Annual Timber Procurement Planning with Bucking Decisions

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Abstract. The problem we consider is a practical multiple-period wood procurement planning problem in the Eastern Canadian context. The forest cut blocks are large and heterogeneous; they have different densities and the diameter of the trees varies. This is a difficult forest management problem because it integrates two related problems: the forest bucking problem with a cut-to-length (CTL) bucking system and the multi-facility supply planning problem. The choice of the areas to harvest in each period and how to harvest them affects the assortments provided to the mills. We must decide which areas to harvest in each period so that the demands of the various wood-processing facilities are satisfied. Moreover, we must indicate how to harvest the different cut blocks according to the bucking priority list and the quantity of harvested logs from each block to transport to the sawmill. In this paper, we extend the procurement model presented in Dems et al. [10] to more detailed multiple-period planning. We develop a mixed integer linear model, and we propose two heuristic approaches that quickly generate an initial feasible schedule of cut blocks. Computational results from an Eastern Canadian forestry company are presented.

Keywords: Forest bucking problem, wood-procurement planning, mixed integer programming.

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1 Introduction

Wood procurement planning (WPP) encompasses a wide range of activities that provide quantities of wood to processing mills (see [2]). The wood supply chain (WSC) involves a complex set of interrelated decisions about harvest scheduling, forest bucking, and supplies to mills [23].

In customer-oriented WPP, it is more important to improve the fit between mill demand and the output of the bucking operations, than to minimize the operational costs [23]. If tree bucking and wood-supply planning are considered separately, some of the supply plans may be infeasible because of the heterogeneity of the forest [7]. In addition, tree bucking is an irreversible process (it is impossible to correct a poor bucking), and it has a direct impact on the end products of the sawmill [7].

In this paper, we extend the procurement model presented in Dems et al. [10] to multiple-period planning. The planning horizon considered is twelve months. Multiple-period planning allows the planner to investigate the impact of temporal variations in demands, log availability, and inventory holdings on the procurement plan.

Our first goal is to find a near-optimal wood-procurement plan, driven by mill demands, within a practical time limit. Given a list of cut blocks to harvest, we must decide which blocks to harvest at each period to satisfy the demands of various wood-processing facilities. The allocation of the cut blocks to harvesting periods affects the assortment produced, since each cut block has a particular mix.

A significant part of the harvesting cost arises because the production yield decreases nonlinearly as the number of product types per cut block increases. Our cost formulation takes this into account (see [10]). This is an important aspect of the decision-making process in forest management [1]. This production level is also affected by the bucking priority list, because of the divergent nature of the bucking process.

Moreover, the model indicates in what quantities the harvested logs from each block should be transported to the mills. The problem includes overall decisions about transportation, storage in the forest, and storage at the mill terminals. There are also a number of restrictions to be considered during harvesting such as the variability of the weather conditions during the year. For example, it is impossible to carry out harvesting and transportation activities during a thaw.

We develop a mixed integer linear model describing the problem. Our objective is to decrease the operational costs (harvesting, transportation, and inventory costs). We achieve this by a better scheduling of the harvesting of cut blocks in different periods and by optimizing the allocation of bucking lists to species. This study is an extension of the research of Dems et al. [10]; see [10] for more details of the bucking problem.
The remainder of this paper is organized as follows. Section 2 presents an overview of the literature, and section 3 introduces the problem. Section 4 presents the mathematical formulation, and in section 5, we present our solution approaches. The data used in our tests and the computational results are introduced in section 6. Finally, section 7 presents concluding remarks and some research perspectives.

2 LITERATURE REVIEW

WSC planning has been discussed in several papers; for a survey, see D’Amours et al. [9]. The WSC includes many decisions and operations. They range from strategic to operational levels of planning, depending on the planning horizon. The times differences for each level are not well defined and may differ from one problem to another. Carlsson et al. [6] present a summary of the strategic, tactical, and operational planning decisions involved in the pulp and paper industry.

Operations research is increasingly used in the development of tools for various forest planning problems ([3],[24],[25]). The methodology developed for some typical problems, at different planning levels in the WSC is reviewed in [20] and [3].

2.1 Integrated wood procurement problem

Operational planning concerns short-term decisions covering one day to about two months (see [13]); it is directly connected to harvest operations. Short-term harvest planning [20] may include harvest planning, transportation problems [13], crew scheduling [16], machine location [12], control of storage in the forest and at the terminals and the use of sorting yards [21], and bucking problems ([15], [18], [17]. In some cases, several activities are integrated within a single model to form a multi-element WPP problem.

WSC planning has helped to improve the performance of forestry companies, but integrating the requirements of different planning problems into the supply chain is still challenging.

Arce et al. [1] proposed a mixed integer linear programming (MIP) model for a harvest planning problem with forest bucking and transport decisions. Their objective was to maximize the total net revenue at the forest level. The bucking patterns are generated using simple heuristic rules, and the number of products bucked per stand is limited; the authors did not consider the impact on the harvesting cost.

Chauhan et al. [8] presented a short-term (e.g., one week) multi-commodity WPP problem. They proposed an extension of the model presented in [7] that takes bucking decisions into account. They used bulk-process-based bucking, which is a simplification of the real bucking process. They were the first to take into account the impact
the number of harvested assortments on the harvesting cost.

Dems et al. [10] proposed an annual procurement plan that respects the harvesting practices used in Eastern Canada. The model incorporates bucking and transportation activities. The bucking optimization is based on the customer demand and generates adequate bucking patterns using a priority-list approach. Furthermore, this approach is based on a simulation of the harvest yields of each bucking scenario for each cut block, whereas the approaches in Chauhan et al. ([7], [8]) compute these yields directly according to the cut-block tree-diameter distributions. The model includes a harvesting cost function that considers the nonlinearity of the harvester productivity function; this is important in forest management [1]. The authors compared different bucking scenarios to help decision makers to develop a more efficient forest procurement system.

The problems presented above consider a single period. Single-period WPP does not consider seasonality, which has a large impact on the WSC. It is therefore not possible to analyze the impact of temporal variations (e.g., weather conditions) on inputs such as the demand, the flow of logs from the cut blocks, and the inventory holdings. Moreover, single-period planning assumes that it is possible to mix logs from different cut blocks since they can be harvested simultaneously. However, in the real problem, there are operational restrictions on the number of cut blocks that can be harvested at a given time.

Epstein et al. [11] proposed a multi-period WPP problem including cut-block scheduling, bucking, and transportation activities. The method relies on a decomposition technique where the bucking patterns are generated in the subproblem and included in the master problem during the optimization process. As noted by many authors, this decomposition approach is theoretically correct and computationally efficient ([19], [22]). However, it is difficult to implement because of the generation of a large number of cutting instructions and the difficulty of subdividing the cut blocks into different stem classes.

Karlsson et al. [16] proposed an MIP model for a WPP problem integrating transportation and annual road-maintenance planning. In this short-term problem, bucking patterns are not considered because the cutting instructions are short-term decisions provided by the harvester’s on-board computer.

Bredström et al. [4] presented an MIP closely related to that of Karlsson et al. [16]. The model integrates the assignment of machines and harvest teams (crews) to harvest areas and the scheduling of the harvest areas during the year for each machine. The two-phase solution approach first solves the assignment and then considers the scheduling.

The problem addressed in this paper is a multi-period multi-commodity WPP problem with multiple sources (cut blocks) and multiple destinations (mills). We generate an annual wood-procurement plan that respects the harvesting practices used
in Eastern Canada. The planning integrates bucking, transportation, and inventory decisions. The model extends that presented in [10] to multiple periods. We use the priority-list approach developed in [10] to generate bucking patterns that are practical and easy to implement.

3 PROBLEM DESCRIPTION

The management of the forest cut blocks (harvesting areas) is centralized and done by the same multifacility forestry company. A strategic five-year plan defining the blocks to harvest is defined at a higher level of planning. The right to harvest the trees of these cut blocks is obtained from the government through timber licences (TL). The company performs the harvesting and transportation activities itself to supply its geographically distributed mills.

**Current planning approach.** The WPP problem is currently solved manually. Typically, the experienced planners rely on data from preceding periods and trial-and-error. They generate the plan using general-purpose tools such as spreadsheets and a geographical information system. It is difficult and time-consuming to generate a plan that meets the requirements of all the stakeholders. These plans generate significant log inventories through mismatches of production and demand.

The major WSC activities included in this project are harvesting, storing logs in dedicated areas, transporting logs from the forest to the mills, and storing logs at the mills.

**Harvesting.** The cut blocks are large and heterogeneous, with important differences in the tree diameters. They are accessible through a road network. The proposed model links activities from two different levels of forest planning: the scheduling of cut blocks (tactical level) and forest bucking (operational level). In this paper, our optimization approach defines the sequence of blocks to be harvested over the twelve periods. We assume that an area can be harvested in at most six contiguous periods. Seasonality has also a large effect on the harvesting operations.

**Forest bucking.** We use the priority-list bucking approach of [10] to generate simple patterns. We allocate logs to each stem section using a priority list. A priority list is a sequence of at least two of at most \( l_{\text{max}} \) allowable log-types obtained from a stem, generated according to simple rules. The position of a product in the priority list depends on its commercial value, length, and minimum small end diameter (MSED). The lowest priority is assigned to the product with the shortest length and smallest MSED, generally a pulp log. A bucking priority list is assigned to each species.
We use the simulation tool FPInterface to predict the yield products from the application of a given bucking priority list to a sample of trees from the cut blocks. This software, designed by FPInnovations, is used to simulate different activities in the forest supply chain.

As in [10], we also consider the effect of a productivity decrease in the harvesting machinery on the harvesting cost. The harvesting cost increases with the number of products bucked per cut block and decreases with the average length of the products. As reported in the literature ([14], [5]), a reduction of 1%–4% in the harvester productivity (respectively 3%–7% in the forwarder productivity) is generated by harvesting a new log type in a cut block. This increases the harvesting cost and leads to complex instructions for the log makers. The resulting harvesting-cost formulation increases the combinatorial complexity of classical WPP. The unit harvesting cost does not depend on the time periods.

Storage in forest areas. Some of the harvested volumes are left at the roadside. Others are transported to mills to be used or stored in their storage areas, depending on the demand. The cost of storage at the cut blocks corresponds to the quality deterioration; this decrease in quality is not as important in the winter. The roadside storage is unlimited but it is not desirable for too many products to stay at the roadside.

Transportation. The transportation cost is a significant portion of the total cost. It depends on the distance between the blocks and the mills as well as on the product type. All transported volumes are delivered to their final destinations. Part of the delivery is used to meet the mill demand, and the remainder is placed in storage, with an associated inventory cost. We place an upper bound on the total volume transported in each period.

Storage at mills. Part of each delivery may be placed in storage areas. No exchange of timber between mills is allowed, and each mill has a given storage capacity. There is an inventory holding cost per cubic meter, which depends on the period.

Decision-support objective. Given the annual demand from a set of geographically distributed mills and the set of forest cut blocks to harvest during the year, we propose a mathematical model for the problem described above. The objective of this paper is to find a near-optimal wood procurement plan for a planning horizon of one year, divided into twelve months, to support Eastern Canadian forestry companies without changing the technologies currently in use (see Fig. 1).
4 MATHEMATICAL FORMULATION

This section presents a formal mathematical model for the general problem. We use the following variables and parameters:

- **Parameters**
  - $B$ Set of forest cut blocks;
  - $U$ Set of mills;
  - $P$ Set of product types;
  - $E$ Set of species;
  - $E_b$ Set of species in block $b$;
  - $B_e$ Set of cut blocks containing species $e$;
  - $R$ Set of priority lists;
  - $P_r$ Set of products in bucking priority list $r$;
  - $I$ Set of schedules;
  - $I_b$ Set of schedules for block $b$;
  - $T$ Set of time periods;
  - $T_b$ Number of months needed to totally harvest block $b$;
\( d_{pe}^u \) Demand at mill \( u \) for product \( p \) of species \( e \) in period \( t \) \((m^3)\);

\( V_{pe}^{br} \) Volume of product \( p \) available when bucking species \( e \) of block \( b \), according to priority list \( r \) (output of simulation);

\( C_{bupe}^T \) Unit transportation cost between block \( b \) and mill \( u \) for product \( p \) of species \( e \) \((\$\/m^3)\);

\( C_{bpet}^{SF} \) Unit inventory cost of product \( p \), species \( e \), in block \( b \) during time period \( t \) \((\$\/m^3)\);

\( C_{upet}^{SU} \) Unit inventory cost of product \( p \), species \( e \), in mill \( u \) during time period \( t \) \((\$\/m^3)\);

\( M \) Large number, for example equal to value of largest cut block’s standing timber.

\( V_{i}^{Max} \) Maximum harvesting capacity per period according to schedule \( i \);

\( V_{t}^{Hi} \) Total harvesting capacity in time period \( t \) \((m^3)\);

\( V_{t}^{Ti} \) Total transportation capacity in time period \( t \) \((m^3)\);

\( V_{u}^{S} \) Inventory capacity of mill \( u \) \((m^3)\);

\( a_{bi} \) Coefficient used to extract information from schedule \( i \). Takes value 1 if block \( b \) is harvested in period \( t \), according to schedule \( i \);

\( NB_{Max} \) Maximum number of blocks to harvest in each period;

Variables

\( \theta_n^b \) Binary: takes value 1 if \( n \) different products are obtained from block \( b \); 0 otherwise;

\( z_{ber}^n \) Binary: takes value 1 if bucking priority list \( r \) is applied to species \( e \) of block \( b \) when \( n \) different products are obtained from \( b \); 0 otherwise;

\( h_{bi} \) Binary: takes value 1 if block \( b \) is allocated to schedule \( i \); 0 otherwise;

\( q_{b}^t \) Proportion of block \( b \), harvested in time period \( t \);

\( y_{ber}^t \) Proportion of block \( b \), harvested in time period \( t \), when bucking species \( e \) of \( b \), using bucking priority list \( r \);

\( x_{bpe}^{ut} \) Flow of product type \( p \), species \( e \) from block \( b \) to mill \( u \) in period \( t \) \((m^3)\);

\( k_{upe}^t \) Volume of product \( p \), species \( e \), used by mill \( u \) in time period \( t \);
\( f_{bpe} \) Stored volume of product \( p \), species \( e \), in block \( b \), at end of period \( t \) \((-m^3)\);
\( s_{upe}^{t} \) Stored volume of product \( p \), species \( e \), in mill \( u \), at end of period \( t \) \((-m^3)\);
\( t_{upe}^{t} \) Orders of product \( p \), species \( e \), used by mill \( u \) in time period \( t \);

### 4.1 Mathematical Model

A mixed-integer linearized mathematical formulation of problem \((P)\) is:

**Model**

\[
(P) \quad \text{Min} \sum_{b \in B} \sum_{e \in E_{b}} \sum_{r \in R} \sum_{p \in P} C_{bren}^{H} V_{pe}^{b} s_{upe}^{t} + \sum_{t \in T} \sum_{b \in B} \sum_{e \in E_{b}} \sum_{p \in P} C_{bupe}^{T} f_{bpe}^{t}
\]

\[
+ \sum_{t \in T} \sum_{b \in B} \sum_{e \in E_{b}} \sum_{p \in P} C_{bpe}^{S} F_{bpe}^{t} + \sum_{t \in T} \sum_{b \in B} \sum_{e \in E_{b}} \sum_{p \in P} \sum_{u \in U} C_{bpe}^{U} s_{upe}^{t} + \sum_{t \in T} \sum_{b \in B} \sum_{e \in E_{b}} \sum_{p \in P} \sum_{u \in U} L_{bpe}^{t} s_{upe}^{t}
\]

subject to

**Harvesting activities**

\[
\sum_{n \in N} \theta_{b}^{n} = 1 \quad \forall b \in B \tag{1}
\]

\[
\sum_{e \in E_{b}} \sum_{r \in R} \gamma_{r} z_{b}^{n} = n \theta_{b}^{n} \quad \forall b \in B \text{ and } \forall n \in N \tag{2}
\]

\[
\sum_{n \in N} \sum_{e \in E_{b}} z_{b}^{n} = 1 \quad \forall b \in B \text{ and } \forall e \in E_{b} \tag{3}
\]

\[
\sum_{n \in N} z_{b}^{n} = \sum_{i \in T} y_{i}^{t}_{b} \quad \forall b \in B, \forall e \in E_{b}, \text{ and } \forall r \in R \tag{4}
\]
Block scheduling constraints

\[
\sum_{b \in B} q^b_t V^b \leq V^H_t \quad \forall t \in T
\] (5)

\[
\sum_{i \in I_b} h_{bi} = 1 \quad \forall b \in B
\] (6)

\[
q^b_t \leq \sum_{i \in I_b} a^t_{bi} h_{bi} \quad \forall b \in B \text{ and } \forall t \in T
\] (7)

\[
q^{t+1}_b \leq q^t_b - \sum_{i \in I_b} a^t_{bi} h_{bi} + 1 \quad \forall b \in B \text{ and } \forall t \in T - 1
\] (8)

\[
q^b_t V^b \leq \sum_{i \in I_b} V^b_{i_{\text{Max}}} h_{bi} \quad \forall b \in B \text{ and } \forall t \in T
\] (9)

\[
\sum_{r \in R} y^t_{ber} = q^t_b \quad \forall b \in B, \forall e \in E_b, \text{ and } \forall t \in T
\] (10)

\[
\sum_{b \in B} \sum_{i \in I_b} a^t_{bi} h_{bi} \leq N B_{\text{Max}} \quad \forall t \in T
\] (11)

Procurement activities

\[
f^t_{bpe} = \sum_{r \in R} y^t_{ber} V^{br}_{pe} - \sum_{u \in U} x^u_{t_{bep}} + f^{t-1}_{bpe} \quad \forall b \in B, \forall e \in E_b, \forall p \in P, \text{ and } \forall t \in T - 0
\] (12)

\[
s^t_{upe} = s^{t-1}_{upe} + \sum_{b \in B} x^u_{t_{bep}} - D^u_{t_{upe}} + t^u_{t_{upe}} \quad \forall u \in U, \forall e \in E_b, \forall p \in P, \text{ and } \forall t \in T - 0
\] (13)

\[
\sum_{e \in E} \sum_{p \in P} s^t_{upe} \leq V^S_u \quad \forall u \in U \text{ and } \forall t \in T
\] (14)

\[
\sum_{b \in B} \sum_{e \in E_b} \sum_{p \in P} \sum_{u \in U} x^u_{t_{bep}} \leq V^T_t \quad \forall t \in T
\] (15)

Binary variables

\[
h_{bi}, \theta^n_{b}, \bar{z}^n_{ber} \in \{0,1\} \quad \forall b \in B, \forall r \in R, \forall e \in E, \forall p \in P, \text{ and } \forall n \in N
\] (16)

Continuous variables

\[
x^u_{t_{bep}}, k^t_{bpe}, s^t_{upe}, y^t_{ber} \geq 0 \quad \forall b \in B, \forall e \in E, \forall p \in P, \forall u \in U, \text{ and } \forall t \in T
\] (17)

The objective function minimizes the total operational costs: the harvesting cost, the transportation cost, the storage cost, and the penalties on the default volumes at the mills (the orders). The unit harvesting cost considers the nonlinearity of the
harvester productivity function. Specifically, it considers the number of different log types harvested per block, which is a delicate aspect of forest management. We calculate the unit harvesting cost \( C_{\text{har}} \) using the Dems et al. approximation.

Constraints (1) and (2) count the number of different log types harvested in each cut block \( n \). Constraint (3) ensures that we have only one \( n \) in each cut block and we use only one bucking list per species per block. Constraint (4) ensures that the sum of the volume proportions of a species using a bucking list \( r \) is equal to one if this priority list is assigned to it and zero otherwise.

Constraints (5) through (11) deal with block scheduling. Constraint (5) limits the total volume harvested per period. Constraint (6) ensures that we assign only one schedule per block. Constraints (7) and (8) ensure the continuity of the harvesting activity of a block once begun. Constraint (9) ensures that the harvested volume does not exceed the associated harvesting schedule capacity. Constraint (10) ensures that the proportion of the volume harvested from each species is equal to the proportion harvested from the block volume. This is an approximation of the real problem, since we consider that the species are uniformly distributed in the block. Constraint (11) limits the number of blocks in which harvesting can occur during a period.

According to FPInovations, we can define five different harvesting capacities. The harvesting capacity is determined by the production capacity of different types of harvesting equipment and harvesting teams \( (m^3/\text{period}) \). In this paper, we do not consider crew scheduling; we consider these different harvesting capacities only to generate harvesting schedules. To generate a schedule for a given cut block harvested according to a given harvesting capacity, we take the ceiling of the division of its standing timber by the associated harvesting capacity. This gives us the duration (the number of periods needed to harvest the whole block). Then, we associate with every cut block (duration) a set of possible harvesting sequences when beginning the harvesting in different time periods of the planning horizon, these represent the set of schedules. The schedules must respect seasonal conditions. If different harvesting capacities give the same schedule, we choose the smallest one.

Constraints (12) to (18) define the procurement activities. Constraints (12) and (13) represent the flow conservation constraints at the forest and the mills. Constraint (14) limits the stocked volume per period in each mill. Constraint (15) limits the total transportation capacity. Constraint (16) ensures that the variables are binary, and constraint (17) is a non-negativity constraint.
5 SOLUTION APPROACHES

We first tried to solve the problem using Cplex directly. We found that Cplex took a long time (more than one day of calculation time) to find an initial solution for some of the instances. This was mostly due to the number of binary variables \( h_{bi} \) dealing with the block harvest scheduling. Therefore, fixing these variables in the initial solution may let Cplex branch differently or use the fixed variables to rapidly find an initial solution. Hence, we needed a strategy to fix these variables in an initial solution (i.e., to assign a schedule to every cut block).

5.1 Approach (1): Relax, fix, and optimize

In this approach, we use Cplex to solve a relaxed problem where the scheduling variables \( h_{bi} \) are continuous, so we can harvest a cut block using fractional schedules. This new problem is easier to solve but does not give feasible solutions. Then, we solve the real problem with some variables fixed to 1 as part of an initial solution. The three steps are:

- **Step 1.** Solve the partially relaxed problem (relax only the \( h_{bi} \) variables).
- **Step 2.** Fix to 1 the \( h_{bi} \) variables that are equal to 1 in the solution of Step 1.
- **Step 3.** Solve the MIP using the fixed variables as part of an initial solution to the problem.

5.2 Approach (2): Construction heuristic

We present a new construction heuristic that assigns a harvesting schedule to every cut block as part of the initial solution; Algorithm 1 presents this heuristic. We tested different methods to assign schedules to blocks and blocks to periods; our heuristic is based on the approach that gave the best results. In the heuristic, we assign blocks to each period according to their order in the list. We then assign to each block the schedule with the lowest harvesting capacity. At each iteration we calculate the residual volume, which is the difference between the maximum possible harvesting volume and the harvesting capacity of each period. At each assignment, we ensure that the limits on the maximum harvesting capacity and the maximum allowable blocks to harvest are respected so that the block schedule is feasible.
Algorithm 1 Construction heuristic

1: Let $\text{List} \_\text{Blocks}$ be the set of blocks $b$, ordered according to decreasing volume $V_b$
2: Let $\text{List} \_\text{Sched}(b)$ be the set of harvesting schedules for block $b$ in decreasing order of duration
3: Let $\text{List} \_\text{Forbid}$ be the set of tabu blocks that cannot be assigned in the current iteration, which is empty at $t = 0$
4: Let $V_t^H$ be the total harvesting capacity in time period $t$
5: Let $V_{b,s}^t$ be the maximum harvesting capacity per period $t$ according to schedule $s$ of block $b$
6: Let $NB_{\text{Max}}$ be the maximum number of blocks to harvest in each period
7: Let $NB_t^t$ be the current number of blocks to harvest in period $t$
8: Let $V_r^t$ be the residual volume, defined as $V_r^t = V_t^H - V_{b,s}^t$
9: $t \leftarrow 0; NB_0^0 \leftarrow 0; V_0^0 \leftarrow V_0^H; T$ is the last time period of the horizon;
10: if $(t \leq T)$ then go to step 13;
11: else STOP.
12: end if
13: if $(NB_t^t \leq NB_{\text{Max}})$ then
  find the first block $b \in \text{List} \_\text{Blocks}$ such that $V_b \leq V_r^t$ and not in $\text{List} \_\text{Forbid}$;
  if no such a block exists; increment $t$; go to step 10;
14: else
  increment $t$; empty $\text{List} \_\text{Forbid}$; go to step 10;
15: end if
16: Assign $b$ to the schedule $s$ from $\text{List} \_\text{Sched}(b)$ with the longest length ($\text{duration}$) and ($t + \text{duration} \leq T$)
17: if no such schedule exists, set $b \in \text{List} \_\text{Forbid}$; go to step 10;
18: Update $V_r^t$ and $NB_t^t$ for all $t' \in [t, t + \text{duration}]$
19: if $(V_r^t' \geq 0$ for all $t' \in [t, t + \text{duration}])$ then
  remove $b$ from $\text{List} \_\text{Blocks}$; go to step 13;
20: else
21:    Set $b \in \text{List} \_\text{Forbid}$;
22:    Fix $V_r^t$ and $NB_t^t$ for all $t' \in [t, t + \text{duration}]$ to their previous values;
23:    Go to step 13;
24: end if
6 COMPUTATIONAL EXPERIMENTS AND DISCUSSION

6.1 Description of data

The forest inventory consists of 30 heterogeneous and mature cut blocks in Eastern Canada. They occupy 3673 ha and a volume of about 580000 m$^3$. Each block is composed of at least two of five different species. Table 2 presents each block, its corresponding area in hectares (ha), and the volume per ha (m$^3$/h) of each species.

We considered potentially twenty-five log-types, varying in terms of species, length, and MSED; see Table 1. We set up sixteen priority lists. The model is solved using the commercial LP package CPLEX v12.5 via its Concert Technology C++ platform.

We define different instances to test the performance of the model under various demand conditions. The instances are generated by varying the volumes of the product mix required per mill. We ensure that the production capacity of each mill is respected every period, a constraint defined by the forestry company.

Also, we consider an upper bound on every product type, which represents the yield of this product when it is considered as the first element of the priority list applied to the whole cutting block. The total demand and the monthly demand are nearly constant for all the scenarios. They are specified in terms of volume (i.e., m$^3$) of the different product types. The average demand is between 5% and 8% less than the total quantity of standing timber, which represents low-value small-diameter logs and branches. The demand for each mill is given per month and product type.

Table 1: Product specifications

<table>
<thead>
<tr>
<th>ProdType</th>
<th>Log length (cm)</th>
<th>MSED (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>502</td>
<td>17</td>
</tr>
<tr>
<td>2</td>
<td>440</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
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6.2 Results and discussion

Table 3 presents the instances (Ins), the solution time (Time) in hours (h), and the optimality gap (Gap) as a percentage. The maximum computational time is set to
Table 2: Cut-block inventories

<table>
<thead>
<tr>
<th>Block</th>
<th>Area (ha)</th>
<th>( VH_E_1 ) (m³/ha)</th>
<th>( VH_E_2 ) (m³/ha)</th>
<th>( VH_E_3 ) (m³/ha)</th>
<th>( VH_E_4 ) (m³/ha)</th>
<th>( VH_E_5 ) (m³/ha)</th>
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</table>

about 24 hours, if no solution with a gap of at most 5% is found. The model contains 44312 constraints and 162901 variables where 73440 are binary.

Table 3 shows that Cplex successfully solved 9 of the 20 problems; the second approach solved 12 of them; and the third approach solved all the problems.
Table 3: Comparison of the three solution approaches

<table>
<thead>
<tr>
<th>Ins</th>
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<th>Approach (2)</th>
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<tr>
<td>20</td>
<td>2.75</td>
<td>14.4</td>
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</table>

*: no solution after 24 h

For the third approach, near-optimal solutions (average gap 3.31%) were found in an average computational time of 10 h. Our experiments show that the third approach finds good solutions for almost all the problems within reasonable time limits. Some of the tests are solved in less than 3 h (see test 1, test 5 and test 13 in Table 3) since Cplex succeeded to repair the heuristic solution (i.e. finds quickly an initial solution using the fixed variables). The CPU time decreases by about 21% (resp. about 32% and almost the same gap) when we use the second (resp. the third) approach; the solution quality is the same.

When evaluating the model, we noticed that the solutions of the different tests correspond to plans that satisfy all the requirements. The majority of the cut blocks are harvested in one or two periods.

7 CONCLUSION

We have presented an integrated multi-period wood procurement problem in the Eastern Canadian context. We have proposed a linearized integer programming model
that aims to minimize the operational costs. Our model integrates the planning of the activities involved in the WSC such as harvesting, transportation, storage in the forest, and storage at the mill terminals. The resulting mixed-integer linearized problem is large. This problem can be solved using Cplex, but the solution time is large. We have proposed two approaches to obtain solutions more quickly. When we applied a construction heuristic to generate an initial schedule for the problem, the model provided near-optimal solutions (average gap < 4%) for a realistically sized problem within a reasonable time limit (10 h). The model places some restrictions on harvesting and transportation, but it does not control events affecting the flow of logs from the forest such as route construction, and it does not control the transportation activities (capacities of vehicles) in detail. It would be interesting to use the model as a basis for short-term harvest planning with crew scheduling. These aspects could be addressed in future research.

References


