging operations. A consolidation port is used for the transshipment operations, while loading and discharging operations are performed at the container's origin and destination nodes. This study assumes a static environment, thus the ship scheduling decisions are omitted.

We refer the reader to Crainic (2003); Christiansen et al. (2004, 2007) for a review of maritime ship routing and related scheduling problems. A review of intermodal transportation optimization problems is given in Crainic and Kim (2007).

Some of the above mentioned models compute approximate transfer volumes of flow and time of occurrence of events, other compute them accurately. Nevertheless, the implementation of these models leads to very large systems that demand a lot of computational power even for small-to-medium size test instances. Therefore, the aim of this paper is to propose a concise model that still provides accurate decision support.

3 Problem statement

The problem that we consider is the design of a scheduled service network for intermodal transportation systems with asset-management considerations. In such systems, a service provider performs river (canal or coastline) navigation and offers services of intermodal container transportation between its terminals and major port areas. It owns or rents handling machinery and vehicles, and employs terminal yards for container storage.

A set of services are offered by the service provider to respond to a given set of shipment orders requiring transportation of containers for import (from the major port areas) and export (to the same areas) directions.

A service is defined by its first and last stops, intermediate stops are not allowed. A good example of such services is a direct barge route between two terminals. Containers can be loaded and unloaded at terminals or transshipped between services. Operation of services requires vehicles that are available in limited quantities. Vehicles differ in speed and capacity, therefore, the corresponding services are different in the same way. A vehicle can perform a sequence of services that starts and ends at the terminal of the vehicle's initial location.

A shipment order is a collection of containers that have the same terminal of origin, the time they become available, the terminal of destination, and the deadline of delivery. It is not obvious that orders are serviced immediately when they become available. They have to be delivered before a deadline, however. Each shipment order can be split and transported by several services to the destination location. We do not require that a shipment order or its parts are picked up and delivered by the same vehicle.

The planning time horizon is defined by an interval of three days to one week length.

We consider costs associated with the selection of services, asset-related costs associated with operating a vehicle, container transportation and handling transshipment costs. The problem, then, consists in selecting services and their schedules, managing the given fleet of vehicles, and determining the routing of the given demand from origins to destinations. The overall goal is to minimize the total transportation costs of distributing the demand through the system using the intermodal transportation services over the given planning time horizon.

4 Problem description

Let us consider a directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ defined by a set of nodes $\mathcal{N}$ and a set of arcs $\mathcal{A} = \{(i,j) : i,j \in \mathcal{N}, i \neq j\}$. Each node of this graph corresponds to a terminal at a port area or a terminal along rivers, canals and coastlines. Each arc represents a potential service connection between two corresponding nodes standing for a road, a canal, a river or a rail connection. Thus, we model a direct service network design problem. A set $\mathcal{V}$ of heterogeneous vehicles represented by trucks and barges is available to the service provider. Vehicles are different in speed and capacity, and the maximal capacity of a vehicle v is given by C^v.

Intermodal transportation is facilitated by the use of a homogeneous set of containers of the same volume and size. We denote the overall set of orders for shipment of containers by $\mathcal{P}$. For a given order p, we identify the node of origin $i = o(p)$, where the order of containers becomes available at time $T_i^{\min p}$, and the node of destination $j = d(p)$, where the order has to be delivered before time $T_j^{\max p}$. Let an integer number D^p represent the number of containers in order p that have to be transported from the node of its origin to the node of its destination. All the other nodes are considered to be transshipment nodes. Therefore, we introduce the demand value w_i^p for each node i and shipment order p as:

$$w_i^p = \begin{cases} D^p & \text{if } i = o(p) \\ -D^p & \text{if } i = d(p) \\ 0 & \text{otherwise.} \end{cases}$$

A service is defined as $[v,(i,j)]$, where $v \in \mathcal{V}$ is the vehicle performing the transportation and $(i,j) \in \mathcal{A}$ is the arc on which the service occurs. The duration of a service $[v,(i,j)]$ coincides with the traveling time of the vehicle v executing the service. The service departs from node i at time $t_i^{D^v}$ and arrives at node j at time $t_j^{A^v}$. The time necessary to execute a service $[v,(i,j)]$ is given by T_{ij}^v, the sailing time of vehicle v between the pair of nodes i and j. The departure time of vehicle v from the node of origin of order p is constrained by the time the containers become available at this node. The arrival time of a vessel at the node of destination of order p is constrained by the deadline of delivery of the order. We allow vehicles to be initially located at any given node, therefore, we designate the node of the initial position of vehicle v as $k(v)$.

We allow containers to be transported by more than one vehicle. A container can, therefore, be involved in four types of container operations: 1) transshipment between two vehicles, 2) stay on board the same vehicle, 3) unload from a vehicle at the destination terminal of the container, 4) load to a vehicle at the origin node of the container. Handling and transshipment operations are executed during a time interval, that we model by the service time. The service time at node i is fixed for all shipment orders and vehicles and is given by s_i.

Our model accounts for two types of vehicle operating costs: fixed and variable. Fixed costs are the expenses independent of the number of transported containers, whereas variable costs change proportionally to the volume of flow on a service. Unit variable transportation cost c_{ij}^v represents the cost incurred to move one container on arc (i, j) by vehicle v. Handling cost per container at node i is denoted by h_i. Fixed cost for using a vehicle v is denoted by b^v. Finally, f_{ij}^v represents the fixed cost of operating a service $[v, (i, j)]$. The objective is to define the least cost transportation plan, including decisions on service scheduling, vehicle and container routing, that satisfies demand, supply and lead time requirements. The outcome transportation plan is subject to the following constraints:

- a sequence of services, executed by the same vehicle, starts and ends at the same terminal,
- demand for shipment orders has to be satisfied,
- capacity of the vehicle executing a service cannot be exceeded,
- container transshipment and vehicle moves have to be synchronized.

The solution to the problem determines:

- a set of services, each being a combination of a direct route between a pair of terminals and vehicle serving this route,
- time of departure and arrival of each service,
- vehicle routing,
- how many containers of each shipment order to transport by each chosen service,
- how many containers of each shipment order to tranship from one vehicle to another,

- terminals where the transshipment takes place.

5 Mathematical model

We define four sets of decision variables associated with the design of the network. Selection boolean variables y_{ij}^{v} describe whether the service $[v,(i,j)]$ performed by a vehicle v on an arc (i,j) is included in the the transportation plan. Vehicle selection boolean variables δ^{v} define whether the vehicle v is used. Boolean variables θ_{i}^{vs} identify the occurrence of container transshipment from vehicle v to vehicle s at node i and boolean variables r_{i}^{pv} identify whether shipment order p is onboard of vehicle v at node i.

Additionally, we define four sets of continuous decision variables. A set of continuous variables x_{ij}^{pv} represents the number of containers of shipment order p transported by service $[v,(i,j)]$. A set of continuous variables q_{i}^{pvs} stands for the amount of containers of order p transshipped from vehicle v to vehicle s at node i. Two sets of continuous variables $t_{i}^{D^{v}}$ and $t_{j}^{A^{v}}$ stand for the departure time of service $[v,(i,j)]$ from node i and its arrival time to node j, respectively.

For each node i, we define sets $\mathcal{N}^{+}(i)$ and $\mathcal{N}^{-}(i)$ of outwards and inwards neighbors of the node, respectively. We define a constant M as a very large number. Further, we describe the objective function and the set of constraints.

5.1 Objective function

The total operational costs to be minimized (1) is the sum of the cost of using a vehicle, the cost of operating the service network, the cost of distributing containers, and the container handling costs.

$$\min \sum_{v\in\mathcal{V}} b^{v}\delta^{v} + \sum_{v\in\mathcal{V}}\sum_{(i,j)\in\mathcal{A}} f_{ij}^{v}y_{ij}^{v} + \sum_{p\in\mathcal{P}}\sum_{v\in\mathcal{V}}\sum_{(i,j)\in\mathcal{A}} c_{ij}^{v}x_{ij}^{pv} + \sum_{p\in\mathcal{P}}\sum_{\substack{v\in\mathcal{V}\\ v\neq s}}\sum_{s\in\mathcal{V}}\sum_{i\in\mathcal{N}} h_{i}q_{i}^{pvs} \tag{1}$$

The service operating costs and the unit variable transportation costs are vehicle- and distance-dependent and are independent of the shipment orders carried on vehicles executing the service. The container handling costs are terminal-dependent, while the vehicle selection costs are vehicle-dependent.

5.2 Design balance constraints

These constraints impose that the number of services entering and leaving a node is the same:

$$\sum_{j \in \mathcal{N}^+(i)} y_{ij}^v - \sum_{j \in \mathcal{N}^-(i)} y_{ji}^v = 0 \qquad \forall i \in \mathcal{N}, \forall v \in \mathcal{V}. \tag{2}$$

5.3 Flow conservation constraints

These constraints guarantee that the difference between the number of containers of commodity p outgoing from node i and ingoing at node i is equal to the value of demand at the node:

$$\sum_{v \in \mathcal{V}} \sum_{j \in \mathcal{N}^+(i)} x_{ij}^{pv} - \sum_{v \in \mathcal{V}} \sum_{j \in \mathcal{N}^-(i)} x_{ji}^{pv} = w_i^p \qquad \forall i \in \mathcal{N}, \forall p \in \mathcal{P}. \tag{3}$$

5.4 Vehicle capacity

These constraints guarantee that the vehicle's capacity is not exceeded:

$$\sum_{p \in \mathcal{P}} x_{ij}^{pv} \leq C^v y_{ij}^v \qquad \forall (i,j) \in \mathcal{A}, \forall v \in \mathcal{V}. \tag{4}$$

5.5 Occurrence of container transshipment

These constraints describe the relation between the occurrence of container transshipment between vehicles v and s at node i and the number of containers of order p transshipped between them at this node. The boolean variables θ_i^{vs} and the continuous variables q_i^{pvs} are linked by the following condition: the container transshipment activity takes place if and only if there are containers to be transshipped from vehicle v to vehicle s at node i. More formally:

$$\sum_{p \in \mathcal{P}} q_i^{pvs} > 0 \text{ if and only if } \theta_i^{vs} = 1 \qquad \forall i \in \mathcal{N}, s, v \in \mathcal{V}.$$

We linearize this condition by the following inequalities:

$$\sum_{p \in \mathcal{P}} q_i^{pvs} \geq M(\theta_i^{vs} - 1) + \epsilon \qquad \forall i \in \mathcal{N}, \forall s, v \in \mathcal{V}, \tag{5}$$

$$\sum_{p \in \mathcal{P}} q_i^{pvs} \leq M\theta_i^{vs} \qquad \forall i \in \mathcal{N}, \forall s, v \in \mathcal{V}, \tag{6}$$

with ϵ being a small positive constant. Typically we consider $\epsilon = 10^{-4}$.

5.6 Definition of container transshipment at a node

These constraints describe the relation between the inflow and the outflow of containers at a node and the transshipment of these containers to other vehicles at this node. Constraint (7) garanties that all containers of the same shipment order p brought by vehicle v to node i (except for the node of destination of order p) are either transshipped to other vehicles ($v \neq s$) or stay on board the same vehicle ($v = s$):

$$\sum_{j \in \mathcal{N}^-(i)} x_{ji}^{pv} = \sum_{s \in \mathcal{V}} q_i^{pvs} \qquad i \neq d(p), \forall p \in \mathcal{P}, \forall v \in \mathcal{V}. \tag{7}$$

Constraint (8) is similar to the previous constraint. It garanties that all the containers belonging to a shipment order p transported by vehicle v from node i (except for the node of origin of order p) are either transshipped to vehicle v from other vehicles ($s \neq v$) or stay on board vehicle v ($s = v$):

$$\sum_{j \in \mathcal{N}^+(i)} x_{ij}^{pv} = \sum_{s \in \mathcal{V}} q_i^{psv} \qquad i \neq o(p) \forall p \in \mathcal{P}, \forall v \in \mathcal{V}. \tag{8}$$

Constraint (9) describes the absence of container transshipment at nodes of order origin and order destination.

$$\sum_{s \in \mathcal{V}} q_i^{pvs} = 0 \qquad i = d(p) \text{ or } i = o(p), \forall p \in \mathcal{P}, \forall v \in \mathcal{V}. \tag{9}$$

5.7 Definition of container flow at the node of its origin and destination

These constraints describe the outflow and the inflow of containers at the node of origin and destination, respectively. The boolean variables r_i^{pv} and the continuous variables x_{ij}^{pv} are linked by the following condition: this order is onboard of vehicle v at node i if and only if the flow of containers of order p leaving its origin node i onboard of vehicle v is positive. More formally:

$$\sum_{j \in \mathcal{N}^+(i)} x_{ij}^{pv} > 0 \text{ if and only if } r_i^{pv} = 1 \qquad i = o(p), \forall p \in \mathcal{P}, \forall v \in \mathcal{V}.$$

We linearize this condition by the following inequalities:

$$\sum_{j \in \mathcal{N}^+(i)} x_{ij}^{pv} \geq M(r_i^{pv} - 1) + \epsilon \qquad i = o(p), \forall p \in \mathcal{P}, \forall v \in \mathcal{V}, \tag{10}$$

$$\sum_{j \in \mathcal{N}^+(i)} x_{ij}^{pv} \leq M r_i^{pv} \qquad i = o(p), \forall p \in \mathcal{P}, \forall v \in \mathcal{V}. \tag{11}$$

Constraints (12) - (13) are similar to the previous constraints. They describe the following condition: if the flow of containers of order p arriving at its destination node i onboard of vehicle v is positive, then this order is onboard of vehicle v at node i. More formally:

$$\sum_{j \in \mathcal{N}^-(i)} x_{ji}^{pv} > 0 \text{ if and only if } r_i^{pv} = 1 \qquad i = d(p), \forall p \in \mathcal{P}, \forall v \in \mathcal{V}.$$

We linearize this condition by the following inequalities:

$$\sum_{j \in \mathcal{N}^-(i)} x_{ji}^{pv} \geq M(r_i^{pv} - 1) + \epsilon \qquad i = d(p), \forall p \in \mathcal{P}, \forall v \in \mathcal{V}, \tag{12}$$

$$\sum_{j \in \mathcal{N}^-(i)} x_{ji}^{pv} \leq M r_i^{pv} \qquad i = d(p), \forall p \in \mathcal{P}, \forall v \in \mathcal{V}. \tag{13}$$

5.8 Synchronization of vehicle calls

In case containers have to be transshipped between vehicles s and v at node i, the synchronization of arrival and departure times of vehicles garanties that containers transported by vehicle v from node i are available for the considered service at the departure time $t_i^{D^v}$ at node i. Neglecting this could give rise to solutions in which a vehicle is asked to start the transportation of containers from node i at time $t_i^{D^v}$ even if these containers arrive to node i later than $t_i^{D^v}$. Thus, in case there is a transshipment of containers from vehicle s to v, we require that the departure time of vehicle v from node i is later than the arrival time of vehicle s to the node i plus the service time at this node. More formally this condition can be written as the following:

$$\text{If } \theta_i^{sv} = 1, \text{ then } t_i^{D^v} - t_i^{A^s} - s_i \geq 0, \qquad \forall i \in \mathcal{N}, s, v \in \mathcal{V}.$$

The linear version of this condition is the following:

$$t_i^{D^v} - t_i^{A^s} - s_i \geq M(\theta_i^{sv} - 1) \qquad \forall i \in \mathcal{N}, \forall v, s \in \mathcal{V}. \tag{14}$$

5.9 Vehicle sequencing constraints

(a) For each chosen service $[v,(i,j)]$ we require that the arrival time of vehicle v to node j occurs not earlier than the departure time of vehicle v (associated with the service) from node i plus the vehicle traveling time between nodes i and j. The linear version of this requirement is stated in constraint (15). The relation between the arrival time of the vehicle to a node, the service time at the node and the departure time of the vehicle from the node is given in (16). Vehicle v departs from node i not earlier than the vehicle service is executed. The same set of constraints describes the initial position of vehicles.

$$t_i^{Dv} + T_{ij}^v - t_j^{Av} \leq M(1 - y_{ij}^v) \qquad \forall (i,j) \in \mathcal{A}, \forall v \in \mathcal{V}, \tag{15}$$
$$t_i^{Dv} \geq t_i^{Av} + s_i \qquad i \neq k(v), \forall v \in \mathcal{V}. \tag{16}$$

(b) The departure time of a vehicle from the node of origin of shipment order p occurs after the time of availability of a commodity at this node. The arrival time of a vehicle to the node of destination of shipment order p must occur before the deadline of order's delivery. This is guarantied by the following inequalities:

$$t_i^{Dv} \geq T_i^{\min p} r_i^{pv} \qquad i = o(p), \forall v \in \mathcal{V}, \forall p \in \mathcal{P}, \tag{17}$$
$$t_i^{Av} \leq T_i^{\max p} + M(1 - r_i^{pv}) \qquad i = d(p), \forall v \in \mathcal{V}, \forall p \in \mathcal{P}. \tag{18}$$

(c) If a vehicle v is used, it should perform a single service emanating from node i:

$$\sum_{j \in \mathcal{N}^+(i)} y_{ij}^v \leq \delta^v \qquad \forall v \in \mathcal{V}, \forall i \in \mathcal{N}. \tag{19}$$

5.10 Integrality and nonnegativity constraints

$$x_{ij}^{pv} \geq 0, \qquad \forall (i,j) \in \mathcal{A}, \forall p \in \mathcal{P}, \forall v \in \mathcal{V}, \tag{20}$$
$$q_i^{pvs} \geq 0, \qquad \forall i \in \mathcal{N}, \forall p \in \mathcal{P}, \forall v, s \in \mathcal{V}, \tag{21}$$
$$t_i^{Av}, t_i^{Dv} \geq 0 \qquad \forall i \in \mathcal{N}, \forall v \in \mathcal{V}, \tag{22}$$
$$y_{ij}^v \in \{0,1\} \qquad \forall (i,j) \in \mathcal{A}, \forall v \in \mathcal{V}, \tag{23}$$
$$\theta_i^{vs} \in \{0,1\} \qquad \forall i \in \mathcal{N}, \forall v, s \in \mathcal{V}, \tag{24}$$
$$r_i^{pv} \in \{0,1\} \qquad \forall i \in \mathcal{N}, \forall v \in \mathcal{V}, \forall p \in \mathcal{P}, \tag{25}$$
$$\delta^v \in \{0,1\} \qquad \forall v \in \mathcal{V}. \tag{26}$$

The scheduled service network design with asset-management consideration and container distribution problem is NP-hard since it belongs to the class of capacitated fixed charge network design problems, which were shown to be NP-hard (Balakrishnan et al., 1997).

5.11 A simple four-node example

In this section, we provide an example to emphasize the necessity of synchronization and transshipment constraints, as well as the influence of these constraints on the scheduled service network design and container distribution plan.

We consider a network with four nodes, where the first three nodes represent canal terminals and the fourth node represents a deep-sea port terminal. All the terminals can store and transship containers. The nodes of the network are interconnected as shown in Figure 1 and transportation in both directions is allowed on links. We suppose that five containers have to be shipped from node 1 to node 4 and another five containers have to be shipped from node 2 to node 4 by means of three available vehicles v_1, v_2 and v_3. For each shipment order we define the time of availability at the node of origin and the deadline for delivery to the node of destination as given in Table 1. Vehicles v_1 and v_2 can carry up to five containers, while vehicle v_3 can carry up to 10 containers. We consider node 1 to be the initial position of vehicle v_1, node 2 to be the initial position of vehicle v_2 and node 3 to be the initial position of vehicle v_3. To illustrate the behavior of the model, we restrict the routes the vehicles can operate as shown in Figure 1. The first vehicle serves the connection between nodes 1 and 3, the second vehicle serves the connection between nodes 2 and 3, and the last vehicle serves the connection between nodes 3 and 4. The time duration of connections between the terminals is indicated on the links in the figure. The service of a vehicle can start any time after its arrival to the terminal, the duration of service at each terminal is equal to 1 hour. For simplicity, in this example, we neglect fixed, variable, and handling costs.

We want to construct a scheduled service network design and container distribution plan, that satisfy the demand (supply) of containers and time requirements. To emphasize the importance and the influence of synchronization and transshipment constraints on the model output, we generate two plans. The first one is based on objective function (1), constraints (2) - (4), (10) - (13), and (15) - (26), where the synchronization and transshipment constraints are omitted. The second transportation plan is generated based on the complete model: equations (1) - (26).

Nodes	1	2	4
Shipment order 1	$T_1^{\min 1} = 6$		$T_4^{\max 1} = 18$
Shipment order 2		$T_2^{\min 2} = 2$	$T_4^{\max 2} = 18$

Table 1: Time of availability and due dates of shipment.

One can easily see that, in order to satisfy the demand, any transportation plan must involve all the three vehicles. The first vehicle must transport five containers from node 1 to node 3, the second vehicle should carry the other five containers from node 2 to node 3, and the last vehicle should carry all ten containers together from node 3 to node 4.

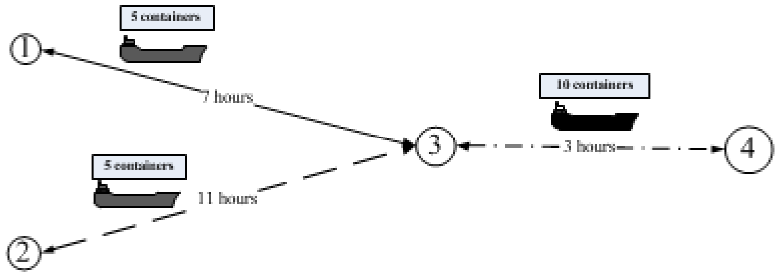

Figure 1: A simple four-node example illustration.

Transportation plan 1								
	Node 1		Node 2		Node 3		Node 4	
Vehicles	$t_1^{Dv_i}$	$t_1^{Av_i}$	$t_2^{Dv_i}$	$t_2^{Av_i}$	$t_3^{Dv_i}$	$t_3^{Av_i}$	$t_4^{Dv_i}$	$t_4^{Av_i}$
v_1	6	20			14	**13**		
v_2			2	25	14	**13**		
v_3					**0**	7	4	3

Table 2: Schedule for transportation plan 1, omitting transshipment and synchronization constraints.

The third node is a transshipment node, where 10 containers are transshipped from two smaller vehicles to a larger one. The schedules for the obtained transportation plans are presented in Tables 2 and 3. Though the network design, the services, and the flows of containers are similar for both transportation plans, the departure and the arrival times of services are different. In the first transportation plan, the first two vehicles leave the corresponding nodes of their initial position at 6 and 2 time units, respectively. This is the time the associated shipment orders become available. First vehicle arrives to the transshipment node after 7 time units and the second vehicle arrives to the transshipment node after 11 time units. The third vehicle leaves the third node at 0 time units, since this node is not associated with an origin or a destination of any shipment order. Therefore, in this scenario, the first and second vehicles arrive to node 3 later than the third vehicle leaves it with commodities that have not yet arrived there. This inconsistency occurs because the vehicle and container moves synchronization was ignored.

The second transportation plan, that is the output of the complete mathematical model (1) - (26), shows that the inclusion of transshipment and synchronization constraints provides consistency in vehicle and container moves.

As in the first schedule, first two vehicles leave the corresponding node of their initial positions at 6 and 2 time units and arrive to the transshipment nodes after 7 and 11

Transportation plan 2								
	Node 1		Node 2		Node 3		Node 4	
Vehicles	$t_1^{D^{v_i}}$	$t_1^{A^{v_i}}$	$t_2^{D^{v_i}}$	$t_2^{A^{v_i}}$	$t_3^{D^{v_i}}$	$t_3^{A^{v_i}}$	$t_4^{D^{v_i}}$	$t_4^{A^{v_i}}$
v_1	6	20			14	**13**		
v_2			2	25	14	**13**		
v_3					**14**	21	18	17

Table 3: Schedule for transportation plan 2 based on complete model.

time units, respectively. In contrast to the the first schedule, the third vehicle leaves the transshipment node at 14 time units, after the service of the second vehicle is finished. The inconsistency in container and vehicle moves observed in the first schedule is removed.

6 Experimental Results

In this section we evaluate the performance of the proposed model with respect to the generated scheduled service network design, container distribution plan, and solution time. Test instances used for the analysis are inspired from the practice of EU canal terminals and short sea shipping carrier companies. We describe the scheme used to generate test instances in Section 6.1. The managerial insights that capture the influence of handling costs, available fleet capacity, and lead time on the performance of the scheduled service network design and container distribution plan are presented in Section 6.2. We evaluate the influence of above mentioned parameters on the percentage of containers transshipped between vehicles, percentage of direct services, and number of vehicles used. In Section 6.3 we report on solution times in response to the size of test instances, i.e., the number of nodes in the network, the number of shipment orders, and the number of available vehicles.

6.1 Instance generation

We generate instances as discussed below. We consider a set of nodes representing canal(river and coastline) terminals and deep-sea port terminals. All of them can store and tranship containers. We assume that the efficiency of handling machinery is the same at every terminal. The duration of servicing a vehicle is equal to 1 hour. A set of shipment orders have to be transported between terminals. Origin-destination pairs of nodes are randomly chosen from pairs of deep-sea port terminal and other terminals. The demand (supply) values for each shipment order are randomly chosen from the interval $[0, 80]$ containers.

Several vehicles, different in speed and capacity, are available for the distribution purposes. The total capacity of all vehicles is always larger than the total demand of all shipment orders.

Transit time between a pair of terminals is proportional to the Euclidean distance between them and also depends on the vehicle operating the route. The coordinates X and Y of the geographical position of terminals are randomly generated from the domain $[-6.8, 10] \times [10, 48]$. We consider three possible vehicle speed profiles: fast, average and slow. To adjust the transit time according to the speed profile, we multiply the Euclidean distance between a pair of terminals with the corresponding multiplier α : $\alpha = 0.8$ for fast, $\alpha = 1.0$ for average and $\alpha = 1.2$ for slow speed profile.

The fixed service selection costs f_{ij}^{v} and the unit variable transportation costs c_{ij}^{v} depend on the capacity of vehicle v and on the duration of the corresponding connection (i, j). To represent the economies of scale, the unit variable transportation costs are set higher for smaller vehicles than for larger vehicles. The handling costs are the same for all terminals. We define the handling cost as 10% of the most expensive connection between a pair of terminals. The time of availability of a shipment order at the node of origin is randomly generated from the interval $[0, 4]$ hours. The deadline for delivery to the node of destination of a shipment order is calculated according to the following formula:

$$T_j^{\max p} = T_i^{\min p} + \beta_1 \cdot \max_{v \in \mathcal{V}} \left[T_{ij}^{v} \right] + \beta_2 \cdot s_i \tag{27}$$
$$\forall p \in \mathcal{P}, i = o(p), j = d(p), \beta_1 = 1.5, \beta_2 = 5.$$

In this formula, we take a multiple of the longest possible duration between a pair of terminals $\beta_1 \cdot \max_{v \in \mathcal{V}}[T_{ij}^{v}]$ and a multiple of possible service time $\beta_2 \cdot s_i$ on the way to the node of order destination. The total capacity of the available fleet is chosen to be 30% larger than the total demand. We refer to an instance generated according to the scheme described above as a basic scenario.

For sensitivity analysis of the solution (Section 6.2) we create problem instances with 5,7 and 10 terminals, 5 vehicles and 5 commodities. We perform 30 experiments in total.

For the evaluation of model performance (Section 6.3) we create instances with 5 and 7 nodes, 5,7, and 10 vehicles, 5, 10, 20, 30 and 50 shipment orders. In total, we generate 20 instances, which we use for 4 tests, where each test has its own parameter setting discussed in Section 6.3. A problem instance is labeled as *npv*, where n is the number of terminals, p is the number of shipment orders, and v is the number of vehicles. The model is implemented in C++ programming language and solved with CPLEX optimization solver (Version 12.4, 32-bit). Numerical experiments were conducted on a 16-core 2.93GHz Intel Xeon computer, with a total of 64GB RAM.

6.2 Analysis of the solution

In this subsection, we study the design of scheduled service network in response to the changes in the value of handling costs, fleet capacity, and length of lead time interval.

We use three system performance indicators to perform the analysis. The first indicator reports on the efficiency of asset management. This indicator calculates the ratio of the number of vehicles used to the total number of vehicles in the fleet. The second performance indicator is the percentage of containers transshipped between vehicles related to the total number of containers transported in the system. This indicator also reports on the level of service consolidation. The last performance indicator is the percentage of direct services out of the total number of services chosen in the model output. By a direct service we mean the direct transportation of a shipment order between its origin and destination.

	$h_i = 0$			$h_i = 0.1 \cdot \max\left[c^v_{kj}\right]$			$h_i = \max\left[c^v_{kj}\right]$			$h_i = 2 \cdot \max\left[c^v_{kj}\right]$		
1	2	3	4	5	6	7	8	9	10	11	12	13
Instances	ratio veh.	Trans. cont.	Direct ser.	ratio veh.	Trans. cont.	Direct ser.	ratio veh.	Trans. cont.	Direct ser.	ratio veh.	Trans. cont.	Direct ser.
n5p5v5	0.6	52%	25%	0.6	52%	25%	0.6	48%	25%	0.6	0%	50%
n7p5v5	0.6	31%	25%	0.6	31%	25%	0.8	0%	0%	0.8	0%	0%
n10p5v5	0.6	52%	0%	0.6	52%	0%	0.6	52%	0%	0.8	42%	0%

Table 4: Model performance for five-, seven-, and ten-terminal networks with respect to handling cost.

Table 4 reports on the percentage of containers transshipped, the percentage of direct services and the ratio of vehicles used for transportation in response to changes in the value of handling costs. The results for the basic scenario are presented in columns five, six and seven. The rest of the results are generated for cases where transshipment costs are not taken into account ($h_i = 0$), are equal to the unit variable transportation costs ($h_i = \max_{v \in \mathcal{V},(k,j) \in \mathcal{A}} \left[c^v_{kj}\right]$), and are considerably higher than the unit variable transportation costs ($h_i = 2 \cdot \max_{v \in \mathcal{V},(k,j) \in \mathcal{A}} \left[c^v_{kj}\right]$).

The number of containers transshipped between vehicles decreases with the increase of handling costs. For five-terminal networks the percentage of containers transshipped between vehicles drops from 52% to 0%, while the rate of handling costs increases from 0 to $2 \cdot \max_{v \in \mathcal{V},(k,j) \in \mathcal{A}} \left[c^v_{kj}\right]$. The same tendency can be observed for seven and ten-terminal networks, the percentage of transshipped containers decreasing from 31% to 0 and from 52% to 42%, respectively.

Note that, the design of the network changes when handling costs increase. For a five-terminals network, the percentage of direct services increases from 25% to 50%, while for seven and ten terminal networks, the ratio of vehicles used for transportation services increases from 0.6 to 0.8.

The increase in handling costs makes transshipment operations less cost efficient, therefore, direct services or sequences of services that are longer and do not include container transshipment provide the least cost transportation plan.

	Available capacity 30%			Available capacity 50%			Available capacity 15%			Available capacity 5%		
Instances	ratio veh.	Trans. cont.	Direct routes	ratio veh.	Trans. cont.	Direct routes	ratio veh.	Trans. cont.	Direct routes	ratio veh.	Trans. cont.	Direct routes
n5p5v5	0.6	52%	33%	0.6	65%	33%	0.6	51%	25%	1	50%	80%
n7p5v5	0.6	31%	33%	0.6	31%	33%	0.8	0%	67%	0.8	0%	75%
n10p5v5	0.6	39%	0%	0.6	39%	0%	0.6	39%	0%	0.8	39%	25%

Table 5: Model performance for five-, seven-, and ten-terminal networks with respect to the available fleet capacity.

Table 5 shows the influence of the fleet capacity on the percentage of containers transshipped between vehicles and the routing decision. For this analysis we keep handling costs as in the basic scenario and vary the capacity of the fleet between 5% to 50% larger than the total demand. Second, third and fourth columns represent the model output for the basic scenario. If the fleet capacity is tight, the number of possibilities to tranship containers between vehicles decreases. As presented in the first line of Table 5, for the five-terminal network, the number of containers transshipped between vehicles drops from 65% to 50%, while the available fleet capacity is decreasing from 50% to 5%. The performance of the system for seven and ten-terminal networks is similar.

Direct services between the origin of the shipment order and its destination become more cost-efficient with the decrease in available fleet capacity, as the remaining idle capacity of vehicles is so small that it does not bring any additional cost savings to use it for transshipped containers from other vehicles. We observe this behavior from the results in Table 5. When the available fleet capacity decreases from 50% to 5%, the percentage of direct services increases from 33% to 80% for the five-terminal network. For seven and ten-terminal network the percentage of direct services increases from 33% to 75% and from 0% to 25%, respectively.

The percentage of direct services also depends on the length of the delivery lead time interval: decreasing it causes the increase of the percentage of direct services. Since in our formulation we require all shipment orders to be delivered on time, tight deadlines for shipment order delivery force vehicles to move directly between nodes of origin and nodes of destination of the shipment orders. This trend is observed in Table 6.

Table 6 describes the influence of the length of the lead time interval on the percentage of the direct services and the percentage of containers transshipped. The generated output for instances with five-, seven- and ten-terminal networks are presented in columns five to ten, respectively. The lead time interval associated with $\beta_1 = 2, \beta_2 = 0$ is larger

	$\beta_1 = 1.5, \beta_2 = 5$			$\beta_1 = 2, \beta_2 = 0$			$\beta_1 = 1, \beta_2 = 3$		
Instances	ratio veh.	Trans. cont.	Direct routes	ratio veh.	Trans. cont.	Direct routes	ratio veh.	Trans. cont.	Direct routes
n5p5v5	0.6	53%	33%	0.8	34%	75%	0.8	31%	100%
n7p5v5	0.6	31%	33%	0.6	17%	33%	0.6	17%	80%
n10p5v5	0.6	52%	0%	0.8	39%	25%	0.8	39%	80%

Table 6: Model performance for five-, seven-, and ten-terminal networks with respect to the length of delivery lead time.

than the one associated with $\beta_1 = 1, \beta_2 = 3$ and smaller than the lead time interval associated with the basic scenario. For five-terminal network, we observe the increase of percentage of direct services from 33% to 80% with shortening of the lead time interval.

On contrary, shortening of lead time interval causes the decrease of the percentage of containers transshipped between vehicles. Since each transshipment operation has a duration of one hour, short lead times make them infeasible. For the remaining networks we observe a similar behavior.

Our numerical experiments suggest that direct services between the origin of an order and its destination happen more often when either the fleet capacity is tight or the handling costs or the lead time interval increase. At the same time, with a decrease in the values of these parameters the number of container transshipments between vehicles increases.

6.3 Computation results

In this subsection we report the results on the solution time required to solve our problem to optimality.

As any capacitated multicommodity service network design problem with side constraints, the proposed problem formulation is hard to solve. For example, an instance with 7 nodes, 7 vehicles, and 5 shipment orders generates 707 binary variables on 294 arcs (full graph). It takes more than ten hours to solve it to optimality. Note that, a transportation network based on a full graph is rarely considered in real-life settings due to economic or geographical reasons. However, even for reduced graph networks, the considered problem is still difficult to solve. We propose several adjustments to the model such that one can solve the resulting problem formulation with a standard commercial solver during a reasonable solution time.

To improve the strength of the linear relaxation, we consider the strong formulation of the problem, in which we include strong forcing constraints as in Andersen et al. (2009a)

$$x_{ij}^{pv} - \min[|w_i^p|, C^v] y_{ij}^v \leq 0 \qquad \forall (i,j) \in \mathcal{A}, \forall p \in \mathcal{P}, \forall v \in \mathcal{V}. \tag{28}$$

Table 7 presents the results of the computational studies for the improved problem formulation. The last column of Table 7 reports on solution times when strong forcing constraints are implemented for the original problem.

Further, we make the formulation tighter by penalizing transshipment events and variables indicating the availability of containers onboard of vehicles at the nodes of their origin and destination. In such a way, we can exclude three sets of constraints (5),(10),(12) from considered problem formulation. We define z as the fixed costs for transshipment of containers at nodes. These costs do not depend on the number of containers being transshipped. Thus, the improved problem formulation is the following:

$$\begin{aligned} \min \sum_{v \in \mathcal{V}} b^v \delta^v + \sum_{v \in \mathcal{V}} \sum_{(i,j) \in \mathcal{A}} f_{ij}^v y_{ij}^v + \sum_{p \in \mathcal{P}} \sum_{v \in \mathcal{V}} \sum_{(i,j) \in \mathcal{A}} c_{ij}^v x_{ij}^{pv} + \\ \sum_{p \in \mathcal{P}} \sum_{\substack{v \in \mathcal{V} \\ v \neq s}} \sum_{s \in \mathcal{V}} \sum_{i \in \mathcal{N}} h_i q_i^{pvs} + \sum_{\substack{v \in \mathcal{V} \\ v \neq s}} \sum_{s \in \mathcal{V}} \sum_{i \in \mathcal{N}} z \theta_i^{vs} + \\ \sum_{p \in \mathcal{P}} \sum_{v \in \mathcal{V}} \sum_{i \in \mathcal{N}} z r_i^{pv}, \end{aligned} \tag{29}$$

subject to constraints (2) - (4), (6) - (9), (11), (13)-(26), and (28).

Introduction of the transshipment fixed costs leads to the discussion of the appropriate values of these costs. The choice of the value for the fixed costs for transshipment is based on the trade-off between the quality of solution and solution time. High values prevent the transshipment events to occur and, consequently, impact the quality of solution. Moderate and low values of the transshipment costs allow solutions that represent reality during reasonable time. We chose three values of fixed transshipment costs to show their impact on the solution time of the problem. First value of z is 10 times greater than the most expensive unit variable transportation costs between a pair of terminals. Though this value is unrealistic, we still use it for the experimental purposes. Second value is 5 times greater than the most expensive unit variable transportation costs between a pair of terminals, and the last value is chosen to be very small and it equals to 0.0001. This value guides the solution procedure and almost does not influence the quality of the solution.

The execution time of the test was limited to 10 hours (36,000 seconds). The gap between the lower and the upper bounds is reported if the required running time exceeds 10 hours and the optimal solution was not found.

The original problem formulation with strong forcing constraints is the hardest to solve, the solution time required to solve this problem for each of the twenty data instances is presented in the last column of Table 7. As presented in the third column of Table 7, among the considered problem variants, the problem for z = 0.0001 is the easiest to solve. We can also observe that the problem for $z = 10 \cdot \max\left[c_{kj}^v\right]$ is slightly easier to solve than the problem for $z = 5 \cdot \max\left[c_{kj}^v\right]$, except for the first five data instances.

Therefore, in industrial settings, one should use real rates for fixed transshipment costs, when they are available. Otherwise, very small artificial values such as $z = 0.0001$ provide reasonable solution times with a small impact on the optimal solution value.

		$z = 10 \cdot \max\left[c_{kj}^v\right]$		$z = 5 \cdot \max\left[c_{kj}^v\right]$		$z = 0.0001$		$z = 0$	
Data instance	Number of arcs	Solution time, s.	Gap, %	Solution time, s.	Gap, %	Solution time, s.	Gap, %	Solution time, s.	Gap, %
n5p5v5	100	3.05	0.00	2.69	0.00	1.74	0.00	120.89	0.00
n5p10v5	100	19.74	0.00	53.37	0.00	31.89	0.00	131.95	0.00
n5p20v5	100	495.63	0.00	287.32	0.00	64.47	0.00	296.63	0.00
n5p30v5	100	1605.11	0.00	1333.39	0.00	493.25	0.00	1848.94	0.00
n5p50v5	100	7821.82	0.00	1521.93	0.00	157.92	0.00	1675.91	0.00
Average	100	1989.07	0.00	639.74	0.00	149.85	0.00	814.78	0.00
n5p5v7	140	16826.08	0.00	2364.39	0.00	17.12	0.00	1893.76	0.00
n5p10v7	140	36000.00	9.51	36000.00	6.33	1155.59	0.00	36000.00	5.08
n5p20v7	140	36000.00	0.65	36000.00	1.16	3239.34	0.00	36000.00	3.86
n5p30v7	140	27139.92	0.00	28385.01	0.00	737.91	0.00	36000.00	4.06
n5p50v7	140	16352.48	0.00	32399.42	0.00	2117.11	0.00	36000.00	1.48
Average	140	26463.70	2.03	27029.76	1.50	1453.41	0.00	29178.75	2.90
n5p5v10	200	36000.00	7.36	36000.00	3.39	889.45	0.00	36000.00	10.46
n5p10v10	200	36000.00	12.14	36000.00	15.06	256.42	0.00	36000.00	19.47
n5p20v10	200	36000.00	5.02	36000.00	3.11	1619.37	0.00	36000.00	16.20
n5p30v10	200	36000.00	5.24	36000.00	5.38	36000.00	2.80	36000.00	14.00
n5p50v10	200	36000.00	3.00	36000.00	6.51	36000.00	9.93	36000.00	14.20
Average	200	36000.00	6.55	36000.00	6.69	14953.10	2.54	36000.00	14.87
n7p5v5	210	930.20	0.00	7766.11	0.00	2763.79	0.00	36000.00	7.24
n7p10v5	210	36000.00	5.87	36000.00	7.00	6130.92	0.00	36000.00	19.06
n7p20v5	210	36000.00	9.15	36000.00	9.90	4111.94	0.00	36000.00	4.18
n7p30v5	210	36000.00	2.66	36000.00	3.73	4443.13	0.00	36000.00	3.45
n7p50v5	210	36000.00	4.26	36000.00	6.01	26295.53	0.00	36000.00	6.98
Average	210	28986.04	4.39	30353.22	5.33	8749.10	0.00	36000.00	8.18

Table 7: Solution time (CPU seconds) and gaps between lower and upper bounds.

We draw the reader's attention that the size of the data instances used for the problem testing represents industrial settings for intermodal transportation, river, canal and coastline navigation. However, due to large computation cost, for larger data instances, decomposition or heuristic methods have to be applied.

7 Conclusions

We have introduced a new variant of scheduled service network design and container freight distribution problem with asset-management considerations, where container transshipments and vehicle synchronization are taken into account. We presented a new concise mathematical model that accurately evaluates the container transshipment volumes and vehicle arrival and departure times that are crucial for river, canal and coastline navigation. This requires, however, the introduction of an additional set of the vehicle synchronization constraints, which control the schedule of container flows.

We tested the model on problem instances inspired from real-world problems faced by EU carrier companies and short-sea shipping companies. Our results indicate that direct services happen more often when the available fleet capacity is tight or the rate of handling costs is increasing. Short lead times require direct services for container distribution as well. With such parameter settings, the percentage of container transshipment between vehicles decreases.

Further research may target model extensions, e.g., allowing vehicles to perform the same route multiple times. The computational difficulty encountered in solving the proposed problem formulation points to the continuous need for development of efficient solution methods for service network design problems with asset-management consideration.

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