Entry, Imperfect Competition, and Futures Market for the Input

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Abstract. We analyze firms’ entry, production and hedging decisions under imperfect competition. We consider an oligopoly industry producing a homogeneous output in which risk-averse firms incur a sunk cost upon entering the industry, and then compete in Cournot with one another. Each firm faces uncertainty in the input cost when making production decision, and has access to the futures market to the edge its random cost. We provide two sets of results. First, under general assumptions about risk preferences, demand, and uncertainty, we characterize the unique equilibrium. In contrast to previous results in the literature (without entry), production and output price depend on uncertainty and risk aversion. In other words, when entry is endogenized, access to the futures market does not lead to separation. Second, to study the effect of access to the futures market on entry and production, we restrict attention to constant absolute risk aversion (CARA) preferences, a linear demand, and a normal distribution for the spot input price. In general, the effect of access to the futures market on the number of firms and production is ambiguous. However, when the values of the model parameters lead to partial hedging, the effect is unambiguous. Under partial hedging, access to the futures market induces more firms to enter the market and each one of them to produce more.

Keywords. Cournot, entry, futures, hedging, imperfect competition.

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1 Introduction

Recent financial literature on firms’ risk management of market risk has focused on the determinants of hedging and the economic value of financial coverage. The two main questions in this literature are: Why do firms hedge? and Does hedging increase the economic value of the firms? Firms’ hedging is explained by managerial risk aversion (Stulz, 1990; Tufano, 1996) or market imperfections such as corporate income taxation (Smith and Stulz, 1985; Graham and Smith, 1999; Graham and Rogers, 2002), financial distress costs (Smith and Stulz, 1985), corporate governance (Dionne and Triki, 2013), investment opportunity costs (Froot et al., 1993; Froot and Stein, 1998), and information asymmetries (DeMarzo and Duffie, 1991). The empirical effect of hedging on firm value is rather mixed (Hoyt and Liebenberg, 2011; Campello et al., 2011).

Another strand of the literature analyzes the joint production and hedging decisions of the firm under output price uncertainty (Holthausen, 1979; Feder et al., 1980). The main result from this literature is that optimal output production is independent of the probability distribution of the output price and the manager’s risk aversion. The distribution of the output price and risk aversion affect only firms’ involvement in futures trading. Hence, with access to the futures market, uncertainty does not introduce any efficiency in production. The same separation result is obtained under perfect competition and input price uncertainty (Holthausen, 1979; Katz and Paroush, 1979; Paroush and Wolf, 1992). Paroush and Wolf (1992) show, however, that the separation result does not hold in the presence of basis risk, while Anderson and Danthine (1981) obtain a similar negative result with production uncertainty. Different extensions have been proposed by considering multiple risky inputs, background risk, and joint output price and input price uncertainty.1

Although there are many contributions regarding firms’ hedging in both literatures, to our knowledge there are few analyses of firms’ hedging behavior under imperfect competition, and none that consider entry in the output

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1See Viaene and Zilcha (1998) for instance. See also Alghalith (2008) for a review of the literature with competitive markets.
We propose to fill the gap by analyzing firms’ entry, production and hedging decisions under imperfect competition. Specifically, we consider an oligopoly industry producing a homogeneous output in which risk-averse firms incur a sunk cost upon entering the output industry, and, then, compete in Cournot with one another. Each firm faces uncertainty in the input cost when choosing production, and has access to the futures market to hedge its random cost. There is only one source of risk in our analysis. One application of our model is the airline market, where it has been verified that Cournot competition is present in empirical investigations of U.S. airline industry (Brander and Zhang, 1990; Fisher and Kamerschen, 2003). In this market, airline companies face future fuel price uncertainty when they make their optimal routes decisions for the next few months, and purchase futures contracts for jet fuel (Morrell and Swan, 2006).

Here, entering or exiting the output market is mainly interpreted as route decisions.

We provide two sets of results. First, under general assumptions about risk preferences, demand, and uncertainty, we show that there exists a unique equilibrium in which a finite number of firms enter the market as long as the sunk cost is not too high (the standard case) or not too low. Indeed, if the

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2There are three notable exceptions for imperfect competition. First, Eldor and Zilcha (1990) study the hedging behavior of an oligopoly under uncertainty in the output sector. However, while the spot price is endogenous (and the firms exercise market power under uncertainty), the futures (or forward) price is exogenous and fixed. In other words, the firms exercise market power in the spot output market, but behave perfectly competitively for the futures market of the same good. In addition, Eldor and Zilcha (1990) do not consider entry, which is our main focus in this paper. Second, in a very different setting, Allaz and Villa (1993) isolate the strategic reasons for using futures contracts. By selling futures contracts, Cournot firms attach a lower value to a high spot price and commit to aggressive behavior on the spot price, which implies more production at a lower price in equilibrium, and thus benefits consumers but not producers. Third, the effect of strategic hedging on Cournot and Bertrand competition is studied in Léautier and Rochet (2012). We compare Léautier and Rochet (2012) with our model and results later in the introduction.

3In this study, we assume that the firms have a concave payoff due to managerial risk aversion. Concavity can be explained by different market imperfections. See Froot et al. (1993) for a discussion.

4Fuel cost represents about 15% of the airlines’ costs. Other costs are usually less volatile so hedging fuel costs guarantees stable profits. Usually, airlines do not hedge business cycle risk. Airline companies can also purchase other derivatives products such as options and even collars. These options would introduce more flexibility for the firm but would not affect the main results of the paper.
cost of entry in the output industry is too low, an infinite number of firms may enter the output industry and engage in speculation in the futures market, which yields the competitive outcome in the real sector. That is, the price of the output is equal to the marginal cost and the firms only make profits from speculating on the input market. We also show that, in contrast to previous results in the literature, production and output price depend on uncertainty and risk preferences. In particular, production and output price depend on the distribution of the spot price and risk aversion. The key element is that the entry decision limits the ability of the firms to adjust their production decisions, which implies that they are no longer independent from uncertainty and risk aversion. One implication is that access to the futures market alters the comparative analysis. If there is no access to the futures market, either a mean-preserving increase in risk or an increase in risk aversion induces firms to produce less. If there is access to the futures market, such changes imply an increase in production.\footnote{The result without financial access is consistent with classical results obtained in a static environment (i.e., without entry decision) for perfect competition (Sandmo, 1971; Batra and Ullah, 1974) and quantity-setting monopoly (Leland, 1972).} In other words, financial access reverses the effect of risk on per-firm production.

The second set of results concern the effect of access to the futures market on entry, production, and prices. To study this effect, we restrict attention to constant absolute risk aversion (CARA) preferences, a linear demand, and a normal distribution for the spot input price. The effect of access to the futures market on the number of firms is ambiguous depending on the value of the futures price and the parameters of the model. Further, the equilibrium number of firms is convex in the futures price when the firms partially hedge. In particular, an increase in the futures price of the input can yield an increase in the number of firms in the output sector. This is due to the fact that an increase in the futures price induces firms to produce less, which reduces the market externality in a Cournot game and induces more firms to enter while hedging their cost. Finally, we show that hedging induces the risk-averse firm to produce more, while speculating reduces production.

As noted, very few articles study the interaction of real and financial
activities when the firms exert some sort of market power. One exception is a recent paper by Léautier and Rochet (2012) which studies the effect of committing to a hedging strategy on production or pricing strategies. Specifically, Léautier and Rochet (2012) considers a two-stage game in which each firm commits to a hedging strategy in the first stage and then chooses production or pricing strategies in the second stage. Like in our model, the firms have market power in the output sector but are perfectly competitive in the input market. There are however main differences in the setups as well as in the issues studied. Regarding the model, Léautier and Rochet (2012) considers a market with a fixed number of firms, each one committing to a hedging strategy before production or pricing strategies. In our model, entry is a decision variable in the first stage whereas hedging and production are chosen simultaneously in the second stage.

Beyond the differences in modeling, we study different and complementary aspects of the link between real and financial activities when the firms exert some sort of market power. Léautier and Rochet (2012) shows that strategic hedging (when used as a strategic commitment device) has a profound effect on the real decisions of the firms. Specifically, under actuarially fair pricing, when the firm commits to a hedging strategy, hedging toughens quantity competition, but softens price competition. We also consider issues related to risk management and real activities but of different nature. Specifically, we show that when the futures price is not actuarially fair, the separation result does not hold in the long-run when market structure is endogenized.\(^6\) We then show the effect of access to the futures market on entry and production decisions.\(^7\)

The paper is organized as follows. Section 2 presents the model and defines the equilibrium. Section 3 states the equilibrium and presents results related to the issue of separation between production (and output price) and

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\(^6\) As noted, separation means that production decisions are independent of uncertainty and risk preferences, and depend only on the futures price (Danthine, 1973; Holthausen, 1979; Feder et al., 1980).

\(^7\) In other words, we show how commitment in entry removes the separation result obtained in the literature (i.e., production strategies depend on uncertainty and risk preferences in the long run) whereas Léautier and Rochet (2012) shows that commitment in hedging has a profound effect on Bertrand or Cournot competition.
uncertainty and risk preferences. Section 4 discusses the effect of access to futures market on entry, production, and output price. Section 5 concludes the paper. Proofs and extensions are found in the Appendix.

2 Model

In this section, we present the model and define the free-entry equilibrium with full access to the futures input market. In the next sections, we analyze the equilibrium. Under a general characterization of the unique equilibrium, we show that the entry decision links production and output price to uncertainty and risk aversion.

2.1 Preliminaries

We embed access to the futures input market in a two-stage entry game. At the first stage, all potential firms decide whether to enter an industry in the output sector. Each entering firm pays an exogenous entry cost $K > 0$. At the second stage, all firms that have entered make production and financial decisions while competing in Cournot in the output sector. The firms face uncertainty in the input price, and have access to perfectly competitive spot and futures input markets. Figure 1 describes the timeline of the model.

We now describe the second stage of the game in detail. In an industry with $J$ firms, firm $j$ produces $q_j \geq 0$ units of output and faces the inverse demand $p = D\left(\sum_{k=1}^{J} q_k\right)$ where $p$ is the output price and $q_k$ is the output sold by firm $k$. The technology to transform the input into the output is assumed to be linear and deterministic. A unit of input can be purchased in the spot market at price $\tilde{S}$, which is unknown at the time of setting output.

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8The case of no entry cost is excluded. In the data, industries with access to and participation in the futures market generally comprise a small number of large firms. See Campello et al. (2011). It is well documented in the literature that large firms hedge (Stulz, 1996).

9We abstract from bankruptcy or solvency problems that could arise after the spot input price is realized. Because we use futures contracts, there is no credit risk in the financial market.

10A tilde sign distinguishes a random variable from its realization.
In addition to the spot market, there is a futures market for the input. A futures contract can be purchased at known price $F$ for delivery of one unit of input.

The decisions of the firm can be summarized by two variables: one related to production and another one related to financial activity. Specifically, firm $j$ sets output $q_j \geq 0$ and chooses the hedge coverage $\omega_j \in \mathbb{R}$ for the random cost so that firm $j$ purchases $(1 - \omega_j)q_j$ units of input in the spot market at the random spot input price $\tilde{S}$, and buys futures contracts at the futures input price $F$ for the remaining $\omega_j q_j$ units of input. Given production and financial decisions, the random profit of firm $j$ when there are $J$ firms in the industry is

$$\pi(J, q_j, \omega_j, \sum_{k \neq j} q_k, \tilde{S}, F) = D(q_j + \sum_{k \neq j} q_k)q_j - \tilde{S}(1 - \omega_j)q_j - F\omega_j q_j$$

where the firms compete in Cournot in the output market, but are price-takers in the (spot and futures) input markets.

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11 In other words, firm $j$ purchases $x_j \equiv (1 - \omega_j)q_j$ units of input in the spot market, and the remaining $y_j \equiv \omega_j q_j$ units are purchased in the futures market. Hence, $q_j = x_j + y_j$ units of output are produced.

12 This situation is representative of industries that participate in the futures input markets. For instance, while airline companies have market power in providing their services, they cannot have an effect on the financial prices of the futures contracts for fuel because many other industries interact in these futures market.
as

\[
\pi(J, q_j, \omega_j, \sum_{k \neq j}^J q_k, \tilde{S}, F) = D\left(q_j + \sum_{k \neq j}^J q_k\right)q_j - Fq_j + (F - \tilde{S})(1 - \omega_j)q_j. \tag{2}
\]

From (2), the profit of a firm is the sum of the real profit under full hedging and the financial payoff from not hedging a fraction \(1 - \omega_j\) of the input, i.e., the terms \(D(q_j + \sum_{k \neq j}^J q_k)q_j - Fq_j\) and \((F - \tilde{S})(1 - \omega_j)q_j\), respectively.

Firms may engage in various types of financial activities. Specifically, firm \(j\) may decide not to access the futures market, i.e., \(\omega_j = 0\). It may also partially hedge (\(\omega_j \in (0, 1)\)) or fully hedge (\(\omega_j = 1\)).\(^{13}\) It may finally engage in two forms of speculation. First, when \(\omega_j < 0\), firm \(j\) sells futures contracts at price \(F\) which are deliverable by purchasing the input in the spot market.\(^{14}\) Second, when \(\omega_j > 1\), firm \(j\) fully hedges, and buys additional units of input in the futures market for resale in the spot market.\(^{15}\) While firms whose main activity is production rarely speculate (e.g., the board often prevents the firm’s managing team from speculating), it occurs and has occurred (Stulz, 1996). For our analysis, it turns out that allowing firms to engage in speculation simplifies the characterization of the equilibrium (i.e., no corner solution), and, more importantly, has no effect on most of our results.\(^{16}\)

### 2.2 Assumptions

Each firm is managed by a risk-averse officer (e.g., the CEO) whose objective is to maximize the firm’s expected utility of profit over output and hedge coverage. There always exists an output price high enough to cover the input cost using both input markets so that trivial cases for which the output

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\(^{13}\) Full hedging means that the input is purchased only in the futures market, whereas, under partial hedging, the input is purchased in both the spot and the futures markets.

\(^{14}\) Consistent with footnote 11, \(\omega_j < 0\) implies that \(x_j > 0, y_j < 0\), so that production is \(q_j = x_j + y_j < x_j\) because some of the input purchased in the spot market is used for delivery via the futures market, while the remaining input is used for production.

\(^{15}\) Consistent with Footnote 11, \(\omega_j > 1\) implies that \(x_j < 0, y_j > 0\).

\(^{16}\) Assuming CARA preferences, a linear demand, and a normally distributed spot price, Appendix E provides a full characterization of the equilibrium when the firms have partial access to the futures market, i.e., the firms may hedge but cannot speculate.
market does not exist are ignored. The next four assumptions hold for the remainder of the paper.

**Assumption 2.1.** The utility function for profit $\pi$ is $u(\pi)$ such that $u' > 0, u'' < 0$.

**Assumption 2.2.** Inverse demand $p = D(Q), Q \equiv \sum_{k=1}^{J} q_k$ is twice continuously differentiable such that

1. $D(0) < \infty$,
2. $D'(Q) < 0$ in the interval for which $p = D(Q) > 0$, and
3. $D''(Q)q_j + D'(Q) < 0$ for all $j$.

**Assumption 2.3.** The p.d.f. of the random spot price $\tilde{S}$ is $\phi(S)$ for $S \in (0, D(0))$.

**Assumption 2.4.** $F \in (0, D(0))$. 

We make two comments regarding our assumptions. First, Assumptions 2.1, 2.2, and 2.3 yield a unique number of firms entering the market in the first stage of the game and ensures the existence of a unique Cournot equilibrium in the second stage. In particular, Condition 3 in Assumption 2.2 ensures that a firm’s best reply function to the total output of the other firms have a nonpositive slope greater than $-1$. Second, Assumption 2.4 implies that no restriction is imposed on the futures price.\(^{17}\) Specifically, in addition to having an actuarially fair futures price, i.e., $F = \mathbb{E}\tilde{S}$ where $\mathbb{E}$ is the expectation operator, the futures market may be either in normal backwardation (i.e., $F < \mathbb{E}\tilde{S}$) or in contango (i.e., $F > \mathbb{E}\tilde{S}$).\(^{18}\)

\(^{17}\)To discard uninteresting cases in which the firms do not produce, the futures price is restricted to be below the reservation price of the output.

\(^{18}\)The futures markets for oil were in contango in 2011. This situation is generally explained by the recent political situation in Arab countries. Other futures markets (e.g., gold and silver) were in normal backwardation during the same period. See http://www.zacks.com/stock/news/57493/Backwardation-and-Contango.
2.3 Definition of Equilibrium

Definition 2.5 provides the free-entry equilibrium with full access to the futures market, i.e., $\omega_j \in \mathbb{R}$. The term free entry means that there is no institutional constraint on firms entering the market, i.e., firms may enter the market in response to profit opportunities. The equilibrium consists of the number of firms entering the industry, $J^*$; the Cournot strategies, $\{q^*(J^*), \omega^*(J^*)\}$; and the output price, $p^*(J^*)$.\footnote{Since the equilibrium is symmetric, the summation operator is no longer needed, i.e., $\sum_{k \neq j} q^*(J^*) = (J^* - 1)q^*(J^*)$.}

Definition 2.5. The tuple $\{J^*, q^*(J^*), \omega^*(J^*), p^*(J^*)\}$ is an equilibrium if

1. For all $j$, given $J^* \geq 1$ and the strategies $\{q^*(J^*), \omega^*(J^*)\}$ of firm $k \neq j$, $q^*(J^*)$ and $\omega^*(J^*)$ solve

$$\max_{q_j \geq 0, \omega_j \in \mathbb{R}} D(0) \int_0^D u(\pi(J^*, q_j, \omega_j, (J^* - 1)q^*(J^*), S, F)) \cdot \phi(S)dS. \quad (3)$$

2. Given $J^* \geq 1$ and $q^*(J^*)$, $p^*(J^*) = D(J^*q^*(J^*))$.

3. Given the strategies $\{q^*(J^*), \omega^*(J^*)\}$, $J^* \geq 0$ is an integer that satisfies

$$D(0) \int_0^D u(\pi(J^*, q^*(J^*), \omega^*(J^*), (J^* - 1)q^*(J^*), S, F)) \cdot \phi(S)dS \geq K \quad (4)$$

for $J^* \geq 1$, and

$$\int_0^D u(\pi(J^* + 1, q^*(J^* + 1), \omega^*(J^* + 1), J^*q^*(J^* + 1), S, F)) \cdot \phi(S)dS < K. \quad (5)$$

From Definition 2.5, Conditions 1 and 2 define the Cournot equilibrium at stage 2 of the game. Condition 3 is related to the entry decision at stage 1. Specifically, the equilibrium number of firms in the industry is such that,
from (4), each entering firm receives an expected utility weakly greater than the entry cost, and, from (5), further entry yields an expected utility strictly smaller than the entry cost.

In our model, the firms do not enter the product market in order to have access to the financial market. Their main expertise is to offer goods and services in the product market. In other words, if the firms do not produce, they have no need or demand for futures contracts. The firms face a risk emanating from the product market and, due to risk aversion, they develop a demand for financial products. However, the firms are not on the supply side of financial markets because they do not have any expertise to enter into the financial market and to become an investment bank or an insurance company. For example, they do not have the actuarial expertise to compute insurance premiums for pure or accident risks or to underwrite debt contracts or derivative products.\(^{20}\) Hence, these firms have no intention or ability to trade financial assets if they do not enter the product market.

### 3 Equilibrium and Separation

In this section, we provide a general characterization of the free-entry equilibrium with full access to the futures market. We also discuss the effect of entry on the separation property as defined in the literature (Danthine, 1973; Holthausen, 1979; Feder et al., 1980; Viaene and Zilcha, 1998).

**Definition 3.1.** There is separation when production and output price are independent of uncertainty and risk preferences.

We show that, whenever the free-entry equilibrium exists and the futures price is not actuarially fair, the entry decision links production and output

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\(^{20}\) According to Freixas and Rochet (2008), banks differ from other firms because they have expertise for managing loans and deposits, for choosing their level of monitoring of different clients and for choosing their level of investment in specific relationships with their clients. These specificities are forms of entry barriers in the banking industry. They also have expertise in risk management of large portfolios of derivatives with market, liquidity, and default risks. For an empirical analysis on scale economies in the provision of underwriting services by banks and related entry barriers in the banking industry, see Santos and Tsatsarinis (2003).
price to the distribution of the spot price as well as risk aversion. We proceed in two steps. We first show that the separation property holds at the second stage of the game, i.e., for a given number of firms. We then show that, once the number of firms is endogenized, the separation property no longer holds because both uncertainty and risk preferences alter market concentration, which, in turn, affects both production and output price.

Proposition 3.2 provides the firm’s production and hedge coverage in the Cournot equilibrium at the second stage of the game, i.e., for a given number of firms. In equilibrium, the firms always produce regardless of the type of financial activity, i.e., \( q^*(J) > 0 \).

**Proposition 3.2.** Suppose that there are \( J \geq 1 \) firms in the industry at the second stage of the game. Then, there exists a unique Cournot-Nash equilibrium. In equilibrium, \( q^*(J) > 0 \) and \( \omega^*(J) \) are defined by

\[
D'(Jq^*(J))q^*(J) + D(Jq^*(J)) - F = 0 \tag{6}
\]

and

\[
\int_0^{D(0)} (F - S) \cdot u'(\Pi^*(J) + (F - S)(1 - \omega^*(J))q^*(J)) \cdot \phi(S)dS = 0, \tag{7}
\]

\( \Pi^* \equiv D(Jq^*(J))q^*(J) - Fq^*(J) \).

**Proof.** Letting \( x_j \equiv (1 - \omega_j)q_j \) be the units of output for which firm \( j \) does not hedge and using (2), (3) is rewritten as

\[
\max_{q_j, x_j} \int_0^{D(0)} u(D(q_j + Q^*_{-j})q_j - Fq_j + (F - S)x_j) \cdot \phi(S)dS. \tag{8}
\]

where \( Q^*_{-j} \equiv (J - 1)q^*(J) \) is the total output of the other firms. Since the Hessian matrix is negative definite,\(^{21}\) firm \( j \)'s best reply is defined by the

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\(^{21}\)See Appendix A.
first-order conditions

\[ q_j : \int_0^{D(0)} (D'(q_j + Q^*_j)q_j + D(q_j + Q^*_j) - F) \]
\[ \times u'(D(q_j + Q^*_j)q_j - Fq_j + (F - S)x_j) \cdot \phi(S)dS = 0 \quad (9) \]

and

\[ x_j : \int_0^{D(0)} (F - S) \cdot u'(D(q_j + Q^*_j)q_j - Fq_j + (F - S)x_j) \cdot \phi(S)dS = 0. \quad (10) \]

First, consider expression (9). Since \( u' > 0 \), it follows that

\[ \int_0^{D(0)} u'(D(q_j + Q^*_j)q_j - Fq_j + (F - S)x_j) \cdot \phi(S)dS > 0. \quad (11) \]

Hence, (9) holds if and only if

\[ D'(q_j + Q^*_j)q_j + D(q_j + Q^*_j) - F = 0 \quad (12) \]

since (12) is not a function of \( S \). From Assumption 2.2, firm \( j \)'s best response function to the output of the other firms has a nonpositive slope larger than \(-1\), i.e., from (12) \( \partial q_j / \partial Q^*_j \in (-1, 0) \). Hence, \( q^*(J) \) is uniquely defined by (12) evaluated at \( q_j = q^*(J) \), which yields (6). Second, consider expression (10). Since \( u'' < 0 \) and given that \( q^*(J) \) exists and is unique, \( x^*(J) \) exists and is unique. Since \( x^*(J) = (1 - \omega^*(J))q^*(J) \), it follows that \( \omega^*(J) \) exists and is uniquely defined by (7).

Using Proposition 3.2, Proposition 3.3 states the separation property at the second stage of the game. That is, the distribution of the spot price and risk preferences have no effect on production and output price. The futures price is the sole driving force for production because, from (6), the marginal revenue of output is equal to the futures price.\(^{22}\) The separation property

\(^{22}\)Note that at stage 2 of the game, the separation property holds unconditionally be-
is consistent with the case of perfect competition either when there is un-
certainty about the output price (Ethier, 1973; Danthine, 1973; Holthausen,
1979; Feder et al., 1980) or the input price (Holthausen, 1979; Katz and
Paroush, 1979; Paroush and Wolf, 1992) as long as there is no other source
of uncertainty (e.g., uncertainty in production or basis risk).

**Proposition 3.3.** From (6), at the second stage of the game, production and
output price are independent of uncertainty and risk aversion.

Having shown that separation occurs when there is no entry, we next
show that, when the firms make a decision on entry, the futures price is
no longer the driving force for the production decision. In fact, there is
always nonseparation because the distribution of the spot price and the utility
function have an effect on the production decision (and, thus, the output
price) through the number of firms entering the industry. We proceed as
follows. We first characterize the number of firms entering the market at the
first stage of the game. We then provide a comparative analysis of the effect
of changes in the distribution of the input spot price as well as changes in
risk preferences on production and output price.

Proposition 3.4 states that there exists a unique free-entry equilibrium
with full access to the futures market as long as the entry cost is not too
high to prevent at least one firm from entering the industry. The entry cost
must also be not too low to ensure a finite number of entering firms.

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cause firms may either hedge or engage in speculation. Assuming CARA preferences, a
linear demand, and a normally distributed spot price, Appendix D shows that, if firms
can only hedge, (i.e., have partial access to the futures market), production and output
price are only conditionally independent of uncertainty and risk preferences. That is, con-
ditional on hedging, production and output price remains independent of uncertainty and
risk preferences. However, the upper bound of the range of futures input prices yielding
hedging is increasing in the mean and variance of the spot input price as well as risk
aversion.
Proposition 3.4. Suppose that
\[
\lim_{J \to \infty} \int_0^{D(0)} u(D(Jq^*(J))q^*(J) - Fq^*(J) + (F - S)(1 - \omega^*(J))q^*(J)) \cdot \phi(S) dS < K
\]
\[
\leq \int_0^{D(0)} u(D(q^*(1))q^*(1) - Fq^*(1) + (F - S)(1 - \omega^*(1))q^*(1)) \cdot \phi(S) dS. \quad (13)
\]

Then, there exists a unique equilibrium with \(1 \leq J^* < \infty\) firms in the industry such that \(J^* = \lfloor t^* \rfloor\) where \(t^*\) is implicitly defined by
\[
\int_0^{D(tq^*(t))} u(D(tq^*(t))q^*(t) - Fq^*(t) + (F - S)(1 - \omega^*(t))q^*(t)) \cdot \phi(S) dS = K \quad (14)
\]
evaluated at \(t = t^*\).

Proof. We need to show that the left-hand side of (14) is strictly decreasing in \(t\).

1. The term \(D(tq^*(t))q^*(t) - Fq^*(t)\) in the left-hand side of (14) is strictly decreasing in \(t\), i.e.,
\[
\frac{\partial (D(tq^*(t))q^*(t) - Fq^*(t))}{\partial t} < 0 \quad (15)
\]
since, from (6), \(\partial q^*(t)/\partial t < 0\) and \(\partial (tq^*(t))/\partial t > 0\).\(^{23}\)

2. Plugging \(x^*(t) \equiv (1 - \omega^*(t))q^*(t)\) into the left-hand side of (14) and applying the envelope theorem, the derivative of the left-hand side of (14)

\^[23\]Given Conditions 2 and 3 of Assumption 2.2 and \(q^*(t) > 0\), differentiating (6) yields
\[
\frac{\partial q^*(t)}{\partial t} = -\frac{D''(tq^*(t))q^*(t) + D'(tq^*(t))}{D''(tq^*(t))q^*(t) + (t + 1)D'(tq^*(t))}q^*(t) < 0. \quad (16)
\]
Hence,
\[
\frac{\partial (tq^*(t))}{\partial t} = \frac{D'(tq^*(t))}{D''(tq^*(t))q^*(t) + (t + 1)D'(tq^*(t))}q^*(t) > 0. \quad (17)
\]
with respect to \( t \) is

\[
\frac{\partial}{\partial t} \int_0^{D(0)} u(D(tq^*(t))q^*(t) - Fq^*(t) + (F - S)x^*(t)) \cdot \phi(S) dS
\]

\[
= \frac{\partial x^*(t)}{\partial t} \int_0^{D(0)} (F - S) \cdot u'(D(tq^*(t))q^*(t) - Fq^*(t) + (F - S)x^*(t)) \cdot \phi(S) dS
\]

\[
+ \int_0^{D(0)} \frac{\partial}{\partial t} (D(tq^*(t))q^*(t) - Fq^*(t))
\]

\[
\times u'(D(tq^*(t))q^*(t) - Fq^*(t) + (F - S)x^*(t)) \cdot \phi(S) dS.
\]

Using (15) and the fact that \( u' > 0 \) implies that (18) is strictly negative.

Since (13) holds, and given (18), it follows that the left-hand side of (14) crosses the \( K \)-line from above only once. Hence, \( t^* \) (as defined by (14)) is unique and so is \( J^* = \lfloor t^* \rfloor \).

Propositions 3.5 and 3.6 state that, when the futures price is not actuari-
ally fair, market concentration depends on uncertainty and risk preferences.\(^{24}\)

In particular, from Proposition 3.5, an increase in the mean of \( \bar{S} \) weakly decreases (weakly increases) the number of firms when the futures market is contango (normal backwardation). Indeed, when the market is contango (normal backwardation), the firms are net buyers (net sellers) on the spot market for the input. Hence, an increase in the mean of the spot price for the input decreases (increases) the expected utility in the second stage, which induces less (more) firms to enter the industry.

A riskier spot input price weakly decreases the number of firms in the industry.\(^{25}\) From Proposition 3.6, an increase in risk aversion also weakly decreases the number of firms in the industry. The similar result comes from \(^{24}\)If \( F = E\bar{S} \), then market concentration is independent of the distribution of the spot price and the utility function.

\(^{25}\)We adopt the expression weakly decrease or weakly increase because \( J^* \) is an integer.
the fact that a mean-preserving increase in risk or an increase in risk aversion both reduce the expected utility in the second stage, which induces less firms to enter the industry.

We begin with the effect of uncertainty on market concentration. To that end, suppose that \( \phi(S) = \psi(S; m, r) \) where an increase in \( m \) implies an increase in the mean of \( \tilde{S} \) whereas an increase in \( r \) implies a mean-preserving increase in the risk of \( \tilde{S} \) in the sense of Rothschild and Stiglitz (1971).

**Proposition 3.5.** Suppose that \( \phi(S) = \psi(S; m, r) \). Then, for \( F \neq \mathbb{E}\tilde{S} \),

1. An increase in the mean of \( \tilde{S} \) weakly decreases (weakly increases) \( J^* \) when \( F > \mathbb{E}\tilde{S} \) (\( F < \mathbb{E}\tilde{S} \)).

2. A mean-preserving increase in the risk of \( \tilde{S} \) weakly decreases \( J^* \).

**Proof.** Since \( J^* = [t^*] \), we use (14) to derive the effect of an increase in \( m \) and \( r \) on \( J^* \). The proof has two steps. First, we establish the sign of \( x^*(t) \). Second, we show the effects of an increase in \( m \) and an increase in \( r \) on the left-hand side of (14), which depends on the sign of \( x^*(t) \).

1. We first sign \( x^*(t) \equiv (1 - \omega^*(t))q^*(t) \). Plugging \( x^*(t) \equiv (1 - \omega^*(t))q^*(t) \) into expression (7) (evaluated at \( J = t \)) yields

\[
\text{cov}[(F - \tilde{S}), u'(\Pi^* + (F - \tilde{S})x^*(t))] + (F - \mathbb{E}\tilde{S})\cdot \mathbb{E}u'(\Pi^* + (F - \tilde{S})x^*(t)) = 0,
\]

(19)

where \( \mathbb{E} \) and \( \text{cov} \) are, respectively, the expectation operator and the covariance operator. Here, \( \Pi^* \equiv D(tq^*(t))q^*(t) - Fq^*(t) \), and \( \mathbb{E}u'(\Pi^* + (F - \tilde{S})x^*(t)) > 0 \).

(a) Suppose first that \( F = \mathbb{E}\tilde{S} \). Then, from (19), it must be that \( x^*(t) = 0 \) so that \( \text{cov}[(F - \tilde{S}), u'(\Pi^* + (F - \tilde{S})x^*(t))] = 0 \).

(b) Suppose next that \( F > \mathbb{E}\tilde{S} \). Then, from (19), it must be that \( x^*(t) > 0 \) so that \( \text{cov}[(F - \tilde{S}), u'(\Pi^* + (F - \tilde{S})x^*(t))] < 0 \).

(c) Suppose that \( F < \mathbb{E}\tilde{S} \). Then, from (19), it must be that \( x^*(t) < 0 \) so that \( \text{cov}[(F - \tilde{S}), u'(\Pi^* + (F - \tilde{S})x^*(t))] > 0 \).
2. Next, plugging \( x^*(t) \equiv (1 - \omega^*(t))q^*(t) \) and \( \phi(S) = \psi(S; m, r) \) into (14) yields

\[
D^{(0)} \int_0^D u(D(tq^*(t))) q^*(t) - Fq^*(t) + (F - S)x^*(t)) \cdot \psi(S; m, r) dS = K \tag{20}
\]
evaluated at \( t = t^* \). Using the sign of \( x^*(t) \), the proof consists in showing that an increase in \( m \) or an increase in \( r \) changes the left-hand side of (20) thereby changing \( t^* \), which in turn changes \( J^* = [t^*] \). To that end, let

\[
\Gamma \equiv D^{(0)} \int_0^D u(D(tq^*(t))) q^*(t) - Fq^*(t) + (F - S)x^*(t)) \cdot \psi(S; m, r) dS \tag{21}
\]
be the left-hand side of (20).

(a) Consider first an increase in \( m \). From (21),\(^{26}\)

\[
\frac{\partial \Gamma}{\partial m} = \frac{\partial q^*(t)}{\partial m} (D'(tq^*(t)) tq^*(t) + D(tq^*(t)) - F) \times \int_0^D u'(D(tq^*(t))) q^*(t) - Fq^*(t) + (F - S)x^*(t)) \cdot \psi(S; m, r) dS
\]

\[+ \frac{\partial x^*(t)}{\partial m} \int_0^D (F - S)u'(D(tq^*(t))) q^*(t) - Fq^*(t) + (F - S)x^*(t)) \cdot \psi(S; m, r) dS
\]

\[+ \int_0^D u(D(tq^*(t))) q^*(t) - Fq^*(t) + (F - S)x^*(t)) \cdot \frac{\partial \psi(S; m, r)}{\partial m} dS \tag{22}
\]

where \( \frac{\partial q^*(t)}{\partial m} = 0 \) due to the separation property stated in Propo-

---

\(^{26}\)Here, the notation \( \frac{\partial \psi(S; m, r)}{\partial m} \) refers to the difference between two p.d.f.'s, i.e., for \( m_1 > m_2 \), \( \psi(S; m_1, r) - \psi(S; m_2, r) \) as used in Laffont (1989).
sition 3.3, and, from (7),
\[
D^{(0)} \int_0^1 (F-S)u'(D(tq^*(t)))q^*(t) - Fq^*(t) + (F-S)x^*(t)) \cdot \psi(S; m, r) dS = 0.
\]

(23)

Hence, (22) simplifies to
\[
\frac{\partial \Gamma}{\partial m} = D^{(0)} \int_0^1 u(D(tq^*(t)))q^*(t) - Fq^*(t) + (F-S)x^*(t)) \cdot \frac{\partial \psi(S; m, r)}{\partial m} dS.
\]

(24)

The sign of (24) is for the moment ambiguous because \(\frac{\partial \psi(S; m, r)}{\partial m}\) may be positive or negative depending on the value for \(S\). Integrating by parts (24) yields
\[
\frac{\partial \Gamma}{\partial m} = u(D(tq^*(t)))q^*(t) - Fq^*(t) + (F-S)x^*(t)) \frac{\partial \psi(S; m, r)}{\partial m} \bigg|_0^{D^{(0)}}
\]
\[
+ x^*(t) \int_0^{D^{(0)}} u'(D(tq^*(t)))q^*(t) - Fq^*(t) + (F-S)x^*(t)) \frac{\partial \psi(S; m, r)}{\partial m} dS.
\]

(25)

where \(\psi(S; m, r)\) is the c.d.f of \(\tilde{S}\) and \(\frac{\partial \psi(S; m, r)}{\partial m}\) refers to the difference between two c.d.f.'s. Since \(\psi(0; m, r) = 0\) and \(\psi(D(0); m, r) = 1\) for all \(m\), the first term in (25) is equal to zero. Moreover, using the definition of first-order stochastic dominance and the fact that an increase in \(m\) induces an increase in the mean of \(\tilde{S}\), it follows that \(\frac{\partial \psi(S; m, r)}{\partial m} < 0\).

i. Suppose first that \(F = \mathbb{E}\tilde{S}\) so that \(x^*(t) = 0\). Then, from (25), \(\partial \Gamma/\partial m = 0\). Hence, from (14), \(t^*\) and thus \(J^* = \lfloor t^* \rfloor\) remain unchanged with a change in \(m\).

ii. Suppose next that \(F > \mathbb{E}\tilde{S}\) \((F < \mathbb{E}\tilde{S})\) so that \(x^*(t) > 0\) \((x^*(t) < 0)\). Then, from (25), \(\partial \Gamma/\partial m < 0\) \((\partial \Gamma/\partial m > 0)\).

Hence, from (14), \(t^*\) and thus \(J^* = \lfloor t^* \rfloor\) are weakly decreasing
(weakly increasing) along with an increase in $m$.

(b) Consider next an increase in $r$. From (21),

$$
\frac{\partial \Gamma}{\partial r} = \frac{\partial q^*(t)}{\partial r} \left( D'(tq^*(t)) + D(tq^*(t)) - F \right)
$$

$$
\times \int_0^{D(0)} u'(D(tq^*(t)))q^*(t) - Fq^*(t) + (F - S)x^*(t)) \cdot \psi(S; m, r) dS
$$

$$
+ \frac{\partial x^*(t)}{\partial r} \int_0^{D(0)} (F - S)u'(D(tq^*(t)))q^*(t) - Fq^*(t) + (F - S)x^*(t)) \cdot \psi(S; m, r) dS
$$

$$
- \int_0^{D(0)} u(D(tq^*(t)))q^*(t) - Fq^*(t) + (F - S)x^*(t)) \cdot \frac{\partial \psi(S; m, r)}{\partial r} dS
$$

(26)

where $\frac{\partial q^*(t)}{\partial r} = 0$ due to the separation property stated in Proposition 3.3, and, from (7),

$$
\int_0^{D(0)} (F - S)u'(D(tq^*(t)))q^*(t) - Fq^*(t) + (F - S)x^*(t)) \cdot \psi(S; m, r) dS = 0.
$$

(27)

Hence, (26) simplifies to

$$
\frac{\partial \Gamma}{\partial r} = \int_0^{D(0)} u(D(tq^*(t)))q^*(t) - Fq^*(t) + (F - S)x^*(t)) \cdot \frac{\partial \psi(S; m, r)}{\partial r} dS.
$$

(28)

The sign of (28) is for the moment ambiguous because $\frac{\partial \psi(S; m, r)}{\partial m}$ may be positive or negative depending on the value for $S$. Inte-
grating by parts (28) yields

\[
\frac{\partial \Gamma}{\partial r} = u(D(tq^*(t)))q^*(t) - Fq^*(t) + (F - S)x^*(t) \frac{\partial \Psi(S; m, r)}{\partial r}\bigg|_0^{D(0)}
\]

\[
+ x^*(t) \int_0^{D(0)} u'(D(tq^*(t)))q^*(t) - Fq^*(t) + (F - S)x^*(t) \frac{\partial \Psi(S; m, r)}{\partial r} dS.
\]

(29)

As in (25), the first term in (29) is equal to zero. We cannot sign directly (29) because, by the definition of a mean-preserving spread, \(\frac{\partial \Psi(S; m, r)}{\partial r}\) may be positive or negative depending on the value of \(S\). Integrating by parts (29) a second time yields

\[
\frac{\partial \Gamma}{\partial r} = x^*(t) \cdot u'(D(tq^*(t)))q^*(t) - Fq^*(t) + (F - S)x^*(t) \left( \int_0^S \frac{\partial \Psi(y; m, r)}{\partial r} dy \right) \bigg|_0^{D(0)}
\]

\[
+ (x^*(t))^2 \cdot \int_0^{D(0)} u''(D(tq^*(t)))q^*(t) - Fq^*(t) + (F - S)x^*(t))
\]

\[
\times \left( \int_0^S \frac{\partial \Psi(y; m, r)}{\partial r} dy \right) dS.
\]

(30)

Since \(\Psi(0; m, r) = 0\) and \(\Psi(D(0); m, r) = 1\) for all \(r\), the first term in (30) is equal to zero. Moreover, using the definition of a mean-preserving spread and the fact that an increase in \(r\) induces a mean-preserving increase in the risk of \(\tilde{S}\), it follows that \(\frac{\partial \Psi(S; m, r)}{\partial r} > 0\) for all \(S < D(0)\).\(^{27}\) From (30), \(\partial \Gamma/\partial r < 0\) since \(x^*(t) \neq 0\) (from \(F \neq E\tilde{S}\)) and \(u'' < 0\). Hence, from (14), \(t^*\) and thus \(J^* = \lfloor t^* \rfloor\) are weakly decreasing along with an increase in \(r\).

\[\square\]

Proposition 3.6 states the effect of increasing risk aversion on the number

\(^{27}\)We recall the integral definition of a mean-preserving spread. Suppose that for \(x \in [a, b]\), \(G(x)\) is a mean-preserving spread of \(H(x)\). Then, \(\int_a^b (G(x) - H(x)) dx = 0\) to preserve the same mean between the two distributions and, for all \(z \in [a, b]\), \(\int_a^z (G(x) - H(x)) dx > 0\) so that \(G(x)\) has more weight in the tails than \(H(x)\).
of firms in the industry. Using the notation in Diamond and Stiglitz (1974),
suppose that \( u(\pi) = v(\pi; \rho) \) with \( v_1 > 0, v_{11} < 0 \) and \( \partial (-v_{11}(\pi; \rho)/v_1(\pi; \rho)) / \partial \rho > 0 \). Hence, an increase in \( \rho \) implies an increase in risk aversion.

**Proposition 3.6.** Suppose that \( u(\pi) = v(\pi; \rho) \) such that \( v_1 > 0, v_{11} < 0 \) and \( \partial (-v_{11}(\pi; \rho)/v_1(\pi; \rho)) / \partial \rho > 0 \). Then, for \( F \neq \mu_S \), an increase in risk aversion weakly decreases \( J^* \).

**Proof.** Plugging \( x^*(t) \equiv (1 - \omega^*(t))q^*(t) \) and \( u(\pi) = v(\pi; \rho) \) into (14) yields

\[
\int_0^{D(0)} v(D(tq^*(t)))q^*(t) - Fq^*(t) + (F - S)x^*(t); \rho) \cdot \phi(S)dS = K \tag{31}
\]
evaluated at \( t = t^* \). Let

\[
\Gamma \equiv \int_0^{D(0)} v(D(tq^*(t)))q^*(t) - Fq^*(t) + (F - S)x^*(t); \rho) \cdot \phi(S)dS \tag{32}
\]
be the left-hand side of (31). From (32),

\[
\frac{\partial \Gamma}{\partial \rho} = \frac{\partial q^*(t)}{\partial \rho} (D'(tq^*(t))tq^*(t) + D(tq^*(t)) - F) \times \int_0^{D(0)} v_1(D(tq^*(t)))q^*(t) - Fq^*(t) + (F - S)x^*(t); \rho) \cdot \phi(S)dS
\]

\[
+ \frac{\partial x^*(t)}{\partial \rho} \int_0^{D(0)} (F - S) \cdot v_1(D(tq^*(t)))q^*(t) - Fq^*(t) + (F - S)x^*(t); \rho) \cdot \phi(S)dS
\]

\[
+ \int_0^{D(0)} \frac{\partial v(D(tq^*(t)))q^*(t) - Fq^*(t) + (F - S)x^*(t); \rho)}{\partial \rho} \cdot \phi(S)dS \tag{33}
\]
where \( \frac{\partial q^*(t)}{\partial \rho} = 0 \) due to the separation property stated in Proposition 3.3, and, from (7),

\[
\int_0^{D(0)} (F - S) \cdot v_1(D(tq^*(t)))q^*(t) - Fq^*(t) + (F - S)x^*(t); \rho) \cdot \phi(S)dS = 0. \tag{34}
\]
Hence, (22) simplifies to

\[ \frac{\partial \Gamma}{\partial \rho} = \int \frac{\partial v(D(tq^*(t)))q^*(t) - Fq^*(t) + (F - S)x^*(t); \rho)}{\partial \rho} \cdot \phi(S)dS. \]  

(35)

Since an increase in \( \rho \) means an increase in risk aversion, it follows that for \( F \neq \mathbb{E}\tilde{S} \), (35) is negative for all \( t \). Hence, from (14), \( t^* \) and thus \( J^* = [t] \) are weakly decreasing in \( \rho \) because a more risk-averse firm requires a higher risk premium to remain in the market.

Proposition 3.7 states that as long as the futures price is not actuarially fair, the separation property does not hold when entry is considered. The negative effect of mean (in a contango situation), riskiness, or risk aversion on the number of firms implies that the remaining firms can exercise more market power. Specifically, when there is access to the futures market, higher riskiness induces each remaining firm to produce more. However, while per-firm production increases along with more riskiness, the number of firms decreases, which is the dominant effect, and the equilibrium output price unambiguously increases along with an increase in the riskiness of the spot input price. The result also holds for an increase in the mean of the spot price in a contango situation or an increase in risk aversion.

**Proposition 3.7.** Suppose that the futures price is not actuarially fair, i.e., \( F \neq \mathbb{E}\tilde{S} \). Then, when entry is endogenized, production and output price depend on uncertainty and risk aversion. In particular,

1. An increase in the mean of \( \tilde{S} \) weakly increases (weakly decreases) \( q^*(J^*) \) and \( p^*(J^*) \) when \( F > \mathbb{E}\tilde{S} \) (\( F > \mathbb{E}\tilde{S} \)).

\(^{28}\)Since \( x^*(t) \neq 0 \) when \( F \neq \mathbb{E}\tilde{S} \), it follows that \( v \) is strictly concave in \( S \). Hence, for any \( \rho_1, \rho_2 : \rho_2 > \rho_1 \) and as long as \( x^*(t) \neq 0 \),

\[
\int v(D(tq^*(t)))q^*(t) - Fq^*(t) + (F - S)x^*(t); \rho_2) \cdot \phi(S)dS
\]

\[
< \int v(D(tq^*(t)))q^*(t) - Fq^*(t) + (F - S)x^*(t); \rho_1) \cdot \phi(S)dS. \quad (36)
\]
2. A mean-preserving increase in the risk of $\tilde{S}$ or an increase in risk aversion weakly increases $q^*(J^*)$ and $p^*(J^*)$.

Proof. From (16) and (17), $q^*(J^*)$ and $p^*(J^*) = D(J^* q^*(J^*))$ are both decreasing in $J^*$. Using Proposition 3.5 and 3.6 yields the results stated in Proposition 3.7.

The result stated in Proposition 3.7 is in sharp contrast to the separation result obtained in the literature in the absence of another source of uncertainty (e.g., uncertainty in production, basis risk). In other words, once firms are allowed to make entry decisions, the futures price is no longer the driving force for the production decision (even with one source of uncertainty). Indeed, conditional on the number of firms, each firm is able to fully adjust production in such a way that it is independent of uncertainty and risk preferences. When firms also make entry decisions, production decisions becomes less flexible. Hence, the endogenization of the number of firms in an industry with sunk cost $K > 0$ yields nonseparation.\(^{29}\)

4 The Effect of Access to Futures Market

In this section, we study the effect of access to the futures market first on entry, then on production and output price. To simplify the discussion, we make the following restrictions. Managers’ risk preferences on profit exhibit constant absolute risk aversion. Output demand is linear and the firms’ beliefs about the spot input price are normally distributed. These restrictions are consistent with Assumptions 2.1, 2.2, and 2.3 except for the fact that the support of the input spot price is the real line. Although the input spot price can be negative, the values of the parameters of the model can be restricted to ensure that the probability of such events be arbitrarily close to zero. Moreover, it turns out that, by assuming a positive mean of the spot input

\(^{29}\)If entry were not costly, the number of firms would be infinity in our case. In the limit, total production and output price would be independent of the distribution of the spot price and risk aversion.
price, equilibrium values for the number of firms, the production, and the output price are always positive.

Formally, our restrictions are as follows. The coefficient of absolute risk aversion is \(\alpha > 0\). Inverse demand is linear, i.e.,

\[
D \left( \sum_{k=1}^{J} q_k \right) = \theta - \gamma \sum_{k=1}^{J} q_k,
\]

where \(\theta, \gamma > 0\) are demand parameters. The input spot price is normally distributed, i.e., \(\tilde{S} \sim N(\mu_S, \sigma_S^2)\), \(\mu_S \in (0, \theta)\). Given our restrictions, the certainty equivalent has a closed-form solution. Using (2), the certainty equivalent of firm \(j\) is

\[
CE \left( J, q_j, \omega_j, \sum_{k \neq j}^{J} q_k \right) = D \left( q_j + \sum_{k \neq j}^{J} q_k \right) q_j - \mu_S (1 - \omega_j) q_j \\
- F \omega_j q_j - \alpha \sigma_S^2 (1 - \omega_j)^2 q_j^2 / 2
\]

as shown in Appendix B.

We also make two simplifications. First, we ignore the fact that \(J^* = \lfloor t^* \rfloor\) is an integer. Specifically, \(\partial J^*/\partial \varphi\) for any parameter \(\varphi\) refers to the derivative of \(t^*\) with respect \(\varphi\). This approximation has no bearing on our results since the number of firms is not bounded between two integers. Hence, changes in \(t^*\) are informative about changes in \(J^*\).\(^{31}\) Second, when determining the number of firms at the first stage of the game, we compare the certainty equivalent (instead of the utility of the certainty equivalent) with the cost of entry. This has no bearing on the qualitative nature of our results.

We first characterize the equilibrium with and without access to the futures market. We then compare the equilibrium values under full access and under no access to the futures market, first on entry, and then on production and output price.

\(^{30}\)In other words, the utility function for profit \(\pi\) is exponential: \(u(\pi) = -e^{-\alpha \pi}\).

\(^{31}\)Moreover, since the equilibrium is symmetric, the summation operator is not present in the equilibrium values.
4.1 Equilibrium Characterization

Access to Financial Market. Proposition 4.1 states the unique free-entry equilibrium with full access to the futures market.

**Proposition 4.1.** For \( F \in (0, \theta) \), there exists a unique equilibrium with \( 1 \leq J^* < \infty \) firms in the industry if and only if

\[
\frac{(F - \mu_S)^2}{2\alpha \sigma_S^2} < K \leq \frac{(\theta - F)^2}{4\gamma} + \frac{(F - \mu_S)^2}{2\alpha \sigma_S^2}. \tag{39}
\]

In equilibrium,

\[
J^* = \frac{\theta - F}{\sqrt{(K - \frac{(F - \mu_S)^2}{2\alpha \sigma_S^2}) \gamma}} - 1 \tag{40}
\]

firms enter the industry. Each firm produces

\[
q^*(J^*) = \sqrt{\left(K - \frac{(F - \mu_S)^2}{2\alpha \sigma_S^2}\right) / \gamma} \tag{41}
\]

at output price

\[
p^*(J^*) = \sqrt{\left(K - \frac{(F - \mu_S)^2}{2\alpha \sigma_S^2}\right) \gamma} + F. \tag{42}
\]

Hedge coverage is

\[
\omega^*(J^*) = 1 - \frac{\sqrt{(F - \mu_S)}}{\alpha \sigma_S \sqrt{K - \frac{(F - \mu_S)^2}{2\alpha \sigma_S^2}}} \tag{43}
\]

**Proof.** See Appendix C. \( \square \)

We now discuss several properties of the equilibrium. From condition (39) in Proposition 4.1, there exists an equilibrium with full access to the futures market as long as the entry cost is not too high to prevent at least one firm from entering the industry. The entry cost must also be not too low to ensure a finite number of entering firms.

Condition (39) is depicted in Figure 2, where \( F \in (0, \theta) \) is on the \( x\)-
axis, and $K > 0$ is on the $y-$axis.$^{32}$ The two convex lines depict the lower and upper bounds in (39). Hence, the darker shaded area between the two curves encompasses the points $\{K, F\}$ for which the equilibrium exists, and, in particular, a finite number of firms enter the industry. Note that entry may occur for all values of $F$, whether the futures market is normal backwardation ($F \in (0, \mu_S)$), actuarially fair ($F = \mu_S$), or contango ($F > \mu_S$). Note as well that, while the upper and lower bounds of (13) depends on the mean and variance of the spot price (and risk aversion), the darker shaded area between the two curves,

$$\int_0^\theta \left(\frac{(\theta - x)^2}{4\gamma} + \frac{(x - \mu_S)^2}{2\alpha \sigma_S^2} - \frac{(x - \mu_S)^2}{2\alpha \sigma_S^2}\right) \, dx = \frac{\theta^3}{12\gamma}$$

$^{32}$To generate Figure 2, we set $\{\theta, \gamma\} = \{7, 1\}$, and $\{\mu_S, \sigma_S^2, \alpha\} = \{2, 1, 1\}$. Although Figure 2 is generated with specific values, the shapes of the curves hold in general. The same comment applies to all figures.
is unaffected by changes in the mean and variance of the spot price as well as risk aversion. In other words an increase in any of these three parameters does not reduce the possibility of entry. Below the lowest convex curve, there is no equilibrium with a finite number of firms. In other words, all potential entrants have an incentive to enter. Because unlimited entry (with \( K > 0 \)) is due to speculation motives, we delay our discussion about the limiting case (i.e., \( J^* \to \infty \)).

Having discussed the condition for entry, we provide information about the types of financial activities in which the firms engage in equilibrium. Proposition 4.2 states that, whenever the equilibrium exists, the firms may hedge or speculate (or both) depending on the structure of the futures market and the value of the sunk cost.

**Proposition 4.2.** Suppose that (13) holds. Then, optimal hedging is

\[
\omega^*(J^*) = 1 - \frac{\sqrt{\gamma}(F - \mu_S)}{\alpha \sigma^2 S \sqrt{K - \frac{(F - \mu_S)^2}{2 \alpha \sigma^2 S}}}
\]  

(45)

Further,

1. For \( F \in (0, \mu_S) \), the firms fully hedge production and, at the same time, speculate by buying in the futures market to sell in the spot market, i.e., \( \omega^*(J^*) > 1 \).

2. For \( F = \mu_S \), the firms fully hedge production, i.e., \( \omega^*(J^*) = 1 \).

3. For \( F \in (\mu_S, \theta) \), there are three exclusive outcomes.

   (a) The firms partially hedge, i.e., \( \omega^*(J^*) \in (0, 1) \).

   (b) The firms do not access the futures markets, i.e., \( \omega^*(J^*) = 0 \).

   (c) The firms speculate by buying in the spot market to sell in the futures market, i.e., \( \omega^*(J^*) < 0 \).

**Proof.** See Appendix C.  

Proof
Figure 3: Financial Activity, Full Access to the Futures Market

Figure 3 illustrates Proposition 4.2 by providing information about the firms’ financial activity when there is a finite and positive number of firms in the industry. If the futures market is in normal backwardation (i.e., $F < \mu_S$), then the firms fully hedge and speculate. That is, the input is purchased only on the futures market, some of which is used for production and the remaining is sold on the spot market. Whenever the futures price is actuarially fair (i.e., $F = \mu_S$), the firms fully hedge. See the dashed vertical line in Figure 3 for which $\omega^*(J^*) = 1$.

A contango futures market (i.e., $F > \mu_S$) yields either partial hedging (with no speculation) or speculation (with no hedging) depending on the value of the sunk cost and the futures price. The division between these two outcomes is depicted by the dashed increasing convex line $K = \frac{(2\gamma + \alpha \sigma_S^2)(F - \mu_S)^2}{2\alpha^2 \sigma_S^4}$, intersecting with the minimum of the upper bound for $K$.

\footnote{Figures 2 and 3 are generated using the same parameter values.}
in (13), i.e., when \( F = \overline{F}_1 \equiv \frac{2v\mu_S + \alpha \sigma_S^2 \theta}{2 \gamma + \alpha \sigma_S^2} \). From Figure 3, in a contango situation, hedging is possible only for lower values of the futures input price, while speculation (buying from the spot market to sell on the futures market) can occur at any futures input price as long as the sunk cost is low enough.

**Remark 4.3.** For \( F \in [\overline{F}_1, \theta] \), hedging is no longer chosen regardless of the value of the sunk cost.

The entry cost influences the type of financial activity. In Figure 3, consider a point \( \{K, F\} \) in the area for partial hedging (i.e., \( \omega^*(J^*) \in (0, 1) \)). A decrease in the sunk cost while keeping the futures input price constant eventually leads to a switch from hedging to speculation. This is due to the fact that a lower \( K \) yields more entry, which reduces profit from selling.

\(^{34}\)The points \( \{K, F\} \) on the dashed increasing line that intersects the upper bound of (13) at its minimum refer to cases for which the firms do not access the futures market, i.e., \( \omega^*(J^*) = 0 \).
the output, and, thus, raises the opportunity cost of hedging (instead of speculating) under contango.

**Remark 4.4.** For \( F \in (\mu_S, \bar{F}_1] \), a lower sunk cost can induce the firms not to hedge, but to engage in speculation instead.

Finally, hedging becomes more likely under contango along with an increase in the variance of the spot input price or risk aversion. This is illustrated in Figure 4, which shows that an increase in the variance of the spot input price moves \( \bar{F}_1 \equiv \frac{2\gamma \mu_S + a \sigma_S^2 \theta}{2\gamma + a \sigma_S^2} \) to the right, which increases the darker shaded area (partial hedging) and reduces the lighter shaded area (speculation).

**Remark 4.5.** For \( F \in [\mu_S, \theta) \), an increase (decrease) in \( \sigma_S^2 \) or \( \alpha \) makes it more likely for hedging (speculation) to occur.

It remains to discuss the limiting case below the lowest convex curve in Figure 2. Specifically, the lighter shaded area in Figure 2 combines the points \( \{K, F\} \) for which entry is always beneficial regardless of the number of firms active in the market. In other words, the stage-2 certainty equivalent is high enough to cover the sunk cost for any number of firms, which yields the case of perfect competition. Due to unlimited entry, the profit from the output sector approaches zero (i.e., the perfect competition outcome drives the output price to the marginal cost), while the firms engage in speculation to generate revenue from the financial sector. Consistent with Figure 2, this is only possible when the futures price is not actuarially fair. From Figure 2, there are two outcomes under the limiting case of perfect competition (i.e., in the lighter shaded area). The firms speculate by selling futures contracts under contango (i.e., \( F > \mu_S \)), while buying them under normal backwardation (i.e., \( F < \mu_S \)). Although \( K > 0 \), speculation on the futures market makes it possible for the output market to approach perfect competition in the limit.

**Proposition 4.6.** For \( F \in (0, \theta) \), \( F \neq \mu_S \), and \( 0 < K \leq \frac{(F - \mu_S)^2}{2a\sigma_S^2} \), \( J^* \to \infty \) yielding the perfectly competitive outcome in the output sector. Further, firms always engage in speculation in the futures market.

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35To generate Figure 4, we set \( \{\theta, \gamma\} = \{10, 1\} \) and \( \{\mu_S, \alpha\} = \{5, 1\} \).
Proof. Suppose that $F \in (0, \theta)$, $F \neq \mu_S$ and $0 < K \leq \frac{(F-\mu_S)^2}{2\alpha\sigma_S^2}$. From (75) in Appendix C, $CE^*(J) = \frac{(\theta-F)^2}{(1+J)^2} + \frac{(F-\mu_S)^2}{2\alpha\sigma_S^2} > K$ for any $J$. Hence, $J^* \rightarrow \infty$. From (70) and (71) in Appendix C, $\lim_{J^* \rightarrow \infty} x^*(J^*) = \frac{F-\mu_S}{\alpha\sigma_S^2}$ and $\lim_{J^* \rightarrow \infty} y^*(J^*) = -\frac{F-\mu_S}{\alpha\sigma_S^2}$, while, from (72) and (74) in Appendix C, $\lim_{J^* \rightarrow \infty} q^*(J^*) = 0$, and $\lim_{J^* \rightarrow \infty} p^*(J^*) = F$.\[\square\]

No Access to Financial Market. Next, we turn to the characterization and discussion of the benchmark equilibrium when the firms have no access to the futures market. Proposition 4.7 characterizes the unique equilibrium. To clarify the analysis, the hat sign is used on equilibrium values when there is no access to the futures market.

Proposition 4.7. Suppose that no firm has access to the futures market, i.e., the constraint $\omega_j = 0$ holds for all $j$. Then, there exists a unique equilibrium with $1 \leq \hat{J}^* < \infty$ if and only if

$$0 < K \leq \frac{(\theta - \mu_S)^2}{2(2\gamma + \alpha\sigma_S^2\gamma)}. \quad (46)$$

In equilibrium,

$$\hat{J}^* = \frac{(\theta - \mu_S)\sqrt{2\gamma + \alpha\sigma_S^2\gamma} - \alpha\sigma_S^2}{\gamma\sqrt{2K}} - 1 \quad (47)$$

firms enter the industry. Each firm produces

$$\hat{q}^*(\hat{J}^*) = \frac{\sqrt{2K}}{\sqrt{2\gamma + \alpha\sigma_S^2}} \quad (48)$$

at output price

$$\hat{p}^*(\hat{J}^*) = \mu_S + \frac{\sqrt{2K(\gamma + \alpha\sigma_S^2)}}{\sqrt{2\gamma + \alpha\sigma_S^2}}. \quad (49)$$

Proof. See Appendix C.\[\square\]

Two comments about Proposition 4.7 are warranted. First, there exists an equilibrium as long as the entry cost is not too high to prevent at least

\[\text{Recall that } q^*(J^*) = x^*(J^*) + y^*(J^*) \text{ where } x^*(J^*) \text{ is the amount of input purchased (or sold) in the spot market and } y^*(J^*) \text{ is the amount of input purchased (or sold) in the futures market.}\]

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one firm from entering the industry.\textsuperscript{37} Condition (46) is depicted in Figure 5, where $F \in (0, \theta)$ is on the $x$-axis, and $K > 0$ is on the $y$-axis. Given that the firms do not access the futures market, the condition is independent of $F$ and the firms enter as long as $K \leq \hat{K} \equiv \left(\frac{(\theta - \mu_S)^2}{2(2\gamma + \alpha \sigma_S^2)}\right)^{\frac{1}{2}}$.\textsuperscript{38}

Second, access to the futures market alters the comparative analysis on the effect of uncertainty and risk preferences stated in Propositions 3.5 and 3.6. Indeed, recall from Propositions 3.5 and 3.6 that a riskier input price or an increase in risk aversion yields more production under access to the futures market. However, without financial access, an increase in risk or risk

\textsuperscript{37}An equilibrium with a finite number of firms exists as long as the sunk cost is strictly greater than zero, otherwise an infinite number of potential firms would enter the industry.

\textsuperscript{38}Unlike the case of full access to the futures market, an increase in the mean or variance of the spot input price, or an increase in risk aversion reduces the possibility of entry. See (44) for the case of full access to the futures market. Indeed, from (46), $\partial \hat{K} / \partial \mu_S < 0$, $\partial \hat{K} / \partial \sigma_S^2 < 0$, $\partial \hat{K} / \partial \alpha < 0$. 

Figure 5: Entry, No Access to the Futures Market
aversion induces the firms to produce less. That is, using (48),
\[ \frac{\partial q^*(j^*)}{\partial \sigma_S^2} < 0, \]
\[ \frac{\partial q^*(j^*)}{\partial \alpha} < 0. \]
In other words, financial access reverses the effect of riskiness and risk aversion on per-firm production.

4.2 Entry

Using Section 4.1, we can now study the effect of access to the futures market on entry. Combining the information of Figures 2 and 5 into Figure 6 shows that (anticipated) access to the futures input market facilitates entry. In particular, for futures prices \( F \in (\mu_S, F_1] \), \( F_1 \equiv \frac{2\gamma \mu_S + \alpha \sigma_S^2 \theta}{2\gamma + \alpha \sigma_S^2} \), partial hedging (without speculation) allows firms to enter for a sunk cost above \( \hat{K} \), which would had been otherwise impossible without access to the futures input market. See area A in Figure 6 such that \( F \in (\mu_S, F_1) \). Moreover, for futures prices \( F \in (0, \mu_S) \) or \( F \in (F_1, \theta) \), speculation induces firms to enter for a sunk cost above \( \hat{K} \). See area A such that \( F \in (0, \mu_S) \) and area B in Figure 6.

**Remark 4.8.** Access to the futures market allows firms to bear a higher sunk cost, i.e., entry of at least one firm is possible for \( K > \hat{K} \). In particular, access to a futures market under partial hedging (and no speculation) can generate higher expected profits, which compensates for a higher fixed cost of entry.\(^{40}\)

While the industry can bear a higher sunk cost, the effect of access to the futures input market on the number of firms is ambiguous. That is, (40) and (47) cannot in general be ordered. To see this, we begin by comparing the number of firms under an actuarially fair futures input price with the number of firms when there is no access to the futures market. Proposition 4.9 states that the number of firms is greater with an actuarially fair futures input price as long as the sunk cost is high enough.

\(^{39}\)The result without financial access is consistent with classical results obtained in a static environment (i.e., without entry decision) for perfect competition (Sandmo, 1971; Batra and Ullah, 1974) and quantity-setting monopoly (Leland, 1972).

\(^{40}\)This situation arises in area A in Figure 6 such that \( F \in (\mu_S, F_1) \).
Proposition 4.9. Suppose that $0 < K < \hat{K} \equiv \frac{(\theta - \mu_S)^2}{2(2\gamma + \alpha\sigma_S^2)}$. Then, $J^*|_{F=\mu_S} > \hat{J}^*$ if and only if

$$\frac{(\theta - \mu_S)^2}{2(\sqrt{2\gamma} + \alpha\sigma_S^2 + \sqrt{2\gamma})^2} < K \leq \frac{(\theta - \mu_S)^2}{2(2\gamma + \alpha\sigma_S^2)}.$$  

(50)

Proof. From (40) and (47), $J^*|_{F=\mu_S} > \hat{J}^*$ if and only if $K > \frac{(\sqrt{2\gamma} + \alpha\sigma_S^2 - \sqrt{2\gamma})^2(\theta - \mu_S)^2}{2\alpha^2\sigma_S^4}$, which is the same as the lower bound in (50). The inequality $\frac{(\theta - \mu_S)^2}{2(\sqrt{2\gamma} + \alpha\sigma_S^2 + \sqrt{2\gamma})^2} < \frac{(\theta - \mu_S)^2}{2(2\gamma + \alpha\sigma_S^2)}$ always holds.

In order to understand why $J^*$ and $\hat{J}^*$ cannot be ordered in general, we need to show how the ordering of (40) and (47) depends on the value of the futures input price. To that end, we first illustrate the pattern graphically. We then study in details the effect of $F$ on $J^*$ and show that due to the

Figure 6: Entry, Full Access vs. No Access to the Futures Market
convex shape of $J^*$ as a function of $F$, $J^*$ can either below or above $\hat{J}^*$.

Consider Figure 7, where $F \in [\mu_S, \theta)$ is on the $x$-axis, while $J^* > 0$ is on the $y$-axis. The convex solid line plots $J^*$ as a function of $F$, which is the general shape of (40). The straight dash-dot line is the number of firms under no access to the futures market. From (47), $\hat{J}^*$ is independent of $F$. When the convex curve intersects the straight line from below at $F = F''$ in Figure 7a, the firms switch from hedging to speculation, i.e., from $\omega^*(J^*) \in (0, 1)$ to $\omega^*(J^*) < 0$.\textsuperscript{41}

Consider first the case in which $J^*|_{F=\mu_S} > \hat{J}^*$ as depicted in Figure 7a. Here, the sunk cost is high in the sense that $K \in (\overline{K}, \hat{K})$, \(\overline{K} \equiv \frac{(\theta - \mu_S)^2}{2(\sqrt{2} \gamma + \alpha \sigma^2_S + \sqrt{2} \gamma)^2}\), as in (50). Note that, as long as the futures input price is close enough to $\mu_S$, hedging yields more firms in the industry. Consider next the case in which $J^*|_{F=\mu_S} < \hat{J}^*$ as depicted in Figure 7b. Here, the sunk cost is low, i.e., $K \in (0, \overline{K})$. Regardless of the futures input price, hedging always yields fewer firms in the industry. While access to the futures market may increase or decrease the number of firms when partial hedging occurs,\textsuperscript{42} it is clear from Figures 7a and 7b that speculation in a contango situation always yields more

\textsuperscript{41}Hence, $F''$ is the largest value of the futures price such that (40) and (47) are equal and $\partial J^*/\partial F > 0$. From (45), $\omega^*(J^*)|_{F=F''} = 0$.

\textsuperscript{42}Recall that $\omega^*(J^*) \in (0, 1)$ when $F \in (\mu_S, F'')$. 

Figure 7: Number of Firms

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firms.\textsuperscript{43}

Having shown graphically that the ordering of \( J^* \) and \( \hat{J}^* \) depend on \( F \), we now provide the derivative of \( J^* \) with respect to \( F \) in Proposition 4.10. Consistent with Figure 7 \( J^* \) first decreases, then increases. We then explain why an increase in \( F \) can lead to a higher number of firms in the industry even when the firms partially hedge (for \( F \in (\mu_S, F''') \)).

**Proposition 4.10.** Suppose that firms have access to the futures market. Then,

1. For \( F \in (0, \mu_S) \), \( \frac{\partial J^*}{\partial F} < 0 \).
2. For \( F \in [\mu_S, \theta) \), \( \frac{\partial J^*}{\partial F} > 0 \) if and only if \( F > \mu_S + \frac{2\alpha\sigma_S^2}{\theta - \mu_S} \).

\textbf{Proof.} Differentiating (40) yields

\[
\frac{\partial J^*}{\partial F} = -\sqrt{\left( K - \frac{(F - \mu_S)^2}{2\alpha\sigma_S^2} \right) \gamma + \frac{(F - \mu_S)(\theta - F)}{2\alpha\sigma_S^2} \left( K - \frac{(F - \mu_S)^2}{2\alpha\sigma_S^2} \right)^{\frac{3}{2}} \sqrt{\gamma}}, \tag{51}
\]

which yields the cases stated in Proposition 4.10. \hfill \Box

Before proceeding with a detailed explanation of this result, note that the positive relationship between the futures price and the number of firms entering the industry may occur not only when firms speculate, but also when firms partially hedge in a contango futures market. See conditions (78) and (80) in Appendix C.\textsuperscript{44}

\textsuperscript{43}Recall that \( \omega^*(J^*) < 0 \) when \( F \in (F''', \theta) \).

\textsuperscript{44}To obtain \( \frac{\partial J^*}{\partial F} > 0 \) when the firms hedge, the following must hold

\[
\frac{(F - \mu_S)(\theta - \mu_S)}{2\alpha\sigma_S^2} > \frac{(2\gamma + \alpha\sigma_S^2)(F - \mu_S)^2}{2\alpha^2\sigma_S^4}. \tag{52}
\]

Rearranging (52) yields

\[
F < \frac{2\gamma\mu_S + \alpha\sigma_S^2\theta}{2\gamma + \alpha\sigma_S^2} \equiv F_1, \tag{53}
\]

which, from Remark 4.3, is a necessary condition on the value of the futures price for hedging to occur. See also Figure 3.
We now provide an explanation for the positive relationship between the futures price and the number of firms entering the industry. Due to strategic interactions, an increase in \( F \) might increase payoffs for given \( J \), which enables more firms to cover the sunk cost, and, thus, enter the industry.\(^{45}\) To show this, we study the effect of \( F \) on the equilibrium certainty equivalent for a given number of firms in the industry \( CE^*(J) \equiv CE(J, q^*(J), \omega^*(J), (J - 1)q^*(J)) \). Indeed, if \( F \) increases \( CE^*(J) \), then \( J^* \) implicitly defined by \( CE^*(J^*) = K \) increases as well. From (75) in Appendix C, the certainty equivalent (with financial access) is

\[
CE^*(J) = \frac{(\theta - F)^2}{(1 + J)^2 \gamma} + \frac{(F - \mu_S)^2}{2\alpha \sigma_S^2}. \tag{54}
\]

Proposition 4.11 states that the firms might not necessarily benefit from a lower futures input price due to a more competitive futures market.\(^{46}\) In other words, the firms’ payoffs are not necessarily decreasing in the futures input price. In fact, \( CE^*(J) \) is convex in \( F \), so that a lower futures input price may lead to a lower certainty equivalent. This effect occurs sometimes when firms partially hedge, and always when firms speculate. Further, it can only occur in a contango situation. In other words, \( CE^*(J) \) is decreasing in \( F \) under normal backwardation and actuarially fair pricing.

**Proposition 4.11.** Suppose that firms have access to the futures market. Then, \( CE^*(J) \) is strictly increasing in \( F \) if and only if

\[
\frac{(1 + J)^2 \gamma \mu_S + 2\alpha \sigma_S^2 \theta}{(1 + J)^2 \gamma + 2\alpha \sigma_S^2} < F < \theta, \tag{55}
\]

**Proof.** Differentiating (54) with respect to \( F \) yields (55).

\[\square\]

The positive relationship between payoff and \( F \) when the firms partially hedge is due to the fact that an increase in the cost of hedging induces firms to decrease output, which can mitigate the effect of increasing output due to

\(^{45}\)Here, the term payoff refers to the equilibrium certainty equivalent.

\(^{46}\)A more competitive futures market might arise in the presence of risk-neutral speculators.
the strategic interaction of the firms.\footnote{The firms partially hedge when $F$ is such that $(1+J)^2\gamma\mu_S + 2\alpha^2\theta^2 < F < (1+J)^2\gamma + 2\alpha^2\theta$.} Specifically, the effect of an increase in the futures input price on the firms’ payoffs is two-fold. First, an increase in $F$ directly decreases the payoffs. Second, there is an indirect effect through the behavior of the firms, i.e., an increase in $F$ induces firms to decrease production. This, in turn, mitigates the externality that the firms have on one another, which may increase their payoffs. Both effects pull in opposite directions and the overall effect is ambiguous. See Appendix F for a formal exposition.

Figure 8 depicts the effect of the futures input price on the equilibrium certainty equivalent resulting from the strategic interaction of the firms in a non-cooperative game. Specifically, Figures 8a and 8b depict the equilibrium certainty equivalent of a firm with contango for an industry with $J = 3$ firms and $J = 4$ firms, respectively.\footnote{The values of the remaining parameters of the model are $\theta = 10$, $\gamma = \mu_S = \sigma_S^2 = \alpha = 1$.} For low futures input prices, the firm hedges. For prices greater than $\overline{F}_J \equiv \frac{(1+J)^2\gamma\mu_S + \alpha^2\theta^2}{(1+J)^2\gamma + \alpha^2\theta}$, the firm produces without hedging its random cost, but speculates.

For the case in which there is no speculation in equilibrium (i.e., $F \in [\mu_S, \overline{F}_J]$), we make an additional comment. In Figure 8a, with $J = 3$, each
firm attains its highest payoffs in a Cournot equilibrium when the price of hedge coverage is actuarially fair, $F = \mu_S$. Here, hedging results in higher payoffs as long as $\mu_S \leq F \leq F'$. However, in Figure 8b, with $J = 4$, $CE^*|_{F=\mu_S}$ is not the highest value. The ambiguous effect of the cost of hedging on the firms’ payoffs implies that a more competitive futures market due in part to risk-neutral speculators might actually have a detrimental effect on payoffs.

4.3 Production and Output Price

We next turn to the effect of access to the futures market on production and output price. Proposition 4.12 states that if firms hedge, then access to the futures input market leads to an increase in production. However, if firms only speculate, then production decreases with access to the futures input market. Hence, access to the futures market dampens the effect of uncertainty on production when the firms hedge, but exacerbates the effect of uncertainty when firms speculate. In addition, when the firms hedge, the result for Cournot markets stated in Proposition 4.12 is consistent with Holthausen (1979) and Feder et al. (1980) for competitive markets.

**Proposition 4.12.** Suppose that firms have full access to the futures market. In equilibrium, if firms hedge (i.e., $\omega^*(J^*) > 0$),

$$\hat{q}^*(\hat{J}^*) < q^*(J^*),$$

while if firms only speculate (i.e., $\omega^*(J^*) < 0$), then $\hat{q}^*(\hat{J}^*) > q^*(J^*)$.

**Proof.** The proof is immediate from comparing (41) with (48).

An important implication from Remark 4.4 and Proposition 4.12 is that a decrease in the sunk cost makes it more likely that the dampening effect does not occur.

**Remark 4.13.** Suppose that firms have full access to the futures market. In equilibrium, in a contango futures market, a decrease in the sunk cost can induce firms to speculate, which exacerbates the effect of uncertainty on production.

49The result holds whether the firms partial hedge (i.e., $\omega^*(J^*) \in (0,1)$), fully hedge (i.e., $\omega^*(J^*) = 1$), or fully hedge and speculate simultaneously (i.e., $\omega^*(J^*) > 1$).
While Proposition 4.12 states that hedging always increases output, the effect of access to the futures market on output price is more complicated because the number of firms entering the industry also depends on whether the firms have access to the futures market. In particular, while hedging increases output, it may also decrease the number of firms. The overall effect on total output, and, thus, output price is then ambiguous. However, Proposition 4.14 states that when the futures price is actuarially fair, access to the futures market reduces the output price.

**Proposition 4.14.** Suppose that firms have full access to the futures market. If the futures price is actuarially fair, then \( \hat{p}^*(\hat{J}^*) > p^*(J^*) \).

*Proof.* Evaluating (42) and (49) at \( F = \mu_S \) yields \( \hat{p}^*(\hat{J}^*) > p^*(J^*) \). \( \square \)

We conclude the analysis by studying the effect of \( F \) on production and output price. Proposition 4.15 states that, depending on the structure of the futures market, per-firm production decreases or increases with a higher futures price.

**Proposition 4.15.** Suppose that firms have full access to the futures market. Then,

1. For \( F \in (0, \mu_S) \), \( \frac{\partial q^*(J^*)}{\partial F} > 0 \).
2. For \( F = \mu_S \), \( \frac{\partial q^*(J^*)}{\partial F} = 0 \).
3. For \( F \in (\mu_S, \theta) \), \( \frac{\partial q^*(J^*)}{\partial F} < 0 \).

*Proof.* Differentiating (41) yields

\[
\frac{\partial q^*(J^*)}{\partial F} = -\frac{(F - \mu_S)}{2\alpha\sigma_S^2} \left( K - \frac{(F - \mu_S)^2}{2\alpha\sigma_S^2} \right)^{-\frac{3}{2}}.
\] (56)

Given (13), the sign of (56) depends on \( F \in (0, \theta) \). \( \square \)

Finally, if firms have full access to the futures market, the overall effect of the futures price on the output price is ambiguous as well. On the one hand, an increase in \( F \) might decrease per-firm production, which increases
the output price. On the other hand, an increase in \( F \) might increase the number of firms, which decreases the output price. Proposition 4.16 states that an increase in \( F \) might result in a lower output price only when the firms engage in speculation.

**Proposition 4.16.** Suppose that firms have access to the futures market. Then \( \frac{\partial p^*(J^*)}{\partial F} < 0 \) if and only if \( \omega^*(J^*) < 0 \).

**Proof.** From (42),

\[
\frac{\partial p^*(J^*)}{\partial F} = - \left( K - \frac{(F - \mu_S)^2}{2\alpha \sigma_S^2} \right)^{-\frac{1}{2}} \frac{(F - \mu_S)}{2\alpha \sigma_S^2} \sqrt{7} + 1. \tag{57}
\]

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From (57), \( \frac{\partial p^*(J^*)}{\partial F} < 0 \) if and only if

\[
K < \frac{(F - \mu_S)^2(2\alpha \sigma_S^2 + \gamma)}{4\alpha^2 \sigma_S^4}, \tag{58}
\]

which implies, from (78) in Appendix C, that \( \frac{\partial p^*(J^*)}{\partial F} < 0 \) if and only if \( \omega^*(J^*) < 0 \). \( \square \)

## 5 Final Remarks

This paper provides an analysis of the firms’ production and hedging decisions under imperfect competition with potential entry. Entry is shown to remove the separation result, i.e., although the firms have access to the futures market, their production decisions depend on uncertainty and risk preferences through the determination of the number of firms in the industry. We also show that the use of futures contracts have an ambiguous effect on the market structure of the industry. For instance, when the futures input price is actuarially fair, access to the futures market increases or decreases the number of entering firms depending on the value of the sunk cost.

To study the interaction between entry and the futures market, we have abstracted from two important aspects. First, we have assumed that the spot
and futures prices were exogenous. However, these prices are determined by markets as well, which, in turn, affects resources allocation, production decisions, and risk-taking. Extending the model to include suppliers of the input along with speculators is an avenue for future research. While the determination of spot and futures prices has already been studied by Turnovsky (1983), the output producers are assumed to be passive, i.e., their demand for the input is given. In fact, output producers are active and forward-looking and, as shown in this paper, their output and input decisions are entwined.

Second, we have ignored the role of financial decisions in deterring entry. It would also be interesting to study how strategic hedging from an incumbent firm may alter the decision entry of a potential entrant. For instance, Maskin (1999) considers a model in which capacity installation by an incumbent firm serves to deter others from entering the industry. Uncertainty about demand or costs forces the incumbent to choose a higher capacity level than it would under certainty. This higher requirement for capacity diminishes the attractiveness of deterrence. It would be interesting to study the incumbent’s incentive to deter entry when it has access to futures markets.

Finally, an empirical extension would be to test the model in the airline industry or any industry with similar characteristics facing Cournot competition. Recent empirical tests on hedging were limited to the effect of different determinants such as CEO risk aversion, convexity of tax function, corporate governance, distress costs, information asymmetry, and the effect of hedging on firm value. To our knowledge, no study has analyzed the effect of hedging on entry. The main empirical question would be: Do airline companies that hedge (or speculate) enter different routes more aggressively? Our theoretical results are ambiguous on this question and an empirical prediction from the model is that airline companies produces less in different routes when futures prices are high, which induces more firms to enter and hedge their fuel cost.
A Hessian Matrix

The Hessian matrix evaluated at \( q_j = q^*(J) \) and \( x_j = x^*(J) \) is

\[
H = \begin{bmatrix}
H_{11} & H_{12} \\
H_{21} & H_{22}
\end{bmatrix}
\] (59)

where, given Assumption 2.1 and 2.2 (i.e., \( u'(\pi) > 0, \ u''(\pi) < 0, \ D'(Q) < 0, \) and \( D''(Q)q_j + D'(Q) < 0 \)),

\[
H_{11} = (D''(q_j + Q^*_{-j})q_j + 2D'(q_j + Q^*_{-j})) \int_0^{D(0)} u'(D(q_j + Q^*_{-j})q_j - Fq_j + (F - S)x_j) \cdot \phi(S) dS
\]

\[
+ \left( D'(q_j + Q^*_{-j})q_j + D(q_j + Q^*_{-j}) - F \right)^2
\]

\[
\times \int_0^{D(0)} u''(D(q_j + Q^*_{-j})q_j - Fq_j + (F - S)x_j) \cdot \phi(S) dS < 0,
\] (60)

\[
H_{22} = \int_0^{D(0)} (F - S)^2 \cdot u''(D(q_j + Q^*_{-j})q_j - Fq_j + (F - S)x_j) \cdot \phi(S) dS < 0,
\] (61)

and, using (9) or (12),

\[
H_{12} = H_{21} = (D'(q_j + Q^*_{-j})q_j + D(q_j + Q^*_{-j}) - F)
\]

\[
\times \int_0^{D(0)} (F - S) \cdot u''(D(q_j + Q^*_{-j})q_j - Fq_j + (F - S)x_j) \cdot \phi(S) dS = 0.
\] (62)

B The Certainty Equivalent

The certainty equivalent is implicitly defined by

\[
\mathbb{E}u(\pi(J, q_j, \omega_j, \sum_{k\neq j}^J q_k, \tilde{S}, F)) = u(CE(J, q_j, \omega_j, \sum_{k\neq j}^J q_k)),
\] (63)
so that, given (2), (37), \( u(\pi) = -e^{-\alpha \pi} \) and \( \tilde{S} \sim N(\mu_S, \sigma^2_\tilde{S}) \)

\[
CE(J, q_j, \omega_j, \sum_{k \neq j}^J q_k) = \mathbb{E}\pi(J, q_j, \omega_j, \sum_{k \neq j}^J q_k, \tilde{S}, F) - \alpha \mathbb{V}\pi(J, q_j, \omega_j, \sum_{k \neq j}^J q_k, \tilde{S}, F)/2, \tag{64}
\]

which yields (38). Here, \( \mathbb{E} \) is the expectation operator and \( \mathbb{V} \) is the variance operator over \( \tilde{S} \).

C Proofs

Proof of Proposition 4.1. We first solve the Cournot equilibrium at stage 2 for a given number of firms \( J \). We then determine the number of firms entering the industry. We finally derive the condition for existence.

1. Cournot equilibrium at stage 2.

We perform a change of variables. Let \( x_j + y_j \equiv q_j \), where \( x_j \equiv (1 - \omega_j)q_j \) is the units of output for which firm \( j \) does not hedge, and \( y_j \equiv \omega_j q_j \) is the units of output for which firm \( j \) hedges (or speculates). Hence, (38) is rewritten as

\[
CE(J, q_j, \omega_j, \sum_{k \neq j}^J q_k) = \left( \theta - \gamma \sum_{k=1}^J (x_k + y_k) \right) (x_j + y_j) - \mu_S(x_j + y_j) - (F - \mu_S)y_j - \alpha \sigma^2_S x_j^2/2. \tag{65}
\]

Using (65) and given that the optimal policies of firm \( k \neq j \) are \( q^*(J) \) and \( \omega^*(J) \), the maximization problem of firm \( j \) is\(^{50}\)

\[
\max_{x_j, y_j \geq 0} (\theta - \gamma (J - 1)(x^*(J) + y^*(J)) - \gamma (x_j + y_j))(x_j + y_j) - \mu_S(x_j + y_j) - (F - \mu_S)y_j - \alpha \sigma^2_S x_j^2/2. \tag{66}
\]

\(^{50}\)Uniqueness is immediate from the assumption of linear demand and convex cost.
From (66), the first-order conditions are

\[
\frac{\partial}{\partial x_j} : \theta - \gamma(J-1)(x^*(J) + y^*(J)) - 2\gamma(x_j + y_j) - \mu_S - \alpha\sigma_S^2 x_j = 0, \\
\frac{\partial}{\partial y_j} : \theta - \gamma(J-1)(x^*(J) + y^*(J)) - 2\gamma(x_j + y_j) - \mu_S - F + \mu_S = 0,
\]

(67)  (68)
evaluated at \(x_j = x^*(J)\) and \(y_j = y^*(J)\).\(^{51}\) Solving (67) and (68) for \(x^*(J)\) and \(y^*(J)\) yields

\[
x^*(J) = \frac{F - \mu_S}{\alpha\sigma_S^2}, \tag{70}
\]
\[
y^*(J) = \frac{\theta - F}{(1 + J)\gamma} - \frac{F - \mu_S}{\alpha\sigma_S^2}. \tag{71}
\]

Hence, from (70) and (71), \(q^*(J) = x^*(J) + y^*(J)\) and \(\omega^*(J) = y^*(J)/q^*(J)\), i.e.,

\[
q^*(J) = \frac{\theta - F}{(1 + J)\gamma} > 0, \tag{72}
\]
\[
\omega^*(J) = 1 - \frac{(1 + J)\gamma(F - \mu_S)}{\alpha\sigma_S^2(\theta - F)}. \tag{73}
\]

Plugging (72) into (37) yields

\[
p^*(J) = \frac{\theta + JF}{1 + J}. \tag{74}
\]

Finally, plugging (72) and (73) into (38) yields the certainty equivalent \(CE^*(J) \equiv CE (J, q^*(J), \omega^*(J), (J - 1)q^*(J))\),

\[
CE^*(J) = \frac{(\theta - F)^2}{(1 + J)^2\gamma} + \frac{(F - \mu_S)^2}{2\alpha\sigma_S^2}. \tag{75}
\]

\(^{51}\)The Hessian matrix

\[
H = \begin{bmatrix}
-2\gamma - \alpha\sigma_S^2 & -2\gamma \\
-2\gamma & -2\gamma
\end{bmatrix} \tag{69}
\]
satisfies the second-order conditions.
2. **Entry decision at stage 1**: Setting (75) equal to \( K \) and solving for \( J = J^* \) yields (40). Plugging (40) into (72), (73), and (74) yields (41), (43), and (42), respectively.

3. **Derivation of expression** (39). From (40), \( J^* \geq 1 \) implies that

\[
K \leq \frac{(\theta - F)^2}{4\gamma} + \frac{(F - \mu_S)^2}{2\alpha \sigma_S^2}, \tag{76}
\]

while, from (75), \( J^* < \infty \) as long as \( CE^*(J) > K \) does not hold for all \( J \geq 1 \), i.e.,

\[
K > \lim_{J \to \infty} CE^*(J) = \frac{(F - \mu_S)^2}{2\alpha \sigma_S^2}. \tag{77}
\]

Combining (76) and (77) yields (39).

**Proof of Proposition 4.2.** Suppose that (39) holds. Using (39) and (43) we obtain:

1. If \( F \in (0, \mu_S) \), then \( \omega^*(J^*) > 1 \).
2. If \( F = \mu_S \), then \( \omega^*(J^*) = 1 \).
3. If \( F > \mu_S \) and

\[
\frac{(2\gamma + \alpha \sigma_S^2)(F - \mu_S)^2}{2\alpha^2 \sigma_S^4} \leq K \leq \frac{(\theta - F)^2}{4\gamma} + \frac{(F - \mu_S)^2}{2\alpha \sigma_S^2}, \tag{78}
\]

then \( \omega^*(J^*) \in (0, 1) \).
4. If \( F \in (\mu_S, \overline{F}_1) \), \( \overline{F}_1 \equiv \frac{2\gamma \mu_S + \alpha \sigma_S^2 \theta}{2\gamma + \alpha \sigma_S^2} \) and

\[
K = \frac{(2\gamma + \alpha \sigma_S^2)(F - \mu_S)^2}{2\alpha^2 \sigma_S^4}, \tag{79}
\]

then \( \omega^*(J^*) = 0 \).
5. If \( F \in (\mu_S, \theta) \) and

\[
\frac{(F - \mu_S)^2}{2\alpha \sigma_S^2} < K < \min \left\{ \frac{(2\gamma + \alpha \sigma_S^2)(F - \mu_S)^2}{2\alpha^2 \sigma_S^4}, \frac{(\theta - F)^2}{4\gamma} + \frac{(F - \mu_S)^2}{2\alpha \sigma_S^2} \right\}, \tag{80}
\]
then \( \omega^*(J^*) < 0 \).

**Proof of Proposition 4.7.** Suppose that \( \omega_j = 0 \) holds for all \( j \). Then, from (66) evaluated at \( y^*(J) = y_j = 0 \), the first-order condition is

\[
\frac{\partial}{\partial x_j} : \theta - \gamma(J - 1)\hat{x}^*(J)) - 2\gamma x_j - \mu S - \alpha \sigma^2 x_j = 0. \tag{81}
\]

Solving (81) for \( \hat{x}^*(J) = \hat{q}^*(J) \) yields

\[
\hat{q}^*(J) = \frac{\theta - \mu S}{(1 + J)\gamma + \alpha \sigma^2 S}. \tag{82}
\]

Plugging (82) into (37) yields

\[
\hat{p}^*(J) = \frac{(\gamma + \alpha \sigma^2 S)\theta + J\gamma \mu S}{(1 + J)\gamma + \alpha \sigma^2 S}. \tag{83}
\]

Finally, plugging (82) into (38) yields the certainty equivalent \( \widehat{CE}^*(J) \equiv \widehat{CE}^*(J, \hat{q}^*(J), 0, (J - 1)\hat{q}^*(J)) \),

\[
\widehat{CE}^*(J) = \frac{2\gamma + \alpha \sigma^2 S)(\theta - \mu S)^2}{2((1 + J)\gamma + \alpha \sigma^2 S)^2}. \tag{84}
\]

Setting (84) equal to \( K \) and solving for \( J = \hat{J}^* \) yields (47). Plugging (47) into (82) and (83) yields (48) and (49), respectively. Finally, we derive the existence condition defined by (46). From (47), \( \hat{J}^* \geq 1 \) implies that

\[
K \leq \frac{(\theta - \mu S)^2}{2(2\gamma + \alpha \sigma^2 S)} \tag{85}
\]

as in (46).
D Partial Access to the Futures Market, Second Stage

In this appendix, we show that, at the second stage of the game, if firms have partial access to the futures market, i.e., hedging but no speculation, then production and output price are only conditionally independent of uncertainty and risk preferences.

Consider the case in which firms have partial access to the futures market, i.e., the constraint $\omega_j \in [0,1]$ holds for all $j$. The subscript $H$ stands for hedging, and no speculation.

**Proposition D.1.** Suppose that $J$ firms have access to the futures market, but cannot speculate, i.e., the constraint $\omega_j \in [0,1]$ holds for all $j$. Then, in equilibrium, each firm supplies

$$q^*_H(J) = \begin{cases} \frac{\theta-F}{(1+J)\gamma}, & 0 < F < \frac{(1+J)\gamma\mu_s + \alpha\sigma^2_S}{(1+J)\gamma + \alpha\sigma^2_S} \\ \frac{\theta-\mu_s}{(1+J)\gamma + \alpha\sigma^2_S} - \frac{(1+J)\gamma\mu_s + \alpha\sigma^2_S}{(1+J)\gamma + \alpha\sigma^2_S} & \leq F < \theta \end{cases}$$

and hedges a fraction

$$\omega^*_H(J) = \begin{cases} 1, & 0 < F \leq \mu_s \\ \frac{(1+J)\gamma(F-\mu_s)}{\alpha\sigma^2_S(\theta-F)} & \mu_s < F < \frac{(1+J)\gamma\mu_s + \alpha\sigma^2_S}{(1+J)\gamma + \alpha\sigma^2_S} \\ 0, & \frac{(1+J)\gamma\mu_s + \alpha\sigma^2_S}{(1+J)\gamma + \alpha\sigma^2_S} \leq F < \theta \end{cases}$$

of its random cost. The equilibrium output price is

$$p^*_H(J) = \begin{cases} \frac{\theta+JF}{1+J} - \frac{(\gamma+\alpha\sigma^2_S)\mu_s + J\gamma\mu_s}{(1+J)\gamma + \alpha\sigma^2_S}, & 0 < F < \frac{(1+J)\gamma\mu_s + \alpha\sigma^2_S}{(1+J)\gamma + \alpha\sigma^2_S} \\ \frac{(1+J)\gamma\mu_s + \alpha\sigma^2_S}{(1+J)\gamma + \alpha\sigma^2_S} & \leq F < \theta \end{cases}$$

**Proof.** Financial participation: $\omega_j \in (0,1]$. From (66), the first-order condi-
tions are
\[
\frac{\partial}{\partial x_j} : \theta - \gamma(J - 1)(x_H^*(J) + y_H^*(J)) - 2\gamma(x_j + y_j) - \mu_S - \alpha \sigma_x^2 x_j = 0, \quad (89)
\]
\[
\frac{\partial}{\partial y_j} : \theta - \gamma(J - 1)(x_H^*(J) + y_H^*(J)) - 2\gamma(x_j + y_j) - \mu_S - F + \mu_S = 0, \quad (90)
\]
evaluated at \(x_j = x_H^*(J)\) and \(y_j = y_H^*(J) \neq 0\).\(^{52}\) Solving (89) and (90) for \(x_H^*(J)\) and \(y_H^*(J)\) yields
\[
x_H^*(J) = \frac{F - \mu_S}{\alpha \sigma_x^2}, \quad (92)
\]
\[
y_H^*(J) = \frac{\theta - F - \frac{F - \mu_S}{(1 + J)\gamma}}{1 + \alpha \sigma_x^2} \neq 0, \quad (93)
\]
if and only if \(0 < F < \frac{(1 + J)\gamma \mu_S + \alpha \sigma_x^2 \theta}{(1 + J)\gamma + \alpha \sigma_x^2} < \theta\) when speculation is not allowed. Hence, from (92) and (93), \(q_H^*(J) = x_H^*(J) + y_H^*(J)\) as in (86) when \(0 \leq F < \frac{(1 + J)\gamma \mu_S + \alpha \sigma_x^2 \theta}{(1 + J)\gamma + \alpha \sigma_x^2}\). Moreover, as in (87) \(\omega_H^*(J) = y_H^*(J)/q_H^*(J)\) for \(\mu_S < F < \frac{(1 + J)\gamma \mu_S + \alpha \sigma_x^2 \theta}{(1 + J)\gamma + \alpha \sigma_x^2}\) and \(\omega_H^*(J) = 1\) for \(0 < F \leq \mu_S\).

**No Financial Participation:** \(\omega_j = 0\). We next consider corner solutions, i.e., \(\omega_H^*(J) = 0\). From (93), \(y_H^*(J) = 0\) if and only if \(\frac{(1 + J)\gamma \mu_S + \alpha \sigma_x^2 \theta}{(1 + J)\gamma + \alpha \sigma_x^2} \leq F < \theta\). Hence, from (66), the first-order condition for \(x_j\) is
\[
\frac{\partial}{\partial x_j} : \theta - \gamma(J - 1)(x_H^*(J) + y_H^*(J)) - 2\gamma(x_j + y_j) - \mu_S - \alpha \sigma_x^2 x_j = 0, \quad (94)
\]
evaluated at \(x_j = x_H^*(J)\) and \(y_j = y_H^*(J) = 0\), so that, for \(\frac{(1 + J)\gamma \mu_S + \alpha \sigma_x^2 \theta}{(1 + J)\gamma + \alpha \sigma_x^2} \leq F < \theta\), \(q_H^*(J) = x^*(J)\) and \(\omega_H^*(J) = 0\) as in (86) and (87).

In view of Proposition D.1, we now provide two results regarding separation. First, when speculation is excluded, we obtain a conditional separation result. That is, Proposition D.2 states that, conditional on hedging (i.e., the futures input price is not too high), production and output price are

\(^{52}\)The Hessian matrix
\[
H = \begin{bmatrix}
-2\gamma - \alpha \sigma_x^2 & -2\gamma \\
-2\gamma & -2\gamma
\end{bmatrix}
\]

satisfies the second-order condition.
independent of risk and risk aversion.

**Proposition D.2.** From (86) and (88), \( q^*_H(J) \) and \( p^*_H(J) \) are conditionally independent of uncertainty and risk preferences. That is, conditional on hedging, i.e., \( 0 < F < \frac{(1+J)\gamma \mu_S + \alpha \sigma^2_S}{(1+J)\gamma + \alpha \sigma^2_S} \), \( \partial q^*_H(J)/\partial \mu_S, \partial q^*_H(J)/\partial \sigma^2_S, \partial q^*_H(J)/\partial \alpha = 0 \) and \( \partial p^*_H(J)/\partial \mu_S, \partial p^*_H(J)/\partial \sigma^2_S, \partial p^*_H(J)/\partial \alpha = 0 \).

Second, Proposition D.3 states that there is no unconditional separation. Indeed, production depends indirectly on \( \alpha \) and \( \sigma^2_S \) via the range of futures input prices that induce access to the futures market for hedging. Specifically, from Proposition D.1, the upper bound of the range of futures input prices yielding hedging is increasing in the variance of the spot input price and risk aversion. This, in turn, increases the likelihood of hedging, and thus dampens the effect of uncertainty on production.

**Proposition D.3.** From (86) and (88), \( q^*_H(J) \) and \( p^*_H(J) \) are unconditionally dependent of the distribution of the spot price and risk preferences.

### E Partial Access to the Futures Market, Entry

In this appendix, we consider an intermediate benchmark model in which access to the futures input market is restricted to hedging, i.e., no speculation. As in Appendix D, we use the subscript \( H \) to indicate that firms may hedge, but cannot speculate. Proposition E.1 states that an equilibrium exists as long as the entry cost is not too high to prevent at least one firm from entering the industry.

**Proposition E.1.** Suppose that firms have access to the futures market, but cannot speculate, i.e., the constraint \( \omega_j \in [0,1] \) holds for all \( j \). There exists a unique equilibrium with \( 1 \leq J^*_H < \infty \) firms in the industry if and only if

\[
0 < K \leq \max \left\{ \frac{\theta - F}{4\gamma}, \frac{(\theta - F)^2}{2\gamma}, \frac{(F - \mu_S)^2}{2(2\gamma + \alpha \sigma^2_S)} \right\}.
\]
Further, the firms hedge, i.e., \( \omega^*_H(J^*_H) \in (0,1) \) when \( F \in [0,F_1], F_1 \equiv \frac{2\gamma S + \alpha \sigma_S^2 \theta}{2\gamma + \alpha \sigma_S^2} \) and
\[
0 < K \leq \left( \frac{\theta - F}{4\gamma} \right)^2 + \left( \frac{F - \mu_S}{2\alpha \sigma_S^2} \right)^2,
\]
and do not hedge, i.e., \( \omega^*_H(J^*_H) = 0 \), when
\[
0 < K \leq \min \left\{ \frac{(2\gamma + \alpha \sigma_S^2)(F - \mu_S)^2}{2\alpha \sigma_S^2}, \frac{(\theta - \mu_S)^2}{2(2\gamma + \alpha \sigma_S^2)} \right\}.
\]

Proposition E.2 characterizes the equilibrium under partial access to the futures market. Note that the equilibrium when the firms may only hedge is a hybrid between no access to the futures input market and full access to the futures input market. For instance, (99) combines both (41) and (48). Hence, all results under hedging (i.e., \( \omega^*_H(J^*_H), \omega^*(J^*) \in (0,1) \)) hold regardless of the possibility of speculating.

**Proposition E.2.** Suppose that firms have access to the futures market, but cannot speculate, i.e., the constraint \( \omega_j \in [0,1] \) holds for all \( j \). In equilibrium,
\[
J^*_H = \begin{cases} 
\frac{\theta - F}{\sqrt{K\gamma}} - 1, & F \in (0,\mu_S), 0 < K \leq \frac{(\theta - F)^2}{4\gamma}, \\
\left( \frac{K - (F - \mu_S)^2}{2\alpha \sigma_S^2} \right)^{\gamma} - 1, & F \in [\mu_S, F_1], \frac{(2\gamma + \alpha \sigma_S^2)(F - \mu_S)^2}{2\alpha \sigma_S^2} < K \leq \frac{(\theta - F)^2}{4\gamma} + \frac{(F - \mu_S)^2}{2\alpha \sigma_S^2}, \\
\frac{(\theta - \mu_S)^2}{\sqrt{2\gamma + \alpha \sigma_S^2}} - 1, & 0 < K \leq \min \left\{ \frac{(2\gamma + \alpha \sigma_S^2)(F - \mu_S)^2}{2\alpha \sigma_S^2}, \frac{(\theta - \mu_S)^2}{2(2\gamma + \alpha \sigma_S^2)} \right\}.
\end{cases}
\]

firms enter the industry. Each firm supplies
\[
q^*_H(J^*_H) = \begin{cases} 
\frac{\sqrt{K/\gamma}}{\sqrt{2\gamma + \alpha \sigma_S^2}}, & F \in (0,\mu_S), 0 < K \leq \frac{(\theta - F)^2}{4\gamma}, \\
\sqrt{\left( \frac{K - (F - \mu_S)^2}{2\alpha \sigma_S^2} \right)} / \gamma, & F \in [\mu_S, F_1], \frac{(2\gamma + \alpha \sigma_S^2)(F - \mu_S)^2}{2\alpha \sigma_S^2} < K \leq \frac{(\theta - F)^2}{4\gamma} + \frac{(F - \mu_S)^2}{2\alpha \sigma_S^2}, \\
\frac{\sqrt{2\gamma + \alpha \sigma_S^2}}{\sqrt{2\gamma + \alpha \sigma_S^2}}, & 0 < K \leq \min \left\{ \frac{(2\gamma + \alpha \sigma_S^2)(F - \mu_S)^2}{2\alpha \sigma_S^2}, \frac{(\theta - \mu_S)^2}{2(2\gamma + \alpha \sigma_S^2)} \right\}.
\end{cases}
\]
and hedges a fraction

$$\omega_H^*(J_H^*) = \begin{cases} 
1, & F \in (0, \mu_S), 0 < K \leq \frac{(\theta-F)^2}{4\gamma}, \\
1 - \frac{\sqrt[3]{F-\mu_S}}{\alpha \sigma_S^4} \sqrt{\left(K - \frac{(F-\mu_S)^2}{2\alpha \sigma_S^2}\right)}, & F \in [\mu_S, F_1], \\
0, & 0 < K \leq \min \left\{ \frac{(2\gamma + \alpha \sigma_S^4)(F-\mu_S)^2}{2\alpha^2 \sigma_S^2}, \frac{(\theta-\mu_S)^2}{2(2\gamma+\alpha \sigma_S^2)} \right\} 
\end{cases}$$

of its random cost. The equilibrium output price is

$$p_H^*(J_H^*) = \begin{cases} 
\sqrt{K \gamma + F}, & F \in (0, \mu_S), 0 < K \leq \frac{(\theta-F)^2}{4\gamma} \\
\sqrt{\left( K - \frac{(F-\mu_S)^2}{2\alpha \sigma_S^2} \right) \gamma + F}, & F \in [\mu_S, F_1], \\
\frac{\gamma K + \alpha \sigma_S^4 + \gamma}{2\gamma + \alpha \sigma_S^2} + \mu_S, & 0 < K \leq \min \left\{ \frac{(2\gamma + \alpha \sigma_S^4)(F-\mu_S)^2}{2\alpha^2 \sigma_S^2}, \frac{(\theta-\mu_S)^2}{2(2\gamma+\alpha \sigma_S^2)} \right\} 
\end{cases}$$

We now provide a detailed proof of Propositions E.1 and E.2.

Proof. Interior Solutions. We first consider interior solutions to (66), i.e., $x_H^*(J), y_H^*(J) > 0$ or $\omega_H^*(J) \in (0,1)$. From (66), the first-order conditions are

$$\frac{\partial}{\partial x_j} : \theta - \gamma(J-1)(x_H^*(J) + y_H^*(J)) - 2\gamma(x_j + y_j) - \mu_S - \alpha \sigma_S^2 x_j = 0,$$

$$\frac{\partial}{\partial y_j} : \theta - \gamma(J-1)(x_H^*(J) + y_H^*(J)) - 2\gamma(x_j + y_j) - \mu_S - F + \mu_S = 0,$$

evaluated at $x_j = x_H^*(J)$ and $y_j = y_H^*(J)$.$^{53}$ Solving (102) and (103) for

$^{53}$The Hessian matrix

$$H = \begin{bmatrix} -2\gamma - \alpha \sigma_S^2 & -2\gamma \\ -2\gamma & -2\gamma \end{bmatrix}$$

satisfies the second-order condition.
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\[ x^*_H(J) \text{ and } y^*_H(J) \text{ yields} \]

\[ x^*_H(J) = \frac{F - \mu_S}{\alpha \sigma_f^2}, \quad (105) \]

\[ y^*_H(J) = \frac{\theta - F}{(1 + J)\gamma} - \frac{F - \mu_S}{\alpha \sigma_f^2} > 0, \quad (106) \]

if and only if \( \mu_S \leq F < \frac{(1 + J)\gamma \mu_S + \alpha \sigma_f^2 \theta}{(1 + J)\gamma + \alpha \sigma_f^2} < \theta \) when speculation is not allowed. Hence, from (105) and (106), \( q^*_H(J) = x^*_H(J) + y^*_H(J) \) and \( \omega^*_H(J) = y^*_H(J)/q^*_H(J) \) as in (86) and (87) when \( \mu_S \leq F < \frac{(1 + J)\gamma \mu_S + \alpha \sigma_f^2 \theta}{(1 + J)\gamma + \alpha \sigma_f^2} \).

**Corner Solutions.** We next consider corner solutions, i.e., \( \omega^*_H(J) = 0 \) and \( \omega^*_H(J) = 1 \). We consider the case \( \omega^*_H(J) = 0 \) first. From (93), \( y^*_H(J) = 0 \) if and only if \( \frac{(1 + J)\gamma \mu_S + \alpha \sigma_f^2 \theta}{(1 + J)\gamma + \alpha \sigma_f^2} \leq F < \theta \). Hence, from (66), the first-order condition for \( x_j \) is

\[ \frac{\partial}{\partial x_j} : \theta - \gamma(J - 1)(x^*_H(J) + y^*_H(J)) - 2\gamma(x_j + y_j) - \mu_S - \alpha \sigma_f^2 x_j = 0, \quad (107) \]

evaluated at \( x_j = x^*_H(J) \) and \( y_j = y^*_H(J) = 0 \), so that, for \( \frac{(1 + J)\gamma \mu_S + \alpha \sigma_f^2 \theta}{(1 + J)\gamma + \alpha \sigma_f^2} \leq F < \theta \), \( q^*_H(J) = x^*_H(J) \) and \( \omega^*_H(J) = 0 \) as in (86) and (87). Plugging (86) into (37) yields

\[ p^*_H(J) = \frac{\theta + JF}{1 + J}. \quad (108) \]

Next, we consider the case \( \omega^*_H(J) = 1 \). Following appendix D, \( \omega^*_H(J) = 1 \) if and only if \( 0 < F \leq \mu_S \) when speculation is not allowed. From (66), the first-order condition for \( y_j \) is

\[ \frac{\partial}{\partial y_j} : \theta - \gamma(J - 1)(x^*_H(J) + y^*_H(J)) - 2\gamma(x_j + y_j) - \mu_S - F + \mu_S = 0 \quad (109) \]

evaluated at \( x_j = x^*_H(J) = 0 \) and \( y_j = y^*_H(J) \), so that for \( 0 < F < \mu_S \), \( q^*_H(J) = y^*_H(J) \) and \( \omega^*_H(J) = 1 \). Solving (109) for \( y^*_H(J) \) yields

\[ y^*_H(J) = \frac{\theta - F}{(1 + J)\gamma}. \quad (110) \]

Plugging (110) into (37) yields the same equilibrium price as in (108).
Plugging (86) and (87) into (38) yields the certainty equivalent \( CE^*_H(J) \equiv CE(J, q^*_H(J), \omega^*_H(J), (J - 1)q^*_H(J)) \),

\[
CE^*_H(J) = \begin{cases} 
\frac{(\theta - F)^2}{(1+J)^{\gamma}}, & 0 < F < \mu_S \\
\frac{(F - \mu_S)^2}{(1+J)^{\gamma} + \frac{(2\gamma + \sigma_S^2)(\theta - \mu_S)^2}{2(1+J)(\gamma + \alpha_S^2\theta)^2}}, & \mu_S \leq F < \frac{(1+J)\mu_S + \alpha_S^2\theta}{(1+J)(\gamma + \alpha_S^2\theta)^2}, \\
\frac{(1+J)\mu_S + \alpha_S^2\theta}{(1+J)(\gamma + \alpha_S^2\theta)}, & \frac{(1+J)\mu_S + \alpha_S^2\theta}{(1+J)(\gamma + \alpha_S^2\theta) \leq F < \theta}
\end{cases},
\]

Setting (111) equal to \( K \) and solving for \( J = J_H^* \) yields (98). Plugging (98) into (86), (87), and (88) yields (99), (100), and (101), respectively.

Next, we derive the inequalities in Proposition E.1. First, note, from Proposition D.1, that partial hedging, i.e., \( \omega^*_H(J) \in (0, 1) \), occurs when \( \mu_S \leq F < \frac{(1+J)\mu_S + \alpha_S^2\theta}{(1+J)(\gamma + \alpha_S^2\theta)^2} \). From (98), plugging \( J_H^* = \frac{\theta - F}{\sqrt{K - \frac{(F - \mu_S)^2}{2\alpha^2\sigma_S^4}}} \) into \( F < \frac{(1+J)\mu_S + \alpha_S^2\theta}{(1+J)(\gamma + \alpha_S^2\theta)} \) and rearranging yields

\[
K > \frac{(2\gamma + \alpha_S^2)(F - \mu_S)^2}{2\alpha^2\sigma_S^4}.
\]

In addition, when there is partial hedging, from (98), \( J_H^* \geq 1 \) implies that

\[
K \leq \frac{(\theta - F)^2}{4\gamma} + \frac{(F - \mu_S)^2}{2\alpha^2\sigma_S^4}.
\]

Hence, from (112) and (113), there exists an equilibrium with entry and partial hedging when

\[
\frac{(2\gamma + \alpha_S^2)(F - \mu_S)^2}{2\alpha^2\sigma_S^4} < K \leq \frac{(\theta - F)^2}{4\gamma} + \frac{(F - \mu_S)^2}{2\alpha^2\sigma_S^4},
\]

as stated in Proposition E.2.\(^{54}\)

Second, from Proposition D.1, there is full hedging, i.e., \( \omega^*_H(J) = 1 \), when

\[
\frac{(1+J)\mu_S + \alpha_S^2\theta}{(1+J)(\gamma + \alpha_S^2\theta) \leq F < \theta}
\]

\[\frac{(2\gamma + \alpha_S^2)(F - \mu_S)^2}{2\alpha^2\sigma_S^4} < (\theta - F)^2 + \frac{(F - \mu_S)^2}{2\alpha^2\sigma_S^4}.
\]

\(^{54}\)The set of \( K \) satisfying (114) is nonempty. Indeed, hedging occurs when \( F < \frac{2\mu_S + \alpha_S^2\theta}{2\gamma + \alpha_S^2\theta} \), which implies that

\[
\frac{(2\gamma + \alpha_S^2)(F - \mu_S)^2}{2\alpha^2\sigma_S^4} < (\theta - F)^2 + \frac{(F - \mu_S)^2}{2\alpha^2\sigma_S^4}.
\]
$0 < F \leq \mu_S$. Furthermore, with full hedging, from (98), $J_H^* \geq 1$ implies that

$$K \leq \frac{(\theta - F)^2}{4\gamma}.$$  \hfill (116)

Hence, from (116), there exists an equilibrium with entry and full hedging when

$$0 < K \leq \frac{(\theta - F)^2}{4\gamma}.$$  \hfill (117)

Third, from Proposition D.1, there is no hedging when $F \geq \frac{(1 + J)\gamma \mu_S + \alpha \sigma_S^2 \theta}{(1 + J)\gamma + \alpha \sigma_S^2}$. From (98), plugging $J_H^* = \frac{(\theta - \mu_S)\sqrt{2\gamma + \alpha \sigma_S^2}}{\gamma \sqrt{2K}} - \frac{\alpha \sigma_S^2}{\gamma} - 1$ into $F \geq \frac{(1 + J)\gamma \mu_S + \alpha \sigma_S^2 \theta}{(1 + J)\gamma + \alpha \sigma_S^2}$, and rearranging yields

$$K \leq \frac{(2\gamma + \alpha \sigma_S^2)(F - \mu_S)^2}{2\alpha^2 \sigma_S^4}.$$  \hfill (118)

In addition, when there is no hedging, from (98), $J_H^* \geq 1$ implies that

$$K \leq \frac{(\theta - \mu_S)^2}{2(2\gamma + \alpha \sigma_S^2)}.$$  \hfill (119)

Hence, from (118) and (119), there exists an equilibrium with entry and no hedging when

$$0 < K \leq \min \left\{ \frac{(2\gamma + \alpha \sigma_S^2)(F - \mu_S)^2}{2\alpha^2 \sigma_S^4}, \frac{(\theta - \mu_S)^2}{2(2\gamma + \alpha \sigma_S^2)} \right\},$$  \hfill (120)

as stated in Proposition E.1.

Finally, there exists a Cournot equilibrium with $J_H^* \geq 1$ as long as (113),
(117) or (119) hold, i.e.,

$$K \leq \max \left\{ \frac{(\theta - F)^2}{4\gamma}, \frac{(\theta - F)^2}{4\gamma} + \frac{(F - \mu_S)^2}{2\alpha\sigma_S^2}, \frac{(\theta - \mu_S)^2}{2(2\gamma + \alpha\sigma_S^2)} \right\},$$  \hspace{1cm} (122)

$$= \frac{(\theta - F)^2}{4\gamma} + \frac{(F - \mu_S)^2}{2\alpha\sigma_S^2},$$  \hspace{1cm} (123)

as stated in Proposition E.1.\(^56\)

\[\Box\]

\textbf{F  The Effect of Futures Price on Certainty Equivalent}

In this appendix, we study the ambiguous effect of \(F\) on \(CE^*(J)\) in a contango structure (i.e., \(F > \mu_S\)). Recall that, for \(F < \mu_S\), \(\partial CE^*(J)/\partial F < 0\). To that end, rewrite (54) as revenue minus cost, i.e.,

$$CE^*(J) = R^*(J) - \Psi^*(J),$$  \hspace{1cm} (124)

where

$$R^*(J) = p^*(J)q^*(J),$$  \hspace{1cm} (125)

$$= \frac{\theta + JF}{1 + J} \cdot \frac{\theta - F}{\gamma(1 + J)}$$  \hspace{1cm} (126)

\(^{55}\)Note that

$$\frac{(\theta - F)^2}{4\gamma} + \frac{(F - \mu_S)^2}{2\alpha\sigma_S^2} > \frac{(\theta - \mu_S)^2}{2(2\gamma + \alpha\sigma_S^2)},$$  \hspace{1cm} (121)

simplifies to \((\alpha\sigma_S^2\theta - (2\gamma + \alpha\sigma_S^2)F + 2\gamma\mu_S)^2 > 0\), which is always true.

\(^{56}\)Uniqueness is immediate from the assumption of linear demand and convex cost.
is the revenue and

\[ \Psi^*(J) = \mu_S q^*(J) + (F - \mu_S)\omega^*(J)q^*(J) + \alpha\sigma^2(1 - \omega^*(J))^2q^2/2, \quad (127) \]

\[ = \mu_S \frac{\theta - F}{\gamma(1 + J)} + (F - \mu_S) \left( \frac{\theta - F}{(1 + J)\gamma} - \frac{F - \mu_S}{\alpha\sigma^2} \right) + \frac{(F - \mu_S)^2}{2\alpha\sigma^2}, \quad (128) \]

is the equilibrium cost. Therefore,

\[ \frac{\partial CE^*(J)}{\partial F} = \frac{\partial R^*(J)}{\partial F} - \frac{\partial \Psi^*(J)}{\partial F}, \quad (129) \]

where

\[ \frac{\partial R^*(J)}{\partial F} = \frac{(J - 1)\theta - 2JF}{\gamma(1 + J)^2} \quad (130) \]

and

\[ \frac{\partial \Psi^*(J)}{\partial F} = -\frac{\mu_S}{\gamma(1 + J)} + \frac{\theta + \mu_S - 2F}{(1 + J)\gamma} - \frac{2(F - \mu_S)}{\alpha\sigma^2} + \frac{(F - \mu_S)}{\alpha\sigma^2}. \quad (131) \]

Consider first the effect of \( F \) on revenue \( R^*(J) \). A higher cost of hedging induces the firms to reduce production, which, in turn, reduces the market externality due to the strategic interaction of the firms. In other words, \( \frac{\partial \Psi^*(J)}{\partial F} q^*(J) + p^*(J)\frac{\partial q^*(J)}{\partial F} > 0 \), or the percentage increase in the equilibrium output price is greater than the percentage decrease in the output. This has the effect of increasing revenue. Formally, from (130), \( \frac{\partial R^*(J)}{\partial F} > 0 \) if and only if \( (J - 1)\theta > 2JF \).

Consider next the effect of \( F \) on cost \( \Psi^*(J) \). The cost may decrease with a higher cost of hedging. Specifically, from (131), the cost of production unambiguously decreases in \( F \) because the firm reduces production. See the first term in (131). The total cost of hedge coverage decreases in \( F \) if and only if \( (\theta - 2F + \mu_S)\alpha\sigma^2 < 2(1 + J)\gamma(F - \mu_S) \). See the second term in (131). Finally, the cost of bearing risk (through the risk premium in (131)) unambiguously increases in \( F \). In general, the cost decreases in
$F$ when $(\theta - 2F)\alpha\sigma_S^2 < (1 + J)\gamma(F - \mu_S)$. The overall effect is stated in Proposition 4.11.

As noted, if $J = 1$, then condition (55) is not satisfied. Indeed, the positive relation between the certainty equivalent and the futures input price is only possible when the firms exercise a negative externality on one another.
References


