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A Lagrangian Relaxation Based Heuristic for Integrated Lumber Supply Chain Tactical Planning

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Abstract. The problem investigated in this paper is focused on integrated tactical planning in the lumber supply chain which is featured as a divergent value chain. In this industry, raw materials (logs) are shipped from forest contractors to sawmills. Then the logs are sawn to finished lumbers and are distributed to the lumber market through different channels. A mixed integer programming (MIP) model is proposed to address harvesting, procurement, production, distribution, and sales decisions in an integrated scheme so as to maximize the total profit. Three decoupled models are also formulated representing, respectively, harvesting and procurement, production, sales and distribution. The proposed MIP model is hard to solve for real-life size instances. As a consequence, a Lagrangian Relaxation (LR) algorithm is developed to solve the proposed integrated model in a reasonable time with high quality results. In order to accelerate the LR algorithm and to obtain a feasible converged solution, a heuristic algorithm is proposed. The latter obtains a high quality lower bound in each iteration of the algorithm based on the most recent upper bound. The benefit of the integrated model is evaluated by comparing the revenue and cost of the integrated model and decoupled models in a realistic-size case study. It is found that substantial improvement can be reached by considering an integrated model. Finally, the solutions quality and the resolution time of the proposed LR heuristic algorithm are evaluated through comparison with a commercial solver, the classical LR, and a time-decomposition heuristic algorithm.

Keywords. Lumber supply chain, integrated tactical planning, decoupled planning, Lagrangian relaxation.

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1. Introduction

1.1. Research motivation

Lumber supply chains incorporate forest, as the supplier, sawmills as the manufacturing entities, different distribution channels, as well as contract and non-contract-based customers. Unlike the traditional manufacturing industry which has a convergent structure (i.e., assembly lines), the lumber supply is characterized by: (i) a divergent structure (i.e., logs are transformed into several products and by-products), (ii) the highly heterogeneous nature of its raw material, and (iii) different manufacturing processes (Gaudreault et al., 2010). Because of these characteristics, lumber supply chain planning represents a major challenge in this industry. There are many ways to deal with such planning problem, using either decoupled or integrated models.

On the one hand, addressing the supply chain tactical planning problem in a decoupled manner may lead to infeasible plans in upstream entities. For example, promising sales amount without considering mills and harvesting capacities in the production and harvesting and procurement planning models may lead to infeasibility in the abovementioned models.

On the other hand, the purpose of an integrated model is to combine supply chain functions with the goal of increasing efficiency and better connecting demand with supply, which can both improve customer service and lower costs. It is worth mentioning that the variable mix of products, in addition to the existence of by-products and other aforementioned features of this supply chain make the integration and the coordination of production, procurement, distribution and sales (demand) planning a difficult task. To the best of our knowledge, less effort has been done in the literature in integrating tactical decisions in the lumber supply chain planning. The problem dealt with in this paper is focused on integrating tactical planning decisions in lumber supply chains that can be stated by the following research questions:

(i) How to integrate all medium-term decisions that different entities of lumber supply chains are dealing with?

(ii) What are the benefits of the integrated model in comparison with decoupled models in lumber supply chains?

(iii) How to solve the resulting complex integrated mathematical model?

By answering the proposed research questions, some important challenges in the tactical lumber supply chain literature will be covered.

In what follows, we first review the literature on lumber supply chain tactical planning, then we summarize the contribution of the article.

1.2. Relevant literature

There are three bodies of literature that are related to our research. First, we will review the tactical planning in the supply chain. Then, we will review the harvesting and procurement planning in the lumber supply chain. Finally, we will present a brief literature review on lumber production planning.

Tactical planning in a supply chain incorporates the synchronized planning of procurement, production, distribution and sale activities, in order to ensure that the customer demand is satisfied by the right product at the right time (Jayashankar et al., 2003). Over the last twenty years, much research has been conducted into the partial integration of the functions in a supply chain (SC) due to the difficulty in their complete integration (Shapiro, 1999). Moreover, SC tactical planning is also addressed in the framework of Sales & Operation planning (S&OP) in the literature. Recent studies consider S&OP as a synchronization mechanism that integrates the demand forecast with supply chain capabilities through coordination of marketing, manufacturing, purchasing, logistics, and financing decisions and activities (Croxton et al., 2002; Feng et al., 2008). Feng et al. (2008) presented a modelling approach in order to quantitatively evaluate the impact of S&OP program before implementation in the context of OSB industry.

Harvesting planning is one of the most important decisions in the lumber supply chain. Two main operations in the forests are harvesting and forwarding. The main important tactical decisions in the forests are the harvesting area (block) selection and bucking over the planning horizon (Bredstrom et al., 2010). Wood procurement models can be tracked back to the early 1960s. Since that time, several models have been developed to address different aspects of wood procurement (Beaudoin et al., 2007). Some of these models have been designed for specific activities such as skidding or transportation (Westerlund et al., 1999; Wightman and Jordan, 1990). Beaudoin et al. (2007) proposed a deterministic model for forest tactical planning. They also assessed the impact of uncertainty into their model and evaluated these uncertainties under alternative tactical scenarios by the aid of simulation. Other models tried to integrate several decisions in forest planning in a single model in order to capture possible synergies between them. As an instance, Burger and Jamnick (1995) integrated harvesting, storage, and transportation decisions. Andalaft et al. (2003) integrated harvesting and road-building decisions.

Karlsson et al. (2004) presented an optimization model for annual harvest planning. Their model includes transportation planning, road maintenance decisions, and control of storage both in the forest and at terminals in mills. Bredstrom et al. (2010) formulated a MIP model to integrate the assignment of machines and harvest teams to harvesting blocks and assigning each machine to the blocks. They proposed a two stage methodology that the first one solved the assignment and the second one tries to schedule. Dems et al. (2014) developed a MIP model for annual timber procurement planning with considering bucking decisions in order to minimize the operational costs such as harvesting, transportation, and inventory costs. In their proposed procurement planning model, they considered a multi-period, multi-product, multiple blocks and multi-mill setting. Chauhan et al. (2009) proposed an integrated approach for harvesting, bucking, and transportation decisions. They assumed a multi-product setting and several mills in a single period planning horizon. They minimized the harvesting and transportation costs in the forest supply. They also developed a heuristic algorithm based on columns generation to solve the procurement models, and another algorithm in order to generate bucking patterns required in the column generation approach. However, to the best of our knowledge, there is no attempt to coordinate the above mentioned decisions with production, distribution, and sales decisions in the lumber SC.

There are several contributions in the literature focused on lumber production planning. Among them, Maness et al. (1993) proposed a MIP to simultaneously determine the optimal bucking and sawing policies based on demand and final product prices. Singer et al. (2007) presented a model for optimizing production planning decisions in the sawmill industry in Chile. They demonstrated the benefit of collaboration in the SC. Kazemi Zanjani et al. (2010a, 2011) proposed a two-stage stochastic programming model and two robust optimization models for sawmill production planning by considering the non-homogeneity of raw materials. Kazemi Zanjani et al. (2010b) proposed a multi-stage stochastic program for sawmill production planning under demand and yield uncertainty.

To summarize, the available research on lumber supply chain only covers the decoupled or partial integrated models. In addition, majority of the existing MIP models are solved with the aid of commercial solvers such as CPLEX. However, solving an integrated tactical planning model in the lumber supply chain which is a large-scale MIP by the aid of a commercial solver is expected to be very time-consuming. This article is trying to fill these gaps as summarized in the following sub-section.

1.3. Contribution and article outline

Based on the existing gaps in the literature in integrating tactical decisions in lumber supply chains, in this paper we aim at integrating harvesting, procurement, production, distribution, and sales decisions in the lumber supply chain so as maximize the total profit of the supply chain. Moreover, three decoupled models are formulated representing, respectively, harvesting and procurement, production, sales and distribution. We also compare the results of the integrated model with decoupled planning models. The integrated model considers all entities of the lumber supply chain, therefore we can claim that this model is more comprehensive than the others in the related literature, and we can cover a gap. Finally, because the latter is a large-scale MIP model, solving this model in a reasonable time is another challenge which will be covered in this paper. We applied the Lagrangian Relaxation (LR) algorithm to solve the proposed MIP model where we faced with two essential issues. The first issue is the resolution time and convergence rate of the LR algorithm, and the second one is the infeasibility of the converged solution obtained by the LR method. Consequently, we propose a LR heuristic algorithm that obtains a high quality lower bound in each iteration of the algorithm based on the most recent upper bound. This method converges the proposed large-scale MIP faster than the classical approach and ensures that the converged solution is feasible.

To summarize, the paper contribution is twofold. Not only a new integrated model is proposed and compared to several decoupled models, but an efficient LR based heuristic is also developed.

This paper is organized as follows: The problem definition and formulation is presented in Section 2. The solution methodology is provided in Section 3. Finally, the numerical results and conclusions are presented in Section 4 and 5, respectively.

2. Problem definition and formulation

2.1. Problem definition

The lumber supply chain entities are summarized as a network in Fig.1. This network includes forests, sawmills, distribution centers (DCs), and customers. The supplier entities (forests) deal with harvesting planning. Through the harvesting operations the trees are cut down and are piled in the harvesting blocks and are thinned. The forwarding operations collect the piles and transfer them to the storage locations on the harvesting blocks or to the adjacent forest roads. These operations are conducted by machines such as harvesters or forwarders. Harvesters fell the trees and cut them into the logs, then forwarders pick up these piles of logs and transfer them to the storage locations. Finally, the logs could be transported to the mill, heating plants or other destinations by trucks. Forests are composed of blocks that contain raw materials (trees) from different species. The availability of each raw material on each block is different during the year. Block, storage, and transportation capacity on each block are other parameters that should be considered in the harvesting planning. Furthermore, maximum number of harvesting and maximum number of blocks in which harvesting can occur are two important factors which we consider in our problem. Furthermore, there are two important factors which should be considered in our problem. The first one is the maximum number of periods over which harvesting can occur in each block, and the second one is the maximum number of blocks in which harvesting can occur during each period.

Sawmills purchase logs from the forest, and then transform them to lumbers as main products and chips or sawdust as by-products. There are three main processes in sawmills: sawing, drying, and finishing. In the sawing process, the logs are cut into different sizes of rough lumbers by different cutting patterns. In the drying process, the lumber moisture contents are reduced by large kiln dryers or air-drying. In the finishing process, the lumbers are planned or surfaced, trimmed and sorted based on customer requirements. According to the demand, some logs are shipped to the distribution centers or directly to customers after sawing process, while others are first sent to drying and finishing processes and then are shipped to distribution centers or customers. Product shipping to customers is carried out by a number of distribution companies that use different transportation modes such as rail, truck, and different vehicle types.

The supply chain serves two different types of customers: contract-based customers such as construction, and furniture manufacturers, and noncontract-based such as pulp & paper industries or spot markets. Contract-based customers sign a contract at an agreed price and quantity for a given planning horizon. Although the contract demand must be satisfied, the enterprise reserve the right of postponing or not satisfying some parts of agreed quantities, because of capacity shortage in the demand period. With a non-contract-based customer the demand may not be satisfied when capacity is not available in the demand period. Unsatisfied demand may be served in a future period as backlog. When there is surplus capacity in sawmills, the spot market is sought to absorb the remaining capacity.

Finally, in this paper, we try to propose an integrated scheme to address harvesting, procurement, production, distribution, and sales decisions in lumber supply chain. The objective is to maximize the global net profit by balancing the sales revenue and supply chain cost, and determine the tactical planning decisions over a planning horizon *T*.

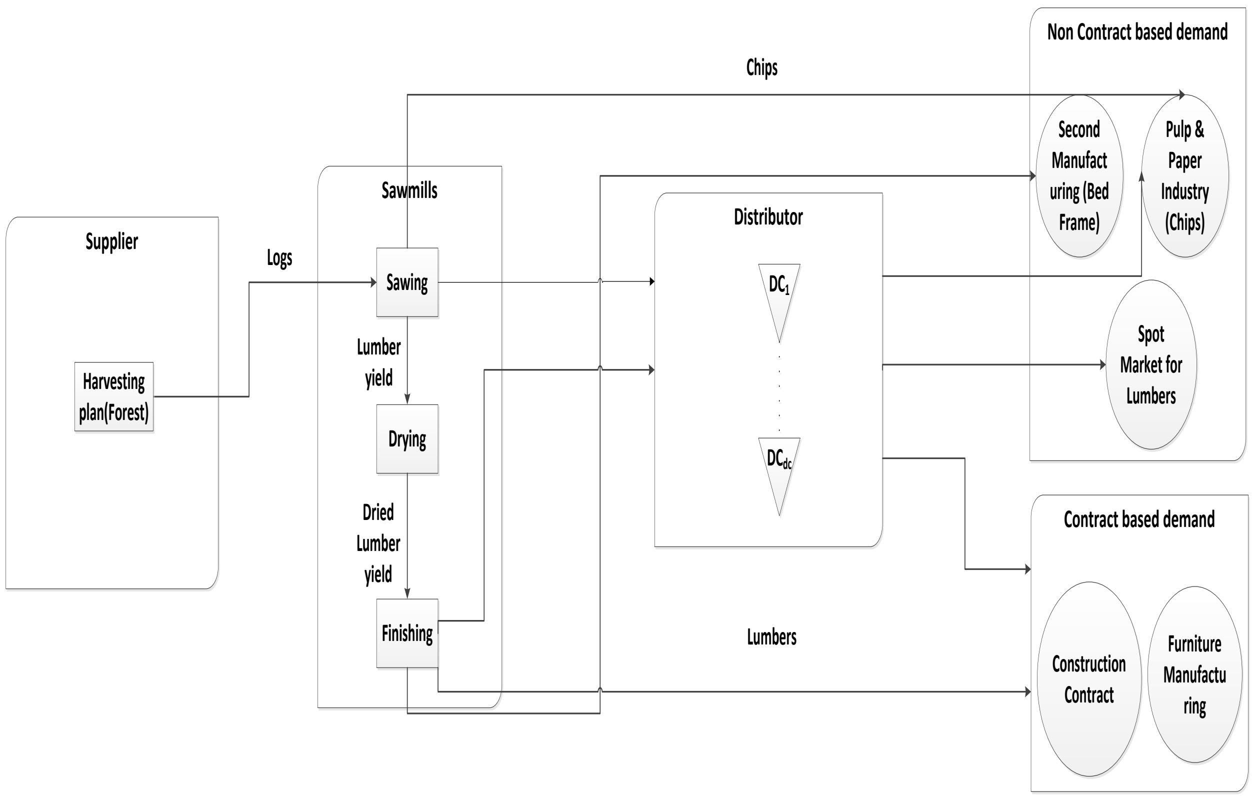

Fig 1. The lumber supply chain network

2.2. Problem formulation

In this section, we first provide a MIP model that represents the integrated harvesting, procurement, production, distribution and sales planning in the lumber supply chain. Then, we develop two classes of decoupled models. The first class incorporates three decoupled models representing, respectively, harvesting & procurement, production, and distribution & sales decisions. The second class involves two decoupled models representing harvesting & procurement, and production & distribution & sales decisions.

2.2.1. Integrated model

In the integrated model, the objective is to maximize the global net profit by balancing the sales revenue and supply chain cost over a planning horizon *T*. The harvesting decisions involve the blocks where the harvesting should occur as well as the proportion of the harvested blocks in different periods of the planning horizon. The procurement decisions include the purchasing quantity of raw material from each block, and the inventory of raw materials in each block. Production decisions incorporate the quantity of lumbers that should be sawn, dried, and finished as well as inventory and backorder quantity of lumbers. Distribution decisions include the shipping quantity of products, the inventory quantity of products in each distribution center, the number of truckload requirement along with the type of vehicle and route. Finally, sales planning involve the amount of sales promised to customers as well as possible backorder quantity of products.

The indices, sets, parameters, and decision variables used in the proposed models are listed in table 12, 13 and 14 in appendix A.

Mixed-integer programming Model

The proposed MIP model is presented in table 1. In the integrated model (1) – (51), constraint (10) ensures that the harvested proportion of a block do not exceed the availability of logs in that block. Constraint (11) describes that if harvesting occurs on a block then we can ensure that raw materials from that block are available. Constraints (12) and (13) correspond to the maximum number of harvesting and maximum number of blocks in which harvesting can occur, respectively. Constraints (14) and (15) correspond to harvesting and transportation capacity from each block to each mill, respectively. Constraint (16) represents the final inventory of raw materials in each block. Constraint (17) formulates the inventory balance of raw materials in each block. Constraint (18) determines the quantity of products which should be processed in sawing, drying and finishing units. The raw material safety stock policies are stated in constraint (19) and the raw material inventory capacity constraint is provided in constraint (20). Constraint (21) describes the raw material supply capacity constraints. Constraint (22) states that the material procured from a supplier must satisfy the contract quantity commitment. Constraints (23) and (35) are the coupling constraints that link the production and sales decisions and determine the maximum amount of production, inventory and backorder in sawing and finishing processes. The backorder quantities are converted into backlogged sales (BS_{it}^{c}) (24) and (36), in order to be used in distribution constraints (40). Constraint (25), (31), and (37) formulates the production capacity constraints in sawing, drying, and finishing units. Constraint (26), (32), and (38) define the warehouse inventory capacity in sawing, drying, and finishing units. The beginning and ending backlog conditions in sawing, drying, and finishing units are described in constraints (27), (33), and (39), respectively. Constraint (28) is a flow conservation constraint for consumed product, inventory and backorder, and calculates the output quantity of products from the sawing unit. Constraint (29) ensures that the total amount of green lumber sawn in the sawing unit will be processed in the drying unit with a specific yield. Constraint (30) links the quantities of green lumber received from the sawing process, the inventory and backorder, and the quantity of dried lumbers. Constraint (34) ensures that the total amount of dried lumbers received from the drying unit will be processed in the finishing unit by considering a specific yield. Constraint (40) links the sales and distribution decisions. Constraints (41) and (42) link the production and distribution decisions. Constraint (43) is the flow balance constraints at a distribution center. Constraint (44) calculates the number of truckload requirements for each vehicle type from each shipping supplier. Constraint (45) and (46) formulate the shipping supplier capacity and the mill dispatch capacity constraints, respectively. Constraint (47) represents the initial inventory of each product in each distribution center. Constraints (48) and (49) describe the sales decisions for contract and non-contract-based demands. In this case, the demand might be accepted and be served in future periods as backorder (I^{-}_{imt}), or, might be rejected. In either case, the backorder amount (BS^{c}_{it}) should not be greater than the sales quantity (S^{c}_{it}) (50). Upon satisfaction of the base amount (48), the company may continue serving the contract demand up to the capacity limit, or switch to serve non-contract demand, whichever is more profitable. Finally, constraint (51) represents the domain constraints.

Table 1. Mixed-integer programming model

$$Max\ Z = R - (C_{harvesting} + C_{stumpage} + C_{transportation} + C_{storage} + C_{procurement} + C_{production} + C_{distribution}) \quad (1)$$

where:

$$R = \sum_{c \in C} \sum_{i \in I} \sum_{t \in T} b^{c}_{it}\, S^{c}_{it} \quad (2)$$

$$C_{harvesting} = \sum_{bl \in BL} \sum_{t \in T} c^{H}_{blt} y_{blt} \left(\sum_{rm \in RM} v_{rm,bl}\right) \quad (3)$$

$$C_{stumpage} = \sum_{rm \in RM} \sum_{bl \in BL} \sum_{t \in T} v_{rm,bl} f_{rm,bl,t} y_{blt} \quad (4)$$

$$C_{transportation} = \sum_{bl \in BL} \sum_{m \in M} \sum_{rm \in RM} \sum_{t \in T} c^{T}_{rm,bl,m,t} X^{bl}_{rm,m,t} \quad (5)$$

Table 1. Continued

$$C_{storage} = \sum_{rm\in RM}\sum_{bl\in BL}\sum_{t\in T} c^{S}_{rm,bl,t} I_{rm,bl,t} \tag{6}$$

$$C_{procurement} = \sum_{bl\in BL}\sum_{rm\in RM}\sum_{m\in M}\sum_{t\in T} m^{bl}_{rm,t}\ X^{bl}_{rm,m,t} + \sum_{rm\in RM}\sum_{m\in M}\sum_{t\in T} h_{rm,m} I_{rm,m,t} \tag{7}$$

$$\begin{aligned} C_{production} = & \sum_{m\in M}\sum_{i\in I}\sum_{t\in T} c_{im}\,(XSW_{imt} + XDR_{imt} + OXF_{imt}) + \\ & \sum_{m\in M}\sum_{i\in I_{SW}}\sum_{t\in T} h1_{im} ISW^{+}_{imt} + \sum_{m\in M}\sum_{i\in I_{DR}}\sum_{t\in T} h2_{im} IDR^{+}_{imt} + \\ & \sum_{m\in M}\sum_{i\in I_{F}}\sum_{t\in T} h3_{im} IF^{+}_{imt} + \sum_{m\in M}\sum_{i\in I_{SW}}\sum_{t\in T} bo1_{im} ISW^{-}_{imt} + \\ & \sum_{m\in M}\sum_{i\in I_{DR}}\sum_{t\in T} bo2_{im} IDR^{-}_{imt} + \sum_{m\in M}\sum_{i\in I_{F}}\sum_{t\in T} bo3_{im} IF^{-}_{imt} \end{aligned} \tag{8}$$

$$\begin{aligned} C_{distribution} = & \sum_{s\in S}\sum_{i\in I}\sum_{r\in R}\sum_{v\in V}\sum_{t\in T}(e^{s}_{irv}\ X^{s}_{irvt} + sh^{s}_{rv} N^{s}_{rvt}) + \\ & \sum_{s\in S}\sum_{i\in I}\sum_{dc\in DC}\sum_{r\in R_{m,dc}}\sum_{v\in V}\sum_{t\in T} tr_{idc}\ X^{s}_{irvt} + \sum_{i\in I}\sum_{dc\in DC}\sum_{t\in T} h_{idc}\, I_{idct} \end{aligned} \tag{9}$$

Subject to:

Harvesting constraints:

$$\sum_{t\in T} y_{blt} \le 1 \quad \forall\, bl \tag{10}$$

$$y_{blt} \le H_{blt} \quad \forall\, bl, t \tag{11}$$

$$\sum_{t\in T} H_{blt} \le l_{bl} \quad \forall\, bl \tag{12}$$

$$\sum_{bl\in BL} H_{blt} \le n_t \quad \forall\, t \tag{13}$$

$$\sum_{bl\in BL}(y_{blt}\ \sum_{rm\in RM} v_{rm,bl}) \le b^{H}_{t} \quad \forall\, t \tag{14}$$

$$\sum_{rm\in RM}\sum_{m\in M}\sum_{bl\in BL} X^{bl}_{rm,m,t} \le b^{T}_{t} \quad \forall t \tag{15}$$

$$I_{rm,bl,T} = 0 \quad \forall\, rm, bl \tag{16}$$

$$I_{rm,bl,t} = I_{rm,bl,t-1} - \sum_{m\in M} X^{bl}_{rm,m,t} + v_{rm,bl}\, y_{blt} \quad \forall\, rm, bl, t \ge 1 \tag{17}$$

Procurement constraints:

$$\sum_{bl\in BL} X^{bl}_{rm,m,t-L^{bl}_{rm}} + I_{rm,m,t-1} - I_{rm,m,t} = \sum_{i\in(I_{SW}\cup I'_{SW})} cu_{rm,i,m}\ XSW_{imt} \quad \forall\, rm, m, t = 1 + L^{bl}_{rm}, \dots, T \tag{18}$$

$$I_{rm,m,t} - ss_{rm,m} \ge 0 \quad \forall\, rm, m, t \tag{19}$$

$$\sum_{rm\in RM} I_{rm,m,t} \le KI_{rm,m} \quad \forall\, rmc, m, t \tag{20}$$

Table 1. Continued

$$\sum_{rm\in RM}\sum_{m\in M} X^{bl}_{rm,m,t} \leq KS^{bl}_{t} \quad \forall\, bl, t \tag{21}$$

$$\sum_{m\in M}\sum_{rm\in RM}\sum_{t\in T} X^{bl}_{rm,m,t} \geq qmin^{bl} \quad \forall\, bl \tag{22}$$

Production constraints:

Sawing process:

$$\sum_{m\in M}(XSW_{imt} + ISW^{+}_{imt-1} - ISW^{-}_{imt-1} - ISW^{+}_{imt} + ISW^{-}_{imt}) \leq \sum_{c\in C} S^{c}_{it} \quad \forall\, i \in I'_{sw}, t \tag{23}$$

$$\sum_{m\in M} ISW^{-}_{imt} = \sum_{c\in C} BS^{c}_{it} \quad \forall\, i \in I'_{sw}, t \tag{24}$$

$$\sum_{i\in(I_{SW}\cup I'_{SW})} p1_{imt} XSW_{imt} \leq Ksw_{mt} \quad \forall m, t \tag{25}$$

$$\sum_{i\in I_{sw}} ISW^{+}_{imt} + \sum_{i\in I'_{SW}} ISW^{+}_{imt} \leq KIsw_{m} \quad \forall\, m, t \tag{26}$$

$$ISW^{-}_{im0} = ISW^{-}_{imT} = 0 \quad \forall\, i \in (I_{sw} \cup I'_{SW}), m \tag{27}$$

$$XSW_{imt} + ISW^{+}_{imt-1} - ISW^{-}_{imt-1} - ISW^{+}_{imt} + ISW^{-}_{imt} = OXSW_{imt} \quad \forall i \in I_{SW}, m, t \tag{28}$$

Drying process:

$$\emptyset_{i'mt} OXSW_{i'mt} = XDR_{imt} \quad \forall\, i' \in I_{SW}, i \in I_{DR}, m, t \tag{29}$$

$$XDR_{imt} + IDR^{+}_{imt-1} - IDR^{-}_{imt-1} - IDR^{+}_{imt} + IDR^{-}_{imt} = OXDR_{imt} \quad \forall i \in I_{DR}, m, t \tag{30}$$

$$\sum_{i\in I_{DR}} p2_{imt} XDR_{imt} \leq Kdr_{mt} \quad \forall\, m, t \tag{31}$$

$$\sum_{i\in I_{DR}} IDR^{+}_{imt} \leq KIdr_{m} \quad \forall\, m, t \tag{32}$$

$$IDR^{-}_{im0} = IDR^{-}_{imT} = 0 \quad \forall\, i \in I_{DR}, m \tag{33}$$

Finishing process:

$$\rho_{i'mt} OXDR_{i'mt} = OXF_{imt} \quad \forall\, i' \in I_{DR}, i \in I_{F}, m, t \tag{34}$$

$$\sum_{m\in M}(OXF_{imt} + IF^{+}_{imt-1} - IF^{-}_{imt-1} - IF^{+}_{imt} + IF^{-}_{imt}) <= \sum_{c\in C} S^{c}_{it} \quad \forall\, i \in I_{F}, t \tag{35}$$

$$\sum_{m\in M} IF^{-}_{imt} = \sum_{c\in C} BS^{c}_{it} \quad \forall\, i \in I_{F}, t \tag{36}$$

$$\sum_{i\in I_{F}} p3_{imt} OXF_{imt} \leq Kf_{mt} \quad \forall\, m, t \tag{37}$$

$$\sum_{i\in I_{F}} IF^{+}_{imt} \leq KIf_{m} \quad \forall\, m, t \tag{38}$$

$$IF^{-}_{im0} = IF^{-}_{imT} = 0 \quad \forall\, i \in I_{F}, m \tag{39}$$

Table 1. Continued

Distribution constraints:

$$\sum_{c\in C}(S_{it}^{c} + BS_{it-1}^{c} - BS_{it}^{c}) = \sum_{s\in S}\sum_{r\in(R_{m,c}\cup R_{dc,c})}\sum_{v\in V} X_{irvt}^{s} \quad \forall\, i,t \tag{40}$$

$$\sum_{m\in M}(XSW_{imt} + ISW_{imt-1}^{+} - ISW_{imt}^{+}) = \sum_{s\in S}\sum_{r\in(R_{m,dc}\cup R_{m,c})}\sum_{v\in V} X_{irvt}^{s} \quad \forall\, i \in I'_{sw}, t \tag{41}$$

$$\sum_{m\in M}(OXF_{imt} + IF_{imt-1}^{+} - IF_{imt}^{+}) = \sum_{s\in S}\sum_{r\in(R_{m,dc}\cup R_{m,c})}\sum_{v\in V} X_{irvt}^{s} \quad \forall\, i \in I_F, t \tag{42}$$

$$\sum_{s\in S}\sum_{r\in R_{m,dc}}\sum_{v\in V} X_{irvt}^{s} + I_{idct-1} - I_{idct} = \sum_{s\in S}\sum_{r\in R_{dc,c}}\sum_{v\in V} X_{irvt}^{s} \quad \forall\, i \in (I'_{SW}\cup I_F), dc, t \tag{43}$$

$$N_{rvt}^{s} \geq \sum_{i\in(I'_{SW}\cup I_F)} \frac{a_i X_{irvt}^{s}}{KV_v} \quad \forall\, s\in S, r, v, t \tag{44}$$

$$\sum_{r\in R} N_{rvt}^{s} \leq KSH_v^{s} \quad \forall\, s\in S, v, t \tag{45}$$

$$\sum_{s\in S}\sum_{r\in(R_{m,dc}\cup R_{m,c})}\sum_{v\in V} N_{rvt}^{s} \leq KD_m \quad \forall\, m,t \tag{46}$$

$$I_{idc0} = 0 \quad \forall\, i, dc \tag{47}$$

Sales constraints:

$$S_{it}^{c} - BS_{it}^{c} \geq dmin_{it}^{c} \quad \forall c\in CC, i\in I_F, t \tag{48}$$

$$S_{it}^{c} \leq d_{it}^{c} \quad \forall c\in C, i, t \tag{49}$$

$$BS_{it}^{c} \leq S_{it}^{c} \quad \forall c\in C, i, t \tag{50}$$

Domain constraints:

$$S_{it}^{c}, BS_{it}^{c}, X_{imt}, OXSW_{imt}, XSW_{imt}, OXDR_{imt}, XDR_{imt}, OXF_{imt}, XF_{imt}, ISW_{imt}^{+}, IDR_{imt}^{+}, IF_{imt}^{+}$$

$$IDR_{imt}^{-}, IF_{imt}^{-}\, X_{irvt}^{s}, I_{idct}, X_{rm,m,t}^{bl}, I_{rm,m,t}, y_{blt}, I_{rm,bl,t}, N_{rvt}^{s} \geq 0$$

$$H_{blt} \in \{0,1\} \quad \forall c, i, t, m, s, r, v, dc, bl, rm \tag{51}$$

2.2.2. Decoupled models

In this section, two classes of decoupled models are considered. The first one considers three sub-models including sales & distribution, production, harvesting & procurement, and the second one considers two sub-models including sales & distribution & production, and harvesting & procurement. As we mentioned before, solving the problem in the decoupled manner may lead to infeasible plans in the upstream echelon. Thus, it is necessary to add extra constraints in each sub-model in order to link sub-models to each other and to ensure the feasibility of each one. Moreover, the output of one sub-model acts as the input of another one. For instance, the outputs of sales & distribution sub-model act as the inputs of production sub-model.

Sales & distribution sub-model

The objective of this model is to maximize the total revenue from sales activities minus the distribution costs as follows:

$$Max\, z = R - C_{distribution} \tag{52}$$

The constraints of this model include constraints (40), and (43)-(51) in the integrated model in addition to the following ones:

$$\sum_{i \in I_F} S_{it}^{c} \leq \sum_{m \in M} Kf_{mt} \quad \forall\, t \tag{53}$$

$$\sum_{i \in (I_{SW} \cup I'_{SW})} S_{it}^{c} \leq \sum_{m \in M} Ksw_{mt} \quad \forall\, t \tag{54}$$

Constraints (53) and (54) enforce the sales and distribution model to control the amount of the sales quantity of each product based on the production capacity of sawing and finishing units. These two constraints are added to the decoupled model in order to ensure the feasibility of promised sales amount to the customer.

Production sub-model

The objective of this model is to minimize the production, inventory, and backlog costs at sawing, drying and finishing units. Also, this model gets the sales and distribution decisions (S_{it}^{c}, I_{idct}) as parameters (input) from the sales & distribution sub-model (52) – (54).

$$Min\, z = C_{production} \tag{55}$$

The constraints of this model involve constraints (23) – (39) in the integrated model in addition to the following ones:

$$\sum_{rm \in RM} \sum_{m \in M} \sum_{i \in (I_{SW} \cup I'_{SW})} cu_{rm,i,m} \, XSW_{imt} \leq b_t^T \quad \forall t \tag{56}$$

$$\sum_{rm \in RM} \sum_{m \in M} \sum_{i \in (I_{SW} \cup I'_{SW})} cu_{rm,i,m} \, XSW_{imt} \leq \sum_{bl \in BL} KS_t^{bl} \quad \forall t \tag{57}$$

$$\sum_{rm \in RM} \sum_{m \in M} \sum_{i \in (I_{SW} \cup I'_{SW})} \sum_{t \in T} cu_{rm,i,m} XSW_{imt} \geq qmin^{bl} \quad \forall bl \tag{58}$$

$$\sum_{m \in M} (OXF_{imt} + IF^{+}_{imt-1} - IF^{+}_{imt}) \geq \sum_{c \in CC} dmin^{c}_{it} \quad \forall\, i \in I_F, t \tag{59}$$

Constraint (56) and (57) enforce the production model to control the production amount based on the supply and transportation capacity of raw material in the forest. Constraint (58) and (59) ensure that the production amount satisfies the minimum purchase quantity of raw material from each block, and minimum contract demand, respectively.

Harvesting & procurement sub-model

The objective of this model is to minimize the harvesting cost, stumpage fee, storage and procurement cost in the forest. Also, this model receives the production quantities of lumber (XSW_{imt} , $OXSW_{imt}$)from the production sub-model (55) – (59) as the input.

$$Min\ Z = C_{harvesting} + C_{stumpage} + C_{transportation} + C_{storage} + C_{procurement} \tag{60}$$

The constraints of this model are the same as constraints (10)-(22) in the integrated model.

Production & sales & distribution sub-model

The objective of this model is to maximize the total revenue from sales activities minus the distribution and production costs.

$$Max\ z = R - (C_{production} + C_{distribution}) \tag{61}$$

The constraints of this model include constraints (23)-(51) in the integrated model, and constraints (56)-(58) from the production model.

3. Solution methodology

The proposed integrated model (1)-(51) in section 2 is a large-scale mixed-integer model. Therefore, solving this problem in a reasonable time is a challenge for realistic-scale problem instances. We implemented two methods to solve the proposed integrated model: CLPLEX 12.3, as well as a LR heuristic method. The goal of the heuristic algorithm is to accelerate the LR algorithm, and to ensure the feasibility of the converged solution.

In the following, we first provide a brief description of LR algorithm. Then we elaborate on the LR heuristic algorithm proposed in the framework of this study.

3.1.Lagrangian Relaxation algorithm

Lagrangian Relaxation is a well-known decomposition approach which is used to solve large-scale MIP models. In this method a set of complicated constraints in the original MIP are relaxed and are added to the objective function in a Lagrangian fashion with associated multipliers. Then, the relaxed model is expected to be easier to solve (Geoffrion, 1974; Wolsey, 1998). The LR algorithm can be presented as follows (Wolsey, 1998):

Consider the following integer programming model:

$$\begin{gathered} z = \max cx \\ Ax \leq b \\ Dx \leq d \\ x \in Z_+^n \end{gathered}$$

Suppose that the constraints $Ax \leq b$ are "nice" in the sense that an integer program (IP) with just these constraints is easy. Thus, if one drops the "complicating constraints" $Dx \leq d$, the resulting relaxation is easier to solve than the original problem IP. For example, in model (1) – (51), constraints (12) and (13) are complicating constraints and by relaxing them the model can be solved faster. However, the resulting bound obtained from the relaxation may be weak, because some important constraints are totally ignored. One way to tackle this difficulty is by LR algorithm.

We consider the problem IP in a slightly more general form:

(IP)

$$\begin{gathered} z = \max cx \\ Dx \leq d \\ x \in X \end{gathered}$$

where $Dx \leq d$ are m complicating constraints.

For any value of $u = (u_1, \dots, u_{1m}) \geq 0$, we define the problem:

(IP (u))

$$\begin{gathered} z = \max cx + u(d - Dx) \\ x \in X \end{gathered}$$

In the integrated model (1)-(51), we relaxed constraints (12)-(13) which are related to the harvesting part of our model, and incorporated them to the objective function by introducing multipliers u_{bl} , and v_t. These two constraints correspond to the maximum number of harvesting and maximum number of blocks in which harvesting can occur, respectively. Thus, the Lagrangian relaxation of model (1)-(51) can be stated as follows:

$$L_{IP}(u,v) = Maximize\,\{\, Z + \sum_{bl \in BL} u_{bl} * \,(l_{bl} - \sum_{t \in T} H_{blt}) + \sum_{t \in T} v_t * (n_t \,- \sum_{bl \in BL} H_{blt}\,)\} \tag{62}$$

Subject to:

(10)-(11) and (14)-(51).

Proposition 1 (Wolsey, 1998). Problem IP (u) is a relaxation of problem IP for all $u \geq 0$.

We see that in IP (u) the complicating constraints are handled by adding them to the objective function with a penalty term $u(d - Dx)$, or in other words, u is the price or dual variable or Lagrange multiplier associated with the constraints $Dx \leq d$.

Problem IP (u) is called a Lagrangian relaxation (sub-problem) of IP with parameter u. As IP (u) is a relaxation of IP, $z(u) \geq z$ and we obtain an upper-bound on the optimal value of IP. To find the best (smallest) upper-bound over the infinity of possible values for u, we need to solve the Lagrangian Dual problem (LD):

$$w_{LD} = \min\{\, z(u) \colon u \geq 0\}$$

Proposition 2 (Wolsey, 1998). If $u \geq 0$,

(i) *x(u)* is an optimal solution of *IP (u)* , and

(ii) $Dx(u) \leq d$, and

(iii) $(Dx(u))_i = \, d_i$ whenever $u_i > 0$ (complementarity), then *x (u)* is optimal in IP.

The LD problem can be solved by the aid of sub-gradient algorithm.

Sub-gradient algorithm for the Lagrangian dual (Wolsey, 1998)

Table 2. Sub-gradient algorithm

Initialization. $u = \, u^0$.

Iteration k. $u = \, u^k$.

Solve the Lagrangian problem IP $(\, u^k)$ *with optimal solution* $x\ (u^k)$.

$$u^{k+1} = max\{u^k \, - \, \mu_k\Big(d - Dx(u^k)\Big), 0\}$$

$$k \, \leftarrow k + 1$$

The vector $d - Dx(u^k)$ is easily shown to be a sub-gradient of $z\,(u)$ at u^k.

At each iteration one takes a step from the present point u^k in the direction opposite to a sub-gradient. The difficulty is in choosing the step length $\{\mu_k\}_{k=1}^{\infty}$.

Theorem 1 (Wolsey, 1998). (a) If $\sum_k \mu_k \rightarrow \infty$, and $\mu_k \rightarrow \infty$ as $k \rightarrow \infty$, then $z\,(u^k) \rightarrow w_{LD}$ the optimal value of LD.

(b) If $\mu_k = \mu_0 \rho^k$ for some parameter < 1 , then $z\,(u^k) \rightarrow w_{LD}$ if μ_0 and ρ are sufficiently large.

(c) If $\overline{w} \geq w_{LD}$ and $\mu_k = \epsilon_k [z(u^k) - \overline{w}] / \|d - Dx(u^k)\|^2$ with $0 < \epsilon_k < 2$, then $z\,(u^k) \rightarrow \overline{w}$, or the algorithm finds u^k with $\overline{w} \geq z\,(u^k) \geq w_{LD}$ for some finite *k*.

This theorem tells us that rule (a) guaranties convergence, but as the series $\{\mu_k\}$ must be divergent (for example $\mu_k = 1/k$), convergence is too slow to be of real practical interest.

Using rule (b), the initial values of μ_0 and ρ must be sufficiently large, otherwise the geometric series $\mu_0 \rho^k$ tends to zero too rapidly, and the sequence u^k converges before reaching an optimal point.

Using rule (c), the difficulty is that a dual upper-bound $\overline{w} \geq w_{LD}$ is typically unknown. It is more likely in practice that a good primal lower-bound $\underline{w} \leq w_{LD}$ is known. Such a lower-bound $\underline{w}$ is then used initially in place of $\overline{w}$. However, if $\underline{w} < w_{LD}$, the term $z\,(u^k) - \underline{w}$ in the numerator of expression for μ_k will not tend to zero, and so the sequences $\{u^k\}$, $\{z(u^k)\}$ will not converge. If such behavior is observed, the value of $\underline{w}$ must be increased. In this article, we used rule (c), because it is shown to be the most efficient rule for updating Lagrangian multipliers.

It is worth mentioning that in order to find the initial LB in the numerical results of this article, we ran CPLEX for 30 minutes, and then the best feasible solution is considered as the initial LB. Moreover, if the gap between the LB and UB is large, the converged solution might be infeasible, as is the case in our problem. On the other hand, if we manage to update this bound as we proceed in the sub-gradient algorithm, we might succeed to speed-up the algorithm and to guaranty the feasibility of the converged solution. This is the motivation behind proposing a LR heuristic provided in the following sub-section.

3.2.Lagrangian Relaxation Heuristic algorithm

As we mentioned before, the classical sub-gradient approach, the lower-bound is considered as a fixed amount. Hence, the classical approach cannot guaranty to obtain a feasible converged solution. In order to guaranty the feasibility of the converged solution in the sub-gradient method, we propose a heuristic to update the lower-bound in each iteration. More precisely, we propose to improve the quality of the lower-bound (LB) based on the most recent upper-bound (UB) obtained at each iteration of the sub-gradient algorithm. The reason is that the quality of the UB is expected to be improved as we proceed in the sub-gradient algorithm. In order to update the LB, after each iteration, we calculate the slack variables corresponding to the relaxed constraints (12) - (13). If the slack is positive, it means that its constraint is satisfied. Hence, we suggest to find those variables in these constraints that already have taken value zero, then fix them in the initial model (1) – (51). Then, by solving the revised model (1) – (51), we can obtain a new feasible solution (LB). The proposed LR heuristic can be summarized in table 3.

Table 3. LR Heuristic algorithm

Step 0 (initialization):

Assign zero to u_{bl} *and* v_t

Assign an initial value to the lower-bound (LB) and assign ∞ to the upper-bound (UB)

Let iteration counter (k) equal to 1

While (the stopping criteria is not satisfied) do

Step 1:

Solve the Lagrangian problem (62) and determine the optimal solutions and $L_{IP}(u, v)$

Step 2:

If $(L_{IP}(u, v) < UB)$ *then* $UB = L_{IP}(u, v)$

Update lower-bound (LB) based on the "lower-bound heuristic algorithm"

Step 3: Update dual multipliers as follows:

$$u_{bl}^{k+1} = max\{u_{bl}^{k} - \varepsilon_k * \frac{L_{IP}^{k}(u,v)-LB}{\|l_{bl}- \sum_{t\in T} H_{blt}\|^2} * (l_{bl} - \sum_{t\in T} H_{blt}), 0\}$$

$$v_{t}^{k+1} = max\{v_{t}^{k} - \varepsilon_k * \frac{L_{IP}^{k}(u,v)-LB}{\|n_t - \sum_{bl\in BL} H_{blt}\|^2} * (n_t - \sum_{bl\in BL} H_{blt}), 0\}$$

$$k = k + 1$$

End-do

It is worth mentioning that the stopping criteria in the LR heuristic algorithm could be the number of iterations or ($\varepsilon_k \rightarrow 0$), but in this article the stopping criterion is defined by the number of iterations (15 iterations).

The heuristic algorithm for updating the LB in the LR algorithm can be summarized in table 4.

Lower-bound Heuristic

Table 4. Lower-bound heuristic algorithm

Step 0:

Calculate $\{slack_{bl} = (l_{bl} - \sum_{t\in T} H_{blt})\ \forall bl\}$ *and* $\{slack_t = (n_t - \sum_{bl\in BL} H_{blt})\ \forall t\}$ *after solving Lagrangian problem in each iteration*

If ($slack_{bl} \geq 0$) *then*

Step 1:

Identify the binary variables which are equal to 0 and fix them in the initial MIP model (1) - (51)

If ($slack_t \geq 0$) *then*

Step 2:

Identify the binary variables which are equal to 0 and fix them in the initial MIP model (1) – (51)

Step 3:

Solve the MIP model (1) - (51) resulted from steps (1) and (2) to obtain new lower-bound ($new\ LB$)

If ($new\ LB > old\ LB$) *then*

Step 4:

Lower-bound for the next iteration in the sub-gradient algorithm (LB) = *new* LB

4. Numerical results

In this section, we first describe the case study, then we provide numerical results derived from solving the integrated model and its comparison with the decoupled ones introduced in section 2.2.2. In the second part, we just focus on the integrated problem, and we present the results of applying the proposed LR heuristic to the large-scale MIP integrated tactical planning model. Finally, we compare the proposed LR heuristic with a time-decomposition algorithm available in the literature.

4.1.Case study

In order to validate the integrated tactical planning model proposed in the context of a lumber supply chain, we need a data set that sufficiently represents a realistic scale sawmill in Canada. The realistic environment which we are studying in this paper consists two sawmills producing 27 product families with using 14 types of raw materials. Products are shipped to 140 customers by 4 outbound shipping suppliers using 5 different vehicle types with via 2 distribution centers and 20 routes (Feng et al., 2008). Capacity of each vehicle type is randomly generated from uniform distribution [3, 25]. Also, we assumed that 50 harvesting blocks are available in the forest during the 12 month planning horizon. The supply capacity of each block in the forest per month is supposed to be 2350 m^3. Maximum number of periods (months) over which harvesting can occur in each block, maximum number of blocks in which harvesting can occur per month are randomly selected from uniform distributions [10, 12] and [1, 6], respectively (Beaudoin et al., 2007). The average volumes of each log class available in each block are randomly generated based on Karlsson, et al. (2004). Total harvesting capacity per month is supposed to be approximately 117,500 m^3. In sawmills, we supposed approximately 750,000 m^3 production capacity per month. Finally, the demand for each type of products is derived from Kazemi Zanjani, et al. (2011).

There are some aspects in harvesting planning such as weather conditions during the year, road maintenance, and crew scheduling that should be taken into consideration. For example, it is not possible to transport the logs from some blocks to mills during winter, because the snow might close some roads, therefore more maintenance or road substitution may lead to changing the harvesting plan. The abovementioned aspects of harvesting were included implicitly in the transportation cost from the blocks to mills. For instance, if a road does not exist, the cost of building that road is included in the transportation cost.

The real-life-size case study defined in this paper results nearly 280,000 continuous and 600 binary variables and nearly 280,000 constraints in the integrated model. All models were coded by CPLEX 12.3 in a Dual-Core CPU 2.80GHz computer with 4.00 GB RAM and windows 7. Furthermore, the LR heuristic was coded in C++ using CPLEX concert technology.

4.2. Results of integrated and decoupled models

The comparison of the integrated model with the decoupled ones is carried out through evaluating the total revenue and the total cost of harvesting, procurement, production and distribution resulted from the integrated and decoupled models (tables 5, 6). In the following tables, Δ denotes the difference between the revenue/cost of integrated and decoupled models. The negative amount of Δ in revenue and costs indicates that the total revenue and costs of the decoupled model is greater and less than the integrated one, respectively. As an instance, the negative amount of Δ for actual revenue in table 5 implies that the actual revenue in the integrated model is less than the decoupled one, and the positive amount of Δ for backlog cost in the same table indicates that the backlog cost in the integrated model is decreased or less than the decoupled model. Furthermore, the actual revenue denotes to the income acquired from selling the products to the customer.

Table 5. Comparison of the integrated model and decoupled class 1

Criteria	Integrated model	Decoupled model	Δ over class 1	Deviation over class 1
Actual revenue	435,359,356	709,780,241	-274,420,885	-39%
Inventory cost at DCs	0	0	0	0%
Transshipment cost at DCs	3,787,256	5,097,326	1,310,070	26%
Inventory cost	27,086,403	29,822,645	2,736,242	9%
Backlog cost	86,258,429	479,162,072	392,903,643	82%
Production cost	28,125,487	26,322,034	-1,803,453	-7%
Harvesting cost	20,849,181	21,153,326	304,145	1%
Procurement cost	4,568,000	4,205,207	-362,793	-9%
Total profit	264,684,600	144,017,631	120,666,969	84%

Table 6. Comparison of the integrated model and decoupled class 2

Criteria	Integrated model	Decoupled model	Δ over class 2	Deviation over class 2
Actual revenue	435,359,356	422,530,613	12,828,743	3%
Inventory cost at DCs	0	0	0	0%
Transshipment cost at DCs	3,787,256	3,594,670	-192,586	-5%
Inventory cost	27,086,403	10,810,574	-16,275,829	-151%
Backlog cost	86,258,429	115,758,944	29,500,515	25%
Production cost	28,125,487	26,829,579	-1,295,908	-5%
Harvesting cost	20,849,181	21,962,604	1,113,423	5%
Procurement cost	4,568,000	4,341,904	-226,096	-5%
Total profit	264,684,600	239,232,338	25,452,262	11%

As already explained, in the decoupled class 1, three sub-models including sales & distribution, production, and harvesting & procurement are considered, while in the decoupled class 2, two sub-models including sales & distribution & production, and harvesting & procurement are considered.

In table 5, we can see the total revenue in the decoupled model is greater than the integrated one. It means that the promised sales in the decoupled model are greater than the integrated one. Because in the decoupled model, sales & distribution and production models are considered separately, the inventory and backorder quantity and their costs in the decoupled model are much higher than the integrated model in order to satisfy the bigger amount of promised sales in the decoupled model. Consequently, a big Δ in inventory and backorder cost can be observed in table 5. On the other hand, in the integrated model, because sales & distribution and production sub-models are considered in an integrated scheme, the backlogged and inventory costs are considerably lower. Also, the production quantity is greater than the decoupled one. Consequently the procurement quantity and costs of raw material are greater in the integrated model. It is worth mentioning that the availability of raw material on each block is different. Hence, the model will satisfy the raw material purchase amount based on the raw material inventory and available harvesting amount in each block. As a consequence, the greater procurement quantity is not necessarily equivalent to the greater harvesting amount or raw material inventory on each block.

In analyzing the results of the integrated and decoupled models, we observed that although the revenue in class 1 was greater than the integrated model, the integrated model made further modifications on sales decisions. In other words, while the overall revenue was reduced, the total inventory and backorder costs were reduced more significantly, resulting in a net profit improvement (84% improvement in the total profit).

Table 6 summarizes the comparison of the integrated model with the decoupled model (class 2). The greater amount of actual revenue (greater amount of sale) in the integrated model generates the greater amounts of inventory, production and distribution costs. However, the backlog costs in the integrated model are considerably lower. Analyzing the rest of costs in table 6 is the same as table 5 explained earlier. Finally, as expected, the integrated model generates the highest profit in comparison with the decoupled model. In this case, the benefit of the integrated model over class 2 is relatively moderate because of the improved performance with integrating production model with the sales & distribution one (84% vs 11%). In the decoupled model (class2), the actual revenue was very close to the integrated actual revenue, but the total costs were greater in the decoupled model resulting a greater total profit in the integrated model.

Table 7. CPU time

Models	Objective function	CPU time (Sec)
Integrated model	264,684,600	17,097
Sales & Distribution model (Class 1)	704,682,915	78
Production model (Class 1)	535,306,751	141
Harvesting & Procurement model (Class 1)	25,358,533	4,368
Sales & Distribution & Production model (Class 2)	265,536,846	303
Harvesting & Procurement model (Class 2)	26,304,508	7379

Table 7 presents the objective functions and resolution time of different models. The proposed sub-models in decoupled classes are linear problems, except for the harvesting & procurement sub-model which is a MIP model. Consequently, the harvesting & procurements sub-models are the most time consuming models in comparison to the other sub-models. Finally as expected, the resolution times of decoupled models are considerably lower than the integrated one. As the CPU time of solving the integrated model is high (around 5 hours), we applied a LR heuristic method described in 3.2, in order to reduce the computation time while getting high quality feasible solutions.

4.3.Results of Lagrangian Relaxation heuristic

The LR heuristic was run using the sub-gradient approach for updating the dual multipliers. The step-size was divided by 2 whenever the upper-bound was not updated in an iteration. As explained earlier, in order to find the initial lower-bound, we ran the original integrated problem for 30 minutes, then the best feasible solution is considered as the initial LB.

Table 8 represents a comparison between the classical LR approach with the LR heuristic method proposed in this paper. The classical approach converges in 11 iterations, but the results are not feasible. On the other hand, the heuristic approach provides a high quality feasible solution in a considerably faster time (the execution time is reduced by approximately 5,000 seconds). Moreover, the resolution time to solve the proposed MIP model is reduced about 12,200 seconds (about 3.5 hours) by applying LR method and the proposed lower-bound heuristic algorithm.

Table 8. Comparison of classical and LR heuristic methods

	Classical Lagrangian	LR heuristic	Gap
Profit	264,653,000	264,624,000	29,000
CPU time (Sec)	9,028	3,921	5,107

Moreover, we implemented the local branching heuristic for the sub-gradient algorithm. In our implementation, first we ran the sub-gradient method without adding the local branching heuristic algorithm, and then we added this algorithm and evaluated the quality of solution and resolution time.

Table 9 presents the results of LR heuristic runs with and without local branching heuristic. The "gap %" denotes the difference between the LR heuristic results and the optimal results obtained with CPLEX. The LBheur represents the results of using the local branching heuristic algorithm. In table 9, "gap" denotes the difference between the optimal and LR heuristic solutions. As it is shown in table 9, the local branching heuristic algorithm improves the optimality gap by 0.003%. However it increases the resolution time by 200 seconds.

Table 9. The LR heuristic results for the original problem

Instances	Objective function	CPLEX	Gap	Gap%	CPU time (Sec)
LR heuristic	264,618,000	264,684,600	66,600	0.025%	4,876
LR heuristic with LBheur	264,624,000	264,684,600	60,600	0.022%	5,095

Table 10 represents the results of applying the proposed LR heuristic on 10 problem instances, while the first instance corresponds to the main case study. In this table, the "LB" and "UB" represents the best lower-bound and upper-bound of LR heuristic approach, respectively. Column "LR heuristic gap%" corresponds to the gap between the UB and LB calculated based on $(\frac{UB-LB}{UB} * 100)$, while column "LR time(Sec)" , " Heuristic time (Sec)", and "Total time (Sec)" represents the time spent by the solver in the Lagrangian sub-problems, lower-bound heuristic algorithm, and the total time of running the LR heuristic method, respectively. Moreover, we provided the "CPLEX results" and the "CPLEX time (Sec)" in table 10 to show the results and CPU time of different instances run with the CPLEX solver. Finally, the "Gap%" field is the relative gap between the best feasible solution found by the LR heuristic method and the optimal solution found by CPLEX, and is calculated by$(\frac{CPLEX\ result-LB}{CPLEX\ result} * 100)$.

As expected, we could not reach to the optimal solution by CPLEX in 5 hours (>5h) in some instances, because of the size of these instances. As a consequence, the "CPLEX results" and "Gap%" fields are represented by N.A. It is important to note that "LB", "UB", and "CPLEX results" are divided by 1000 in table 10.

As it can be observed in table 10, the LR heuristic proposed in section 3 provides high quality solutions with small (negligible) optimality gaps in a reasonable time. Hence, it can be concluded that the proposed lower-bound heuristic has improved the classical LR algorithm specially regarding the long convergence time and infeasibility of converged solution in the classical LR method.

Table 10. The LR heuristic results for different instances

Instance	LB	UB	LR heuristic Gap%	LR time(Sec)	Heuristic time(Sec)	Total time(Sec)	CPLEX time(Sec)	CPLEX results	Gap%
1	264,624	264,950	0.12%	2,275	1,646	3,921	17,097	264,684	0.023
2	429,115	429,439	0.08%	681	1,418	2,099	7,465	429,214	0.023
3	375,406	375,668	0.07%	2,151	1,554	3,705	7,616	375,444	0.01
4	524,317	524,361	0.01%	2,776	1,113	3,889	12,806	524,345	0.005
5	191,875	191,905	0.02%	1,424	805	2,229	>5h	N.A.	N.A.
6	352,000	353,153	0.33%	1,190	1,805	2,895	>5h	N.A.	N.A.
7	560,347	560,406	0.01%	633	1,644	2,277	>5h	N.A.	N.A.
8	244,833	244,973	0.06%	2,350	3,470	5,820	>5h	N.A.	N.A.
9	221,278	221,408	0.06%	3,605	2,321	5,926	>5h	N.A.	N.A.
10	47,972	48,116	0.3%	13,178	1,447	14,625	>5h	N.A.	N.A.

4.4.Results of time decomposition algorithm

In this section we compare the performance of the propose LR heuristic with a "time-decomposition" algorithm as described in the appendix B. Table 11 summarizes the results.

Table 11. Time decomposition heuristic algorithm results

	Time decomposition	LR heuristic	Gap
Profit	263,057,370	264,624,000	1,566,630
CPU time (Sec)	3,328	3,921	593

With comparing the results of the time-decomposition heuristic algorithm and the LR heuristic, we can observe that the quality of the LR heuristic algorithm is better than the time-decomposition algorithm in terms of the objective function. We can conclude that although the time decomposition algorithm solves the model slightly faster, the LR heuristic algorithm obtains higher quality feasible solution with the optimality gap of 0.61% vs 0.025%.

5. Conclusions

In this paper, we proposed a MIP model to address harvesting, procurement, production, distribution, and sales decisions in lumber supply chain in an integrated scheme. Also, three decoupled models were also formulated representing, respectively, harvesting and procurement, production, sales and distribution. The benefit of the integrated model was evaluated by comparing the integrated model and decoupled models in terms of total revenue and costs based on a realistic-scale industrial case study. It was observed that substantial improvement can be reached by using an integrated model rather than a decoupled model. In reality, enterprises are not owned by one company. Each company works to satisfy its own objectives. Thus, despite of the expected improvement of the integrated model, we will face some challenges in integrating the decisions in realistic environments. Consequently, collaboration among lumber supply chain partners will help to achieve the predictable results obtained by the integrated model.

Moreover, in order to overcome the complexity of the integrated model for real-size instances, we applied a LR heuristic algorithm. In order to accelerate the sub-gradient algorithm in the LR problem and to obtain a feasible converged solution, we applied a heuristic algorithm to update the lower-bound in each iteration. Our computational results revealed high quality solutions and reduction in resolution time by applying the proposed LR heuristic algorithms. Finally, future research will consider uncertainties of parameters such as demand, production yield, and log supply into the proposed integrated model.

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Appendix A

Table 12. Sets

M	Set of manufacturing mills
I_{SW}	Set of products produced by sawing process that are transferred to drying unit (such as lumbers)
I'_{SW}	Set of products produced by sawing process (such as chips and green lumbers)
I_{DR}	Set of products produced by drying process
I_F	Set of products produced by sawing, drying and finishing processes (such as finished product)
I	Set of end products ($I = I'_{SW} \cup I_F$)
T	Set of time periods
C	Set of customers
CC	Set of contract customers
NC	Set of non-contract customers
RM	Set of raw materials
DC	Set of distribution centers
V	Set of vehicles
R	Set of all routes
S	Set of outbound shipping suppliers
$R_{m,dc}$	Set of routes from mills to distribution centers
$R_{dc,c}$	Set of routes from distribution centers to customers
$R_{m,c}$	Set of routes from mills to customer directly
BL	Set of harvesting blocks

Table 13. Parameters

b_{it}^c	Sales price of product i to customer c in period t
d_{it}^c	Forecasted demand of customer c for product i in period t
$dmin_{it}^c$	Minimum demand from customer c for product i in period t
$Ksw_{mt}, Kdr_{mt}, Kf_{mt}$	Production capacity of mill m in period t at sawing, drying and finishing units
$p1_{imt}, p2_{imt}, p3_{imt}$	Capacity consumption for producing product i at mill m in sawing, drying and finishing units during period t
c_{im}	Unit production cost to produce product i at mill m
$h1_{im}, h2_{im}, h3_{im}$	Inventory cost of product i at sawing, drying and finishing units of mill m
$bo1_{im}, bo2_{im}, bo3_{im}$	Backlog cost of product i at sawing, drying and finishing units of mill m
$KIsw_m, KIdr_m, KIf_m$	Warehouse inventory capacity of mill m at sawing, drying and finishing units
ρ_{imt}	Average yield of product i processed at finishing unit of mill m in period t
$\emptyset_{imt}$	Average yield of product i processed at drying unit of mill m in period t
sh_{rv}^s	Shipping fixed cost of supplier s on route r using vehicle type v
e_{irv}^s	Shipping variable cost of supplier s for product i on route r using vehicle type v
h_{idc}	Inventory holding cost for unit quantity of product i at distribution centre dc
a_i	Vehicle capacity absorption coefficient per unit of product i

Table 13. Continued

tr_{idc}	Transshipment cost of product i through distribution centre *dc*
KSH_v^s	Shipping capacity of supplier *s* with vehicle *v*
KV_v	Capacity of vehicle type *v*
KD_m	Expedition capacity of mill *m*
$cu_{rm,i,m}$	Consumption of raw material *rm* for producing unit quantity of product i at mill m
$KI_{rm,m}$	Inventory capacity of raw material category *rm* at mill *m*
KS_t^{bl}	Supply capacity of block *bl* in period *t*
$qmin^{bl}$	Minimum contract purchase quantity from block *bl*
$ss_{rm,m}$	Safety stock of raw material *rm* at mill *m*
$m_{rm,t}^{bl}$	Unit purchase cost of raw material *rm* from block *bl* in period *t*
$h_{rm,m}$	Inventory holding cost of raw material *rm* at mill *m*
L_{rm}^{bl}	Lead time of procuring raw material *rm* from block *bl*
c_{blt}^H	Unit cost to harvest block *bl* during period *t*
$c_{rm,bl,t}^S$	Unit cost to store raw material *rm* on block *bl* during period *t*
$f_{rm,bl,t}$	Stumpage fee for raw material *rm* on block *bl* during period *t*
$c_{rm,bl,m,t}^T$	Unit cost to transport raw material *rm* from block *bl* to mill *m* during period *t*
l_{bl}	Maximum number of periods over which harvesting can occur in block *bl*
n_t	Maximum number of blocks in which harvesting can occur during period *t*
$v_{rm,bl}$	Volume of raw material *rm* available on block *bl*
b_t^H , b_t^T	Total harvesting and transportation capacity in period *t*
u_{bl}^k , v_t^k	Lagrangian multipliers in iteration *k*
ε_k	Step size modifier in iteration *k*

Table 14. Decision variables

$X_{rm,m,t}^{bl}$	Purchasing quantity of raw material rm from block bl by mill m in period t
$I_{rm,m,t}$	Inventory of raw material rm at mill m at the end of period t
$I_{rm,bl,t}$	Inventory of raw material rm in block bl at the end of period t
y_{blt}	Proportion of harvested block bl in period t
H_{blt}	Binary variable (if harvesting occurs on block bl during time period t)
$OXSW_{imt}$	Quantity of product i that should be transferred from sawing to drying unit of mill *m* in period *t*
XSW_{imt}	Quantity of product i which should be sawn at sawing unit of mill *m* in period *t*
XDR_{imt}	Quantity of product i which should be processed at drying unit of mill *m* in period *t*
$OXDR_{imt}$	Quantity of product i which should be transferred from drying to finishing unit of mill *m* in period *t*
OXF_{imt}	Quantity of product i which should be transferred from finishing unit of mill *m* in period *t*
$ISW_{imt}^+, IDR_{imt}^+, IF_{imt}^+$	Inventory quantity of product *i* at sawing, drying and finishing units of mill *m* in period *t*
$ISW_{imt}^-, IDR_{imt}^-, IF_{imt}^-$	Backlog quantity of product *i* at sawing, drying and finishing units of mill *m* in period *t*
X_{irvt}^s	Shipping quantity of product i with shipping supplier s on route *r* with vehicle *v* in period *t*
I_{idct}	Inventory quantity of product i in distribution center *dc* at the end of period *t*
N_{rvt}^s	Number of truckload requirement from shipping supplier s on route *r* with vehicle *v* in period *t*
S_{it}^c	Sales quantity of product *i* to customer *c* in period *t*
BS_{it}^c	Backlogged quantity of product *i* to customer *c* in period *t*

Appendix B

Time-decomposition algorithm

In the proposed integrated model, all binary and continuous variables of the model depend on time. The idea of time-decomposition algorithm is dividing the planning horizon into smaller intervals that would be considered as the new planning horizon. This method has been used in the literature for solving large-scale MIP problems (e.g., Ouhimmou et al. (2008)). Based on this algorithm, we divide the initial problem to different sub-problems corresponding to each sub-planning horizon. For example, in our case study, we divided the initial problem to 12 sub-problems. In each iteration of this algorithm, we solve the sub-problem and find the solutions. Then, we can fix some variables in the current sub-problem solution to their values and them as new constraints to the next sub-problem. The new added constraints might reduce the set of feasible solutions of the new sub-problem. It is important to mention that we fix only binary variables that are equal to 1, which means that other variables include continuous and null binary variables might take other values in the solution of the new sub-problem. The main steps of the heuristic algorithm are described in table 15. Let P_i is the mixed integer problem related to period i.

Table 15. Time-decomposition algorithm

Step 0:

Divide the planning horizon into n equal intervals (i)

Define $i = 1$

Step 1:

Solve P_i and get the binary variables which are equal to 1

If ($i \leq n$) then

Step 2:

2.1. Identify the binary variables which are equal to 1 and fix them in the problem P_{i+1}

2.2. Solve P_{i+1}

2.3. $i = i + 1$

As an instance, suppose $i=3$ and $bl=4$, so there are twelve (3×4) binary variables such as $\{H_{11}, H_{12}, H_{13}, \ldots, H_{43}\}$. After solving P_3, we observe that $H_{12} = H_{23} = H_{33} = H_{41} = 1$, and the other binary variables are zero. Therefore, we should add four new constraints to P_4 and solve it ($H_{12}=1$, $H_{23}= 1$, $H_{33}= 1$ and $H_{41}= 1$).