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Continuous Approximation Models for the Fleet Replacement and Composition Problem

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Abstract. Many goods distribution companies need to determine which vehicles to purchase and when. The resulting fleet is then routed to serve a set of geographically dispersed customers. In this paper we study the fleet replacement and composition problem while explicitly accounting for vehicle routing costs. In particular, we account for vehicle purchasing cost, maintenance cost, salvage revenue and routing costs. The latter is modelled via continuous approximation. We consider a finite planning horizon, throughout which we optimize the fleet replacement and composition. The resulting fleet is used to serve a subset of customers in a rectangular service region. We assume that unserved customers are outsourced at a cost. We study the problem for homogeneous as well as heterogeneous fleets, present formulations for the special case of a single period, and extend them to construct formulations for multiple periods. We provide theoretical properties of our models and their solutions. Finally, we derive and present managerial insights based on a series of computational experiments.

Keywords: Vehicle routing problem, continuous approximation, fleet composition, fleet sizing, fleet management, fleet replacement.

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1. Introduction

Many goods distribution companies operate a fleet of vehicles on a daily basis to serve a set of geographically dispersed customers. The strategic decisions that pertain to the fleet replacement and composition i.e., the types and numbers of vehicles purchased, used and sold, are fundamentally important as they determine the cost of sustaining the fleet, as well as the total available vehicle capacity to meet customer demand. Therefore, coupling fleet replacement and composition decisions with routing costs results in a comprehensive fleet management framework. In this paper we study the multi-period fleet replacement and composition problems while simultaneously accounting for vehicle routing decisions.

Given a fleet composition, efficient use of the vehicles on a daily basis is essential. Minimizing the daily operational costs are often achieved through solving the Vehicle Routing Problem (VRP), which aims to find feasible vehicle routes that satisfy customer demand at a minimum cost. The VRP has been studied extensively (Toth and Vigo, 2014; Laporte, 2009). Several papers have addressed the issue of fleet sizing at the operational level, i.e., determining the fleet size required on a particular day with a given demand (Hall and Racer, 1995; Powell and Carvalho, 1998). However, significantly less research has been carried out on the strategic fleet replacement and composition problem. We note that while detailed operational decisions are taken on a daily basis, fleet investment decisions are made in a time frame of several years. Therefore, long term fleet composition decisions are out of the scope of the operational fleet sizing literature.

Incorporating very detailed operational decisions within the strategic fleet composition problem is impractical, due to the difficulty of accurately forecasting the data. For a related set of problems, Francis and Smilowitz (2006) advocate the development of continuous approximation models when the data being used is aggregated. Data aggregation smooths out minor variations in input parameters, which are less critical in strategic planning. In addition, aggregate data is easier to forecast than detailed data and continuous approximations yield simplified models that are straightforward to reproduce. The wide use of continuous approximation models in freight distribution (Langevin et al., 1996) provides yet another incentive for their application to our problem. For these reasons, we opt to use a continuous approximation to model our problem.

The scientific contribution of this paper is the development of strategic models for fleet replacement and composition problem. The model takes vehicle purchasing cost, maintenance cost, salvage revenue and routing costs into account. The latter is modelled via continuous approximation based upon the seminal works of Daganzo (1984a,b, 2005). We consider a rectangular service region and determine the optimal fleet replacement and composition to service a subset of customers over a finite planning horizon. As is common in fleet sizing, we also consider the option of outsourcing customer service (Hoff et al., 2010). To the best of our knowledge, continuous approximation models have not been utilized for the vehicle fleet replacement and composition problem.

Considering homogeneous set of vehicles, we first introduce and model the single period Fleet Sizing and Routing Problem (FSR), which we use as a stepping stone for designing the model for the homogeneous multi-period fleet replacement and composition problem (MRC). Finally we introduce the heterogeneous multi-period fleet replacement and composition problem (HMRC). While the latter problem encompasses the first two problems as special cases, we see merit in presenting the three problems as each has a practical value.

The remainder of this paper is organized as follows. In Section 2 we survey the relevant literature. In Section 3 we develop the model for the FSR, which determines the optimal number of vehicles to be used in a single period. We develop the MRC model in Section 4 and we present the HMRC model in Section 5. Our computational results are presented in Section 6. Concluding remarks and future research directions are provided in Section 7.

2. Literature review

There are only three papers that have attempted to solve the joint problem of fleet sizing and routing, which we briefly review below. The authors of these studies did not name their problem with an acronym. For the sake of brevity, we provide acronyms for each problem that best fit the title of the paper they are studied.

The earliest study that we are aware of is by Beaujon and Turnquist (1991), where the authors develop a model for the Fleet Sizing and Vehicle Allocation Problem (FSVAP), in which the objective is to determine an optimal initial fleet size, the locations of vehicle pools, and the movements of the fleet over a planning horizon. Their model involves random variables to account for the nondeterministic nature of transportation requests. They also develop an expected value formulation and a network approximation model. Their models do not account for fleet sizing decisions other than the initial period.

Koo et al. (2005) attempt to solve the Fleet Sizing and Vehicle Routing Problem (FSVRP) that arises within a container terminal. They provide a heuristic algorithm that determines routing decisions through tabu search, and increases the fleet size by one if no feasible routing can be found. Their planning horizon is limited to a single day, and they do not consider purchasing costs, maintenance cost, or salvage revenue.

Finally, Li and Tao (2010) study the Fleet Sizing and Vehicle Transfer Problem (FSVTP) of a car rental company that operates in two cities. The fleet size is determined in the beginning of the planning horizon and is kept constant until the end. The routing decisions are limited to the number of vehicles to be transferred between two cities at the end of every working day.

There are three main topics in the literature that are related to our problem. The first topic is vehicle replacement, which aims at finding optimal replacement cycles for the vehicles in a fleet. The second topic is fleet sizing, in which the objective is to find the number and mix of vehicles required to serve a number of customers with minimum total cost. Finally, the third topic is VRP, which models the daily operational problem of determining minimum cost routes through which the customers are served. In what follows, we present a set of studies from each of these topics, in the order given above.

2.1 Literature on Vehicle Replacement

Vehicle replacement strategies have been traditionally based on finding optimal replacement cycles while accounting for purchasing cost, maintenance cost, and resale cost (e.g. Eilon et al. (1966), Christer and Goodbody (1980), and Scarf and Bouamra (1999)). Redmer (2009) elaborated the notion of optimal replacement cycles by accounting for a time-decreasing utilization intensity of vehicles by distribution companies. While such models may be suitable for equipment used for stable operations, they do not capture the complexity and costs entailed by operating vehicles in distribution activities.

Suzuki and Gregory (2005) emphasize that using optimal replacement cycles implicitly assumes that companies can apply this pattern to all of their vehicles, irrespective of any cash availability. Therefore, the long-term solution of optimal replacement cycle is not necessarily implementable. The authors also develop a Multi-Year Planning Model for Vehicle Replacement (MYPVR). The model considers vehicle annual maintenance, insurance and down-time costs. Customer demand is assumed to be constant and must be met by a fixed number of vehicles throughout a finite planning period. Thus, the operating costs may vary solely with the age of the vehicle. The model is solved through well-known integer programming algorithms, and sensitivity analysis is performed on the relaxed model where the integrality constraints are eliminated.

2.2 Literature on Fleet Sizing

Fleet sizing, as described by Dejax and Crainic (1987), aims to solve the problem of finding the optimal number of vehicles needed to satisfy demand with respect to a criterion, often to minimize the total cost, given the demand for loaded trips and their characteristics. While this definition is broad, the research in this area does not consider less-than-truckload distribution activities, i.e., no routing decisions are involved. Furthermore, fleet sizing problems consider leasing or outsourcing customer demand, in the event that the vehicle capacity is not sufficient to serve all customer demand. Hall and Racer (1995) mention that, in the retailing and manufacturing sectors, private fleet operators often use common carriers for distribution outside their normal area of coverage, or for distribution to locations that do not generate an adequate volume of shipments to justify private carriage. We adopt a similar assumption in this study.

List et al. (2003) argue that vehicles are generally long-lived assets, and there is intrinsic uncertainty about the demands that they will serve over their lifetime. The authors therefore introduce the problem of Fleet Planning under Uncertainty (FPU), which they handle via a robust optimization model. Also, the authors assume that customer demand might be deferred or delayed with adequate costs. Wu et al. (2005) study fleet sizing in the context of the truck-rental industry. Considering a multi period planning horizon, in which the demand requires loaded truck movements over time, the authors introduce the Rental Fleet-Sizing (RFS) problem. The model optimizes operational decisions, including customer demand allocation and empty truck repositioning, along with asset purchase and sale decisions over space and time. To solve large instances the authors develop a two-phase approach based on Benders decomposition and Lagrangian relaxation.

Zak et al. (2008) study a Multiobjective Heterogeneous Fleet Sizing Problem (MHFSP) arising in road freight transportation, involving multiple types of vehicles. They focus on three criteria, utilization of vehicles, value of subcontracted jobs, and the utilization of technical back-up facilities. Drawing upon queuing theory, they opt to use an M/M/n/0 model, where each queue corresponds to a type of vehicle. Their model does not involve routing decisions, and does not account for multiple periods.

Feng and Figliozzi (2013) analyze the Heterogeneous Fleet Sizing Problem (HFSP) for electric vehicles over a planning period. They develop an integer programming formulation to minimize the lifecycle costs of the fleet, including the purchase, energy, maintenance, and emission costs as well as salvage revenue. They assume that the demand for a period is given as the total mileage of vehicles in the period, and the miles to be traveled by a vehicle of a given type and age is known in advance. Based on these assumptions, they use a demand satisfaction constraint that forces a number of vehicles to cover the mileage in every period.

2.3 Literature on Vehicle Routing

The aforementioned literature does not account for vehicle routing decisions, which is faced in practice by distribution companies. The operations of such companies yield routes with different lengths. As mentioned by Suzuki and Gregory (2005) and Suzuki (2008), vehicle maintenance cost is usually computed on a per-mile basis. This practice implies that the maintenance cost incurred by a carrier for a given route is partially determined by the total distance of the route. Therefore, coupling routing decisions with the fleet size decisions is a valid extension for a number of companies operating routes with variable sizes.

The VRP has been studied for more than 50 years, and throughout the years a number of realistic extensions of the VRP have been proposed (Laporte, 2009). A good example is the Vehicle Routing Problem with Time Windows (VRPTW), in which each customer needs to be served within his time window. In most VRPTW papers surveyed by Bräysy and Gendreau (2005a,b), the objective is to find the minimum number of required vehicles while adhering to capacity constraints. Golden et al. (1984) introduced The Fleet Size and Mix Vehicle Routing Problem (FSMVRP), which has been studied recently by Baldacci et al. (2008) and Jabali et al. (2012). It extends the VRP by considering a heterogeneous fleet in which vehicles have different fixed costs. The FSMVRP assumes an unlimited fleet size and thus includes operational fleet sizing decisions. The Periodic Vehicle Routing Problem (PVRP) is a generalization of the classic VRP in which vehicle routes must be constructed over multiple days (Francis and Smilowitz, 2006; Francis et al., 2008). During each day within the planning period, a fleet of capacitated vehicles travels along routes that begin and end at a single depot. Hence, the PVRP is a multi-period problem in which no vehicle related fixed cost are considered.

The VRP literature is based on the assumption that all demand points are known. Therefore, the models focus on operational (often daily) planing. Capturing daily distribution activities in long-term planing problems requires aggregating the data, which is often captured by continuous approximation models. Research on continuous approximation models for routing problems was pioneered by Beardwood et al. (1959) in the context of the Traveling Salesman Problem (TSP). Daganzo (1984a) has developed a simple and intuitive formula for computing the total travel time in the VRP when the depot is not necessarily located in the area that contains the customers. This approximation was later validated by Robusté et al. (1990). Continuous approximation models were applied to a number of VRP variants, including the VRPTW (Daganzo, 1987a,b; Figliozzi, 2008a; Davis and Figliozzi, 2013), and the PVRP.

Problem	Period	Fleet size	Hetero-geneous	Routing	Customer service	Vehicle mainte-nance	Vehicle replace-ment	References
VRP	single	fixed	no	yes	all	no	no	†Daganzo (1984a) *Laporte (2009)
VRPTW	single	vari-able	no	yes	all	no	no	*Bräysy and Gendreau (2005a) †Figliozzi (2008b)
FSR	single	vari-able	no	yes	not all	no	no	
FSVAP	multi	vari-able	no	yes	all	no	no	†Francis and Smilowitz (2006) *Francis et al. (2008)
PVRP	multi	fixed	no	yes	all	no	no	†Francis and Smilowitz (2006) *Francis et al. (2008)
FSMVRP	single	vari-able	yes	yes	all	no	no	*Baldacci et al. (2008) †Jabali et al. (2012)
FPU	multi	vari-able	yes	no	not all	no	yes	List et al. (2003)
RFS	multi	vari-able	yes	no	all	yes	yes	Wu et al. (2005)
MYPVR	multi	fixed	no	no	all	yes	yes	Suzuki and Gregory (2005)
MRC	multi	vari-able	no	yes	not all	yes	yes	
HMRC	multi	vari-able	yes	yes	not all	yes	yes	

* *Literature survey on the problem*
† *Continuous approximation based model*

Table 1: Summary of related literature

The continuous approximation models cited above require all customers to be served. In contrast, the number of vehicles in the FSR, MRC, and HMRC are decision variables and these models do not necessarily require all customers to be served. We conclude this section by presenting Table 1, which summarizes the main characteristics of the main models presented in this section.

3. The single period fleet sizing and routing problem

The FSR consists of determining the number of vehicles required for serving a subset of customers in a given service region. The problem arises when a company must decide which customers it should service. Specifically, this model is suitable for distribution companies delivering goods to a large number of customers with fairly homogeneous demand.

We consider the routing costs of servicing area A, which has a uniform demand density of δ. We start by recalling the continuous approximation model for the VRP in Section 3.1. Building on this formulation, we proceed by developing the FSR model in Section 3.2.

3.1 A continuous approximation model for the vehicle routing problem

Daganzo (1984a) developed a simple and intuitive formula for approximating the total travel time for the VRP. The formula is valid when the depot is not necessarily located in the area that contains the customers. Using the Euclidean distance metric, the formula can be expressed as

$$T(n) \approx 2\frac{n\rho}{Q} + 0.57\sqrt{nA}. \tag{1}$$

Given a fleet of vehicles, each of which having sufficient capacity to serve Q customers with unit demand, Equation (1) expresses an approximation for the total distance traveled to serve n customers by $\frac{n}{Q}$ vehicles. The average distance between the depot and the customers is ρ, and demand density is δ. The total distance in Equation (1) consists of two components; the first corresponding to the line haul cost related to the distance between the depot and the customers and the second is the transverse cost related to the distance traveled between customers.

The constants in Equation (1) were derived assuming $Q > 6$ and $n > 4Q^2$. Under these conditions, it is best to divide the service area A into rectangular shaped zones elongated towards the depot, where each zone is serviced by a single vehicle. Figure 1 provides an example of a rectangle service area A.

The optimal width of each zone can be computed as $w = \sqrt{\frac{6.7}{\delta}}$, and the optimal length as $l = \frac{Q}{\sqrt{\delta 6.7}}$. For the sake of simplicity we assume that an integer number of zones can fit in A. The transportation cost for each zone is comprised of the line haul cost and the transverse cost, required for visiting customers within the zone. Figure 2 shows an example of a vehicle route serving customers within a zone.

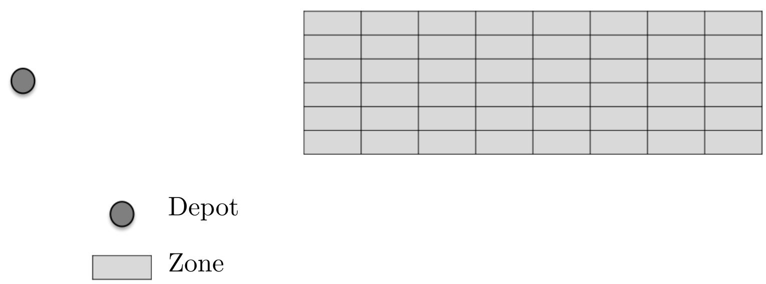

Figure 1: Partition of service area A into zones

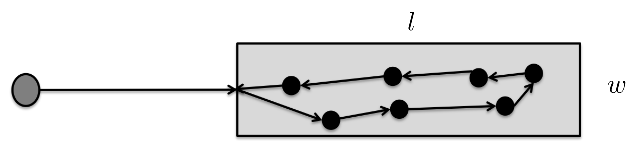

Figure 2: Vehicle route over two strips in a rectangle

3.2 Single period fleet sizing routing problem formulation

The FSR consists of determining the number of vehicles required to service $A' \subset A$, such that the total cost is minimized. Therefore, contrary to classical VRP models, the service area (and consequently the number of vehicles) are decision variables. We assume that the transportation cost per unit distance is α and an outsourcing cost β is incurred for every unserved customer. We note that α may include a maintenance cost per mile. A purchasing cost ξ is associated with using a vehicle. The average cost of serving a client increases with A', thus it is profitable to increase A' as long as the average cost of serving a client is less than β.

We denote the length of the horizontal and vertical boundaries of A by L and W, respectively. We assume that the depot is located at height $\frac{W}{2}$ with respect to the bottom left corner of A. Let $a = \frac{L}{l}$ and $b = \frac{W}{w}$. Furthermore, we denote the zone in the i^{th} column and in the j^{th} row as π_{ij}. Thus, $1 \leq i \leq a$ and $1 \leq j \leq b$. Figure 3 shows an example where A' is comprised of zones $\pi_{13}, \pi_{14}, \pi_{23}$ and π_{24}.

For each zone we approximate the term ρ in Equation (1) by the distance from the depot to the centre of a served zone. Thus, we introduce r_{ij} that denotes the distance from the depot to the centre of zone π_{ij}.

We now formulate the FSR as a mixed integer linear programming model. Let us define

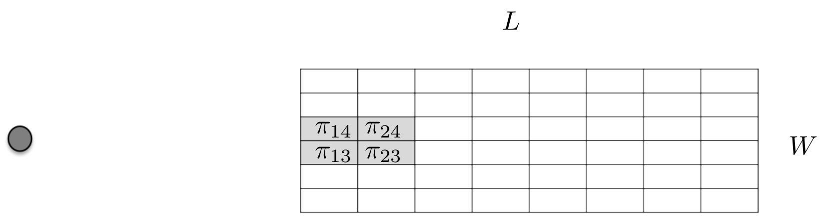

Figure 3: Possible coverage of service area A

the following decision variables:

$$z_{ij} = \begin{cases} 1 & \text{if zone } \pi_{ij} \text{ is served,} \\ 0 & \text{otherwise.} \end{cases}$$

Since $A = \frac{n}{\delta}$ we rewrite the transverse distance component of Equation (1) as $0.57\, n\sqrt{1/\delta}$. As this is dependent on the number of customers, we approximate the transverse distance per vehicle is $0.57\, Q\sqrt{1/\delta}$. We assume this distance applies to all served zones. The FSR model is then:

$$\text{(FSR)} \quad \text{Minimize} \quad \sum_{i=1}^{a}\sum_{j=1}^{b}\left(\xi + 2\alpha\, r_{ij} + 0.57\alpha\, Q\sqrt{1/\delta}\right) z_{ij} + \beta \sum_{i=1}^{a}\sum_{j=1}^{b} Q(1 - z_{ij}) \qquad (2)$$

subject to

$$z_{ij} \in \{0, 1\} \quad (i = 1, \ldots, a; j = 1, \ldots, b). \qquad (3)$$

Note that this model can be solved to optimality in polynomial time by a greedy algorithm that assigns $z_{ij} = 1$ if $\xi + \alpha 2 r_{ij} + 0.57\alpha\, Q\sqrt{1/\delta} \leq \beta Q$ and $z_{ij} = 0$ otherwise.

4. The homogeneous multi-period fleet replacement and composition problem

Given a planning horizon of T periods, the MRC aims to find the number of vehicles to purchase, sell, and operate in each period. The MRC is distinctly different from the models in the literature since it considers a number of periods with interdependent decisions, while accounting for vehicle routing costs at each period. The MRC is suitable for companies that deliver goods to a large number of customers, whose demand can be forecast over the planning horizon, and is relatively homogeneous within a period.

4.1 Detailed description of the MRC model

Unless stated otherwise, all assumptions and definitions from Section 3 hold. We assume that a vehicle is always purchased in the beginning of a period, and is therefore available for use during its first year of operation. Let each vehicle have a life cycle of K periods, which implies that a vehicle purchased at the beginning of period t must be sold by the beginning of period $t+K$. As defined in the previous section, the cost of purchasing a new vehicle is ζ. A cost F_k is incurred for having a vehicle of age k, i.e., for using a vehicle for its k^{th} period. The cost F_k is defined as the vehicle maintenance cost, and may include any other annual time-dependent cost. A vehicle of age k can be sold for S_k. Finally, we assume that the forecasted customer density for period t is δ_t, so that the potential number of customers to serve at period t is expressed as

$$n_t = \delta_t LW \qquad (t = 1, \ldots, T). \tag{4}$$

As previously mentioned, the customer density influences the dimension of the resulting zones. Therefore, we redefine the total number of horizontal and vertical zones in period t to be a_t and b_t, respectively. The width and length of the zones for each period t is defined as

$$w_t = \sqrt{\frac{6.7}{\delta_t}} \qquad \text{and} \qquad l_t = \frac{Q}{\sqrt{\delta_t 6.7}}. \tag{5}$$

We recall that the transportation cost per unit of distance is α, and the cost of not serving a customer is β. These cost parameters are independent of t, and thus remain constant throughout the planning horizon. Furthermore, we assume these costs are calibrated to represent annual costs. The maintenance cost is computed as

$$F_k = \begin{cases} 0 & \text{for } k = 0, \\ \gamma k & \text{otherwise.} \end{cases}$$

We assume that the maintenance cost for the first year of the vehicle's operation is included in the purchasing cost ζ. Therefore, the vehicle maintenance cost is incurred starting from the vehicle's second year of operation, and increases linearly by a factor of γ for each year of operation. Finally, let S_k denote the salvage value in period k, where

$$S_k = \begin{cases} 0 & \text{for } k = 0, \\ \omega - \mu k & \text{otherwise.} \end{cases}$$

Where ω is the potential selling price of a vehicle at zero, and is less than the purchasing cost ζ. This definition implies that the selling price decreases with a depreciation factor μ each year.

The decision variables for the MRC are:

d_t the number of vehicles available for customer service at period t;

p_t the number of vehicles purchased in the beginning of period t;

s_t the number of vehicles sold in the beginning of period t;

y_{kt} the number of available vehicles of age k in period t;

e_{kt} the number of vehicles of age k sold in the beginning of period t;

$$z_{ijt} = \begin{cases} 1 & \text{if zone } \pi_{ij} \text{ is served at period } t, \\ 0 & \text{otherwise;} \end{cases}$$

We proceed with defining the objective function of the MRC. We assume that period T is dedicated only to selling vehicles, i.e., no vehicles are purchased at period T. Since the problem concerns long-term planning, we account for the discounted costs and revenues, and define ϕ as the discount rate. We now describe the cost components that constitute the objective function.

The vehicle purchasing cost is

$$\zeta \sum_{t=1}^{T-1} (1+\phi)^{-t} p_t.$$

The vehicle maintenance cost is

$$\sum_{t=1}^{T-1} \sum_{k=1}^{K-1} (1+\phi)^{-t} \gamma k y_{kt}.$$

The salvage revenue of the vehicles is

$$-\sum_{t=1}^{T}\sum_{k=1}^{K}(1+\phi)^{-t}(\omega-\mu k)e_{kt}.$$

The transportation and outsourcing costs are similar to those presented in Section 3 and are

$$\alpha\sum_{t=1}^{T-1}\sum_{i=1}^{a_t}\sum_{j=1}^{b_t}(1+\phi)^{-t}\big(2r_{ij}+0.57\,Q\sqrt{1/\delta_t}\big)z_{ijt}+\beta\sum_{t=1}^{T-1}\sum_{i=1}^{a_t}\sum_{j=1}^{b_t}(1+\phi)^{-t}Q\,(1-z_{ijt}).$$

The formulation of the MRC is as follows.

$$\begin{aligned}
\text{(MRC)}\qquad & \text{Minimize } \zeta\sum_{t=1}^{T-1}(1+\phi)^{-t}p_t+\sum_{t=1}^{T-1}\sum_{k=1}^{K-1}(1+\phi)^{-t}\gamma k y_{kt}\\
& - \sum_{t=1}^{T}\sum_{k=1}^{K}(1+\phi)^{-t}(\omega-\mu k)e_{kt}\\
& + \alpha\sum_{t=1}^{T-1}\sum_{i=1}^{a_t}\sum_{j=1}^{b_t}(1+\phi)^{-t}\big(2r_{ij}+0.57\,Q\sqrt{1/\delta_t}\big)z_{ijt}\\
& + \beta\sum_{t=1}^{T-1}\sum_{i=1}^{a_t}\sum_{j=1}^{b_t}(1+\phi)^{-t}Q(1-z_{ijt}) \qquad (6)
\end{aligned}$$

subject to

$$\begin{aligned}
d_1 &= p_1 && && (7)\\
y_{k1} &= 0 && (k=1,\dots,K), && (8)\\
e_{k1} &= 0 && (k=1,\dots,K+1), && (9)\\
s_1 &= 0, && && (10)\\
d_t &= d_{t-1}+p_t-s_t && (t=2,\dots,T), && (11)\\
\sum_{i=1}^{t}p_i &\le \sum_{i=2}^{\min\{T,t+K\}} s_i && (t=1,\dots,T), && (12)\\
y_{K,t} &= 0 && (t=1,\dots,T), && (13)\\
e_{kt} &= y_{k-1,t-1}-y_{k,t} && (t=2,\dots,T;k=1,\dots,K), && (14)\\
s_t &= \sum_{k=1}^{K}e_{kt} && (t=2,\dots,T), && (15)\\
d_t &= \sum_{k=0}^{K-1}y_{kt} && (t=2,\dots,T), && (16)
\end{aligned}$$

$$\sum_{k=0}^{K} y_{kt} \geq \sum_{i=1}^{a_t} \sum_{j=1}^{b_t} z_{ijt} \qquad (t = 1, \ldots, T), \tag{17}$$
$$d_t, p_t, s_t \in \{0, 1, \ldots\} \;\; (t = 1, \ldots, T),$$
$$y_{kt}, e_{kt} \in \{0, 1, \ldots\} \;\; (k = 1, \ldots, K; t = 1, \ldots, T),$$
$$z_{ijt} \in \{0, 1\} \;\; (t = 1, \ldots, T, i = 1, \ldots, a_t; j = 1, \ldots, b_t;).$$

Constraint (7) reflects our assumption that zero vehicles are available in the beginning of the planning horizon and consequently we can only use the vehicles we purchase. However, this can be easily altered to include an initial fleet size. Constraints (8)–(10) ensure that no vehicles older than one year of age are used, available or sold in the beginning of the planning horizon. Constraints (11) state that the number of available vehicles at periods $t \geq 2$ equals the number of available vehicles in $t-1$ plus the purchased vehicles at t minus the number of vehicles sold at t. Constraints (12) ensure that vehicles purchased at t are sold by $K+t$, we recall that that a vehicle may not be sold in its year of purchase, i.e., $k = 0$. Constraints (13) dictate that no vehicles of age K are available for service. Constraints (14) state that for $t \geq 2$ the number of vehicles of age k sold in period t equals the difference between the number of available vehicles in $t-1$ of age $k-1$ and the number of vehicles in t of age k. We note that constraints (14) may also be viewed as

$$y_{kt} = y_{k-1,t-1} - e_{kt} \quad (t = 2, \ldots, T; k = 1, \ldots, K),$$

implying that that the number of available vehicles of age k in period t equals the difference between the number of available vehicles of age $k-1$ in period $t-1$ and the number of vehicles of age k sold in period t. The number of vehicles sold at period t is then given by (15) and similarly, the number of vehicles available for customer service at period t is given by (16). Finally, constraint (17) sets the upper bound on the number of vehicles used to serve customers in period t as the number of available vehicles in that period.

4.2 Theoretical properties of the MRC model

In what follows we present three theoretical observation stemming from the MRC. In Proposition 4.1 we develop the conditions on the cost parameters which necessitate serving all customers, i.e., no outsourcing is performed. This is useful for applications where serving all customers is mandatory. Under the condition that no outsourcing is performed, in Proposition 4.2 we show that if vehicles are sold in a certain period then the older vehicles must be sold with priority if $\phi = 0$. This can be denoted as the First–In–First–Out (FIFO) property. In Proposition 4.3, we deal with the situation where $\phi > 0$.

Proposition 4.1. *Let $\delta_{t'} \leq 1$ be the minimum customer density in the planing horizon. Furthermore, let $r' = \sqrt{(\frac{W}{2} - w)^2 + (r + L - \frac{l}{2})^2}$ and*

$$\eta = \frac{\zeta + \gamma(K-1) - \omega + K\mu + 2r'}{Q} + 0.57\sqrt{1/\delta_{t'}}.$$

If $\beta > \eta$ then all zones must be served in all periods.

Proof. The cost of serving a zone with a vehicle is comprised of:

- The vehicle purchasing cost, which is less than or equal to ζ, since a vehicle may or may not be purchased for serving the zone.
- The vehicle maintenance cost, which is less than or equal to $\gamma(K-1)$.
- The salvage revenue of the vehicles, which is greater than or equal to $\omega - \mu K$.
- Since r' is equal to the largest line haul distance, the transportation cost is less than or equal to $2r' + 0.57Q\sqrt{1/\delta_{t'}}$.

  Since

  $$(1+\phi)^{-t}\beta Q \geq (1+\phi)^{-t}[\zeta + \gamma(K-1) - (\omega - \mu K) + 2r' + 0.57Q\sqrt{1/\delta_{t'}}]$$

  holds for any t by the condition stated above, we conclude that all zones must be served in all periods.

$\square$

Proposition 4.2. *If $\phi = 0$, $\beta > \eta$ and if vehicles are sold in a certain period then the older vehicles are sold with priority.*

Proof. In a given period, consider two vehicles v_1 and v_2, which were bought in periods t_1 and t_2, with $t_2 = t_1 + \epsilon$, where ϵ is a positive integer. Suppose that one of the vehicles is sold in period t and the other is sold in period t' such that $t_2 < t < t'$. The maintenance cost of vehicles v_1 and v_2 until period $t-1$ are equal to

$$\sum_{k=t_1+1}^{t-1} \gamma(k - t_1) + \sum_{k=t_2+1}^{t-1} \gamma(k - t_2).$$

Case (i): If vehicle v_1 is sold at t and v_2 is sold at t', then in the periods between t and t' the maintenance cost and salvage revenue are

$$= \sum_{k=t-t_2}^{t'-t_2-1} \gamma k - (\omega - \mu(t-t_1)) - (\omega - \mu(t'-t_2))$$

$$= \gamma(t-(t_1+\epsilon)) + \gamma(t-(t_1+\epsilon)+1) \cdots + \gamma(t'-(t_1+\epsilon)-1) - (\omega-\mu(t-t_1)) - (\omega-\mu(t'-(t_1+\epsilon)))$$

Case (ii): If vehicle v_2 were to be sold at t and v_1 were to be sold at t', then in the periods between t and t' the maintenance cost and salvage revenue are

$$= \sum_{k=t-t_1}^{t'-t_1-1} \gamma k - (\omega - \mu(t-t_2)) - (\omega - \mu(t'-t_1))$$

$$= \gamma(t-t_1) + \gamma(t-t_1+1) \cdots + \gamma(t'-t_1-1) - (\omega - \mu(t-(t_1+\epsilon)) - (\omega - \mu(t'-t_1))$$

Subtracting the cost of Case (ii) from Case (i) yields,

$$-\gamma(t'-t)\epsilon$$

Since $t'-t>0$, we infer that the cost of Case (i) is less than the cost of Case (ii). As $\beta > \eta$, all zones must be visited in all periods. Therefore, the non discounted transportation cost is fixed throughout the planning horizon and thus does not influence the sales priority of the vehicles. We conclude that if vehicles are sold in a certain period then the older vehicles are sold with priority. □

Proposition 4.2 implies that in the situation of no outsourcing and $\phi = 0$, the FIFO property holds.

Proposition 4.3. *Given $\phi > 0$ and $\beta > \eta$, the condition $\gamma > \mu$ is sufficient to ensure that if vehicles are sold in a certain period, then the older vehicles are sold with priority.*

Proof. In a given period, consider two vehicles v_1 and v_2 that were bought in periods t_1 and t_2 respectively, with $t_2 = t_1+\epsilon$, where ϵ is a positive integer. Suppose that one of the vehicles is sold in period $t > t_2$ and the other is sold in period $t' = t+\sigma$, where σ is a positive integer.

Case (i): If vehicle v_1 is sold at t and v_2 is sold at t', then the salvage revenue of selling v_1 and v_2 are

$$(1+\phi)^{-t}(\omega - \mu(t-t_1)) + (1+\phi)^{-t'}(\omega - \mu(t'-(t_1+\epsilon)))$$

Case (ii): If vehicle v_2 were to be sold at t and v_1 were to be sold at t', then the salvage revenue of selling v_2 and v_1 are

$$(1+\phi)^{-t}(\omega-\mu(t-(t_1+\epsilon)))+(1+\phi)^{-t'}(\omega-\mu(t'-t_1))$$

The revenue of Case (i) minus the revenue of Case (ii) yields

$$-\mu(1+\phi)^{-t}\epsilon+\mu(1+\phi)^{-t'}\epsilon$$

$$=-\mu(1+\phi)^{-t}\epsilon+\mu(1+\phi)^{-t}\epsilon(1+\phi)^{-\sigma}$$

$$=\mu(1+\phi)^{-t}\epsilon[(1+\phi)^{-\sigma}-1] \tag{18}$$

The maintenance cost of vehicles v_1 and v_2 until period $t-1$ are equivalent for Case (i) and Case (ii), and are equal to

$$\sum_{k=t_1+1}^{t-1}(1+\phi)^{-k}\gamma(k-t_1)+\sum_{k=t_2+1}^{t-1}(1+\phi)^{-k}\gamma(k-t_2).$$

Case (i): If vehicle v_1 is sold at t and v_2 is sold at t', then in the periods between t and t' the maintenance cost of v_2 is

$$\sum_{k=t-t_2}^{t'-t_2-1}(1+\phi)^{(-k-t_2)}\gamma k$$

Case (i): If vehicle v_2 is sold at t and v_1 is sold at t', then in the periods between t and t' the maintenance cost of v_1 is

$$\sum_{k=t-t_1}^{t'-t_1-1}(1+\phi)^{(-k-t_1)}\gamma k$$

Subtracting the maintenance cost of Case (ii) from Case (i) yields,

$$\sum_{k=t-t_1}^{t'-t_2-1}(1+\phi)^{(-k-t_2)}\gamma k-\sum_{k=t-t_1}^{t'-t_1-1}(1+\phi)^{(-k-t_1)}\gamma k$$

$$=(1+\phi)^{-t}\gamma(t-(t_1+\epsilon))+(1+\phi)^{-(t+1)}\gamma(t-(t_1+\epsilon)+1)\cdots+(1+\phi)^{-(t'-1)}\gamma(t'-(t_1+\epsilon)-1)$$

$$-(1+\phi)^{-t}\gamma(t-t_1)+(1+\phi)^{-(t+1)}\gamma(t-t_1+1)\cdots+(1+\phi)^{-(t'-1)}\gamma(-t'-t_1-1)$$

$$=-(1+\phi)^{-t}\gamma\epsilon-(1+\phi)^{-t-1}\gamma\epsilon\cdots-(1+\phi)^{-(t'-1)}\gamma\epsilon$$

$$= -(1+\phi)^{-t}\gamma\epsilon - (1+\phi)^{-t-1}\gamma\epsilon \cdots - (1+\phi)^{-(t+\sigma-1)}\gamma\epsilon$$

$$= -(1+\phi)^{-t}\gamma\epsilon[1 + (1+\phi)^{-1} \ldots (1+\phi)^{-(\sigma-1)}] \tag{19}$$

Therefore, subtracting Equation (18) from Equation (19) yields

$$-(1+\phi)^{-t}\gamma\epsilon[1 + (1+\phi)^{-1} \ldots (1+\phi)^{-(\sigma-1)}] - \mu(1+\phi)^{-t}\epsilon[(1+\phi)^{-\sigma} - 1]$$

In order for the FIFO to hold, the following must hold

$$-(1+\phi)^{-t}\gamma\epsilon[1 + (1+\phi)^{-1} \ldots (1+\phi)^{-(\sigma-1)}] - \mu(1+\phi)^{-t}\epsilon[(1+\phi)^{-\sigma} - 1] < 0$$

$$-\gamma[1 + (1+\phi)^{-1} \ldots (1+\phi)^{-(\sigma-1)}] - \mu[(1+\phi)^{-\sigma} - 1] < 0$$

$$\gamma[1 + (1+\phi)^{-1} \ldots (1+\phi)^{-(\sigma-1)}] > -\mu[(1+\phi)^{-\sigma} - 1]$$

$$\gamma[1 + (1+\phi)^{-1} \ldots (1+\phi)^{-(\sigma-1)}] > \mu[1 - (1+\phi)^{-\sigma}]$$

$$\frac{\gamma}{\mu} > \frac{[1 - (1+\phi)^{-\sigma}]}{[1 + (1+\phi)^{-1} \ldots (1+\phi)^{-(\sigma-1)}]} \tag{20}$$

Since $[1 - (1+\phi)^{-\alpha}] < 1$ and $1 + (1+\phi)^{-1} \ldots (1+\phi)^{-(\sigma-1)} > 1$ then the right hand side of Equation (20) is less than one. As $\gamma > \mu$ we conclude that Equation (20) holds, and therefore the cost of Case (i) is less than Case(ii).

□

Proposition 4.3 implies that in the situation of no outsourcing and if $\gamma > \mu$ then the FIFO property holds.

5. The heterogeneous multi-period fleet replacement and composition problem

In this section we extend the MRC to account for a heterogeneous fleet of vehicles, these have different capacities and costs. We recall that the optimal width of each zone is $w = \sqrt{\frac{6.7}{\delta}}$, and the optimal length is $l = \frac{Q}{\sqrt{\delta 6.7}}$. Note that w is independent of Q, but the length of the served area by a vehicle depends on its capacity.

5.1 Detailed description of the HMRC model

We now present the heterogeneous version of the problem. Given H vehicle types, let Q^h denote the capacity of vehicle type $h \in \{1, ..., H\}$. We assume that vehicle types are labeled based on an ascending order with respect to their capacity, i.e., $Q^1 < Q^2, \ldots, < Q^H$. Let Q_d be a common divisor of $Q^1, \ldots, Q^H$, and let θ^h denote the divisor of Q^h resulting in Q_d, i.e., $Q^h = \theta^h Q_d$. We partition the service region into zones corresponding to having a homogeneous fleet with vehicles of capacity Q_d. We assume that each vehicle type may visit an integer number of zones, up to its capacity.

To formulate the HMRC, we extend the definitions of the parameters $\zeta, \gamma, \omega, \mu, \alpha$ and the decision variables d_t, p_t, s_t, y_{kt} and e_{kt} by adding an index h to denote the type of vehicle they pertain to. To model the service areas of the different types of vehicles we define the following decision variables.

$$g_{ijt}^{qh} = \begin{cases} 1 & \text{if zone } \pi_{ij} \text{ is the first zone served in period } t \\ & \text{by vehicle type } h \text{ that serves } q \text{ horizontally consecutive zones,} \\ 0 & \text{otherwise.} \end{cases}$$

$$x_{ijt} = \begin{cases} 1 & \text{if zone } \pi_{ij} \text{ is outsourced at period } t, \\ 0 & \text{otherwise;} \end{cases}$$

We note that if vehicle type h is used to its full capacity and begins serving zone π_{ij} in t then $g_{ijt}^{\theta^h h} = 1$. In this case the vehicle is serving zones $\pi_{i,j}, \pi_{i+1,j}, \ldots \pi_{i+\theta^h-1,j}$ in t. However, if the vehicle is used less than its full capacity then $q < \theta^h$, where q corresponds to an integer number of zones. In this case $g_{ijt}^{qh} = 1$ then the vehicle serves zones $\pi_{i,j}, \pi_{i+1,j}, \ldots \pi_{i+q-1,j}$ in t. We accordingly introduce r_{ij}^q as a parameter denoting the distance from the depot to the centre of the service zone comprised of the aggregation of $\pi_{ij}, \pi_{i+1,j} \ldots \pi_{i+q-1,j}$, .i.e., the latter are considered as a single zone.

We illustrate the modelling approach with an example in Figure 4. Consider a row comprised of seven zones and suppose that the capacity of vehicle type 4 is 40 and that each zone has a demand of 10, i.e., $Q_d = 10$. Suppose that $g_{3jt}^{34} = 1$ for a given j and t. This implies that while the vehicle may serve up to four horizontally consecutive zones it was decided that it would serve three zones starting from zone π_{3j}, i.e., the vehicle would serve zones π_{3j}, π_{4j} and π_{5j} consecutively on the same trip.

π_{1j}	π_{2j}	π_{3j}	π_{4j}	π_{5j}	π_{6j}	π_{7j}

Figure 4: HMRC zone allocation example

The formulation of the HMRC is as follows.

$$
\begin{aligned}
\text{(HMRC)} \qquad & \text{Minimize} \sum_{h=1}^{H}\sum_{t=1}^{T-1}(1+\phi)^{-t}\zeta^h p_t^h + \sum_{h=1}^{H}\sum_{t=1}^{T-1}\sum_{k=1}^{K-1}(1+\phi)^{-t}\gamma^h k y_{kt}^h \\
- & \sum_{h=1}^{H}\sum_{t=1}^{T}\sum_{k=1}^{K}(1+\phi)^{-t}(\omega^h-\mu^h k)e_{kt}^h \\
+ & \sum_{h=1}^{H}\sum_{q=1}^{\theta^h}\sum_{t=1}^{T-1}\sum_{i=1}^{a_t}\sum_{j=1}^{b_t}(1+\phi)^{-t}\alpha^h\left(2r_{ij}^q+0.57\,Q_d\sqrt{1/\delta_t q}\right)g_{ijt}^{qh} \\
+ & \beta\sum_{h=1}^{H}\sum_{t=1}^{T-1}\sum_{i=1}^{a_t}\sum_{j=1}^{b_t}(1+\phi)^{-t}Q_d x_{ijt} \qquad (21)
\end{aligned}
$$

subject to

$$
\begin{aligned}
d_1^h &= p_1^h && (h=1,\dots,H) && (22)\\
y_{k1}^h &= 0 && (k=1,\dots,K;h=1,\dots,H), && (23)\\
e_{k1}^h &= 0 && (k=1,\dots,K+1,h=1,\dots,H), && (24)\\
s_1^h &= 0 && (h=1,\dots,H), && (25)\\
d_t^h &= d_{t-1}^h+p_t^h-s_t^h && (t=2,\dots,T;h=1,\dots,H), && (26)\\
\sum_{i=1}^{t}p_i^h &\le \sum_{i=1}^{\min\{T,t+K\}} s_i^h && (t=1,\dots,T;h=1,\dots,H), && (27)\\
y_{K,t}^h &= 0 && (t=1,\dots,T;h=1,\dots,H), && (28)\\
e_{kt}^h &= y_{k-1,t-1}^h-y_{k,t}^h && (t=2,\dots,T;k=1,\dots,K;h=1,\dots,H), && (29)\\
s_t^h &= \sum_{k=1}^{K}e_{kt}^h && (t=2,\dots,T,h=1,\dots,H), && (30)\\
d_t^h &= \sum_{k=0}^{K-1}y_{kt} && (t=2,\dots,T,h=1,\dots,H), && (31)\\
\sum_{k=0}^{K}y_{kt}^h &\ge \sum_{q=1}^{\theta^h}\sum_{i=1}^{a_t}\sum_{j=1}^{b_t}g_{ijt}^{qh} && (t=1,\dots,T;h=1,\dots,H), && (32)
\end{aligned}
$$

$$\sum_{h=1}^{H} \sum_{q=1}^{\theta^h} \sum_{u=1}^{q} g_{i-q+u,j,t}^{qh} + x_{ijt} = 1 \qquad (t = 1, \ldots, T; i = 1, \ldots, a_t; j = 1, \ldots, b_t;), \tag{33}$$

$$\sum_{h=1}^{H} \sum_{q=1}^{\theta^h} g_{ijt}^{qh} \leq 1 \qquad (t = 1, \ldots, T, i = 1, \ldots, a_t; j = 1, \ldots, b_t), \tag{34}$$

$$d_t^h, p_t^h, s_t^h \in \{0, 1, \ldots\} \;\; (t = 1, \ldots, T; h = 1, \ldots, H), \tag{35}$$

$$q_{kt}^h, e_{kt}^h \in \{0, 1, \ldots\} \;\; (t = 1, \ldots, T; k = 1, \ldots, K; h = 1, \ldots, H), \tag{36}$$

$$g_{ijt}^{qh} \in \{0, 1\} \;\; (t = 1, \ldots, T; i = 1, \ldots, a_t; j = 1, \ldots, b_t; \; h = 1, \ldots, H; q = 1, \ldots, h),$$

$$x_{ijt} \in \{0, 1\} \;\; (t = 1, \ldots, T, i = 1, \ldots, a_t; j = 1, \ldots, b_t;).$$

The objective function (21) as well as constrains (22)–(31) are direct translations from the MRC model. Constraints (32) ensure that the total number of vehicles of type h used in period t is bounded by the total number of available vehicles of type h in period t. Constraints (33) enforce that each zone in each period is either served by a vehicle or outsourced. Specifically, the first summation in these constraints sums all vehicle combinations that may cover zone π_{ij} in period t, we note that if $i - q + u < 1$ then no term is added in the summation. Constraints (34) state that each zone may be served by at most one vehicle type covering q consecutive zones.

6. Computational study

In this section we illustrate the use of the presented models by performing various sensitivity analyses. The results from these analyses will provide guidelines for fleet management. Our experiments are focused on the HMRC model, as it encompasses the other models as special cases. The HMRC model is coded in C++ and solved using IBM ILOG CPLEX 12.6. All experiments are performed on an Intel(R) Xeon(R) CPU X5675 with 12-Core 3.07 GHz and 96 GB of RAM. Since the CPU times for all models are below 5 seconds we do not report computation times. In what follows we present a base case, based on which we study the effect of various model components.

6.1 Base case

We consider a service region A with $L = 219.9$ and $W = 40.9$, with an area of 9000. We consider a planning horizon of $T = 10$, the distance r between the depot and the closest point to the depot in A to the depot is 60. The customer density δ_t is 0.4 t. The discount rate ϕ is set to 5% and $\beta = 1500$. We consider three vehicle types, the characteristics of which are summarized in Table 2.

Vehicle type (h)	Q^h	ζ^h	α^h	γ^h	ω^h	μ^h
1	10	70,000	1	300	60,000	5000
2	20	120,000	2	600	100,000	10000
3	30	178,865	3	900	148,865	15000

Table 2: Vehicle characteristics for the base case

We emphasize that aside from ζ^h and ω^h, the characteristics of each vehicle are linearly related to its capacity. This relationship will be relaxed in the subsequent sections. Each vehicle can be used for a maximum of 5 years, thus $K = 6$. We set Q^d to be 10, thus $\theta^1 = 1, \theta^2 = 1$ and $\theta^3 = 3$. As a result, we have $a_t = 36$ for all t and $b_t = 10$ for all t.

t	1	2	3	4	5	6	7	8	9	10
p_t^2	168	0	0	0	0	168	0	0	0	0
p_t^3	8	0	0	0	0	8	0	0	0	0
s_t^2	0	0	0	0	0	168	0	0	0	168
s_t^3	0	0	0	0	0	8	0	0	0	8

Table 3: Solution for the base case

The solution to the base case is summarized in Table 3. We note that for each period we have a fleet comprised of 168 vehicles of type 2 and 8 vehicles of type 3. As each of these vehicle types can serve two and three zones respectively, the combined capacity of the fleet is sufficient to serve all 360 zones. All vehicles purchased at the beginning of period one are sold at the beginning of period 6. An equivalent fleet is purchased at the beginning of period six which is then sold in the beginning of period six. We infer that given the cost parameters it is worth operating vehicles for as long as possible. Since we consider the discounted values and purchase costs are higher than the salvage value, it is more beneficial to postpone the purchase of a vehicle fleet as long as possible. Therefore, the fleet is renewed in period six.

The assignment of vehicles to zones in the base case solution for $t = 1$ is illustrated in Figure 5. The assignment is identical for periods 2 to 9. All vehicles are used at their full capacity, i.e., vehicle type 2 serves two zones while vehicle type 3 serves three zones. Despite the fact that $\alpha^2 < \alpha^3$, vehicles of type 3 are assigned to further zones. This is due to the fact that a vehicle type is used to serve a block of zones, and this aggregation may yield nonintuitive assignments. For example in Figure 5, it is cheaper to serve zones $(1,1)$ to $(6,1)$ by two vehicles of type 3 and zones $(1,3)$ to $(6,3)$ by three vehicles of type 2, as opposed to serving zones $(1,1)$ to $(6,1)$ by three vehicles of type 2 and zones $(1,3)$ to $(6,3)$ by two vehicles of type 3.

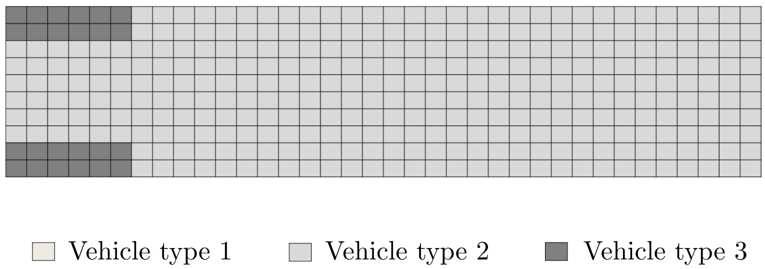

Figure 5: Solution for the base case for $t = 1$

6.2 Effect of transportation cost per unit distance

In a series of tests, we have investigated the effect of the transportation cost per unit distance α^h. We experimented with various values of α^1, while fixing all other parameters to their respective values in the base case. The results for $\alpha^1 = 0.8, 0.6, 0.4$ and 0.2 are summarized in Table 4. We observe that reducing α^1 to 0.8 or 0.6 does not yield a different solution than that of the base case. When α^1 is reduced to 0.4 or 0.2, the solution is substantially different from that of the base case in that vehicles of type 1 are purchased instead of a number of vehicles of type 2. Specifically, for $\alpha^1 = 0.4$ for $t = 1$, we purchase 100 vehicles of type 1 and 118 (instead of 168) of vehicle type 2.

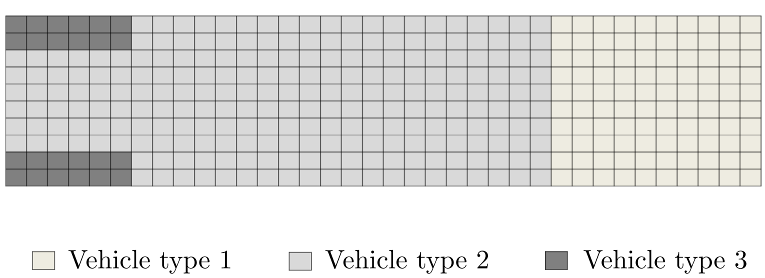

Figure 6: Solution for the base case with $\alpha^1 = 0.4$ for $t = 1$

The assignment of vehicles to zones for the base case with $\alpha^1 = 0.4$ is illustrated in Figure 6. The assignment is identical for periods 2 to 9. All vehicles are use to their full capacity.

ψ	t	1	2	3	4	5	6	7	8	9	10
0.8	p_t^2	168	0	0	0	0	168	0	0	0	0
	p_t^3	8	0	0	0	0	8	0	0	0	0
	s_t^2	0	0	0	0	0	168	0	0	0	168
	s_t^3	0	0	0	0	0	8	0	0	0	8
0.6	p_t^2	168	0	0	0	0	168	0	0	0	0
	p_t^3	8	0	0	0	0	8	0	0	0	0
	s_t^2	0	0	0	0	0	168	0	0	0	168
	s_t^3	0	0	0	0	0	8	0	0	0	8
0.4	p_t^1	100	0	0	0	0	100	0	0	0	0
	p_t^2	118	0	0	0	0	118	0	0	0	0
	p_t^3	8	0	0	0	0	8	0	0	0	0
	s_t^1	0	0	0	0	0	100	0	0	0	100
	s_t^2	0	0	0	0	0	118	0	0	0	118
	s_t^3	0	0	0	0	0	8	0	0	0	8
0.2	p_t^1	180	0	0	0	0	180	0	0	0	0
	p_t^2	78	0	0	0	0	78	0	0	0	0
	p_t^3	8	0	0	0	0	8	0	0	0	0
	s_t^1	0	0	0	0	0	180	0	0	0	180
	s_t^2	0	0	0	0	0	78	0	0	0	78
	s_t^3	0	0	0	0	0	8	0	0	0	8

Table 4: Computational results for several values of α^1

The 100 vehicles of type 1 are used to serve a block of zones towards the end of the service area, from zone (27,1) to zone (36,10). The assignment of vehicles of type 2 is identical to what it was in the base case. Thus, we conclude that lowering the transportation cost per unit distance of smaller vehicles increases their numbers in fleet. Moreover, such vehicles are likely to be assigned to further zones.

6.3 Effect of the maintenance cost

In a second series of experiments we have assessed the impact of γ^h on the solutions. We define γ_b^h to be the base case value for γ^h, we changed the value γ^h in the experiments by using $\gamma^h = \gamma_b^h + 10,00\theta^h\psi$. In Table 5 we present the results for $\psi = 1, \ldots 6$. Therefore, in these experiments we increased the γ^h for all h using the formula $\gamma^h = \gamma^1\theta^h$.

The results in Table 5 indicate that the increase in γ^h yields the same fleet composition at every t as in the base case. However, the increase in γ^h causes vehicles to be replaced earlier than in the base case. The fleet purchased in $t = 1$ is sold in $t = 6$ in the base case, while the same fleet is sold in period 5 for $\psi = 1$. As ψ increases vehicles tend to be sold earlier.

ψ	t	1	2	3	4	5	6	7	8	9	10
1	p_t^2	168	0	0	0	168	0	0	0	0	0
	p_t^3	8	0	0	0	8	0	0	0	0	0
	s_t^2	0	0	0	0	168	0	0	0	0	168
	s_t^3	0	0	0	0	8	0	0	0	0	8
2	p_t^2	168	0	0	168	0	0	168	0	0	0
	p_t^3	8	0	0	8	0	0	8	0	0	0
	s_t^2	0	0	0	168	0	0	168	0	0	168
	s_t^3	0	0	0	8	0	0	8	0	0	8
3	p_t^2	168	0	0	168	0	0	168	0	0	0
	p_t^3	8	0	0	8	0	0	8	0	0	0
	s_t^2	0	0	0	168	0	0	168	0	0	168
	s_t^3	0	0	0	8	0	0	8	0	0	8
4	p_t^2	168	0	168	0	168	0	168	0	0	0
	p_t^3	8	0	8	0	8	0	8	0	0	0
	s_t^2	0	0	168	0	168	0	168	0	0	168
	s_t^3	0	0	8	0	8	0	8	0	0	8
5	p_t^2	168	0	168	0	168	0	168	0	168	0
	p_t^3	8	0	8	0	8	0	8	0	8	0
	s_t^2	0	0	168	0	168	0	168	0	168	168
	s_t^3	0	0	8	0	8	0	8	0	8	8
6	p_t^2	168	0	168	0	168	0	168	0	168	0
	p_t^3	8	0	8	0	8	0	8	0	8	0
	s_t^2	0	0	168	0	168	0	168	0	168	168
	s_t^3	0	0	8	0	8	0	8	0	8	8

Table 5: Computational results for several values of γ^h

6.4 Effect of customer outsourcing

In a third series of experiments we explored the influence of the outsourcing cost β on the solution, and we experimented with lower values of β, with respect its value in the base case. These lower values yield solutions in which outsourcing is used.

Table 6 shows the fleet composition resulting from β values of 500, 495, 490, and 485. Naturally, as the outsourcing decreases the the number of vehicles in the fleet decreases. As in the base case solution in all cases in Table 6, we use only vehicles of type 2 and 3. However, as β decreases the number of vehicles of type 2 decreases. Moreover, the fleet size and composition are not stable throughout the planning horizon. For $\beta = 500$ we observe two fleet composition patterns, one with 50 vehicles of type 2 and 9 vehicles of type 3, which holds for all periods aside from period five. The second pattern consists of 100 vehicles of type 2 and 20 vehicles of type 3, this pattern applies to period five. Similar patterns can be observed for $\beta = 490$ and $\beta = 485$. Whereas for $\beta = 495$ we observe three fleet composition patterns. The first with 30 vehicles of type 2 and eight vehicles of type 3, this pattern holds for periods one to four. In period 5 the fleet composition consists of 68 vehicles of type 2 and eighteen vehicles of type 3. In periods 6 to 9 the fleet composition consists of 30 vehicles of type 2 and 10 vehicles of type 3. Essentially, the existence of several patterns stems from the fact that the vehicles are purchased in two time periods and are kept for as along as possible. Therefore, vehicles purchased in period 1 are kept in service for 5 years, while to guarantee that the vehicles salvaged at period 10 do serve the maximum number of years they must be purchased in period five. This situation leads to having a greater number of vehicles in period 5.

β	t	1	2	3	4	5	6	7	8	9	10
500	p_t^2	50	0	0	0	50	0	0	0	0	0
	p_t^3	10	0	0	0	10	0	0	0	0	0
	s_t^2	0	0	0	0	0	50	0	0	0	50
	s_t^3	0	0	0	0	0	10	0	0	0	10
495	p_t^2	38	0	0	0	30	0	0	0	0	0
	p_t^3	8	0	0	0	10	0	0	0	0	0
	s_t^2	0	0	0	0	0	38	0	0	0	30
	s_t^3	0	0	0	0	0	8	0	0	0	10
490	p_t^2	18	0	0	0	18	0	0	0	0	0
	p_t^3	8	0	0	0	8	0	0	0	0	0
	s_t^2	0	0	0	0	0	18	0	0	0	18
	s_t^3	0	0	0	0	0	8	0	0	0	8
485	p_t^3	10	0	0	0	10	0	0	0	0	0
	s_t^3	0	0	0	0	0	10	0	0	0	10

Table 6: Computational results for several values of β

Figures 7–9 illustrate the solution for $\beta = 495$, where uncoloured zones are outsourced.

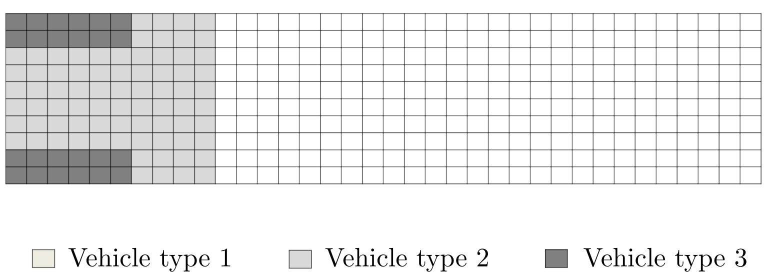

Figure 7: Solution for the base case with $\beta = 495$ for $t = 1, \ldots, 4$

6.5 Effect of customer density

The customer density in the base case was set to 0.4. In order to explore the effect of customer density on the solutions we experimented with values 0.2, 0.3, 0.5, and 0.6, which cover deviations from the base case value of 0.4 in both directions. Considering the same

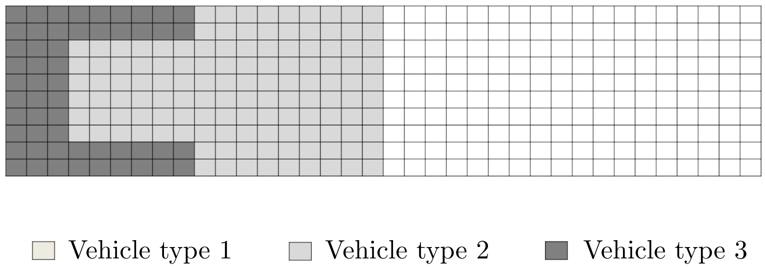

Figure 8: Solution for the base case with $\beta = 495$ for $t = 5$

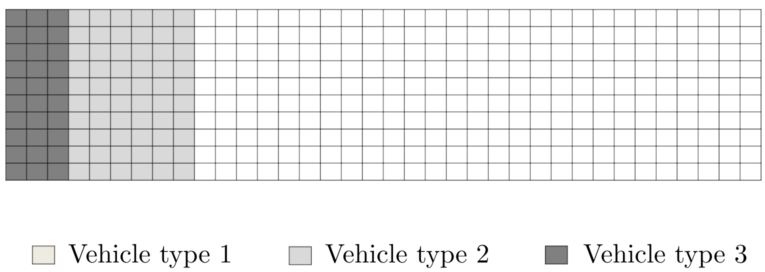

Figure 9: Solution for the base case with $\beta = 495$ for $t = 6, \ldots, 9$

service region A, changing the density yields a different number of zones, which we rounded to the closest integer.

In Table 7 we present the results for $\delta = 0.2, 0.3, 0.5$, and 0.6. As in the base case the fleet composition is stable throughout $t = 1 \ldots 9$. Furthermore, the fleet is renewed in period six in all settings. For $\delta = 0.2, 0.3$ and 0.4 the fleet is comprised of more vehicles of type 2 than type 3. However, for higher densities, i.e., 0.5 and 0.6, the fleet is comprised of more vehicles of type 3 than type 2. Thus we conclude that as the customer density increases we tend to use larger vehicles.

In Figure 10 we show the solution for $\delta = 0.6$. For this density $a_t = 42$ for all t and $b_t = 12$ for all t. In this case the medium vehicles are generally allocated to the further away zones, with the exception of the zones within (40,6) and (42,7). This can be explained by the same reasoning presented for the base case, for a similar situation.

δ	t	1	2	3	4	5	6	7	8	9	10
0.2	p_t^2	77	0	0	0	0	77	0	0	0	0
	p_t^3	7	0	0	0	0	7	0	0	0	0
	s_t^2	0	0	0	0	0	77	0	0	0	77
	s_t^3	0	0	0	0	0	7	0	0	0	7
0.3	p_t^2	126	0	0	0	0	126	0	0	0	0
	p_t^3	9	0	0	0	0	9	0	0	0	0
	s_t^2	0	0	0	0	0	126	0	0	0	126
	s_t^3	0	0	0	0	0	9	0	0	0	9
0.5	p_t^2	22	0	0	0	0	22	0	0	0	0
	p_t^3	132	0	0	0	0	132	0	0	0	0
	s_t^2	0	0	0	0	0	22	0	0	0	22
	s_t^3	0	0	0	0	0	132	0	0	0	132
0.6	p_t^2	12	0	0	0	0	12	0	0	0	0
	p_t^3	168	0	0	0	0	168	0	0	0	0
	s_t^2	0	0	0	0	0	12	0	0	0	12
	s_t^3	0	0	0	0	0	168	0	0	0	168

Table 7: Computational results for several values of δ

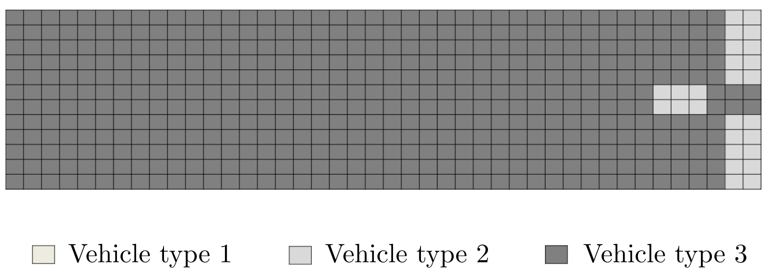

Figure 10: Solution for the base case with $\delta = 0.6$

7. Conclusion and further research

We have introduced a multi-period fleet replacement and composition model that simultaneously considers vehicle purchasing costs, maintenance costs, salvage revenues and routing costs. This problem is of strategic importance to distribution companies who operate a fleet of vehicles on a long term basis. The main contribution of this paper lies in developing a continuous approximation model for the multi-period fleet replacement and composition problem.

Our model uses a continuous approximation to evaluate the routing costs in a rectangular service region with a large number of customers. Considering a homogeneous fleet, we develop the single period fleet sizing and routing model, the solution to which can be achieved in polynomial time. Building upon this problem, we develop the homogeneous multi-period fleet replacement and composition problem. We developed several theoretical properties for the resulting model. Namely, we propose values for the customer outsourcing cost that guarantee that all zones in all period are served by vehicles. For this case, and when the discount factor is zero we show that if vehicles are sold in a certain period then the older vehicles are sold with priority. Furthermore, considering a discount factor greater than zero, we show that the condition $\gamma > \mu$ is sufficient to ensure that if vehicles are sold in a certain period, then the older vehicles are sold with priority.

Finally we introduce the heterogeneous multi-period fleet replacement and composition problem. We formulate the resulting problem in an efficient manner that enabled solving the test instances with minimal computational effort. We studied a base case in which no outsourcing is performed, and conducted several sensitivity analyses with respect to its fundamental parameters. Namely transportation costs, maintenance cost, outsourcing cost, and customer density. In all experiments vehicles' capacity was fully utilized. A reduction in the transportation cost of the smallest vehicle results in purchasing more vehicles of this type, and allocating them to relatively far zones. As the maintenance cost increases vehicles tend to be sold earlier. In most cases, with the exception of the first period and the last period of operation, vehicles are sold and purchased in the same period. However, for the case where some zones are outsourced this observation does not hold. Thus, we observe situations where vehicles are purchased before older vehicles are sold. Furthermore, while the fleet composition was relatively stable in most experiments, for the case where outsourcing is used we observed several fleet composition patterns throughout the planning horizon. Finally, we observed that for higher customer densities more larger vehicles are used in closer zones.

We believe that the models presented in this paper can facilitate sound strategic decision making by enabling the evaluation of multiple scenarios. The contributions from this paper open several avenues for future research. A natural extension lies in exploring more elaborate customer density structures. Finally, the models could be extended to consider environmental aspects such as CO_2 emissions.

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