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An Inventory Model with Recovery and Environment Considerations

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Abstract. Current environmental regulations and economic conditions force organizations to limit Greenhouse Gas (GHG) emissions. Since inventories have proven their crucial role in supply chains, the aim of this paper is to study the impact of inventory control in reducing the environmental damage of an organization. Modeled as a Markov Decision Process (MDP), we dealt with a stochastic recovery inventory system considering an infinite-horizon, and a cap-and-trade mechanism. In numerical examples, we compute optimum replenishment remanufacturing and manufacturing quantities, and characterize the structure of the optimal inventory policy. We show that there is a direct link between carbon credit price and inventory policies. Moreover, there is a carbon credit price from which the organization prefers to lost sales than to invest in carbon credits. Ultimately, we extend our results towards a finite-horizon context, and discuss the impact of environmental strategies in replenishment decisions.

Keywords. Inventory control, green supply chain, closed-loop supply chain, emissions trading, Markov Decision Processes (MDP).

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1 Introduction

Several factors, including the increasing environmental deterioration, and legislation, have forced organizations to redesign their methods of supply chain management under a financial, ecological and social perspective. In particular, product recovery and Greenhouse Gas (GHG) reduction policies are gaining increasing attention from researchers in order to minimize the environmental impact of supply chain management.

Product recovery and GHG reduction policies are studied by green supply chain management (GrSCM). GrSCM is defined by Srivastava (2007) as the integration of the environmental thinking into supply-chain management, including product design, material sourcing, manufacturing processes, delivery, as well as end-of-life management of the product after its useful life ends. This paper focuses primarily on recovery activities and GHG reduction policies. Motivations for recovery include the reduction of raw material costs, the decrease of waste disposal cost, and in some cases laws enacted by government agencies. Meanwhile, GHG reduction is mainly motivated by new environmental laws and regulations. This is the case of the Western Climate Initiative (WCI) program launched in 2010. It aims to reduce the GHG level of 2005 by 15% in 2020. The program is based on a cap-and-trade scheme. Under this policy, the total quantity of emissions generated by the regulated industries within a period must be below an emission-cap. Hence, emitters might reduce, buy, or sell their emissions; driven by a trade-off between costs and environmental performance. Several Canadian provinces as well as some U.S. states have already joined the WCI program.

The particular case of GrSCM studying systems which merge forward and reverse flows is known as Closed-loop supply chains (CLSC). Therefore, contrary to popular belief CLSC are not sustainable supply chains by definition (Quariguasi Frota Neto et al., 2009). In CLSC, inventory plays a key role in managing traditional and reverse flows, and it has already been studied by some authors. The authors van der Laan et al. (2004), studied replenishment decisions in CLSC. Later on, Ahiska and King (2010) considered setup costs and different lead-times. However, while the introduction of recovery strategies might bring benefits to companies, higher complexity is faced by inventory models which have to deal with multiple reuse options, random returns and convergent flows (Fleischmann, 2001).

GHG reduction policies such as: 1) emission tax, 2) direct-cap, and 3) cap-and-trade, have also been the subject of multiple studies. Recently, Chaabane et al. (2010) studied the integration of emission trading on the design of supply chains. Later on, Benjaafar et al. (2013) gave several managerial insights emphasizing the importance of operational decisions on emission reduction. However, despite progress on environmental policies understating, thus far neither the impact of environmental constraints on inventory policies nor the impact of inventory control in environmental performance have been defined.

In practice, CLSC systems are sophisticated structures since they must support flows in both directions. While the inclusion of environmental policies adds complexity to the system, inventory control might play an important role in achieving environmental objectives. To our knowledge, few publications have dealt with this problem. Surprisingly, most of them focus on deterministic demand despite the stochastic behavior of most processes. Our study contributes to the understating of the relationship between environmental policies and CLSC inventory control by developing of an environmental stochastic inventory model integrating a CLSC structure and a cap-and-trade scheme. Furthermore, using Markov Decision Process (MDP), we characterize the optimal inventory policy for some numerical examples that allow us to gather information about the behavior of inventory policies facing environmental constraints. Hence, the purpose of this paper is: first to provide a stochastic inventory model for remanufactured items under the inclusion of an emissions trading scheme; second, to derive insights through numerical examples about the impact of emissions trading constraints on the structure of the inventory policies; and third, to determine the impact of inventory policies on environmental performance. The rest of the paper is organized as follows: in Section 2, we present a literature review. Section 3 describes the proposed inventory model. Numerical results are given in Section 4. Section 5, extend our results to a finite-horizon scenario. Conclusion and further work are presented in Section 6.

2 Literature review

Our research is focused on two streams: 1) period-review CLSC inventory control, and 2) environmental inventory control. In recent years, literature on CLSC inventory control has become more prolific and it has been studied from a continuous and periodic point of view as in traditional inventory control. Nevertheless, for purposes of this paper, we are only focusing on periodic-review systems. Moreover, we only discuss the most relevant studies for our paper.

Initial research on a periodic-review approach with random demand and returns was published in 1978. Using dynamic programming, Simpson (1978) characterized the optimal periodic policy involving product recovery. Latter policy is defined by three parameters per period: 1) the manufacturing-up-to-level (S_p), 2) the remanufacturing-up-to-level (S_r) and 3) the disposal-down-to-level (U). Each parameter denotes the trigger to produce, remanufacture, and dispose, respectively. Inderfurth (1997) extended the above model to the inclusion of lead-times. Kiesmüller and Scherer (2003) revised the work of Inderfurth (1997) and derived a method for the exact computation of S_p, S_r and U under a single product system with a finite horizon. Since exact computation leads to high computation effort, the authors developed two approximation methods. The authors van der Laan et al. (2004), extended the model presented by Inderfurth (1997). A hybrid system (S_p, S_r, U) under finite horizon with different lead-times, demand, and returns was introduced. Ahiska and King (2010) extended the model presented by van der Laan et al. (2004). The authors considered non-zero remanufacturing and manufacturing setup costs and different lead-time structures. Modeling the system as a discrete-time MDP, the authors were capable to characterize the optimal policy. Hence, for the given scenario, the optimal policy is composed by four parameters: 1) the reorder level for manufacturing (s_p), 2) the manufacturing-up-to-level (S_p), 3) the reorder level for remanufacturing (s_r) and 4) the minimal quantity to remanufacture (q_r).

The second stream of research involves environmental policies on supply chain decision, in particular, inventory control. Sheu et al. (2005) present a Multi-objective Linear Program (MOLP) seeking to maximize the economic profit of the organization in an environmental context. Chaabane et al. (2010) addressed the inclusion of the carbon market into a supply chain. They dealt with the problem with a MOLP. Finally, Diabat and Simchi-Levi (2009) treated the minimization of environmental impacts with the inclusion of a carbon constraint. A mixed integer programming technique that determines the structure of a supply chain was developed. Regarding literature under the context of inventory control, the study of Bonney and Jaber (2010) proposed an extension of the Economic Order Quantity (EOQ) model, the "Enviro-EOQ." In addition to traditional costs, they considered disposal and emission costs from transport. The authors concluded that when the environmental costs are introduced, the size of the lot is greater than the one provided by the traditional EOQ model. The work of Arslan and Turkay (2010) also extended the EOQ model, yet towards the integration of the sustainable concept. In their work, they presented five environmental management approaches to make the inclusion: 1) direct accounting, 2) carbon tax, 3) direct cap, 4) cap-and-trade and 5) carbon offsets. Hua et al. (2011) included an environmental damage cost in the development of their model. Using a deterministic approach, they carried out an extension of the EOQ model. The authors determined the effect of the economic lot size, carbon price, emissions and legislation in the total cost. The study of Chen et al. (2011) also focused on the EOQ model. Latter study is based on the traditional objective function, subject to an emission-cap. The authors proved that a cap is effective only when it is small enough to trigger a change in the quantity to order. Bouchery et al. (2011) presented an extension of the EOQ model, named "the Sustainable Order Quantity" (SOQ) model. A multi-objective formulation coupled with an iterative method which allows interaction with decision makers, is presented. Chen and Monahan (2010) analyzed the impact of environmental policies in inventory levels. Based on a stochastic model with random demand and environmental impacts over a finite horizon, the authors determined the optimal inventory policies.

The stochastic scenario has been studied by Song and Leng (2012). The authors investigated the newsvendor problem under a carbon emission tax, a direct-cap and emission trading scheme. A recent work on green inventory presented by Rosič and Jammernegg (2013) explores companies' decisions considering transport carbon emission. Based on the newsvendor framework, the author presents a basic dual outsourcing model. Finally, Benjaafar et al. (2013) provide a series of insights that highlight the impact of operational decisions on carbon emissions, and the importance of operational models in evaluating the impact of different regulatory policies, and in assessing the benefits of investment in more carbon efficient technologies.

Although literature related to sustainability and inventory management is increasing, to the best of our knowledge, there are no papers dealing with the inclusion of environmental policies into a stochastic CLSC inventory model. Based on the model presented by Ahiska and King (2010), we introduce environmental considerations into inventory control. Through numerical examples based on the aluminum industry, we derive insights to characterize the optimal inventory policy. Thus, the aim of this paper is to provide a stochastic inventory model under the inclusion of emission trading, as well as to determine the implication of emission trading constraints in the structure of inventory policies.

3 Problem Definition

In the sequel we study a single-item, two-echelon system with returns subject to a cap-and-trade program. Latter program allow to buy up to a maximum of carbon credits in case of exceeding the emission-cap, and to sell up to a maximum of allowances, only if a β-emission reduction from the previous period has been achieved. The system, illustrated by Figure 1, is a periodic-review process modeled in discrete time. It considers two finite capacity stocking points: 1) remanufacturable and 2) serviceable inventories. The holding costs in the remanufacturable (serviceable) inventory are h^R (h^S) per unit hold per period. In most practical situations $h^R \leq h^S$, since in serviceable products, a value has been added. Environmental implications from holding activities are not considered, since they are considered negligible compared to the manufacturing and remanufacturing environmental impact.

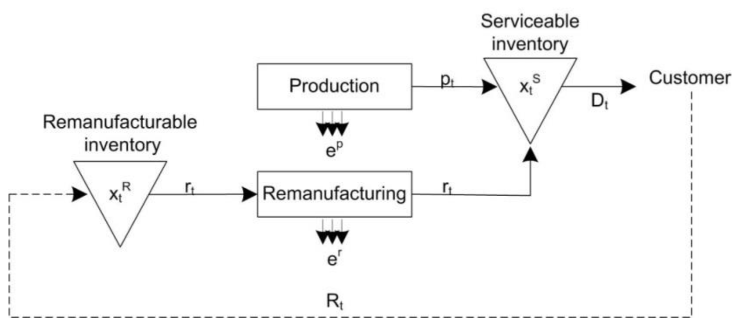

Figure 1: Remanufacturing System

Remanufacturable inventory is replenished by returns. All recovered products meet quality standards for reuse. The remanufacturing process has limited capacity and a single period lead time, which increases the serviceable inventory level at the end of the period. At each period there is a minimal proportion α of the remanufacturable inventory that must be remanufactured at each period. The economic and environmental contributions of remanufacturing are denoted, respectively, by C_r and e^r per quantity remanufactured. Serviceable inventory is also replenished by manufactured products. The manufacturing process, which uses virgin material, has limited capacity and a single period lead time as remanufacturing. In consequence, manufacturing also raises the inventory level at the end of the period. The manufacturing cost consists of a variable cost C_p per product. The manufacturing environmental contribution is denoted by e^p which is the amount of emissions generated per quantity produced.

The problem is modeled as a MDP. The dynamics of the system are the following: at the beginning of a period, inventories are revised, and remanufacturing and manufacturing decision are made. Then, throughout the period, demands and returns are observed. We assume that demands D_t and returns R_t in each period t are independent, non-negative, and discrete random variables with a probability distribution $\phi(i) = \Pr[D_t = i]$ and $\phi(j) = \Pr[R_t = j]$, respectively. Demand and return rates remain unchanged from period to period. Furthermore, demands that cannot be fulfilled immediately are backordered up to a maximal quantity κ^v, above which sales are lost. In addition, disposal of returns is only considered when remanufacturable inventory capacity is exceeded since disposal is only relevant for excessively high return rates (Teunter and Vlachos, 2002). Ultimately, holding costs, penalties (lost-sale and backorder), as well as environmental impacts are considered at the end of the period. The objective is to characterize the policy that will determine at each period, the quantity of products to remanufacture (r_t) and to manufacture (p_t), minimizing the total cost while respecting an emission trading program.

3.1 Notation

Notations used throughout this paper are the following:

Parameters

κ^r	Remanufacturing capacity.
κ^p	Manufacturing capacity.
κ^S	Serviceable inventory capacity.
κ^{aR}	Recoverable inventory capacity.
κ^v	Maximum amount of backlog allowed.
κ^e	Maximum amount of credits allowed to buy or to sell.
$\phi(i)$	$\Pr[D_t = i]$.
$\phi(j)$	$\Pr[R_t = j]$.
E^c	Emission-cap (limit on carbon emissions).
e^r	Amount of carbon emissions per remanufactured product.
e^p	Amount of carbon emissions per manufactured product.
α	Minimal proportion of recoverable inventory to remanufacture per period.
β	Minimal emission reduction between period *t* and *t+1* to allow selling of carbon credits at period *t.*

Costs

h^S	Holding cost per unit of serviceable product per period.
h^R	Holding cost per unit of remanufacturable product per period.
v	Shortage cost per unit.
C_r	Remanufacturing cost per unit.
C_p	Manufacturing cost per unit.
C_d	Disposal cost per unit.
C_{ls}	Lost sale cost per unit.
τ_r	Remanufacturing setup cost.
τ_p	Manufacturing setup cost.
${C_c}^+$	Carbon credit purchase price.
${C_c}^-$	Carbon credit selling price.
C^{e}	Cost per emissions generated.

Random Variables

D_t	Stochastic demand in period *t*.
R_t	Stochastic returns in period *t*.

Decision Variables

p_t	Quantity of products manufactured in period *t*.
r_t	Quantity of products remanufactured in period *t*.
${C_t}^+$	Carbon credits bought in period *t*.
${C_t}^-$	Carbon credits sold in period *t*.

3.2 Problem Formulation

The system state is characterized by the remanufacturable inventory level ${x_t}^{\mathrm{R}}$, the serviceable inventory level ${x_t}^{\mathrm{S}}$, the number of carbon credits e_t possessed by the company, and the number of emissions generated at the previous period ϖ_t. Consequently, the state space $\mathcal{S}$ is defined by {[0, κ^S]×[0, κ^{aR}]×[0, E^c] ×[0,$E^c + \kappa^e$]}. Then, the state of the system at the beginning of a period is given as: $s_t \coloneqq ({x_t}^{\mathrm{S}}, {x_t}^{\mathrm{R}}, , e_t, \varpi_t)$. The action space $\mathcal{A}(s_t)$, corresponds to the set of all possible decisions satisfying the constraints, given the system state s_t. The decisions are a combination of the decision to manufacture [0, κ^p], to remanufacture [0, κ^r], and to buy or sell allowances [0, κ^e]×[0, κ^e]. Generally, decisions $d_{s_t}(\pi)$ are specified for each state $s_t \in \mathcal{S}$ according to a policy π. For a given problem, there might be several possible policies denoted by the set Π. We consider a stationary policy only. Hence, decisions are determined by the current state of the system, regardless of time.

Transition from state s_t to state s_{t+1} will depend of the set of decisions $d_{s_t}(\pi) := (p_t, r_t, {C_t}^+, {C_t}^-)$ taken according to the policy π as well as random variables (demand and returns) associated with their corresponding probabilities. For the studied system, the determination of the transition probability matrix is relatively straightforward; it is defined by the joint probability of the demand and returns, i.e.: $P_\pi(s_t, s_{t+1}) = \sum_{j=0}^{\infty}\sum_{i=0}^{\infty} \Pr[D_t = i]\Pr[R_t = j]$.

The transition from state $s_t$to state s_{t+1} where $s_{t+1} :=(x_{t+1}{}^{\mathrm{S}}, x_{t+1}{}^{\mathrm{R}}, ,e_{t+1} ,\varpi_{t+1})$, is given by expressions 1 to 4.

$$x_{t+1}^R = \min\{j - r_t + x_t^R, \kappa^{\mathrm{aR}}\} \tag{1}$$
$$x_{t+1}^S = \min(\max(x_t^s - i, -\kappa^v) + p_t + r_t, \kappa^S) \tag{2}$$
$$e_{t+1} = e_t - \eta_t\left(x_t{}^{\mathrm{S}}, x_t{}^{\mathrm{R}}, p_t ,r_t\right) - C_t{}^- + C_t{}^+ \tag{3}$$
$$\varpi_t = \eta_t\left(x_t{}^{\mathrm{S}}, x_t{}^{\mathrm{R}}, p_t ,r_t\right) \tag{4}$$

In particular, e_{t+1} is composed by the emission of the previous period e_t minus the emissions associated with the actions taken $\eta_t(\cdot)$ plus the quantity of allowances bought and sold $(C_t{}^+, C_t{}^-)$.

A policy will be defined according to the expected reward function $f_\pi(s_t)$. This is defined by: 1) manufacturing and remanufacturing costs, 2) holding costs, and penalties, and 3) environmental contributions.

Manufacturing and Remanufacturing costs. Expressions 5 and 6 give the manufacturing and manufacturing cost, respectively. They consider a setup cost, if applicable, per batch and a quantity-related cost.

$$\delta_t(p_t) = \begin{cases} \tau_p + C_p p_t & p_t > 0 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

$$\gamma_t(r_t) = \begin{cases} \tau_r + C_r r_t & r_t > 0 \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

Holding costs and penalties. Let $H_t(x_t^R, r_t)$ denote the expected holding and disposal costs for remanufacturable inventory. A holding cost h^R per unit will be charged for all returned products remaining in the inventory at the end of the period. In addition, if the remanufacturable inventory level exceeds its capacity κ^{aR}, surplus products are disposed at a cost C_d per unit disposed.

$$H_t(x_t^R, r_t) \;=\; h^R \sum_{j=0}^{\kappa^{aR}+r_t-x_t^R} (x_t^R + j - r_t)\,\phi(j) + \; C^d \sum_{j>\kappa^{aR}+r_t-x_t^R}^{\infty} \left(x_t^R + j - (\kappa^{aR} + r_t)\right)\phi(j) \tag{7}$$

The expected holding costs and penalties for serviceable products $\mathrm{L}_t(x_t^S, r_t, p_t)$, are given by expression 8. It is a function of: 1) the holding cost h^S that is charged to all serviceable products remaining at the inventory at the end of the period; 2) the expected shortage cost, a penalty v will be charged to the sum of unfilled demands lower than the maximal quantity of backorder allowed κ^v, and 3) the expected lost-sale cost given by a lost-sale penalty C_{ls} associated with the unfilled demand going above κ^v.

$$\mathrm{L}_t(x_t^S, r_t, p_t) = \; h^S \sum_{i=0}^{x_t^S} [x_t^S - i + p_t + r_t]^+ \phi(i) + v \sum_{i>x_t^S}^{x_t^S+\kappa^v} (i - x_t^S)\phi(i) + C_{\mathrm{ls}} \sum_{i>x_t^S+\kappa_v}^{\infty} (i - x_t^S)\phi(i) \tag{8}$$

Where $[x]^+ = \max\{\mathrm{x},0\}$.

Environmental Contributions. The total amount of emissions generated over period *t* for the set of activities (p_t ,r_t) is defined by: $\eta_t(x_t{}^{\mathrm{S}}, x_t{}^{\mathrm{R}}, p_t ,r_t)$.

$$\eta_t(x_t{}^{\mathrm{S}}, x_t{}^{\mathrm{R}}, p_t ,r_t) = e^p p_t + e^r r_t \tag{9}$$

Let $\varrho(x_t{}^{\mathrm{S}}, x_t{}^{\mathrm{R}}, e_t, p_t ,r_t)$, denote the cost of the emissions generated. The first term represents the expected cost of the emissions generated. The second and third terms represent the expected quantity of allowances to buy or to sell, respectively. The value of C^{e}, $C_c{}^+$, and $C_c{}^-$ will depend on the environmental policy. A cap-and-trade scheme must assume only an emission cost (C^{e}) equals zero.

$$\varrho(x_t{}^{\mathrm{S}}, x_t{}^{\mathrm{R}}, e_t, p_t ,r_t) = \;\; C^{\mathrm{e}}\eta_t(x_t{}^{\mathrm{S}}, x_t{}^{\mathrm{R}}, p_t ,r_t) + C_c{}^+ C_t{}^+ - C_c{}^- C_t{}^- \tag{10}$$

Finally, let $f_\pi(x_t{}^{\mathrm{S}}, x_t{}^{\mathrm{R}}, e_t, \varpi_t)$, denote the expected cost when the system is operated under the policy $\pi \in \Pi$ given the state of the system $(x_t{}^{\mathrm{S}}, x_t{}^{\mathrm{R}}, e_t, \varpi_t)$, at the beginning of period *t*. The objective is to determine the policy $\pi \in \Pi$ that minimizes the total expected cost while satisfying the constraints.

$$f_\pi(x_t{}^{\mathrm{S}}, x_t{}^{\mathrm{R}}, e_t, \varpi_t) = \quad \delta_t(p_t) + \gamma_t(r_t) + H_t(x_t^R, r_t) + \quad \mathrm{L}_t\big(x_t^S, r_t, p_t\big) + \varrho(x_t{}^{\mathrm{S}}, x_t{}^{\mathrm{R}}, e_t, p_t\ , r_t) \tag{11}$$

The above model is subject to the following constraints: remanufacturing must be greater or equal to the minimal quantity to reuse α.

$$r_t \geq \alpha x_t{}^{\mathrm{R}} \tag{12}$$

Remanufacturing and production decisions must not exceed capacities. Furthermore, they are integers and non-negative.

$$r_t \leq \min(x_t{}^{\mathrm{R}}, \kappa^r) \tag{13}$$

$$p_t \leq \kappa^p \tag{14}$$

$$r_t, p_t \geq 0 \text{ and integer} \tag{15}$$

Carbon allowances $(C_t{}^+, C_t{}^-)$ must be less than the maximum amount granted. Moreover, allowances can be sold only when the amount of emissions generated have been reduced in the previous period.

$$C_t{}^+ \leq \kappa^e \tag{16}$$

$$C_t{}^- \leq \kappa^e y \tag{17}$$

$$\frac{\varpi_t - \varpi_{t+1}}{\varpi_t} \geq \beta y \tag{18}$$

$$\mathrm{y} \geq 0, y \leq 1, \text{and integer} \tag{19}$$

Emissions banked at each period must be less than the emission-cap.

$$e_t \leq E^c \tag{20}$$

Finally, state variables x_t^R and e_t must be non-negative.

$$x_t^R \geq 0, e_t \geq 0 \tag{21}$$

4 Case Study

Primary aluminum production uses a significant amount of electricity; nevertheless progress is being made to reduce its emissions (e.g.: nowadays 30% of produced aluminum is derived from recycled products). Moreover, since aluminum processing is one of the industries most affected by environmental regulations, our numerical analysis is based on the aluminum industry.

The basic scenario was built according to cases studies found in literature. Primary aluminum billets could come from two different sources: recycled products and raw bauxite. Throughout this paper, billets made of raw bauxite are assigned to the manufacturing process. On the other hand, billets made of recycled material are made through the remanufacturing process. The values used in the basic scenario are presented in the Table1; original values are multiple of 10.

Parameter	Value	Units
C_p	130	\$/tAl
C_r	90	\$/tAl
C_d	0	\$/tAl
h^S	15	\$/tAl
h^R	1.6	\$/tAl
v	115	\$/tAl
C_{ls}	179	\$/tAl
τ_p	0	\$
τ_r	0	\$
${C_c}^+$	1.36	\$/tCO_2
${C_c}^-$	1.32	\$/tCO_2
e^p	2	tCO_2/tAl
e^r	1	tCO_2/tAl
E^c	8	tCO_2
κ^e	2	tCO_2
α	0.1	-
β	0.2	-

Table 1: Cost and emission factors

The rest of parameters, shown in Table 2, were mostly taken from Ahiska and King (2010).

Parameter	Value	Units
κ^p	50	tAl
κ^r	20	tAl
κ^S	8	tAl
κ^{aR}	4	tAl
κ^v	1	tAl

Table 2: Parameters

We consider demand and returns distributed in the following way:

$$\Phi(i) = \Pr[D_t = i] = \begin{cases} \dfrac{i}{20}, & 1 \le i \le 4 \\ \dfrac{9-i}{20}, & 4 < i \le 4 \\ 0, & \text{otherwise} \end{cases}$$

$$\Phi(j) = \Pr[R_t = j] = \begin{cases} \dfrac{j+1}{9}, & 0 \le j \le 2 \\ \dfrac{5-j}{9}, & 2 < j \le 4 \\ 0, & \text{\&otherwise} \end{cases}$$

4.1 Baseline Scenario

The baseline scenario consists in determining the best remanufacturing and manufacturing decisions without taking account of their environmental contributions. The latter is considered as the current state of many industries, where the only constraint to preserve is the minimal quantity to remanufacture. Then, it is evident that the optimal cost and the GHG emission are not dependent of the emission-cap (E^c) and since the baseline scenario does not consider selling or purchasing carbon credits, the value of ${C_c}^+$ and ${C_c}^-$ neither have an implication in the cost.

Through the scenario studied when remanufacturing is cheaper than manufacturing, remanufacturing is preferred over manufacturing. Decisions are characterized by a policy (s^b, S^b, q^b).

A policy (s^b, S^b, q^b) works in the following way: remanufacturable and serviceable inventory levels are seen at the beginning of a period. If ${x_t}^{\mathrm{S}}$ is less than the reorder level s^b, it is remanufactured the minimum value between q^b and ${x_t}^{\mathrm{R}}$. Considering the manufacturing actions the quantity to manufacture is $q^b - {x_t}^{\mathrm{R}} - r_t$ units. Whether the serviceable inventory ${x_t}^{\mathrm{S}}$ is greater or equal than the reorder level s^b, it is remanufactured the minimum between $S^b - {x_t}^{\mathrm{S}}$ and ${x_t}^{\mathrm{R}}$. Then, if the ${x_t}^{\mathrm{S}}$ is still less than S^b, there are manufactured $S^b - {x_t}^{\mathrm{S}} - r_t$ units. With an optimal cost of 639.95 and an environmental impact of 6.65tCO_2, this characterization has an expected deviation of 0.05% from the optimal cost. The parameters values of the policy (s^b, S^b, q^b) correspond to the values (4,9,7), similar to the first case. Policy (s^b, S^b, q^b) is summarized as:

$$r_t = \begin{cases} \min(q^a, x_t^{\mathrm{R}}), & x_t^{\mathrm{S}} < s^b \\ \min(x_t^{\mathrm{R}}, S^b - x_t^{\mathrm{S}}), & x_t^{\mathrm{S}} \geq s^b, x_t^{\mathrm{R}} < S^b - x_t^{\mathrm{S}} \\ 0, & \text{otherwise} \end{cases}$$

$$p_t = \begin{cases} q^b - r_t, & x_t^{\mathrm{S}} < s^b \\ S^b - x_t^{\mathrm{S}} - r_t, & \text{otherwise} \end{cases}$$

We can see that when an environmental contribution is not considered, the structure of the inventory policy is easy to recognize and it could be expressed with a minimal quantity of parameters. Based on the GHG emissions generated through the baseline scenario, in the following section four different emission-cap values (E^c=2, 3, 4, 5) have been tested to evaluate the implications of an environmental-cap in the structure of an environmental policy.

4.2 Cap-and-Trade Scenarios

Through the cap-and-trade scheme three emission-caps are tested (E^c=2, 3, 4, 5).

When a cap-and-trade mechanism is considered, the inventory policy is characterized by three inventory policies: 1) a (s^d, S^d, r^d) policy, 2) a (s^e, S^e, r^e, q^e) policy and 3) a (ε') policy.

Through a (s^d, S^d, r^d) policy, the manufacturing process is not used. Nevertheless, remanufacturing decision is taken in the following way. The remanufacturable and serviceable inventory levels are seen at the beginning of a period. If the x_t^{R} level is less than the reorder level s^d, the minimal quantity ε is remanufactured; however, if x_t^{R} is greater or equal to r^d, all x_t^{R} is remanufactured. On the other hand, if x_t^{S} is greater than the reorder level s^d, $\min(x_t^{\mathrm{R}}, S^d - x_t^{\mathrm{S}})$ units are remanufactured; nevertheless if the minimal quantity ε is greater than $S^d - x_t^{\mathrm{S}}$ then ε units are remanufactured. Decision could be summarized as:

$$r_t = \begin{cases} x_t^{\mathrm{R}}, & x_t^{\mathrm{S}} < s^d, x_t^{\mathrm{R}} \geq r^d \\ \min(x_t^{\mathrm{R}}, S^d - x_t^{\mathrm{S}}), & x_t^{\mathrm{S}} \geq s^d, \varepsilon < S^d - x_t^{\mathrm{S}} \\ \varepsilon, & \text{otherwise} \end{cases}$$

$$p_t = 0$$

The value of ε denotes the minimum value between: 1) the minimal quantity of remanufactured items necessary to reduce the quantity of credits e_t to the emission-cap E^c and 2) the minimal proportion α to remanufacture.

In a policy (s^e, S^e, r^e, q^e) remanufacturing decision are taken in the same way as in a (s^d, S^d, r^d) policy. Nonetheless, the policy (s^e, S^e, r^e, q^e) does manufacturing. Manufacturing is used if $e_t \geq E^c - 1$, where $q^e - r_t$ are manufactured; otherwise, there is no need of manufacturing. Thus, the above policy is described as follows:

$$r_t = \begin{cases} x_t^{\mathrm{R}}, & x_t^{\mathrm{S}} < s^e, x_t^{\mathrm{R}} \geq r^e \\ \min(x_t^{\mathrm{R}}, S^e - x_t^{\mathrm{S}}), & x_t^{\mathrm{S}} \geq s^e, \varepsilon < S^e - x_t^{\mathrm{S}} \\ \varepsilon, & \text{otherwise} \end{cases}$$

$$p_t = \begin{cases} q^e - r_t, & e_t \geq E^c - 1 \\ 0, & \text{otherwise} \end{cases}$$

Through the (ε') policy, it is only remanufactured the minimal quantity to satisfy the α-minimal remanufacturing proportion. Hence,

$$r_t = \lceil \alpha x_t^{\mathrm{R}} \rceil$$
$$p_t = 0$$

Characterization of the cap-and-trade scenario results in a deviation of the optimal cost on the range [0.00%, 0.27%] with average value of 0.07% and a standard deviation of 0.06. The deviation from the optimal cost caused by the use of a baseline scenario policy (s^b, S^b, q^b) on an environmental scenario has an average expected value of 1.18% and a standard deviation of 0.75.

4.3 **Analysis of Results**

Regarding studied scenarios, under a baseline case the inventory policy is simpler than under an environmental scenario, where it is not obvious the characterization of inventory policies structure. In later policies, remanufacturing and manufacturing quantities are strongly dependent on carbon credit stock e_t, emission-cap and carbon credit price. Furthermore, even if the emission-cap is high the inventory policy changes in order to reduce the cost by selling carbon credits.

Figure 2 shows the cost by emission-cap. It is seen that the cost does not change no matter the emission-cap. However, comparing the emission-cap scenario to the baseline, there is a significant increase on cost. It is important to notice that the cost could be defined by an increasing function with an inflection point in the allowance price of \$40.8. This point represents the change between filling most of demand and an increase on shortage. Hence, it means that when the allowance price is greater than \$40.8 the company prefers to lost sales than invest on carbon credits.

The expected emissions and lost sales per emission-cap scenario are presented in Figures 3 and 4. Over the emission-cap scenarios, the system tries to remanufacture the maximal possible quantity when the credit cost is cheap (i.e. [1.36-34]).

Given the remanufacturing cost of \$90, the company is capable to afford the purchase of allowances up to an allowance price of \$81.60. According to this behavior, the lost sales increase from \$102. This same behavior is shared by the remanufacturable inventory level. Emissions rate and serviceable inventory level present the opposite behavior, where the values are constant from \$1.36 to \$81.60 or \$40.80 depending of remanufacturing cost, and after the maximal value the emission rate and the inventory level decrease.

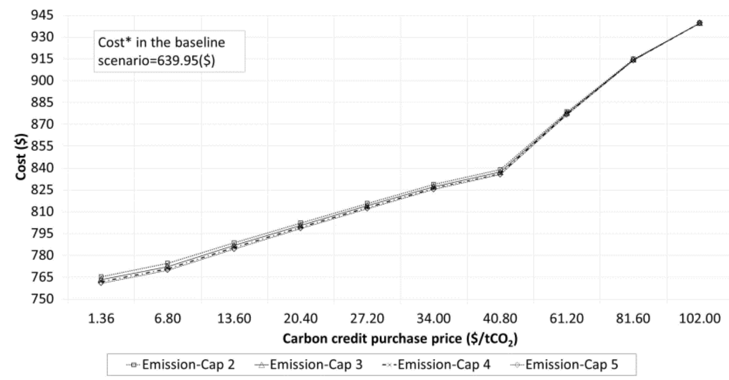

Figure 2.: Variation in cost due to the emission cap.

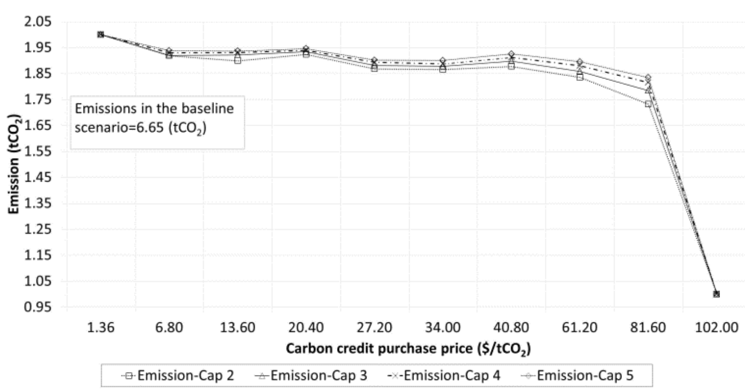

Figure3.: Expected emissions (tCO_2) per scenario

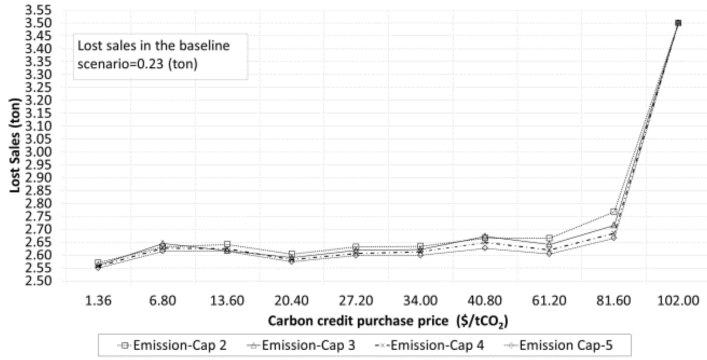

Figure 4: Expected lost sales per scenario

5. Finite Horizon

We extend the model presented in Section 3, over a finite horizon scenario. Adapting the model to fit a finite horizon context leads to better knowledge of the impact of environmental policies and it and can contribute to improving inventory policies. The conversion to a finite horizon is straightforward.

We study a scenario with s_0 ≔(5,3,7 ,0), being s_0 the initial state and the planning horizon equals to 12 periods. We use the parameters: $E^c = 7$, $\kappa^e = 3$, $\kappa^r = 5$, $\kappa^p = 3$, $C_p = 130$, $C_r = 90$, ${C_c}^- = 1.32$ and ${C_c}^+ = 1.36$. The remaining parameters correspond to the values presented in Table 1 and 2. The results suggest a reduction in manufacturing replenishment quantities, dropping in average 23% the amount of emissions generated, but increasing costs by 3%. However, 20% of cost increment results from the carbon credit purchase, thus operational costs are only raised by 2.5%.

Due to the restricted size of state and action spaces, the results are not enough to determine a significant change on remanufacturing replenishment quantities. Then, despite there is evidence that inventory policies are affected by environmental policies, it is required a deeper study implicating bigger instances. Besides, since decisions are affected by initial and final states an extended number of scenarios should be studied.

6. Conclusions and Further work

In this paper, we have addressed the inclusion of a carbon trading scheme on a stochastic periodic-review inventory model with remanufacturing. We formulate this problem using the Markovian Decision Process. Then, based on optimal remanufacturing and manufacturing quantities we characterize the inventory policies. From a methodological standpoint, Markov Decision models have proved to be a powerful tool on inventory control when analytical policies appear limited to define.

Considering that the aluminum industry is one of the industries most influenced by environmental regulations, our numerical analysis was based on it. Hence, for this particular case we find the optimal remanufacturing and manufacturing quantities for two scenarios: 1) a baseline scenario where emissions are not considered and 2) a cap-and-trade scenario, for which we characterized the inventory policies. We proved that inventory policies change from the traditional to the environmental scenario. Then, while under a traditional scenario the inventory policy is simpler, under a environmental scenario is more difficult to characterize the structure of the inventory policy. Moreover, we proof that if an organization does not change its inventory policy, and continues to use a baseline scenario policy even if an emission-cap is applied, the company risks to lose money and the quantity will more important conforming the carbon credit price increases.

Finally, we introduce a finite-horizon scenario. The results suggest an impact of environmental regulations on replenishment policies. Nevertheless, bigger instances must be studied to generalize the results. Hence, this study provides a good explanation why companies must change their managerial decision strategies. This problem was formulated as a minimization problem since we were looking to analyze the impact on cost from the inclusion on environmental constraints. Moreover, we believe the results are a good indicator that green inventory policies represent a promising area for further research. Thus, the results presented in this paper provide a first step towards a better understanding of how inventory policies react to the integration of environmental practices. Several directions could be considered for extending these results. For the finite-horizon case, it is necessary to study bigger instances and extend the numerical examples. From a practical point of view, it will be also interesting to study a revenue maximization approach, where sales distribution varies according to the environmental implication of the companies. The latter will mean improvements in profitability that will encourage environmental implication of managers in a wide range of industries, even if they are not influenced by legislations.

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