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A Network Model for Capped Distance-Based Tolls

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Abstract. Toll road operators and other toll facility stakeholders require analysis tools to estimate the ridership and projected income for an increasing variety of tolling schemes. Some tolling schemes commonly considered include distance-based tolls as well as derived schemes such as charging a maximum toll (or cap) for the use of the facility or minimum toll, if the distance based toll is less than this value. In addition, different entry ramp tolls are considered, which may be added to a distance-based toll and additionally subjected to the toll cap value. In order to meet these requirements a new model formulation and algorithm for distance-based toll modelling is developed. It uses the toll cost per link, which may be distance dependent, together with minimum and a maximum value of the tolls paid. The model is based on the addition of a set of temporary links to the network, which inherit the tolls and the delays of the original links. The method presented in this paper is general and self-contained. The model may be easily extended for other types of nonlinear toll schemes. A proof is provided for the equivalence of the modified and original network formulations. This new method is illustrated with a small example and a case of capped distance-based toll modelling on a network originating from practice. In order to solve the resulting multi-class network equilibrium model, a multi-threaded bi-conjugate variant of the linear approximation method has been adapted for the particular toll structure considered

Keywords. Toll highways, nonlinear tolls, multi-class network equilibrium with tolls.

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1. Introduction

Distance-based tolls charge the road users proportionally to the distance of their trip. Some feasibility studies for current toll road projects require the ability to model different variants of distance-based tolls, which is the way that some toll road operators establish the fees for usage. In some instances this way of charging tolls increases the fairness in toll payment and prevents the overuse of short free trips. For example, in Taiwan the average length of short trips free of charge made between booths is 10 km, account for ~51% of all daily traffic compose mostly of light vehicles (Chen and Wen (2013)). Such distance-based tolls have been successfully implemented in Australia, Canada, and Taiwan for light vehicles and in Germany for heavy trucks. There are plans to implement such tolls in other European Union countries and Russia. Several papers considered the impact of such tolls on the user behaviour, mostly using statistical methods (see May and Milne (2000), Wen and Tsai (2005), Balwani and Singh (2009), Joua et. al (2012), Meng et. al (2012), Natzel et. al (2011)).

In practice, different variants of distance-based tolls are evaluated, such as capped maximum toll, minimum toll and different fees per entry ramp. Under the capped maximum toll scheme toll road users pay a fee for up to a certain number of kilometres in monetary units. Minimum distance-based tolls require a minimum payment, regardless of the distance traveled. Since all the distance based detection systems have entry/exit ramps readings, some models require the ability to model additional flat fees on top of the distance-based tolls, which may depend as well on the entry ramp chosen. An example of the toll cost along a path is given in Figure 1. Note that, even though the toll presented in Figure 1 is linear, the method described in this paper extends to all link-wise monotonically increasing functions.

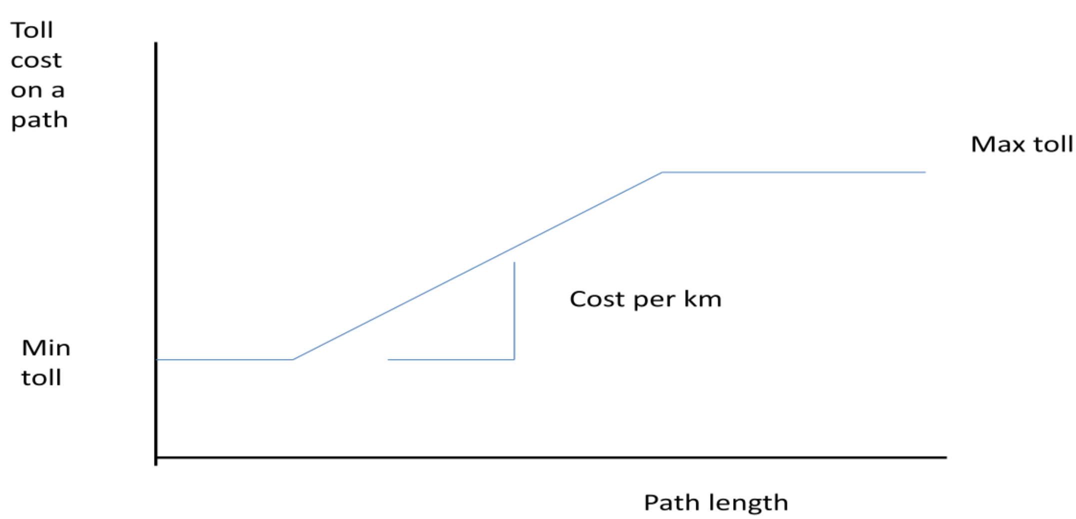

Figure 1 – An example of the toll cost along a path

In the absence of capping, distance-based tolls are just link additive; that is, their contribution along a path is the sum of the tolls corresponding to each link belonging to the path. These tolls can be modelled by classic traffic equilibrium methods by solving for the user equilibrium (see Wardrop (1952)) with a fixed link cost term added to the objective function (see for example Larsson and Patriksson (1998), Hagstrom (1998), Florian (2006)). In the presence of capping, distance-based tolls are not link additive, that is, the cost of a trip does not depend only on the links traveled but also on the path chosen. General algorithms for the non-additive traffic equilibrium problem have been considered in several papers, using path enumeration (see for example Bernstein and Gabriel (1997), Loa and Chenb (2000), Chen et. al (2010), Qian et. al (2013)). Of particular interest is Lawphongpanich and Yin (2011), who consider this problem for piecewise linear toll functions with a path-space formulation, along with conditions which allow to convert the problem into a link-based formulation. Their method requires the solution of a path generation sub-problem, which is not trivial. The method presented in this paper relies on a network construction and facilitates its solution by any method for computing network equilibrium flows.

In practice, distance-based tolls can be modelled by a method referred to as "ramp-to-ramp tolls", which avoids the enumeration of the complete paths between the origin and destination. Yang et. al (2004) describes such an algorithm, without a formal proof of its convergence and correctness.

The ramp-to-ramp approach from Yang et. al (2004) can model distance-based tolls under the condition that there are no alternative tolled roads between an entry-exit ramp pair, and there are no entry-exit ramp pairs in between that are not tolled. In such a case, the proportionality between the cost and the traveled distance would be ambiguous since for each entry/exit pair only one cost can be specified. In addition, the ramp-to-ramp method requires preprocessing of the toll scheme in order to obtain ramp-to-ramp toll costs for all the possible combinations of entry/exit ramps, as well as the additional network construction.

This article presents a new model and algorithm for distance-based toll modelling which directly uses the toll cost per link instead of the cost of entry/exit ramp pairs. Similar to the ramp-to-ramp method, the algorithm is based on the construction of additional links, which span the toll road, with the significant difference that the original links are kept as well and are used during the equilibration process. In order to solve the static traffic assignment problem on this augmented network, a multithreaded version of the bi-conjugate linear approximation method (Mitradjieva and Lindberg (2013)) has been adapted for the particular network structure considered. The new method is illustrated with an example of capped distance-based toll modelling on a simple application and another which originates from practice.

The paper is organized as follows. The next section introduces the notation used and the model formulation. Section 3 explains the conversion of the non-additive path cost problem into an additive one via a network construction, along with proofs of correctness. Section 4 describes the main steps of the method. In section 5 a small numerical example is given. In section 6 a large-scale application is given, where the influence of the toll cap is illustrated with an application for the Sydney M2 toll highway. Section 7 offers some conclusions.

2. Model formulation

In order to formulate the traffic assignment problem in the presence of distance-based tolls the following notation is used. A road network $G=(N,A)$ consists of nodes n, $n \in N$ and directed arcs (also called links) a, $a \in A$ which may carry vehicular traffic. The demand for origin-destination (O-D) pair i, $i \in I \subset N \times N$ is denoted by g_i. These demands use paths k, with $k \in K_i$, where K_i is the set of paths used for travel between O-D pair i. The vector of flow on links a is denoted by $v = v_a, a \in A.$. In its simplest form, the cost c_a on arc a is the sum of the travel time function, denoted as $s_a(v_a)$, and a toll t_a that is converted into time units from its money value τ_a by a function which is assumed monotonically increasing. Also, it is assumed that all the link volume-delay functions $s_a(v_a)$ are monotonically increasing.

All the paths taken by the vehicles between an O-D pair i form a set denoted by K_i. The set of all the paths in the network is the union of all the paths $K = \cup_{i \in I} K_i$.

The time of a path $k \in K_i$ is link additive

$$s_k = \sum_{a \in A} \delta_{ak} s_a(v_a), \tag{1}$$

where

$$\delta_{ak} = \begin{cases} 1 & \text{if } a \in k \\ 0 & \text{otherwise} \end{cases}. \tag{2}$$

If $t^{\max}$ is the toll cap, the toll along a path is

$$t_k = \min(t^{\max}, \sum_{a \in A} \delta_{ak} t_a). \tag{3}$$

Hence, the cost of a path is

$$c_k = \sum_{a \in A} \delta_{ak} s_a(v_a) + \min(t^{\max}, \sum_{a \in A} \delta_{ak} t_a). \tag{4}$$

The path flows satisfy flow conservation and non-negativity

$$\begin{gathered}\sum_{k\in K_i} h_k = g_i \\ h_k \geq 0, \quad k \in K_i\,, \quad i \in I\end{gathered}\,. \tag{5}$$

The link flows are

$$v_a = \sum_{i\in I}\sum_{k\in K_i} \delta_{ak} h_k\ . \tag{6}$$

The network equilibrium flows (Wardrop (1952)) satisfy

$$u_i\ = \min_{k\in K_i}\left(c_k\right), \quad i \in I \tag{7}$$

and

$$c_k^* - u_i\ = \begin{cases} 0 & if \quad h_k^* > 0 \\ \geq 0 & if \quad h_k^* = 0 \end{cases}, \tag{8}$$

where * denotes the equilibrium values for path travel times and path flows. This is a network equilibrium model with non-additive costs due to the capped tolls.

3. Conversion into a link additive model

In order to formulate the distance-based toll network equilibrium model it is assumed, without loss of generality, that the toll road forms a contiguous stretch of links; if this not the case, the tolled facility can be split into several disjoint sequence of links, each including a stretch of road that is tolled. For the sake of simplicity, node turn penalties are not considered.

Consider now the sub-network shown in Figure 2 where links *a*, *b*, *c*, and *d* are subject to distance-based tolls, proportional to their length.

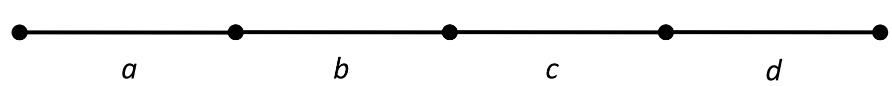

Figure 2 – Sub-network with distance-based tolls

In the absence of toll caps, the toll paid by a road user traveling through links *a* to *d* is the sum of the toll paid on links *a*, *b*, *c*, and *d*, that is, $t_{a\to d} = t_a + t_b + t_c + t_d$. If the tolls are capped to $t^{\max}$, the toll paid is $t_{a\to d} = \min\left(t^{\max}, t_a + t_b + t_c + t_d\right)$.

Hence, the classical network equilibrium formulation which adds a toll to each link is no longer valid since the tolls are non additive. In order to convert non-additive tolls to additive tolls a network construction is used. All the possible paths on the toll highway are enumerated in order to take into account the path dependent cost due to the toll cap. Since tolled facilities consist of a relatively small number of links of the network, the enumeration of all the paths needs to be done only for the links of the tolled facility, as shown in Figure 3. (See also Yang et. al (2004)).

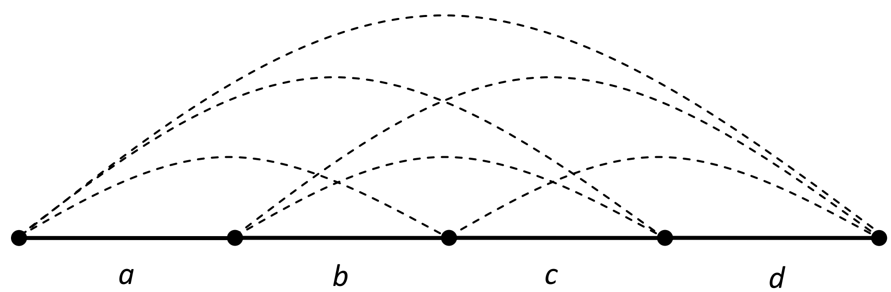

Figure 3 - Additional links constructed on top of the sub-network

The links inherit the travel times of the spanned stretch of the road, and the capped toll costs. Unlike the method described in Yang et. al (2004), the original tolled links of the network are not removed since a single tolled link can be used part of a larger path. Note that the cost of a trip that uses the original tolled links without a toll cap is implicitly compared with the same trip with a toll cap via this construction. If the toll cap applies then the path used will be the one that has been subject to a toll cap since it is of lesser cost.

Since the toll cost on the additional links is known *a priori* and is fixed during an assignment, we transformed in this way a model with non-additive path costs into a model with additive path costs on an augmented network. A demonstration that the two problems are equivalent is given in the following paragraphs.

The augmented network is indicated by $\tilde{G}=\left(N,\tilde{A}\right)$ where $\tilde{A}=A\cup D$ is the set of the base network links and D is the set of the additional links. Additional links are assigned a two-letter index, while links that belong to the base network A are referred to with only one index. For example, an additional links which spans a toll road from link *j* to link *k* is denoted by $jk\in D$. In order to distinguish between toll and non-toll links, the set of toll links denoted by T , $T\subset A$.

Assuming that all the toll links form a contiguous stretch numbered from 1 to *n*, the cost c_a of a toll link $a\in T$, $1\leq a\leq n$, can be expressed as:

$$c_a = s_a\left(\sum_{1\le j<a<k\le n} v_{jk} + v_a\right) + t_a, \quad j \ne k\,. \tag{9}$$

Note that the cost of a toll link $a \in T$ is computed by using the total flow on that link, which includes all the trips that use it:

$$v_a^{total} = \sum_{1\le j<a<k\le n} v_{jk} + v_a, \quad j \ne k\,. \tag{10}$$

For example, the cost of the toll link *b* from Figure 2 can be expressed as:

$$c_b = s_b\left(v_{ab} + v_{ac} + v_{ad} + v_{bc} + v_{bd} + v_b\right) + t_b \tag{11}$$

With the same notations, the cost of an added link, which spans the original network links from link *l* to *m*, $1 \le l < m \le n$ is:

$$c_{lm} = \sum_{a=l}^{m} s_a\left(\sum_{1\le j\le a\le k\le n} v_{jk} + v_a\right) + t_{\widehat{lm}}, \quad j \ne k\,. \tag{12}$$

Since the toll of an added link can be written as

$$t_{\widehat{lm}} = \min\left(t^{\max}, \sum_{a=l}^{m} t_a\right), \tag{13}$$

equation (12) can be rewritten as

$$c_{lm} = \sum_{a=l}^{m} s_a\left(\sum_{1\le j\le a\le k\le n} v_{jk} + v_a\right) + \min\left(t^{\max}, \sum_{a=l}^{m} t_a\right), \quad j \ne k\,, \tag{14}$$

where $s_a(\cdot)$ is the cost function of link $a \in T$, v_{jk} is the flow of the additional link spanning links *j* to *k*, v_a is the flow of the base link.

The single-class network equilibrium problem on the augmented network $\hat{G}$ can be written as:

$$\min\left(\sum_{a\in A-T} \int_0^{v_a} \left(s_a(x) + t_a\right)dx + \sum_{a\in T} \int_0^{v_a} \left(s_a(x) + t_a\right)dx + \sum_{\hat{a}\in D} \int_0^{v_{\hat{a}}} \left(s_{\hat{a}}(x) + t_{\hat{a}}\right)dx\right) \tag{15}$$

This is a classic traffic equilibrium problem with fixed costs added to some links, with two exceptions. First, as indicated in (12), an added link $\hat{a}, \hat{a} \in D$ does not have its own cost function, but inherits the ones from the spanned base links. Since these functions are simple sums of monotonically increasing functions and the toll value is fixed, this dependency does not influence the optimization problem (15). The second and third terms of the objective function are non-separable since the cost of a link does not depend only on the flow of that link as indicated in (9). The objective function to optimize from (15) is well defined if and only if the Jacobian matrix of the cost functions is symmetric everywhere (see Ortega and Rheinboldt (1970) or Larsson and Patriksson (1998)). The terms of the Jacobian matrix related to links $a \in A-T$ are symmetric since s_a depends only on v_a. In order to show that the terms of the Jacobian matrix are symmetric for links $a \in T$ and $\frown{a} \in D$, one has to show that

$$\text{a) } \frac{\partial s_{ij}}{\partial v_{kl}} = \frac{\partial s_{kl}}{\partial v_{ij}} \quad , \tag{16}$$

$$\text{b) } \frac{\partial s_{a}}{\partial v_{ij}} = \frac{\partial s_{ij}}{\partial v_{a}} \quad , \tag{17}$$

where $a \in T$, $ij \in D,\ lk \in D,\ 1 \le i < j \le n,\ 1 \le l < k \le n$, assuming that the tolled links $a \in T$ are contiguously numbered from 1 to n.

According to (10) and (12),

$$\frac{\partial s_{ij}}{\partial v_{kl}} = \frac{\partial}{\partial v_{kl}} \sum_{i \le a \le j} s_a\left(v_a^{total}\right) = \frac{\partial}{\partial v_{kl}} \sum_{i \le a \le j} s_a\left(\sum_{1 \le b \le a \le c \le n} v_{bc} + v_a\right),\ b \ne c . \tag{18}$$

Recall from (10) that the partial derivatives are evaluated with respect to the total flow on a toll link. The total flow of term s_a on v_{kl} depends on the relative position of the four indices $i,\ j,\ l,\ k$ in the integer interval $[1,n]$. If the integer intervals $[i,j]$ and $[k,l]$ are disjoint, there is no dependency, therefore the partial derivative is zero. If they are not disjoint, there are as many base link cost terms as the number of overlapping indices in the intersection of the two integer intervals. Therefore:

$$\frac{\partial s_{ij}}{\partial v_{kl}} = \sum_{a \in [i,j] \cap [k,l]} s_a{}'\left(v_a^{total}\right) = \frac{\partial s_{kl}}{\partial v_{ij}}, \tag{19}$$

such that (16) follows.

According to (9), using the same integer interval argument

$$\begin{cases} \dfrac{\partial s_a}{\partial v_{ij}} = s_a{}'\left(v_a^{total}\right) & if\ \ a \in [i,j] \\ \dfrac{\partial s_a}{\partial v_{ij}} = 0 & if\ \ a \notin [i,j] \end{cases} . \tag{20}$$

Using (10) and (12) we get

$$\begin{cases} \dfrac{\partial s_{ij}}{\partial v_a} = s_a{}'\left(v_a^{total}\right) & if \;\; a \in [i, j] \\ \dfrac{\partial s_{ij}}{\partial v_a} = 0 & if \;\; a \notin [i, j] \end{cases}, \tag{21}$$

such that (17) follows.

Under the above assumptions, one can show that the equilibrium flows on $G = (N, A)$ with capped distance-based tolls are equivalent to the equilibrium flows in $G = (N, A)$ with fixed costs as computed in (13). Assume that equilibrium flows have been computed in $G = (N, A)$ by solving the minimization from (15). There may be a non-zero flow through the toll links. In order to traverse a contiguous toll stretch $l_1 \to l_2 \to ... \to l_n$, the flow can be split into two: a part that uses the additional link $\widetilde{l_1 l_n} \in D$ and the rest that uses a path containing at least one link from the base network $l_1 \in A$ denoted here by $l_1 \to l_{2,n}$. At equilibrium, the cost of the two paths must be equal. According to (14), the only difference between the cost of the two paths is due to the toll term, which satisfies

$$\sum_{l=l_1}^{l_n} t_l \geq \min\left(t^{\max}, \sum_{l=l_1}^{l_n} t_l\right). \tag{22}$$

If equality holds, one can shift the entire flow from path $l_1 \to l_{2,n}$ to link $\widetilde{l_1 l_n}$ without influencing the equilibrium. In the case of strict inequality, the flow on path $l_1 \to l_{2,n}$ must be zero according to the user equilibrium condition. Therefore, at equilibrium in $\tilde{G} = (N, \tilde{A})$, there are no flows that can parse a contiguous stretch of more than one toll link in the base network, but all this flow will be "absorbed" by the additional links. Since these links are virtual links corresponding to the splitting of trips on the base network based on their toll value, it follows that the two problems are equivalent.

4. Algorithm description

Briefly, the steps of the solution algorithm may be summarized as follows:

- Given $G = (N, A)$ construct $G = (N, A)$ by adding the fictitious links in D;
- From toll links in T compute the toll on fictitious links in D;
- Solve the traffic equilibrium problem in $G = (N, A)$ with tolls as additional link cost

- From the flows on fictitious links in D compute the flows on the toll links in $T \in G = (N, A)$.

The last step of the algorithm, after computing the equilibrium flows in G, consist in recovering the traffic flow on the toll links by accumulating the flow from the additional links. Assuming that the toll links form a set $T \in A$ with their corresponding links in the augmented network $T \in A$, then the total flow of a toll link in T is:

$$v_{a \in T} = \sum_{1 \leq j \leq a \leq k \leq n} v_{jk} + v_{a \in T}, \;\; i \neq j \,. \tag{23}$$

The algorithm was implemented[1] by using a multi-threaded bi-conjugate variant of the linear approximation. The solution has been extended for multiple traffic classes as well as for multiple toll facilities, each of them with different cap values. Some computational results are reported in the next sections.

5. A small numerical example

The algorithm used to solve the minimization problem from (15) is a parallelized application of the bi-conjugate variant of the linear approximation method. The direction of descent for the added artificial links is computed as a function of only the arc flow, while keeping fixed the values of all the other flows that contribute to its travel time.

The influence of the capped distance-based tolls is illustrated on the small network shown in Figure 4, which has only one origin-destination pair with a demand of 100. Simple BPR volume delay functions are assigned to all the links. The top links are subject to distance-based tolls, which are all set to the same value (10 generalized cost units/unit of distance).

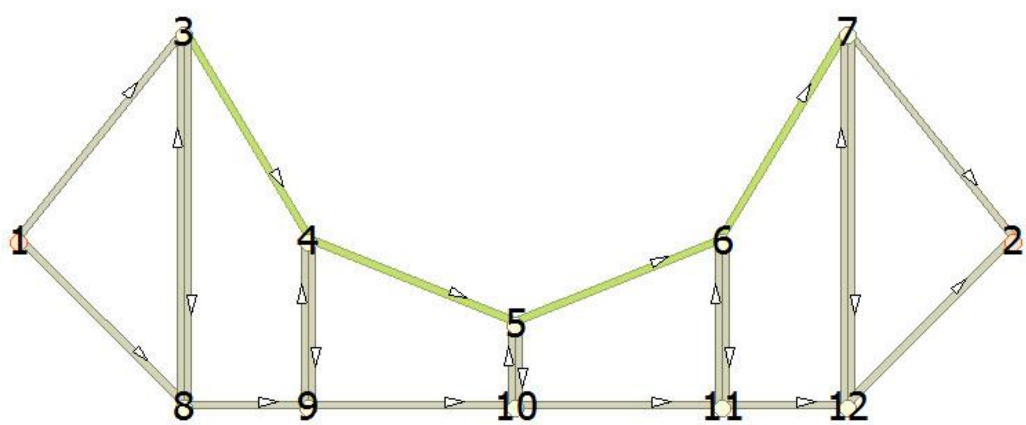

Figure 4 - Simple network with tolled links in green

[1] Implemented in the Emme 4 software package INRO(2013)

The proposed method was applied on five different scenarios, which share the same network and toll values but with different cap value applied to the tolls: 0, 10, 20, 30, and 40. In Figures 5 to 8, scenario comparisons are showing with the difference in link flows between the scenario with 0 cap (no toll paid) and the others. The green bars show the flow that shifted from the tolled path 1→3→4→5→6 to the non-tolled path 1→8→9→10→11 because the former becomes more attractive when a lower toll cap is applied. Red bars indicate an increase in the flow.

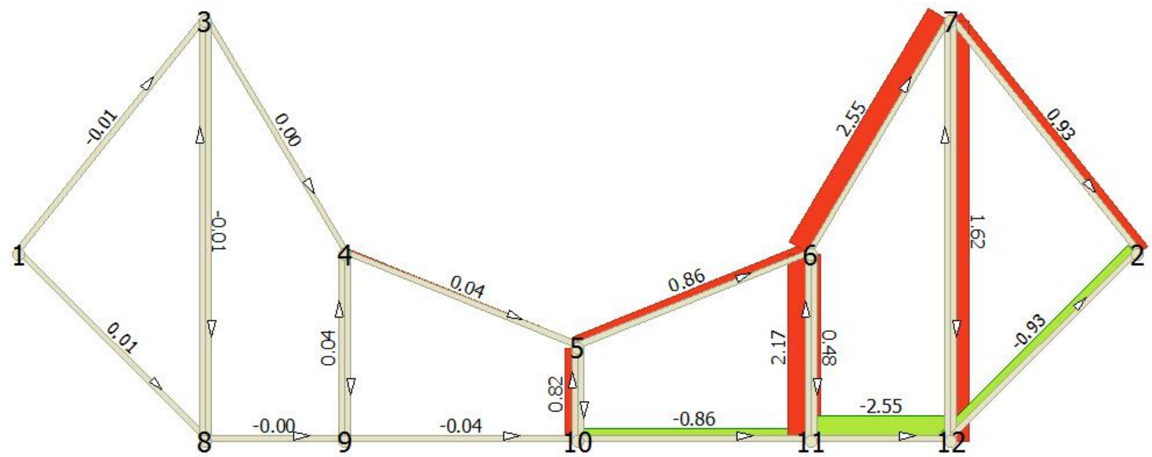

Figure 5 - Link flow comparison with toll cap 0 vs. 10 cost units

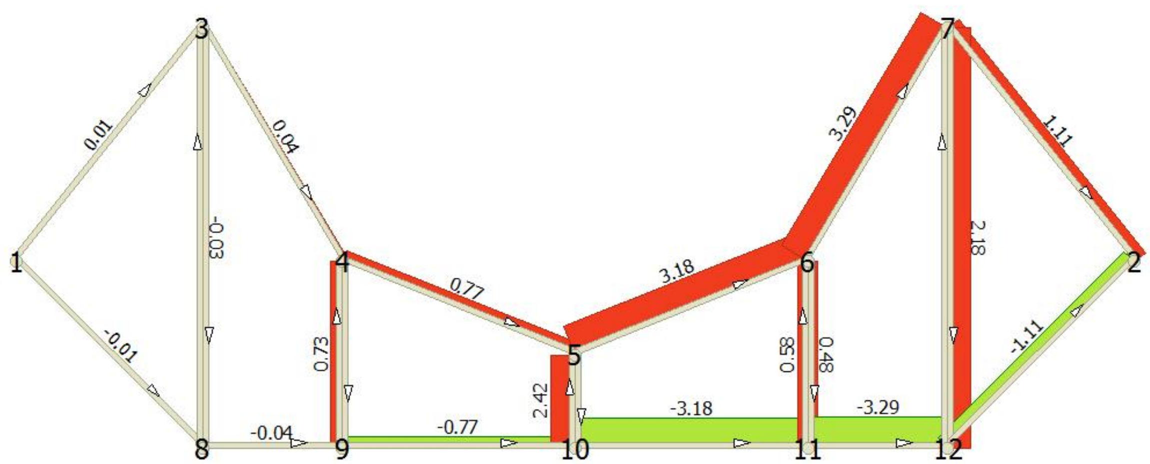

Figure 6 - Link flow comparison with toll cap 0 vs. 20 cost units

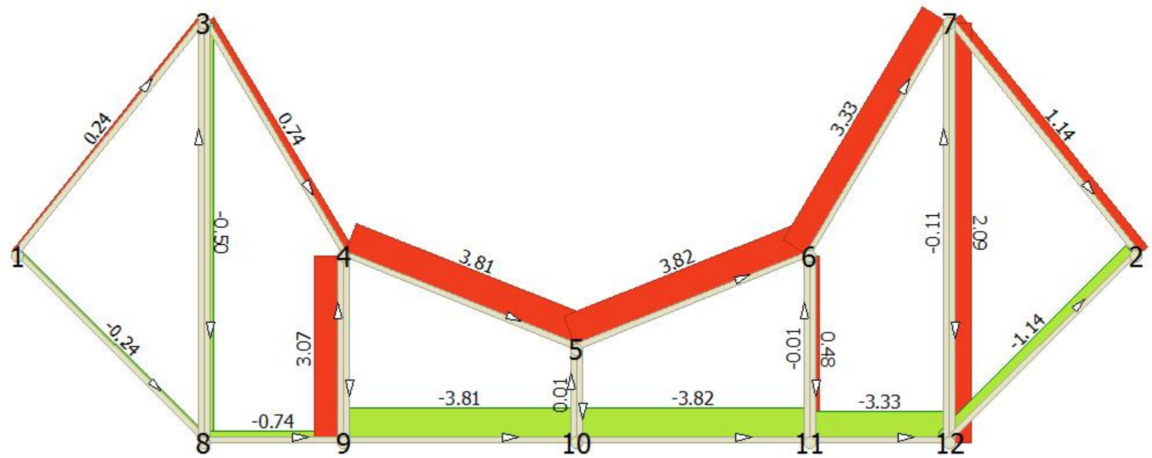

Figure 7 - Link flow comparison with toll cap 0 vs. 30 cost units

Figure 8 - Link flow comparison with toll cap 0 vs. 40 cost units

6. Capped distance-based toll study on Sydney M2 highway

The method was applied in the evaluation different the toll cap values for the eastbound M2 toll highway in Sydney, Australia, shown in blue in Figure 9[2].

[2] By the courtesy of INRO.

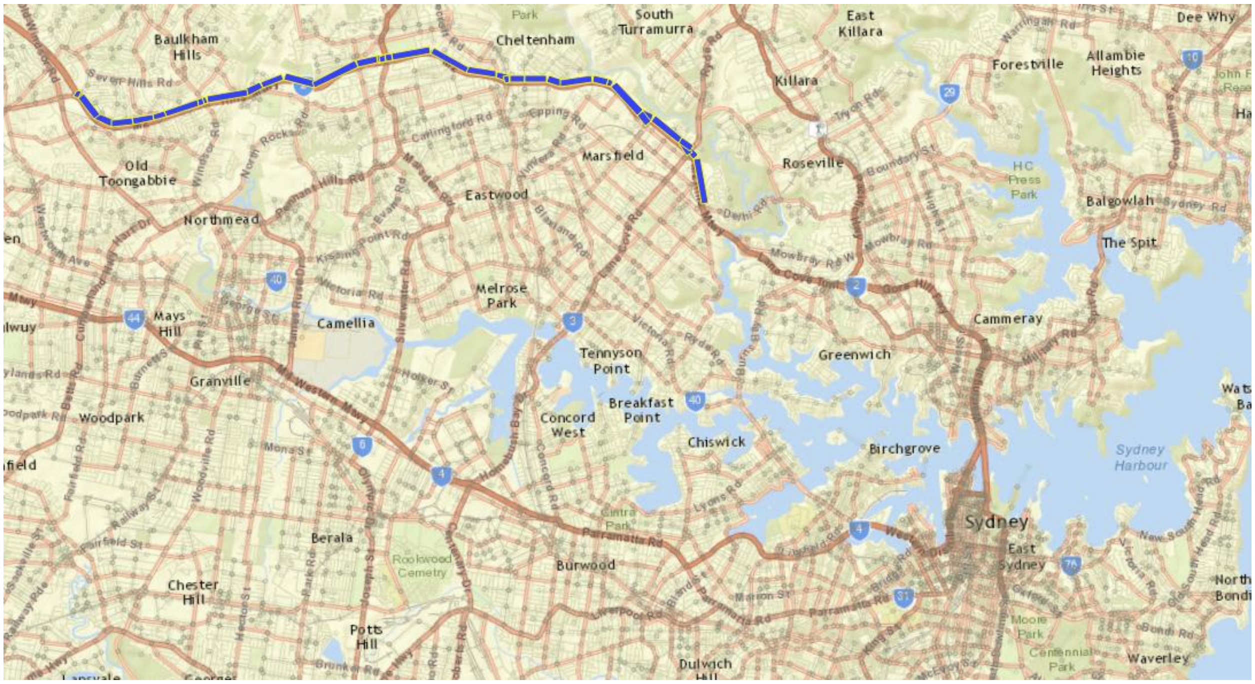

Figure 9 – The M2 toll highway is shown in blue on the map

This application uses 30 classes of traffic that correspond to different values of time for the private car and truck modes. These 30 classes were derived from continuous value of time distributions, which are shown in Figure 10.

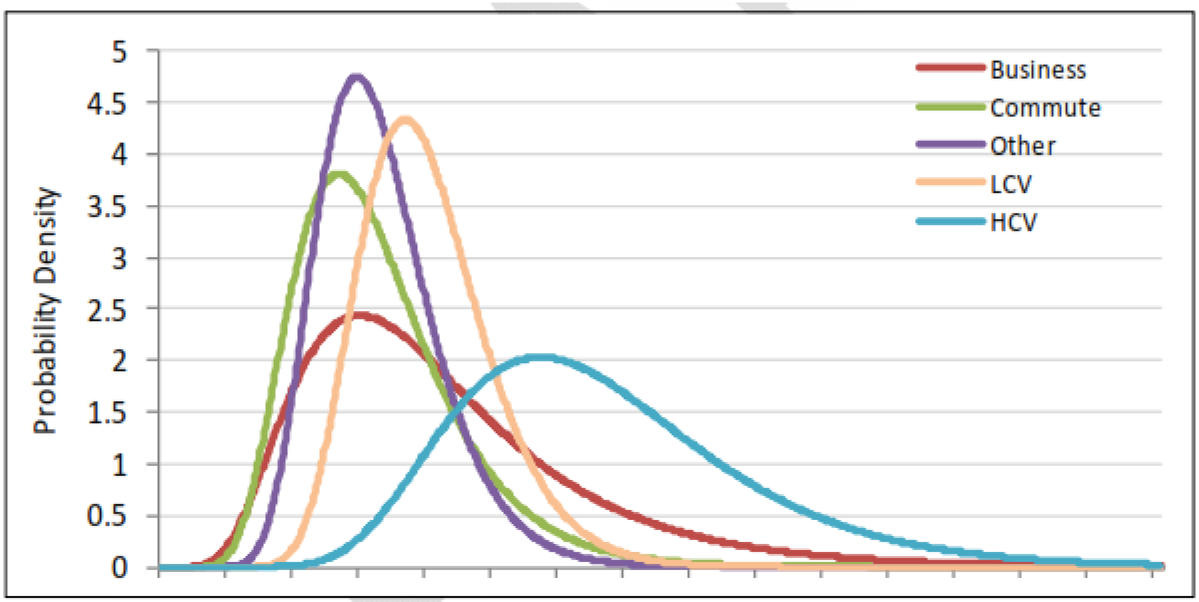

Figure 10 – Lognormal distributions of the value of time by user class

The computational results reported here use 10 different cap values, varying from 0$ to 60$ (0% to 1000% from the prevailing 6$ toll cap, respectively). The 10 corresponding equilibrium assignments were executed using a relative gap of ~10^{-4} as a stopping criterion. The results obtained are shown for several links of the M2 highway. The monotonically decreasing volumes on four links are plotted in Figure 11.

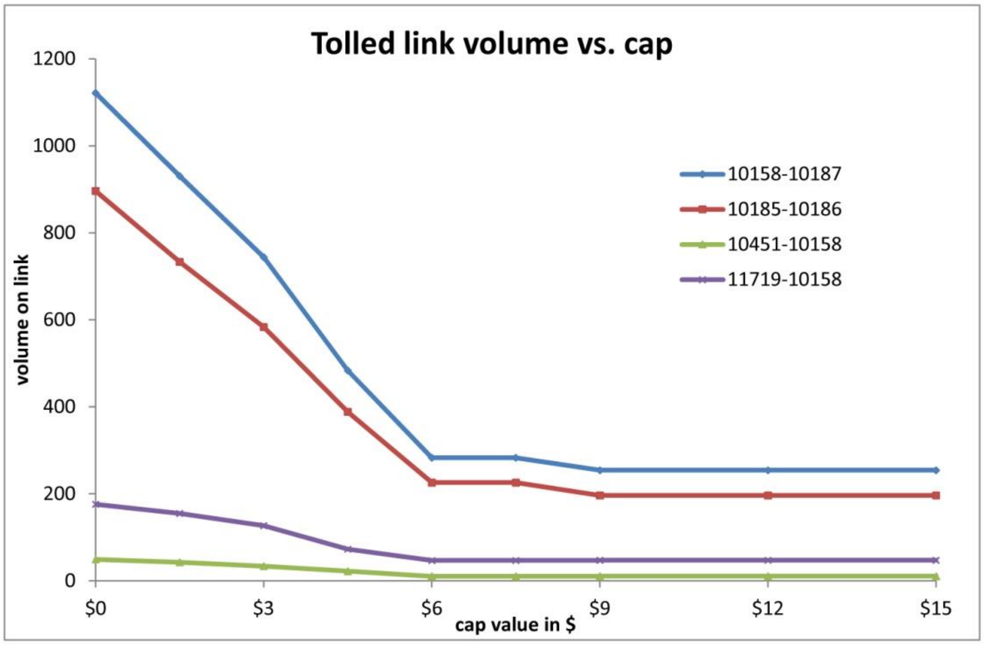

Figure 11 – The impact of cap value over link volume on the toll highway

The toll cap that is in effect at the time that these analyses were done was 6.00$. There are no plans to change this value. It should be mentioned that these analyses are just done for illustrative purposes and do not correspond to any results that were obtained in a study of this toll highway. The results are easily interpreted to conclude that additional revenues may be realized by increasing the toll cap since certain classes of users with a high value of time are nearly indifferent to the value of the toll cap

7. Conclusions

The method presented in this paper for computing distance-based tolls with minimum and maximum cap values has the advantage that it can be solved by any algorithm that computes network equilibrium flows. Other significant contributions in the literature that have addressed network equilibrium flows with non-additive tolls resort to far more complex algorithmic solutions or are restricted to linear dependent distance toll. The proof that is presented in this paper does not relay on the linearity of the toll function of distance; the only requirements are link-wise dependency via monotonically increasing functions. Moreover, the same proof can be straightforward applied in order to show that the equilibrium problem solved in Yang et. al (2004) is in fact equivalent to the original equilibrium problem with non-additive path cost for networks with entry–exit based tolls.

8. Acknowledgements

We would like to express our appreciation to SKM Australia who permitted us to report the computational results on the M2 toll highway of the Sydney network model used for planning.

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