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The Fleet Size and Mix Location-Routing Problem with Time Windows: Formulations and a Heuristic Algorithm

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Abstract. This paper introduces the fleet size and mix location-routing problem with time windows (FSMLRPTW) which extends the location-routing problem by considering a heterogeneous fleet and time windows. The main objective is to minimize the sum of vehicle fixed cost, depot cost and routing cost. To this end, we have developed integer programming formulations and a family of valid inequalities to strengthen these. Furthermore, we present a powerful hybrid evolutionary search algorithm (HESA) to solve the problem. The HESA successfully combines several metaheuristics and offers a number of new advanced efficient procedures tailored to handle heterogeneous fleet dimensioning and location decisions. We evaluate the strengths of the proposed formulations with respect to their ability to find optimal solutions. We also investigate the performance of the HESA. Extensive computational experiments on new benchmark instances have shown that the HESA is highly effective on the FSMLRPTW.

Keywords: Location-routing, heterogeneous fleet, time windows, mixed integer programming, genetic algorithm.

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1. Introduction

The design of distribution networks is critical for most companies because it usually requires a major capital outlay. Two major types of decisions intervene in this process, namely determining the locations of depots, and designing vehicle routes supplying customers from these depots. In the classical Facility Location Problem (FLP) (Balinski, 1965), it is assumed that each customer is served individually through a direct shipment, which makes sense when customer demands are close to the vehicle capacity. However, there exist several situations where customer demands can be consolidated. In such contexts, the FLP and Vehicle Routing Problem (VRP) should be solved jointly. The idea of combining location and routing decisions was put forward more than fifty years ago (Von Boventer, 1961) and has given rise to a rich research known as the Location-Routing Problem (LRP) (see, e.g., the surveys of Laporte (1988); Min et al. (1998); Nagy and Salhi (2007); Prodhon and Prins (2014); Albareda Sambola (2015)). Applications of the LRP arise in areas as diverse as food and drink distribution, parcel delivery and telecommunication network design.

Many algorithms, mostly heuristics, have been developed for the LRP and its variations over the past fifty years, including some population metaheuristics (Prins et al., 2006b; Duhamel et al., 2010). Neighborhood-based metaheuristics have become popular in recent years (Prins et al., 2006a; Duhamel et al., 2011). An adaptive large neighborhood search (ALNS) algorithm was proposed by Hemmelmayr et al. (2012) for the two-echelon VRP and the LRP.

According to Hoff et al. (2010), heterogeneous fleets are more common in real-world distribution problems than homogeneous ones. For example, TNT Express, one of the world's largest transportation companies, which employs around 11,000 people and delivers in excess of 100 million items per year in the United Kingdom, has more than 70 express delivery depots strategically located throughout the United Kingdom and Ireland, three sorting hubs and three national contact centres (TNT, 2014). This company also operates a fleet of more than 3,500 heterogeneous vehicles in the United Kingdom alone (TNT, 2014). To our knowledge, a number of studies indirectly consider heterogeneous fleets in an LRP context but without taking time windows into account (Ambrosino et al., 2009; Wu et al., 2010). Berger et al. (2007) only consider a distance constraint. Therefore, combining heterogeneous fleets and time windows in the LRP is done here for the first time.

We believe there is methodological interest in solving the Fleet Size and Mix Location-Routing Problem with Time Windows (FSMLRPTW). This problem extends the Fleet Size and Mix VRP with Time Windows (FSMVRPTW) by considering depot locations decision. It combines two problems: the LRP and the FSMVRPTW. The Fleet Size and Mix VRP was introduced by Golden et al. (1984) which is the FSMLRPTW with a single depot and very wide time windows. This problem and a number of its variations are reviewed by Baldacci et al. (2008) and Baldacci and Mingozzi (2009).

The contributions of this paper are threefold. First, we introduce the FSMLPRTW as a new LRP variant. The second contribution is to develop integer programming formulations and to present a family of valid inequalities in order to strengthen the formulation. Our third contribution is to develop a hybrid evolutionary search algorithm (HESA) with the introduction of several algorithmic procedures specific to the FSMLRPTW. Namely, we introduce the location-heterogeneous adaptive large neighborhood search (L-HALNS) procedure equipped with a range of several new operators as the main Education procedure within the search. We also propose an Initialization procedure to create initial solutions, and a Partition procedure for offspring solutions. Finally, we develop a new diversification scheme through the Mutation procedure of solutions.

The remainder of this paper is structured as follows. Section 2 formally defines the problem and provides integer programming formulations together with valid inequalities. Section 3 presents a detailed description of the HESA. Computational experiments are provided in Section 4, and conclusions follow in Section 5.

2. The fleet size and mix location-routing problem with time windows

This section first defines the FSMLRPTW and introduces several integer programming formulations for the problem. Valid inequalities are then presented to strengthen the formulations.

2.1. Notation and problem definition

The FSMLRPTW is defined on a complete directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$, where $\mathcal{N} = \mathcal{N}_0 \cup \mathcal{N}_c$ is a set of nodes in which $\mathcal{N}_0$ and $\mathcal{N}_c$ represent the potential depot and customer nodes, respectively, and $\mathcal{A} = \{(i,j) : i \in \mathcal{N}, j \in \mathcal{N}\} \setminus \{(i,j) : i \in \mathcal{N}_0, j \in \mathcal{N}_0, i \neq j\}$ is the set of arcs. Each arc $(i,j) \in \mathcal{A}$ has a nonnegative distance c_{ij}. Here, the terms distance, travel time and travel cost are used in interchangeably. Each customer $i \in \mathcal{N}_c$ has a positive demand q_i. A storage capacity D^k and a fixed opening cost g^k are associated with each potential depot $k \in \mathcal{N}_0$. The index set of vehicle types is denoted by $\mathcal{H}$. Let Q^h and f^h denote the capacity and fixed dispatch cost of a vehicle of type $h \in \mathcal{H}$. Furthermore, s_j corresponds to the service time of node $j \in \mathcal{N}_c$, which must start within the time window $[a_i, b_i]$. If a vehicle arrives at customer $i \in \mathcal{N}_c$ before time a_i, it waits until a_i to starts servicing the customer.

In the FSMLRPTW, one considers a fleet of vehicles with various capacities and vehicle-related costs, as well as a set of potential depots with opening costs, a set of customers with known demands and time windows. The FSMLRPTW consists of opening a subset of depots, assigning customers to them and determining a set of vehicle routes such that all vehicles start and end their routes at their depot, each customer is visited exactly once by a vehicle within a prespecified time window, and the load of each vehicle does not exceed its capacity. The problem minimizes the total cost which is made up of three components: the depot operating cost, the fixed cost of the vehicles, and the total travel costs of the routes. It is assumed that these costs are scaled over the same time horizon.

2.2. Integer programming formulations

To formulate the FSMLRPTW, we define the following decision variables:

$$x_{ij}^h = \begin{cases} 1 & \text{if vehicle of type } h \text{ travels directly from node } i \text{ to node } j \ ((i,j) \in \mathcal{A}, h \in \mathcal{H}), \\ 0 & \text{otherwise;} \end{cases}$$

$$y_k = \begin{cases} 1 & \text{if depot } k \text{ is opened } (k \in \mathcal{N}_0), \\ 0 & \text{otherwise;} \end{cases}$$

$$z_{ik} = \begin{cases} 1 & \text{if customer } i \text{ is assigned to depot } k \ (i \in \mathcal{N}_c, k \in \mathcal{N}_0), \\ 0 & \text{otherwise;} \end{cases}$$

u_{ij}^h: the total load of vehicle of type h after visiting node i and directly going to node j $(i, j \in \mathcal{N}, h \in \mathcal{H})$.

t_j^h : the time at which service starts at node j with vehicle of type h $(j \in \mathcal{N}, h \in \mathcal{H})$.

The formulation is as follows:

$$\text{Minimize} \sum_{k \in \mathcal{N}_0} g_k y_k + \sum_{h \in \mathcal{H}} \sum_{k \in \mathcal{N}_0} \sum_{i \in \mathcal{N}_c} f^h x_{ki}^h + \sum_{h \in \mathcal{H}} \sum_{(i,j) \in \mathcal{A}} c_{ij} x_{ij}^h \tag{1}$$

subject to

$$\sum_{h \in \mathcal{H}} \sum_{j \in \mathcal{N}} x_{ij}^h = 1 \qquad i \in \mathcal{N}_c \tag{2}$$

$$\sum_{h \in \mathcal{H}} \sum_{j \in \mathcal{N}} x_{ji}^h = \sum_{h \in \mathcal{H}} \sum_{j \in \mathcal{N}} x_{ij}^h \qquad i \in \mathcal{N} \tag{3}$$

$$\sum_{h \in \mathcal{H}} \sum_{j \in \mathcal{N}} u_{ji}^h - \sum_{h \in \mathcal{H}} \sum_{j \in \mathcal{N}} u_{ij}^h = q_i \qquad i \in \mathcal{N}_c \tag{4}$$

$$u_{ij}^h \leq Q^h x_{ij}^h \qquad i \in \mathcal{N}_0, j \in \mathcal{N}, i \neq j, h \in \mathcal{H} \tag{5}$$

$$\sum_{h \in \mathcal{H}} \sum_{j \in \mathcal{N}_c} u_{kj}^h = \sum_{j \in \mathcal{N}_c} z_{jk} q_j \qquad k \in \mathcal{N}_0 \tag{6}$$

$$\sum_{h \in \mathcal{H}} \sum_{j \in \mathcal{N}_c} u_{jk}^h = 0 \qquad k \in \mathcal{N}_0 \tag{7}$$

$$u_{ij}^h \leq (Q^h - q_i) x_{ij}^h \qquad i \in \mathcal{N}_c, j \in \mathcal{N}, h \in \mathcal{H} \tag{8}$$

$$u_{ij}^h \geq q_j x_{ij}^h \qquad i \in \mathcal{N}, j \in \mathcal{N}_c, h \in \mathcal{H} \tag{9}$$

$$\sum_{i \in \mathcal{N}_c} q_i z_{ik} \leq D_k y_k \qquad k \in \mathcal{N}_0 \tag{10}$$

$$\sum_{k\in\mathcal{N}_0} z_{ik} = 1 \qquad i \in \mathcal{N}_c \tag{11}$$

$$x_{ij}^h + \sum_{q\in\mathcal{H},q\neq h}\sum_{l\in\mathcal{N},j\neq l} x_{jl}^q \leq 1 \qquad i \in \mathcal{N}, j \in \mathcal{N}_c, i \neq j, h \in \mathcal{H} \tag{12}$$

$$\sum_{h\in\mathcal{H}} x_{ik}^h \leq z_{ik} \qquad k \in \mathcal{N}_0, i \in \mathcal{N}_c \tag{13}$$

$$\sum_{h\in\mathcal{H}} x_{ki}^h \leq z_{ik} \qquad k \in \mathcal{N}_0, i \in \mathcal{N}_c \tag{14}$$

$$\sum_{h\in\mathcal{H}} x_{ij}^h + z_{ik} + \sum_{m\in\mathcal{N}_0,m\neq k} z_{jm} \leq 2 \qquad k \in \mathcal{N}_0, (i,j) \in \mathcal{N}_c, i \neq j \tag{15}$$

$$t_i^h - t_j^h + s_i + c_{ij} \leq M(1 - x_{ij}^h) \qquad i \in \mathcal{N}, j \in \mathcal{N}_c, i \neq j, h \in \mathcal{H} \tag{16}$$

$$a_i \leq t_i^h \leq b_i \qquad i \in \mathcal{N}, h \in \mathcal{H} \tag{17}$$

$$x_{ij}^h \in \{0,1\} \qquad (i,j) \in \mathcal{N}, h \in \mathcal{H} \tag{18}$$

$$z_{ik} \in \{0,1\} \qquad k \in \mathcal{N}_0, i \in \mathcal{N}_c \tag{19}$$

$$y_k \in \{0,1\} \qquad k \in \mathcal{N}_0 \tag{20}$$

$$u_{ij}^h \geq 0 \qquad (i,j) \in \mathcal{N}, h \in \mathcal{H} \tag{21}$$

$$t_i^h \geq 0 \qquad i \in \mathcal{N}_c, h \in \mathcal{H}. \tag{22}$$

The objective function (1) minimizes the total cost including depot fixed cost, vehicle fixed cost and variable travel cost. Constraints (2) and (3) are degree constraints; in particular constraints (2) guarantee that each customer must be visited exactly once, and constraints (3) ensure that entering and leaving arcs to each node are equal. Constraints (4) imply that the demand of each customer is satisfied. Constraints (5) mean that the total load on any arc cannot exceed the capacity of the vehicle traversing it. Constraints (6) ensure that the total load of each vehicle is equal to the total demand of customers assigned to it. Constraints (7) state that the load on a vehicle returning to each depot must be equal to zero. Contraints (8) and (9) are bounding constraints for load variables. Constraints (10) guarantee that total load of a depot cannot exceed its capacity. Constraints (11) and (12) ensure that each customer is assigned to only one depot and to only one vehicle, respectively. Constraints (13)–(15) forbid illegal routes, i.e., routes that do not start and end at the same depot. The validity of the constraints (13)–(15) was proven by Karaoglan et al. (2011). Constraints (16) and (17), where M is a large number, enforce the time window restrictions. Constraints (18)–(22) define the domains of the decision variables.

Formulation (1)–(22) is valid for the FSMLRPTW and is denoted by E_1. Other valid formulations can be derived from E_1. Before defining these formulations, we provide several variations of E_1 by aggregating some of the variables or disaggregating some of the constraints. The reason is to either reduce the size of the formulation through aggregation of variables or to tighten the linear relaxation bound by disaggregating some of the constraints. First, the aggregation of the u_{ij}^h variable, denoted by f_{ij}, is obtained as follows:

$$\sum_{j \in \mathcal{N}} f_{ji} - \sum_{j \in \mathcal{N}} f_{ij} = q_i \qquad i \in \mathcal{N}_c \tag{23}$$

$$f_{ij} \leq \sum_{h \in \mathcal{H}} Q^h x_{ij}^h \qquad i \in \mathcal{N}_0, j \in \mathcal{N}, i \neq j \tag{24}$$

$$\sum_{j \in \mathcal{N}_c} f_{kj} = \sum_{j \in \mathcal{N}_c} z_{jk} q_j \qquad k \in \mathcal{N}_0 \tag{25}$$

$$\sum_{j \in \mathcal{N}_c} f_{jk} = 0 \qquad k \in \mathcal{N}_0 \tag{26}$$

$$f_{ij} \leq \sum_{h \in \mathcal{H}} (Q^h - q_i) x_{ij}^h \qquad i \in \mathcal{N}_c, j \in \mathcal{N} \tag{27}$$

$$f_{ij} \geq q_j \sum_{h \in \mathcal{H}} x_{ij}^h \qquad i \in \mathcal{N}, j \in \mathcal{N}_c. \tag{28}$$

Proposition 1. Let (f, x) be a solution satisfying constraints (23)–(28) and let (u, x) be a solution satisfying constraints (2), (4)–(9), where f, u and x are the respective vectors for variables f_{ij}, u_{ij}^h and x_{ij}^h. Then, for each feasible (u, x), there exists a feasible (f, x) and vice versa.

Proof. Due to (2), there exists an $h^* \in \mathcal{H}$ for which $x_{ij}^{h^*} = 1$ and due to (8), there exists a $u_{ij}^{h^*} \geq 0$, implying $x_{ij}^h = u_{ij}^h = 0$ for all other $h \in \mathcal{H} \setminus \{h^*\}$. We now define f_{ij} as the total load of the vehicle while traversing arc $(i, j) \in \mathcal{A}$, which, using the arguments just stated, is linked to the original variables in the following way:

$$f_{ij} = \sum_{h \in \mathcal{H}} u_{ij}^h \qquad i, j \in \mathcal{N}. \tag{29}$$

We can now derive constraints (23)–(28) from (4)–(9) by using definition (29). Constraints (23)–(28) are satisfied, since they are the same as (4)–(9), respectively. □

Second, the aggregation of t_i^h variable is

$$t_i - t_j + s_i + c_{ij} \leq M(1 - \sum_{h \in \mathcal{H}} x_{ij}^h) \qquad i \in \mathcal{N}, j \in \mathcal{N}_c, i \neq j \tag{30}$$

$$a_i \leq t_i \leq b_i \qquad i \in \mathcal{N}. \tag{31}$$

Proposition 2. Constraints (30)–(31) and (16)–(17) are equivalent.

Proof. Due to (16) there exists $h^* \in \mathcal{H}$ for which $x_{ij}^{h^*} = 1$, i.e., $x_{ij}^h = 0$ for all $h \neq h^*$, $h \in \mathcal{H}$. Define

t_j : the time at which service starts at node j ($j \in \mathcal{N}$).

An equivalent definition is

$$t_j = \sum_{h \in \mathcal{H}} t_j^h \qquad j \in \mathcal{N}. \tag{32}$$

We can now derive constraints (30)–(31) from (16)–(17) by using definition (32). Constraints (30)–(31) are satisfied, since they are the same as (16)–(17), respectively. □

Third, the disaggregation of the balance constraints (3)

$$\sum_{j \in \mathcal{N}} x_{ji}^h = \sum_{j \in \mathcal{N}} x_{ij}^h \qquad h \in \mathcal{H}, i \in \mathcal{N}. \tag{33}$$

Proposition 3. Constraints (33) are equivalent to constraints (3) and (12).

Proof. Given that i, j, k are on the same route, assume $h^1, h^2 \in \mathcal{H}$, $x_{ij}^{h^1} = 1$ and $x_{jk}^{h^2} = 1$, which is valid for (3). But constraints (12) along with (3) only allows the use of the same vehicle on a route, i.e, $x_{ij}^{h^1} = 1$ and $x_{jk}^{h^1} = 1$. We can now derive constraints (33) from (3). Using (33) also makes (12) redundant. Constraints (33) ensure that $x_{ij}^{h^1} = 1$ and $x_{jk}^{h^1} = 1$ without using (12). Constraints (33) are satisfied, since they are the same as (3), respectively. □

We now define the following four variations of E_1 as follows:

1) A formulation E_2 obtained by using Proposition 1 is to minimize (1), subject to (2), (33), and (4)–(22).

2) A formulation E_3 obtained by removing constraints (6) is to minimize (1), subject to (2)–(5) and (7)–(22).

3) A formulation E_4 obtained by using Proposition 1 and Proposition 2 is to minimize (1), subject to (2)–(3), (23)–(28), (10)–(15), (30)–(31) and (18)–(22).

4) A formulation E_5 obtained by using Proposition 1, Proposition 2 and Proposition 3 is to minimize (1), subject to constraints (2), (33), (23)–(28), (10)–(15), (30)–(31) and (18)–(22).

2.3. Valid inequalities

We make use of four polynomial size valid inequalities. These were used by several authors for variants of LRPs. The first inequalities are special case of classical subtour elimination constraints for the Traveling Salesman Problem (Dantzig et al., 1954) for two nodes as follows:

$$x_{ij}^h + x_{ji}^h \leq 1 \qquad i, j \in \mathcal{N}_c, h \in \mathcal{H}. \tag{34}$$

Constraints (34) break subtours involving two customers only. The second valid inequality, which was used by Labbé et al. (2004) for plant-cycle location problem is as follows:

$$z_{ik} \leq y_k \qquad i \in \mathcal{N}_c, k \in \mathcal{N}_0. \tag{35}$$

Constraints (35) impose that customer i cannot be assigned to depot k if the latter is not open. The next valid inequality, described in Achuthan (2003), is adapted for the

FSMLRPTW and is presented is as follows:

$$\sum_{h\in\mathcal{H}}\sum_{k\in\mathcal{N}_0}\sum_{i\in\mathcal{N}_c} x_{ki}^h \geq r_\alpha(\mathcal{N}_c). \quad (36)$$

Constraints (36) provide a lower bound on the number of routes originating from depots where $r_\alpha(\mathcal{N}_c) = \lceil \sum_{i\in\mathcal{N}_c} q_i / \max_{h\in\mathcal{H}} Q^h \rceil$ and $\lceil \bullet \rceil$ is the smallest integer larger than $\bullet$. The fourth and final valid inequality is

$$\sum_{k\in\mathcal{N}_0} y_k \geq y_{min}. \quad (37)$$

Constraints (37) bounds the number of opened depots from below, where y_{min} is taken as the minimum number of k opened depots satisfying $\sum_{k\in\mathcal{N}_0} D^k \geq \sum_{i\in\mathcal{N}_c} q_i$ after the depots have been sorted in non-decreasing order of their capacity.

3. Description of the Hybrid Evolutionary Search Algorithm

This section describes the proposed Hybrid Evolutionary Search Algorithm, called HESA, that has been developed to solve the FSMLRPTW. This algorithm builds on several powerful evolutionary based metaheuristic algorithms (see Koç et al., 2014a,b; Vidal et al., 2014). In this paper, we additionaly introduce the location-heterogeneous adaptive large neighborhood search (L-HALNS) procedure which is used as a main Education component in the HESA. Furthermore, we develope an Initialization to create initial solutions and a Partition procedure to split offspring solutions into routes. Finally, we propose a new Mutation procedure to diversify the solutions.

The general structure of the HESA is sketched in Algorithm 1. The initial population is generated by using the Initialization procedure (line 1). Two parents are selected (line 3) through a binary tournament process, where a crossover operation creates a new offspring C (line 4), which then undergoes the Partition procedure (line 5). The Education procedure uses the L-HALNS operators to educate offspring C and inserts it back into the population (line 6). The probabilities associated with the L-HALNS operators used in the

Education procedure are updated by an Adaptive Weight Adjustment Procedure (AWAP) (line 7). The Intensification procedure which is based on the L-HALNS is run on elite solutions (line 8). As new offsprings are added, the population size n_a which is limited by $n_p + n_o$, changes over the iterations. The constant n_p denotes the size of the population initialized at the beginning of the algorithm and the constant n_o is the maximum allowable number of offspring that can be inserted into the population. If the population size n_a reaches $n_p + n_o$ at any iteration, then a survivor selection mechanism is applied (line 9). The Mutation procedure is applied to a randomly selected individual from the population with probability p_m (line 10). When the number ϖ of iterations without improvement in the incumbent solution is reached, the HESA terminates (line 11). For further implementation details on the ALNS based education, parent selection, crossover, AWAP, intensification and survivor selection sections the reader is referred to Koç et al. (2014a).

Algorithm 1 The general framework of the HESA

1: Initialization: initialize a population with size n_p
2: **while** number of iterations without improvement $< \varpi$ **do**
3:     *Parent selection*: select parent solutions P_1 and P_2
4:     *Crossover*: generate offspring C from P_1 and P_2
5:     Partition: partition offspring C into routes
6:     Education: educate C with L-HALNS and insert into population
7:     AWAP: update probabilities of the L-HALNS operators
8:     Intensification: intensify on elite solutions with L-HALNS
9:     *Survivor selection*: if the population size n_a reaches $n_p + n_o$, then select survivors
10:     Mutation: diversify a random solution with probability p_m
11: **end while**
12: Return best feasible solution

In what follows, we detail the algorithmic features specifically designed for the FSML-RPTW. Section 3.1 presents the Initialization procedure of the population. The Partition procedure is described in Section 3.2. Section 3.3 presents the offspring Education procedure. Finally, the Mutation procedure is desribed in Section 3.4.

3.1. Initialization

We use a three-phase Initialization procedure to generate an initial population. The first and second phases generate the initial solution while the third phase creates other solutions. In the first phase, customers are assigned to the depots. Initially, the closest depot according to its distance is calculated for each customer, and the customers are listed in non-increasing order of distance. Each customer is then assigned to its closest depot starting from the top of the list, without violating the depot capacities. These steps are applied repeatedly to the residual customers who could not be assigned to their closest depots due to depot capacity constraints. In this case they are assigned to their closest feasible depot. In the second phase, routes are constructed for each depot by applying the Clarke and Wright (1964) algorithm and by selecting the largest vehicle type for each route. In the third phase, new individuals are created by applying the L-HALNS operators to the initial solution until the initial population size reaches n_p. A removal operator is randomly selected from a list of diversification based removal operators and a greedy insertion with a noise function operator is used as an insertion operator (see Section 3.3). Both of these operators are used in order to diversify the initial population. The number of nodes removed is randomly chosen from the initialization interval $[b_l^i, b_u^i]$ in the destroy phase. The interval is defined by a lower and an upper bound calculated as a percentage of the total number of nodes in a given instance.

3.2. Partition

We introduce a Partition procedure to be used in the L-HALNS (see Section 3.3). The objective of this procedure is to split the solution into routes after the parent selection and the crossover phases during the algorithm. Before applying this procedure, each solution is represented as a permutation of customers and can therefore be viewed as a list L_r to be used for removals. This procedure includes two phases. In the first phase, all customers of the newly generated solution are put into L_r. In the second phase, we use an intensification-based insertion operator (greedy insertion operator) to insert the customers of L_r to their best possible position. The Partition procedure yields a feasible solution for the FSMLRPTW which is inserted into the population.

3.3. Education

At every iteration, the Education procedure is applied to the newly generated offspring to improve its quality. The ALNS used as a way of educating the solutions, as in Koç et al. (2014a), is basically an improvement heuristic. It consists of two procedures: removal, followed by insertion. In the removal procedure, n' nodes are iteratively removed by the intensification based removal operators and placed in the removal list L_r, where n' is in the interval $[b_l^e, b_u^e]$ for the destroy operators. In the insertion procedure, the nodes of L_r are iteratively inserted into a least-cost position in the incomplete solution.

Here, we introduce the Location-HALNS (L-HALNS) which integrates fleet sizing and the location decisions within the removal and insertion operators. This procedure differs from the Heterogeneous ALNS (HALNS) developed by Koç et al. (2014b) to educate the solutions in the context of a heterogeneous fleet because the latter did not account for the location decisions. When a node is removed, we check whether the resulting route can be served by a smaller vehicle, and we also check whether the related depot has any customer already assigned to it. We then update the solution accordingly. If inserting a node requires additional vehicle capacity or requires opening a new depot, we then consider the option of using larger vehicles or the option of opening the least cost depot not yet open. More formally, for each node $i \in \mathcal{N}_0 \backslash L_r$, let f^h be the vehicle fixed cost and let g^k be the depot cost associated with this node. Let $\Delta(i)$ be the distance saving obtained as a result of using a removal operator on node i. Let g_r^{k*} be the depot fixed cost and let f_r^{h*} be the vehicle fixed cost after the removal of node i, i.e., g_r^{k*} is modified only if node i is the only node of depot k and f_r^{h*} is modified only if the route containing node i can be served by a smaller vehicle when removing node i. The savings in depot fixed cost and vehicle fixed cost can be expressed as $g^k - g_r^{k*}$ and $f^h - f_r^{h*}$, respectively. Thus, for each removal operator, the total savings resulting from removing node $i \in \mathcal{N}_0 \backslash L_r$, denoted $RC(i)$, is calculated as follows:

$$RC(i) = \Delta(i) + (g^k - g_r^{k*}) + (f^h - f_r^{h*}). \tag{38}$$

In the destroyed solution, the insertion cost of node $j \in L_r$ after node i is defined as $\Omega(i,j)$ on a given node $i \in \mathcal{N}_0 \backslash L_r$. Let g_a^{k*} be the depot fixed cost and let f_a^{h*} be the vehicle fixed cost after the insertion of node i, i.e., g_r^{k*} is modified only if node i requires to open a new depot, or f_r^{h*} is modified only if the route containing node i necessitates the use of a larger capacity vehicle after inserting node i. The cost differences in depot fixed cost and vehicle fixed cost can be expressed as $g_r^{k*} - g^k$ and $f_a^{h*} - f^h$, respectively. Thus, the total insertion cost of node $i \in \mathcal{N}_0 \backslash L_r$ is $IC(i)$, for each insertion operator is

$$IC(i) = \Omega(i,j) + (g_r^{k*} - g^k) + (f_a^{h*} - f^h). \tag{39}$$

Figure 1 provides an example of the removal and insertion phases of the L-HALNS procedure.

3.3.1. Diversification based removal operators

The first and second diversification based removal operators were applied by Hemmelmayr et al. (2012), and the third one was used by Ropke and Pisinger (2006a). We introduce the fourth one as a new operator specific to the FSMLRPTW.

1. Depot closing removal (DR): This operator randomly selects an open depot and closes it. The operator removes all customers of this depot. The DR operator also randomly selects a closed depot and opens it.

2. Depot opening removal (DOR): The DOR operator randomly opens a closed depot. The n' customers removed from the solution are those which are closest to this depot according to travel cost.

3. Random removal (RR): This operator randomly selects n' customers and puts them into the removal list.

4. Depot distance removal (DDR): The DSR operator is based on the DR operator but differs from it by the criterion employed to open a closed depot. To open a depot, this operator selects a closest depot with respect to a removed one.

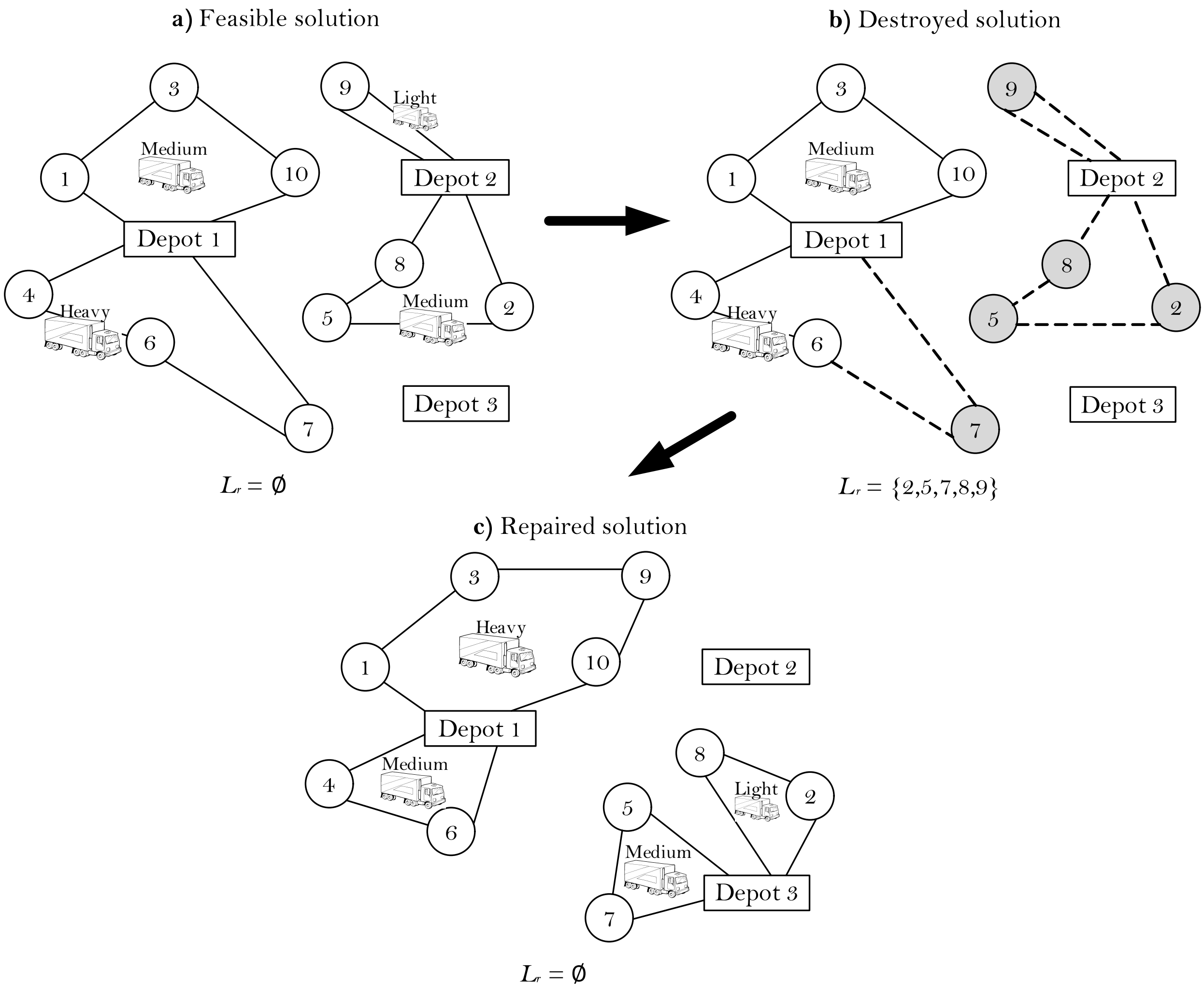

Figure 1: Illustration of the L-HALNS procedure

3.3.2. Intensification based removal operators

We now present the eight intensification based removal operators used in our algorithm. The first seven were used in several papers (Ropke and Pisinger, 2006a,b; Demir et al., 2012; Koç et al., 2014a), whereas the last one is new.

1. Neighborhood removal (NR): The general idea behind the NR operator is to remove n' customers from routes that are extreme with respect to the average distance of a route.

2. Worst distance removal (WDR): The WDR operator iteratively removes n' high cost customers, where the cost is based on the distance.

3. Worst time removal (WTR): This operator is a variant of the WDR operator. For each node, costs are calculated, depending on the deviation between the arrival time and the beginning of the time window. The WTR operator iteratively removes n' customer from the solution which has the largest deviation.

4. Shaw removal (SR): The SR operator aims to remove a set of n' similar customers. The similarity between two customers is based on distance, time, proximity and demands.

5. Proximity-based removal (PBR): This operator is a special case of SR which is solely based on distance.

6. Time-based removal (TBR): The TBR operator is another variant of SR. The selection criterion of a set of customers is solely based on time.

7. Demand based-removal (DBR): The DBR operator is yet another variant of SR which is solely based on demand.

8. Depot closing removal (DCR): The aim of the DCR operator is to calculate the utilization efficiency of each depot $\Phi(k)$ $(k \in \mathcal{N}_0)$ and remove from solution the one with the least $\Phi(k)$ value. $\Phi(k)$ is expressed as the ratio of the total demand of depot k over the capacity of depot k:

$$\Phi(k) = \frac{\sum_{i \in \mathcal{N}_c} q_i z_{ik}}{D^k}. \tag{40}$$

3.3.3. Insertion operators

Two insertion operators (Ropke and Pisinger, 2006a,b; Demir et al., 2012; Koç et al., 2014a) are used in the repair phase of the EDUCATION procedure.

1. Greedy insertion operator (GIO): This operator finds the best possible insertion position for all nodes in L_r while the cost calculation is based on distance. The insertion process is iteratively applied to all nodes in L_r.

2. Greedy insertion with noise function operator (GINFO): The GINFO operator is a variant of the GIO operator that extends it by allowing a degree of freedom in selecting the best possible insertion position for a node.

3.4. MUTATION

We introduce a MUTATION procedure to increase the population diversity over the iterations (e.g., Nagata et al., 2010). This procedure is applied with probability p_m. An individual C different from the elite individuals is randomly selected. A randomly selected diversification based removal operator and the GINFO operator are then used in order to diversify the selected individual C. These two operators are applied to change the positions of a specific number of nodes, which are chosen from the interval $[b_l^m, b_u^m]$ of removable nodes.

4. Computational experiments

In this section, we present the results of computational experiments performed in order to assess the performance of the formulations and the HESA. All experiments were conducted on a server with one gigabyte RAM and Intel Xeon 2.6 GHz processor. We used CPLEX 12.5 with its default settings as the optimizer to solve the integer programming formulations. The HESA was implemented in C++.

Section 4.1 describes the benchmark instances and the experimental settings. The aim of the computational experiments is threefold: (i) to analyze the effect of the metaheuristic components (Section 4.2), (ii) to evaluate the formulation in terms of solving the FSMLRPTW to optimality on small-size instances (Section 4.3), and (iii) to compare and test the integer programming formulation which is integrated with valid inequalities and the HESA on small-medium-and large-size instances (Section 4.4).

4.1. Benchmark instances

Benchmark data sets for the FSMLRPTW were generated by considering the data set described by Liu and Shen (1999) for the Fleet Size and Mix VRP with Time Windows (FSMVRPTW) and derived from the classical Solomon (1987) VRP with Time Windows instances with 100 nodes. These sets include 56 instances, split into a random data set R, a clustered data set C and a semi-clustered data set RC. The sets R1, C1 and RC1 have a short scheduling horizon and small vehicle capacities, in contrast to the sets R2, C2 and RC2 which have a longer scheduling horizon and larger vehicle capacities. Liu and Shen (1999) also introduced three cost structures, namely A, B and C, and several vehicle types with different capacities and fixed vehicle costs for each of the 56 instances. In our data sets, we have used the cost structure A and generated small-size (10-15-20-25-30-customer) as well as medium and large-size (50-75-100-customer) instances. We have selected the first customers of each data sets to generate the instances, i.e., the first 10 customers, 15 customers, and so on, of each Liu and Shen instance.

We followed the similar procedure as Prodhon (2006) and Karaoglan et al. (2012) to generate our depot characteristics. To this end, new depots features were added to each instance of Liu and Shen, as shown in Appendix A.1. The coordinates, depot capacities and fixed costs of these depots were randomly generated. However, these data ensure the opening of at least two depots for each instance. In the original Solomon instances there is only one depot; we have used this depot's time windows for other depots as well, i.e., in each data set, time windows are same for all depots.

All algorithmic parametric values were set as in Koç et al. (2014a), where an extensive meta-calibration procedure was used to generate effective parameter values for the FSMVRPTW.

4.2. Sensitiviy analysis of method components

This section compares four versions of the HESA, the details of which can be found in Table 1. We present four sets of experiments on randomly selected 100-customer instances; C101, C203, R101, R211, RC105 and RC207.

Table 1: Sensitivity analysis experiment setup

Version	EDUCATION	INTENSIFICATION	MUTATION
(1)	No	No	No
(2)	Yes	No	No
(3)	Yes	Yes	No
HESA	Yes	Yes	Yes

Table 2 presents the best results of ten runs for each of four versions. The columns display the instance type, the total cost, percentage deterioration in solution quality (Dev) of the three versions with respect to the HESA, and the computation time in seconds (Time). The row named Avg shows the average results. These results clearly indicate the benefit of including the EDUCATION, INTENSIFICATION and MUTATION procedures within the HESA. The HESA is consistently superior to all other versions on all instances. Version (1) performs worse than all other three versions. The superiority of version (3) over version (2) confirms the usefulness of the INTENSIFICATION procedure in the algorithm. The computation times for all versions are of similar magnitude.

Table 2: Sensitivity analysis of the HESA components

Instance	Version (1)			Version (2)			Version (3)			HESA	
	Total cost	Dev	Time	Total cost	Dev	Time	Total cost	Dev	Time	Total cost	Time
C101	89091.74	4.37	274.19	87091.74	2.17	279.34	86590.81	1.61	286.38	85199.09	297.32
C203	199501.19	4.33	286.24	196801.71	3.02	294.13	192841.36	1.03	301.14	190864.00	308.09
R101	43941.23	4.91	271.39	42841.17	2.47	276.41	42640.26	2.01	282.51	41782.20	292.25
R211	185112.41	5.44	235.29	180119.73	2.81	240.13	178152.37	1.74	244.09	175051.00	247.03
RC105	41640.21	3.96	249.13	41340.21	3.27	271.31	40840.17	2.08	276.91	39990.10	281.31
RC207	183071.91	4.26	280.17	179270.13	2.23	290.43	178274.36	1.68	294.37	175280.00	300.42
Avg		4.54	266.07		2.66	275.29		1.69	280.90		287.74

4.3. Performance of the formulations

The formulations E_1, E_2, E_3, E_4 and E_5 are examined in terms of their ability to solve the FSMLRPTW to optimality on small-size (20-, 25-, and 30-customer) instances. To analyze the computational results, we used the following performance measures: the deviation (Dev) and computation time in seconds (Time) averaged over all instances for each instance set (over a total of 840 experiments), and the number of optimal solutions ($\#Op$) obtained within one hour of computation time. Dev is the percentage deviation between the Upper Bound (UB) and the best-known Lower Bound (LB), i.e., 100 (UB − LB)/ UB. The upper bound is the optimal or best known solution obtained by solving the formulations.

Table 3 presents comparative average results over the five formulations. The first column displays the instance sets, and the following two columns show the number of customers $|\mathcal{N}_c|$ and the number of depots $|\mathcal{N}_0|$, respectively.

The results shown in Table 3 indicate that formulation E_4 performs better than the other models in terms of reaching optimal solutions within one hour of computation time. Formulation E_4 yields 27 optimal solutions out of 56 instances for $|\mathcal{N}_c| = 20$, $|\mathcal{N}_0| = 5$, 13 out of 56 instances for $|\mathcal{N}_c| = 25$, $|\mathcal{N}_0| = 5$, and nine out of 56 instances for $|\mathcal{N}_c| = 20$, $|\mathcal{N}_0| = 5$ within the given time limit of one hour. In terms of computation time, E_4 provides on average lower computation time than the other formulations.

Table 3: Average results of the formulations

Instance set	$\|\mathcal{N}_c\|$	$\|\mathcal{N}_0\|$	E_1			E_2			E_3			E_4			E_5		
			Dev	Time	$\#Op$	Dev	Time	$\#Op$	Dev	Time	$\#Op$	Dev	Time	$\#Op$	Dev	Time	$\#Op$
C1 (9)	20	5	0.07	2245.17	4	0.07	1978.42	5	0.08	2417.07	5	0.08	2505.74	4	0.08	2467.02	3
C2 (8)	20	5	0.07	2245.17	7	0.00	844.86	7	0.42	2782.58	2	0.00	573.49	8	0.00	359.59	8
R1 (12)	20	5	0.29	2947.26	3	0.28	2936.22	3	0.32	3019.03	2	0.21	3066.77	3	0.20	3019.31	2
R2 (22)	20	5	0.01	2059.15	7	0.01	1975.49	7	0.27	3291.85	1	0.00	751.98	11	0.00	894.84	10
RC1 (8)	20	5	0.17	3228.86	1	0.18	3251.18	1	0.25	3208.96	1	0.17	3222.55	1	0.17	3192.03	1
RC2 (8)	20	5	0.07	3600.00	0	0.06	3600.00	0	0.07	3600.00	0	0.07	3600.00	0	0.06	3600.00	0
Avg (Total)			0.11	2720.94	(22)	0.10	2431.03	(23)	0.23	3053.25	(11)	0.09	2286.76	(27)	0.09	2255.46	(24)
C1 (9)	25	5	0.12	2669.77	3	0.11	2480.23	3	0.16	3600.00	0	0.14	3207.57	1	0.14	3432.45	1
C2 (8)	25	5	0.02	1607.91	5	0.05	1932.03	5	0.31	3151.28	1	0.00	1215.17	6	0.00	1120.62	6
R1 (12)	25	5	0.44	3302.73	1	0.43	3302.31	1	0.32	3151.80	2	0.35	3300.77	1	0.37	3301.73	1
R2 (22)	25	5	0.17	3325.51	1	0.11	3057.42	2	0.24	3308.48	1	0.03	2597.39	4	0.06	2957.59	2
RC1 (8)	25	5	0.22	3600.00	0	0.22	3289.92	1	0.23	3305.74	1	0.20	3204.17	1	0.21	3367.11	1
RC2 (8)	25	5	0.10	3600.00	0	0.07	3600.00	0	0.10	3600.00	0	0.09	3600.00	0	0.07	3600.00	0
Avg (Total)			0.18	3017.65	(10)	0.16	2943.65	(12)	0.23	3352.88	(5)	0.14	2854.18	(13)	0.14	2963.25	(11)
C1 (9)	30	5	0.32	3600.00	0	0.26	3264.60	1	0.32	3600.00	0	0.32	3600.00	0	0.33	3600.00	0
C2 (8)	30	5	0.02	2276.50	5	0.00	1478.03	6	0.25	2975.67	2	0.10	1545.67	6	0.12	2083.98	5
R1 (12)	30	5	0.48	3305.80	1	0.50	3317.96	1	0.72	3310.17	1	0.49	3303.81	1	0.47	3303.81	1
R2 (22)	30	5	0.21	3600.00	0	0.14	3434.65	1	0.33	3600.00	0	0.08	3159.86	2	0.07	3087.20	2
RC1 (8)	30	5	0.32	3600.00	0	0.28	3600.00	0	0.34	3600.00	0	0.38	3600.00	0	0.32	3600.00	0
RC2 (8)	30	5	1.26	3600.00	0	0.75	3600.00	0	1.01	3600.00	0	0.8	3600.00	0	0.67	3600.00	0
Avg (Total)			0.43	3330.38	(6)	0.32	3115.87	(9)	0.49	3447.64	(3)	0.36	3134.89	(9)	0.33	3212.50	(8)

4.4. Comparative Analysis

We now present a comparative analysis of the results of the HESA and of the formulation E_4 integrated with valid inequalities, denoted by E_4^v. Each instance was solved once with the HESA, and once with E_4^v. For E_4^v, a common time limit of three hours was imposed on the solution time for all instances. For the HESA, ten separate runs are performed for each instance, the best one of which is reported.

Tables 4 and 5 summarize the average results of the HESA compared with E_4^v. For detailed results, the reader is referred to Appendix A. In Tables 4 and 5, the columns display the LP relaxation value of E_4^v and percent deviations (Dev) of the solution values found by E_4^v with respect to the HESA. In the Time column, "*" denotes that the instance was not solved to optimality within three hours.

Table 4 shows that the HESA finds almost the same solutions as those of E_4^v but in a substantially smaller amount of time on the small-size FSMLPRTW instances. The average time required by E_4^v to solve 10-, 15-, 20-, 25- and 30-customer instances to optimality are 31.76, 3229.94, 5925.18, 8109.69 and 9410.01 seconds, respectively. For HESA, the respective statistics are 3.18, 3.95, 6.53, 8.95 and 13.04 seconds.

As can be observed from Table 5, the results clearly indicate that the HESA runs quickly, also for medium and large-size instances. In particular, the algorithm requires 90.51, 167.90 and 299.02 seconds of average computation time to solve 50, 75 and 100-customer instances, respectively. The HESA is able to produce considerably better results than E_4^v does in three hours. The improvements in solution values can be as high as 11.16%, with an average of 1.94% for the 50-customer instances. Similarly, the average improvement is 14.14% for the 75-customer instances, the highest value sitting at 60.21%. The results are even more striking for the 100-customer instances where the average total cost reduction obtained was 88.65% compared to E_4^v. In case of 50-, 75- and 100-customer instances E_4^v was not able to find optimal solutions for 166 instances out of 168 within three hours.

Table 4: Average results on small-size instances

Instance set	$\lvert\mathcal{N}_c\rvert$	$\lvert\mathcal{N}_0\rvert$	E_4^v			HESA		
			LP relaxation	Total cost	Time	Total cost	Time	Dev
C1 (9)	10	3	82504.85	82674.64	0.38	82674.64	3.57	0.00
C2 (8)	10	3	191118.19	192201.84	2.07	192201.84	3.31	0.00
R1 (12)	10	3	39382.04	39466.34	128.64	39466.34	3.01	0.00
R2 (11)	10	3	175618.55	176157.95	0.96	176157.95	3.18	0.00
RC1 (8)	10	3	35614.85	35716.90	10.77	35716.90	3.19	0.00
RC2 (8)	10	3	173434.98	173622.24	14.76	173622.24	2.82	0.00
Avg					31.76		3.18	0.00
C1 (9)	15	4	81889.59	82059.92	5.51	82059.92	4.51	0.00
C2 (8)	15	4	186126.81	187193.63	9.52	187193.63	3.72	0.00
R1 (12)	15	4	39590.64	39702.34	6328.34	39702.41	3.86	0.00
R2 (11)	15	4	174662.67	175190.12	98.72	175190.12	3.88	0.00
RC1 (8)	15	4	35843.17	35961.05	3469.07	35961.05	3.60	0.00
RC2 (8)	15	4	173626.06	173826.21	9496.51	173826.21	4.15	0.00
Avg					3229.94		3.95	0.00
C1 (9)	20	5	82236.26	82435.86	5263.90	82450.45	6.25	0.01
C2 (8)	20	5	186141.23	187224.06	601.82	187224.06	6.46	0.00
R1 (12)	20	5	37782.90	38139.76	8220.94	38139.37	6.18	0.00
R2 (11)	20	5	174699.40	175239.55	1682.87	175238.39	6.34	−0.01
RC1 (8)	20	5	36075.96	36242.45	9507.20	36246.40	7.04	0.01
RC2 (8)	20	5	173864.43	174079.45	10800.00*	174078.67	7.20	−0.01
Avg					5925.18		6.53	0.00
C1 (9)	25	5	82556.95	82767.35	8404.21	82782.57	9.33	0.02
C2 (8)	25	5	186314.82	187234.75	2919.09	187234.75	8.45	0.00
R1 (12)	25	5	37930.66	38306.32	9081.98	38287.22	9.00	−0.05
R2 (11)	25	5	174795.80	175313.88	7563.09	175311.64	8.96	−0.01
RC1 (8)	25	5	36252.02	36414.34	9571.79	36430.18	9.03	0.04
RC2 (8)	25	5	174075.96	174301.53	10800.00*	174300.67	8.82	−0.01
Avg					8109.69		8.95	−0.01
C1 (9)	30	5	82758.85	83096.43	9656.17	83115.24	13.84	0.02
C2 (8)	30	5	186488.20	187253.95	5519.71	187252.10	12.56	−0.05
R1 (12)	30	5	38126.33	38543.11	9902.81	38495.67	11.75	−0.12
R2 (11)	30	5	174930.96	175371.60	9478.50	175356.43	14.31	−0.01
RC1 (8)	30	5	36474.96	36715.80	10800.00*	36717.50	12.69	0.00
RC2 (8)	30	5	36199.06	36555.58	10800.00*	36572.23	13.18	0.05
Avg					9410.01		13.04	−0.02

Table 5: Average results on medium and large-size instances

Instance set	$\|\mathcal{N}_c\|$	$\|\mathcal{N}_0\|$	E_4^v			HESA		
			LP relaxation	Total cost	Time	Total cost	Time	Dev
C1 (9)	50	7	41386.56	41875.93	9644.33	41910.79	93.09	0.08
C2 (8)	50	7	92446.10	93394.71	9450.78	92822.45	99.21	−0.20
R1 (12)	50	7	38870.22	39722.87	10800.00*	39488.63	93.32	−0.59
R2 (11)	50	7	175518.42	176329.57	10800.00*	175991.78	92.70	−0.19
RC1 (8)	50	7	19574.34	22807.06	10800.00*	20485.74	76.55	−11.18
RC2 (8)	50	7	87065.42	88183.79	10800.00*	87536.04	85.62	−0.74
Avg					10421.52		90.51	−1.94
C1 (9)	75	8	82857.15	83662.53	10800.00*	83479.50	173.35	−0.22
C2 (8)	75	8	188792.53	192980.52	10800.00*	189374.75	165.39	−7.04
R1 (12)	75	8	38056.29	64165.06	10800.00*	39939.26	172.11	−60.21
R2 (11)	75	8	174160.82	178597.02	10800.00*	174601.27	161.90	−2.29
RC1 (8)	75	8	38233.86	39523.52	10800.00*	38768.89	174.04	−1.95
RC2 (8)	75	8	173677.24	176872.38	10800.00*	174367.88	160.14	−1.44
Avg					10800.00*		167.91	−14.14
C1 (9)	100	10	84345.45	86863.29	10800.00*	85234.48	307.82	−1.91
C2 (8)	100	10	190077.54	330436.31	10800.00*	190862.38	308.28	−89.06
R1 (12)	100	10	39151.72	166296.26	10800.00*	40972.43	289.95	−306.22
R2 (11)	100	10	174794.11	185688.94	10800.00*	175171.82	292.88	−6.00
RC1 (8)	100	10	39186.42	49304.23	10800.00*	39889.18	306.05	−23.65
RC2 (8)	100	10	174430.65	268749.92	10800.00*	175057.75	294.88	−54.05
Avg					10800.00*		299.02	−88.65

5. Conclusions

This paper has introduced the FSMLRPTW, a complex integrated logistics problem which, to our knowledge, was studied here for the first time, and described formulations and a hybrid evolutionary search algorithm. Computational results on a new set of benchmark instances of up to 100 nodes and 10 potential depots were presented, which indicate that the proposed algorithm is able to identify solutions within 0.05% of optimality for small size instances and yields better solutions for larger instances as compared to an off-the-shelf solver. The running times of the algorithm are so that it can be used in practical applications.

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Appendix

Table A.1 presents the characteristics of the FSMLRPTW instances. Tables A.2 to A.9 present the detailed results on all benchmark instances for the FSMLRPTW instances.

Table A.1: The FSMLRPTW benchmark instances

Data set	Depot	Cost	Cap_{10}	Cap_{15}	Cap_{20}	Cap_{25}	Cap_{30}	Cap_{50}	Cap_{75}	Cap_{100}	X	Y
C1	1	40000	100	200	210	320	400	610	850	990	40	50
	2	45000	90	170	205	280	410	580	810	800	64	13
	3	42000	110	210	220	300	370	620	820	900	35	79
	4	41000	–	150	190	220	360	540	860	850	44	57
	5	48000	–	–	215	290	390	550	880	840	29	40
	6	50000	–	–	–	–	–	600	900	970	18	82
	7	38000	–	–	–	–	–	590	830	1000	63	93
	8	49000	–	–	–	–	–	–	870	910	85	8
	9	47000	–	–	–	–	–	–	–	930	11	63
	10	46000	–	–	–	–	–	–	–	780	37	17
C2	1	90000	110	220	240	340	430	640	880	950	40	50
	2	100000	100	180	230	290	410	610	840	800	8	95
	3	120000	120	210	240	310	380	630	830	1010	91	46
	4	95000	–	160	210	230	360	660	880	970	35	43
	5	105000	–	–	230	320	420	580	900	920	20	69
	6	97000	–	–	–	–	–	620	930	990	51	100
	7	115000	–	–	–	–	–	640	870	1030	29	28
	8	112000	–	–	–	–	–	–	890	930	60	43
	9	99000	–	–	–	–	–	–	–	870	98	97
	10	117000	–	–	–	–	–	–	–	890	96	42
R1	1	20000	80	150	160	300	330	490	750	960	35	35
	2	19000	95	110	140	280	280	440	690	750	10	54
	3	22000	85	130	140	270	300	420	610	910	52	56
	4	21000	–	160	150	290	310	510	650	820	46	60
	5	18000	–	–	170	310	270	500	760	720	81	24
	6	23000	–	–	–	–	–	390	740	790	11	59
	7	24000	–	–	–	–	–	460	590	1000	94	40
	8	17000	–	–	–	–	–	–	550	800	78	77
	9	25000	–	–	–	–	–	–	–	790	88	66
	10	20000	–	–	–	–	–	–	–	890	72	49
R2	1	85000	100	170	190	305	340	530	780	1020	35	35
	2	90000	105	120	160	290	300	480	730	810	92	77
	3	94000	90	140	150	300	310	470	650	720	82	17
	4	89000	–	180	160	315	330	530	670	790	25	82
	5	100000	–	–	190	310	280	520	790	890	64	17
	6	92000	–	–	–	–	–	450	750	1070	100	87
	7	97000	–	–	–	–	–	480	640	740	10	72
	8	87000	–	–	–	–	–	–	630	700	3	51
	9	99000	–	–	–	–	–	–	–	1100	8	99
	10	96000	–	–	–	–	–	–	–	760	64	60
RC1	1	18000	160	240	250	310	420	700	860	1050	40	50
	2	19000	180	250	260	270	370	680	790	900	15	52
	3	17000	150	230	220	320	390	710	780	1090	40	3
	4	21000	–	210	240	290	430	640	870	850	24	93
	5	26000	–	–	230	340	400	630	840	790	50	76
	6	24000	–	–	–	–	–	640	730	940	62	60
	7	23000	–	–	–	–	–	670	800	970	79	100
	8	19000	–	–	–	–	–	–	850	1180	10	95
	9	20000	–	–	–	–	–	–	–	900	80	11
	10	25000	–	–	–	–	–	–	–	1020	87	75
RC2	1	86000	180	270	320	420	420	720	900	1300	40	50
	2	91000	190	260	300	360	370	710	830	1200	86	37
	3	87000	160	240	270	380	390	740	800	900	23	94
	4	99000	–	230	290	370	430	690	890	800	55	100
	5	96000	–	–	310	410	400	680	860	1080	28	92
	6	100000	–	–	–	–	–	670	760	780	68	52
	7	85000	–	–	–	–	–	710	830	1090	72	19
	8	94000	–	–	–	–	–	–	870	1240	34	61
	9	93000	–	–	–	–	–	–	–	900	26	88
	10	97000	–	–	–	–	–	–	–	1100	61	44

Table A.2: Results on the 10-customer instances

Instance set	$\|\mathcal{N}_c\|$	$\|\mathcal{N}_0\|$	E_4^v			HESA		
			LP relaxation	Total cost	Time	Total	Time	Dev
C101	10	3	82504.85	82675.76	0.04	82675.76	4.59	0.00
C102	10	3	82504.85	82673.74	0.68	82673.74	4.91	0.00
C103	10	3	82504.85	82673.74	0.68	82673.74	2.83	0.00
C104	10	3	82504.85	82673.74	1.46	82673.74	2.60	0.00
C105	10	3	82504.85	82675.76	0.04	82675.76	3.82	0.00
C106	10	3	82504.85	82675.76	0.03	82675.76	3.96	0.00
C107	10	3	82504.85	82675.76	0.03	82675.76	4.32	0.00
C108	10	3	82504.85	82673.74	0.17	82673.74	2.03	0.00
C109	10	3	82504.85	82673.74	0.25	82673.74	3.06	0.00
C201	10	3	191118.19	192212.53	0.06	192212.53	4.87	0.00
C202	10	3	191118.19	192205.36	5.04	192205.36	1.57	0.00
C203	10	3	191118.19	192205.36	5.03	192205.36	1.94	0.00
C204	10	3	191118.19	192195.93	4.97	192195.93	3.57	0.00
C205	10	3	191118.19	192207.74	0.19	192207.74	4.51	0.00
C206	10	3	191118.19	192195.93	0.07	192195.93	3.22	0.00
C207	10	3	191118.19	192195.93	0.16	192195.93	3.13	0.00
C208	10	3	191118.19	192195.93	1.00	192195.93	3.69	0.00
R101	10	3	39382.04	39523.07	0.11	39523.07	1.18	0.00
R102	10	3	39382.04	39459.59	100.29	39459.59	4.53	0.00
R103	10	3	39382.04	39459.59	100.31	39459.59	3.80	0.00
R104	10	3	39382.04	39446.25	195.61	39446.25	4.18	0.00
R105	10	3	39382.04	39498.13	0.20	39498.13	1.99	0.00
R106	10	3	39382.04	39459.59	204.49	39459.59	4.18	0.00
R107	10	3	39382.04	39459.59	204.84	39459.59	1.99	0.00
R108	10	3	39382.04	39446.25	202.89	39446.25	2.33	0.00
R109	10	3	39382.04	39483.11	3.78	39483.11	1.97	0.00
R110	10	3	39382.04	39452.56	33.73	39452.56	4.48	0.00
R111	10	3	39382.04	39462.11	205.15	39462.11	1.40	0.00
R112	10	3	39382.04	39446.25	292.26	39446.25	4.11	0.00
R201	10	3	175618.55	176202.33	0.17	176202.33	4.04	0.00
R202	10	3	175618.55	176152.08	0.76	176152.08	2.65	0.00
R203	10	3	175618.55	176152.08	0.76	176152.08	3.54	0.00
R204	10	3	175618.55	176147.33	1.40	176147.33	4.91	0.00
R205	10	3	175618.55	176175.45	0.67	176175.45	3.17	0.00
R206	10	3	175618.55	176147.33	1.45	176147.33	2.20	0.00
R207	10	3	175618.55	176147.33	1.44	176147.33	1.12	0.00
R208	10	3	175618.55	176147.33	0.95	176147.33	2.77	0.00
R209	10	3	175618.55	176162.07	0.41	176162.07	3.81	0.00
R210	10	3	175618.55	176152.08	1.84	176152.08	4.99	0.00
R211	10	3	175618.55	176152.08	0.75	176152.08	1.79	0.00
RC101	10	3	35614.85	35766.20	1.47	35766.20	4.55	0.00
RC102	10	3	35614.85	35704.15	12.61	35704.15	1.98	0.00
RC103	10	3	35614.85	35704.15	12.60	35704.15	3.71	0.00
RC104	10	3	35614.85	35704.15	6.50	35704.15	1.83	0.00
RC105	10	3	35614.85	35731.49	36.21	35731.49	2.28	0.00
RC106	10	3	35614.85	35716.72	6.38	35716.72	4.94	0.00
RC107	10	3	35614.85	35704.15	3.39	35704.15	4.12	0.00
RC108	10	3	35614.85	35704.15	7.02	35704.15	2.08	0.00
RC201	10	3	173434.98	173626.44	2.62	173626.44	2.28	0.00
RC202	10	3	173434.98	173622.36	8.67	173622.36	2.04	0.00
RC203	10	3	173434.98	173622.36	8.67	173622.36	4.61	0.00
RC204	10	3	173434.98	173620.76	15.82	173620.76	1.62	0.00
RC205	10	3	173434.98	173623.73	8.32	173623.73	1.89	0.00
RC206	10	3	173434.98	173620.76	5.30	173620.76	3.34	0.00
RC207	10	3	173434.98	173620.76	56.11	173620.76	3.72	0.00
RC208	10	3	173434.98	173620.76	12.57	173620.76	3.07	0.00

Table A.3: Results on the 15-customer instances

Instance set	$\|\mathcal{N}_c\|$	$\|\mathcal{N}_0\|$	E_4^v			HESA		
			LP relaxation	Total cost	Time	Total	Time	Dev
C101	15	4	81889.59	82060.38	0.36	82060.38	5.56	0.00
C102	15	4	81889.59	82059.67	5.10	82059.67	4.09	0.00
C103	15	4	81889.59	82059.67	14.04	82059.67	5.66	0.00
C104	15	4	81889.59	82058.48	14.50	82058.48	3.34	0.00
C105	15	4	81889.59	82060.38	0.66	82060.38	5.35	0.00
C106	15	4	81889.59	82060.38	0.25	82060.38	3.76	0.00
C107	15	4	81889.59	82060.38	0.63	82060.38	2.15	0.00
C108	15	4	81889.59	82060.38	3.71	82060.38	5.98	0.00
C109	15	4	81889.59	82059.54	10.38	82059.54	4.73	0.00
C201	15	4	186126.81	187199.20	0.24	187199.20	2.64	0.00
C202	15	4	186126.81	187190.12	11.17	187190.12	4.56	0.00
C203	15	4	186126.81	187190.12	12.93	187190.12	4.26	0.00
C204	15	4	186126.81	187190.12	26.52	187190.12	3.58	0.00
C205	15	4	186126.81	187194.86	2.38	187194.86	3.31	0.00
C206	15	4	186126.81	187194.86	3.17	187194.86	2.84	0.00
C207	15	4	186126.81	187194.86	9.43	187194.86	5.18	0.00
C208	15	4	186126.81	187194.86	10.29	187194.86	3.37	0.00
R101	15	4	39590.64	39753.66	0.32	39753.66	3.61	0.00
R102	15	4	39590.64	39714.93	3465.17	39714.93	2.24	0.00
R103	15	4	39590.64	39714.93	9902.70	39714.93	2.17	0.00
R104	15	4	39590.64	39691.17	10800.00*	39692.00	4.28	0.00
R105	15	4	39590.64	39748.26	14.69	39748.26	3.56	0.00
R106	15	4	39590.64	39692.47	3703.37	39692.47	4.92	0.00
R107	15	4	39590.64	39692.47	8024.12	39692.47	2.40	0.00
R108	15	4	39590.64	39678.30	10800.00*	39678.30	4.91	0.00
R109	15	4	39590.64	39699.94	239.23	39699.94	4.38	0.00
R110	15	4	39590.64	39683.72	10800.00*	39683.72	5.37	0.00
R111	15	4	39590.64	39692.47	9580.68	39692.47	2.49	0.00
R112	15	4	39590.64	39665.72	8609.80	39665.72	5.95	0.00
R201	15	4	174662.67	175234.54	0.30	175234.54	4.97	0.00
R202	15	4	174662.67	175188.92	131.56	175188.92	4.12	0.00
R203	15	4	174662.67	175188.92	82.87	175188.92	3.80	0.00
R204	15	4	174662.67	175188.16	300.89	175188.16	2.03	0.00
R205	15	4	174662.67	175198.70	0.48	175198.70	2.94	0.00
R206	15	4	174662.67	175188.16	60.86	175188.16	2.43	0.00
R207	15	4	174662.67	175188.16	125.80	175188.16	4.40	0.00
R208	15	4	174662.67	175186.29	146.41	175186.29	5.54	0.00
R209	15	4	174662.67	175185.29	88.86	175185.29	3.01	0.00
R210	15	4	174662.67	175188.92	138.54	175188.92	5.39	0.00
R211	15	4	174662.67	175155.24	9.32	175155.24	4.08	0.00
RC101	15	4	35843.17	36000.10	68.82	36000.10	3.28	0.00
RC102	15	4	35843.17	35959.01	403.37	35959.01	2.44	0.00
RC103	15	4	35843.17	35954.04	1735.09	35954.04	4.44	0.00
RC104	15	4	35843.17	35951.15	7053.61	35951.15	2.61	0.00
RC105	15	4	35843.17	36003.22	6482.74	36003.22	3.35	0.00
RC106	15	4	35843.17	35950.90	842.69	35950.90	4.96	0.00
RC107	15	4	35843.17	35936.06	8415.18	35936.06	5.27	0.00
RC108	15	4	35843.17	35933.94	2751.09	35933.94	2.45	0.00
RC201	15	4	173626.06	173836.24	372.04	173836.24	3.63	0.00
RC202	15	4	173626.06	173829.47	10800.00*	173829.47	5.14	0.00
RC203	15	4	173626.06	173829.47	10800.00*	173829.47	2.48	0.00
RC204	15	4	173626.06	173823.01	10800.00*	173823.01	2.17	0.00
RC205	15	4	173626.06	173829.47	10800.00*	173829.47	5.64	0.00
RC206	15	4	173626.06	173824.84	10800.00*	173824.84	4.36	0.00
RC207	15	4	173626.06	173819.37	10800.00*	173819.37	4.90	0.00
RC208	15	4	173626.06	173817.77	10800.00*	173817.77	4.91	0.00

Table A.4: Results on the 20-customer instances

Instance set	$\lvert\mathcal{N}_c\rvert$	$\lvert\mathcal{N}_0\rvert$	E_4^v			HESA		
			LP relaxation	Total cost	Time	Total	Time	Dev
C101	20	5	82236.26	82436.10	1.87	82436.10	5.91	0.00
C102	20	5	82236.26	82436.10	1382.38	82436.10	8.04	0.00
C103	20	5	82236.26	82436.10	10800.00*	82436.10	4.30	0.00
C104	20	5	82236.26	82435.04	10800.00*	82566.30	7.41	0.16
C105	20	5	82236.26	82436.10	2766.31	82436.10	5.36	0.00
C106	20	5	82236.26	82436.10	3.39	82436.10	4.58	0.00
C107	20	5	82236.26	82436.10	21.17	82436.10	6.91	0.00
C108	20	5	82236.26	82436.10	10800.00*	82436.10	6.89	0.00
C109	20	5	82236.26	82435.04	10800.00*	82435.04	6.87	0.00
C201	20	5	186141.23	187227.98	0.68	187227.98	5.00	0.00
C202	20	5	186141.23	187221.32	143.12	187221.32	8.58	0.00
C203	20	5	186141.23	187219.23	1655.04	187219.23	5.03	0.00
C204	20	5	186141.23	187218.28	2657.51	187218.28	7.23	0.00
C205	20	5	186141.23	187227.98	21.85	187227.98	4.27	0.00
C206	20	5	186141.23	187227.98	112.96	187227.98	4.50	0.00
C207	20	5	186141.23	187227.03	87.66	187227.03	8.10	0.00
C208	20	5	186141.23	187222.64	135.71	187222.64	8.95	0.00
R101	20	5	37782.90	39130.26	3.58	39130.26	6.47	0.00
R102	20	5	37782.90	39055.61	10800.00*	39055.61	4.47	0.00
R103	20	5	37782.90	37973.97	10800.00*	37965.50	5.91	−0.02
R104	20	5	37782.90	37931.84	10800.00*	37918.90	6.96	−0.03
R105	20	5	37782.90	38054.09	221.14	38054.09	6.75	0.00
R106	20	5	37782.90	37965.26	10800.00*	37993.40	4.00	0.07
R107	20	5	37782.90	37940.97	10800.00*	37947.90	4.50	0.02
R108	20	5	37782.90	37927.36	10800.00*	37915.90	8.88	−0.03
R109	20	5	37782.90	37931.63	1226.56	37931.63	7.41	0.00
R110	20	5	37782.90	37931.63	10800.00*	37931.63	8.51	0.00
R111	20	5	37782.90	37920.97	10800.00*	37920.97	4.44	0.00
R112	20	5	37782.90	37913.51	10800.00*	37906.70	5.87	−0.02
R201	20	5	174699.42	175334.77	13.84	175335.00	6.83	0.00
R202	20	5	174699.40	175253.40	895.21	175253.40	6.29	0.00
R203	20	5	174699.40	175265.96	10800.00*	175253.00	6.12	−0.01
R204	20	5	174699.40	175210.76	1169.83	175210.76	5.76	0.00
R205	20	5	174699.41	175242.43	2.47	175242.43	5.50	0.00
R206	20	5	174699.40	175225.04	170.27	175225.04	8.00	0.00
R207	20	5	174699.40	175225.04	3394.52	175225.04	5.81	0.00
R208	20	5	174699.40	175207.73	228.61	175207.73	6.40	0.00
R209	20	5	174699.41	175224.09	25.81	175224.09	4.39	0.00
R210	20	5	174699.40	175237.25	1028.92	175237.25	8.88	0.00
R211	20	5	174699.40	175208.53	782.13	175208.53	5.78	0.00
RC101	20	5	36075.96	36274.22	457.60	36288.50	5.30	0.04
RC102	20	5	36075.96	36241.99	10800.00*	36241.99	8.59	0.00
RC103	20	5	36075.96	36229.55	10800.00*	36236.60	7.17	0.02
RC104	20	5	36075.96	36229.32	10800.00*	36229.32	7.02	0.00
RC105	20	5	36075.96	36271.24	10800.00*	36281.50	6.87	0.03
RC106	20	5	36075.96	36248.47	10800.00*	36248.47	8.25	0.00
RC107	20	5	36075.96	36229.15	10800.00*	36229.15	7.59	0.00
RC108	20	5	36075.96	36215.68	10800.00*	36215.68	5.53	0.00
RC201	20	5	173864.43	174093.90	10800.00*	174093.90	6.05	0.00
RC202	20	5	173864.43	174086.74	10800.00*	174086.74	8.81	0.00
RC203	20	5	173864.43	174087.25	10800.00*	174081.00	8.14	−0.01
RC204	20	5	173864.43	174069.80	10800.00*	174069.80	7.86	0.00
RC205	20	5	173864.43	174086.74	10800.00*	174086.74	6.56	0.00
RC206	20	5	173864.43	174082.07	10800.00*	174082.07	5.03	0.00
RC207	20	5	173864.43	174064.56	10800.00*	174064.56	7.90	0.00
RC208	20	5	173864.43	174064.56	10800.00*	174064.56	7.26	0.00

Table A.5: Results on the 25-customer instances

Instance set	$\lvert\mathcal{N}_c\rvert$	$\lvert\mathcal{N}_0\rvert$	E_4^v			HESA		
			LP relaxation	Total cost	Time	Total	Time	Dev
C101	25	5	82556.95	82767.06	5.94	82767.06	9.16	0.00
C102	25	5	82556.95	82767.06	3941.41	82767.06	10.76	0.00
C103	25	5	82556.95	82766.82	10800.00*	82773.80	9.91	0.01
C104	25	5	82556.95	82764.87	10800.00*	82902.10	10.35	0.17
C105	25	5	82556.95	82767.06	10800.00*	82767.06	9.75	0.00
C106	25	5	82556.95	82767.06	6890.55	82767.06	11.80	0.00
C107	25	5	82556.95	82767.06	10800.00*	82767.06	6.61	0.00
C108	25	5	82556.95	82767.06	10800.00*	82767.06	8.68	0.00
C109	25	5	82556.95	82772.10	10800.00*	82764.90	6.96	−0.01
C201	25	5	186314.82	187237.51	1.13	187237.51	11.31	0.00
C202	25	5	186314.82	187231.15	217.90	187231.15	9.45	0.00
C203	25	5	186314.82	187231.15	10800.00*	187231.15	7.06	0.00
C204	25	5	186314.82	187232.33	10800.00*	187232.33	10.29	0.00
C205	25	5	186314.82	187237.51	25.17	187237.51	6.24	0.00
C206	25	5	186314.82	187237.51	333.59	187237.51	8.77	0.00
C207	25	5	186314.82	187233.45	248.47	187233.45	7.80	0.00
C208	25	5	186314.82	187237.35	926.44	187237.35	6.71	0.00
R101	25	5	37930.66	39242.52	8.97	39242.52	8.24	0.00
R102	25	5	37930.66	39163.49	10800.00*	39153.10	8.13	−0.03
R103	25	5	37930.66	38144.61	10800.00*	38126.50	10.35	−0.05
R104	25	5	37930.66	38111.15	10800.00*	38074.50	9.99	−0.10
R105	25	5	37930.66	38225.10	999.73	38225.10	10.69	0.00
R106	25	5	37930.66	38168.91	10800.00*	38151.70	7.54	−0.05
R107	25	5	37930.66	38075.13	10775.00	38069.40	11.55	−0.02
R108	25	5	37930.66	38040.62	10800.00*	38040.62	6.54	0.00
R109	25	5	37930.66	38150.69	10800.00*	38154.90	8.60	0.01
R110	25	5	37930.66	38114.13	10800.00*	38100.50	10.30	−0.04
R111	25	5	37930.66	38126.42	10800.00*	38058.30	6.39	−0.18
R112	25	5	37930.66	38113.11	10800.00*	38049.50	9.73	−0.17
R201	25	5	174795.80	175439.55	32.28	175439.55	9.61	0.00
R202	25	5	174795.80	175361.14	10800.00*	175361.14	7.58	0.00
R203	25	5	174795.80	175331.29	10775.00	175325.00	7.00	−0.01
R204	25	5	174795.80	175267.13	10800.00*	175267.13	9.10	0.00
R205	25	5	174795.80	175319.60	567.39	175319.60	8.53	0.00
R206	25	5	174795.80	175294.33	10800.00*	175294.33	10.14	0.00
R207	25	5	174795.80	175295.41	10800.00*	175284.00	10.34	−0.01
R208	25	5	174795.80	175256.29	1588.22	175256.29	7.30	0.00
R209	25	5	174795.80	175296.89	5431.05	175296.89	8.43	0.00
R210	25	5	174795.80	175326.94	10800.00*	175320.00	11.40	−0.01
R211	25	5	174795.80	175264.11	10800.00*	175264.11	9.13	0.00
RC101	25	5	36252.02	36450.48	974.29	36484.40	8.09	0.09
RC102	25	5	36252.02	36432.10	10800.00*	36447.40	8.48	0.04
RC103	25	5	36252.02	36381.15	10800.00*	36403.40	11.18	0.06
RC104	25	5	36252.02	36378.72	10800.00*	36418.00	8.54	0.11
RC105	25	5	36252.02	36468.03	10800.00*	36489.50	10.76	0.06
RC106	25	5	36252.02	36413.53	10800.00*	36415.20	8.21	0.00
RC107	25	5	36252.02	36381.34	10800.00*	36392.40	9.16	0.03
RC108	25	5	36252.02	36409.38	10800.00*	36391.10	7.79	−0.05
RC201	25	5	174075.96	174326.81	10800.00*	174326.81	7.98	0.00
RC202	25	5	174075.96	174319.65	10800.00*	174319.65	6.77	0.00
RC203	25	5	174075.96	174309.48	10800.00*	174309.48	11.03	0.00
RC204	25	5	174075.96	174285.70	10800.00*	174281.00	6.32	−0.01
RC205	25	5	174075.96	174318.88	10800.00*	174318.88	9.16	0.00
RC206	25	5	174075.96	174295.72	10800.00*	174295.72	11.03	0.00
RC207	25	5	174075.96	174280.19	10800.00*	174278.00	11.92	−0.01
RC208	25	5	174075.96	174275.82	10800.00*	174275.82	6.33	0.00

Table A.6: Results on the 30-customer instances

Instance set	$\|\mathcal{N}_c\|$	$\|\mathcal{N}_0\|$	E_4^v			HESA		
			LP relaxation	Total cost	Time	Total	Time	Dev
C101	30	5	82758.85	83097.51	10800.00*	83097.50	14.16	0.00
C102	30	5	82758.85	83097.51	505.56	83109.40	17.76	0.01
C103	30	5	82758.85	83098.28	10800.00*	83109.40	12.33	0.01
C104	30	5	82758.85	83094.88	10800.00*	83229.20	15.51	0.16
C105	30	5	82758.85	83097.51	10800.00*	83097.51	14.21	0.00
C106	30	5	82758.85	83097.51	10800.00*	83097.51	15.38	0.00
C107	30	5	82758.85	83094.88	10800.00*	83094.88	10.70	0.00
C108	30	5	82758.85	83094.88	10800.00*	83100.60	11.04	0.01
C109	30	5	82758.85	83094.88	10800.00*	83101.20	13.48	0.01
C201	30	5	186488.20	187255.37	1.86	187255.37	11.09	0.00
C202	30	5	186488.20	187249.00	10800.00*	187249.00	11.17	0.00
C203	30	5	186488.20	187263.79	10800.00*	187249.00	11.56	−0.01
C204	30	5	186488.20	187245.82	10800.00*	187245.82	10.66	0.00
C205	30	5	186488.20	187255.37	34.37	187255.37	12.78	0.00
C206	30	5	186488.20	187255.37	543.48	187255.37	16.63	0.00
C207	30	5	186488.20	187251.31	377.96	187251.31	13.87	0.00
C208	30	5	186488.20	187255.56	10800.00*	187255.56	12.75	0.00
R101	30	5	38126.33	39421.28	33.74	39421.28	10.72	0.00
R102	30	5	38126.33	39356.38	10800.00*	39341.30	10.62	−0.04
R103	30	5	38126.33	38346.59	10800.00*	38330.30	13.11	−0.04
R104	30	5	38126.33	38337.89	10800.00*	38278.40	10.11	−0.16
R105	30	5	38126.33	38451.92	10800.00*	38442.10	10.83	−0.03
R106	30	5	38126.33	38457.92	10800.00*	38353.70	10.77	−0.27
R107	30	5	38126.33	38357.13	10800.00*	38295.40	12.79	−0.16
R108	30	5	38126.33	38326.78	10800.00*	38257.70	11.83	−0.18
R109	30	5	38126.33	38396.41	10800.00*	38355.90	10.24	−0.11
R110	30	5	38126.33	38351.22	10800.00*	38321.90	13.68	−0.08
R111	30	5	38126.33	38363.48	10800.00*	38285.10	15.04	−0.20
R112	30	5	38126.33	38350.33	10800.00*	38265.00	11.21	−0.22
R201	30	5	174930.96	175537.23	628.20	175542.00	13.23	0.01
R202	30	5	174930.96	175502.53	10800.00*	175443.00	15.93	−0.03
R203	30	5	174930.96	175386.34	10800.00*	175360.00	11.43	−0.02
R204	30	5	174930.96	175283.96	10800.00*	175283.96	11.37	0.00
R205	30	5	174930.96	175382.37	6435.25	175382.37	12.45	0.00
R206	30	5	174930.96	175370.08	10800.00*	175344.00	15.26	−0.01
R207	30	5	174930.96	175327.23	10800.00*	175303.00	14.98	−0.01
R208	30	5	174930.96	175283.11	10800.00*	175273.00	14.42	−0.01
R209	30	5	174930.96	175338.33	10800.00*	175338.33	15.22	0.00
R210	30	5	174930.96	175353.04	10800.00*	175353.04	15.09	0.00
R211	30	5	174930.96	175323.38	10800.00*	175298.00	18.00	−0.01
RC101	30	5	36474.96	36803.52	10800.00*	36814.10	14.93	0.03
RC102	30	5	36474.96	36766.79	10800.00*	36786.90	13.19	0.05
RC103	30	5	36474.96	36696.26	10800.00*	36684.60	13.31	−0.03
RC104	30	5	36474.96	36667.39	10800.00*	36697.90	13.36	0.08
RC105	30	5	36474.96	36746.25	10800.00*	36748.20	10.20	0.01
RC106	30	5	36474.96	36688.56	10800.00*	36688.56	11.40	0.00
RC107	30	5	36474.96	36699.96	10800.00*	36664.60	13.51	−0.10
RC108	30	5	36474.96	36657.64	10800.00*	36655.10	11.61	−0.01
RC201	30	5	36199.06	36585.08	10800.00*	36578.50	13.03	−0.02
RC202	30	5	36199.06	36543.55	10800.00*	36539.90	13.79	−0.01
RC203	30	5	36199.06	36540.45	10800.00*	36606.90	10.67	0.18
RC204	30	5	36199.06	36565.95	10800.00*	36556.30	12.14	−0.03
RC205	30	5	36199.06	36548.96	10800.00*	36548.96	15.53	0.00
RC206	30	5	36199.06	36554.74	10800.00*	36584.10	14.00	0.08
RC207	30	5	36199.06	36528.09	10800.00*	36528.09	11.64	0.00
RC208	30	5	36199.06	36577.81	10800.00*	36635.10	14.66	0.16

Table A.7: Results on the 50-customer instances

Instance set	$\|\mathcal{N}_c\|$	$\|\mathcal{N}_0\|$	E_4^v			HESA		
			LP relaxation	Total cost	Time	Total	Time	Dev
C101	50	7	41386.56	41865.26	398.97	41892.30	70.96	0.06
C102	50	7	41386.56	41859.80	10800.00*	41859.80	103.47	0.00
C103	50	7	41386.56	41968.98	10800.00*	41922.80	61.00	−0.11
C104	50	7	41386.56	41890.98	10800.00*	42000.50	114.55	0.26
C105	50	7	41386.56	41852.69	10800.00*	41852.69	81.52	0.00
C106	50	7	41386.56	41853.79	10800.00*	41853.79	95.53	0.00
C107	50	7	41386.56	41851.28	10800.00*	41903.50	106.71	0.12
C108	50	7	41386.56	41850.25	10800.00*	41895.50	105.26	0.11
C109	50	7	41386.56	41890.31	10800.00*	42016.20	98.83	0.30
C201	50	7	92446.10	92880.93	6.21	92880.93	109.43	0.00
C202	50	7	92446.10	93384.81	10800.00*	92825.00	62.39	−0.60
C203	50	7	92446.10	93869.63	10800.00*	92818.60	118.84	−1.13
C204	50	7	92446.10	93470.84	10800.00*	92770.40	115.62	−0.76
C205	50	7	92446.10	93391.05	10800.00*	92876.60	115.28	−0.55
C206	50	7	92446.10	93391.05	10800.00*	92811.40	66.99	−0.62
C207	50	7	92446.10	93378.45	10800.00*	92798.30	117.47	−0.63
C208	50	7	92446.10	93390.88	10800.00*	92798.40	87.68	−0.64
R101	50	7	38870.22	40443.15	10800.00*	40443.15	83.30	0.00
R102	50	7	38870.22	40440.99	10800.00*	40328.30	95.11	−0.28
R103	50	7	38870.22	41424.71	10800.00*	40195.20	83.30	−3.06
R104	50	7	38870.22	39368.54	10800.00*	39155.50	112.85	−0.54
R105	50	7	38870.22	39380.48	10800.00*	39356.20	113.54	−0.06
R106	50	7	38870.22	39459.31	10800.00*	39263.10	119.24	−0.50
R107	50	7	38870.22	39443.09	10800.00*	39152.30	92.82	−0.74
R108	50	7	38870.22	39322.58	10800.00*	39157.10	104.15	−0.42
R109	50	7	38870.22	39322.03	10800.00*	39250.70	72.21	−0.18
R110	50	7	38870.22	39411.26	10800.00*	39211.80	61.42	−0.51
R111	50	7	38870.22	39255.20	10800.00*	39179.70	84.72	−0.19
R112	50	7	38870.22	39403.04	10800.00*	39170.50	97.18	−0.59
R201	50	7	175518.42	176218.56	10800.00*	176218.56	86.87	0.00
R202	50	7	175518.42	177013.02	10800.00*	176088.00	91.59	−0.53
R203	50	7	175518.42	176249.23	10800.00*	175995.00	93.72	−0.14
R204	50	7	175518.42	175917.21	10800.00*	175866.00	118.70	−0.03
R205	50	7	175518.42	176200.21	10800.00*	176061.00	90.21	−0.08
R206	50	7	175518.42	176081.07	10800.00*	176000.00	117.02	−0.05
R207	50	7	175518.42	176815.10	10800.00*	175932.00	119.13	−0.50
R208	50	7	175518.42	175962.50	10800.00*	175855.00	63.31	−0.06
R209	50	7	175518.42	176089.81	10800.00*	175995.00	69.61	−0.05
R210	50	7	175518.42	176374.18	10800.00*	175994.00	82.15	−0.22
R211	50	7	175518.42	176704.38	10800.00*	175905.00	87.41	−0.45
RC101	50	7	19574.34	21031.12	10800.00*	20978.50	83.08	−0.25
RC102	50	7	19574.34	21334.83	10800.00*	20892.30	63.81	−2.12
RC103	50	7	19574.34	38142.81	10800.00*	20778.80	74.35	−83.57
RC104	50	7	19574.34	20274.62	10800.00*	20020.00	105.47	−1.27
RC105	50	7	19574.34	20991.89	10800.00*	20889.10	72.03	−0.49
RC106	50	7	19574.34	20325.56	10800.00*	20223.20	76.47	−0.51
RC107	50	7	19574.34	20291.25	10800.00*	20085.60	61.74	−1.02
RC108	50	7	19574.34	20064.39	10800.00*	20018.40	75.42	−0.23
RC201	50	7	87065.42	87586.41	10800.00*	87641.90	94.32	0.06
RC202	50	7	87065.42	87992.95	10800.00*	87546.10	65.43	−0.51
RC203	50	7	87065.42	88339.08	10800.00*	87558.60	114.14	−0.89
RC204	50	7	87065.42	89274.98	10800.00*	87456.90	61.20	−2.08
RC205	50	7	87065.42	87784.60	10800.00*	87530.90	76.40	−0.29
RC206	50	7	87065.42	87989.08	10800.00*	87619.80	69.17	−0.42
RC207	50	7	87065.42	88092.32	10800.00*	87469.30	104.49	−0.71
RC208	50	7	87065.42	88410.89	10800.00*	87464.80	99.78	−1.08

Table A.8: Results on the 75-customer instances

Instance set	$\lvert\mathcal{N}_c\rvert$	$\lvert\mathcal{N}_0\rvert$	E_4^v			HESA		
			LP relaxation	Total cost	Time	Total	Time	Dev
C101	75	8	82857.15	83309.83	10800.00*	83376.60	187.36	0.08
C102	75	8	82857.15	83343.28	10800.00*	83389.00	185.65	0.05
C103	75	8	82857.15	84468.47	10800.00*	83321.10	157.32	−1.38
C104	75	8	82857.15	84670.05	10800.00*	83766.60	159.44	−1.08
C105	75	8	82857.15	83311.92	10800.00*	83489.80	164.07	0.21
C106	75	8	82857.15	83307.33	10800.00*	83372.90	184.56	0.08
C107	75	8	82857.15	83308.44	10800.00*	83632.90	179.85	0.39
C108	75	8	82857.15	83450.51	10800.00*	83411.10	176.41	−0.05
C109	75	8	82857.15	83792.90	10800.00*	83555.50	165.48	−0.28
C201	75	8	188792.53	189585.40	10800.00*	189401.00	163.23	−0.10
C202	75	8	188792.53	192152.42	10800.00*	189371.00	161.08	−1.47
C203	75	8	188792.53	193453.62	10800.00*	189360.00	174.90	−2.16
C204	75	8	188792.53	200531.39	10800.00*	189328.00	141.79	−5.92
C205	75	8	188792.53	189582.69	10800.00*	189400.00	144.57	−0.10
C206	75	8	188792.53	190731.82	10800.00*	189376.00	180.62	−0.72
C207	75	8	188792.53	193275.65	10800.00*	189390.00	170.03	−2.05
C208	75	8	188792.53	194531.18	10800.00*	189372.00	186.91	−2.72
R101	75	8	38056.29	40600.20	10800.00*	40609.90	189.28	0.02
R102	75	8	38056.29	40913.94	10800.00*	40437.50	162.13	−1.18
R103	75	8	38056.29	176468.82	10800.00*	40252.60	147.57	−338.40
R104	75	8	38056.29	40584.42	10800.00*	40256.90	188.58	−0.81
R105	75	8	38056.29	40643.62	10800.00*	40368.50	172.37	−0.68
R106	75	8	38056.29	103148.58	10800.00*	40295.60	157.16	−155.98
R107	75	8	38056.29	81617.99	10800.00*	40332.90	173.12	−102.36
R108	75	8	38056.29	61602.75	10800.00*	40234.80	156.94	−53.11
R109	75	8	38056.29	39933.80	10800.00*	38825.00	171.10	−2.86
R110	75	8	38056.29	42288.53	10800.00*	38715.10	182.79	−9.23
R111	75	8	38056.29	61206.47	10800.00*	40281.90	186.65	−51.95
R112	75	8	38056.29	40971.58	10800.00*	38660.40	177.68	−5.98
R201	75	8	174160.82	175775.01	10800.00*	174859.00	152.31	−0.52
R202	75	8	174160.82	179505.22	10800.00*	174767.00	145.66	−2.71
R203	75	8	174160.82	182253.80	10800.00*	174601.00	171.49	−4.38
R204	75	8	174160.82	177907.86	10800.00*	174472.00	173.79	−1.97
R205	75	8	174160.82	176368.20	10800.00*	174659.00	141.53	−0.98
R206	75	8	174160.82	176973.52	10800.00*	174597.00	178.87	−1.36
R207	75	8	174160.82	178021.12	10800.00*	174524.00	173.78	−2.00
R208	75	8	174160.82	178926.32	10800.00*	174445.00	150.08	−2.57
R209	75	8	174160.82	179798.66	10800.00*	174563.00	176.27	−3.00
R210	75	8	174160.82	179194.44	10800.00*	174605.00	150.63	−2.63
R211	75	8	174160.82	179843.07	10800.00*	174522.00	166.47	−3.05
RC101	75	8	38233.86	39061.10	10800.00*	38940.60	172.15	−0.31
RC102	75	8	38233.86	39221.20	10800.00*	38832.50	174.34	−1.00
RC103	75	8	38233.86	39820.43	10800.00*	38717.80	183.92	−2.85
RC104	75	8	38233.86	39830.82	10800.00*	38677.30	158.68	−2.98
RC105	75	8	38233.86	39258.60	10800.00*	38866.70	167.90	−1.01
RC106	75	8	38233.86	39318.93	10800.00*	38795.00	183.80	−1.35
RC107	75	8	38233.86	39692.26	10800.00*	38676.20	165.33	−2.63
RC108	75	8	38233.86	39984.80	10800.00*	38645.00	186.19	−3.47
RC201	75	8	173677.24	174872.25	10800.00*	174546.00	159.46	−0.19
RC202	75	8	173677.24	177692.77	10800.00*	174332.00	162.91	−1.93
RC203	75	8	173677.24	178208.59	10800.00*	174373.00	173.16	−2.20
RC204	75	8	173677.24	179071.17	10800.00*	174241.00	185.53	−2.77
RC205	75	8	173677.24	175681.10	10800.00*	174444.00	152.35	−0.71
RC206	75	8	173677.24	175218.43	10800.00*	174419.00	147.26	−0.46
RC207	75	8	173677.24	178097.38	10800.00*	174346.00	144.99	−2.15
RC208	75	8	173677.24	176137.37	10800.00*	174242.00	155.45	−1.09

Table A.9: Results on the 100-customer instances

Instance set	$\|\mathcal{N}_c\|$	$\|\mathcal{N}_0\|$	E_4^v			HESA		
			LP relaxation	Total cost	Time	Total	Time	Dev
C101	100	10	84345.45	85199.09	10800.00*	85199.09	297.32	0.00
C102	100	10	84345.45	86605.08	10800.00*	85209.00	304.48	−1.64
C103	100	10	84345.45	88128.63	10800.00*	85190.80	305.06	−3.45
C104	100	10	84345.45	90622.34	10800.00*	85509.90	319.80	−5.98
C105	100	10	84345.45	85258.60	10800.00*	85208.50	319.99	−0.06
C106	100	10	84345.45	85461.36	10800.00*	85204.90	309.38	−0.30
C107	100	10	84345.45	85306.67	10800.00*	85167.30	307.85	−0.16
C108	100	10	84345.45	85570.18	10800.00*	85222.80	294.20	−0.41
C109	100	10	84345.45	89617.66	10800.00*	85198.00	312.34	−5.19
C201	100	10	190083.07	191687.14	10800.00*	190876.00	271.94	−0.42
C202	100	10	190074.71	195858.81	10800.00*	190865.00	259.04	−2.62
C203	100	10	190075.95	304096.37	10800.00*	190864.00	308.09	−59.33
C204	100	10	190074.71	1154483.31	10800.00*	190848.00	337.23	−504.92
C205	100	10	190077.15	194711.89	10800.00*	190863.00	304.44	−2.02
C206	100	10	190084.61	202111.00	10800.00*	190861.00	321.63	−5.89
C207	100	10	190075.31	199617.46	10800.00*	190856.00	330.40	−4.59
C208	100	10	190074.82	200924.51	10800.00*	190866.00	333.46	−5.27
R101	100	10	39159.76	42095.43	10800.00*	41782.20	292.25	−0.75
R102	100	10	39146.09	105133.04	10800.00*	41577.10	324.69	−152.86
R103	100	10	39149.89	225357.73	10800.00*	41398.40	293.22	−444.36
R104	100	10	39151.07	227609.71	10800.00*	41269.90	289.51	−451.52
R105	100	10	39154.42	81860.12	10800.00*	41522.20	250.56	−97.15
R106	100	10	39145.82	226936.85	10800.00*	41397.50	291.75	−448.19
R107	100	10	39145.98	230534.40	10800.00*	41329.90	339.02	−457.79
R108	100	10	39148.67	227135.02	10800.00*	40574.50	336.96	−459.80
R109	100	10	39161.60	106140.17	10800.00*	40135.40	247.20	−164.46
R110	100	10	39146.09	68339.42	10800.00*	40052.30	267.95	−70.63
R111	100	10	39145.98	228378.35	10800.00*	40614.10	296.14	−462.31
R112	100	10	39165.22	226034.92	10800.00*	40015.60	250.16	−464.87
R201	100	10	174794.14	180128.14	10800.00*	175457.00	329.90	−2.66
R202	100	10	174794.09	188288.16	10800.00*	175303.00	256.58	−7.41
R203	100	10	174794.09	186985.40	10800.00*	175186.00	289.80	−6.74
R204	100	10	174794.09	184455.99	10800.00*	175037.00	297.46	−5.38
R205	100	10	174794.09	186185.59	10800.00*	175237.00	339.93	−6.25
R206	100	10	174794.14	187873.20	10800.00*	175168.00	268.04	−7.25
R207	100	10	174794.09	185389.11	10800.00*	175096.00	270.70	−5.88
R208	100	10	174794.09	180572.09	10800.00*	175010.00	322.05	−3.18
R209	100	10	174794.09	188632.91	10800.00*	175156.00	288.98	−7.69
R210	100	10	174794.14	188607.92	10800.00*	175189.00	311.21	−7.66
R211	100	10	174794.14	185459.83	10800.00*	175051.00	247.03	−5.95
RC101	100	10	39180.69	40800.29	10800.00*	40111.60	323.52	−1.72
RC102	100	10	39186.00	44712.85	10800.00*	39940.70	328.25	−11.95
RC103	100	10	39199.13	70169.17	10800.00*	39828.90	316.82	−76.18
RC104	100	10	39181.17	67342.83	10800.00*	39727.00	317.03	−69.51
RC105	100	10	39203.16	42222.87	10800.00*	39990.10	281.31	−5.58
RC106	100	10	39176.91	42502.09	10800.00*	39912.40	323.17	−6.49
RC107	100	10	39176.91	44066.07	10800.00*	39827.50	275.89	−10.64
RC108	100	10	39187.37	42617.67	10800.00*	39775.20	282.44	−7.15
RC201	100	10	174428.16	177634.48	10800.00*	175461.00	282.26	−1.24
RC202	100	10	174429.52	181651.48	10800.00*	175284.00	278.25	−3.63
RC203	100	10	174428.48	185169.73	10800.00*	175271.00	313.55	−5.65
RC204	100	10	174435.83	873972.63	10800.00*	173214.00	260.57	−404.56
RC205	100	10	174432.47	183169.93	10800.00*	175409.00	317.51	−4.42
RC206	100	10	174427.49	184800.00	10800.00*	175333.00	318.89	−5.40
RC207	100	10	174426.83	179681.12	10800.00*	175280.00	300.42	−2.51
RC208	100	10	174436.39	183920.02	10800.00*	175210.00	287.60	−4.97

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