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Solving a Vendor-Managed Inventory Problem Arising in the Distribution of Bottled Water in Morocco

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Abstract. In this paper, we solve a real-life distribution problem faced by a Moroccan bottled water company dealing with a combination of inventory and distribution decision. To manage its distribution process, the company uses a vendor-managed inventory system, which means that the supplier controls the inventory at the customers. This problem is known as the Inventory-Routing Problem (IRP) in which both transportation and inventory costs are simultaneously minimized. Our real-life problem contains several types of bottled water that must be shipped from a supplier to a set of regional depots and wholesalers; the former are served by routing and the latter by full truckloads. The inventory costs are paid at both the plant and at the customers, and shipments are performed by a fleet of homogeneous vehicles. We propose a branch-and-cut algorithm for this problem which consists of an IRP with multiple products and multiple vehicles. Computational tests were carried out on fifteen real-life based instances. The results show that significant savings can be obtained by using our approach with respect to the current company practice.

Keywords. Inventory-routing problem, branch-and-cut, vendor managed inventory, bottled water, multi-vehicle, multi-product.

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1 Introduction

Moroccans have experienced a significant increase in the standard of living and purchasing power in recent years. Consumption has grown rapidly, which has led to intense competition between several national and international brands. Bottled water is one of the sectors that have evolved very quickly. According to official sources in the Ministry of Industry and Trade of Morocco, sales of bottled water have tripled between 2004 and 2012, as their turnover has increased from \$58 million to \$180 million, as depicted in Figure 1.

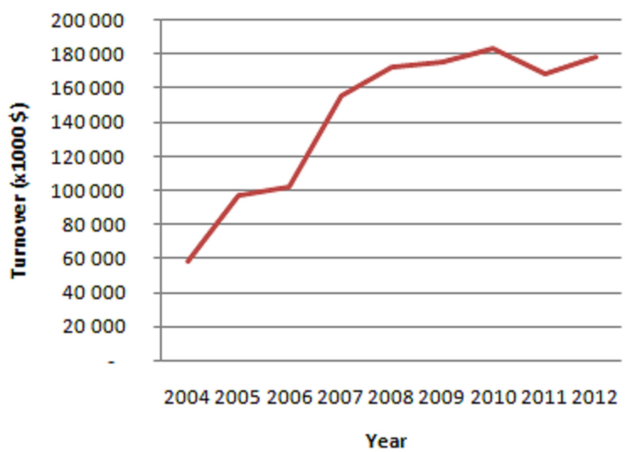

Figure 1: Turnover of the Moroccan bottled water industry
Source: Moroccan Ministry of Industry and Trade

Each company has developed its logistics networks by creating regional depots and developing partnerships with resellers and transporters to ensure a maximum coverage of the national territory, taking into account the significant variation in consumption from one region to another and the geographical dispersion of the covered population. These factors make the supply chain management of bottled water very difficult to solve, particularly at the transportation and inventory management levels.

Transportation costs have increased significantly recently since the Moroccan government started a progressive elimination of its subsidies on the price of fuel. Inventory management has also experienced difficulties due to very high seasonal demands. Indeed, in the summer high demand causes stockouts while in the winter overstock problems appear. The major challenges of bottled water companies are mainly to minimize transportation and inventory costs while avoiding stockouts and overstocks. These companies are in a permanent quest to find a good tradeoff between service level and costs.

In this paper, we investigate the case of a Moroccan company specialized in the production and distribution of bottled water. Our industrial partner deals with a total of four different types of product and the plant produces more than 300 pallets per day, carrying plastic bottles of various sizes. A local warehouse is capable of holding up to 1000 pallets, which is currently small given the needs of the company, making inventory management a challenge. Different types of products are distributed from a single factory located in the north of Morocco to a set of more than two dozen large customers spread throughout the country. These customers are classified as regional depots, belonging to the company, and wholesalers. A map of the north of Morocco depicting the locations of the plant and all customers is provided in Figure 2.

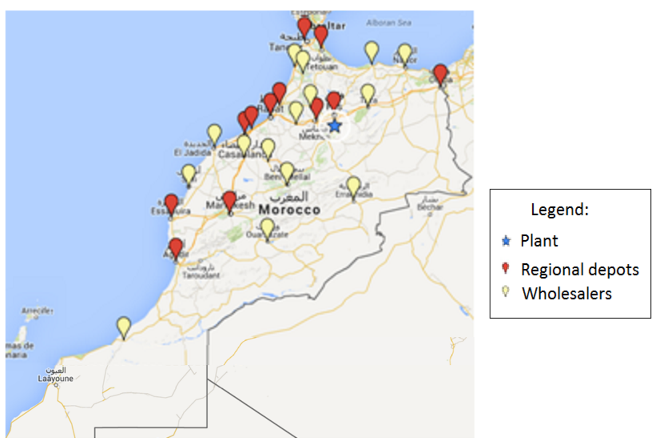

Figure 2: Map of the north Morocco and location of the plant and customers
Source: Google Maps (October 2014)

The sales volume of our industrial partner has increased significantly in the last five years, which has led the company to create new depots and develop new contracts with wholesalers and transporters. Contracts with wholesalers state that the company has to manage their inventory, however, the resulting costs are charged to them. Contracts with transporters state that the company charters trucks according to its needs and pays for the distance travelled.

Production levels are decided at an early stage and are not subject to operational changes. The company plans its distribution manually based on the experience of planners, but also takes into account demand forecasts and inventory levels at customers. However, since the inventory capacity at the plant is limited, the company is forced to ship the excess quantity to the regional depots depending on the inventory level and the storage capacity of each one. As a result, transportation and inventory costs can become very high given that production is fixed. Indeed, the company must review its planning approach in order to eliminate stockouts and overstock and optimize transportation and inventory costs.

Our problem can be defined as a rich multi-vehicle multi-product inventory-routing problem (IRP). The IRP combines the simultaneous optimization of vehicle routes and inventory control, effectively solving the operational problem arising from a vendor-managed inventory strategy. The IRP was formally introduced more than 30 years ago by Bell et al. [6] and has since given rise to many practical and technical contributions. In what follows we review the most relevant to our case. For a recent survey of IRPs, see Coelho et al. [14].

Many papers have studied the single commodity IRP [4, 12, 13, 10], but the case with several products which is the object of this study is relatively recent and significantly more difficult. From a technical perspective, Coelho and Laporte [11] have developed an exact branch-and-cut algorithm applicable to small instances, whereas other authors have proposed a variety of heuristics. Thus Moin et al. [28] have developed a hybrid genetic algorithm, Mjirda et al. [26, 27] have put forward a two phase variable neighborhood search, and Huang and Lin [21] have proposed an ant colony optimization algorithm for a similar problem with demand uncertainty.

Many real-life applications arise in the maritime industry namely in the distribution of several types of fuel and gases by compartmentalized ships [5, 7, 30, 9, 15, 20, 32, 35, 36, 37]. Non-maritime applications include the distribution of perishable products [16, 17], the transportation of gases by tanker trucks [6], the automobile components industry [2], the electrical products with multiple depots [33], fuel delivery [31, 34] and the distribution of grocery and food products [25, 24, 22, 17, 8, 23]. A review of practical IRP applications can be found in Andersson et al. [3].

The main contribution of this paper is to solve a real-life and rich IRP arising in the bottled water industry in Morocco. We extend existing formulations and propose new valid inequalities that exploit the structure of the problem faced by our industrial partner. We are then able to propose high quality solutions to our partner, which significantly outperform their current solutions. As a subproduct of our research, we provide new insights into the problem and the solution method.

The remainder of the paper is organized as follows. In Section 2 we formally describe the problem and propose an integer linear programming formulation for it. In Section 3, we present the branch-and-cut algorithm we have developed to solve the problem. Computational experiments are presented in Section 4, followed by conclusions in Section 5.

2 Problem description and formulation

In order to formulate the problem by means of an integer linear programming model we define it on an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{0, \ldots, n\}$ is the vertex set and $\mathcal{E} = \{(i,j), i < j\}$ is the edge set. Vertex 0 represents the depot while the remaining vertices of $\mathcal{V}' = \mathcal{V}_1 \cup \mathcal{V}_2$ are partitioned into a set $\mathcal{V}_1$ of regional depots and a set $\mathcal{V}_2$ of wholesalers. Each location $i \in \{0\} \cup \mathcal{V}_1$ incurs inventory holding cost h_i per period, and has an inventory capacity C_i. The company distributes a set $\mathcal{M} = \{1, \ldots, M\}$ of products, which are all measured in terms of number of pallets. The length of the planning horizon is p, typically six days, measured in discrete time periods $t \in \mathcal{T} = \{1, \ldots, p\}$. The quantity of product m made available at the depot in period t is r^{mt}. An unlimited fleet of vehicles $\mathcal{K}$ is available. An upper bound K on the number of vehicles is the number

n of customers. We then identify each vehicle $k \in \mathcal{K} = \{1, \dots, K\}$, each with capacity Q_k (in number of pallets). A routing cost c_{ij} is associated with edge $(i, j) \in \mathcal{E}$. The inventories are not allowed to exceed the holding capacity and cannot be negative. At the beginning of the planning horizon the decision maker knows the current inventory level of each product m I_i^{m0} at each location i, and receives the information on the demand d_i^{mt} of each location i for each product m and period t. We assume that the quantities q_i^{mkt} received by location i of product m from vehicle k in period t can be used to meet its demand in that period. The objective of the problem is to minimize the total routing and inventory holding costs while meeting the demands for each product at each location in each period.

The model works with the following decision variables. Integer undirected routing variables x_{ij}^{kt} are equal to the number of times that edge $(i, j) \in \mathcal{E}$ is used on the route of vehicle k in period t; binary variables y_i^{kt} are equal to one if and only if location $i \in \mathcal{V}$ is visited by vehicle k in period t; integer variables I_i^{mt} represent the inventory level of product m at location i at the end of period t; integer variables q_i^{mkt} represent the quantity of product m delivered to location i by vehicle k in period t. The problem can then be formulated as follows:

$$\text{minimize} \sum_{i \in \{0\} \cup \mathcal{V}_1} \sum_{m \in \mathcal{M}} \sum_{t \in \mathcal{T}} h_i^m I_i^{mt} + \sum_{(i,j) \in \mathcal{E}} \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} c_{ij} x_{ij}^{kt}, \tag{1}$$

subject to

$$I_0^{mt} = I_0^{m,t-1} + r^{mt} - \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{V}'} q_i^{mkt} \quad m \in \mathcal{M} \quad t \in \mathcal{T} \tag{2}$$

$$I_i^{mt} = I_i^{m,t-1} + \sum_{k \in \mathcal{K}} q_i^{mkt} - d_i^{mt} \quad m \in \mathcal{M} \quad i \in \mathcal{V}' \quad t \in \mathcal{T} \tag{3}$$

$$\sum_{m \in \mathcal{M}} I_i^{mt} \leq C_i \quad i \in \mathcal{V} \quad t \in \mathcal{T} \tag{4}$$

$$\sum_{m \in \mathcal{M}} \sum_{k \in \mathcal{K}} q_i^{mkt} \leq C_i - \sum_{m \in \mathcal{M}} I_i^{m,t-1} \quad i \in \mathcal{V}' \quad t \in \mathcal{T} \tag{5}$$

$$q_i^{mkt} \leq C_i y_i^{kt} \quad i \in \mathcal{V}' \quad m \in \mathcal{M} \quad k \in \mathcal{K} \quad t \in \mathcal{T} \tag{6}$$

$$\sum_{i \in \mathcal{V}'} \sum_{m \in \mathcal{M}} q_i^{mkt} \leq Q_k y_0^{kt} \quad k \in \mathcal{K} \quad t \in \mathcal{T} \tag{7}$$

$$\sum_{m \in \mathcal{M}} q_i^{mkt} = Q_k y_i^{kt} \quad i \in \mathcal{V}_2 \quad k \in \mathcal{K} \quad t \in \mathcal{T} \tag{8}$$

$$x_{ij}^{kt} = 0 \quad i \in \mathcal{V}_2 \quad j \in \mathcal{V}' \quad k \in \mathcal{K} \quad t \in \mathcal{T} \tag{9}$$

$$\sum_{j \in \mathcal{V}, i<j} x_{ij}^{kt} + \sum_{j \in \mathcal{V}, j<i} x_{ji}^{kt} = 2y_i^{kt} \quad i \in \mathcal{V} \quad k \in \mathcal{K} \quad t \in \mathcal{T} \tag{10}$$

$$\sum_{i \in \mathcal{S}} \sum_{j \in \mathcal{S}, i<j} x_{ij}^{kt} \leq \sum_{i \in \mathcal{S}} y_i^{kt} - y_g^{kt} \quad \mathcal{S} \subseteq \mathcal{V}' \quad k \in \mathcal{K}$$
$$t \in \mathcal{T} \quad g \in \mathcal{S} \tag{11}$$

$$I_i^{mt} \in \mathbb{N} \quad i \in \mathcal{V} \quad m \in \mathcal{M} \quad k \in \mathcal{K} \quad t \in \mathcal{T} \tag{12}$$

$$q_i^{mkt} \in \mathbb{N} \quad i \in \mathcal{V}' \quad m \in \mathcal{M} \quad k \in \mathcal{K} \quad t \in \mathcal{T} \tag{13}$$

$$x_{0j}^{kt} \in \{0,1,2\} \quad j \in \mathcal{V}' \quad k \in \mathcal{K} \quad t \in \mathcal{T} \tag{14}$$

$$x_{ij}^{kt} \in \{0,1\} \quad i,j \in \mathcal{V}' \quad k \in \mathcal{K} \quad t \in \mathcal{T} \tag{15}$$

$$y_i^{kt} \in \{0,1\} \quad i \in \mathcal{V} \quad k \in \mathcal{K} \quad t \in \mathcal{T}. \tag{16}$$

The objective function (1) minimizes the total inventory and routing costs. Constraints (2) and (3) define inventory conservation at the supplier and the customers respectively. Constraints (4) and (5) impose maximal inventory level at the supplier and the customers. Constraints (6) link the quantities delivered to the routing variables. In particular, they only allow a vehicle to deliver any products to a customer if the customer is visited by this vehicle. Constraints (7) ensure the vehicle capacities are respected. Constraints (8) and (9) apply only to the wholesalers and impose full truckload deliveries. Hence, they enforce a round trip strategy to the wholesalers. Constraints (10) and (11) are degree constraints and subtour elimination constraints, respectively. Constraints (12)−(16) enforce integrality and non-negativity conditions on the variables.

This formulation can be strengthened by adding the following known valid inequalities:

$$x_{0j}^{kt} \leq 2y_j^{kt} \quad j \in \mathcal{V}' \quad k \in \mathcal{K} \quad t \in \mathcal{T} \tag{17}$$

$$x_{ij}^{kt} \leq y_i^{kt} \quad i, j \in \mathcal{V} \quad k \in \mathcal{K} \quad t \in \mathcal{T} \tag{18}$$

$$y_i^{kt} \leq y_0^{kt} \quad i \in \mathcal{V}' \quad k \in \mathcal{K} \quad t \in \mathcal{T} \tag{19}$$

$$\sum_{k \in \mathcal{K}} \sum_{l=1}^{t} y_i^{kl} \geq \left\lceil \left[\sum_{l=1}^{t} d_i^{ml} - I_i^{m0} \right] / \min\{Q, C_i\} \right\rceil \quad i \in \mathcal{V}' \quad m \in \mathcal{M} \quad t \in \mathcal{T}. \tag{20}$$

Constraints (17) and (18) are referred to as logical inequalities. They enforce the condition that if the supplier is the successor of a customer in the route of vehicle k in period t, i.e., $x_{i0}^{kt} = 1$ or 2, then i must be visited by the same vehicle, i.e., $y_i^{kt} = 1$. A similar reasoning is applied to customer j in inequalities (18). Constraints (19) include the supplier in the route of vehicle k at the period t if any customer is visited by that vehicle in that period. Constraints (20) ensure that customer i is visited at least the number of times corresponding to the right-hand side of the inequality. This inequality is only valid if the fleet is homogeneous (i.e., $Q_k = Q \ \forall k \in \mathcal{K}$). It was originally developed for the single-vehicle case by Archetti et al. [4] and was later extended to the multi-vehicle case by Coelho and Laporte [10].

To improve constraints (20), we propose new cuts adapted to our context:

$$\sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} y_0^{kt} \geq \left\lceil \left[\sum_{i \in \mathcal{V}'} \sum_{m \in \mathcal{M}} \sum_{t \in \mathcal{T}} d_i^{mt} - \sum_{i \in \mathcal{V}'} \sum_{m \in \mathcal{M}} I_i^{m0} \right] / Q \right\rceil. \tag{21}$$

This inequality takes all the demand, discounts the initial inventories, and checks how many vehicles are needed to satisfy it. Its round up to the nearest integer number of vehicles. This number is the minimum required number of vehicles to be dispatched. However this constraint can be further strengthened four two reasons. First, any initial inventory can be used to satisfy the demand even for different products. Second, because the wholesalers receive full truckloads. We propose the following cuts to derive a stronger formulation:

$$\sum_{k\in\mathcal{K}}\sum_{l=1}^{t} y_i^{kl} \geq \left\lceil \left\lfloor \max_{m\in\mathcal{M}} \left\{ \sum_{l=1}^{t} d_i^{ml} - I_i^{m0} \right\} \right\rfloor / Q \right\rceil \qquad i \in \mathcal{V}' \quad t \in \mathcal{T}. \tag{22}$$

These constraints work as follows: in their right-hand side, they compute the minimum number of vehicles to satisfy the demand up to period t by each product. Then take the maximum one (some products might not need to be delivered, but others do, so there will be a visit to customer i because of the critical product). In the left-hand side, we count how many visits are needed to that customer until period t.

To try to overcome the second issue raised above, which concerns full truckload delivery to wholesalers, we count how many vehicles are needed to satisfy each type of customer: either wholesalers or regular customers. To achieve this, we propose the following inequalities:

$$\begin{aligned}\sum_{k\in\mathcal{K}}\sum_{l=1}^{t} y_0^{kl} \geq \max\left\{0, \left\lceil \left\lfloor \sum_{i=1}^{n_1}\sum_{m\in\mathcal{M}} (\sum_{l=1}^{t} d_i^{ml} - I_i^{m0}) \right\rfloor / Q \right\rceil \right\} \quad + \\ \sum_{i=n_1+1}^{n} \max\left\{0, \left\lceil \left\lfloor \sum_{m\in\mathcal{M}} (\sum_{l=1}^{t} d_i^{ml} - I_i^{m0}) \right\rfloor / Q \right\rceil \right\} t \in \mathcal{T}.\end{aligned} \tag{23}$$

The first term of the right-hand side computes the minimum number of vehicles to satisfy each regional depot up to period t, and the second term computes the minimum number of vehicles to satisfy the wholesalers.

Finally, we also tighten this formulation by imposing the following symmetry breaking constraints valid for the case where the vehicle fleet is homogeneous, which is verified in our case:

$$y_0^{kt} \leq y_0^{k-1,t} \quad k \in \mathcal{K}\backslash\{1\} \quad t \in \mathcal{T} \tag{24}$$

$$y_i^{kt} \leq \sum_{j<i} y_j^{k-1,t} \quad i \in \mathcal{V}' \quad k \in \mathcal{K}\backslash\{1\} \quad t \in \mathcal{T}. \tag{25}$$

Constraints (24) ensure that vehicle k cannot leave the depot if vehicle $k-1$ is not used. This symmetry breaking rule is then extended to the customer vertices by constraints (25) which state that if customer i is assigned to vehicle k in period t, then vehicle $k-1$ must serve a customer with an index smaller than i in the same period. These constraints were also used by Coelho and Laporte [10] and are inspired from those proposed by Fischetti et al. [18] for the capacitated vehicle routing problem and by Albareda-Sambola et al. [1] for a plant location problem.

3 Branch-and-cut algorithm

For very small instance sizes, the model presented in Section 2 can be fully described, and all variables and constraints can be explicitly generated. It can then be solved by feeding it directly into a powerful integer linear programming branch-and-bound solver. However, for instances of realistic size, the number of subtour elimination constraints (11) is too large to allow full enumeration of all constraints and these must be dynamically generated throughout the search process. The exact algorithm we present is a branch-and-cut scheme in which subtour eliminations constraints are only generated and added to the program whenever they are found to be violated. It works as follows. At a generic node of the search tree, a linear program containing the model with a subset of the subtour elimination constraints is solved, a search for violated inequalities is performed, and those found to be violated are added to the current program which is then reoptimized. This process is reiterated until a feasible or dominated solution is reached, or until there are no more cuts to be added. At this point, branching on a fractional variable occurs. We provide a sketch of the branch-and-cut-and-bound scheme in Algorithm 1.

Algorithm 1 Branch-and-cut algorithm

At the root node of the search tree, generate and insert all valid inequalities into the program.
$z^* \leftarrow \infty$.
Termination check:
if there are no more nodes to evaluate **then**
    Stop with the incumbent and optimal solution of cost z^*.
else
    Select one node from the branch-and-bound tree.
end if
Subproblem solution: solve the LP relaxation of the node and let z be its cost.
if the current solution is feasible **then**
    if $z \geq z^*$ **then**
        Go to termination check.
    else
        $z^* \leftarrow z$.
        Update the incumbent solution.
        Prune nodes with lower bound larger than or equal to z^*.
        Go to termination check.
    end if
end if
Cut generation:
if the solution of the current LP relaxation violates any cuts **then**
    Identify connected components as in Padberg and Rinaldi [29].
    Determine whether the component containing the supplier is weakly connected as in Gendreau et al. [19].
    Add violated subtour elimination constraints (11).
    Go to subproblem solution.
end if
Branching: branch on one of the fractional variables.
Go to the termination check.

4 Computational experiments

Our industrial partner operates a single plant producing four products: 0.33L, 0.5L, 1.5L, and 5L bottles and producing respectively 90, 80, 100 and 154 pallets per day. Once production is finished, products are stored at the plant's warehouse with a storage capacity of 1000 pallets. The holding cost of a pallet at the plant is about 2 MAD (Moroccan Dirhams) per day. Products are shipped to a set of customers that are divided in two groups. The first one contains 12 regional depots belonging to the company, with an average daily inventory holding cost of 2.7 MAD per pallet. The second group consists of 15 wholesalers supporting themselves their inventory holding costs. In periods of moderate consumption, the daily demand of all customers is about 300 pallets per day spread over a six days horizon. However, a large disparity in demand is observed among regions which led to a difference in storage capacity between customers that range from 100 to 1000 pallets. It should be noted that each pallet contains only one type of product. We consider a maximal number of 27 homogeneous vehicles with a capacity of 22 pallets, which largely exceeds the requirement of the company. All transportation costs were provided by suppliers of our industrial partner.

All computations were carried out on a grid of Intel XeonTM processors running at 2.66 GHz with up to 48 GB of RAM installed per node, with the Scientific Linux 6.1 operating system. A single thread was used. The algorithms just described were coded in C++ and we used IBM Concert Technology and CPLEX 12.5 as the MIP solver. In order to assess the efficiency of our approach, we have performed different computational experiments on real-life based instances.

Our industrial partner provided us with one real instance, called M1, which is associated with a typical period of moderate consumption. Based on the provided moderate instance M1, we generated two instances H1 and L1, for high and low demand levels, respectively, which was valid by our partner. For each customer i in M1, we have calculated its average demand μ_i related to the whole horizon. Thereafter, we considered, in consultation with

Table 1: Assessment of the performance of valid inequalities (22) and (23)

Instance	Without cuts (22) and (23)			With cuts			Best combined		
	UB	LB	gap(%)	UB	LB	gap (%)	UB	LB	gap(%)
L1	**728430**	590735	18.90	**728430**	**590879**	18.88	728430	590879	18.88
L2	**676261**	**597798**	11.60	**676261**	597791	11.60	676261	597798	11.60
L3	**715433**	601979	15.85	**715433**	**602005**	15.85	715433	602005	15.85
L4	**698316**	**601941**	13.80	698572	591062	15.38	698316	601941	13.80
L5	702544	**601953**	14.31	**693887**	600468	13.46	693887	601953	13.24
M1	599334	565521	5.64	**599136**	**565569**	5.60	599136	565569	5.60
M2	603470	572071	5.20	**603302**	**573208**	4.98	603302	573208	4.98
M3	**599760**	566943	5.47	599814	**567796**	5.33	599760	567796	5.32
M4	597089	565215	5.33	**597071**	**565675**	5.25	597071	565675	5.25
M5	603917	566470	6.20	**603670**	**567523**	5.98	603670	567523	5.98
H1	**633440**	586205	7.45	634380	**589658**	7.04	633440	589658	6.91
H2	635631	583743	8.16	**634928**	**584841**	7.88	634928	584841	7.88
H3	**638192**	**587660**	7.91	640501	586343	8.45	638192	587660	7.91
H4	633783	**577551**	8.87	**629890**	576349	8.50	629890	577551	8.30
H5	629853	**575461**	8.63	**624342**	572637	8.28	624342	575461	7.82
Average	646363	582749	9.55	645307	582120	9.50	645070	583301	9.29

Table 2: Decomposition of the total cost

Instance	Transportation cost	(in %)	Inventory cost	(in %)
L1	682366	93.68	46064	6.32
L2	630322	93.21	45939	6.79
L3	669364	93.56	46069	6.44
L4	652408	93.43	45908	6.57
L5	647978	93.38	45909	6.62
M1	557692	93.08	41444	6.92
M2	561442	93.06	41860	6.94
M3	558196	93.07	41564	6.93
M4	555622	93.06	41449	6.94
M5	562042	93.10	41628	6.90
H1	601664	94.98	31776	5.02
H2	602254	94.85	32674	5.15
H3	605670	94.90	32522	5.10
H4	597846	94.91	32044	5.09
H5	591766	94.78	32576	5.22
Average	605109	93.80	39962	6.20

our industrial partner, $2\mu_i$ and $\mu_i/2$ as average demands associated with H1 and L1 respectively. Finally, demands associated with L1 and H1 were obtained by performing a simple extrapolation. Three families, L, M, and H were randomly generated from L1, M1 and H1 respectively, with a total of five instances per group.

We then consulted the planner hired by our partner to understand and replicate his decision process. His algorithm consists of shipping a full-truckload (FTL) to any customer whose associated demand cannot be satisfied from the stock. To avoid situations in which the plant exceeds its own inventory, the planner must make FTL deliveries to depots while ensuring that their storage capacities are sufficient to enable their reception. It is noteworthy that very often these deliveries are not motivated by a real demand of the depots, but by the limited capacity of the plant. This algorithm was easily implemented and we were then able to evaluate how the company would operate on each of our 15 instances. Our implementation of this algorithm yielded the same solution for the real instance M1 as the one generated by the company.

We have fixed the maximal computational time to four hours. We present in Table 1 the associated results with and without the new cuts (22) and (23) in order to assess their efficiency. We note that in 73% of cases, these cuts have helped identify the best known solution, while not generating them would not have yielded the best known solution in more than 53% of the cases. By examining Table 1, the difference observed between the relatives gaps of the two approaches is in 87% of cases in favor of applying cuts (22) and (23).

Looking at Table 1, instances related to the low level of demand (family 1) are paradoxically more expensive than the other two (families M and H). This can be explained by the fact that the plant can make shipments to customers even if this is not justified by their demand. This implies that the stocks of some customers are saturated and therefore the inventory cost is naturally higher than those of M and H. We have decomposed the total cost into its transportation and inventory holding components to confirm this observation. These figures are shown in Table 2.

Table 3: Comparison with the company's solutions

Instance	Our solutions	Company's solutions	Difference in %
L1	**728430**	730451	0.28
L2	**676261**	730088	7.37
L3	**715433**	730695	2.09
L4	**698316**	730518	4.41
L5	**693887**	730131	4.96
M1	**599136**	709035	15.50
M2	**603302**	708283	14.82
M3	**599760**	712258	15.79
M4	**597071**	709285	15.82
M5	**603670**	709529	14.92
H1	**633440**	697724	9.21
H2	**634928**	694010	8.51
H3	**638192**	696918	8.43
H4	**629890**	685265	8.08
H5	**624342**	685400	8.91
Average	**645071**	710639	9.27

Finally, we compare our solutions with those currently employed by our industrial partner. This comparison is presented in Table 3. We note that for all instances, our approach performs better the one proposed by the company by 9.27% on average, and up to 15.37%. As we can observe from Table 3, the difference between the two approaches is significant for the M and H instances. Our decisions related to transportation are the basic reason for this improvement since we create multi-customer vehicle routes as opposed to simple back and forth FTL deliveries.

5 Conclusion

We have presented a complex real-life problem arising in the distribution of bottled water in which one seeks to minimize the sum of transportation and inventory costs. We have presented an integer formulation which was strengthened through the generation of new cuts developed to our specific context, and we have proposed a branch-and-cut algorithm for solving it. The results indicate that the cuts contribute to improving the solutions, and that our approach outperforms that of the company on all instances.

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