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Gregory Rix
Louis-Martin Rousseau
Gilles Pesant

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Bureaux de Montréal :
Université de Montréal
Pavillon André-Aisenstadt
C.P. 6128, succursale Centre-ville
Montréal (Québec)
Canada H3C 3J7
Téléphone : 514 343-7575
Télécopie : 514 343-7121

Bureaux de Québec :
Université Laval
Pavillon Palasis-Prince
2325, de la Terrasse, bureau 2642
Québec (Québec)
Canada G1V 0A6
Téléphone : 418 656-2073
Télécopie : 418 656-2624

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A Transportation-Driven Approach to Annual Harvest Planning

Gregory Rix[1,2,*], Louis-Martin Rousseau[1,2], Gilles Pesant[1,3]

[1] Interuniversity Research Centre on Enterprise Networks, Logistics and Transportation (CIRRELT)

[2] Department of Mathematics and Industrial Engineering, Polytechnique Montréal, P.O. Box 6079, Station Centre-Ville, Montréal, Canada H3C 3A7

[3] Department of Computer Engineering, Polytechnique Montréal, P.O. Box 6079, Station Centre-Ville, Montréal, Canada H3C 3A7

Abstract. Supply chain planning in the forestry industry includes a wide range of decisions, with time horizons ranging from real-time operational problems to long-term strategic problems. When forest companies plan over a length of approximately one year, referred to as the tactical stage of planning, the decisions commonly made are the schedules of forest sites to be visited by harvest teams in order to produce enough volume to meet all demands over the horizon, and also the allocation of this volume to the different demand points. This allocation allows for an estimation of transportation costs, with more detailed routing and scheduling decisions left for operational planning. The problem described in this article generalizes this tactical problem to include routing decisions, and hence falls into the classes of production-routing problems and pickup-and-delivery problems. This formulation was motivated by an industrial partner, whose goal is to ensure that they have a reliable source of permanent fleet drivers. In order to do this, they must be able to guarantee a variety of different schedules to several trucking contractors whom they hire drivers from, and harvest team scheduling has been identified as more flexible in order to accommodate this requirement. Additionally, significant savings in transportation costs can arise from determining a plan that emphasizes the creation of backhaul opportunities of a heterogeneous set of products. We model this problem as a mixed integer program and develop an effective branch-and-price based heuristic capable of generating solutions to medium sized problems in reasonable execution time. Compared to a decomposed and sequential optimization scheme that more accurately represents current industry practice, this methodology is able to fulfil higher demand levels while decreasing transportation costs by an average of $1.41 per cubic meter, or 12.4%.

Keywords. Forestry, transportation, inventory-routing problem, pickup-and-delivery problem, integer programming, column generation.

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* Corresponding author: Greg.Rix@cirrelt.ca

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1 Introduction

Canada has approximately 400 million hectares of tree cover, and the forest sector contributed to 1.1% of national gross domestic product (*Natural Resources Canada* 2014). With 146.7 million cubic meters of harvest in 2011, transportation expenses represent a multi-billion dollar expense for Canadian forestry companies (*Canadian Council of Forest Ministers* 2014). In the context of an economy of this scale, small relative reductions in transportation costs can represent substantial savings. Therefore, the use of optimization models and decision support systems is of high importance, and in recent years research initiatives pursuing these models have been highly prioritized.

We present here a problem that arises in the Canadian forestry industry in tactical planning; in our context this refers a planning horizon of one fiscal year. The decisions commonly made at this level of forest planning are the schedules of harvest teams, storage decisions, and the allocation each month of volume from supply points to demand points. This is a demand-driven problem based on the needs of a heterogeneous set of products at the mills to be served, and a host of industry-specific constraints must be respected in this plan.

While most tactical plans consider these production and allocation decisions, little emphasis is placed on the routing and scheduling decisions that will be encountered by planners in short term operational planning. The formulation proposed in this article considers generalizing the tactical model to include routing decisions, and we list three reasons for considering this more robust plan. First, a critical component of operations in Canadian forestry companies is to guarantee a variety of different driver schedules to their trucking contractors throughout the year. In Canada, the demand for drivers is high with multiple industries competing for services; hence delivering these schedules is a necessity in order to ensure a reliable source of permanent drivers. Second, transportation costs represent a very significant portion of the total cost of the wood supply chain, with 36% being a reported average in the Canadian context (Audy et al., 2012). Therefore optimizing backhaul opportunities is a major priority when scheduling wood procurement at the operational level, and we look to measure the potential savings if we can plan the harvest to optimize future backhaul opportunities. Third, most companies have their wood delivered via a heterogeneous truck fleet, thus necessitating synchronizing transportation decisions with harvest planning with respect to the length into which harvested timber is cut.

While it is very time consuming to formulate a complete annual plan by hand, this is what is done in many companies. Moreover, it is often the case that a plan must be revised due to unexpected events. Therefore the goal was to create a decision support system (DSS) that can be used to formulate a complete plan in a short computational time, with the option to easily modify inputs to generate several different scenarios if needed. Detailed reports must be given in the form of an Excel workbook, including the schedules and volumes produced by each harvest team, and the set of schedules assigned to each trucking contractor throughout the year.

Our contributions in this paper are a model and methodology for solving the problem. We first model this generalized tactical problem as a mixed integer program (MIP), and give two related formulations. To our knowledge these are the first formulations applied to the forest supply chain that enforce routing decisions in a tactical harvest planning problem. We next develop a branch-and-price heuristic capable of generating quality solutions in a reasonable time limit. We compare this methodology to a decomposed approach that first schedules the harvest while allocating flows to contractors, and then iteratively generates schedules for the trucks. This allows us to measure the benefits of incorporating these routing decisions at this stage of planning.

In Section 2, we describe the problem in more detail. In Section 3, we discuss related work. In Section 4, we present a mathematical formulation of the problem. Section 5 gives the details behind the branch-and-price heuristic used to solve this problem. In Sections 6 and 7 we discuss the case studies that motivated this problem and the computational results. Finally, Section 8 concludes the paper.

2 Problem Definition

We consider the following activities of the value chain: harvesting and forwarding in the forest, roadside wood inventory, transportation, and mill inventory. Additionally, we include the use of intermediate storage locations (remote pits) where wood can be stored before arriving to the mill. These pits usually act as demand nodes in the winter, having their inventory replenished from the forest areas. Then in the spring and summer, when it is more difficult or impossible to traverse much of the transportation network with heavy log-trucks, the pits act as supply nodes serving the mills.

The planning horizon over which we work is one year, with monthly demands and inventory requirements at each mill and pit. These demands exist for each of a set of different log assortments, which may differ with respect to diameter, species, quality, freshness, or other characteristics. All volumes are measured in cubic meters and are based on estimates made by the company.

The total forest management area is partitioned into a set of planning units, and the units that will be harvested are pre-selected at the start of the planning horizon rather than the total available forest management area; this is to avoid creaming of the supply points. If this was not the case, and supply was significantly greater than demand, an optimization model would always choose the supply points that are closest and the average distances would increase over time. In practice, supply is generally chosen to be up to 25% greater than demand, to allow for fluctuations that may arise over the year.

Each planning unit contains a set of subunits which differ in terms of seasonal availability throughout the year. Our harvest decisions are made at the subunit level. Subunits may also differ from each other in terms of their priority: it is important to harvest high priority subunits as soon as possible for any number of reasons, including conflict with caribou hunting seasons or the need to coordinate with other industries such as oil mining. Any roadside inventory in the forest that exists prior to the start of the year must be hauled by the end of the year.

A set of harvest teams is defined, and each harvest team has a capacity measured in cubic meters of harvest per month. This may vary per month due to seasonal access restrictions and holiday time. When a team is assigned to a subunit, it must not leave until the entire standing volume is harvested. Additionally, each assortment can be harvested in a choice of lengths ranging from 32 foot to full tree lengths, and the team must be told the proportion of harvest to produce in each length. We emphasize here that this optimization model only considers lengths as they relate to the transportation constraints, and that mill demands can be satisfied with any length of timber.

Several trucking contractors are used to transport the harvested timber. Each contractor has a set of schedules, defined by the truck class, the shift length in hours, and the cumulative working days each month (based on the number of drivers and the days worked by each driver). Each shift assigned to a particular schedule must start and end at the location representing the home base of the contractor that month, and alternate between supply nodes (forests or pits) and demand nodes (pits or mills) before ending the shifts. For model feasibility purposes, we define each shift by a maximum and minimum target shift length as opposed to a single number. Every truck class is only compatible with specific log assortments and lengths, and has a fixed capacity in cubic meters.

The harvest teams and trucking contractors are each assigned a priority: it is most important to give the high priority contractors their desired workload over the course of the year, with the remaining necessary work assigned to lower priority contractors.

3 Literature Review

Our problem formulation is a variant of the inventory-routing problem (IRP), a well studied problem that integrates inventory management and vehicle routing decisions and has been adapted to many industries. Coelho, Cordeau, et al. (2013) give a recent survey and classify the current IRP literature according to a number of criteria, a few of which we will discuss. While the majority of IRPs considered in literature deal with a homogeneous vehicle fleet, or in many cases a single vehicle, heterogeneous vehicle fleets have been considered. Coelho and Laporte (2012) formulated this problem, and gave a branch-and-cut procedure for its resolution.

Coelho, Cordeau, et al. (2013) further classify three basic problem structures: one-to-one, one-to-many, and many-to-many. As in our problem sites can serve as a source or as a destination for any commodity, it is classified as a many-to-many structure. This is a less-studied variant; Ramkumar et al. (2012) considered a many-to-many problem involving multiple commodities and give a MIP formulation. A column generation based approach was also been successfully applied to a many-to-many IRP in maritime logistics (Christiansen and Nygreen, 2005).

Van Anholt et al. (2013) introduced the inventory-routing problem with pickups and deliveries (IRPPD) in the context of replenishment of automated teller machines, and utilized a branch-and-cut algorithm to resolve the problem. The IRPPD combines the features of the IRP and the well-studied pickup-and-delivery problem (Berbeglia et al., 2007), which is commonly adapted to vehicle routing problems in forestry due to the inherent pickup-and-delivery nature of the industry.

It is natural to think that integrating further elements of the supply chain can lead to even better performance, and Chandra and Fisher (1994) were among the first to include production decisions within the IRP, producing operating cost reductions from 3 to 20%. This problem is classified as the production-routing problem (PRP). Adulyasak et al. (2013) gave strong formulations of this problem in the multi-vehicle context, and solved with an adaptive large neighborhood search heuristic to find initial solutions, followed by a branch-and-cut procedure. Other recent methodological focuses in this field have included tabu search (Bard and Nananukul, 2009) and branch-and-price (Bard and Nananukul, 2010).

Within the forest products industry, D'Amours et al. (2008) present an overview of different planning problems and review the contributions in an operational research (OR) setting. They distinguish between the strategic, tactical and operational planning levels, and comment that while operational problems must be solved in minutes or even seconds, strategic and tactical problems can be solved over a period of up to several hours. For this reason, while heuristics, meta-heuristics and network methods are generally used in operational planning; MIP and stochastic programming based methods are often used to solve tactical and strategic problems.

Tactical models in forest management are commonly used to decide where and when to harvest, which team to use in each harvest decision made, and where and when to transport and store the harvested timber. These plans are made up to 5 years in advance, but are often re-evaluated annually when doing budget projections for the following year. Rönnqvist (2003) gives a simple MIP model incorporating these decisions in which the objective function measures two costs: the cost of harvesting a forest area by a team in a specific period, and the cost of delivering each unit of wood from a forest site to a mill. Gémieux (2009) gives a recent survey on wood procurement planning and outlines a MIP model in the Canadian context.

Karlsson et al. (2004) consider an annual harvest planning problem that arises in Sweden, in which inventory management and road openings and closings must be managed. To solve the MIP model, they utilize a variable fixing heuristic in which they iteratively solve LP relaxations, at each iteration fixing binary variables with a fraction value in a chronological order until a feasible solution is found.

Bredström et al. (2010) consider a tactical problem in which machines must be scheduled to plan the harvest, and also include the minimization of their movement in the objective function. They do not, however, include transportation or inventory costs of the timber. They utilize a two-phase approach in which they first assign machines to forest locations, and then schedule each machine to minimize their moving costs.

Similar models appear in planning over a shorter time horizon, including more operational details to increase efficiency. Karlsson et al. (2003) consider a harvest planning model over a period of 4 to 6 weeks, in which harvest teams are scheduled, transportation and inventory are managed, and additionally the management and maintenance of roads must be considered to yield a feasible solution.

Mitchell (2004) gives a very detailed description of operational harvest scheduling in the Australian and New Zealand context. Road maintenance and management are not considered in this model; however a key term of their objective is to maximize profitability by incorporating the revenue associated with each potential log type produced. They utilize a branch-and-price scheme by pricing variables that represent harvest crew schedules via dynamic programming.

Epstein et al. (1999) present OPTICORT, created for use in the Chilean forest sector. They additionally include machine assignment and bucking decisions in a short term (3 months) harvesting model. OPTICORT uses a MIP model, solved by column generation.

Gerasimov et al. (2013) integrate harvesting and transportation decisions into a single DSS for use in the Russian forestry sector. On an extensive transportation network, optimal paths are generated heuristically and used to influence the routing of both harvest teams and trucks, solved via a dynamic programming algorithm. Potential cost savings of 14 to 25% were reported.

We note that while many tactical models do minimize transportation costs, it is done under the assumption that the timber will be transported in an out-and-back fashion. However in the operational level of planning when more detailed transportation plans are determined, log-trucks are routed in order to take advantage of backhaul opportunities, while also considering loader capacity at supply and demand nodes. The problem of creating these plans is commonly called the log-truck scheduling problem (LTSP), or sometimes the synchronized log-truck scheduling problem (SLTSP) when trucks must share loading equipment at supply or demand points. Audy et al. (2012) provide a recent survey in this field, covering both methodologies and implementations into DSSs.

Hachemi et al. (2014) and Rix et al. (2014) included these routing and scheduling decisions in similar problem formulations in a tactical setting, with supply availability predetermined but loader assignment, wood allocation and inventory decisions remaining to be solved: a generalization of the IRPPD to the forest products industry. In the former article, successive MIPs are solved with the first assigning loaders to supply points by day and determining the destinations of full truckloads of wood, and the second determining the routing and scheduling of the vehicles. In the latter article, a similar formulation is not solved in phases, but instead the problem is reformulated via Dantzig-Wolfe decomposition and solved with column generation, with the generated columns (representing daily vehicle schedules) solved through dynamic programming.

4 Model Formulation

The model consists of input data, decision variables, an objective function, and constraints. The input data appears in Tables 1 through 3 and the decision variables are listed in Table 4.

4.1 Objective Function

Our objective function contains 7 components that contribute to the total cost of a solution. The first and second components are the real costs associated with transportation and storage. The third component is a penalty associated with the total volume produced by each harvest team compared to their desired production. The fourth component is a penalty associated with unsatisfied requested hours on each trucking schedule. The fifth through seventh components are the penalty costs associated with failure to meet demand, failure to meet inventory requirements, and the costs associated with discarding wood at the end of the planning horizon by failing to meet the freshness constraints. These components are listed in Table 5.

4.2 Constraints

All of the constraints of the model are listed in this section.

Table 1: Input Sets

Notation	Representation
$P = \{1, 2, \ldots, \|P\|\}$	Set of planning periods
$P' = \{1, 2, \ldots, \|P\| + 1\}$	Set of planning periods including dummy period for next horizon
F	Set of forest planning units
S_f	Set of subunits of forest f
$S = \bigcup_{f \in F} S_f$	Set of all subunits
M	Set of mills
R	Set of remote pits
K	Set of log assortments
L_k	Set of lengths for assortment k
$N = M \cup R \cup S$	Set of all nodes
$N^{in} = M \cup R$	Set of all demand nodes
$N^{out} = S \cup R$	Set of all supply nodes
H	Set of all harvest teams
T	Set of all trucking schedules
Θ_t	Set of all feasible routes for schedule t
$\Theta = \bigcup_{t \in T} \Theta_t$	Set of all routes

Table 2: Input Data

Notation	Representation
v_{sk}	Volume of assortment k in subunit s available to harvest
$v_s = \sum_{k \in K} v_{sk}$	Volume of all assortments in subunit s available to harvest
d_{mkp}	Demand of assortment k at mill m in period p
i_{nkl}	Initial inventory of assortment k in length l at node n
i_n^{max}	Maximum capacity at demand node n
i_{nkp}^{min}	Minimum capacity of assortment k at demand node n in period p
c_n	Loader capacity at demand node n in loads per period
c_{hp}	Harvesting capacity of team h in period p
$C_h = \sum_{p \in P} c_{hp}$	Target production volume for team h over horizon
e_{hsp}	Number of periods for team h to harvest subunit s if commencing in period p
a_{hsp}	Binary parameter equals 1 iff team h can harvest subunit s in period p based on seasonal availability of subunit and team
$\alpha_{hsp} = \prod_{i=p}^{\min\{\lvert P\rvert, p+d_{hsp}-1\}} a_{hsp}$	Binary parameter equals 1 iff team h can begin harvest of subunit s in period p
β_s	Latest period in which subunit s can have remaining standing volume
c_{tkl}	Capacity of assortment k of length l on truck used in schedule t
b_{tp}	Binary parameter equals 1 iff schedule t is available in period p
h_t^{min}	Minimum hours per shift for schedule t
h_t^{max}	Maximum hours per shift for schedule t
d_{tp}	Cumulative requested working days for schedule t in period p
$\rho_{\theta n_1 n_2 kl}$	Number of trips on route θ carrying assortment k in length l from node n_1 to node n_2
h_θ	Shift length (in hours) of route θ

Table 3: Costs and Penalties

Notation	Representation
$\gamma_t^{transport}$	Per hour cost of operating (driving, loading, unloading) truck on schedule t
$\gamma_{np}^{holding}$	Cost per m^3 of inventory at node n entering period p
$\gamma_h^{harvest}$	Cost per m^3 of shortfall from desired horizon volume of harvest team h
γ_t^{truck}	Cost per unsatisfied shift for trucking schedule t
γ_{mkp}^{demand}	Cost per m^3 of missed demand of assortment k at mill m in period p
$\gamma_{nkp}^{inventory}$	Cost per m^3 of missed minimum inventory of assortment k at demand node n in period p
$\gamma_{nkl}^{freshness}$	Cost per m^3 of discarded product k of length l at supply node n

Table 4: Variables

Notation	Representation
$x_{n_1 n_2 klpt} \in \mathbb{R}_{\geq 0}$	Volume of flow of assortment k of length l from supply node n_1 to demand node n_2 in period p on schedule t
$w_{nklp} \in \mathbb{R}_{\geq 0}$	Volume of assortment k of length l stored at node n entering period p in P'
$\hat{d}_{mklp} \in \mathbb{R}_{\geq 0}$	Volume of length l used to fill demand at mill m of assortment k in period p
$d'_{mkp} \in \mathbb{R}_{\geq 0}$	Volume of missed demand of assortment k at mill m in period p
$w'_{nkp} \in \mathbb{R}_{\geq 0}$	Volume of missed inventory of assortment k at demand node n in period p
$f'_{nkl} \in \mathbb{R}_{\geq 0}$	Discarded volume of length l of assortment k at supply node n at the end of the planning horizon
$y_{hsp} \in \mathbb{B}$	Equals 1 iff harvest team h commences harvesting subunit s in period p
$\hat{v}_{skl} \in [0, 1]$	Proportion of harvested volume of assortment k from subunit s cut into length l
$v_{hspkl} \in \mathbb{R}_{\geq 0}$	Volume of assortment k of length l produced by harvest team h at subunit s in period p
$z_s \in \mathbb{R}_{\geq 0}$	Period in which subunit s has all remaining standing volume harvested
$L_{np} \in \mathbb{R}_{\geq 0}$	Number of trucks loaded and/or unloaded at node n in period p
$q_{\theta p} \in \mathbb{Z}_{\geq 0}$	Number of times route θ is traversed in period p

Table 5: Objective Function Components

Name	Formula
$Z^{transport}$	$\sum_{p \in P} \sum_{\theta \in \Theta} \gamma_t^{transport} h_\theta q_{\theta p}$
$Z^{storage}$	$\sum_{n \in N} \sum_{k \in K} \sum_{l \in L_k} \sum_{p \in P'} \gamma_{np}^{storage} w_{nklp}$
$Z^{harvest}$	$\sum_{h \in H} \gamma_h^{harvest} \left(C_h - \sum_{s \in S} \sum_{p \in p} \sum_{k \in K} \sum_{l \in L_k} v_{hspkl} \right)$
Z^{truck}	$\sum_{t \in T} \sum_{p \in P} \gamma_t^{truck} \left(d_{tp} - \sum_{\theta \in \Theta_t} q_{\theta p} \right)$
Z^{demand}	$\sum_{m \in M} \sum_{k \in K} \sum_{p \in P} \gamma_{mkp}^{demand} d'_{mkp}$
$Z^{inventory}$	$\sum_{n \in N^{in}} \sum_{k \in K} \sum_{p \in P} \gamma_{nkp}^{inventory} w'_{nkp}$
$Z^{freshness}$	$\sum_{n \in N^{out}} \sum_{k \in K} \sum_{l \in L} \gamma_{nkl}^{freshness} f'_{nkl}$

4.2.1 Inventory

Constraints (1) fix the initial inventories at every node. Constraints (2) and (3) impose the minima and maxima at each mill and pit each period.

$$w_{nkl1} = i_{nkl}, \forall n \in N, \tag{1}$$

$$\sum_{l \in L_k} w_{nklp} + w'_{nkp} = i_{nkp}^{min}, \forall n \in N^{in}, k \in K, p \in P, \tag{2}$$

$$\sum_{k \in K} \sum_{l \in L_k} w_{nklp} \leq i_n^{max}, \forall n \in N^{in}, p \in P. \tag{3}$$

4.2.2 Flow Conservation

Constraints (4) through (6) are flow conservation constraints at mills, forests, and pits.

$$w_{mklp} + \sum_{n \in N^{out}} \sum_{t \in T} x_{nmklpt} - \hat{d}_{mklp} = w_{mkl(p+1)}, \qquad (4)$$

$$\forall m \in M, k \in K, l \in L_k, p \in P,$$

$$w_{sklp} - \sum_{n \in N^{in}} \sum_{t \in T} x_{snklpt} + v_{spkl} = w_{skl(p+1)}, \qquad (5)$$

$$\forall s \in S, k \in K, l \in L_k, p \in P,$$

$$w_{rklp} + \sum_{s \in S} \sum_{t \in T} x_{srklpt} - \sum_{m \in M} \sum_{t \in T} x_{rmklpt} = w_{rkl(p+1)}, \qquad (6)$$

$$\forall r \in R, k \in K, l \in L_k, p \in P.$$

4.2.3 Stockout

Constraints (7) and (8) impose that we never have stockouts at forests and penalize stockout at mills: a mill must always be able to meet its demand, else a penalty is accrued, and a forest can never supply more than it has in inventory entering the given period.

$$\sum_{l \in L_k} \hat{d}_{mklp} + d'_{mkp} = d_{mkp}, \forall m \in M, k \in K, p \in P, \qquad (7)$$

$$\sum_{n \in N^{in}} \sum_{t \in T} x_{snklpt} \leq w_{sklp}, \forall s \in S, k \in K, l \in L_k, p \in P. \qquad (8)$$

4.2.4 Freshness

Constraints (9) are freshness constraints that impose that all roadside inventory at the start of the planning horizon must be hauled by the end of the horizon, else a penalty is accrued.

$$\sum_{n_2 \in N^{in}} \sum_{p \in P} \sum_{t \in T} x_{n_1 n_2 klpt} + f'_{n_1 kl} = w_{n_1 kl1}, \forall n_1 \in N^{out}, k \in K, l \in L_k. \qquad (9)$$

4.2.5 Harvest

Constraints (10) impose that a subunit can only be harvested once and by one team.

$$\sum_{h\in H}\sum_{p\in P} y_{hsp} \leq 1, \forall s \in S. \tag{10}$$

Constraints (11) impose that there be only one harvest team per forest unit at a time.

$$\sum_{h\in H}\sum_{s\in S_f}\sum_{i=0}^{\min\{p-1,e_{hsp}-1\}} y_{hs(p-i)} \leq 1, \forall f \in F, p \in P. \tag{11}$$

Constraints (12) impose that a contractor can only a harvest a subunit when allowed.

$$y_{hsp} \leq \alpha_{hsp}, \forall h \in H, s \in S, p \in P. \tag{12}$$

Constraints (13) impose that a team can only harvest one subunit at a time.

$$\sum_{s\in S}\sum_{p'=0}^{p} \mathbb{1}(p' + e_{hsp'} - 1 \geq p) y_{hsp'} \leq 1, \forall h \in H, p \in P. \tag{13}$$

Constraints (14) impose that if harvested, a subunit must be either fully cleaned or be cleaned up to contractor capacity until the end of the horizon.

$$\sum_{p\in P}\sum_{k\in K}\sum_{l\in L_k} v_{hspkl} \geq \sum_{p\in P} \min\left(v_s, (|P| - p + 1)c_{hp}\right) y_{hsp}, \forall h \in H, s \in S. \tag{14}$$

Constraints (15) impose that the harvested volume from a subunit is bounded by team capacity.

$$\sum_{k\in K}\sum_{l\in L_k} v_{hspkl} \leq c_{hp} \sum_{p'=0}^{p} \mathbb{1}(p' + e_{hsp'} - 1 \geq p) y_{hsp'}, \forall h \in H, s \in S, p \in P. \tag{15}$$

Constraints (16) impose that the total horizon volume from a subunit is bounded by what is available.

$$\sum_{h\in H}\sum_{p\in P}\sum_{l\in L_k} v_{hspkl} \leq v_{sk}, \forall s \in S, k \in K. \tag{16}$$

Constraints (17) impose that if nothing is harvested, the team cannot be assigned.

$$\sum_{p'=0}^{p} \mathbb{1}(p' + e_{hsp'} - 1 \geq p) y_{hsp'} \leq \sum_{k \in K} \sum_{l \in L_k} v_{hspkl}, \forall h \in H, s \in S, p \in P. \quad (17)$$

Constraints (18) impose that a subunit's assortment can only be cut into a single length.

$$\sum_{l \in L_k} \hat{v}_{skl} \leq 1, \forall s \in S, k \in K. \quad (18)$$

Constraints (19) impose that all lengths not assigned to that subunit cannot be produced.

$$\sum_{h \in H} \sum_{p \in P} v_{hspkl} \leq v_{sk} \hat{v}_{skl}, \forall s \in S, k \in K, l \in L_k. \quad (19)$$

Constraints (20) impose that no length is produced if nothing is harvested.

$$\hat{v}_{skl} \leq \sum_{h \in H} \sum_{p \in P} v_{hspkl}, \forall s \in S, k \in K, l \in L_k. \quad (20)$$

Constraints (21) disallow the situation of harvest teams producing at full capacity during periods i and $i+2$ in the same subunit, but partial capacity during period $i+1$, in order to optimize contractor satisfaction. Thus we force a contractor to work at full capacity during all intermediate periods:

$$\sum_{k \in K} \sum_{l \in L_k} \sum_{p'=p+1}^{p+e_{hsp}-2} v_{hsp'kl} \geq \left(\sum_{p'=p+1}^{p+e_{hsp}-2} c_{hp'} \right) y_{hsp}, \forall h \in H, s \in S, p \in P, e_{hsp} \geq 3. \quad (21)$$

Constraints (22) link the z_s variables, that determine the period in which a subunit is fully harvested, to the formulation. Constraints (23) then bound these variables when necessary.

$$2|P| \left(1 - \sum_{h \in H} \sum_{p \in P} y_{hsp} \right) + \sum_{h \in H} \sum_{p \in P} (p + e_{hsp} - 1)\, y_{hsp} = z_s, \forall s \in S, \quad (22)$$

$$z_s \leq \beta_s, \forall s \in S. \quad (23)$$

4.2.6 Trucking

Constraints (24) impose that there must be enough trucks working to accommodate flow. We emphasize that this is modeled as an inequality due to volumes being expressed in cubic meters rather than truckloads due to the heterogeneous truck fleet.

$$\sum_{t \in T} \sum_{\theta \in \Theta_t} c_{tkl} \rho_{\theta n_1 n_2 kl} q_{\theta p} \geq x_{n_1 n_2 klpt}, \quad \forall n_1 \in N^{out}, n_2 \in N^{in}, k \in K, l \in L_k, p \in P. \tag{24}$$

Constraints (25) respect the schedule maximum each period.

$$\sum_{\theta \in \Theta_t} q_{\theta p} \leq d_{tp}, \forall t \in T, p \in P. \tag{25}$$

Constraints (26) through (28) determine and constrain loader usage.

$$\sum_{\theta \in \Theta} \sum_{k \in K} \sum_{l \in L_k} \sum_{n \in N^{out}} \rho_{\theta nmkl} q_{\theta p} = L_{mp}, \forall m \in M, p \in P, \tag{26}$$

$$\sum_{\theta \in \Theta} \sum_{k \in K} \sum_{l \in L_k} \left(\sum_{s \in S} \rho_{\theta srkl} + \sum_{m \in M} \rho_{\theta rmkl} \right) q_{\theta p} = L_{rp}, \forall r \in R, p \in P, \tag{27}$$

$$L_{np} \leq c_n, \forall n \in N^{in}, p \in P. \tag{28}$$

4.2.7 Mathematical Formulation

The complete mathematical formulation of the model, which we denote as problem (P1), is then the minimization of the objective function

$$Z = Z^{storage} + Z^{transport} + Z^{harvest} + Z^{truck} + Z^{demand} + Z^{inventory} + Z^{freshness}$$

subject to constraints (1) through (28).

4.3 A Reformulation for More Accurate Harvest Planning

A key issue that arose during preliminary experimentation is that the current formulation does not allow a team to work in more than one subunit in a single period, regardless of whether they have the remaining capacity to do so. Therefore we allow harvesting of any subunit to be done in one of exactly two patterns: where teams produce at partial capacity during either the terminating or commencing period of harvest of a single subunit. In these periods of partial capacity, the team is free to work in up to 2 different subunits. During all other periods, the team is working at full capacity. We introduce two new families of variables, respectively y^1_{hsp} and y^2_{hsp}, that represent the two aforementioned patterns. Constraints (29) link the new families of variables to the variables y_{hsp}.

$$y_{hsp} = y^1_{hsp} + y^2_{hsp}, \forall h \in H, s \in S, p \in P. \tag{29}$$

Constraints (30) and (31) enforce the appropriate production for the given patterns:

$$\sum_{k \in K} \sum_{l \in L_k} v_{hspkl} \geq c_{hp} y^1_{hsp}, \forall h \in H, s \in S, p \in P, \tag{30}$$

$$\sum_{k \in K} \sum_{l \in L_k} v_{hs(p+e_{hsp}-1)kl} \geq c_{h(p+e_{hsp}-1)} y^2_{hsp}, \forall h \in H, s \in S, p \in P. \tag{31}$$

Finally, we replace constraints (13) and (14) with constraints (32) and (33), stipulating that a team can be in up to 2 subunits in a given period,

$$\sum_{s \in S} \sum_{p'=0}^{p} \mathbb{1}(p' + e_{hsp'} - 1 \geq p) y_{hsp'} \leq 2, \forall h \in H, p \in P, \tag{32}$$

but only 1 when producing at full capacity.

$$\sum_{s \in S} \left(\sum_{p'=0}^{p} \mathbb{1}(p' + e_{hsp'} - 2 \geq p) y^1_{hsp'} + \sum_{p'=0}^{p-1} \mathbb{1}(p' + e_{hsp'} - 1 \geq p) y^2_{hsp'} \right) \leq 1, \forall h \in H, p \in P. \tag{33}$$

It is then trivial to determine whether the team harvesting subunits s_1 in period p and s_2 in period $p_2 = p + d_{hs_1p} - 1$ has the capacity to harvest both remainder volumes in the period p_2. We define the binary parameter $g_{hs_1s_2p}$

to this effect.

$$g_{hs_1s_2p} = \begin{cases} 1 & \left(v_{s_1} + v_{s_2} - \sum_{p'=p}^{p_2-1} c_{hp'} - \sum_{p'=p_2+1}^{p_2+e_{hs_2p_2}-1} c_{hp'} \right) \leq c_{hp_2} \\ 0 & \text{Otherwise.} \end{cases}$$

Constraints (34) allow a team producing at partial capacity to move from one subunit to another, provided the total production is sufficiently low.

$$y^1_{hs_1p} + y^2_{hs_2(p+e_{hs_1p}-1)} \leq 1 + g_{hs_1s_2p}, \forall h \in H, s_1, s_2 \in S, p \in P. \quad (34)$$

Constraints (35) are required to enforce harvest team capacity per period; in this formulation we must sum these constraints over all subunits in order to account for teams potentially working in multiple harvest locations per period.

$$\sum_{s\in S}\sum_{k\in K}\sum_{l\in L_k} v_{hspkl} \leq c_{hp}, \forall h \in H, p \in P. \quad (35)$$

We denote the problem (P2) as the minimization of the objective function Z subject to constraints (1) through (12) and (15) through (35).

5 Methodology

The biggest obstacle in formulating the model in this matter is the exponential number of variables representing log-truck routes. Hence we use a branch-and-price based methodology in which we start with an empty pool of routes and generate improving ones a priori. The column generation procedure is adapted from the one used by Rix et al. (2014).

5.1 Initial Restricted Problem

We first relax the problem (P1) to a linear model to be solved via column generation. Since our initial route set Θ is empty, we additionally relax constraints (24) to a soft constraint and give any violation a large penalty in the objective function. This restricted master problem is denoted (P').

After solving the linear relaxation of (P'), we store the dual values associated with constraints (24) through (27); which we denote $\lambda_{n_1n_2klpt}$, π_{tp},

σ^M_{mp} and σ^R_{rp}, respectively. Our search then begins for negative reduced cost columns with which to enrich the model to improve the objective value of the optimal solution. We propose to find these columns by performing a set of dynamic programming algorithms: one for period p, and for each truck schedule t.

5.2 Enriching the Model with Column Generation

To solve these subproblems, we must first construct a space-time network, which we denote $G_{tp} = (N_{tp}, A_{tp})$, for the given schedule and period. We discretize the time dimension, whose horizon ranges from 0 to h^{max}_t, into ω intervals of length $\delta = h^{max}_t/\omega$. We denote this discretized time dimension $I = \{i_0, i_1, \ldots, i_\omega\}$.

We define the network with vertex set

$$V_{tp} = source \bigcup sink \bigcup ((N^{in} \bigcup N^{out}) \times I),$$

where the source and sink nodes correspond to a geographical location where the contractor's trucks are situated. The source node has outgoing arcs to all nodes N^{out}, with the arc originating at time i_0. Similarly, the sink node has incoming arcs from all arcs in N^{in} such that the minimum route length h^{min}_t is respected. The arc set is then $A_{tp} = A_{source} \bigcup A_{sink} \bigcup A_l \bigcup A_u$ where A_{source}, A_{sink}, A_l, and A_u represent out-of-source, into-sink, loaded driving (including loading and unloading time), and unloaded driving arcs, respectively. The cost $c_{n_1n_2}$ of each arc (n_1, n_2) is then easily calculated as a function of per hour operating costs and trucking penalties of that schedule and the distance of the arc.

However in calculating the reduced cost of a route, we modify these arc costs as follows:

$$c_{n_1n_2} \leftarrow \begin{cases} c_{n_1n_2} - \pi_{tp} & (n_1, n_2) \in A_{source}, \\ c_{n_1n_2} & (n_1, n_2) \in A_{sink}, \\ c_{n_1n_2} & (n_1, n_2) \in A_u, \\ c_{n_1n_2} - \lambda_{n_1n_2klpt} - \sigma^M_{n_2p} & (n_1, n_2) \in A_l, n_1 \in S, n_2 \in M, \\ c_{n_1n_2} - \lambda_{n_1n_2klpt} - \sigma^R_{n_1p} - \sigma^M_{n_2p} & (n_1, n_2) \in A_l, n_1 \in R, n_2 \in M, \\ c_{n_1n_2} - \lambda_{n_1n_2klpt} - \sigma^R_{n_2p} & (n_1, n_2) \in A_l, n_2 \in S, n_2 \in R, \end{cases}$$

where we associate with each loaded driving arc (n_1, n_2) in A_l the assortment k and length l that maximize $\lambda_{n_1n_2klp}$. Any feasible route can then be expressed as a source-to-sink path in this network, with the reduced cost of this route equal to the cost of the path.

We note that this network has a clear topological ordering, which is a chronological ordering with ties broken arbitrarily. To find negative reduced cost routes to add to the master problem, we therefore utilize the standard label setting algorithm given by Cormen et al. (1990), in which we associate with each node n a label $[pred_n, RC_n]$ which denotes the predecessor node of n and the length (reduced cost) of the shortest path to n. All nodes only hold one label at any time, except the sink node which holds a set Υ of labels that holds all paths of negative reduced cost. For any schedule t and period p, we provide the details of this algorithm in Algorithm 1. Lines 1 through 4 initialize the labels. Lines 5 through 12 push through the network and update labels as required.

Algorithm 1 Shortest Path Dynamic Programming Algorithm

1: **for all** n in N_{tp} **do**
2:     $pred_n \leftarrow null$
3:     $RC_n \leftarrow \infty$
4: $RC_{source} = 0$
5: **for all** n_1 in N_{tp} following the topological ordering **do**
6:     **for all** (n_1, n_2) in A_{tp} **do**
7:         **if** $n_2 = sink$ **and** $RC_{n_1} + c_{n_1 n_2} < 0$ **then**
8:             $\Upsilon \leftarrow \Upsilon \bigcup \{[n_1, RC_{n_1} + c_{n_1 n_2}]\}$
9:         **if** $RC_{n_2} < RC_{n_1} + c_{n_1 n_2}$ **then**
10:             $RC_{n_2} \leftarrow RC_{n_1} + c_{n_2 n_2}$
11:             $pred_{n_2} \leftarrow n_1$

Thus at every master iteration we store the dual values of constraints (24) through (27), and then solve $|T||P|$ subproblems. All negative reduced cost routes are stored and the columns are added to the master problem. We iterate through this process until no negative reduced cost routes remain or another stopping criterion is achieved.

5.3 Column Pool Management

At each iteration, upon the resolution of all subproblems, the most general method would be to add all negative reduced cost columns found to the LP. However many of these routes will prove unnecessary and remain non-basic until the algorithm terminates. As managing the column pool can require a significant amount of computation time when the pool is very large, we utilize two methods to control the size of the column pool. First, at each iteration we simply added the best (most negative reduced cost) 200 columns found. Second, upon passing a predetermined upper limit on pool size, columns are eliminated randomly until a lower limit is achieved (set to 70% of the upper limit).

5.4 Heuristic Branch-and-Price

In order to solve our problem to optimality, we would have to embed our column generation procedure into a branch-and-bound tree (Barnhart et al., 1998). However we choose to more quickly find integer feasible solutions through the use of an efficient heuristic branching method motivated by Prescott-Gagnon et al. (2009).

We impose branching decisions on the harvest variables y_{hsp} by fixing the one with the largest fractional value to 1 upon the resolution of an LP. We do not fix variables to 0 as this does not significantly modify the problem. Moreover we do not allow backtracking: branching decisions cannot be reversed. We continue this process until none of these variables that remain unfixed remain with value greater than a parameter ψ in $[0, 1]$.

For the formulation (P2), the branching strategy is analogous on the variables y^i_{hsp}. As an addendum, preliminary experimentation found that the resolution of the linear programs slowed considerably when all constraints (34) (cardinality $|H||S|^2|P|$) were initially added to the model. Therefore we only enforce the relevant constraints that become tight after enforcing a branching decision.

To terminate the algorithm and generate an integer feasible solution, we then enforce integrality constraints on all remaining variables that are integral in the MIP formulation, and solve the resulting problem using a branch-and-bound solver.

6 Decomposed Approach

To assess the benefit of incorporating routing decisions in tactical planning, we compare the methodology to an implementation that more accurately reflects the current industry practice. We first derive a model that represents the annual harvest plan, giving schedules to the harvest teams, allocating flow to transportation fleets by month, and managing monthly demands and inventory levels. This is a MIP model including most of the variables and constraints of the model developed in Section 4.

For this first phase, we eliminate the $q_{\theta p}$ variables from the above formulations. Let $d(n_1, n_2)$ represent the cycle time of a truck from supply point n_1 to demand point n_2 and back, including loading and unloading times. We replace the $Z^{transport}$ and Z^{truck} terms in the objective function by their flow-based approximations:

$$Z_H^{transport} = \sum_{n_1 \in N^{out}} \sum_{n_2 \in N^{in}} \sum_{k \in K} \sum_{l \in L_k} \sum_{p \in P} \sum_{t \in T} \gamma_t^{transport} \frac{d(n_1, n_2)}{c_{tkl}} x_{n_1 n_2 klpt},$$

$$Z_H^{truck} = \sum_{t \in T} \sum_{p \in P} \gamma_t^{truck} \left(d_{tp} - \sum_{n_1 \in N^{out}} \sum_{n_2 \in N^{in}} \sum_{k \in K} \sum_{l \in L_k} \frac{d(n_1, n_2)}{c_{tkl} h_t^{max}} x_{n_1 n_2 klpt} \right).$$

We similarly replace constraints (25) through (27):

$$\sum_{n_1 \in N^{out}} \sum_{n_2 \in N^{in}} \sum_{k \in K} \sum_{l \in L_k} \frac{d(n_1, n_2)}{c_{tkl} h_t^{max}} x_{n_1 n_2 klpt} \leq d_{tp}, \forall p \in P, t \in T, \quad \text{(25H)}$$

$$\sum_{k \in K} \sum_{l \in L_k} \sum_{t \in T} \sum_{n \in N^{out}} \frac{1}{c_{tkl}} x_{nmklpt} = L_{mp}, \forall m \in M, p \in P, \quad \text{(26H)}$$

$$\sum_{k \in K} \sum_{l \in L_k} \sum_{t \in T} \frac{1}{c_{tkl}} \left(\sum_{s \in S} x_{srklpt} + \sum_{m \in M} x_{rmklpt} \right) = L_{rp}, \forall r \in R, p \in P. \quad \text{(27H)}$$

We denote the resulting harvest-flow model by $(PH1)$, in which we minimize the objective function

$$Z_H = Z^{storage} + Z_H^{transport} + Z^{harvest} + Z_H^{truck} + Z^{demand} + Z^{inventory} + Z^{freshness}$$

subject to constraints (1) through (24), (25H) through (27H), and (28). Analogously to problem $(P2)$, we define $(PH2)$ to be the minimization of Z_H subject to constraints (1) through (12), (15) through (23), (25H) through (27H), and (28) through (35).

After solving $(PH1)$ or $(PH2)$, we can solve the vehicle routing decisions on a rolling horizon basis, as is the case in practice. We let $(x^*_{n_1 n_2 klpt})$ denote the optimal values of the corresponding variables. We then solve a problem $(PR)_p$ for each period p to determine the optimal routing plan based on the given flow. This is solved by the minimization of $Z_{Rp} = Z_p^{transport} + Z_p^{truck}$ where $Z_p^{transport}$ and Z_p^{truck} are the terms of $Z^{transport}$ and Z^{truck} that represent a single period. We then define constraints (35) to fix total wood flow to the optimal values determined in the annual harvest plan:

$$\sum_{t \in T} x_{n_1 n_2 klpt} \leq \sum_{t \in T} x^*_{n_1 n_2 klpt}, \forall n_1 \in N_{out}, n_2 \in N^{in}, k \in K, l \in L. \qquad (36)$$

After solving each routing problem per period, the cumulative objective value is equal to:

$$Z^{storage} + Z^{harvest} + Z^{demand} + Z^{inventory} + Z^{freshness} + \sum_{p \in P} Z_{Rp}.$$

We emphasize that the decomposed methodology of this section more accurately represents the current industry practice, but is in many cases superior to this manual practice. For our purposes, it represents a point of comparison to measure the resulting savings from implementing a routing-based methodology over an annual time horizon.

7 Case Studies

This project was motivated by several case studies provided by *FPInnovations* (2014), a Canadian not-for-profit organization which carries out scientific research and technology transfer for the Canadian forest industry. Three case studies were built out of previous years' historical data provided by an industrial partner in western Canada, which we denote by A, B and C. In all cases, the demand points to be served are a single mill and 4 remote pits. This demand is of 2 log assortments, deciduous and conifer, each of which can be cut into 3 different lengths: 32 foot, 37 foot, and full tree.

Harvested volumes for 8 harvesting contractors, with varying availability and target production each month of the year, were cumulated over the year to generate the information to be used for available supply. Gross harvested volumes ranged from 1.1 to 1.5 million cubic meters, with an additional 0.3 to 0.5 million cubic meters of roadside inventory at the start of the planning year. This represents roughly 20000 to 30000 truckloads, depending on the truck configuration used. Approximately 75% to 85% of the supply is deciduous, with the remainder being conifer.

Average loaded and empty driving times between all supply and demand points were generated from the forest supply chain control platform *FPInterface* (2014), developed by FPInnovations. Cycle times were generally between 3 and 8 hours.

Demand consumption and inventory requirements were not as readily available in previous years, but were simulated based upon forecasted consumption for the following year, and scaled to match the gross supply (standing and roadside) information of the year being optimized.

The same approach was used to generate the requirements of the 7 driver profiles, with varying monthly available working days of shift lengths ranging from 12 to 16 hours and 5 unique truck configurations. The monthly availabilities of the following year were scaled to match the supply and demand information of the previous years.

Storage and transportation costs are easy to measure, and vary from 0 to 2 $\$/m^3$ and 70 to 100 $\$/hr$, respectively. However the other penalties in the objective function are more difficult to measure, but their setting will dramatically affect the final solution. Both harvest and transportation contractors fell into 2 priority classes, and hence the lower priority contractors were assigned a penalty of 0. Based on discussion with industry decision makers and preliminary sensitivity analysis, the other penalties were assigned as follows:

$$\begin{aligned}
\gamma_h^{harvest} &= \$2/m^3, \\
\gamma_t^{truck} &= \$(0.5)(\gamma_t^{transport} h_t^{max})/\text{day}, \\
\gamma_{mkp}^{demand} &= \$60/m^3, \\
\gamma_{mkp}^{inventory} &= \$50/m^3, \\
\gamma_{mkp}^{freshness} &= \$50/m^3.
\end{aligned}$$

8 Computational Results

The program was modeled in C++, with Gurobi 5.6.2 used as a solver of the master problem. For all linear programs, we utilized the included barrier optimizer in order to generate interior solutions and hence more useful dual values. All other Gurobi parameters were set to the default setting.

We chose to discretize the subproblem into intervals of 20 minutes, as that is approximately the degree of accuracy to which we can measure driving distances. All experimentation was done on an Intel Core i7, 2.67 GHz processor with 4.0 GB of memory, with time limit set to 20 minutes.

For each case study we scaled the demand and inventory requirements to represent a percentage of the total supply, iterating over 80% to 90% to 100%, though we note that 80% most accurately reflects the current industry practice. For each of these problem sets we applied 4 methodologies, allowing for both harvest team scheduling formulations (1 and 2) and both the branch-and-price (BP) and decomposed (D) methodologies.

Solution quality was measured based on several key performance indicators. The total objective value was of course important, as well as the total spent on transportation. The percentage of the demand and inventory requirements that were attained, and the percentage of desired work given to both high priority harvest and trucking contractors were measured. To compare Formulation 2 to Formulation 1 with respect to harvest scheduling, the difference between the work levels of high priority harvest contractors was calculated. Cumulating the total volume of hauled wood over the course of the year allows for expression of the transportation cost in dollars per cubic meter delivered, and for each case study and formulation the improvement of the branch-and-price formulation over the decomposed approach was measured in both absolute and relative terms. All results appear in Table 6.

It is clear that incorporating routing decisions into the harvest planning model allows us to attain a higher percentage of the requested demand and inventory levels, as the decomposition attains lower levels in all cases. Moreover, by linking these decisions into a single model, the savings generated in transportation costs from planning the harvest to emphasize backhaul routes for the trucks over the planning horizon are significant, with an average value of $1.41 per cubic meter or 12.4%.

With respect to driver satisfaction, in all scenarios the branch-and-price approach gives an average of 98.4% of the requested shifts to high priority drivers; hence the allocated flow generated can then be easily assigned as a guarantee of work over the planning horizon. We note that in two out of three case studies, the decomposed methodology does not perform significantly worse in this regard. However in case study C, the decomposed methodology is outperformed due to poor decisions made in the tactical planning phase with respect to the assignment of flow to months in which the combination of driving distance and wood product are incompatible with the

Table 6: Experimental Results

Case Study	Demand Levels (%)	Harvest Scheduling Formulation	Solution Methodology	Objective Value (million $)	Transportation Cost (million $)	Demand Attained (%)	Inventory Levels Attained (%)	Priority 1 Trucking Attained (%)	Priority 1 Harvest Attained (%)	Formulation 2 Improvement (%)	Annual Haul (m^3)	Transportation Cost ($\$/m^3$)	Improvement over Decomposition ($\$/m^3$)	Improvement over Decomposition (%)
A	80	1	BP	21.968	18.459	99.94	99.98	99.98	88.26	-	1,539,612	11.99	1.51	12.63
A	80	2	BP	21.908	18.569	99.94	99.99	100	97.42	9.16	1,539,517	12.06	1.84	15.23
A	80	1	D	38.504	18.647	92	95.7	100	90.36	-	1,380,920	13.50	-	-
A	80	2	D	39.660	19.161	91.59	95.04	99.97	94.71	4.35	1,378,647	13.90	-	-
A	90	1	BP	29.096	21.548	95.71	99.99	99.81	88.88	-	1,714,409	12.57	1.20	9.55
A	90	2	BP	29.697	21.293	95.23	99.56	99.98	90.81	1.93	1,699,018	12.53	1.21	9.67
A	90	1	D	45.294	20.418	84.94	96.59	98.11	86.9	-	1,482,841	13.77	-	-
A	90	2	D	46.886	20.562	85.54	95.73	95.59	89.17	2.27	1,496,061	13.74	-	-
A	100	1	BP	42.878	21.294	84.11	98.85	99.98	86.15	-	1,704,574	12.49	1.34	10.74
A	100	2	BP	43.161	21.285	83.67	98.9	100	92.48	6.33	1,686,244	12.62	0.97	7.70
A	100	1	D	62.867	19.996	72.51	96.02	96.27	85.72	-	1,445,512	13.83	-	-
A	100	2	D	62.436	19.504	72.16	94.21	95.46	78.22	-7.50	1,434,679	13.59	-	-
B	80	1	BP	20.730	17.422	99.99	100.00	99.93	75.68	-	1,470,925	11.84	1.13	9.58
B	80	2	BP	20.643	17.377	99.92	99.97	99.93	81.37	5.69	1,477,908	11.76	1.22	10.36
B	80	1	D	24.190	18.342	97.84	98.96	99.99	73.87	-	1,413,189	12.98	-	-
B	80	2	D	23.811	18.465	97.45	99.98	99.99	71.67	-2.19	1,423,037	12.98	-	-
B	90	1	BP	32.099	17.166	86.54	99.73	99.99	75.68	-	1,465,857	11.71	1.43	12.22
B	90	2	BP	31.181	17.614	87.91	99.94	99.94	78.16	2.48	1,500,811	11.74	1.03	8.79
B	90	1	D	35.519	18.583	84.68	99.62	100.00	76.82	-	1,414,056	13.14	-	-
B	90	2	D	35.529	18.613	84.79	99.22	100.00	72.24	-4.58	1,457,706	12.77	-	-
B	100	1	BP	42.951	17.456	76.71	99.71	99.94	75.19	-	1,497,923	11.65	1.51	12.98
B	100	2	BP	42.597	17.517	77.02	99.77	99.96	78.16	2.96	1,494,002	11.72	1.37	11.72
B	100	1	D	46.986	18.635	74.56	98.91	100.00	73.85	-	1,415,364	13.17	-	-
B	100	2	D	48.085	17.958	73.83	98.03	100.00	75.20	1.34	1,370,956	13.10	-	-
C	80	1	BP	19.204	17.127	99.98	99.43	97.24	87.27	-	1,683,723	10.17	1.54	15.13
C	80	2	BP	25.137	16.430	93.56	98.26	89.18	85.61	-1.66	1,568,948	10.47	2.97	28.40
C	80	1	D	30.273	17.805	91.45	98.40	83.30	85.31	-	1,520,460	11.71	-	-
C	80	2	D	35.875	19.782	88.79	97.25	86.76	70.40	-14.91	1,471,222	13.45	-	-
C	90	1	BP	33.847	18.409	88.56	97.76	98.25	88.47	-	1,702,256	10.81	0.60	5.56
C	90	2	BP	26.178	19.298	94.70	98.77	99.46	97.81	9.34	1,816,003	10.63	1.86	17.47
C	90	1	D	36.842	19.080	86.93	97.18	94.06	86.45	-	1,671,492	11.41	-	-
C	90	2	D	58.773	16.872	69.60	95.94	85.49	63.04	-23.41	1,351,531	12.48	-	-
C	100	1	BP	46.528	17.934	78.34	97.64	94.54	85.00	-	1,735,904	10.33	1.32	12.80
C	100	2	BP	45.590	19.277	79.67	97.67	92.94	92.59	7.60	1,758,843	10.96	1.44	13.15
C	100	1	D	54.377	18.869	75.43	96.02	87.30	87.64	-	1,619,111	11.65	-	-
C	100	2	D	49.601	21.425	79.40	96.38	93.70	89.21	1.58	1,727,622	12.40	-	-

drivers working those months. This situation further illustrates the need to have more detailed vehicle routing decisions taken into account in tactical planning.

Extending the formulation to account for mid-period harvest team movement allows for an increase in the annual harvested volume assigned to high priority harvest teams. In our branch-and-price based formulation, over all case studies and demand scenarios, this average volume assigned is increased by 4.9%. Though the additional variables and constraints present in this formulation do make the model more computationally difficult, without a significant change in the other costs and penalties of the resulting solution, the trade off appears to be beneficial to the industry decision makers.

9 Conclusions

We have introduced a tactical harvest planning model that, unlike prior models in the industry, incorporates vehicle routing decisions along with allocation wood flow decisions in the transportation constraints. This problem was modeled as a mixed integer linear program, and solved via a branch-and-price heuristic with columns generated by a branch-and-price heuristic. The generated columns represent vehicle routes and are generated via dynamic programming, and the branching decisions are made on the harvest teams, with no backtracking in the search tree.

This has been implemented in a decision support system for use by our research partner FPInnovations, and tested on case studies built from 3 years of historical data of a Canadian forest company. Under an array of demand scenarios, and compared to a decomposed and sequential optimization scheme representing the current industrial practice, we are able to meet a higher proportion of demand and inventory requirements, while decreasing transportation costs by an average of 12.4%.

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