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Abstract. This article develops a game-theoretical model to deal with the protection of facilities, in the context of the capacitated fixed-charge location and capacity acquisition problem. A set of investment alternatives are available for direct protection of facilities. Furthermore, extra-capacity of neighbouring functional facilities can be used after attacks to avoid the backlog of demands and backorders. The proposed model considers a non-cooperative two-period game between the players, and an algorithm is presented to determine the equilibrium solution and the optimal defender strategy under capacity constraints. A method is developed to evaluate the utilities of the defender and the attacker. The benefit of the proposed approach is illustrated using a numerical example. The defence strategy of our model is compared to other strategies, and the obtained results indicate clearly the superiority of our model in finding the best trade-off between direct protection investment and extra-capacity deployment.

Keywords. Capacity, protection, facility, attack, damage, game theory, vulnerability, contest.

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1. Introduction

Critical infrastructures such as supply networks represent enormous investments consecrated to the distribution of goods and services. These investments require large capital outlays. Even a minor disruption can degrade the system performance. Reductions of capacity can introduce significant delays in getting back to the planned production schedule and inflict substantial losses. Such supply networks can be victim of different threats, such as accidental failures, natural catastrophes, terrorist attacks and sabotages, fire, industrial accidents, tsunamis, earthquakes, floods and cyclones. Recent events have shown that if one or more entities of a critical logistical network (key facilities, bottlenecks, critical links, etc.) are damaged due to an accident or an intentional attack, the network is paralyzed and the damage would be enormous, resulting in a negative impact at the social, political and economic levels. Attacks can also have a serious impact on the health and safety or the system effective functioning.

There is a relevant literature on the defence of networks infrastructures against intentional attacks. In [1], the authors analyze the strategic defence and attack of complex networks and systems with components in series, parallel, interlinked, interdependent, independent, or combinations of these. The authors of [2] have recently developed a method based on a Monte Carlo simulation approach for evaluating the expected damage related to nodes deprivation of supply of commodities in multi-commodity networks as a consequence of intentional attack on arbitrarily chosen network links. In [3], the authors showed that scale free networks are robust against random failures but fragile to intentional attacks. In [4], critical locations susceptible to terrorist attacks are determined by decision makers on base of geographic regions classification. In [24] the authors study the effects of intentional attacks on transportation networks of two arcs that are subject to traffic congestion. The authors of [27] provide optimal protection configurations for a network with components vulnerable to an interdictor with potentially different attacking strategies. Ref. [28] develops an ordinal optimization based method to identify top contributors to power networks failure when considering cascade failure events.

There is also a mature literature on facility design with probabilistic failure of components [5,29,30]. However, the possibility of intentional strikes or attacks is not normally taken into account in this literature, except in [6] where the authors developed a game-theoretical model to protect facilities against intentional attacks in the context of the *uncapacitated* fixed-charge location problem. This article deals with the protection of network logistic facilities in the context of the *capacitated* fixed-charge location and capacity acquisition problem (CFL & CAP). Facility location and capacity acquisition are of vital importance to supply chain management [7-9]. Here, we consider the CFL & CAP to deal with resource allocation of protection and allocation of the extra-capacity among facilities.

The aim of the CFL & CAP is to decide simultaneously on the optimal location and capacity size of each new facility to be established [8-13]. In this problem, we are given a set of customer locations with known demands and a set of potential facility locations. If we decide to locate a facility with a chosen capacity at a candidate site, we incur a known fixed location cost. There is a known unit cost of shipping between each candidate facility site and each customer location. The problem is to find the locations of the facilities and the shipment pattern between the facilities and the customers, to minimize the sum of the facility location and shipment costs, subject to constraints that all demands must be served, facilities capacities must not be exceeded, and customers can only be served from open facilities.

For the protection of network logistic facilities in the context of the CFL & CAP, we consider that not only a set of investment alternatives are available for "direct" protection of facilities, but also extra-capacities of neighbouring functional facilities can be used after attacks for "indirect" protection. Extra-capacity is among the different strategies to deal with the risk of uncertain production capacity. It can be used, after a capacity shock, to quickly bring back the production on schedule and to avoid the backlog of demands [14]. In the case of demand growth, facilities of supply network might hold extra-capacity against demand variability [15]. In our case extra-capacity of neighbouring functional facilities is used after attacks, in order to satisfy all customers demand and to avoid the backorders.

The CFL & CAP problem assumes that, once constructed, the facilities chosen will always operate as planned. However, if a facility is attacked, it may become unavailable and customers must be served from others functional facilities of the supply network that are farther than their regular facilities, but subject to constraints that functional facilities capacities must not be exceeded. To satisfy customers demand and to avoid backorders, the amount produced by disabled facility must be allocated optimally among the functional facilities. This reduces the cost of delayed production after attacks, but may lead to excessive additional transportation costs. The strategic decision dealt with here is how to allocate optimally the protective resources and the extra-capacity among the facilities, knowing that these facilities are exposed to external intentional attacks. In other words, given a set of investments alternatives for protecting the facilities and set of extra capacities, we want to determine how much to invest optimally in direct protection of facilities and in indirect protection by extra capacity, while taking into account that both the defender and the attacker are fully optimizing agents. The idea of using extra-capacity to indirectly protect supply networks against intentional attack is used in this paper to develop a game-theoretic model with the objective of finding the best trade-off between direct investments in protection and indirect protection by extra-capacities deployment.

The remainder of the paper is organized as follows. Section 2 presents the mathematical model of capacitated fixed-charge location and capacity acquisition problem. Section 3 formulates the studied problem as a two-period non-cooperative game. Section 4 evaluates the players' utilities. Section 5 develops an algorithm to solve the game. Section 6 presents a numerical example. Section 7 concludes the paper.

2. The facility location and capacity acquisition problem

The CFL & CAP considers the problem of locating facilities to minimize the sum of the facility location costs, the costs of capacity acquisition associated with the size of open facility and the shipping costs from open facilities to customers subject to constraints that all demands must be served, facility capacities must not be exceeded, and customers can only be served from open facilities [9]. The CFL &CAP has been widely studied in the literature and applied in a variety of domains, and is known to be *NP*-hard [16].

Let n denote the number of customers (indexed by j) and m denote the number of alternative facility locations (indexed by i). The following notations are also used in the mathematical model:

I the set of candidate facility locations, indexed by i,

J the set of customer locations, indexed by j,

g_i the fixed cost of locating a facility at candidate site i,

$h_i(.)$ the total capacity acquisition cost at facility i,

c_{ij} the unit cost of shipping between candidate facility site i and customer location j,

D_j the demand at customer location j,

CAP_i the maximum capacity that can be built-in at candidate site i.

The decision variables are:

Z_{ij} the quantity shipped from candidate facility site i to customer location j,

Y_i 1 if a facility is to be located at candidate site i, and 0 otherwise.

The problem can be modeled as follows:

Minimize $$\sum_{i=1}^{m}\left[g_iY_i + h_i\left(\sum_{j=1}^{n}Z_{ij}\right)+\sum_{j=1}^{n}c_{ij}Z_{ij}\right], \tag{1}$$

Subject to $$\sum_{i=1}^{m}Z_{ij}=D_j \quad j=1,\ 2,\ ...,\ n, \tag{2}$$

$$0\le Z_{ij}\le Y_iD_j \quad i=1,\ 2,\ ...,\ m,\quad j=1,\ 2,\ ...,\ n, \tag{3}$$

$$\sum_{j=1}^{n}Z_{ij}\le \mathrm{CAP}_i \quad i=1,\ 2,\ ...,\ m. \tag{4}$$

$$Y_i\in\{0,\ 1\} \quad i=1,\ 2,\ ...,\ m. \tag{5}$$

The objective function (1) minimizes the sum of the fixed facility location costs, the costs of capacity acquisition associated to open facility, and the shipments or transportation costs. Constraint (2) ensures that each customer's demand will be fully satisfied. Constraint (3) is a simple non-negativity constraint and it guarantees that customers receive shipments only from open facilities. Capacity constraint (4) ensures that an open facility i does not supply more than its capacity CAP_i, and constraint (5) requires the location variables to be binary.

We assume that $h_i(.)$ is a linear function, then the form of the objective function will become $\sum_{i=1}^{m}\left[g_iY_i + Ac_i\sum_{j=1}^{n}Z_{ij}+\sum_{j=1}^{n}c_{ij}Z_{ij}\right]$, where Ac_i is the capacity acquisition cost at facility location per unit.

To hedge against facilities unavailability, it is possible to acquire at the beginning capacities that are higher than the optimal values obtained from the model (1)-(5). Our objective is to find the best defence and capacity acquisition strategies, knowing that the facilities are subject to intentional attacks. As already explained in the introduction, given a set of investments alternatives for protecting the facilities and set of extra capacities, we want to determine how much to invest optimally in direct protection of facilities and in indirect protection by extra capacity, while taking into account that both the defender and the attacker are fully optimizing agents. The next section presents our game-theoretic model developed to find the best trade-off between direct investments in protection and indirect protection by extra-capacities deployment.

3. Problem formulation using game theory

We consider a system containing *m* facilities (targets) designed using the optimization model (1)-(5). Our model considers two players: the defender and the attacker. We are given a set of investment alternatives for protecting the facilities (we call this direct protection), and a set of extra-capacity options for each facility (we call this indirect protection). The objective is to determine how to allocate optimally (direct) protective resources and (indirect) extra-capacities among the facilities taking into account the attacker strategy.

For the attacker, there are also many ways to attain the facilities performance with a set of investment alternatives. We consider a two-period game between the defender who selects a strategy in the first period that minimizes the maximum loss that the attacker may cause in the second period. This means that the defender determines in the first period, its 2*m* free choice variables simultaneously and independently, and the attacker determines in the second period, its *m* free choice variables simultaneously and independently. For each player strategy, a utility function is associated to each game conclusion.

The defender maximizes his utility by minimizing the expected damage of the system and the investment expenditure incurred to extra-capacity and to protect the system. The attacker maximizes his utility also, calculated as the expected damage minus the attacks expenditures. To better understand, the game, the defence and attack choices are detailed in what follows.

3.1. The defender

We consider that for each facility i there exists a set of β_i available types of protections against the identified threat. Each protection type is indicated by index p ($p = 0, 1, 2, \ldots, \beta_i$). $p = 0$ means that no defence is used. Let $\overline{F_{ip}}$ be the defender investment expenditure in dollar terms. We express $\overline{F_{ip}}$ as the product of two parts, F_{ip} and f_{ip} . That is, $\overline{F_{ip}} = f_{ip}F_{ip}$ where F_{ip} is an investment effort incurred by the defender at unit cost f_{ip} (f_{ip}>0) to protect a facility located at site i using protection type p.

We consider a vector **P** = $\left(\pi_i\right)$ which represents a protection strategy of the m facilities such as π_i takes values from p = 1, 2, …, β_i. For example, **P** = $\begin{pmatrix}2 & 1 & 3\end{pmatrix}$ means that we have facility 1 is protected using type 2 protection, facility 2 is protected using type 1 protection and facility 3 is protected using type 3 protection. To each protection strategy **P**, corresponds a vector of investments **F** = $\left(F_{i\pi_i}\right)$. For example, when P =$\begin{pmatrix}2 & 1 & 3\end{pmatrix}$, we have **F** = $\begin{pmatrix}F_{12} & F_{21} & F_{33}\end{pmatrix}$.

Let λ_{ip} be a binary variable which is equal to 1 if a protection of type p is selected for facility i, and 0 otherwise. We assume that only one type of protection of facility i is used:

$$\sum_{p=1}^{\beta_i} \lambda_{ip} = 1, \quad \forall i. \tag{6}$$

Let ρ_i be the number of available extra-capacity options for each facility i. Let e (e = 1, 2, …, ρ_i) be an index that indicates each extra-capacity option. The defender incurs an investment of proportion of the capacity acquired τ_{ie} associated of the facility located at site i using extra-capacity option e. Let C_i^* denote the capacity acquired. We assume that $h_i(.)$ the capacity acquisition cost at facility location is a linear function. We then consider that the investment of extra-capacity CE_{ie} associated of the facility located at site i using extra-capacity option e, measured in dollar terms, is given by $CE_{ie} = Ac_i \tau_{ie} C_i^*$, where Ac_i is the capacity acquisition cost at facility location i per unit.

We represent an extra-capacity strategy of the n facilities by a vector **E** = $\left(\theta_i\right)$, θ_i takes values from e = 0, 1, …, ρ_i . For example, **E** = $\begin{pmatrix}2 & 2 & 2\end{pmatrix}$ means that option 2 extra-capacity is selected for the 3 facilities.

To each extra-capacity strategy **E**, corresponds a vector of investments **T** = $\left(\tau_{i\theta_i}\right)$. For example, when **E** = $\begin{pmatrix}2 & 2 & 2\end{pmatrix}$, we have **T**=$\left\{\tau_{12},\ \tau_{22},\ \tau_{32}\right\}$.

Let us introduce a binary variable ξ_{ie} which is equal to 1 if an extra-capacity of type *e* is selected for facility *i*. Assuming that one type of extra-capacity is used, we have:

$$\sum_{e=0}^{\rho_i} \xi_{ie} = 1, \ \forall i. \tag{7}$$

3.2. The attacker

The attacker seeks to attack the system to ensure that it does not function reliably. He (she) has a set of α_i available attack actions against any facility *i*. Each attack type is indicated by index *g* (*g* = 0, 1, 2, …, α_i). *g* = 0 indicates the absence of an attack. Analogously, the attacker incurs an effort Q_{ig} at unit cost q_{ig} to attack facility located at site *j* using attack action *g*. The inefficiency of investment is q_{ig}, and $1/q_{ig}$ is the efficiency. Its investment expenditure, in dollar terms, is $\overline{Q_{ig}} = q_{ig} Q_{ig}$,, where q_{ig} >0.

We consider a vector **G** = (ω_i) which represents an attack strategy against the *m* facilities, ω_i takes values from *g* = 1, 2, …, α_i. For example, **G** = $\begin{pmatrix} 3 & 1 & 3 \end{pmatrix}$ means that we have 2 facilities that can be attacked using attacks of type 3 and facility 2 that can be attacked using attack of type 1.

Let μ_{ig} be a binary variable which is equal to 1 if an attack of type *g* is used for facility *i*, and 0 otherwise. We assume that only one type of attack of facility *i* is used:

$$\sum_{g=1}^{\alpha_i} \mu_{ig} = 1, \ \forall i. \tag{8}$$

We suppose that successful or failed attacks against different facilities are independent. We also assume that each facility can be attacked by the attacker only once, and that many facilities can be attacked at the same time.

3.3. Vulnerability of facilities

In game theory, interaction between 2 conflicted players (here the defender and the attacker) can be modeled by introducing the concept of the contest success function commonly used in the rent seeking literature [17, 18, 19, 20]. The vulnerability or the probability of a successful attack on facility *i* is defined by its destruction probability $v_{pg}(i)$. The vulnerability of the attacked facility is usually determined by the ratio form of the attacker–defender contest success function [21, 22, 23]. The vulnerability of any facility *i* by a contest success function is:

$$v_{pg}(i) = \frac{\left(Q_{ig}\right)^{\varepsilon_i}}{\left(F_{ip}\right)^{\varepsilon_i} + \left(Q_{ig}\right)^{\varepsilon_i}}, \tag{9}$$

where $\partial v_{pg}(i) / \partial Q_{ig} > 0$, $\partial v_{pg}(i) / \partial F_{ip} < 0$, and $\varepsilon_i \geq 0$ is a parameter that expresses the intensity of the contest. We assume that ε_i does not depend on *p* and *g*.

On the one hand, if the attacker exerts high offensive effort, it is likely to win the contest which is expressed by high vulnerability; on the other hand, if the defender exerts high defensive effort, it is likely to win the contest which is expressed by low vulnerability [18, 19, 21, 23].

3.4. The game

Having the vulnerabilities of facilities as functions of the attacker's and the defender's efforts, both agents can estimate the expected damage caused by the attack for any possible distribution of these efforts. The defender's objective is to maximize its utility function by minimizing the expected damage and weighing against protection and extra-capacity expenditures. The attacker's objective is to maximize the expected damage while weighing against the attacks expenditures [25]. Facilities are usually built over time by the defender. The attacker takes it as given when he (she) chooses his (her) attack strategy. Therefore, we consider a two-period min-max game where the defender invests in the first period, and the attacker moves in the second period. This means that the defender selects a strategy in the first period that minimizes the maximum loss that the attacker may cause in the second period. The utilities of each player are evaluated in the next section, while the game will be solved with backward recursion, in which the second period is solved first in Section 5.

4. Evaluation of the players' utilities

The damage caused by an attack is associated with two kinds of damage.

Damage 1:

The expected cost required for restoring the attacked facilities. If R_i is the cost required to restore the attacked facility i, this cost depends on the defence and attack strategies **P** and **G**, and it is given by:

$$C_R(\mathbf{P},\mathbf{G}) = \sum_{i=1}^{m}\sum_{g=0}^{\alpha_i}\sum_{p=1}^{\beta_i} \lambda_{ip}\mu_{ig}\nu_{pg}\,(i)R_i\,. \tag{10}$$

Damage 2:

The second expected cost is associated with two terms:

- The cost incurred because of the increasing in transportation cost after attacks. When one or several facilities are unavailable, to avoid the backlog of demands and backorders, available extra-capacities of neighbouring functional facilities are used. Adding the available extra-capacity to initial capacity, customers could be served and may receive shipments from these facilities, which are anyway farther away (subject to constraints that their total capacity must not be exceeded). As a matter of fact, the transportation cost will increase as customers will be reassigned.
- The backorder cost incurred when the demands cannot be satisfied. This will happen either when the entire system is disabled, or when even the available extra-capacities are not enough to fulfil the demands.

While damage 1 has been expressed by equation (10), the evaluation of damage 2 terms is provided in what follows.

Each facility can be either Disabled or Functional. Let S = $\{S_k\}$ be a set of possible combinations when considering all facilities. For m facilities, there are 2^m possible combinations. For example for three facilities denoted by Fac1, Fac2 and Fac3, there are 8 possible combinations for the facilities (k = 0, 1, …, 7) as illustrated in Table 1.

Let us denote by Ct_k the cost incurred because of the increase in transportation cost, *i.e.* the cost under combination S_k minus the cost in a normal situation and B_k the backorder cost when the combination is S_k. Let us denote by B_{img} the brand image of the company, YD_k the annual unmet demand and $\overline{Ac}$ the average of the capacity acquisition costs per unit. We suppose that the backorder denoted by B_k is computed per week, based on 20% of the average of the capacity acquisition costs and it is given by $B_k = B_{img} + (0.20\,\overline{Ac}\;\; YD_k)/52$. Here, 52 is the number of weeks during 1 year. The evaluation of the cost Ct_k and the backorder cost B_k for all combinations except combination 0 where all facilities are functional, may require the solution of (2^m -1) optimisation model (1)-(5). Each solution of the optimisation model (1)-(5) corresponds to a combination S_k. In the Table 1, if we consider attack combination 2 where only facilities 1 and 3 are operational, the model (1)-(5) is solved to determine the cost Ct_2 and B_2. The cost Ct_k and the backorder cost B_k are related to all the possible outcomes. Let denote by Ω the binary variable, which is equal to 1 if the cost Ct_k incurred because of the increase in transportation cost or 0 if the backorder B_k cost is incurred. Let us denote by $T_k(\mathbf{E})$ which can be the cost Ct_k or the backorder cost B_k such as $T_k(\mathbf{E}) = \Omega\, Ct_k(\mathbf{E}) + (1-\Omega)B_k$. Let $\Delta C_{pg\mathbf{E}}\left(k\right)$ be an attack outcomes, with p, g and $\mathbf{E}$ given and k varying from 0 to 2^m -1. Table 1 illustrates the calculation of the attack outcomes $\Delta C_{pg\mathbf{E}}\left(k\right)$. For example if we consider attack combination 2 where facility 2 is disabled and facilities 1 and 3 are operational, we have $T_2(\mathbf{E}) = \Omega\, Ct_2(\mathbf{E}) + (1-\Omega)B_2$ and $\Delta C_{pg\mathbf{E}}\left(2\right) = T_2(\mathbf{E})\nu_{pg}\left(2\right)$.

Table 1
Possible combinations: case of three facilities

Combination of disabled facilities	Index k	Attack outcome $\Delta C_{pg\mathbf{E}}(k)$
All facilities are Functional	0	$\Delta C_{pg\mathbf{E}}(0)=0$
Only Fac1 is Disabled	1	$\Delta C_{pg\mathbf{E}}(1)=T_1(\mathbf{E})\nu_{pg}(1)$
Only Fac2 is Disabled	2	$\Delta C_{pg\mathbf{E}}(2)=T_2(\mathbf{E})\nu_{pg}(2)$
Only Fac3 is Disabled	3	$\Delta C_{pg\mathbf{E}}(3)=T_3(\mathbf{E})\nu_{pg}(3)$
Fac1 and Fac2 are Disabled, Fac3 is Functional	4	$\Delta C_{pg\mathbf{E}}(4)=$ $T_1(\mathbf{E})\nu_{pg}(1)(1-\nu_{pg}(2))+T_2(\mathbf{E})\nu_{pg}(2)(1-\nu_{pg}(1))+$ $T_4(\mathbf{E})\nu_{pg}(1)\nu_{pg}(2)$
Fac1 and Fac3 are Disabled, Fac2 is Functional	5	$\Delta C_{pg\mathbf{E}}(5)=$ $T_1(\mathbf{E})\nu_{pg}(1)(1-\nu_{pg}(3))+T_3(\mathbf{E})\nu_{pg}(3)(1-\nu_{pg}(1))+$ $T_5(\mathbf{E})\nu_{pg}(1)\nu_{pg}(3)$
Fac2 and Fac3 are Disabled, Fac1 is Functional	6	$\Delta C_{pg\mathbf{E}}(6)=$ $T_2(\mathbf{E})\nu_{pg}(2)(1-\nu_{pg}(3))+T_3(\mathbf{E})\nu_{pg}(3)(1-\nu_{pg}(2))+$ $T_6(\mathbf{E})\nu_{pg}(2)\nu_{pg}(3)$
All facilities are Disabled	7	$\Delta C_{pg\mathbf{E}}(7)=$ $T_1(\mathbf{E})\nu_{pg}(1)(1-\nu_{pg}(2))(1-\nu_{pg}(3))+$ $T_2(\mathbf{E})\nu_{pg}(2)(1-\nu_{pg}(1))(1-\nu_{pg}(3))+$ $T_3(\mathbf{E})\nu_{pg}(3)(1-\nu_{pg}(1))(1-\nu_{pg}(2))+$ $T_4(\mathbf{E})\nu_{pg}(1)\nu_{pg}(2)(1-\nu_{pg}(3))+$ $T_5(\mathbf{E})\nu_{pg}(1)\nu_{pg}(3)(1-\nu_{pg}(2))+$ $T_6(\mathbf{E})\nu_{pg}(2)\nu_{pg}(3)(1-\nu_{pg}(1))+$ $T_7(\mathbf{E})\nu_{pg}(1)\nu_{pg}(2)\nu_{pg}(3)$

The expected cost associated with the transportation cost increase and the backorder cost is given by:

$$TB(\mathbf{P},\mathbf{G},\mathbf{E}) = \sum_{k=1}^{2^m-1} \Delta C_{pg\mathbf{E}}(k). \tag{11}$$

The expected damage is then:

$$D(\mathbf{P},\mathbf{G},\mathbf{E}) = \sum_{i=1}^{m}\sum_{g=0}^{\alpha_j}\sum_{p=1}^{\beta_j} \lambda_{ip}\mu_{ig}\nu_{pg}(i)R_i + \sum_{k=1}^{2^m-1} \Delta C_{pg\mathbf{E}}(k) \tag{12}$$

The defender expected utility is:

$$\begin{aligned} U_d(\mathbf{P},\mathbf{G},\mathbf{E}) &= -D(\mathbf{P},\mathbf{G},\mathbf{E}) - \sum_{i=1}^{m}\sum_{p=1}^{\beta_i} \lambda_{ip}\overline{B_{ip}} - \sum_{i=1}^{m}\sum_{e=0}^{\rho_i} \xi_{ie}CE_{ie} \\ &= -\left(\sum_{i=1}^{m}\sum_{g=0}^{\alpha_i}\sum_{p=1}^{\beta_i} \lambda_{ip}\mu_{ig}\nu_{pg}(i)R_i + \sum_{k=1}^{2^m-1} \Delta C_{pg\mathbf{E}}(k)\right) - \sum_{i=1}^{m}\sum_{p=1}^{\beta_i} \lambda_{ip}\overline{B_{ip}} - \sum_{i=1}^{m}\sum_{e=0}^{\rho_i} \xi_{ie}CE_{ie} \end{aligned} \tag{13}$$

The attacker expected utility is:

$$\begin{aligned} U_a(\mathbf{P},\mathbf{G},\mathbf{E}) &= D(\mathbf{P},\mathbf{G},\mathbf{E}) - \sum_{i=1}^{m}\sum_{g=0}^{\alpha_i} \mu_{ig}\overline{Q_{ig}} \\ &= \sum_{i=1}^{m}\sum_{g=0}^{\alpha_i}\sum_{p=1}^{\beta_i} \lambda_{ip}\mu_{ig}\nu_{pg}(i)R_i + \sum_{k=1}^{2^m-1} \Delta C_{pg\mathbf{E}}(k) - \sum_{i=1}^{m}\sum_{g=0}^{\alpha_i} \mu_{ig}\overline{Q_{ig}} \end{aligned} \tag{14}$$

5. Solution of the game

We analyze a two period game where the defender moves in the first period, and the attacker moves in the second period [18, 26]. This means that the defender selects a strategy in the first period that minimizes the maximum loss that the attacker may cause in the second period. In order to find the equilibrium, the game is solved with backward induction in which the second period is solved first using the following algorithm, which is an adaptation of the algorithm in [6] to take into account the extra-capacity dimension as a mean for indirect protection:

1) Inputs

- A system of m facilities ($i = 1, 2, \ldots, m$) located by solving the optimisation model (1)-(5).
- A set of β_ι protection types for each facility i ($p = 1, 2, \ldots, \beta_i$).
- A set of α_i attack types per facility i ($g = 0, 1, \ldots, \alpha_i$).
- A set of ρ_i extra-capacity options per facility i ($e = 0, 1, 2, \ldots, \rho_i$).
- *Parameters*:
    - Protection investment efforts F_{ip};
    - Unit costs of protection efforts f_{ip};
    - Attack investment efforts Q_{ig};
    - Unit costs of attack efforts q_{ig};
    - Capacity acquired C_i^*;
    - Capacity acquisition cost per unit Ac_i;
    - Proportion of the capacity acquired τ_{ie};
    - Contest intensities ε_i; and
    - Restoration costs R_i.

2) Initialization

Assign $U_{\min} = \infty$ ($U_{\min}$ is the defender minimal utility);

Assign $U_{\max} = 0$ ($U_{\max}$ is the attacker maximal utility).

3) Determination of the optimal attack strategy (*i.e.*, the strategy that maximises the attacker utility)

For each protection strategy **P** = $\left(\pi_i\right)$

3.1. For each attack strategy **G** = $\left(\omega_i\right)$

3.1.1. Construct a matrix **λ** = $\left(\lambda_{ip}\right)$ such as

$$\lambda_{ip} = \begin{cases} 1 & if \quad \pi_i = p \\ 0 & otherwise \end{cases}, \text{ and } \sum_{p=1}^{\beta_i} \lambda_{ip} = 1, \ \forall i;$$

3.1.2. Construct a matrix **μ** = $\left(\mu_{ig}\right)$ such as

$$\mu_{ig} = \begin{cases} 1 & if \quad \omega_i = g \\ 0 & otherwise \end{cases}, \text{ and } \sum_{g=0}^{\alpha_i} \mu_{ig} = 1, \ \forall i;$$

3.1.3. Determine the matrix **ν**(**P**,**G**) = $\left(\nu_{pg}(i)\right)$ such as each element $\nu_{pg}(i)$ is evaluated by using equation (9);

3.1.4. Calculate the costs C_R(**P**,**G**) by using equations (10);

3.1.5. For each extra-capacity strategy **E** = $\left(\theta_i\right)$

3.1.5.1. For each combination $k = 1, 2, \ldots, 2^m - 1$

3.1.5.1.1. Solve the model (1)-(5) to determine the cost T_k(**E**);

3.1.5.2. Calculate the expected cost *TB*(**P**,**G, E**) by using equation (11);

3.1.5.3. Calculate the attacker utility U_a (**P**,**G, E**) by using equation (14);

3.1.5.3.1. If U_a (**P**,**G, E**) > U_{max} assign U_{max} = U_a (**P**,**G, E**), $\mathbf{G}_{\text{opt}}$ = **G =** $\left(\omega_i^{opt}\right)$, $\mathbf{Q}_{\text{opt}}$ = $\left(Q_{i\omega_i^{opt}}\right)$;

4) Determination of the optimal defence strategy (*i.e.*, maximising the defender utility)

For each protection strategy **P** = $\left(\pi_i\right)$

4.1. Assign $\boldsymbol{\mu}_{\text{opt}}$ = $\left(\mu_{ig}\right)$ such as

$$\mu_{ig} = \begin{cases} 1 & if \quad \omega_i^{opt} = g \\ 0 & otherwise \end{cases};$$

4.2. Determine the matrix **ν**(**P**,$\mathbf{G}_{\text{opt}}$) = $\left(\nu_{p\omega_i^{opt}}(i)\right)$ such as each element $\nu_{pg}(i)$ is evaluated by using equation (9) with **λ**=$\left(\lambda_{ip}\right)$ and under attack strategy $\mathbf{G}_{\text{opt}}$, *i.e.*,

$$\nu_{p\omega_i^{opt}}(i) = \frac{\left(Q_{i\omega_i^{opt}}\right)^{\varepsilon_i}}{\left(F_{ip}\right)^{\varepsilon_i} + \left(Q_{i\omega_i^{opt}}\right)^{\varepsilon_i}};$$

4.3. Calculate the costs C_R(**P**,$\mathbf{G}_{\text{opt}}$) by using equations (10) (under attack strategy $\mathbf{G}_{\text{opt}}$);

4.4. For each extra-capacity strategy **E** = $\left(\theta_i\right)$

4.4.1. For each combination $k = 1, 2, \ldots, 2^m - 1$

4.4.1.1. Solve the model (1)-(5) to determine the cost T_k(**E**);

4.4.2. Calculate the expected cost TB(**P**,$\mathbf{G}_{\text{opt}}$,**E**) by using equation (11);

4.4.3. Calculate the defender utility U_d (**P**,$\mathbf{G}_{\text{opt}}$,**E**) by using equation (13);

4.4.4. If -U_d (**P**,$\mathbf{G}_{\text{opt}}$,**E**) < U_{min} assign U_{min} = -U_d (**P**,$\mathbf{G}_{\text{opt}}$,**E**),

$$\mathbf{P}_{\text{opt}} = \mathbf{P} = \left(\pi_i^{opt}\right),\ \mathbf{F}_{\text{opt}} = \left(F_{i\pi_i^{opt}}\right),\ \mathbf{E}_{\text{opt}} = \mathbf{E} = \left(\theta_i^{opt}\right),\ \mathbf{T}_{\mathbf{opt}} = \mathbf{T} = \left(\tau_{i\theta_i}^{opt}\right).$$

6. Illustrative example

In this section, a simple example is presented to illustrate the model. The defender optimal strategy obtained is compared to some defence strategies. The issues of limited budgets and the influence of contest intensities are also discussed. The model is used to find the best trade-off between direct investments in protection and extra-capacities deployment.

6.1. Input data

We consider 3 facilities and 5 demand nodes. Table 2 presents the maximum capacity and the fixed costs of locating facilities. Table 3 provides the yearly demands, and Table 4 gives the unit costs of producing and shipping between facility sites and customer locations. We assume that the total capacity acquisition cost at facility location, $h_i(.)$, is a linear function. Table 5 gives the capacity acquisition cost at facility location per unit.

When there is no attack, the optimal CFL & CAP solution for this problem is shown in Table 6 and pictured in Figure 1. This solution entails a fixed cost of $6,300,000 and a transportation cost of $3,812,000 over year.

Table 2
Maximum capacity and fixed cost of locating a facility at candidate site *i*

Site *i*	Fixed cost (in $)	Maximum Capacity
1	2,100,000	45,000
2	2,400,000	61,200
3	1,800,000	38,700

Table 3
Demand per year at customer location

Customer location *j*	Demand
1	25,000
2	21,000
3	13,000
4	11,000
5	10,500

Table 4
Unit cost (in $) of shipping from candidate facility site *i* to customer location *j*

	Site *i*	1	2	3
Customer location *j*	1	48	62	72
	2	55	50	66
	3	60	44	52
	4	65	58	48
	5	70	65	44

Table 5
Capacity acquisition cost at facility location per unit

Site i	Cost (in \$) per unit (Ac_i)
1	11
2	12
3	10

Table 6
Optimal CFL & CAP solution

	Site i	1	2	3
Customer location j	1	25,000	0	0
	2	0	21,000	0
	3	0	13,000	0
	4	0	0	11,000
	5	0	0	10,500
Capacity acquired C_i^*		25,000	34,000	21,500
Cost (in \$) of capacity acquired		275,000	408,000	215,000

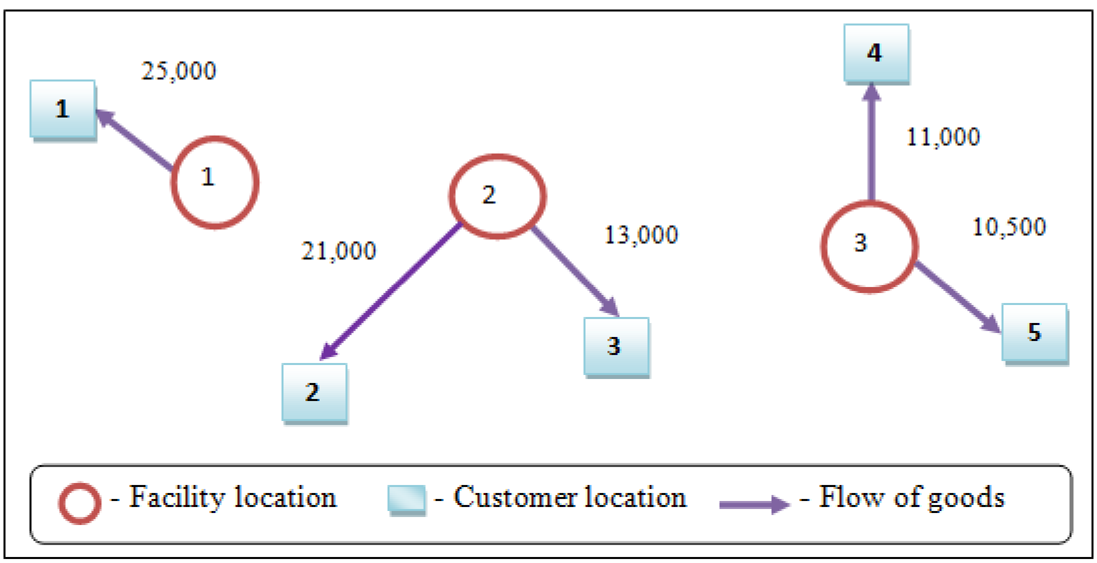

Fig. 1. The optimal CFL & CAP solution

Table 7 provides the protection investment efforts F_{ip}, and the unit costs of protection efforts f_{ip}. Analogously, the attack investment efforts Q_{ig} and the unit costs of attack efforts are given in Table 8. Table 9 presents the extra-capacity parameters. Table 10 gives the restoration costs R_i by week. We consider that all contest intensities are equal to 1, except in Section 6.5.

Table 7
Defence parameters

Protection types p	Unit costs f_{ip}	Protection efforts F_{ip}
1	70	100
2	555	280
3	200	130

Table 8
Attacker parameters

Attack types m	Unit costs q_{ig}	Attack efforts Q_{ig}
1	220	30
2	150	25
3	350	54

Table 9
Extra-capacity parameters

Extra-capacity options e	Proportion of the capacity acquired τ_{ie}
1	32%
2	68%
3	80%

Table 10
Restoration costs of disabled facilities (in $)

Disabled facility i	Restoration costs R_i
1	16,000
2	18,000
3	11,000

6.2. Evaluation of the costs Ct_k and B_k for all combinations of disabled facilities

We consider the case of **E** = (0 0 0) and **E** = (2 1 3) to evaluate the costs Ct_k and B_k for all combinations of disabled facilities as illustrated in Table 11 and Figure 2. If for example extra-capacity strategy is **E** = (0 0 0), the annual unmet demand is 25,000 for combination of disabled facilities S_1 (facility 1 is disabled by an attack), and 34,000 for combination S_2, etc. In this case, the backorders are incurred. If we assume that the cost of the brand image of the company (B_{img}) is \$200,000 and the average of the capacity acquisition costs at facility location per unit $\overline{Ac}$ is \$11, then the backorder B_1 is equal to $200{,}000+\left(25{,}000\times 11\times 0.20\right)/52=200{,}055$. If for example extra-capacity strategy is **E** = (2 1 3), when facility 1 is disabled by an attack, the quantity shipped from disabled facility to customers can be assigned to functional facilities 2 and 3. Recall that Ct_k is the cost incurred because of the increase in transportation cost when the combination is S_k, *i.e.* the cost under combination S_k by solving the new resulting CFL & CAP which leads to an optimal solution minus the cost in a normal situation. Since the restoration time is one week, each cost Ct_k is evaluated as the excess in transportation cost during this week. For example, when extra-capacity strategy is **E** = (2 1 3) and facility 1 is disabled, the cost Ct_1 is given by the cost under combination $S1$ (\$ 4,277,200) minus the cost (\$ 3,812,000) in a normal situation during one week. That is, Ct_1 is equal to (4,277,200 - 3,812,000)/52= \$8,946. Here, 52 is the number of weeks during 1 year.

Table 11
The costs Ct_k and B_k for all possible combinations S_k per week for **E** = (0 0 0) and **E** = (2 1 3)

k	Disabled facilities	**E** = (0 0 0)		**E** = (2 1 3)	
		B_k (in \$)	Ct_k (in \$)	B_k (in \$)	Ct_k (in \$)
1	1	200,055	0	0	8,946
2	2	200,075	0	0	4,865
3	3	200,047	0	0	7,377
4	1, 2	200,130	0	211,508	0
5	1, 3	200,102	0	209,807	0
6	2, 3	200,122	0	210,600	0
7	1, 2, 3	200,177	0	222,163	0

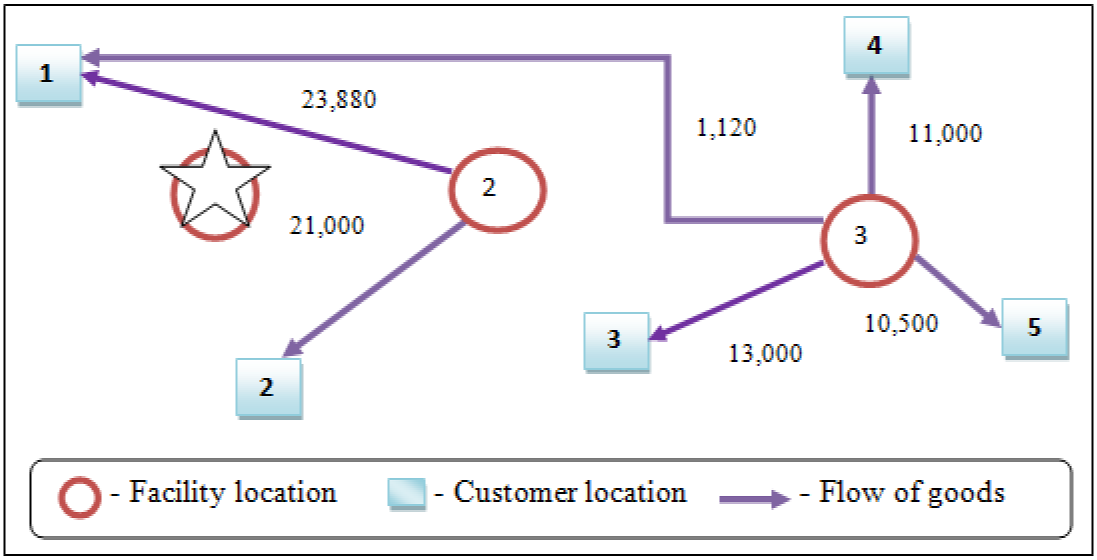

Fig. 2. The Optimal CFL & CAP solution if facility 1 is disabled and the extra-capacity option is

$\mathbf{E} = (2 \quad 1 \quad 3)$

6.3. Determination of the optimal attack strategy

The attacker strategy that maximises his utility by applying step 3 of the algorithm (*i.e*, the second period of the game is solved first) is $\mathbf{G}_{\text{opt}} = (3 \quad 3 \quad 1)$. This means that facilities 1and 2 are disabled using type 3 attacks; and facility 3 is disabled using type 1 attack. The maximum loss is \$1,393,785 and the corresponding attacker utility is U_{max} = \$1,349,385.

6.4. Determination of the optimal defender strategy

By applying step 4 of the algorithm and to find the equilibrium, taking into account the attacker strategy above (*i.e*, the attacker maximizes his utility), the first period of the game is solved in order to maximize the defender utility and consequently to allocate optimally the protective resources and extra-capacity among the facilities. The obtained solution corresponds to the following strategy:

- $\mathbf{P}_{\text{opt}} = (3 \quad 1 \quad 3)$: This means that facilities 1 and 3 are protected using type 3 protections; and facility 2 is protected using type 1 protection.
- $\mathbf{E}_{\text{opt}} = (2 \quad 1 \quad 3)$: This means that facility 1 extra-capacity is 68% of the facility 1 capacity acquired, facility 2 extra-capacity is 32% of the facility 2 capacity acquired and 80% of the facility 3 capacity acquired is the facility 3 extra capacity. The total cost of extra-capacity is \$489,560.

The loss is \$459,915 and the corresponding defender utility is U_{min} = \$1,008,475.

6.5. The optimal defender and attacker strategy as a function of the contest intensity

Figure 3 shows graphically the defender and attacker utility as a function of the contest intensity. We remark again that, the greatest contest intensity is more beneficial for the defender than for the attacker.

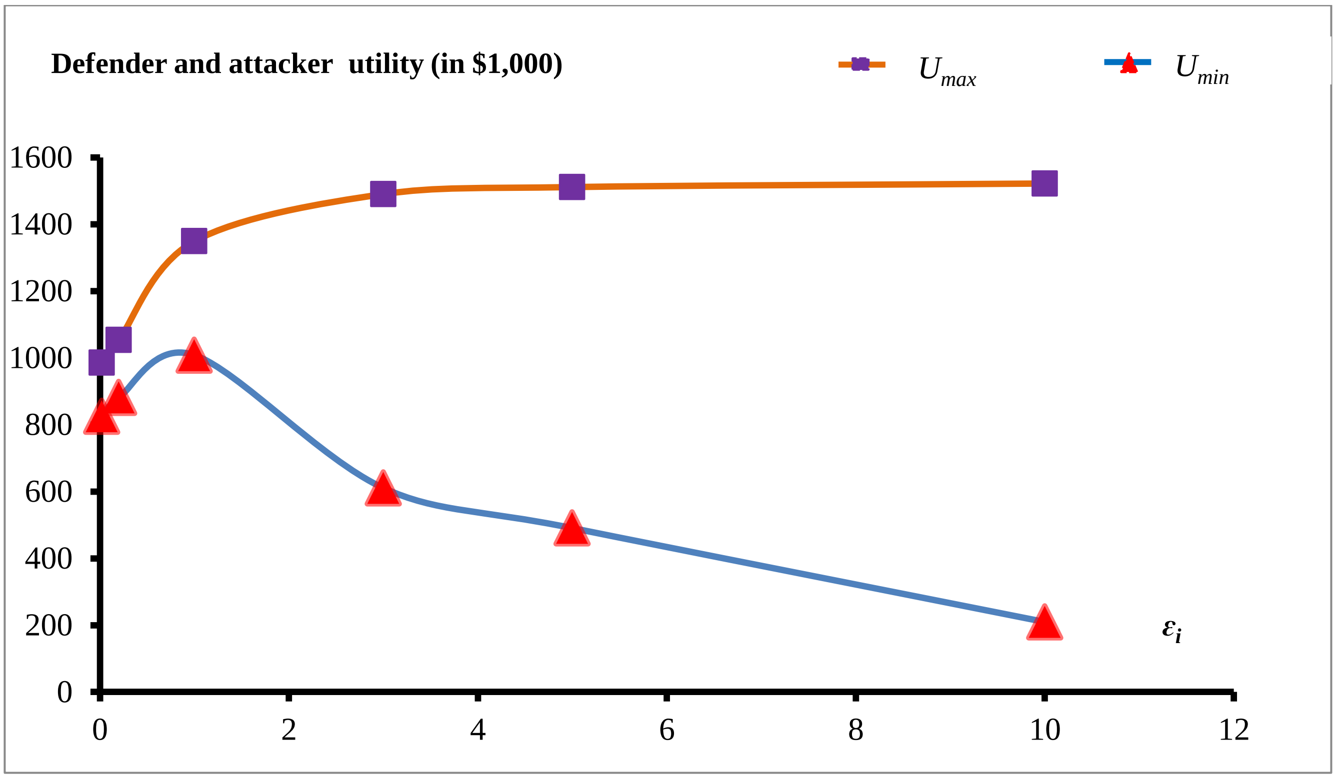

Fig. 3. Defender and attacker utility as a function of the contest intensity

6.6. Comparison

We compare the defender optimal strategy obtained by our model with some defence strategies to show that our method is better than others, when the attacker tries to maximize its utility. In our example, the facilities that have higher fixed costs are ranked as follows (see Table 2): Fac2, Fac1 and

Fac3. On the other hand, the protection types (*p*) are ranked according to their investment expenditures as follows (see Table 7): 2, 3, 1 and the extra-capacity options (*e*) are ranked according to their investment expenditures as follows (see Table 9): 1, 2, 3. The strategies considered in this comparison are as follows:

- *Strategy 1: Protection of facilities by more expensive protection types and higher extra-capacity*

  In this strategy, we consider that each facility is protected using type 2 protection and the highest extra-capacity available. That is, the protection strategy corresponds to $\mathbf{P}_1 = (2 \quad 2 \quad 2)$ and the extra-capacity strategy corresponds to $\mathbf{E}_1 = (3 \quad 3 \quad 3)$. The corresponding defender utility is U_{min} =$1,383,477.

- *Strategy 2: Protection of facilities by more expensive protection types and using lower extra-capacities*

  We consider that each facility is protected using type 2 protection and the lowest extra-capacity available. That is, the protection strategy corresponds to $\mathbf{P}_2 = (2 \quad 2 \quad 2)$ and the extra-capacity strategy corresponds to $\mathbf{E}_2 = (1 \quad 1 \quad 1)$. The corresponding defender utility is U_{min} =$1,549,525.

- *Strategy 3: Protection of facilities by cheaper protection types and using higher extra-capacities*

  We consider here that all facilities are protected using type 1 protections and using the lowest extra-capacities. That is, the protection strategy corresponds to $\mathbf{P}_3 = (1 \quad 1 \quad 1)$ and the extra-capacity strategy corresponds to $\mathbf{E}_3 = (3 \quad 3 \quad 3)$. The corresponding defender utility is U_{min} =$1,432,768.

- *Strategy 4: Protection of facilities by cheaper protection types and using lower extra-capacities*

  We consider that all facilities are protected using type 1 protections and using an the lowest extra-capacities available. That is, the protection strategy corresponds to $\mathbf{P}_4 = (1 \quad 1 \quad 1)$ and the extra-capacity strategy corresponds to $\mathbf{E}_4 = (1 \quad 1 \quad 1)$. The corresponding defender utility is U_{min} =$1,680,310.

- *Strategy 5: Facilities with higher fixed costs are protected by more expensive protection types and using lower extra-capacity*

This means that we consider that: facility 1 is protected using type 3 protection and using extra-capacity option 2; facility 2 is protected using type 2 protection and extra-capacity option 1; facility 3 is protected using type 1 protection and extra-capacity option 3. That is, the protection strategy corresponds to $\mathbf{P}_4 = (3 \quad 2 \quad 1)$ and the extra-capacity strategy corresponds to $\mathbf{E}_4 = (2 \quad 1 \quad 3)$. The corresponding defender utility is U_{min} =\$1,109,842.

The obtained results indicate that the defender strategy obtained by our model is:

- 37.19% better than Strategy 1;
- 53.63% better than Strategy 2;
- 42.07 % better than Strategy 3;
- 66.62% better than Strategy 4; and
- 10.05% better than Strategy 5.

Our model gives better results as it is designed to find the best trade-off between direct protection investment and extra-capacity deployment, unlike the above five strategies.

6.7. Limited budget

We assume that the defender strategy optimization problem is solved for a limited defender's budget. Table 12 presents the best trade-off between extra-capacity options and protection strategies for different budgets.

We remark from this table that for a lower protection budget, the expected damage is indeed higher. The investment size in facility protection (including the use of extra-capacity) affects the facilities defence. In fact, for each increase in the defender budget, it is important to quantify the resulting reduction in the expected damage, in order to decide if it is worth investing more in protecting facilities and using extra-capacity.

Table 12

Obtained defence and extra-capacity strategies for different budgets ($\varepsilon_j = 1$)

Defender budget (in $)	Protection cost (in $)	Extra capacity Cost (in $)	Expected damage (in $)	Defence strategy $\mathbf{P}_{opt}$	Extra-capacity strategy $\mathbf{E}_{opt}$
25,000	21,000	0	1,393,785	(1 1 1)	(0 0 0)
50,000	40,000	0	1,329,245	(1 3 1)	(0 0 0)
100,000	59,000	0	1,177,915	(3 1 3)	(0 0 0)
265,000	59,000	0	1,177,915	(3 1 3)	(0 0 0)
360,000	59,000	0	1,177,915	(3 1 3)	(0 0 0)
400,000	59,000	326,400	740,708	(3 3 1)	(0 3 0)
450,000	78,000	326,400	700,787	(3 3 3)	(0 3 0)
500,000	78,000	326,400	700,787	(3 3 3)	(0 3 0)
550,000	207,400	326,400	506,770	(3 2 3)	(0 3 0)
600,000	59,000	489,560	459,915	(3 1 3)	(2 1 3)
650,000	59,000	489,560	459,915	(3 1 3)	(2 1 3)
700,000	59,000	489,560	459,915	(3 1 3)	(2 1 3)

The expected damage cost as a function of the defender budget (*i.e*, budget for the protection and for using the extra-capacity) is presented in Figure 4. This curve is drawn by evaluating the optimal protection and extra-capacity strategy for each budget, considering that the attacker chooses the harmful strategy. This curve shows graphically the relationship between the investment sizes of protection and extra-capacity and the effect on facilities defence. From this curve, we remark that the budget greater than $600,000 cannot reduce the expected damage.

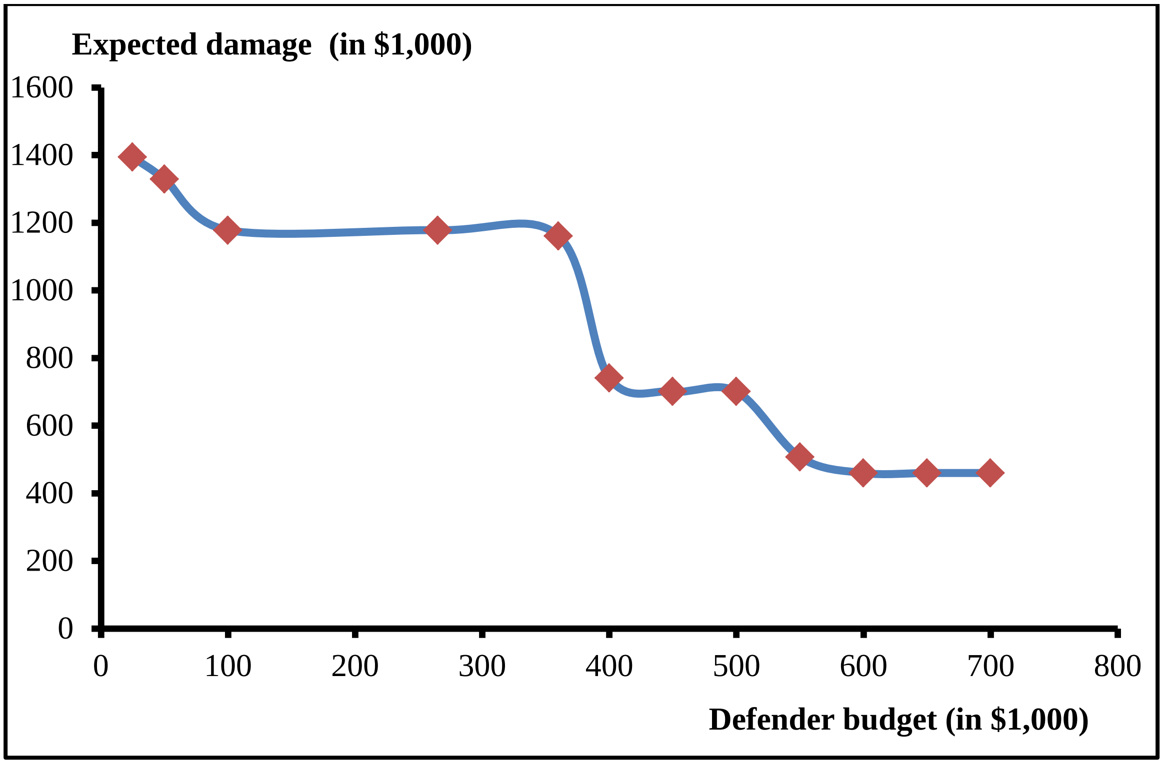

Fig. 4. Expected damage costs as a function of defender budget ($\varepsilon_j = 1$)

7. Conclusion

This article deals with the protection of supply network in the context of the *capacitated* fixed-charge location and capacity acquisition problem. Facility location and capacity acquisition are of vital importance to supply chain management. We consider that not only a set of investment alternatives are available for direct protection of facilities, but also extra-capacities of neighbouring functional facilities can be used after attacks for indirect protection. The strategic decision dealt with is how to allocate optimally the protective resources and the extra-capacity among the facilities, knowing that these facilities are exposed to external intentional attacks. The idea of using extra-capacity to indirectly protect supply networks against intentional attack is then used to develop a game-theoretic model, with the objective of finding the best trade-off between direct investments in protection and indirect protection by extra-capacities deployment. In this model, a non-cooperative two-period game is analyzed. The attacker tries to maximize, in the first period, his utility function by maximizing the expected damage with several alternatives of attack. On the other hand, the defender attempts to minimize, in the second period, the expected damage caused by attacks with several alternatives of facilities protection and extra-capacity options and consequently to maximize his utility function, where resources for direct protection and extra-capacities are limited. A method is developed to evaluate the utilities of the players. The expected costs evaluated by our method include the cost incurred because of the increasing in transportation cost after attacks, the cost necessary to restore disabled facilities and the backorder cost. An algorithm is presented for determining the equilibrium solution and the optimal defender strategy. The defence strategy obtained by our model is compared to five strategies, and the obtained results indicate clearly the superiority of our model in finding the best trade-off between direct protection investment and extra-capacity deployment. The developed approach gives important managerial insights for the protection of located facilities under capacity constraints, while using extra-capacity options for protection purpose.

We are currently working on the modeling and analysis of interdependencies between facilities while considering multi-echelon supply chain networks.

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Annex

Nomenclature

n	number of customers in the system
m	number of facilities in the system
i	jth potential facility location, $i = 1, 2, \ldots, m$
j	ith demand location, $j = 1, 2, \ldots, n$
g_i	fixed cost of locating a facility at candidate site i
$h_i(.)$	total capacity acquisition cost at facility i
c_{ij}	unit cost of shipping between candidate facility site i and customer location j
D_j	demand at customer location j
CAP_i	maximum capacity that can be built-in at candidate site i
Z_{ij}	quantity shipped from candidate facility site i to customer location j
Y_i	binary variable, which is equal to 1 if a facility is to be located at candidate site i, and 0 otherwise
β_i	number of protection types for facility i
p	index of protection type, $p = 1, 2, \ldots, \beta_i$
F_{ip}	investment effort to protect a facility located at site i using protection type p
f_{ip}	unit cost of effort to protect a facility located at site i using protection type p
$\overline{F_{ip}}$	investment expenditure to protect a facility located at site i using protection type p
π_i	value from $p = 1, 2, \ldots, \beta_i$
π_i^{opt}	optimal defence strategy value from $p = 1, 2, \ldots, \beta_i$
P	vector of protection strategy, $\mathbf{P}=\left(\pi_i\right)$
$\mathbf{P}_{\mathrm{opt}}$	vector of the optimal protection strategy, $\mathbf{P}_{\mathrm{opt}} = \left(\pi_i^{opt}\right)$
F	vector of investments to protection strategy **P, F**$= \left(F_{i\pi_i}\right)$
$\mathbf{F}_{\mathrm{opt}}$	vector of investments to protection strategy $\mathbf{P}_{\mathrm{opt}}$, $\mathbf{F}_{\mathrm{opt}} = \left(F_{i\pi_i^{opt}}\right)$
$F_{i\pi_i}$	element of investments vector **F**

$F_{i\pi_i^{opt}}$ element of investments vector $\mathbf{F}_{\text{opt}}$

λ_{ip} binary variable which is equal to 1 if a protection of type p is used for facility i

$\boldsymbol{\lambda}$ matrix, $\boldsymbol{\lambda} = \left(\lambda_{ip}\right)$

ρ_i number of extra-capacity options for each facility i

e index of extra-capacity options, $e = 1, 2, \ldots, \rho_i$

τ_{ie} proportion of the capacity acquired associated of the facility located at site i using extra-capacity option e

C_i^* capacity acquired associated of the facility located at site i

Ac_i capacity acquisition cost at facility location i per unit

CE_{ie} investment of extra-capacity associated of the facility located at site i using extra-capacity option e, $CE_{ie} = Ac_i \tau_{ie} C_i^*$

$\mathbf{E}$ vector of extra-capacity strategy, $\mathbf{E} = \left(\theta_i\right)$

θ_i values from $e = 0, 1, \ldots, \rho_i$

$\mathbf{T}$ vector of investments to each extra-capacity strategy $\mathbf{E}$, $\mathbf{T} = \left(\tau_{i\theta_i}\right)$

ξ_{ie} binary variable which is equal to 1 if an extra-capacity option e is selected for facility i

α_i number of attack types against any facility i

g index of attack type ($g = 0, 1, \ldots, \alpha_i$)

Q_{ig} attack effort to attack facility located at site i using attack action g

q_{ig} unit cost to attack facility located at site i using attack action g

$\overline{Q_{ig}}$ investment expenditure to attack facility located at site i using attack action g

ω_i value from $g = 0, 1, \ldots, \alpha_i$

ω_i^{opt} value from g of the optimal attack strategy

$\mathbf{G}$ vector of attack strategy, $\mathbf{G}=\left(\omega_i\right)$

$\mathbf{G}_{\text{opt}}$	vector of the optimal attack strategy, $\mathbf{G}_{\text{opt}} = \left(\omega_i^{opt}\right)$
$\mathbf{Q}_{\text{opt}}$	vector of attack effort of the optimal attack strategy, $\mathbf{Q}_{\text{opt}} = \left(Q_{i\omega_i^{opt}}\right)$
$Q_{i\omega_i^{opt}}$	element of attack effort vector $\mathbf{Q}_{\text{opt}}$
μ_{ig}	binary variable which is equal to 1 if a type g attack is used for facility i
$\boldsymbol{\mu}$	matrix, $\boldsymbol{\mu} = \left(\mu_{ig}\right)$
$\boldsymbol{\mu}_{\text{opt}}$	matrix, $\boldsymbol{\mu}_{\text{opt}} = \left(\mu_{ig}\right)$
$\nu_{pg}(i)$	destruction probability of a facility i
$\nu_{p\omega_i^{opt}}(i)$	destruction probability of a facility i for the optimal defence strategy
$\boldsymbol{\nu}(\mathbf{P},\mathbf{G})$	matrix, $\boldsymbol{\nu}(\mathbf{P},\mathbf{G}) = \left(\nu_{pg}(i)\right)$
$\boldsymbol{\nu}(\mathbf{P},\mathbf{G}_{\text{opt}})$	matrix, $\boldsymbol{\nu}(\mathbf{P},\mathbf{G}_{\text{opt}}) = \left(\nu_{p\omega_i^{opt}}(i)\right)$
ε_i	parameter that expresses the intensity of the contest concerning facility i
$C_R(\mathbf{P},\mathbf{G})$	expected cost required to restore the attacked facilities which depends on $\mathbf{P}$ and $\mathbf{G}$
$C_R(\mathbf{P},\mathbf{G}_{\text{opt}})$	expected cost required to restore the attacked facilities which depends on $\mathbf{P}$ and $\mathbf{G}_{\text{opt}}$
R_i	cost required to restore the attacked facility i
k	combinations index, $(k = 0,\ldots,2^m - 1)$
S_k	combinations of disabled and functional facilities for the facilities
S	set of combinations of disabled and functional facilities, $S = \{S_k\}$
Ct_k	cost incurred because of the increase in transportation cost when the combination is S_k
$\overline{Ac}$	average of the capacity acquisition costs per unit
B_{img}	brand image of the company
YD_k	annual unmet demand
B_k	backorder cost when the combination is S_k
Ω	binary variable, which is equal to 1 if the cost Ct_k incurred and 0 if the backorder B_k cost is incurred
$\Delta C_{pg\text{E}}(k)$	attack outcomes of combination k which depends on p, g and $\mathbf{E}$

$TB(\mathbf{P},\mathbf{G},\mathbf{E})$	expected cost associated with the transportation cost increase and the backorder cost which depends on **P, G** and **E**
$D(\mathbf{P},\mathbf{G},\mathbf{E})$	expected damage which depends on **P, G** and **E**
$U_d(\mathbf{P},\mathbf{G},\mathbf{E})$	defender expected utility which depends on **P, G** and **E**
$U_d(\mathbf{P},\mathbf{G}_{\mathrm{opt}},\mathbf{E})$	defender expected utility which depends on **P**, $\mathbf{G}_{\mathrm{opt}}$ and **E**
$U_a(\mathbf{P},\mathbf{G},\mathbf{E})$	attacker expected utility which depends on **P, G** and **E**
$U_{\min}$	defender minimal utility
$U_{\max}$	attacker maximal utility