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Agent-Based Simulation of Multi-Round Timber Combinatorial Auction

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Abstract. This paper presents a simulation-based analysis of a multiple-round timber combinatorial auction in the timber industry. Currently, most timber auctions are single unit auctions (i.e., each forest stand is sold separately). However, other types of auctions could be applied in order to take advantage of the various needs of the bidders with respect to species, volumes and quality. This study aims to analyze the use of combinatorial auction to this specific context using a simulation approach. Various number of auctions per year, periodicity, lot size, and number of bidders are considered as parameters to setup the different market configurations. The outcomes of both combinatorial auction and single unit auction are compared with respect to different setup configurations. In particular, this analysis shows that combinatorial auction can bring more profit for both seller and buyer when the market is less competitive.

Keywords: Timber auction, combinatorial auction, learning strategy, multi-agent simulation.

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1 Introduction

Several problems plagued the timber market in Québec in recent years: decrease in forest activities, drop in lumber sales, and constrained access to wood supplies. One of the issues relates specifically to the long-term exclusive timber licences: the wood supplies of a timber licence may not always match the company's needs at a specific time. Consequently, allocating timber access to suitable companies for a defined time period has been one of the main problems for the Québec government. In the new forest regime now in place in Québec, auctions are applied in order to assign 25% of the timber lots that were previously assigned through licencing. This new regime results in a more flexible access to timber, as well as a price index that will be used to establish timber licence prices.

There exist two main types of auctions for timber allocations: single-unit auctions and combinatorial auctions. On the one hand, in a single-unit auction, the seller aims to sell the whole lot to one bidder. Currently, most timber auctions use the single-unit method because of its ease of implementation. When the winner of each lot is announced, the winner has a specific time period to access the lot according to its need. The Québec government uses single-unit auctions to allocate timber access to buyers who are willing to pay more for entire lots.

On the other hand, timber combinatorial auctions may have some advantages over single-unit auctions. These more complex auctions allow bidders to bid on any combination (bundle) of items according to their needs. Here, items are not geographically defined forest stands. They can be defined, for example, as specific volumes (lots) of a specific mix of species and quality within a given stand. Therefore, mixed forest stands can be sold to potential users. In order to identify the winners, the auctioneer must compute the highest value of the bundles. Once identified, the winners must agree on a specific time to harvest the stand in which they share the access.

In this paper, timber combinatorial auctions are studied as an interaction procedure between an auctioneering agent and several bidding agents to assign timber quantities. The auctioneer, i.e. government agents, announces several different lots with defined types of products (i.e., mixed of species and quality) to the market on a regular basis through combinatorial auctions. The bidders (i.e., forest companies and entrepreneur agents) offer sealed bids for any bundle of products of the lots that are announced at each round. The bidders are not allowed to change their offers after submitting their bids. As mentioned above, the auctioneer chooses the winners according to the highest value of the bundles, and the bidders must agree on a specific time to harvest the stand. In order to study these interactions and this type of procedure, a simulation model was developed.

The simulation model needs a framework to follow the dynamics of the auctions systems over the course of several rounds. Agent-based modeling was used to design and implement different agent behaviours. In other words, it was used to simulate realistic bidding agents, auctioneer agent, and auction mechanisms, including realistic bidding patterns and auctioneer's winner determination process.

This paper, which extends Farnia *et al*. (2013), explores how the outcomes of combinatorial auctions in terms of selling price per m^3 (revenue of the seller) and bidders' target achievement can change in different setup configurations. In addition, the study compares the revenue stream of the seller and the target achievement of the bidders of combinatorial auctions and single-unit auctions using several simulations.

The remainder of the paper is organised as follows. In Section 2, the theoretical background is presented in details. The research objective is described in Section 3. In Section 4, the simulated multi-period timber combinatorial auction model is described. Results and discussions including four different experiments are presented in Section 5. Finally, Section 6 concludes the paper and presents a brief overview of future research.

2 Theoretical background and research objectives

Limited natural resources such as oil, mineral rights, radio spectrum, and timber have been studied regarding policies and mechanisms used to allocate and value them. Timber allocation and pricing is of specific interest in Québec, as well as in several jurisdictions in the world. Many auction models have been used to solve timber allocation problems, (Mead, 1967; Hansen, 1985; Paarsch, 1991; Elyakime *et al*., 1994, 1997; Baldwin *et al.*, 1997; Athey and Levin, 2001; Haile, 2001; Athey *et al.*, 2011). These studies are mostly related to the areas of competition and information, comparing open and closed auctions, collusion, and reserve price. Others, described below, are more specifically related to our study in multiple-round combinatorial timber auction.

Farnia *et al.* (2013) studied multiple-round single-unit timber auctions using simulation. The authors studied the effects of several bidding patterns on the auction outcome. Their results suggests that the adaptive and learning bidding patterns have the best outcome. They also analyzed how this outcome is affected by different setup configurations.

Cramton *et al.* (2006) described a timber combinatorial auction model as a method of assigning products to buyers. However they did not compare the single unit auction with combinatorial auction. In a timber combinatorial auction, there are several types of products (e.g., species) for sale, and bidders can bid on any bundle of these products. The bundle of bids that maximizes their combined value defines the winning bids. Combinatorial auctions can be used in many applications. Resource allocation is a type of problem that can be solved through combinatorial auction, e.g., allocating airport landing time periods to airlines (i.e. Rassenti *et al*., 1982; Ghassemi Tari and Alaei, 2013; Wang and Dargahi, 2013). Combinatorial auctions are also useful for scheduling problems of loading cranes in maritime terminals (i.e. Brewer, 1999; De Vries and Vohra, 2003; Cramton *et al.*, 2006; Jung and Kim, 2006).

Multiple-round auctions are a series of any type of auctions that are announced sequentially. At each round, one or many auctions are announced simultaneously. Once a round is completed and the winners identified, the auctioneer announces (potentially after a delay) new auctions (Grossklags *et al*., 2000). In the case of a multiple-round combinatorial auction, each auction includes several types of products and bidders bid sequentially on any bundle of products. This type of auction is used in Lau *et al.* (2007) in order to solve a large-scale scheduling problem. In their model, to schedule jobs to the bidders, each job agent submits a determined list of jobs using multiple-round combinatorial auction. Along the same line, Kwon *et al.* (2005) proposed a multiple-round combinatorial auction for truckload procurement that can be beneficial for both carriers and shippers by assigning better service allocations.

Multi-agent simulation is an efficient method to simulate and analyze auction systems (Vidal, 2007). It allows investigating the complex interactions among different kinds of agents and can handle different types of large-scale data, a common occurrence in combinatorial auctions. Moreover, investigating bidder's behavior and winner selection can be performed as modeling the multi-agent technology platform. One of the challenges in designing and simulating auction systems is the randomness of the parameters of the simulated auction model (Shoham and Leyton-Brown, 2009). We are aware of few papers that have been published regarding combinatorial auction using agent-based systems. For example, Kutanglui and Wu (2001) used combinatorial auction as an autonomous distributed scheduling system.

Finally, Farnia *et al.* (2015) presents a time-based timber combinatorial auction. To the basic timber criteria (i.e., quality and species), the authors add a timetable as a criterion in order to address harvest operations coordination directly in the winner determination process. However, experiments are limited to simulations of one combinatorial auction at a time, in order to study the impacts of various behaviors on the auction outcome. To our knowledge, there is no study of multiple-round timber combinatorial auctions. Also, as the literature reviewed, there are not any studies about the efficiency comparison of multiple-round combinatorial auctions and multiple–round single unit auctions. This paper aims to fill this gap.

3 Research objectives

In this paper, several configurations of multiple-round combinatorial auctions are studied using agent-based simulation. The basic simulation model consists in several sets of combinatorial auctions that are announced sequentially in multiple rounds. Within each round, each set of auctions is announced simultaneously. In this paper, the first goal is to investigate the effect of several configurations of the basic model on the outcome of the combinatorial auctions, i.e. price per m^3, and on the achievement level of the purchase volume target of the bidders. Furthermore, the goal of this study is to analyze the difference between multiple-round combinatorial auctions and multiple- round single-unit auctions in several configurations of the basic combinatorial auction model.

4 Multiple-round timber combinatorial auction

The proposed simulation model involves three components: an auctioneer, a seller, and bidders. The auctioneer, or government agency, manages the announcement and controls the auction as well as the Attribution process. The seller, the State in our case, offers timber products to the market. The bidders are the auction participants.

In this model, combinatorial auctions are used to sell forest stands. We assumed that each forest stand consists of four different species. The species are divided into softwood and hardwood, each with two levels of qualities. Therefore there are four different types of products, including softwood of quality one (s1), softwood of quality two (s2), hardwood of quality one (h1), and hardwood of quality two (h2). In order to make the model more realistic, each lot has other specifications that make them different from each other. These specifications include the location of the forest stand and the volume of each species. The location of each forest stand is randomly defined during simulation, while their lot size (i.e., size of forest stand) is defined at the start of the simulation. The volumes by species are randomly defined according to the lot size.

There are three types of bidders in this model: softwood mills, hardwood mills, and entrepreneurs. This segmentation is done according to the supply need of the bidders. Softwood mills require softwood, and include lumber mills and paper mills. Hardwood mills are interested in hardwoods. Entrepreneurs are interested in both softwood and hardwood products. In our model, it is considered that softwood mills mostly bid for the bundles that include s1 and s2, where s1 is softwood with quality 1 and s2 is softwood with quality 2. These bundles are considered as s1, s2, s1s2, and s1s2h1h2. Large softwood mills might bid for s1, s2, or any bundle that contains s1 and s2, while small softwood mills might bid only for either s1 or s2. If a softwood mill wins bundle s1s2h1h2, the mill resale the undesired volumes to other mills. The illustrated example also applies for hardwood mills that may be interested in h1, h2, h1h2, and s1s2h1h2. The entrepreneurs are also interested in any bundles of s1, s2, h1, and h2. In order to simplify the simulation model, and particularly the winner determination process, as well as to reduce the number of combinations that significantly increases running time, entrepreneurs only bid for s1, s2, s1s2, h1, h2, h1h2, and s1s2h1h2 bundles.

The parameters of bidder agents include the type of bidder, its location, its capacity per year (which defines their total need), supply needs for each species per year, and their own market prices (i.e., the selling price of their own products). In this simulation model, we also define several parameters that define the configuration of the auctions and the auction environment. These parameters include: number of auctions per year, auction periodicity, number of bidders and lot size. Other parameters of the model are either random within a realistic range, or fixed.

The general simulation procedure is the following. The products of several forest stands are sold over the simulation horizon (one year) via combinatorial auctions. The products of each stand are sold simultaneously during one single combinatorial auction. In other words, each stand is sold individually with one combinatorial auction, which is announced and processed individually. At the start of the simulation, the auctioneer announces several combinatorial auctions simultaneously. So each combinatorial auction concerns one specific forest stand, and bidders bid on any bundle of that stand. The auctioneer announces the available products specifying the volume of each species and the location of the corresponding stand. In order to simplify the simulation of the auction, the reserve price of each bundle is also announced, although in the case study on which we base our simulation this information remains private. For each auction, the bidders bid on any combination of products composing the stand. After receiving the bids, the auctioneer chooses the winners according to a winner determination algorithm, which is explained in section 4.2. Because the auction is combinatorial, each stand may be assigned to one or more bidders. After announcing the winners, the bidders must coordinate harvest operations. The supply needs of the winners are updated according to the product allocation, which affects their behaviour in the remaining rounds of auction. Therefore, bidder agents must be able to adapt their bidding behaviour over the course of multiple-round auctions. This adaptive learning bidding approach is presented in section 4.1.

4.1 Bidding approach

In order to have realistic simulation, the bidding pattern inspired by Farnia *et al.* (2013) uses an adaptive learning approach in the context of a single unit auction. This approach combines two types of behavior: a learning behavior and an adaptive behavior. Similarly to Farnia *et al.* (2013), we compare the adaptive learning behavior with other realistic approaches. On the one hand, the learning behavior considers the history (i.e., past rounds of the auction) to define a bidding function, which aim is to avoid over-bidding. This behavior was deemed the most profitable for the bidder. On the other hand, the adaptive behavior considers the bidder's current needs and the time left to fulfill the remaining needs. This bidding behavior was the most capable of fulfilling supply needs. As mentioned in section 3, the bidders must value each bundle of products in the lot under consideration. The sets and indexes to calculate the value of each bundle are indicated in **Table 1**.

Table 1: Sets and indexes of the model

j	bidders
$i \in G$	set of lots
$S \subseteq G$	bundles of lots
$MP_{j,S}$	maximum price of bidder j for bundle S
$NP_{j,S}$	minimum price of bidder j for bundle S
RP_S	reserve price of bundle S
V_S	volume of bundle S
MRP_S	average revenue of the final products made from S; includes the revenue of the direct resale of undesired species.
HC_S	harvest cost of bundle S
$D_{j,S}$	distance of the bundle *S* to bidder *j*
TC_S	transportation cost of bundle S
$PC_{j,S}$	average processing cost of converting the bundle *S* into the final product of bidder *j*
$PR_{j,S}$	minimum profit that bidder j is willing to gain from bundle S
$ND_{j,S}$	need of bidder *j* at time *t* for bundle *S*

The bidders need to calculate their maximum and minimum values for each bundle. The minimum value of bidder *j* for bundle S is considered to be equal to the reserve price of bundle S. The reserve price of bundle S is equal to the sum of the reserve prices of the species including in that bundle. The maximum value of bidder *j* for bundle S is shown in equation (1).

$$MP_{j,S} = V_S(MRP_S - HC_S - D_{j,S}TC_S - PC_{j,S} - PR_{j,S}) \tag{1}$$

As described in Farnia *et al.* (2013) the bidders may face several combinatorial auctions at each round of the auction. Therefore the bidders should consider a decision method to decide on which auction to participate. The decision problem is described as the following binary integer programming.

maximize:

$$\sum_{S} \frac{V_S}{D_{j,S}}\, x_S \tag{2}$$

subject to:

$$MP_{j,S}\, x_S > RP_S \qquad \forall\, S \tag{3}$$

$$\sum_{S} V_S x_S < ND_{j,S} \tag{4}$$

$$x_S = \{0,1\} \qquad \forall\ S \tag{5}$$

In this mathematical programming, bidder j assigns a weight to each desirable bundle at each round. In order to maximize the volume while minimizing the distance to obtain the bundle, the bidder defines a weight as the volume of the bundle over the distance of the bundle to the bidder's mill (equation 2). The first constraint (3) confirms that the bidder considers only the feasible bundles (the maximum price of the bidder is higher than the reserved price of the bundle). In order to avoid bidding on more items than needed, constraint (4) ensures that the sum of the selected bundles is less than the bidder's need, which also considers a small buffer to account for lost bids. Equation (5) shows the binary constraint.

The valuation function of the bidders' bidding approach is described in equation (6), as suggested in Farnia *et al.* (2013).

$$v_{j,S} = \alpha\left(\frac{MP_{j,S} - NP_{j,S}}{2}\right) * tanh\left(\frac{f_1(y)}{f_2(d)} * \frac{f_{j3}(n)}{f_{j4}(c)} - 2\right) + \beta\left(\frac{MP_{j,S} + NP_{j,S}}{2}\right) + (1-\ \beta)y_{j,S} \tag{6}$$

In this equation, $v_{j,S}$ is the valuation function of bidder j for bundle S, $f_1(y)$ is the total duration of the procurement process to achieve a target supply volume (i.e. a year), $f_2(d)$ is the duration of the remaining time at any specific moment in the simulation to achieve that target. Next, $f_{j4}(c)$ and $f_{j3}(n)$ are respectively the target supply volume of bidder j, and the remaining volume at any specific moment in the simulation to achieve the target supply volume. The first two elements of equation (6) represent the adaptive part of the valuation function. The target supply volume of a bidder is a global target (for all species).

$y_{i,S}$ is the learning part of the adaptive learning behaviour. For example, if S consists of species s1 and s2, $y_{i,S}$ for these species is shown in equation (7). We estimate the coefficients of this equation using a regression on the history of the auction. More details can be found in Farnia *et al* (2013). At each round, the bidders consider the winning history and extract the information including the location of the winner and the winning price for each bundle. As each bundle is considered separately, there are less and less data available to compute equation 7. This may affect the capacity of the learning part to anticipate a valid bidding price. Eventually, when not enough data is available, the bidders estimate the bidding price of a bundle according to the price history of single species.

More specifically, the bidders estimate the coefficients (β_0, β_1, $\beta_2 \ldots$) of the regression function at each round and for each bundle, adding the most recent information about the winning bids. Next, once the coefficients are estimated, and using the information of the current auction, the bidder anticipates the value $y_{j,s1s2}$ of the bundle, with $x_{j,s1}$ and $x_{j,s2}$ being the volume of s1 and the volume of s2 in bundle s1s2, respectively, using equation (7). Next, the offer of bidder $v_{j,s1,s2}$ is calculated with equation (6).

$$y_{j,s1s2} = \beta_0 + \beta_1 D_{j,s1s2} + \beta_2 x_{j,s1} + \beta_3 x_{j,s2} \tag{7}$$

In Equation (6), α and β are defined as coefficients that vary within the interval [0;1]. In this model the bidders randomly behave according to the different values of α and β. The valuation function is purely adaptive if $\alpha = \beta = 1$. When $\alpha = \beta = 0$, the valuation function is strictly based on learning.

4.2 Winner determination

This section describes the winner determination algorithm for the simulated combinatorial auction. As previously explained in Section 3, softwood mills are interested in softwoods (*s1* and *s2*). According to their size and their supply needs, the mills are interested in *s1*, *s2*, *s1s2*, and *s1s2h1h2* (i.e. whole forest stand) bundles. For example if the mill is smaller, it will make an offer for only *s1* or *s2*, while a larger mill, or one needing more supplies, will also make an offer for *s1s2*. Mills which do not wish to collaborate with others at the time of harvest, will also make an offer for *s1s2h1h2*. That way, if they win the bundle, they will control harvest operation planning. Similarly, hardwood mills make offers for *h1*, *h2*, *h1h2*, and *s1s2h1h2* bundles. Entrepreneurs can also make offers for these bundles, as their mission is to sell timbers to all kinds of mills. The following algorithm shows the winner determination procedure that is triggered within the simulation to determine the solution of each auction.

Let

J be number of bidders;
$v_{j,s1}$ *be bidder j's* value of s1;
$v_{j,s2}$ *be bidder j's* value of s2;
$v_{j,h1}$ *be bidder j's* value of h1;
$v_{j,h2}$ *be bidder j's* value of h2;
$v_{j,s1s2}$ *be bidder j's* value of s1s2;
$v_{j,h1h2}$ *be bidder j's* value of h1h2;
$v_{j,s1s2h1h2}$ *be bidder j's* value of s1s2h1h2;
w_{s1} *be the winner of s1;*
w_{s2} *be the winner of s2;*
w_{h1} *be the winner of h1;*
w_{h2} *be the winner of h2;*

```
[
vs1=vs2=vh1=vh2=vs1s2=vh1h2=0

for j=1 to J do
      vs1 = max (vs1, vj,s1)
      vs2 = max (vs2, vj,s2)
      vh1 = max (vh1, vj,h1)
      vh2 = max (vh2, vj,h2)
      vs1,s2 = max (vs1,s2, vj,s1,s2)
      vh1,h2 = max (vh1,h2, vj,h1,h2)
      vs1s2h1,h2 = max (vs1s2h1,h2 , vj,s1s2h1h2)
end for

If vs1 + vs2 > vs1s2 then
   ws1 = j | vs1=vj,s1 and j ∈ [1,J]
   ws2 = j | vs2=vj,s2 and j ∈ [1,J]
   vs1s2 = vs1 + vs2
else
   ws1 = ws2 = j | vs1s2 = vj,s1s2 and j ∈ [1,J]
end if

If vh1 + vh2 > vh1h2 then
   wh1 = j | vh1=vj,h1 and j ∈ [1,J]
   wh2 = j | vh2=vj,h2 and j ∈ [1,J]
   vh1h2 = vh1 + vh2
else
   wh1 = wh2 = j | vh1h2 = vj,h1h2 and j ∈ [1,J]
end if
```

If $v_{s1s2} + v_{h1h2} < v_{s1s2h1h2}$ **then**
$w_{s1} = w_{s2} = w_{h1} = w_{h2} = j \mid v_{s1s2h1h2} = v_{j,s1s2h1h2}$ *and* $j \in [1,J]$
end if
]

The outcome of this simple procedure selects w_{s1}, w_{s2}, w_{h1} *and* w_{h2}, which are the winners of species s1, s2, h1, and h2 respectively. The winners of each species can be similar or different. This procedure only aims to identify the highest value of a fix and small number of combinations. In reality, the number of combinations can be much higher and required more advanced algorithms.

5 Results and discussion

Four experiments using the simulated model are presented in this section. We will first examine the outcomes of the simulation in terms of price per m^3 for the seller and then in terms of target achievement for the bidders. In both cases we will first present the results for combinatorial auctions in various settings. They are then compared with those obtained with single-unit auctions.

In this study, the first objective is to evaluate how combinatorial auctions can benefit the seller in different setup configurations (Experiment 1). Next, target achievement is analyzed in order to know how much the buyer can fulfill its needs through the combinatorial auction (Experiment 2). We then aim at comparing the revenue (i.e., price per m^3) generated in both the combinatorial and single-unit auctions (Experiment 3). Finally, the fourth objective is the compare the ability of companies to achieve their target supply needs with both combinatorial and single-unit auctions (Experiment 4).

5.1 Experiment 1 – Price per m^3 in combinatorial auctions

This part of the experiments describes a sensitivity analysis of the impacts of several parameters on the price per m^3 of the combinatorial auction. The parameters that can be changed in the model are the number of auctions per year, the auction periodicity, the lot size, and the number of bidders. In order to assess the impacts of these parameters on the revenue, three levels (low, medium and high) are defined for each parameter. The levels for the number of auctions per year are 100, 250, and 400. The periodicity levels are defined as 7, 15 and 30 days. The three lot sizes are 10,000 15,000 and 20,000 m^3. Finally, the number of bidders is set to 100, 150, and 200. These values are inspired by actual data from timber auctions in Quebec. In this study, within each simulation, lot size is the same for all lots. Therefore, there is no possible scale economy to be gained from large lots. This limitation does not affect the general results, although it limits our ability to properly evaluate the impacts of simultaneous auctions with variable lot sizes.

All 3^4 (81) configurations of these parameter levels were tested. For each configuration, the experiments are repeated 25 times for a total of 2,025 experiments. Table 2 shows the analysis of variance for the price per m^3 resulting from the combinatorial auctions. In order to simplify the analysis, these studies only analyze the effects of each parameter and all combinations of any two independent variables. The ANOVA studies show a R^2 above 0.80, which indicates a reasonable level of statistical certainty. The results show that the only significant parameters are the number of auctions, the lot size, the number of bidders, and the combination of number of auctions X lot size.

Figure 1 shows the effects of each parameter. As it is shown, when the number of auctions increases, the price per m^3 decreases due to more supply on the market. The price per m^3 diminishes when the lots get larger, again because of more supply. When number of bidders increases, the price per m^3 goes up.

Table 2: Analysis of variance for price per^3 from the combinatorial auctions

Number of observations = 2025 R^2 =0.800016

Root MSE = 0.104238 Adjusted R^2 =0.796803

Source	***Partial SS***	***MS***	***F***	***Prob > F***
Model	*86.5861*	*2.70582*	*249.0248*	***<.0001***
Number of Auctions	*0.327819*	*0.16391*	*15.0851*	***<.0001***
Periodicity	*0.016152*	*0.008076*	*0.7432*	*0.4757*
Lot Size	*0.168619*	*0.084309*	*7.7593*	***0.0004***
Number of Bidders	*10.991764*	*5.495882*	*505.8035*	***<.0001***
Number of Auctions # Periodicity	*0.021667*	*0.005417*	*0.4985*	*0.7369*
Number of Auctions # Lot Size	*0.114439*	*0.02861*	*2.633*	***0.0327***
Number of Auctions # Number of Bidders	*0.014781*	*0.003695*	*0.3401*	*0.851*
Periodicity # Lot Size	*0.011365*	*0.002841*	*0.2615*	*0.9027*
Periodicity # Number of Bidders	*0.046794*	*0.011698*	*1.0766*	*0.3664*
Lot Size # Number of Bidders	*0.044701*	*0.011175*	*1.0285*	*0.3911*
Residual	*21.64437*	*0.01087*		
Total	*108.23046*			

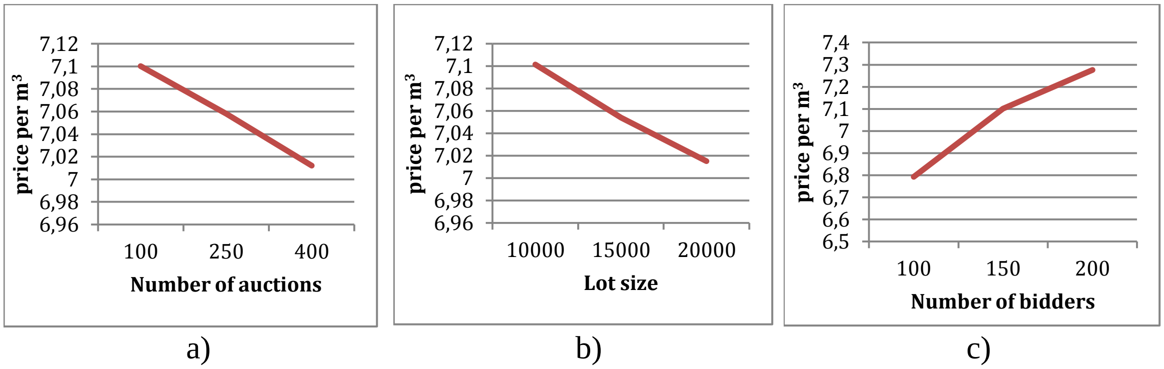

Figure 1: Price per m^3 from the combinatorial auctions; (a) impact of the number of auctions; (b) impact of the lot size; (c) impact of the number of bidders.

Figure 2 describes how a pair of independent variables affects the outcome of the auction. In particular, this figure shows that when the number of auctions increases, the price per m^3 decreases, even if lot size changes, but the price is lower when lot size gets larger. The reason for this result is that both number of auctions and lot size significantly increase supply. Therefore, the combination of these two parameters affects the price per m^3.

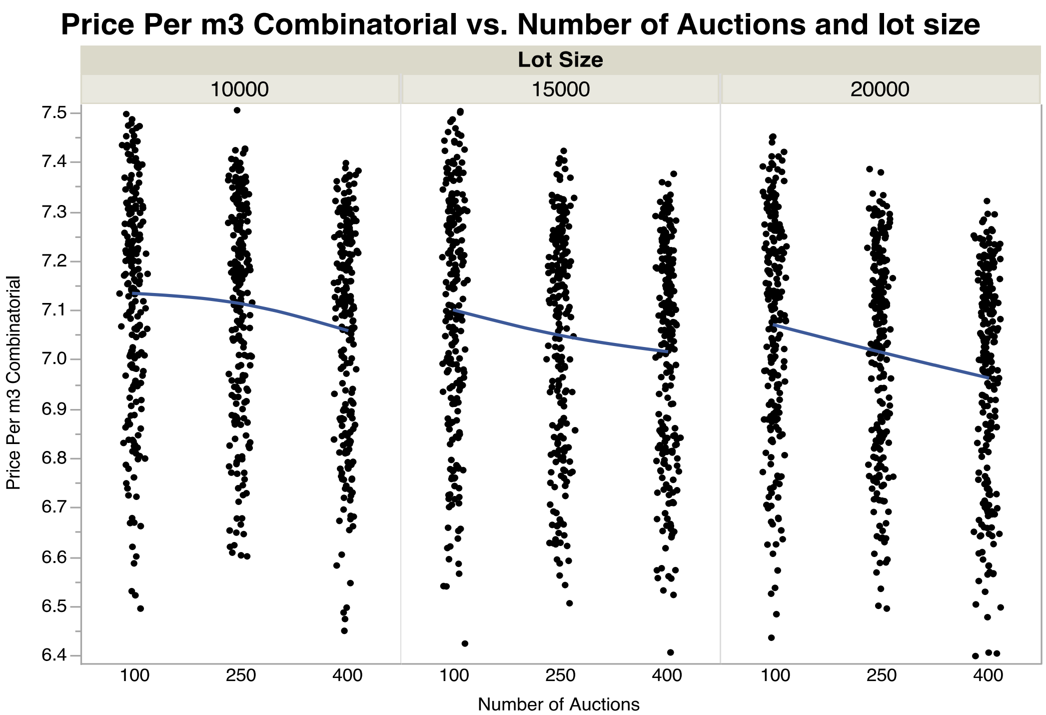

Figure 2: Comparative analysis of the price per m^3 from the combinatorial auctions: combined effect of number of auctions and lot size

5.2 Experiment 2 – Bidders' target achievement with combinatorial auctions

Target achievement is a dependent variable that represents the ability of the bidders to achieve their target supply volume. This experiment investigates the impacts of a combinatorial auction on this variable. Again, we analyzed the effect of the same parameters (number of auctions, periodicity, lot size, and number of bidders) on the target achievement of the bidders. Table 3 shows the analysis of variance of the parameters on the target achievement. As it is shown in this table, the parameters contribute to explaining more than 98% of the variance of the target achievement. All the analyzed parameters have a significant effect.

Table 3: Analysis of variance for the target achievement of bidders with combinatorial auctions

Number of observations = *2025* *R^2 =0.984379*
Root MSE = *0.022816* *Adjusted R^2 =0.984128*

Source	***Partial SS***	***DF***	***MS***	***F***	***Prob > F***
Model	*65.345546*	*32*	*2.04205*	*3922.827*	***<.0001***
Number of Auctions	*6.8862696*	*2*	*3.443135*	*6614.349*	***<.0001***
Periodicity	*0.0279949*	*2*	*0.013997*	*26.8894*	***<.0001***
Lot Size	*1.0181216*	*2*	*0.509061*	*977.9187*	***<.0001***
Number of Bidders	*0.053689*	*2*	*0.026844*	*51.5689*	***<.0001***
Number of Auctions # Periodicity	*0.2343491*	*4*	*0.058587*	*112.5476*	***<.0001***
Number of Auctions # Lot Size	*2.2101542*	*4*	*0.552539*	*1061.441*	***<.0001***
Number of Auctions # Number of Bidders	*2.9138346*	*4*	*0.728459*	*1399.388*	***<.0001***
Periodicity # Lot Size	*0.036632*	*4*	*0.009158*	*17.5927*	***<.0001***
Periodicity # Number of Bidders	*0.0523244*	*4*	*0.013081*	*25.1291*	***<.0001***
Lot Size # Number of Bidders	*0.8238443*	*4*	*0.205961*	*395.6564*	***<.0001***
Residual	*1.036946*	*1992*	*0.00052*		
Total	*66.382493*	*2024*			

To illustrate the analysis of variance, Figure 3 shows the effects of single parameters. Number of auctions and lot size show a positive impact, while periodicity and number of bidders have a negative impact on the target achievement. The reasons for the positive impact is that there is more timbers through the auction system, therefore, the bidders can win more wood supply to fulfill their needs. Similarly, when number of bidders rises, the auction becomes more competitive and the chance of winning drops. When periodicity is higher, there are more auctions at each round. Therefore, the bidders have more auctions to decide at each round. The bidders tend to bid only on the amount they need, since they think they may win; hence, they might loose some opportunities by not participating in other auctions. Therefore, it is probable that bidders win lower volumes when there are more auctions in one round.

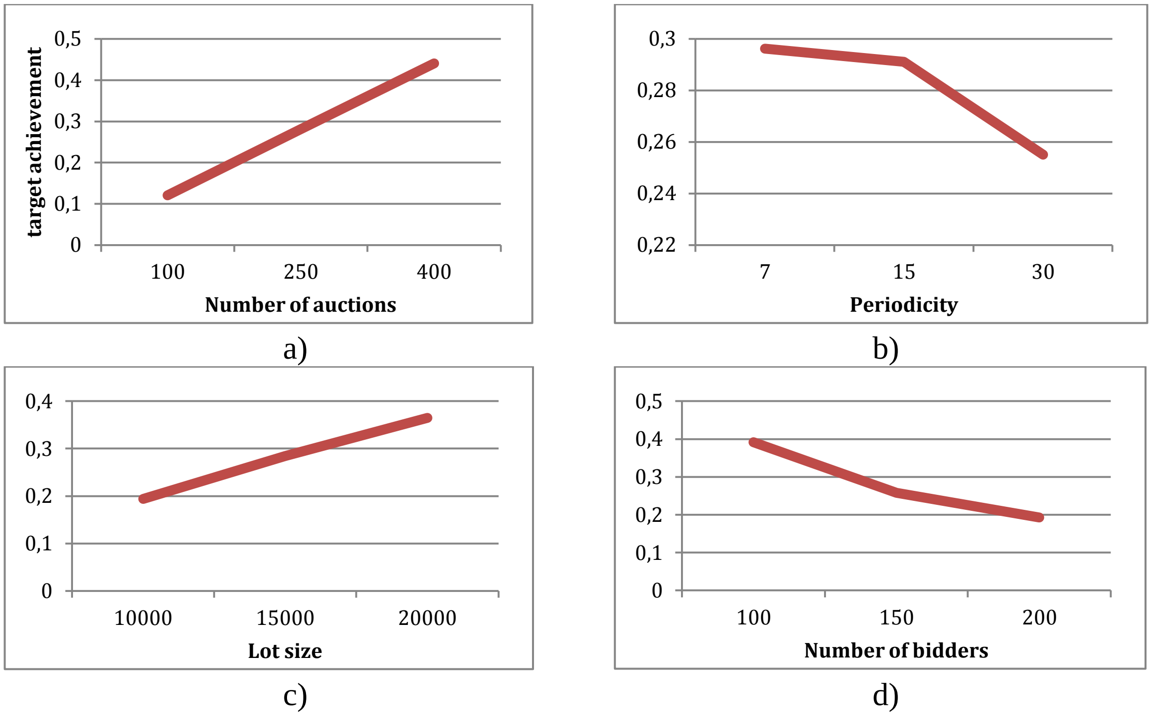

Figure 3: analysis of the target achievement of bidders from combinatorial auctions: (a) effect of the number of auctions; (b) effect of periodicity; (c) effect of lot size; (d) effect of the number of bidders

Figure 4 shows the comparative analysis of target achievement. As shown, any combination of two parameters has a significant effect on target achievement of combinatorial auction.

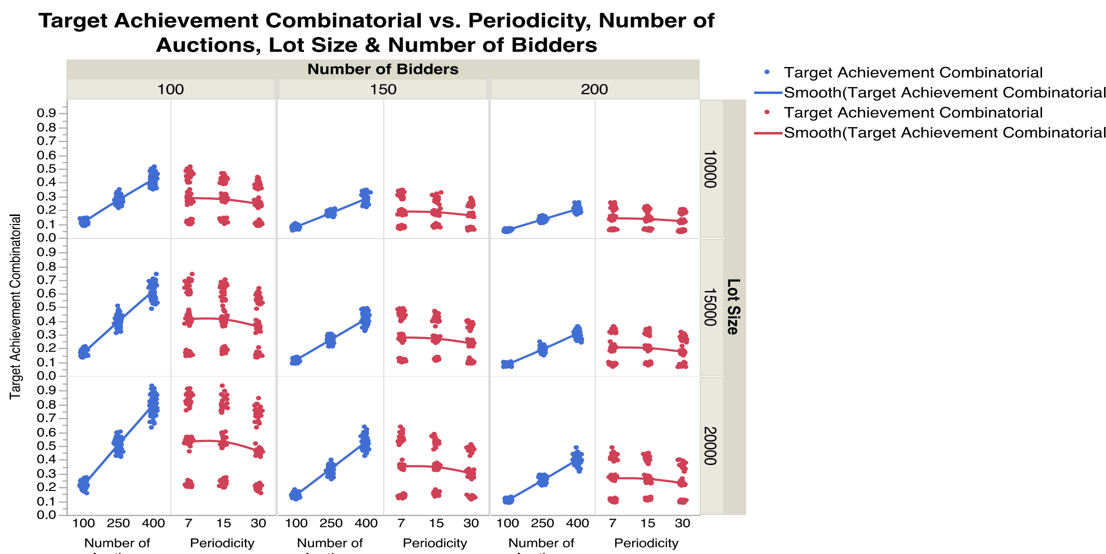

Figure 4: Comparative analysis of the target achievement of bidders from combinatorial auctions: combined effects of each two parameter.

5.3 Experiment 3 – Price per m^3 – comparison between combinatorial and single-unit auctions

In this section, we compare the combinatorial auction and the single-unit auction and analyse specifically the impact on revenue (i.e., price per m^3) using different setup configurations. We define this impact as the gain of using combinatorial auctions expressed as a percentage of the average price per m^3 of single unit auction ((price per m^3 of combinatorial auction / price per m^3 of single unit auction - 1) * 100). In order to perform a sensitivity analysis, we again consider the number of auctions, periodicity, lot size, and the number of bidders. Figure 5 displays the impact of these parameters.

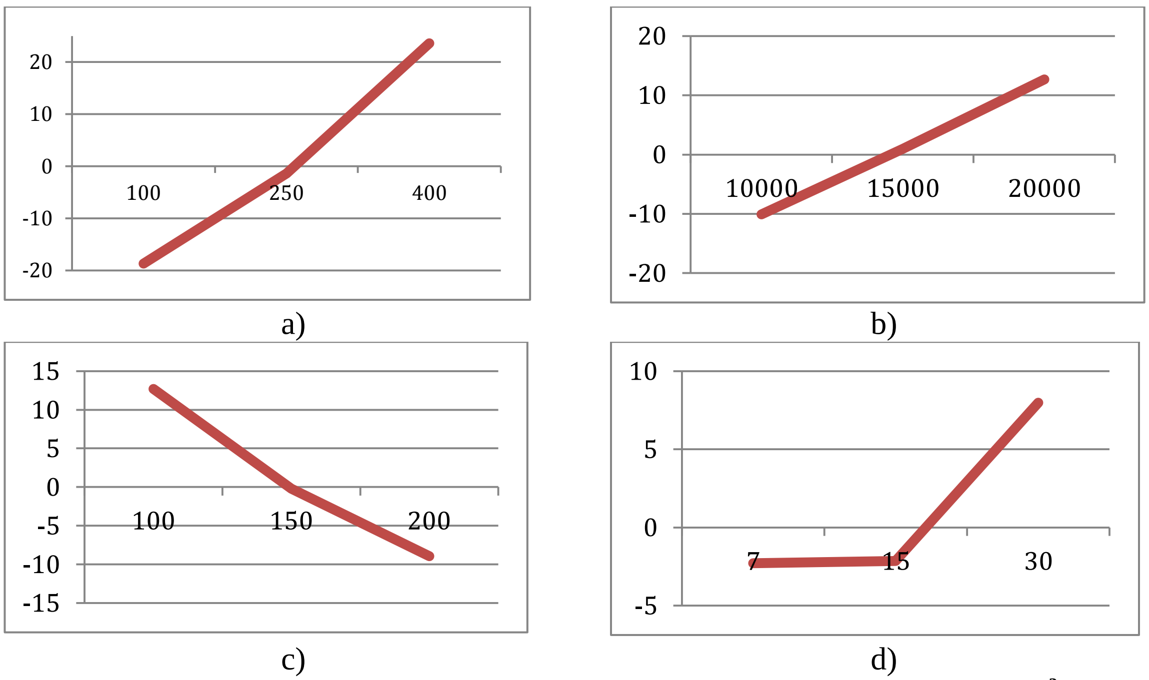

Figure 5: percentage change (combinatorial auction over single-unit auction) in price per m^3 as a function of (a) the number of auctions (b) lot size (c) the number of bidders, and (d) periodicity

First, the gain increases as the number of auctions rises. In other words, the combinatorial auction is preferable to the single-unit auction as the number of auctions increases. Similarly, the combinatorial auction is preferable to single-unit auction, when there are fewer bidders. The results show that the difference between combinatorial auction and single-unit auction is higher, when the number of auctions and the number of bidders are in non-equivalent market situation. In other words, when there is a considerable difference between potential supply and potential demand, the difference between two auctions is higher. For example, when there are 250 auctions and 150 bidders in the market there is not considerable difference between the outcomes of two types of auctions. The comparative analysis of these parameters is detailed in experiment 4 along with comparative analysis of the effects of the parameters on target achievement.

5.4 Experiment 4 – Target achievement of bidders – comparison between combinatorial and single-unit auctions

The impact on the target achievement of bidders between combinatorial and single-unit auctions is measured as the target achievement increase from using combinatorial auction expressed as a percentage of the target achievement of single unit auction ((target achievement of combinatorial auction / target achievement of single unit auction -1) * 100). Figure 6 presents the impact of each parameter on target achievement.

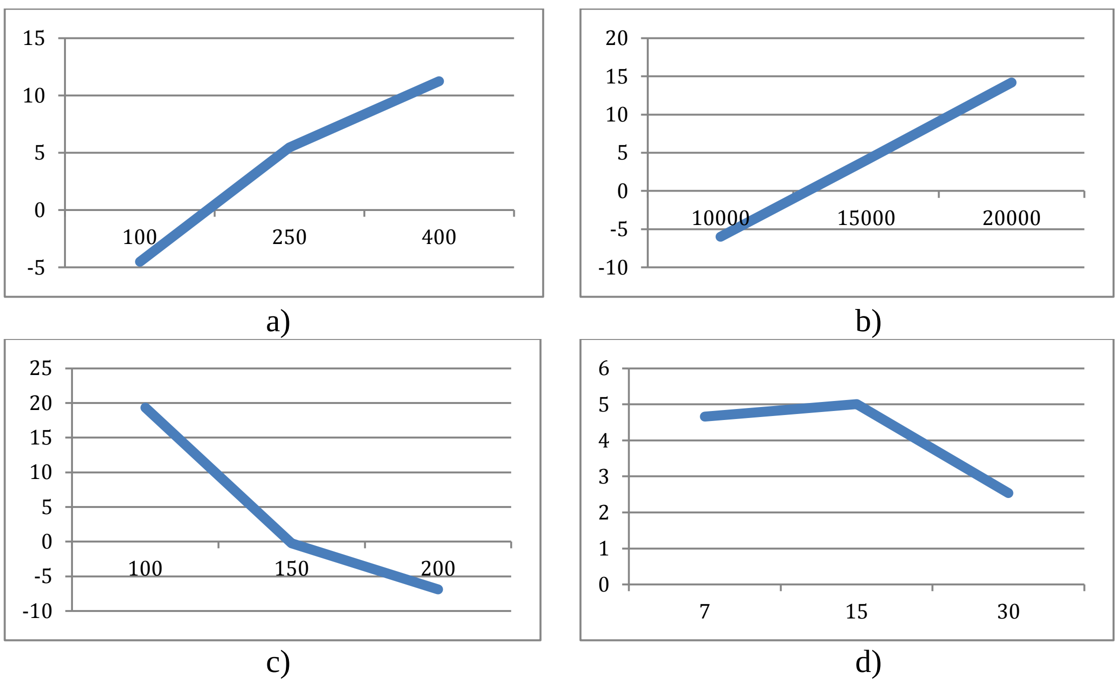

Figure 6: percentage change (combinatorial auction over single-unit auction) in target achievement as a function of (a) the number of auctions (b) lot size (c) the number of bidders, and (d) periodicity

The combinatorial auction is preferable to single-unit auction when the number of auctions is high and the number of bidders is low. Therefore, the combinatorial auction is better than single-unit auction for bidders, when the market is less competitive. Similarly, bidders will prefer combinatorial auction when lot size is larger, or there are more items in the market.

Figure 7 presents the comparative analysis of the impact on sale price and target achievement with any two combinations of auction design parameters (i.e., the number of bidders and the number of auctions, the number of bidders and lot size, the number of bidders and periodicity, the number of auctions and lot size, the number of auctions and periodicity, and lot size and periodicity). The results show that the impact on sale price increases when there are more auctions, while lots are large and there are fewer bidders. Figure 7 illustrates that the auctioneer can obtain higher price through combinatorial auction, when the market is less competitive (i.e., more demand and less supply). As it is shown, the combinatorial auction is better (in terms of both sale price and target achievement) in situations when there are more auctions, few rounds of auction (higher periodicity), lager lots, and fewer bidders.

Comparing the comparative analysis of any two combinations of parameters between the sale price and the target achievement (e.g. figure 7(a) and figure 7(b)), it can easily be observed in Figure 7 that this correlation is positive, which tends to show that both objectives can be achieved simultaneously. For example, figure 7(a) and figure 7(b) present that combinatorial auction is preferable over single-unit auction when the combination of “the number of bidders” and “the number of auctions” are 100 and 250, 100 and 400, and 150 and 400 auctions per year.

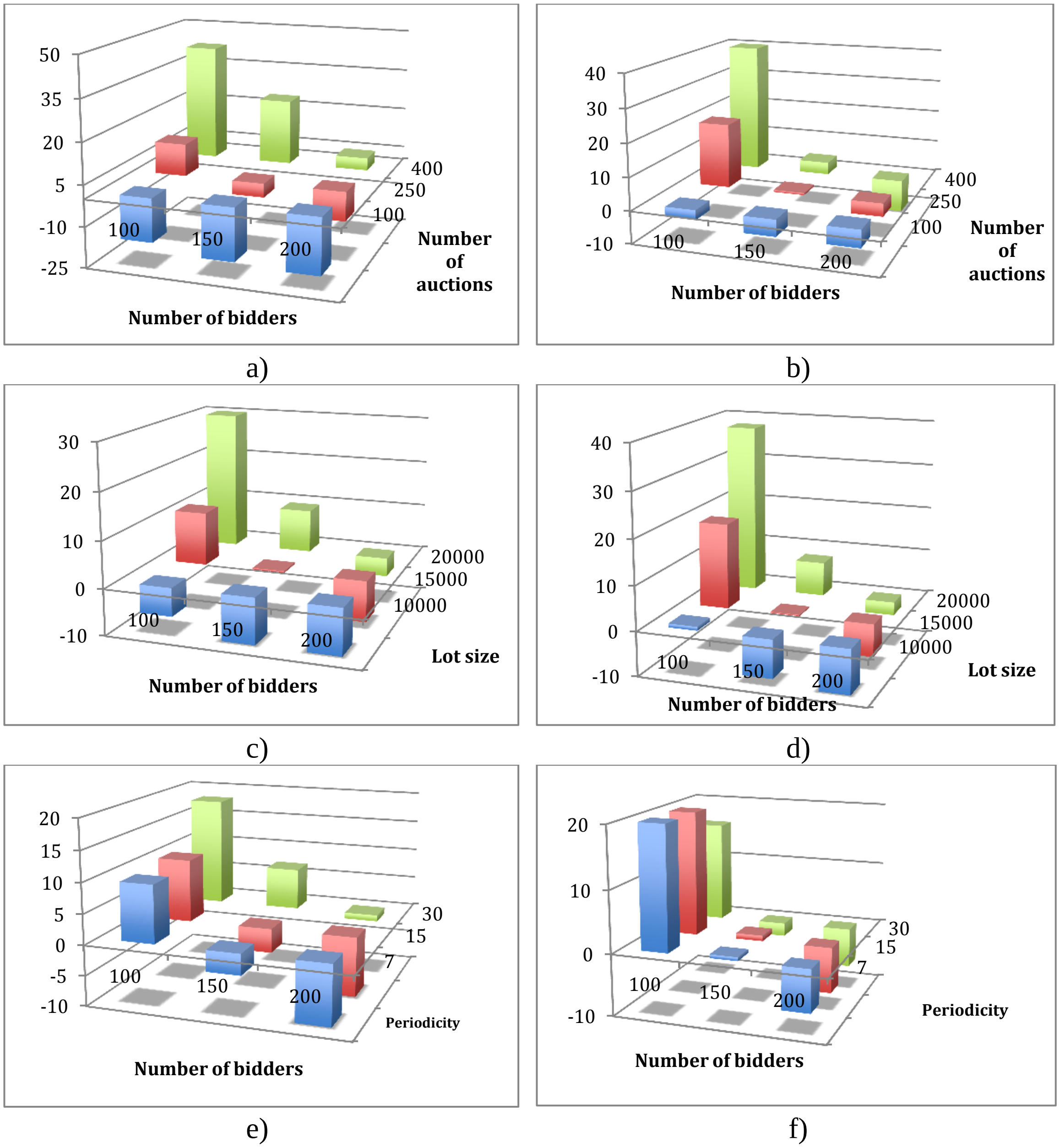

Figure 7(part 1): Comparative analysis of the impacts on sale price and target achievement (Part 1). (a) Price per m^3: combined effects of number of bidders and number of auctions. (b) Target achievement: combined effects of number of bidders and number of auctions. (c) Price per m^3: combined effects of number of bidders and lot size. (d) Target achievement: combined effects of number of bidders and lot size. (e) Price per m^3: combined effects of number of bidders and periodicity. (f) Target achievement: combined effects of number of bidders and periodicity.

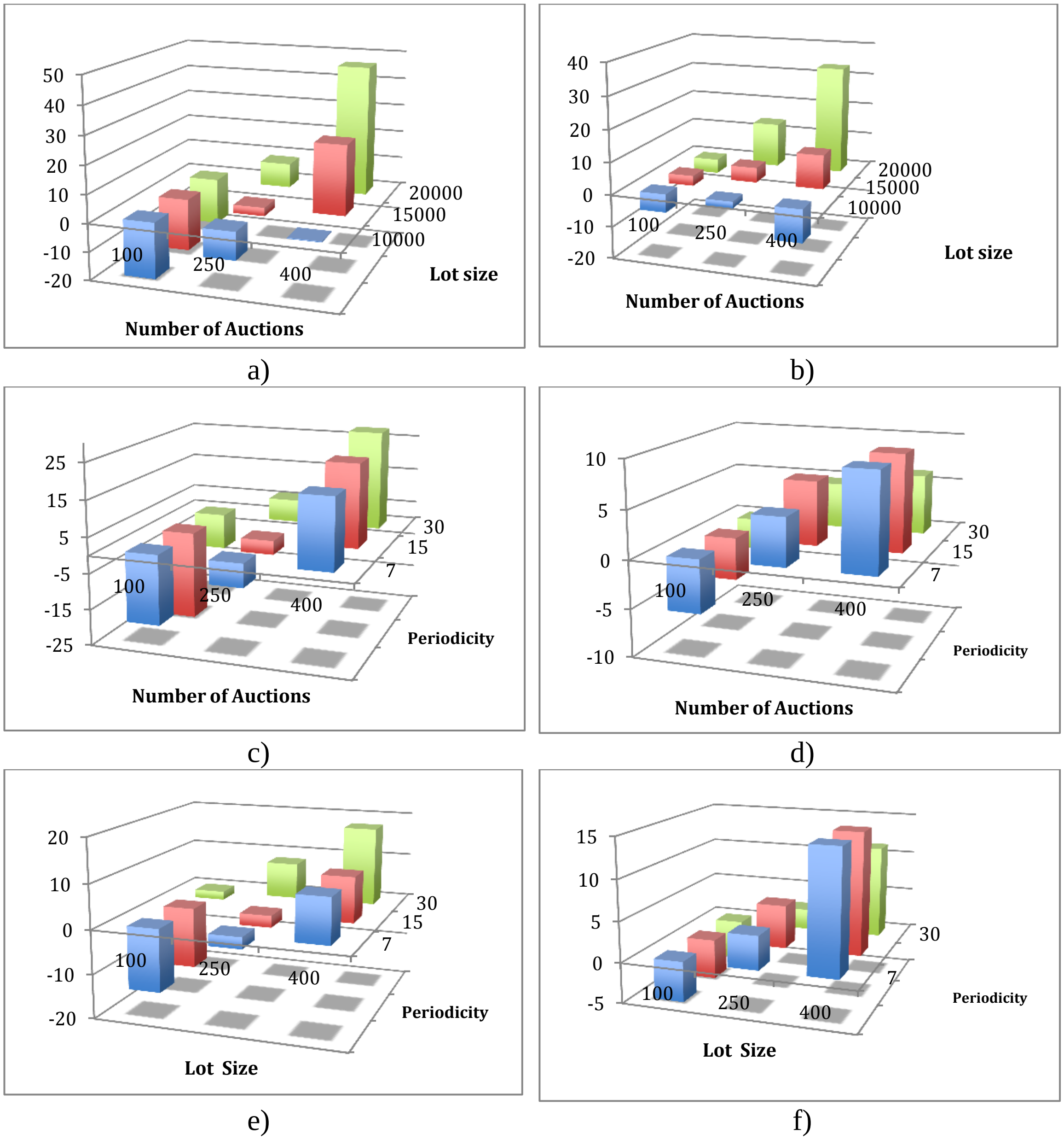

Figure 7 (part 2): Comparative analysis of the impact on sale price and target achievement (Part 2). (a) Price per m^3: combined effects of number of auctions and lot size. (b) Target achievement: combined effects of number of auctions and lot size. (c) Price per m^3: combined effects of number of auctions and periodicity. (d) Target achievement: combined effects of number of auctions and periodicity. (e) Price per m^3: combined effects of lot size and periodicity. (f) Target achievement: combined effects of lot size and periodicity.

6 Conclusion and future studies

This paper presents a study of a multiple-round timber combinatorial auction using a multi-agent simulation platform. The performance of the auction was experimented in various setup configurations, to confirm that the simulated model provides realistic outcomes. Two main indicators were proposed in order to measure the performance of the model. Sale price per m^3 evaluates the auctioneer capability to gain generate revenue from the auctions. Target achievement evaluates the bidder capability to fulfill their needs from auctions. We also analyzed the indicators for validating the performance of combinatorial auction, and the performance of the comparison of combinatorial auction with single-unit auction.

Our results show that the design auction parameters that effect on the sale price of combinatorial auction are the number of auctions, the number of bidders and lot size. In combinatorial auction price per m^3 is higher when the market is less competitive. The bidders' target achievement also can be effected by the number of auctions, the number of bidders, periodicity, and lot size and combined effects of each two parameter.

The sensitivity analysis of the auctions comparisons also illustrates that the combinatorial auction is preferable over single-unit auction in less competitive situations. The reason is that in combinatorial auction when there are more auctions and fewer bidders, the bidders can bid on variety of bundles of the species, which are related to their needs, without bidding on the species that they don't need. In other words, in combinatorial auction the bidders can fulfill their needs with a combination of auctions, while in single unit auction they must bid on fewer auctions and win species that they don't necessarily need.

Both objectives of timer auction (sale price and target achievement) can be achieved simultaneously following the results of the comparative analysis of any two combinations of parameters. That means the combinatorial auction is better than single unit auction in terms of both objectives when there are more auctions, few rounds of auction (higher periodicity), lager lots, and fewer bidders.

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