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Multi-Period Collection of Recyclable Materials in a Multi-Compartment Vehicle under Uncertainty

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Abstract. We address the problem of collecting recyclable materials, from locations distributed over a rather large territory with different population densities and land utilization, over a multi-period planning horizon under stochastic daily accretion rates of materials at collection locations. The recyclable materials are collected jointly by multi-compartment vehicles, but must be handled separately, due to different methods for reuse of the materials. Deposited materials may overflow the capacity at collection locations under certain location-specific conditions, but vehicle and depot capacities must be enforced. The recyclable materials are transported to treatment facilities when necessary. The aim is to minimize the operation cost over the planning horizon, while avoiding violations of capacity constraints for vehicles and reducing as much as possible the capacity overflow at collection points. We present a two-stage stochastic programming formulation with simple recourse, where collection decisions make up the first stage. We propose single- and multi-period management policies to address the daily scheduling and transportation issues, and embed them into a rolling horizon procedure. We introduce a large set of instances, and present the results of a comprehensive analysis of the proposed methodology, including the investigation of the impact of sensor and information systems providing information on material levels at collection points. Computational experiments show the efficiency of the proposed methodology to address various problem settings of realistic dimensions. They also show that the multi-period policy generally outperforms the single-period policy and both policies benefit from sensor and information systems availability.

Keywords: Recyclable material collection, multi-compartment multi-period vehicle routing, inventory routing, stochastic demand, two-stage model, management policy, rolling time horizon, sensor and information system.

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1. Introduction

Collection of recyclable materials is an important activity that makes it possible to reduce our environmental footprint by turning waste into valuable materials. Society has become increasingly concerned about environmental issues in recent years, and this has led to both increased pressure on the collection and recycling services and an accentuated focus on this field of research.

Other than the usual economic (cost minimization) and operational (vehicle and storage capacities) concerns, organizations providing the collection and recycling services face a continuously increasing set of operational and service requirements and constraints, e.g., the presence of materials with different handling requirements, an uneven distribution of the quantities of materials to be collected over the territory covered, the uncertainty relative to these quantities at each collection location and for each day of operation, the citizen tolerance, or lack thereof, of overflowing collection recipients, and so on and so forth. As the focus, importance, and complexity of these issues increase, so does the need to design good policies for when and how to collect recyclable materials. Yet, as emphasized in a recent overview of transportation of waste and reusable materials from an Operations Research point of view (Beullens et al., 2010) 1) research in reverse logistics systems is very inadequate and sparse compared to that in (normal) forward logistics, and 2) more research needs to be conducted with the goal of making reverse logistics systems efficient. The authors also point out that there is a particular lack of models and methods for multi-period routing problems with combined collection in multi-compartment vehicles.

The purpose of this paper is to address several of these challenges and thus contribute to filling up this gap. We focus on a general problem of collecting different types of recyclable materials, from locations distributed over a rather large territory with different population densities and land utilization, over a multi-period planning horizon under uncertainty conditions.

The materials are collected jointly by a multi-compartment vehicle, but must be handled separately, due to different requirements and procedures for reuse. The daily rate of deposited materials at the various locations is not known with certainty, and only becomes known when visiting each collection location. Sensor and information technology could be deployed, however, to alleviate some of the consequences of this uncertainty. The capacity of the collecting vehicles is not allowed to be exceeded, but the deposit capacities at various locations can be overflown under certain location-specific conditions. The collected materials are brought back to a depot and are stored until the storage capacity for a given type is reached, at which time that particular material is transported to a treatment facility.

There is no regularity in the way and rate people deposit recyclable materials and, thus, there are no regularity requirements on routes, scheduling of visits to collection locations, and transportation to the treatment facilities. Furthermore, there is no obligation to collect all the materials present at a location when the vehicle visits it. No time windows are imposed on activities. The collection plan for any day is thus unique and non recurrent. It must take into account, however, the citizens' disapproval of overfilled recipients at collection locations, in particular when materials pile up in the streets or within sight of important sites. Their dislike varies with the location and the degree of overfilling. The decision maker must decide each day the collection locations to visit, what to collect at each location, the vehicle route servicing the selected locations, as well as when to deliver the collected materials to the treatment facilities. Collectively, these decisions make up the daily operations plan, which aims for system efficiency and the satisfaction of the citizens' concerns. In putting together the daily operation plan, the decision maker also needs to address the uncertainty regarding the levels of the different types of material present at the various locations, levels that evolve over time with the quantities of materials people deposit and the previous collecting activities. The objective is thus economic and service efficiency not only for the current day, but also "globally" for a given look-ahead planning horizon.

Point estimates are generally used for future material levels and are assumed in this paper as well. On the other hand, sensor and information systems may reveal the material levels at collection locations at the beginning of each day, information that can be used for planning. Notice that such systems do not eliminate the uncertainty of the system as the behavior of people and the rate of deposit for the following days is still uncertain. The impact of the deployment of such systems has been seldom, if at all, investigated for the class of problems we address and we contribute such a study in this paper.

To summarize, we consider the problem where multi-compartment vehicles are used for the simultaneous collection of several types of recyclable materials that must be handled separately over a finite non-periodic planning horizon. The recyclable materials are picked up at several collection locations and transported to the treatment facilities when necessary. The problem is further complicated by 1) stochastic daily accretion rates of materials at collection locations; 2) the variation in time of the quantities of materials available at each location, due to the fluctuating accretion rates and the consequences of the activities of previous days, as a subset of locations is visited at each vehicle departure and not all materials present at a visited location are collected; 3) the variation in time of the available capacity of the vehicle containers, due to non-daily visits to treatment facilities; 4) the possibility that deposited materials may overflow the capacity of the respective collection points under certain location-specific conditions. The aim is to minimize the operation cost over the planning horizon, while avoiding violations of capacity constraints for vehicles and reducing as much as possible the capacity overflow at collection points.

Our main contributions are:

1. The formulation of a full mathematical model for the considered optimization problem. The formulation takes the form of a two-stage stochastic programming model with simple recourse, where collection decisions make up the first stage;

2. Two deterministic daily management policies that can be used by the decision maker to address the problem;

3. A global rolling-horizon procedure that aims to mange operations over a planning horizon, where the collection plan at a given time period is determined by using one of the daily management policies;

4. The investigation of the impact and value of the availability of sensor and information systems providing information on material levels at collection locations;

5. The introduction of a large and complete set of instances, which we used for our experiments and a comprehensive analysis of the proposed methodology given the absence or presence of a sensor and information system.

The remainder of the paper is organized as follows. The problem description and the related literature are presented in the next section. The main modeling ideas and the full mathematical formulation are the topic of Section 3. Section 4 introduces the management policies for addressing the problem at a given time period, followed by more detailed descriptions in Sections 5 and 6. Implementation details, instances, and results of computational experiments are presented in Section 7. Finally, conclusions and directions for further research are given in Section 8.

2. Problem description and literature review

Various types of waste are suitable for recycling. They can be divided into two main material groups: breakable, e.g., glass, bottles, and jam jars, and non-breakable, e.g., paper, cardboard, cans, and plastic bottles. The first group must be handled with extra care to avoid extensive shattering, which would have a negative impart on the recycling process. Both groups of materials may be transformed into new products at treatment facilities designated for that purpose. Notice that biodegradable materials, e.g., food, could fit into the second-type definition. Yet, as they must not be mixed with the other types, biodegradable waste is usually handled through separate bins both at collection points and on vehicles. Considering this third material type would be important for an actual application, but it does not modify the methodology presented in this paper. For clarity of presentation reasons, we thus consider two types only, identified as *glass* (g) for breakable and *paper* (p) for the non-breakable materials.

The collection of recyclable materials is performed within a predetermined operational area such as a city, region, or county. Environmentally-conscious citizens interested in reducing waste as much as possible voluntarily deposit the recyclable part of waste. In return, they expect that further handling of the materials is done in a suitable way, such that their effort is not made in vain. To ease accessibility and increase the degree of recycling, several collection points are organized within the operational area. They can be located in a variety of locations with respect to the population density and the immediate neighborhood social, cultural, economic and touristic characteristics, such as the city center, residential zones, near a highway, along a countryside road, etc. Small bins for collection, called *cubes*, are located at each collection point. To keep the recyclable materials separate, cubes are dedicated to each type. For the same reason, the combined collection of the materials is performed with multi-compartment vehicles; these compartments are movable and are called *containers*. The vehicles are equipped with a crane to handle the offloading of the materials and the emptying of the cubes.

The rate of deposited materials in the cubes varies daily and, in the absence of a sensor and information system, it is not known with certainty when the daily collection plan is built in the morning. Hence, the true fill level of material in a cube only becomes known when visiting the collection location. Due to economic and vehicle-capacity reasons, only a subset of locations is visited on any given day. The selection of these locations and the route servicing them make up the daily operations plan. Cubes may be allowed to overflow under certain conditions, while the capacity of the vehicle containers is not allowed to be exceeded. There is no obligation to empty all cubes at a visited collection location. A cube cannot be partially emptied, however. Therefore, the selection of the cubes to empty is part of the operations plan and the fill level of the cubes at any location and given day depends on the operations performed the previous days.

The vehicle starts and ends each collection route at a depot where an extra container for each type of material is kept. When the vehicle returns to the depot, the crane offloads the non-breakable materials from the vehicle into the proper extra container while, due to the risk of shattering, this is not possible for the breakable materials. Instead, when a container for breakable materials on the vehicle is sufficiently full, it is swapped with the associated extra one. Once two containers of the same type are full, they are transported to the relevant treatment facility. Due to the limited amount of material, such trips to the treatment facilities are not needed on a daily basis.

The available, *residual* capacity in a container on the vehicle for glass thus depends heavily on the work performed during the preceding days. Likewise, once the extra container at the depot for paper is full, the capacity available in the container on the vehicle also depends on previous collections. Thus, the capacity of the vehicle is time-dependent.

The sequence of the events and decisions on a given day is as follows. First, the decision maker schedules the visits to the treatment facilities and collection locations for the day based on information available and empirical knowledge. Next, the planned transportation to the treatment facilities is performed, if any, and then the planned collection route is executed. A visit to a treatment facility starts and ends at the depot. The containers brought to the treatment facility are serviced, i.e., emptied, thus becoming again available for work.

A collection route normally visits many collection locations and starts and ends at a depot. The different materials are collected simultaneously on the same route. The emptying of cubes is preformed with the crane on the vehicle. The crane makes it possible to lift up a cube and remove its bottom, such that the material in the cube is emptied into the proper container on the vehicle. The cube is then returned to its place. A cube is therefore always emptied completely. The procedure is referred to as the *service of a cube* and has a duration that depends on which cube is emptied. Emptying the first cube at a location takes the longest time since the crane needs to be set up. A second cube with another type of material can be emptied slightly faster while any additional cubes are the least time consuming. The time to service a location depends on the number of cubes and number of different types of material.

The operations plan for the current day aims to minimize the operation cost, while avoiding violating the vehicle capacity and restricting as much as possible the cube overflow at collection points. Given the uncertainty in the rates of deposit of recyclable materials into cubes and the dependence on the activities performed in previous days of both the container residual capacities and the amount of materials present at collection locations, the objective is to propose management policies that build efficient daily operation plans while accounting for the uncertainty and time dependency inherent to the system. An associated objective is to evaluate the potential impact of a sensor and information system that could provide accurate information on the fill levels at the time the operations plan is built.

The *multi-period, multi-product, multi-compartment collection* problem we consider belongs to the large and varied family of node vehicle routing problems. It combines characteristics found in the Vehicle Routing (VRP) and the Capacitated Vehicle Routing (CVRP) problems with multi-product pick-up and delivery, and in the Inventory Routing Problem (IRP). We refer to Andersson et al. (2010) and Coelho et al. (2014b) for general overviews of the IRP literature. Neither setting addresses coherently our problem, however, which is further complicated by the transportation to treatment facilities, the special vehicle capacity constraints, and the uncertainty of the accretion rates.

A number of authors address real-world waste and recyclable material cases where collection is performed at nodes (collection locations), e.g., Angelelli and Speranza (2002) study a periodic problem in Italy, Kim et al. (2006) consider a collection problem with time windows in North America, Teixeira et al. (2004) study different types of waste which are collected separately in Portugal, and Kara and Önüt (2010) address collection of paper for recycling in Turkey. Our problem setting differs in characteristics, since the different types of materials are collected simultaneously over a non-periodic time horizon without time windows regulations. However, our problem generalizes the one introduced in Bogh et al. (2014), and further studied in Elbek and Wøhlk (2015), who consider a real-world collection of recyclable materials in Denmark.

With regards to the information systems presence, Bertazzi et al. (2013) and Coelho et al. (2014a) have addressed IRPs where demand is revealed over time, but neither of these two papers considers multi-compartment vehicles and transportation to treatment facilities. Delivery and collection problems with multi-compartment vehicles are studied in Coelho and Laporte (2015) and Muyldermans and Pang (2010), but without the special capacity constraints that characterize our problem.

3. Modeling

This section is dedicated to the model formulation of the multi-period, multi-product, multi-compartment general combined collection problem of breakable and non-breakable recyclable materials defined previously. We first discuss the main components of the problem and model: the information and decision processes and the modeling of uncertainty in the rate of material deposits into the cubes available at the various collection locations (Section 3.1); the representation of the material deposit process, introducing the decision variables of the formulation, as well as the modeling of the citizens' concerns with respect to the cube overflow (Section 3.2); and the treatment of the complex container capacity restrictions, particularly for breakable materials (Section 3.3).

The resulting mathematical formulation is presented last (Section 3.4). It takes the form of a two-stage stochastic programming model with simple recourse and routing and multi-capacity constraints for cubes and containers. The objective is to minimize the total cost, including routing, service, transportation, and overfilling penalty costs, over a given planning horizon. For simplicity and without loss of generality, we provide the formulation for the case of a single vehicle, as it is common in the IRP literature. This model extends the deterministic one introduced in Elbek and Wøhlk (2015) by explicitly modeling the uncertainty in accretion rates. A summary of all notation used in the paper may be found in the Annex C.9.

3.1. Modeling the uncertainty

Operations are taking place "continuously", that is every working day and they are supposed to continue to be performed for the foreseeable future. As no operation regularity can be assumed, in theory, one should consider an infinite planning horizon. This is clearly unreasonable, not only due to methodological and data-availability/forecasting challenges, but also because such a representation does not reflect reality. Indeed, only decisions for the immediate future are essential for carrying out the operations. One actually needs the operations plan for "today" and an estimation of its consequences for the following "days".

We thus build the model for a finite time horizon H, divided into $|H|$ time periods of equal length (e.g., days). Let $t \in \mathcal{T}' = \{0, 1, \ldots, |H|\}$ be a time instant (e.g., the beginning of the t^{th} day) and $\mathcal{T} = \mathcal{T}' \setminus \{0\}$, where $t = 1$ represents the present moment and $t = 0$ represents the previous.

Let $\mathcal{L}$ be the set of all collection locations. Let $\mathcal{N}$ be the set of recycling materials, where $n \in \mathcal{N}$ represents one type of material, which can be breakable or non-breakable. We use the expression *family of cubes*, noted $\mathcal{N}(l)$ to refer to the cluster of cubes for a material of type $n \in \mathcal{N}$ present at location $l \in \mathcal{L}$.

The type of operations and the stochastic nature of the problem yield numerous moments in time and space when information becomes available and where a decision can be taken and an action initiated. All decisions are not necessarily made by the same people, nor at the same time. A proper description of the information and decision process is given bellow and it justifies our decisions regarding the modeling approach.

Two "layers" of information update have to be considered. The first takes place at the depot prior to the selection of the operation plan for the day. On the one hand, it involves the capacity levels of the containers. The residual capacity following the operations of the previous day is observed. Then, if one decides to transport some materials to the treatment facility, the information regarding the capacity available in the associated containers (which are empty after the trip to the treatment facility) is updated. On the other hand, the information regarding the fill levels of the cubes is also updated at that moment. When a sensor and information system is in place, this information is assumed to be up to date and precise (because collection is performed at the beginning of a day, before significant new quantities of materials can be deposited). Otherwise, the update of the estimation of the fill levels is performed based on the activities of the previous day and the evaluation of the (distributions) of the rates of material deposits into the cubes. The route is then normally built.

A second layer of information update take place every time we visit a collection location since the true fill level of materials in the cubes is revealed, and the residual capacity of the containers on the vehicle is updated every time a cube is emptied. The decision process associated to these updates concerns how the route is built, *a priori* at the beginning of the day or dynamically while in route. In the latter case, at each visited cube we can make a decision where to go next, that is, which cube to visit next, the alternative decision being to go back to the depot. This implies that, at each cube (and at the depot) there is a stage where information is received and it is possible to take an action. As one normally visits many cubes on a route, this would yield many stages for each day, leading to a very large and complex multi-stage formulation.

The problem setting we address is different, however. The collection plan and the vehicle route are built a priori. The locations to visit and the cubes to empty are thus decided before the route starts. Then, at each cube, the fill levels of the cube and containers are observed, the former being later used at the depot to update the information. The only "decision" left once a cube is serviced is whether to continue to the next cube on the planned route or return to the depot, finishing the collection for the day, because all containers are full.

We therefore propose a two-stage model with simple recourse, modeling the relations between the stochastic fill levels and the cube and container capacities through penalty costs for overfilling. The first stage represents the decisions taken at the depot regarding the scheduling of the collection plan, i.e., the treatment facilities and the cubes to visit each day. In the second stage, the he planned route is performed, the cube fill levels are revealed, and the penalty cost of overfilling is paid.

Notice that the introduction of overfilling penalty costs implies that feasibility for cubes and containers capacity constraints can always be obtained. Furthermore, penalty costs can make it possible to achieve the desired behavior of the system while accounting for the citizens' concerns.

3.2. Collection locations and cubes

Let $\mathcal{G}$ and $\mathcal{P}$ be the set of cubes for glass ans paper, respectively, and let $\mathcal{M} = \mathcal{G} \cup \mathcal{P}$. Let the node set $\mathcal{V}$ represent all cubes (at all locations) $\mathcal{M}$ and the depot $\{0\}$.

To represent the route of the vehicle, we introduce *routing variables* y^t_{ij}, equal to one if the vehicle travels from node j to node i, and zero otherwise, $\forall i, j \in \mathcal{V}, j < i, t \in \mathcal{T}$. The routing variables y^t_{i0} can be equal to one, two, or zero, where $y^t_{i0} = 2$ represents a tour which visits cube i only, $\forall i \in \mathcal{M}, t \in \mathcal{T}$.

The quantity of material deposited into cube i at the period t is given by the *stochastic parameter* r_{it} for all $i \in \mathcal{M}$ and $t \in \mathcal{T}'$. The *collection decision variable* x_{it}, for all $i \in \mathcal{M}$ and $t \in \mathcal{T}'$, represents the quantity of material collected from cube i at time t. F_{it} then represents the quantity of material in cube i at time t and is given by

$$F_{it} = F_{i(t-1)} - x_{i(t-1)} + r_{i(t-1)} \quad i \in \mathcal{M}, t \in \mathcal{T}.$$

Cubes cannot be partially emptied, therefore

$$x_{it} = \begin{cases} F_{it} & \text{if cube } i \text{ is emptied at } t \in \mathcal{T}, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

We introduce binary variables θ_{it}, equal to one if and only if node i is visited at $t \in \mathcal{T}$, $\forall i \in \mathcal{V}$. Thereby, $\theta_{it} = 1$ indicates that one visits and collects the amount of material $x_{it} = F_{it}$ from cube i at time t, $\forall i \in \mathcal{M}$, whereas $\theta_{0t} = 1$ indicates that a collection route is performed at time t.

As the system aims to avoid overfilling the cubes, one would prefer the fill level F_{it} not to exceed the capacity U_i of cube i, that is, $F_{it} \leq U_i$ for all $i \in \mathcal{M}, t \in \mathcal{T}$. However, given that F_{it} is a stochastic variable, as it contains r_{it}, we cannot guarantee that overfilling will never occur. Moreover, it is always interesting, particularly from a management point of view, that planning models and tool provide the capability to evaluate economic versus service-quality trade offs. Hard capacity constraints are not easily amenable to this end. We therefore introduce *overfill slack variables* z_{it} to capture the quantities of excess materials in cubes, $z_{it} \geq 0, i \in \mathcal{M}, t \in \mathcal{T}$. The fill level restrictions can then be formulated as soft capacity constraints $F_{it} - z_{it} \leq U_i, i \in \mathcal{M}, t \in \mathcal{T}$, and penalties can be attached to the overfill levels.

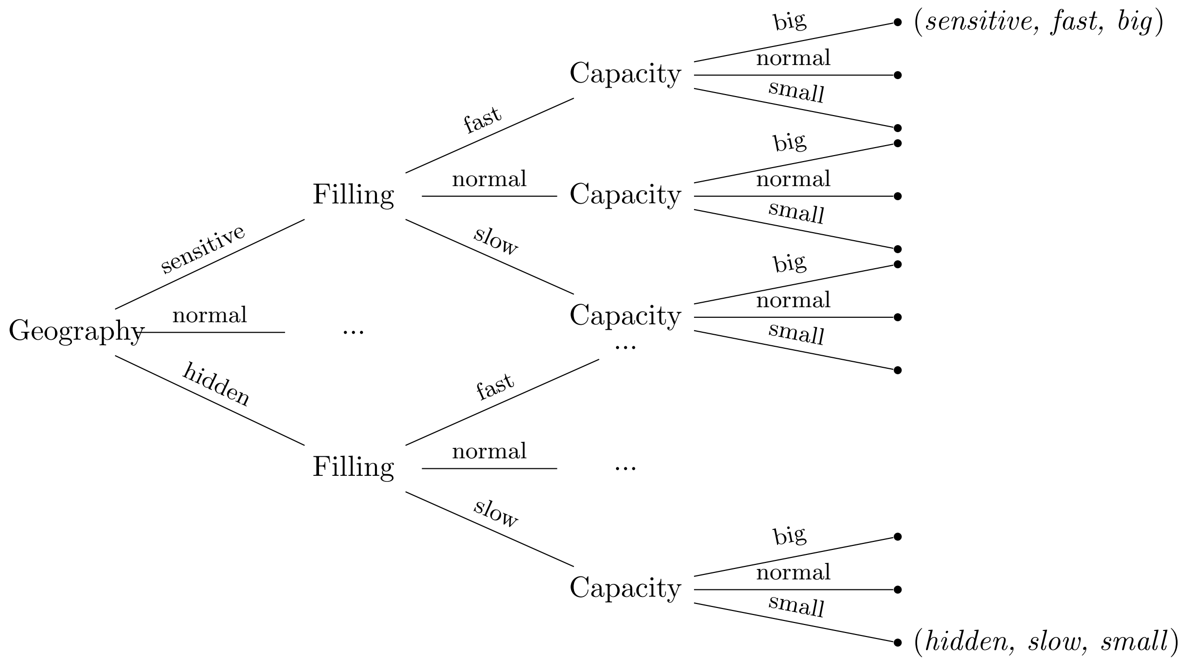

Figure 1: Subsets of families of cubes

One of the main modeling issues concerns the capturing of the citizens' disapproval of overfilled cubes, their degree of inconvenience/unhappiness varying particularly with the location of the cubes, and the degree of overflow. The families of cubes at the different collection locations in the operation area have different characteristics with regard to filling, capacity, and geographic position. We thus classify the families of cubes based on these criteria and introduce penalties for excess materials into the objective function.

The citizens have different preferences according to the properties of a family of cubes. To illustrate, consider that overflowing at a family of cubes located next to a significant site will create a stir, whereas if located out of view in a rural or less-visited area (e.g., at the back of shopping mall), the disturbance created by overfilling will be considerably smaller. Similarly, suppose that a collection schedule does not account for the fact that some cubes get filled faster than others, then the citizens would sense that insufficient attention is paid to those popular and fast filled families of cubes. The same could be said regarding families of cubes with large capacity: if overfilling cannot be avoided even with several cubes, the collection plan would not by acceptable for the citizens.

We therefore define a classification of families of cubes aiming to measure the importance of a given family of cubes for the planning process, according to its location, how fast the cubes fill up, and finally the number of cubes at the collection location, i.e., its capacity. We define three sets for geography: *hidden*, *normal*, and *sensitive*. The subset *sensitive* contains families of cubes located in the city center, the subset *hidden* contains families of cubes which are located out of sight, e.g., at a pull-up in the countryside, whereas the subset *normal* contains the families of cubes located in between. Three sets are also defined for the filling: *slow*, *normal*, and *fast*, using the same filling classification of Elbek and Wøhlk (2015), where the subset *fast* contains the families of cubes which accumulate an amount of material corresponding to at least 30 % of the capacity within a fixed length planning period. Among the remaining families of cubes, the subset *normal* contains those where the amount of material accumulated is at least 20 % of the capacity. The subset *slow* contains the remaining families of cubes. Finally, we introduce three sets for the capacity: *small*, *normal*, and *big*. Families of cubes with *big* capacity are families with more than two cubes, families of cubes with *normal* capacity are families with two cubes, and families of cubes with *small* capacity are families with only one cube. This results in a total of 27 subsets as shown in Figure 1 (a complete tree is illustrated in the Annex D.5). Notice that for all families of cubes at the same collection location, geography is the same, but filling and capacity can be different. The classification is used in the development of the collection plan, sets being predetermined to facilitate planning.

The penalty cost for overfilling cube $i \in \mathcal{M}$ is defined as ρ_i, yielding a penalty cost for the family of cubes of material of type n at location l defined as $\rho_l(\mathrm{n}) = \sum_{i \in \mathcal{N}(l)} \rho_i$. The value of the cube penalty cost is given according to the subset to which the family of cubes belongs, thus there are 27 different values for the penalty cost of overfilled cubes. We define the normal family based penalty cost at location l as $\rho_l(\mathrm{n}) = A2c_{0l}$, which is applied for the subset: normal geography, normal filling, and normal capacity (*normal, normal, normal*), where A is a large number and c_{0l} is the transportation cost from the depot to collection location $l \in \mathcal{L}$. As widely used in the literature, the intuition of the penalty cost is to estimate the cost of a solo visit to the location. For the other subsets, the penalty cost is multiplied by a parameter larger than 1 if the subset is more important and multiplied by a parameter smaller than 1 if the subset is less important. The parameters in the most important subset (*sensitive, fast, big*) and the least important subset (*hidden, slow, small*) are determined by computational experiments. The penalty costs for the subsets “in between” are established using a Fibonacci scaling factor. Thus, a family of cubes with a large capacity being filled rapidly and located close to an important site in a city center gets a higher penalty cost than a family of cubes with small capacity which gets filled slowly at a less conspicuous collection location.

3.3. Containers

The residual capacities $C_t(\mathrm{g})$ and $C_t(\mathrm{p})$ available in the containers on the vehicle at time t are highly dependent on the time when the materials are transported to the treatment facilities. If we transport the material to a treatment facility at time t, then the capacity available in the containers will be equal to the capacity of an empty container C. For glass, an empty container can also be obtained by swapping with an empty container at the depot. But, due to the possibility of simply offloading paper, the capacity $C_t(\mathrm{p})$ will be equal to C as long as the extra container for paper at the depot is not full. At other times, the capacities $C_t(\mathrm{g})$ and $C_t(\mathrm{p})$ depend on previously performed collections. This means that the capacity available at time t is given by the capacity available at time $t-1$ minus the total quantity of collected material at time $t-1$. For glass, the capacity $C_t(\mathrm{g})$ can be described as

$$C_t(\mathrm{g}) = \begin{cases} C & \text{if an empty container is swapped at time } t, \\ C_{(t-1)}(\mathrm{g}) - \sum_{i \in \mathcal{G}} x_{i(t-1)} \quad t \in \mathcal{T} & \text{otherwise.} \end{cases}$$

An empty-container swap may be performed only if one is available at the depot or if we visited the treatment facility at time t. The case for collecting paper is different due to the possibility to offload such materials at the depot. The capacity $C_t(\mathrm{p})$ may therefore be described as

$$C_t(\mathrm{p}) = \begin{cases} C & \text{if the container at the depot is not full at time } t, \\ C_{(t-1)}(\mathrm{p}) - \sum_{i \in \mathcal{P}} x_{i(t-1)} \quad t \in \mathcal{T} & \text{otherwise.} \end{cases}$$

Given that we have two containers for paper (one on the vehicle and one at the depot), the total capacity available for paper can be described as $C_t(\dot{\mathrm{p}})$, where $C_t(\dot{\mathrm{p}})$ is equal to $2C$ if we visited the treatment facility at time t, otherwise $C_t(\dot{\mathrm{p}}) = C_{(t-1)}(\dot{\mathrm{p}}) - \sum_{i \in \mathcal{P}} x_{i(t-1)}$. Hence, the capacity $C_t(\mathrm{g})$ can also be declared as $C_t(\mathrm{p}) = \min\{C, C_t(\dot{\mathrm{p}})\}$.

The *binary swapping variables* $\eta_t(\mathrm{g})$ are introduced to control the swap of glass containers, $\eta_t(\mathrm{g})$ being equal to 1 if and only if containers are swapped at time t. A trip to the treatment facility is required when the containers have been swapped twice. The *binary tour variables* $\eta_t(\mathrm{p})$ control the trips to the treatment facility for paper. $\eta_t(\mathrm{p})$ is equal to 1 if and only if the vehicle goes to the treatment facility at time t.

The quantities of glass and paper collected at time t should not exceed $C_t(\mathrm{g})$ and $C_t(\mathrm{p})$, respectively. Yet, these quantities are uncertain and, again, we cannot guarantee strict enforcement of capacity restrictions. We therefore introduce *overfill slack variables* $\delta_t(\mathrm{g})$ and $\delta_t(\mathrm{p})$ in the capacity constraints to capture the excess quantities of materials in containers. The capacity constraints for the containers are thereby given as

$$\sum_{i \in \mathcal{G}} x_{it} - \delta_t(\mathrm{g}) \leq C_t(\mathrm{g}) \text{ and } \sum_{i \in \mathcal{P}} x_{it} - \delta_t(\mathrm{p}) \leq C_t(\mathrm{p}), \quad t \in \mathcal{T}.$$

The excess material in containers is minimized through penalties in the objective function, where the corresponding overfilling penalty represents the cost of extra capacity. The penalty cost for overfilling a container of material of type n is defined as $\rho(\mathrm{n}) = B(\max_{l \in \mathcal{L}}\{2c_{0l}\} + k(\mathrm{n}))$, where $k(\mathrm{n})$ is the transportation cost of a round trip to the treatment facility for material of type n and B is a parameter used to adjust the resulting value. Thus, the penalty cost is the cost of a detour to the depot from the collection location farthest away plus a trip to the associated treatment facility. Hence, it is a worst-case penalty cost.

3.4. Two-stage model

Several cost components are considered, transportation and service costs for cubes (routes) and containers (treatment facilities), as well as the penalty costs for overflowing cubes and containers.

Let c_{ij}, $\forall i \in \mathcal{V}$ be the transportation cost of traveling between nodes i and j. The service cost w_{ij} of emptying cube i after cube j is defined as

$$w_{ij} = \begin{cases} w^1 & \text{if cubes } i \text{ and } j \text{ are at the same collection location and contain the same type of material,} \\ w^2 & \text{if cubes } i \text{ and } j \text{ are at the same location but contain different types of material,} \\ w^3 & \text{otherwise,} \end{cases}$$

where $0 < w^1 < w^2 < w^3$ as described in Section 2.

Let $k(\mathrm{g})$ and $k(\mathrm{p})$ be the transportation cost of a round trip to the treatment facility for glass and paper, respectively. The service costs $w(\mathrm{g})$ or $w(\mathrm{p})$ are incurred every time containers for glass and paper, respectively, are emptied at the treatment facility.

Let $\mathcal{S}$ be the set of scenarios describing the uncertainty. Then, r^s_{it} is the stochastic quantity of material deposited into cube $i \in \mathcal{M}$ at time $t \in \mathcal{T}$ in scenario $s \in \mathcal{S}$, and p^s is the probability of scenario $s \in \mathcal{S}$. The two-stage formulation then becomes:

$$\begin{aligned}
\min \quad & \sum_{i\in\mathcal{V}} \sum_{j\in\mathcal{V}, j<i} \sum_{t\in\mathcal{T}} (c_{ij} + w_{ij}) y^t_{ij} + \sum_{s\in\mathcal{S}} \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{M}} \rho_i p^s z^s_{it} \\
& + \tfrac{1}{2}(k(\mathrm{g}) + w(\mathrm{g})) \sum_{t\in\mathcal{T}} \eta_t(\mathrm{g}) + \rho(\mathrm{g}) \sum_{s\in\mathcal{S}} \sum_{t\in\mathcal{T}} p^s \delta_t(\mathrm{g})^s \\
& + (k(\mathrm{p}) + w(\mathrm{p})) \sum_{t\in\mathcal{T}} \eta_t(\mathrm{p}) + \rho(\mathrm{p}) \sum_{s\in\mathcal{S}} \sum_{t\in\mathcal{T}} p^s \delta_t(\mathrm{p})^s
\end{aligned}$$

$$\text{s.t.:} \quad F^s_{it} = F^s_{i(t-1)} - x^s_{i(t-1)} + r^s_{i(t-1)} \qquad \forall i \in \mathcal{M}, t \in \mathcal{T}, s \in \mathcal{S} \tag{2a}$$

$$F^s_{it} - z^s_{it} \leq U_i \qquad \forall i \in \mathcal{M}, t \in \mathcal{T}, s \in \mathcal{S} \tag{2b}$$

$$x^s_{it} \geq (1 - \theta_{it})(-U_i - z^s_{it}) + F^s_{it} \qquad \forall i \in \mathcal{M}, t \in \mathcal{T}, s \in \mathcal{S} \tag{2c}$$

$$x^s_{it} \leq F^s_{it} \qquad \forall i \in \mathcal{M}, t \in \mathcal{T}, s \in \mathcal{S} \tag{2d}$$

$$x^s_{it} \leq (U_i + z^s_{it})\theta_{it} \qquad \forall i \in \mathcal{M}, t \in \mathcal{T}, s \in \mathcal{S} \tag{2e}$$

$$\sum_{i\in\mathcal{G}} x^s_{it} - \delta_t(\mathrm{g})^s \leq C_t(\mathrm{g})^s \cdot \theta_{0t} \qquad \forall t \in \mathcal{T}, s \in \mathcal{S} \tag{2f}$$

$$C_t(\mathrm{g})^s = \Big(C_{(t-1)}(\mathrm{g})^s - \sum_{i\in\mathcal{G}} x^s_{i(t-1)}\Big)(1 - \eta_t(\mathrm{g})) + C \cdot \eta_t(\mathrm{g}) \qquad \forall t \in \mathcal{T}, s \in \mathcal{S} \tag{2g}$$

$$C_t(\mathrm{g})^s \leq C \qquad \forall t \in \mathcal{T}', s \in \mathcal{S} \tag{2h}$$

$$\sum_{i\in\mathcal{P}} x^s_{it} - \delta_t(\mathrm{p})^s \leq \min\{C, C_t(\dot{\mathrm{p}})^s\} \cdot \theta_{0t} \qquad \forall t \in \mathcal{T}, s \in \mathcal{S} \tag{2i}$$

$$C_t(\dot{\mathrm{p}})^s = \Big(C_{(t-1)}(\dot{\mathrm{p}})^s - \sum_{i\in\mathcal{P}} x^s_{i(t-1)}\Big)(1 - \eta_t(\mathrm{p})) + 2C \cdot \eta_t(\mathrm{p}) \qquad \forall t \in \mathcal{T}, s \in \mathcal{S} \tag{2j}$$

$$C_t(\dot{\mathrm{p}})^s \leq 2C \qquad \forall t \in \mathcal{T}', s \in \mathcal{S} \tag{2k}$$

$$\sum_{j \in \mathcal{V}, j<i} y_{ij}^t + \sum_{j \in \mathcal{V}, j>i} y_{ji}^t = 2\theta_{it} \qquad \forall i \in \mathcal{V}, t \in \mathcal{T} \tag{2l}$$

$$\sum_{i \in \mathcal{B}} \sum_{j \in \mathcal{B}, j<i} y_{ij}^t \leq \sum_{i \in \mathcal{B}} \theta_{it} - \theta_{bt} \qquad \forall \mathcal{B} \subseteq \mathcal{M}, b \in \mathcal{B}, t \in \mathcal{T} \tag{2m}$$

$$F_{it}^s \geq 0, \;\; z_{it}^s \geq 0, \;\; x_{it}^s \geq 0, \;\; r_{it}^s \geq 0 \qquad \forall i \in \mathcal{M}, t \in \mathcal{T}', s \in \mathcal{S} \tag{2n}$$

$$\delta_t(\mathrm{g})^s \geq 0, \;\; \delta_t(\mathrm{p})^s \geq 0 \qquad \forall t \in \mathcal{T}, s \in \mathcal{S} \tag{2o}$$

$$C_t(\mathrm{g})^s \geq 0, \;\; C_t(\dot{\mathrm{p}})^s \geq 0 \qquad \forall t \in \mathcal{T}', s \in \mathcal{S} \tag{2p}$$

$$\eta_t(\mathrm{g}) \in \{0,1\}, \;\; \eta_t(\mathrm{p}) \in \{0,1\} \qquad \forall t \in \mathcal{T} \tag{2q}$$

$$\theta_{it} \in \{0,1\} \qquad \forall i \in \mathcal{V}, t \in \mathcal{T} \tag{2r}$$

$$y_{ij}^t \in \{0,1\} \qquad \forall i \in \mathcal{M}, j \in \mathcal{M}, j < i, t \in \mathcal{T} \tag{2s}$$

$$y_{i0}^t \in \{0,1,2\} \qquad \forall i \in \mathcal{M}, t \in \mathcal{T} \tag{2t}$$

The objective function defines the minimization of routing, service, and penalty costs for cubes and containers. Constraints (2a) are the bookkeeping (inventory) constraints on the fill level in the cubes. Constraints (2b) constrain the fill level in the cube by its capacity. The following constraints, (2c)–(2e), state that the quantity of material x_{it}^s emptied from the cube is equal to the quantity of material F_{it}^s in the cube as outlined in equation (1). Constraints (2f) and (2i) constrain the total quantity of collected materials by the residual container capacity. Constraints (2g) and (2j) are the bookkeeping constraints for the residual capacity available in the containers. The limits of the residual container capacity are emphasized in constraints (2h) and (2k). Constraints (2l) and (2m) are the degree and subtour elimination constraints, respectively. Constraints (2n)–(2t) enforce the integrality or non-negativity conditions on variables.

As indicated earlier, the model can easily be extended to handle more materials. When a material can be mixed with an already handled material and transported to the same treatment facility, e.g. general glass and wine bottles, the same cubes and containers can be used for collection of both materials. The corresponding stochastic parameter r_{it}^s would have to be adjusted to represent the sum of the materials deposited into cube i. When the material, e.g. organic waste, requires separate cubes and containers, the cube set $\mathcal{M}$ has to be enlarged. Furthermore, constraints (2f)–(2h) have to be duplicated when the material is of the breakable type, while constraints (2i)–(2k) are to be duplicated when non-breakable materials are added. The associated parameters and variables (e.g., F_it, x_it, etc.) and the objective function are adjusted correspondingly. Finally, notice that the problem is NP-hard since it contains a Traveling Salesman Problem (TSP) and the CVRP is a special case.

4. Solution approach

In this section, we detail the proposed solution approach for the considered collection problem. Let us first recall that a solution to the problem can be built over time by determining the collection plan to implement at each time period defined in the planning horizon. It should be further emphasised that when a particular time period t is reached, the only collection plan that needs to be established is the current one. Therefore, to solve the collection problem, we develop a solution approach that applies different management policies in a rolling horizon framework. As defined in Powell (2011), given the available information in the state at a time period t, a policy is a set of rules that determine a decision (or a set of decisions) to be made. Consequently, the proposed management policies are used to determine the collection plan at time t. The obtained plan is applied and the overal outcome of the operations as well as the realized fill levels are observed. Following a rolling horizon framework, this process is then repeated for the next time period and onward.

Two approximate policies are developed in this paper. These policies are obtained by limiting the problem's horizon, aggregating the decision stages, and by generating point estimates for the stochastic parameters. We introduce a *myopic* and a *look-ahead* policy. In the former, decisions made at time t are based solely on the current state of the system and the future consequences of the decisions are not considered. In the latter, the decisions made at time t take the consequences of future decisions and states into consideration. The look-ahead policy explicitly solves the problem over a shorter planning horizon by combining an approximation of future information (fill levels in cubes and containers) with an approximation of future actions (visits to collection locations and treatment facilities). Hence, in the look-ahead policy, decisions are established for the following time periods: $t, t+1, \ldots, t+k$. However, the decisions associated to $t' > t$ are simply used to assess the quality of the decisions to be made in period t.

We refer to the myopic policy as the *single-period policy* since it only determines the collection plan for one period. The look-ahead policy is identified as the *multi-period policy* given that collection plans for several periods are established. The single-period policy serves as a policy benchmark considering that no effort is spent on computing the consequences of the decisions taken for the present period. The main advantages of this policy are that it can easily be implemented in practice and the computational effort to perform it is limited. As for the multi-period policy proposed, it uses a more accurate cost function to evaluate the presently taken decisions. As such, it is a more computationally demanding policy to use. However, since approximations of both the current and projected costs of decisions are considered in the planning process, it is expected to produce a better overall solution for the problem. Recently, Powell (2014) has stated that look-ahead policies are particularly useful for time-dependent problems, especially in cases where forecasts, that evolve over time, are used. Furthermore, the author argues that deterministic look-ahead models can be effecient even when applied to stochastic problems.

Table 1: Settings for the policies

	Are information systems available?	
	No	Yes
Single-period policy	SN	SY
Multi-period policy	MN	MY

Table 1 displays the four settings we intend to investigate: single-period policy without sensor and information systems available (SN), single-period policy with sensor and information systems available (SY), multi-period policy without sensor and information systems available (MN), and multi-period policy with sensor and information systems available (MY). A certain level of knowledge is assumed known regarding how the daily accretion rates vary for all collection locations. We define μ_l(n) as the mean and σ_l(n) as the standard deviation for the family of cubes for material of type n at location l. It is further assumed that the accretion rates follow normal distributions. These distributions are used to make forecasts for the fill levels in the cubes. If information systems are present, these forecasts can be updated every day. Hence, it is expected that the performance of the defined policies will vary according to whether or not sensor and information systems are available to produce the forecasts.

Finally, the proposed policies are implemented using an order-up-to-level strategy (as known from inventory control) for all collection locations. The use of such a strategy reduces the number of daily decisions to be made, i.e., all cubes at a given location are emptied each time the location is visited. It should be noted that this approach implies a fixed service cost for each location. In order to further facilitate the daily collection planning, we introduce service levels to define the moment when a family of cubes, and the associated location, should be included in the set of collection locations to visit. The motivation for this is that the exact knowledge of when the cubes at a location were last emptied can be combined with the forecasts of the accretion rates, to produce an appropriate estimate for the time of the next visit to the considered location. We therefore define a service level for each subset of families of cubes to guide the selection of the next time for a visit. The values of the service levels should reflect the importance of the subsets. Similar to the penalty costs, the values of the service levels for the most and the least important subsets are determined by numerical calibration, while the values for the subsets in between are established using a Fibonacci scaling factor. The different policies proposed are detailed in the next two sections of the paper.

5. Single-period policy

In the single-period policy, decisions regarding the routing, transportation, and collection operations are exclusively based on the current levels of the materials in the families of cubes. At time t, all families of cubes with fill levels greater than or equal to the family-dependent threshold $s_{\mathcal{N}(l)}$ are selected to be emptied. Inspired by classical inventory management, $s_{\mathcal{N}(l)}$ is computed as $s_{\mathcal{N}(l)} = \sum_{i\in\mathcal{N}(l)} U_i - \big(\mu_l(\mathrm{n}) + v_\alpha\sigma_l(\mathrm{n})\big)$, for all $n \in \mathcal{N}$ and $l \in \mathcal{L}$. Considering that α is the probability of the cubes overfilling at a collection location, $1-\alpha$ defines the service level considered, where v_α is the corresponding quantile. The fill levels can either be established using the sensor and information systems, or they can be estimated. Then, the family of cubes $\mathcal{N}(l)$ at location l is to be emptied at time t if its fill level is at least $s_{\mathcal{N}(l)}$, in which case location l is selected to be included in the current route. The route is established by solving a TSP.

The total quantity of materials to be collected at time t can be calculated exactly when sensor and information systems are available, otherwise, an estimate is used. When this quantity is above the available capacity in the vehicle's containers, then either the containers are swapped, or, the vehicle goes to the associated treatment facility prior to performing the route at time t. Once the route has been carried out, the non-breakable materials of type n from the container on the vehicle are partly, or fully, offloaded to the associated container at the depot, provided that it is not full. The capacity $C_{(t+1)}(\mathrm{n})$, which will be available for the next period, is updated, $\forall n \in \mathcal{N}$. In addition, overfilled cubes and containers at time t are reported and penalties are calculated. Due to the finite time horizon, a check for overfilled cubes at the end of the time horizon is performed. The overall structure of the single-period policy is shown in Algorithm 1. The following sections include detailed descriptions of the SN and SY policies, respectively, as the differences between the two are vital.

Algorithm 1: Overview of the single-period policy

Initialization;
Classification of families of cubes;
while $t \leq H$ **do**
  Select collection locations for visiting at time t;
  Create the route at time t;
  Transport materials to the treatment facilities, if necessary;
  Perform the route at time t;
  Update capacities and calculate penalties;
  Consider next t;
end
Check for overfilled cubes at the end of the time horizon;

5.1. Outline of SN

Estimating unknown information is particularly important when information systems are unavailable. The estimate of the quantity of material of type n at collection location l for time t is computed as

$$\sum_{i \in \mathcal{N}(l)} \hat{F}_{it} = \mu_l(\mathrm{n}) \cdot (t - \tilde{t}_l) + z_\alpha \sqrt{\sigma_l(\mathrm{n})^2 \cdot (t - \tilde{t}_l)}, \quad \forall n \in \mathcal{N}, \tag{3}$$

where $\tilde{t}_l \in \mathcal{T}'$ represents the time of the last visit to location l and z_α corresponds to the 95 %–quantile of the accretion rate distribution. The value $\sum_{i \in \mathcal{N}(l)} \hat{F}_{it}$ is defined as a point estimate of $\sum_{i \in \mathcal{N}(l)} F_{it}$, and serves to incorporate the stochastic nature of the accretion rates in the solution process. For all $n \in \mathcal{N}$ and $l \in \mathcal{L}$, if $\sum_{i \in \mathcal{N}(l)} \hat{F}_{it} \geq s_{\mathcal{N}(l)}$, then the family of cubes of material of type n at location l is selected to be emptied at time t. In the absence of information systems, the penalty costs for cubes are only discovered when the vehicle arrives at the locations. Thus, the total penalty for cubes is not known prior to the route being performed at time t. As for the estimate of the total quantity of material of type n to be collected from the locations on route $\mathcal{R}_t$ at time t, it is computed as

$$\sum_{i \in n} \hat{x}_{it} = \sum_{l \in \mathcal{R}_t} (\mu_l(\mathrm{n}) \cdot (t - \tilde{t}_l)) + z_\alpha \sqrt{\sum_{l \in \mathcal{R}_t} (\sigma_l(\mathrm{n})^2 \cdot (t - \tilde{t}_l))}, \quad \forall n \in \mathcal{N}. \tag{4}$$

Value $\sum_{i \in n} \hat{x}_{it}$ is a point estimate of $\sum_{i \in n} x_{it}$, that considers the uncertainty related to the amount of the material to be collected at time t. If the estimated total quantity of materials to be collected is greater than or equal to the available capacity (i.e., $\sum_{i \in n} \hat{x}_{it} > C_t(\mathrm{n}), \forall n \in \mathcal{N}$), then either the containers are swapped at the depot, or the vehicle goes to the treatment facility before performing the route. The penalty costs $\rho(\mathrm{n})$, for all $n \in \mathcal{N}$, are added when the materials of type n collected at time t exceed the capacity of the associated container.

5.2. Outline of SY

When sensor and information systems are available, the exact fill levels for all families of cubes are known at time t. If $\sum_{i \in \mathcal{N}(l)} F_{it} \geq s_{\mathcal{N}(l)}$, then the family of cubes of type n at collection location l will be selected to be emptied at time t. Assuming that the information systems are correct and precise, the penalty cost for cubes at time t is zero, unless there exists a location l such that $\sum_{i \in \mathcal{N}(l)} F_{it} > \sum_{i \in \mathcal{N}(l)} U_i$. This implies that, even when using information systems, cubes can still be overfilled. However, such cubes will be emptied on the first day of overfilling, considering that $s_{\mathcal{N}(l)} \leq \sum_{i \in \mathcal{N}(l)} U_i$. In this case, the total penalty cost for cubes is known prior to route $\mathcal{R}_t$ being performed at time t. Hence, there are no surprises when the route is performed. The total quantity of materials for collection at time t can be calculated exactly since information systems provide the actual fill levels. If the estimated quantity of material of type n for collection at time t exceeds the available capacity, i.e., $\sum_{i \in n} x_{it} = \sum_{l \in \mathcal{R}_t} \sum_{i \in \mathcal{N}(l)} F_{it} > C_t(\text{n})$, the containers are either swapped (if possible), or, the vehicle goes to the associated treatment facility before beginning the route. Furthermore, no unnecessary visits to the treatment facilities are performed, given that both the available capacity and the total amount of materials to be collected are known exactly. Finally, the penalty costs $\rho(\text{n})$, for all $n \in \mathcal{N}$, are known prior to the execution of the route, since the information systems eliminate the uncertainty concerning the quantity of materials to collect from the selected locations.

6. Multi-period policy

The multi-period policy creates a k-period collection plan, $k < H$, where fill level and cost forecasts are used to support decisions and optimize the overall collection plan. A k-period fill level forecast is computed for each family of cubes on the basis of the assumed distributions and then used as a proxy for the unknown fill levels. The problem is then solved heuristically as a deterministic problem, resulting in a k-period collection plan. Using this plan, the first-period solution at time t is implemented. Afterwards, time $t+1$ is considered and the process is reiterated in a rolling horizon framework. We now detail the general improvement heuristic proposed to solve the problem using a multi-period policy (Section 6.1) and we define the specific MN and MY policies that are used (Sections 6.2 and 6.3, respectively).

6.1. Overview of the improvement heuristic

The problem is solved using the heuristic originally introduced in Elbek and Wøhlk (2015). The overall procedure starts by constructing an initial solution to the collection problem over the H considered periods (the details of how this solutions is obtained are provided below). For each time period t in the horizon, a look-ahead policy is then applied. To do so, the heuristic first generates a k-period forecast for each family of cubes. The Variable Neighborhood Search (VNS), proposed in Elbek and Wøhlk (2015), is then applied to improve the collection plan defined over the next k periods. When the VNS procedure completes its search, the obtained solution is implemented for the current time period t. This entails that a visit to the treatment facility is performed, if necessary, and the obtained route at time t is executed. At this point, the container capacities are updated and the penalty costs are also calculated. Furthermore, each visited collection location is rescheduled in a route at a subsequent time period. Once all of this is done, the heuristic proceeds to the following period. The overall structure of the proposed heuristic is summarized in Algorithm 2.

When applying the VNS procedure in this context, an important challenge to address is how to properly assess the future effects on the overall total cost associated to the modifications made to the current k-period collection plan. In an effort to improve the manner in which these modifications are evaluated, an objective function is defined using two components: the direct and the future costs. The following elements are included in the direct costs: the routing cost, the service cost for

Algorithm 2: Heuristic defined using a multi-period policy

Initialization;
Classification of families of cubes;
Insertion of all collection locations in routes in $\mathcal{T}$;
while $t \leq H$ **do**
  Create a k-period fill level forecast for each family of cubes in the k-period;
  Improve the collection in the k-period using VNS;
  Transport materials to the treatment facilities, if necessary;
  Execute the route at time t;
  Update capacities and calculate penalties;
  Insert each visited location in a route at a time $\hat{t} > t$, $\hat{t} \in \mathcal{T}$;
  Consider next t;
end
Check for overfilled cubes at the end of the time horizon.

the collection locations and treatment facilities, the penalty cost for overfilled cubes and containers, and the transportation cost to the treatment facilities in the k-period. As for the future costs, they include the estimated costs Q_{lt} of visiting the locations $l \in \mathcal{L}$ at a given time $t \in \mathcal{T}$, and the estimated costs $Q(\mathrm{n})$ of visiting, as planned, the treatment facilities for the materials $n \in \mathcal{N}$. The reasoning behind the use of these future costs is that if a location is visited earlier than necessary, then additional visits to this location may be necessary towards the end of the time horizon. The same observation can be made in the case of the treatment facilities. In addition, a specific location that is positioned far from the rest may be hard to include in a future route without incurring a high cost. As a rolling horizon framework is applied to solve the problem, these effects are not captured by the direct costs. Therefore values Q_{lt} and $Q(\mathrm{n})$ are used to assess these future costs for collection locations and treatment facilities, respectively.

Let the day $t_l^\dagger \in \mathcal{T}$ represent the last possible day to empty the cubes at collection location $l \in \mathcal{L}$ to avoid overfilling with a probability of 95 %. The future cost for location $l \in \mathcal{L}$ on day $t \in \mathcal{T}$, $t \leq t_l^\dagger$, is computed as follows:

$$Q_{lt} = \frac{t_l^\dagger - t}{t_l^\dagger - \tilde{t}_l} \cdot (\gamma_l + \phi_l), \tag{5}$$

where γ_l is the average routing cost from location l to its ten nearest neighbors multiplied by two, to represent a route. Hence, the future cost Q_{lt} will increase if location l is positioned far away from the other locations. The parameter ϕ_l represents the service cost, of emptying all cubes, for location l. Therefore, the first term in (5) is the fraction of extra visits needed to service collection location l, while the second term is an estimate for the cost of such an additional visit.

As for the future cost for the treatment facility for material of type $n \in \mathcal{N}$, it is computed as follows:

$$Q(\mathrm{n}) = \frac{\zeta(\mathrm{n})}{C} \cdot \frac{k(\mathrm{n}) \cdot w(\mathrm{n})}{2}, \tag{6}$$

where $\zeta(\mathrm{n})$ is the estimated quantity of the material to be collected in the k-period following the latest planned visit to the treatment facility. Intuitively, $\zeta(\mathrm{n})$ is the amount of material which has to be transported to the treatment facility at a later point in the time horizon. Therefore, the first term in (6) is the fraction of extra visits needed to the treatment facility and the second term is an estimate for the cost of such an additional visit. The future costs are included in the evaluation of the collection plan to provide an incentive to delay the emptying of the cubes and containers as long as possible. For further details regarding these future costs, the reader is referred to Elbek and Wøhlk (2015), where it is shown that the inclusion of these costs leads to better solutions for the collection problem.

We now detail the steps of Algorithm 2 and provide a brief description of the VNS procedure of Elbek and Wøhlk (2015) and how it is applied in the present context. To obtain a starting solution, all families of cubes are first classified based on the criteria in Section 3.2. Collection locations to be visited are then inserted in the time horizon according to their corresponding service levels (defined by the classification) and their associated thresholds $s_{\mathcal{N}(l)}$. This is done in the following way: $\forall n \in \mathcal{N}$, find the day where the quantity of material n in the family of cubes at location l is at least $s_{\mathcal{N}(l)}$, the location is then inserted on the earliest day identified.

The creation of the forecasts for the k-period is dependent on the availability, or not, of information systems (details being provided in Sections 6.2 and 6.3). The VNS procedure is then used to improve the collection plan in the k-period. The layout of the procedure is shown in Algorithm 3 in the Annex. As input to the VNS, the current solution for the k-period is given, and the improvement process is applied until a time limit or a stop criterion is reached. The four shakes in the VNS part consist of block moves with 2–5 locations. All shakes perform one block move, where a block (i.e., a sequence of locations) is moved from a route on day t' to a route on day t'' in the k-period. The best block of a given size in the route on day t' is found and moved to the best position in the route on day t'' in the k-period, where days t' and t'' are randomly chosen in the k-period and $t' \neq t''$. The best block and best position are defined as the ones that produce the largest gain when removing the block form from day t' and inserting it at the specific position on day t''. Once a shake has been performed, a local search is launched using three neighborhoods: Swap, Move, and 2-opt exchange. The Swap neighborhood includes moves where two locations are exchanged between a route on day t' and a route on day t'' in the k-period, $t' \neq t''$. The two locations are inserted at the positions in the routes where the insertion costs are minimized. Considering all locations on day t', the best swap is implemented if an improvement can be obtained; this is done for all t' in the k-period. In the second neighborhood, a single location can be moved from a route on day t' to a route on day t'' in the k-period, $t' \neq t''$. The location is placed in the route at the selected day t'' in the position that minimizes the insertion cost. For all locations on day t' the move is performed if an improvement can be obtained; this is done for all t' in the k-period. Using the last neighborhood, 2-opt exchanges are performed on all routes in the k-period until no further improvements can be obtained.

When applying the solution approach by Elbek and Wøhlk (2015) in the present context, it should be noted that we eliminate the early and late limits when moving a collection location in the time horizon and introduce penalty costs instead. Whenever the VNS performs a move which results in either cubes or containers overfilling, the corresponding penalty cost is added. This implies that all possible moves are considered feasible, but a penalty cost is applied for overfilling. It should be noted that, shakes will often result in the penalties being increased given that these consequences are not considered when evaluating such modifications in the solution. However, moves in the local search are performed while minimizing the total cost defined by the modified objective function (including both the direct and future costs). Once the improvement process is completed and prior to the route being performed, the treatment facilities are visited, if necessary. Thereafter, the non-breakable materials are offloaded at the depot and the capacities available for the next period are updated. Furthermore, overfilled cubes and containers at time t are reported and penalties are calculated. Finally, the visited collection locations are inserted in routes in $\mathcal{T}$ again as in the initialization step.

6.2. Outline of MN

The forecasts of the quantities of materials at the collection locations $l \in \mathcal{L}$ for the time periods $t, t+1, ..., t+k$ in the k-period are computed as in Equation (3). The penalty cost for the families of cubes is applied if $\sum_{i\in\mathcal{N}(l)} \hat{F}_{it'} > \sum_{i\in\mathcal{N}(l)} U_i$ for $t' = t, t+1, ..., t+k$ in the k-period collection plan. The forecast for the total quantity of materials to be collected from the locations scheduled at the time periods $t, t+1, ..., t+k$ is computed as in Equation (4). Once again, if the forecast is above the available capacity at time t (i.e., $\sum_{i\in n} \hat{x}_{it} > C_t(\mathrm{n})$, $\forall n \in \mathcal{N}$), then either the containers are swapped, or, the vehicle returns to the treatment facilities. The penalty cost $\rho(\mathrm{n})$, for a container of type n, is applied at the time periods $t, t+1, ..., t+k$, if the forecast of the quantity of material to be collected exceeds the capacity C. The exact penalty costs for cubes and containers at time t are discovered once the route is performed.

6.3. Outline of MY

In the presence of sensor and information systems, the real fill levels for all families of cubes are known at time t. Hence, the quantity of material of type n at collection location l at time t, $\sum_{i\in\mathcal{N}(l)} F_{it}$, is exactly known for all $n \in \mathcal{N}$, whereas the forecast for time $t' = t+1, ..., t+k$ is computed as

$$\sum_{i\in\mathcal{N}(l)} \hat{F}_{it'} = \sum_{i\in\mathcal{N}(l)} F_{it} + \mu_l(\mathrm{n}) \cdot (t'-t) + z_\alpha \sqrt{\sigma_l(\mathrm{n})^2 \cdot (t'-t)}, \quad \forall n \in \mathcal{N}.$$

Notice that the forecasts of the fill levels at the time periods $t+1, ..., t+k$ become more reliable when sensor and information systems are used (i.e., $\sum_{i\in\mathcal{N}(l)} F_{it}$ are part of the computation of the forecasts). The penalty cost for the families of cubes is applied if $\sum_{i\in\mathcal{N}(l)} F_{it} > \sum_{i\in\mathcal{N}(l)} U_i$ for time t, or, whenever $\sum_{i\in\mathcal{N}(l)} \hat{F}_{it'} > \sum_{i\in\mathcal{N}(l)} U_i$ for the time periods $t' = t+1, ..., t+k$ in the k-period collection plan, $\forall n \in \mathcal{N}$. In this case, cubes can still overfill even if information systems are available. However, these cubes are likely to be serviced on the first day of overfilling.

As for the total quantity of materials to be collected at time t, it can be calculated exactly, considering that information systems provide the real fill levels. Hence, the total quantity of material of type n to be collected at time t is known (i.e., $\sum_{i\in n} x_{it}$), whereas the forecast for the total quantity of material of type n to be collected on route $\mathcal{R}_{t'}$ at time $t' = t+1, ..., t+k$ is computed as

$$\sum_{i\in n} \hat{x}_{it'} = \sum_{i\in\mathcal{N}(l), l\in\mathcal{R}_{t'}} F_{it} + \sum_{l\in\mathcal{R}_{t'}} (\mu_l(\mathrm{n}) \cdot (t'-t)) + z_\alpha \sqrt{\sum_{l\in\mathcal{R}_{t'}} (\sigma_l(\mathrm{n})^2 \cdot (t'-t))}, \quad \forall n \in \mathcal{N}.$$

Once again, it should be noticed that these forecasts become more reliable with information systems. As is performed in all policies, at time t, if the total quantity of materials to be collected is above the available capacity, $\sum_{i\in n} x_{it} > C_t(\mathrm{n})$, $\forall n \in \mathcal{N}$, then either the containers are swapped, or, the vehicle goes to the treatment facilities. In the planning process, the penalty cost $\rho(\mathrm{n})$ for a container of type n is applied if $\sum_{i\in n} x_{it} > C$ at time t, or, whenever $\sum_{i\in n} \hat{x}_{it'} > C$ at time $t' = t+1, ..., t+k$ in the collection plan. Finally, in this case, the exact penalty costs for cubes and containers at time t are known prior to the route being performed.

7. Computational experiments

We now present the various computational experiments conducted to assess the efficiency of the various policies developed to solve the considered problem. We begin in Section 7.1, by presenting the test instances used in our experiments. In Section 7.2, we describe the implementation and tuning details for the solution procedures. Finally, in Section 7.3, we analyze the computational experiments conducted to compare the single- and multi-period policies in both the absence (Section 7.3.1) and presence (Section 7.3.2) of sensor and information systems; we then assess the impact of using such systems within the proposed policies (Sections 7.3.3).

7.1. Test instances

We start by presenting the base case, which is inspired by an actual application presented in Elbek and Wøhlk (2015). This base case, which is used to generate the instance set for our computational experiments, includes 211 collection locations with 240 cubes for glass and 198 cubes for paper that are geographically positioned both in cities and rural areas. The majority of these locations have one cube for each material type, but in some cases, up to six cubes are located together. Each cube has a capacity of $1.65\,\mathrm{m}^3$. The accretion rates of materials are assumed to be normally distributed with a mean and standard deviation derived from historical data and empirical knowledge for each family of cubes. The service cost w_{ij} for a collection location is 3 for emptying the first cube, 2 for emptying the first cube of the other type of material, and 1 for emptying any additional cube. The costs $k(\mathrm{g})$ and $k(\mathrm{p})$ for the transportation to the treatment facilities for glass and paper are 246 and 98, respectively. Furthermore, the service costs $w(\mathrm{g})$ and $w(\mathrm{p})$, which are set to 15, are added when visits are performed to the treatment facilities. All distances and costs are given in minutes.

Several test instances were generated. The number of considered collection locations $|\mathcal{L}|$ is set to 50, 100, 150, and 211, locations being selected randomly from the 211 locations included in the base case. In addition to the base case, we have generated five instances of each size. For the instances with 50 locations, the capacity of an empty container C is equal to $15\,\mathrm{m}^3$, for the instances with 100 and 150 locations, C is equal to $25\,\mathrm{m}^3$, and for the instances with 211 locations, C is equal to $36\,\mathrm{m}^3$.

We aimed to investigate the performance of the policies under different degrees of uncertainty. Considering the accretion rate distributions, we let the term *normal* mean refer to the mean value defined in the base case. We generated a higher mean (*high*) by multiplying the *normal* mean by an integer x for each family of cubes, where $x = U[1, 5]$. As for the standard deviations, the term *normal* refers to a standard deviation of $5\,\%$ of the mean, as defined in the base case. As for the higher standard deviation (*high*), it is defined as y % of the mean for each family of cubes, where y is an integer value such that $y = U[5, 25]$. Considering the two types of materials, we have four possible settings for both the means and the standard deviations: *normal*(g), *high*(g), *normal*(p), and *high*(p) for glass and paper, respectively. Six filling settings were selected for the accretion rate distributions (i.e., Filling I–VI). These settings are shown in Table 2. Therefore, a total of 96 instances were used to perform the computational experiments. Finally, five realizations were randomly generated for each accretion rate, yielding a total of 480 instances.

7.2. Implementation and tuning details

The proposed policies were coded in C++ in Microsoft Visual Studio 2010 and all experiments were carried out on computers equipped with two Intel(R) Xeon(R) X5675 processors running at 3.07GHz, and 96Go of RAM.

Table 2: Settings for the accretion rate distributions

		σ			
		normal(g) *normal*(p)	*normal*(g) *high*(p)	*high*(g) *normal*(p)	*high*(g) *high*(p)
μ	*normal*(g) *normal*(p)	Filling I			Filling II
	normal(g) *high*(p)		Filling V		
	high(g) *normal*(p)			Filling VI	
	high(g) *high*(p)	Filling III			Filling IV

To apply the proposed policies, both the overfilling penalty costs (for cubes and containers) and the service levels first need to be fixed. The values for these parameters were determined for each proposed policy by performing a set of preliminary tests.

Let us recall that the overfilling penalty costs for cubes are fixed according to their associated subset type. As previously defined, assuming that a family of cubes at collection location l are in the median subset (*normal, normal, normal*), then the overfilling penalty cost is defined as $\rho_l(\mathrm{n}) = A2c_{0l}$, where parameter A is a scaling factor. The following values were considered for A: 1, 10, 200, 500, and 1000. Once the scaling factor A in $\rho_l(\mathrm{n})$ is fixed, the overfilling penalty costs for a family of cubes belonging to the other subset types are defined as: $\rho_l(\mathrm{n})$ multiplied by a parameter, which is either greater than or equal to one, when the subset is more important than the median, or less than or equal to one, otherwise. To fix these parameters, specific values are assigned to the most important subset, i.e., (*sensitive, fast, big*), and the least important subset, i.e., (*hidden, slow, small*). As for the rest of the subsets, their values are obtained by applying a Fibonacci scaling factor in the range defined by (*sensitive, fast, big*) and (*hidden, slow, small*). The specific values from 2 to 10 (obtained with a step size of 0.5), and from 0.1 to 0.5 (obtained with a step size of 0.025), were respectively considered for the (*sensitive, fast, big*) and (*hidden, slow, small*) subsets.

Finding an appropriate value for parameter A is important, given that this value sets the overall scale of the overfilling penalty costs for cubes. The preliminary tests showed that if these costs are too low, then the quantify of overfilled materials can be arbitrarily large. The tests conducted also confirmed that the use of subset specific values for the penalty costs reduces the risk of overfilling at higher priority collection locations. Moreover, in the case where the cubes of an important location overfill, the quantity of excess materials is usually low. All of these observations are in accordance with the expectations of citizens regarding the emptying of cubes.

As for the overfilling penalty costs for containers, let us recall that, for the material of type n, it is defined as $\rho(\mathrm{n}) = B(\max_{l\in\mathcal{L}}\{2c_{0l}\} + k(\mathrm{n}))$, where B is again a scaling factor. The values 1, 10, 200, 500, and 1000 were also considered for parameter B. It was observed in the preliminary tests that a value of 1 is appropriate in the case of single-period policies. However, a larger value is warranted for the multi-period policies. In this case, a lower scale for these costs may cause the multi-period policy to deliberately produce collection plans with overfilled containers. As a result of the preliminary tests, the specific values found for the parameters that define the overfilling penalty costs for cubes and containers to implement the policies proposed are reported in the Annex B.7.

The second set of parameters to tune is the service levels used for the various cube subsets. Let us recall that, for a family of cubes at collection locations l, the service threshold is defined as $s_{\mathcal{N}(l)} = \sum_{i\in\mathcal{N}(l)} U_i - \big(\mu_l(\mathrm{n}) + v_\alpha \sigma_l(\mathrm{n})\big)$. Parameter v_α is the quantile associated to the event of the family of cubes at the location not overfilling with a probability of $1-\alpha$ (i.e., the service level). In the preliminary tests conducted, specific service level values were tested for the different cube subsets used for each proposed policy. The values from 96 % to 99.8 % (obtained with a step size of 0.2 %) and from 93 % to 97 % (obtained with a step size of 0.2 %), were investigated for the service levels of the most important subsets and the least important subsets, respectively. It should be noted that we also explored the possibility of setting a single service level value for all subsets. In this case, the following values were tested: 95 %, 96.5 %, 98 %, and 99.99 %. However, the best results were obtained using subset specific values in each policy. These values are reported in the Annex B.8.

The VNS procedure developed in Elbek and Wøhlk (2015) (Algorithm 3 in the Annex) is applied in the case of the multi-period policy using the parameter values originally identified. Therefore, the length of the k-period is set to 11 days and the stopping criteria of the procedure is defined as either reaching a time limit of 10 seconds or 1000 iterations without any improvements.

Finally, three time horizon lengths, $H = 30$, 182, and 365 days, were considered for each of the 480 instances. Furthermore, given the randomness involved in the algorithm, the results reported are based on the averages obtained over 10 repetitions.

7.3. Computational results

The results of the computational experiments are summed up in Tables 3–6 and analyzed in the next three subsections. For each policy, the numerical results are aggregated according to the number of collection locations and the settings for the accretion rate distributions. Furthermore, the key performance measures used in the analysis, *Cost*, *Cube*, and *Container*, group the results for the two material types considered (glass and paper). The *Cost* measures include the total operational costs (column *Operation*), consisting of the routing costs, the service costs for cubes and the transportation and service costs for the treatment facilities, detailed in columns *Routing*, *Service* and *Facility*, respectively. The *Cube* and *Container* measures report the number of emptied units (column *No. emptied*), their associated fill levels in percentage of total capacity (column *Full(%)*), the number of units that overflowed (column *No. overfilled*) and the amount of overflowed materials in percentage of total capacity (column *Overflow(%)*). The last line of each table reports the average values of the different measures over all instances.

The results are analyzed as follows: we first numerically compare the four policies developed in the absence (Section 7.3.1) and in the presence (Section 7.3.2) of sensor and information systems. We then assess the impact of sensor and information system technology on the two policies (Section 7.3.3).

7.3.1. Performance of the policies without sensor and information systems

The results for policies SN and MN are reported in Tables 3 and 4. When examining the results, one first observes that the average filling level of the containers, when they are transported to the treatment facilities, is lower when the SN policy is applied compared to the MN policy

(87.31 % versus 92.84 %). As a consequence of this, the average number of trips performed to the treatment facilities is higher in the case of SN (125.83 compared to 109.13), as well as the average transportation and service costs incurred for these trips (12461 versus 10520). Overall, the fact that the policy MN evaluates future effects on the overall total cost appears beneficial, as it produces, throughout the time horizon, a more cost efficient schedule of visits to the treatment facilities.

When analyzing the values of the *Cube* measures, the opposite trend is observed. When SN is compared to MN, on average, fewer cubes are emptied (2725.16 versus 4006.51), their associated service cost is lower (6157 versus 8966) and the cubes are significantly more filled (67.49 % versus 47.49 %). Considering the number of overfilled cubes and containers, the values are generally higher when using SN compared to MN. In terms of the amount of overflowed materials, it is, on average, higher for containers in the case of SN (4.48 % versus 0.36 %), whereas the opposite observation is made in the case of cubes. For cubes, the average amount of overflowed materials is higher for MN, albeit the difference is much less pronounced (1.90 % compared to 0.78 %). As for the *Cost* measures, the MN policy produces solutions that considerably reduce the routing costs when compared to the solutions obtained using the SN policy (on average, 17910 versus 26876). By allowing visits to be rescheduled in the considered k-period collection plan, the MN policy is able to improve the visit assignments throughout the time horizon. All in all, when sensor and information systems are not available, lower operational costs are achieved when the multi-period policy is used compared to the single-period policy.

When analyzing the results associated to the different instance sizes, one first notices, as expected, that both the costs and the number of overfilled cubes increase with the number of collection locations in the instances. However, the amount of overfilled materials in the cubes remains stable. In addition, the relative operational cost per location actually decreases as the instance size increases. For example, considering policy MN and the instances with the filling setting VI, on average, the relative operational costs per location are 575.86, 403.04, 388.75 and 300.96 for the instances with 50, 100, 150 and 211 collection locations, respectively.

Finally, when examining the results related to the filling settings, not surprisingly, the number of overfilled cubes increases in the cases of II, IV, V, and VI (where the standard deviations are higher than the base case). In addition, it is expected that the overall volume of materials to be collected will increase in the settings where the mean values for the accretion rates are higher than the base case. We thus observe that in the filling settings III–VI, the number of emptied cubes and containers is significantly larger than in the cases of I–II. This effect is also clearly observed in the associated cost components.

7.3.2. Performance of the policies with sensor and information systems

The results for policies SY and MY are reported in Tables 5 and 6. The cost components and key performance measures when comparing SY to MY exhibit the same tendencies as previously observed in the comparison between SN and MN. The filling degrees for the cubes are 16–31 percentage points higher under SY than under MY, whereas the filling degrees for the containers are up to 14 percentage points higher under MY than under SY as in the comparison of MN and SN. The percentages of overflowed material in the cubes are higher under MY than under SY, but the difference is less than 1.6 percentage point, while in the comparison of MN and SN the difference is up to 2 percentage points. Regarding the containers, the overflowed materials are up to 11.6 percentage points higher under SY than under MY, whereas in the comparison of SN and MY the difference is only up to 10.3 percentage points. To summarize, the multi-period policy generally preforms better than the single-period policy with regard to the key performances at a

Table 3: Results for the SN policy

No.	Filling	Cost				Cube				Container			
locations	type	Operation	Routing	Service	Facility	No. emptied	Full(%)	No. overfilled	Overflow(%)	No. emptied	Full(%)	No. overfilled	Overflow(%)
50	I	11651	7532	831	3287	360.47	83.58	1.31	0.23	33.63	92.76	0.00	0.00
	II	11966	7786	864	3316	373.75	80.70	1.45	0.66	34.11	92.00	0.04	6.10
	III	35854	20788	3225	11841	1398.29	68.93	0.73	0.27	120.05	86.64	0.61	6.24
	IV	38582	22721	3842	12019	1673.03	58.80	1.53	1.47	120.37	86.22	0.37	5.83
	V	25321	17456	2406	5459	1069.88	58.74	1.45	1.03	71.36	91.79	0.09	10.37
	VI	31517	18759	2719	10039	1181.83	55.67	1.79	1.34	86.00	88.93	0.39	3.06
100	I	20095	13750	1816	4528	815.17	83.39	2.52	0.20	46.37	93.00	0.00	0.00
	II	20745	14270	1899	4576	853.04	79.86	3.04	0.72	46.91	92.81	0.07	0.05
	III	53638	30495	6739	16404	3037.48	70.30	2.27	0.31	165.87	83.94	0.35	6.19
	IV	57886	33340	7759	16788	3507.21	61.16	2.45	1.53	168.08	83.60	0.19	3.87
	V	40980	27804	5507	7670	2498.20	57.41	2.77	0.96	100.61	90.08	0.21	6.50
	VI	46026	27683	5458	12885	2398.16	57.00	2.88	1.21	111.01	87.45	0.23	2.34
150	I	27103	17774	2635	6694	1161.21	83.02	3.48	0.25	68.11	90.92	0.01	0.02
	II	27693	18252	2725	6716	1200.71	80.05	4.79	0.81	68.16	91.11	0.01	0.07
	III	73706	38184	10188	25334	4490.75	69.30	2.61	0.27	255.36	80.02	8.12	9.48
	IV	78097	40330	11411	26356	5075.97	61.11	4.33	1.19	259.92	78.94	6.79	8.04
	V	52801	33590	7872	11339	3553.08	56.73	4.37	1.28	148.05	87.04	1.65	8.27
	VI	65663	35556	8883	21224	3930.09	52.39	4.99	1.19	180.51	83.72	5.37	10.93
211	I	32395	22124	3742	6529	1652.87	82.94	4.87	0.20	66.67	92.70	0.00	0.00
	II	33224	22749	3903	6572	1726.20	79.53	7.53	0.75	67.07	92.08	0.07	0.16
	III	80666	43898	13641	23127	5908.00	68.69	4.07	0.32	231.20	80.33	1.13	3.35
	IV	88725	48135	15965	24625	7055.20	61.07	7.93	0.98	249.33	78.62	1.40	5.11
	V	62064	39565	11361	11137	5204.13	55.47	6.47	0.87	145.60	86.75	0.40	6.60
	VI	75451	42472	12380	20599	5279.13	53.89	8.20	0.78	175.47	83.95	1.47	4.89
Avg.		45494	26876	6157	12461	2725.16	67.49	3.66	0.78	125.83	87.31	1.21	4.48

Table 4: Results for the MN policy

No.	Filling	Cost				Cube				Container			
locations	type	Operation	Routing	Service	Facility	No. emptied	Full(%)	No. overfilled	Overflow(%)	No. emptied	Full(%)	No. overfilled	Overflow(%)
50	I	8603	4496	1039	3068	453.83	67.29	0.87	1.04	31.85	85.21	0.02	0.08
	II	8864	4670	1097	3098	479.97	64.57	0.73	1.20	32.15	85.51	0.09	0.25
	III	31239	15525	5435	10278	2392.01	42.23	0.72	0.89	106.69	94.94	0.26	0.23
	IV	34896	18083	6350	10463	2800.98	36.70	1.57	2.58	107.29	94.12	0.60	0.58
	V	20647	11825	3641	5181	1635.74	39.94	0.63	1.45	67.59	91.59	0.18	0.54
	VI	28793	15566	4661	8565	2056.63	33.39	0.70	2.12	74.56	94.02	0.57	0.65
100	I	14353	7645	2431	4277	1115.92	61.27	1.39	1.01	44.27	86.97	0.03	0.09
	II	14642	7760	2551	4331	1171.44	58.61	1.07	1.56	44.73	84.88	0.16	0.25
	III	45730	21068	10789	13873	4906.74	44.20	2.05	1.58	144.65	94.40	0.48	0.17
	IV	50747	24362	12124	14261	5518.93	39.28	3.64	2.84	146.46	93.81	1.14	0.58
	V	32086	17123	7844	7120	3585.07	40.36	2.48	2.56	93.09	95.66	0.48	0.53
	VI	40304	20463	9077	10764	4037.60	34.28	2.07	3.13	94.52	95.56	0.93	0.55
150	I	18772	9135	3476	6161	1566.76	61.63	2.04	1.31	63.97	94.93	0.10	0.08
	II	19093	9283	3616	6194	1627.53	59.46	1.58	1.42	64.06	93.84	0.26	0.23
	III	62914	26713	15865	20336	7091.61	44.13	3.28	2.02	212.02	94.21	0.93	0.18
	IV	72400	32643	18731	21026	8409.34	37.25	3.52	2.96	214.12	92.84	2.02	0.55
	V	41898	20949	10774	10176	4880.43	41.41	4.67	2.33	132.44	95.52	0.84	0.59
	VI	58313	27371	14490	16452	6482.78	32.23	3.76	3.18	143.55	95.29	1.85	0.64
211	I	20663	9851	4744	6068	2134.85	63.74	3.35	1.16	63.08	94.63	0.07	0.08
	II	21281	10190	4989	6102	2245.77	60.50	2.48	1.47	63.45	94.09	0.34	0.24
	III	65112	27907	18539	18666	8116.35	49.96	3.79	1.20	194.09	93.35	0.61	0.27
	IV	73297	32370	21093	19834	9283.78	46.38	7.71	1.79	208.27	92.45	1.45	0.36
	V	49368	24461	14781	10126	6802.93	42.78	6.37	2.25	132.21	95.53	0.85	0.40
	VI	63503	30382	17053	16067	7359.37	38.25	4.23	2.49	140.13	94.76	1.42	0.50
Avg.		37397	17910	8966	10520	4006.51	47.49	2.70	1.90	109.13	92.84	0.65	0.36

lower actual operation cost, when sensor and information systems are available. However, we can observe that even with sensor and information systems available and an algorithm which considers several days in the planning process, overfilled cubes and containers can not be avoided, but the number of units is on average lower than 0.3.

Table 5: Results for the SY policy

No.	Filling	Cost				Cube				Container			
locations	type	Operation	Routing	Service	Facility	No. emptied	Full(%)	No. overfilled	Overflow(%)	No. emptied	Full(%)	No. overfilled	Overflow(%)
50	I	11415	7339	812	3265	351.16	84.96	0.17	0.05	33.44	92.65	0.00	0.00
	II	11472	7407	817	3248	353.36	84.50	0.45	0.19	33.36	92.85	0.00	0.00
	III	35172	20351	3111	11709	1350.97	71.33	0.52	0.22	118.99	87.33	0.67	10.81
	IV	35495	20550	3202	11743	1393.88	68.83	1.67	0.98	117.97	87.62	0.57	9.25
	V	24269	16645	2215	5408	984.16	63.63	1.13	0.55	70.64	92.24	0.11	8.92
	VI	29945	17646	2494	9805	1081.03	60.58	1.09	0.81	84.16	89.49	0.44	7.93
100	I	19922	13627	1783	4511	800.04	84.84	0.35	0.11	46.21	93.14	0.00	0.00
	II	20101	13779	1800	4523	807.69	83.97	0.75	0.33	46.35	93.33	0.00	0.00
	III	52723	29975	6511	16237	2933.55	72.68	0.99	0.35	164.51	84.57	0.39	5.10
	IV	54786	31507	6872	16406	3088.23	69.18	2.91	1.58	164.53	84.83	0.37	7.73
	V	39126	26504	5003	7618	2266.45	63.03	2.59	0.98	99.73	90.72	0.24	5.52
	VI	43849	26281	4970	12598	2184.92	62.42	2.40	1.05	108.77	88.09	0.17	5.79
150	I	26965	17722	2590	6652	1141.20	84.33	0.47	0.08	67.87	91.51	0.00	0.00
	II	26951	17716	2601	6635	1145.76	83.46	1.23	0.18	67.52	91.81	0.00	0.00
	III	72538	37605	9828	25104	4329.57	71.78	1.32	0.31	253.28	80.74	8.87	8.78
	IV	74058	38127	10084	25847	4459.84	69.08	4.80	0.97	255.49	80.09	6.67	9.01
	V	50457	32173	7113	11171	3200.93	62.59	3.23	1.06	145.47	88.07	1.59	8.90
	VI	61892	33396	7798	20698	3424.69	60.22	4.12	1.13	176.48	84.35	5.23	8.87
211	I	31952	21801	3674	6477	1622.40	84.34	0.20	0.23	66.27	92.45	0.00	0.00
	II	32300	22081	3709	6510	1638.13	83.50	2.40	0.24	66.67	91.36	0.00	0.00
	III	79234	43189	13092	22953	5666.20	71.52	2.27	0.16	229.87	80.48	0.60	9.86
	IV	84549	45885	14326	24338	6312.93	68.10	6.20	1.12	246.53	80.12	0.87	11.65
	V	59104	37931	10205	10967	4646.53	61.92	4.60	0.83	143.47	88.28	0.53	4.31
	VI	71413	40299	10872	20242	4642.47	61.16	4.60	0.45	172.80	84.15	1.47	8.58
Avg.		43737	25814	5645	12278	2492.75	72.17	2.10	0.58	124.18	87.93	1.20	5.46

We conclude by emphasizing that the MY policy offers within our experiments the best performance compared to the other policies. Thus,MY achieves the lowest (47.63 %) average filling degree for the cubes, and the highest (93.32 %) utilization of container capacity. Moreover, the lowest number of overfilled cubes and containers and the overall lowest operation cost are also achieved under the use of MY. Figure 2 illustrates this performance for the instances with 211 locations, by displaying the average values of the operation cost, number of overfilled cubes, and number of overfilled containers achieved by the four policies for these instances; results are grouped according to the accretion rate distributions.

7.3.3. Impact of technology on management policies

We turn to the analysis of the impact of sensor and information systems on the performance of the management policies proposed. We expect more information to have a positive impact on performances and the results support this hypothesis. But the benefits are not uniformly high for all cases.

The results for the single-period policy, Tables 3 and 5, thus show that a decrease in costs, overfilled cubes, and the number of emptied cubes and containers can be obtained by introducing sensor and information technology. Figure 3 illustrates these results by plotting the the average cost and the average numbers of overfilled cubes and containers for instances grouped by the number of locations.

The number of overfilled containers and the quantity of overflowed material in cubes and containers are approximately the same on average. With respect to costs, it appears possible to save

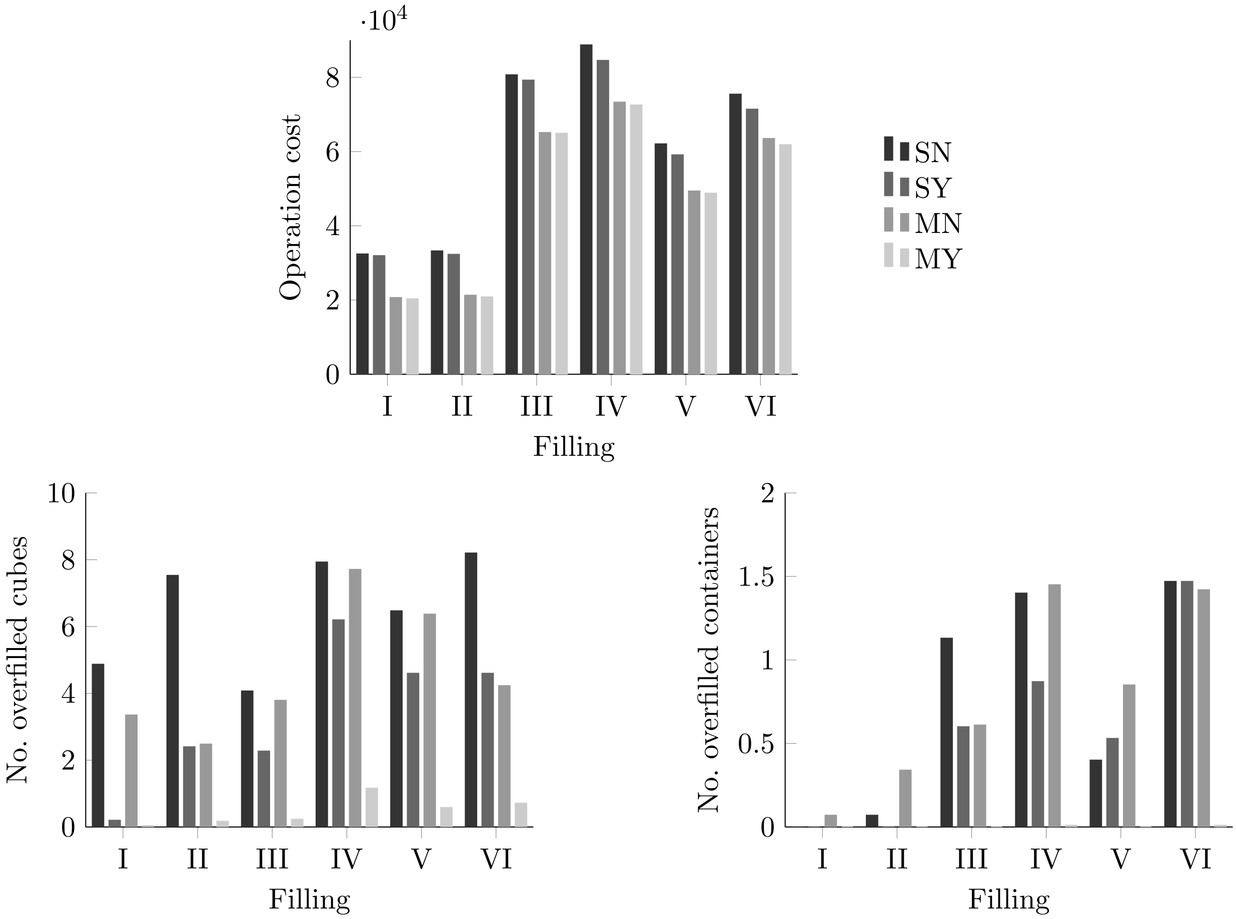

Figure 2: Key performance indicators for MY policy on 211-location instances

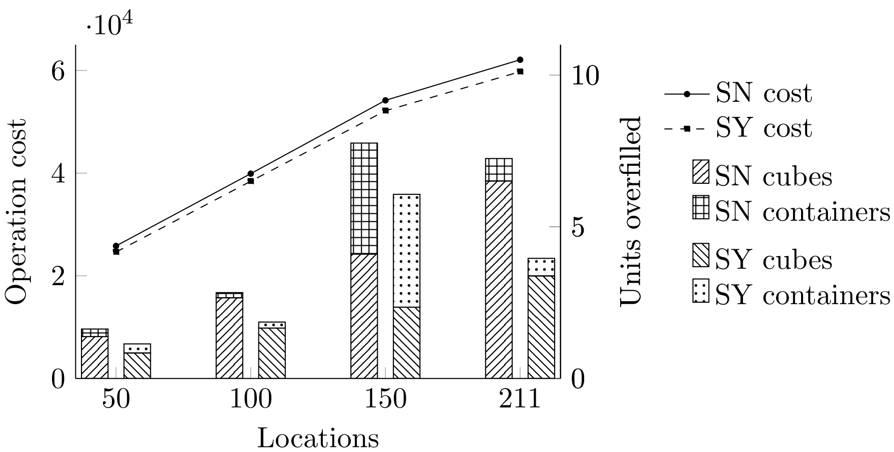

Figure 3: Comparison of SN and SY performance indicators

Table 6: Results for the MY policy

No.	Filling	Cost				Cube				Container			
locations	type	Operation	Routing	Service	Facility	No. emptied	Full(%)	No. overfilled	Overflow(%)	No. emptied	Full(%)	No. overfilled	Overflow(%)
50	I	8243	4149	1029	3064	449.44	68.32	0.01	0.08	31.90	88.54	0.001	0.001
	II	8445	4286	1084	3074	474.84	65.40	0.05	0.59	32.04	81.73	0.001	0.003
	III	31002	15280	5469	10253	2404.76	41.85	0.06	0.24	106.51	95.05	0.004	0.001
	IV	34643	17942	6410	10292	2815.25	35.91	0.27	1.55	105.86	95.33	0.008	0.001
	V	19947	11209	3573	5166	1599.86	40.69	0.13	0.89	67.25	91.88	0.005	0.002
	VI	27911	14914	4641	8356	2044.35	33.24	0.21	1.83	72.93	94.58	0.017	0.001
100	I	13936	7235	2411	4289	1106.36	61.83	0.05	0.15	44.49	89.06	0.000	0.000
	II	14288	7440	2539	4309	1165.57	59.10	0.07	0.30	44.67	85.25	0.001	0.001
	III	45302	20676	10812	13814	4914.66	43.97	0.14	0.48	144.24	94.75	0.004	0.001
	IV	50427	24198	12221	14008	5560.17	38.84	0.45	2.17	144.17	95.12	0.004	0.001
	V	31255	16404	7750	7101	3540.36	40.92	0.40	1.11	92.64	95.87	0.000	0.000
	VI	40098	20323	9220	10555	4099.07	33.80	0.36	2.06	92.95	96.59	0.005	0.001
150	I	18452	8773	3487	6191	1570.58	61.50	0.03	0.27	64.47	94.02	0.003	0.001
	II	18664	8863	3618	6184	1628.04	59.45	0.08	0.33	64.20	93.81	0.001	0.001
	III	62593	26446	15925	20223	7121.85	43.93	0.17	0.40	211.18	94.51	0.011	0.001
	IV	69864	30977	18278	20610	8210.21	38.01	0.87	1.89	210.81	94.08	0.009	0.001
	V	41722	20754	10865	10103	4924.53	41.08	0.37	1.28	131.18	96.12	0.003	0.001
	VI	56476	26184	14270	16022	6379.83	32.71	0.65	2.01	140.24	96.62	0.016	0.001
211	I	20310	9462	4744	6103	2132.03	63.62	0.04	0.12	63.67	93.67	0.000	0.000
	II	20819	9744	4980	6094	2241.13	60.76	0.17	0.30	63.49	94.53	0.000	0.000
	III	64915	27718	18578	18619	8136.61	49.66	0.23	0.54	193.89	93.42	0.000	0.000
	IV	72528	31887	21087	19554	9298.69	46.49	1.16	1.35	205.65	93.31	0.007	0.001
	V	48766	23962	14746	10058	6760.3	43.01	0.58	1.12	131.09	96.11	0.000	0.000
	VI	61838	29300	16760	15779	7226.97	39.11	0.71	2.05	137.92	95.66	0.013	0.001
Avg.		36768	17422	8937	10409	3991.89	47.63	0.30	0.96	108.23	93.32	0.005	0.001

up to 8 % on the operation cost using sensor and information systems. Notably, the largest savings are generally obtained when the filling rates display high standard deviations.

We conclude that for the single-period policy, more information is beneficial but the improvement in performance measures is relatively small, except when citizen deposit rates vary significantly.

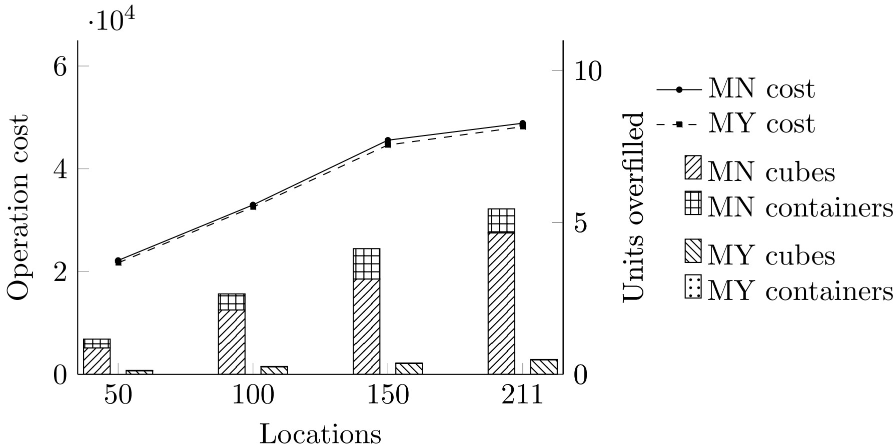

Figure 4: Comparison of MN and MY performance indicators

The trend is generally similar for the multi-period policy, Tables 4 and 6 and Figure 4, and so are the conclusions.

Introducing sensor and information technology leads to a decrease in the operation (4.7 %) and routing costs, as well as a generally lower service cost for cubes and lower transportation and service costs for the treatment facilities. We again observe that the cost benefit is larger in filling situations with high variability. Interestingly, though, the improvement is lower in the case of the multi-period policy than the one observed for the single-period. On the other hand, the number of overfilled cubes and containers, and the associated amount of overflowed materials, are significantly lower when the technology is present.

The general conclusion therefore is that a better overall performance is achieved when sensor and information systems are available. A more significant gain can be obtained, however, by using a more refined management policy. Thus, the cost saving (up to 8 %) achieved through additional information is relatively small compared to the savings (up to 36 %) that can be obtained by applying the multi-period policy instead of the single-period policy.

8. Conclusion

We addressed a general problem of collecting multiple-type recyclable materials, from locations distributed over a rather large territory with different population densities and land utilization, over a multi-period planning horizon under uncertainty conditions. Multi-compartment vehicles are used for the simultaneous collection of several types of recyclable materials, which must, however, be handled separately. The problem is further complicated by stochastic daily accretion rates of materials at collection locations and the non-recurrent, non-periodic nature of the resulting demand and operations. Complex capacity restrictions, due to the nature of the materials handled, the possibility that deposited materials may overflow the capacity of some collection points under certain location-specific conditions, and the possible presence of sensor and information technology providing information on the accumulation levels at the collection points add to the difficulty of the problem. The aim is to minimize the operation cost over the planning horizon, while avoiding violations of capacity constraints for vehicles and reducing as much as possible the capacity overflow at collection points.

We proposed a mathematical formulation for this complicated problem, which takes the form of a two-stage stochastic programming model with simple recourse, where collection decisions make up the first stage. We have also proposed two deterministic daily management policies that can be used to address the problem. The single-period policy creates a 1-period collection plan, while the multi-period policy looks ahead and yields a k-period collection plan. Each of these policies may be used in the proposed rolling-horizon procedure that aims to mange operations over a given planning horizon.

We have investigated the efficiency and behavior of these methods by applying them to a large set of instances, which we proposed and which represent a broad array of situations. The analysis emphasized the good performance of the proposed methodology. In particular, it showed the superiority in terms of various performance measures of the multi-period policy over the single-period one. The investigation also showed the positive impact of sensor and information technology on cost reductions and efficiency increase.

Directions for future research include further consideration of the handling uncertainty, both in algorithmic terms and in the planning process. Enlarging the scope of the methodology by, e.g., relaxing the assumption that all cubes are emptied at a visited location, is also part of our plans.

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AppendixA. VNS to improve the collection plan

The VNS meta-heuristic, shown in Algorithm 3, contains four neighborhoods (k_{max}) in the VNS part and three neighborhoods (h_{max}) in the VND part (local search). We use the current solution x for **the k-period** as input to the VNS and the improvement proceeds until a time limit (t_{max}) or a stop criterion (b_{max}) is reached. When searching for a better solution, shakes are performed on a solution x', which is a replicate of x. The total cost $f(x)$ is defined by the objective function including future cost.

Algorithm 3: Improvement with VNS

```
Data: x, k_max, h_max, t_max, b_max
time = 0, it = 0;
while time < t_max or it < b_max do
    k = 1;
    while k < k_max do
        x' = x;
        if k = 1 then x' ← Block move size 2 end;
        if k = 2 then x' ← Block move size 3 end;
        if k = 3 then x' ← Block move size 4 end;
        if k = 4 then x' ← Block move size Rand(2,5) end;
        h = 1;
        while h < h_max do
            x'' = x';
            if h = 1 then x'' ← Swap locations on different routes, ∀t' in the k-period end;
            if h = 2 then x'' ← Move a location to another route, ∀t' in the k-period end;
            if h = 3 then x'' ← 2-opt exchange on a route, ∀t' in the k-period end;
            if f(x'') < f(x') then x' = x'', h = 1 else h = h + 1 end;
        end
        if f(x') < f(x) then x = x', k = 1 else k = k + 1 end;
    end
    Update time and it;
end
```

AppendixB. Parameters for the subsets of the families of cubes (geography, filling, capacity)

Table B.7: Parameters in the penalty costs

	Values			
	SN	SY	MN	MY
(*sensitive, fast, big*)	8.500	6.500	8.500	6.500
(*sensitive, fast, normal*)	7.675	5.895	7.675	5.895
(*sensitive, fast, small*)	5.127	4.027	5.127	4.027
(*sensitive, normal, big*)	3.550	2.870	3.550	2.870
(*sensitive, normal, normal*)	2.575	2.155	2.575	2.155
(*sensitive, normal, small*)	1.975	1.715	1.975	1.715
(*sensitive, slow, big*)	1.600	1.440	1.600	1.440
(*sensitive, slow, normal*)	1.375	1.275	1.375	1.275
(*sensitive, slow, small*)	1.225	1.165	1.225	1.165
(*normal, fast, big*)	1.150	1.110	1.150	1.110
(*normal, fast, normal*)	1.075	1.055	1.075	1.055
(*normal, fast, small*)	1.075	1.055	1.075	1.055
(*normal, normal, big*)	1.000	1.000	1.000	1.000
(*normal, normal, normal*)	1.000	1.000	1.000	1.000
(*normal, normal, small*)	1.000	1.000	1.000	1.000
(*normal, slow, big*)	0.840	0.840	0.840	0.840
(*normal, slow, normal*)	0.710	0.710	0.710	0.710
(*normal, slow, small*)	0.630	0.630	0.630	0.630
(*hidden, fast, big*)	0.580	0.580	0.580	0.580
(*hidden, fast, normal*)	0.550	0.550	0.550	0.550
(*hidden, fast, small*)	0.530	0.530	0.530	0.530
(*hidden, normal, big*)	0.520	0.520	0.520	0.520
(*hidden, normal, normal*)	0.510	0.510	0.510	0.510
(*hidden, normal, small*)	0.505	0.505	0.505	0.505
(*hidden, slow, big*)	0.505	0.505	0.505	0.505
(*hidden, slow, normal*)	0.500	0.500	0.500	0.500
(*hidden, slow, small*)	0.500	0.500	0.500	0.500
A for cubes	100	100	10	1000
B for containers	1	1	500	100

Table B.8: Service levels

	Values			
	SN	SY	MN	MY
(*sensitive, fast, big*)	99.99	97.00	96.60	99.40
(*sensitive, fast, normal*)	99.94	96.94	96.54	98.95
(*sensitive, fast, small*)	99.39	96.39	95.99	98.01
(*sensitive, normal, big*)	99.05	96.05	95.65	97.44
(*sensitive, normal, normal*)	98.84	95.84	95.44	97.08
(*sensitive, normal, small*)	98.71	95.71	95.31	96.86
(*sensitive, slow, big*)	98.63	95.63	95.23	96.72
(*sensitive, slow, normal*)	98.58	95.58	95.18	96.64
(*sensitive, slow, small*)	98.55	95.55	95.15	96.59
(*normal, fast, big*)	98.53	95.53	95.13	96.55
(*normal, fast, normal*)	98.52	95.52	95.12	96.53
(*normal, fast, small*)	98.51	95.51	95.11	96.52
(*normal, normal, big*)	98.51	95.51	95.11	96.52
(*normal, normal, normal*)	98.50	95.50	95.10	96.50
(*normal, normal, small*)	98.44	95.44	95.04	96.48
(*normal, slow, big*)	97.89	94.89	94.49	95.38
(*normal, slow, normal*)	97.55	94.55	94.15	94.70
(*normal, slow, small*)	97.34	94.34	93.94	94.28
(*hidden, fast, big*)	97.21	94.21	93.81	94.02
(*hidden, fast, normal*)	97.13	94.13	93.73	93.86
(*hidden, fast, small*)	97.08	94.08	93.68	93.76
(*hidden, normal, big*)	97.05	94.05	93.65	93.70
(*hidden, normal, normal*)	97.03	94.03	93.63	93.66
(*hidden, normal, small*)	97.02	94.02	93.62	93.64
(*hidden, slow, big*)	97.01	94.01	93.61	93.62
(*hidden, slow, normal*)	97.01	94.01	93.61	93.62
(*hidden, slow, small*)	97.00	94.00	93.60	93.60

AppendixC. Summary of notation

Table C.9: Overview of Notation

Index Sets and Indices	
$\mathcal{L}$	The set of all collection locations
l	A site in $\mathcal{L}$
$\mathcal{M}$	The set of all cubes
$\mathcal{V}$	The set of all cubes and the depot ($M \cup \{0\}$)
$\mathcal{N}$	The set of recycling materials

The table continues

Table C.9: Overview of Notation (continued)

n	A type of recycling material in $\mathcal{N}$, which can be breakable or non-breakable
$\mathcal{G}$, $\mathcal{P}$	The sets of glass and paper cubes, respectively
$\mathcal{N}(l)$	The family of cubes of type $n \in \mathcal{N}$ at location $l \in \mathcal{L}$
$\mathcal{G}(l)$, $\mathcal{P}(l)$	The family of glass and paper cubes at location $l \in \mathcal{L}$
$\mathcal{T}$, $\mathcal{T}'$	The set of the discrete time instants excluding / including time zero
t	A time instant in $\mathcal{T}$ or $\mathcal{T}'$
$\mathcal{S}$	The set of scenarios
s	A scenario in $\mathcal{S}$
$\mathcal{R}_t$	The route at time $t \in \mathcal{T}$
Scalars and Parameters	
H	The finite time horizon
C	The capacity of an empty container
U_i	The capacity of cube $i \in \mathcal{M}$
r_{it}	The quantity of material deposited in cube $i \in \mathcal{M}$ at time $t \in \mathcal{T}'$
c_{ij}	The transportation cost between $i \in \mathcal{V}$ and $j \in \mathcal{V}$
$k(\mathrm{n})$	The transportation cost of a round trip to the treatment facility for material of type $n \in \mathcal{N}$
$k(\mathrm{g})$, $k(\mathrm{p})$	The transportation cost of a round trip to the treatment facility for glass and paper, respectively
γ_l	The average transportation cost from location $l \in \mathcal{L}$ to its ten nearest neighbors multiplied by two
w_{ij}	The service cost for emptying cube $i \in \mathcal{M}$ after cube $j \in \mathcal{M}$
$w(\mathrm{n})$	The service cost at the treatment facility for material $n \in \mathcal{N}$
$w(\mathrm{g})$, $w(\mathrm{p})$	The service cost at the treatment facility for glass and paper
ϕ_l	The service cost for location $l \in \mathcal{L}$ where all cubes are emptied
ρ_i	The penalty cost for overfilling cube $i \in \mathcal{M}$
$\rho_l(\mathrm{n})$	The penalty cost for overfilling the family of cubes of material of type $n \in \mathcal{N}$ at location $l \in \mathcal{L}$
$\rho(\mathrm{n})$	The penalty cost for overfilling the container of material of type $n \in \mathcal{N}$
$\rho(\mathrm{g})$, $\rho(\mathrm{p})$	The penalty cost for overfilling the container for glass / paper
$s_{\mathcal{N}(l)}$	A threshold determining when a family of cubes of type $n \in \mathcal{N}$ at location $l \in \mathcal{L}$ is emptied
$\mu_l(\mathrm{n})$	The mean of the daily accretion rate for the family of cubes of type $n \in \mathcal{N}$ at location $l \in \mathcal{L}$
$\mu_l(\mathrm{g})$, $\mu_l(\mathrm{p})$	The mean of the daily accretion rate for the family of glass (paper) cubes at location $l \in \mathcal{L}$
$\sigma_l(\mathrm{n})$	The standard deviation of the daily accretion rate for the family of cubes of type $n \in \mathcal{N}$ at location $l \in \mathcal{L}$
$\sigma_l(\mathrm{g})$, $\sigma_l(\mathrm{p})$	The standard deviation of the daily accretion rate for the family of glass (paper) cubes at location $l \in \mathcal{L}$
p^s	The probability of scenario $s \in \mathcal{S}$

The table continues

Table C.9: Overview of Notation (continued)

Variables	
θ_{it}	Binary. Visit to node $i \in \mathcal{V}$ at time $t \in \mathcal{T}$
$\eta_t(\mathrm{g})$	Binary. Swap of glass containers at time $t \in \mathcal{T}$
$\eta_t(\mathrm{p})$	Binary. Visit to treatment facility for paper at time $t \in \mathcal{T}$
y_{ij}^t	Route traversal $\forall i, j \in \mathcal{V}, j < i, t \in \mathcal{T}$
x_{it}	The quantity of material collected from cube $i \in \mathcal{M}$ at time $t \in \mathcal{T}'$
F_{it}	The quantity of material in cube $i \in \mathcal{M}$ at time $t \in \mathcal{T}'$
z_{it}	The quantity of excess material in cube $i \in \mathcal{M}$ at time $t \in \mathcal{T}$
$C_t(\mathrm{n})$	The capacity available in the container for material of type $n \in \mathcal{N}$ at time $t \in \mathcal{T}'$
$C_t(\mathrm{g})$, $C_t(\mathrm{p})$	The capacity available in the container for glass (paper) at time $t \in \mathcal{T}'$
$C_t(\dot{\mathrm{p}})$	The total available capacity in the paper containers at time $t \in \mathcal{T}'$
$\delta_t(\mathrm{g})$, $\delta_t(\mathrm{p})$	The quantity of excess glass (paper) in the container at time $t \in \mathcal{T}$
$\zeta(\mathrm{n})$	The estimated quantity of material of type $n \in \mathcal{N}$ which has to be collected in the planning period after the latest planned visit to the treatment facility
$\zeta(\mathrm{g})$, $\zeta(\mathrm{p})$	The estimated quantity of glass (paper) which has to be collected in the planning period after the latest planned visit to the treatment facility
$Q(\mathrm{n})$	Future cost of visiting the treatment facility for material of type $n \in \mathcal{N}$ as planned in the planning period
$Q(\mathrm{g})$, $Q(\mathrm{p})$	Future cost of visiting the treatment facility for glass (paper) as planned in the planning period
Q_{lt}	Future cost of visiting location $l \in \mathcal{L}$ at time $t \in \mathcal{T}$

AppendixD. Complete tree of the subsets of the families of cubes

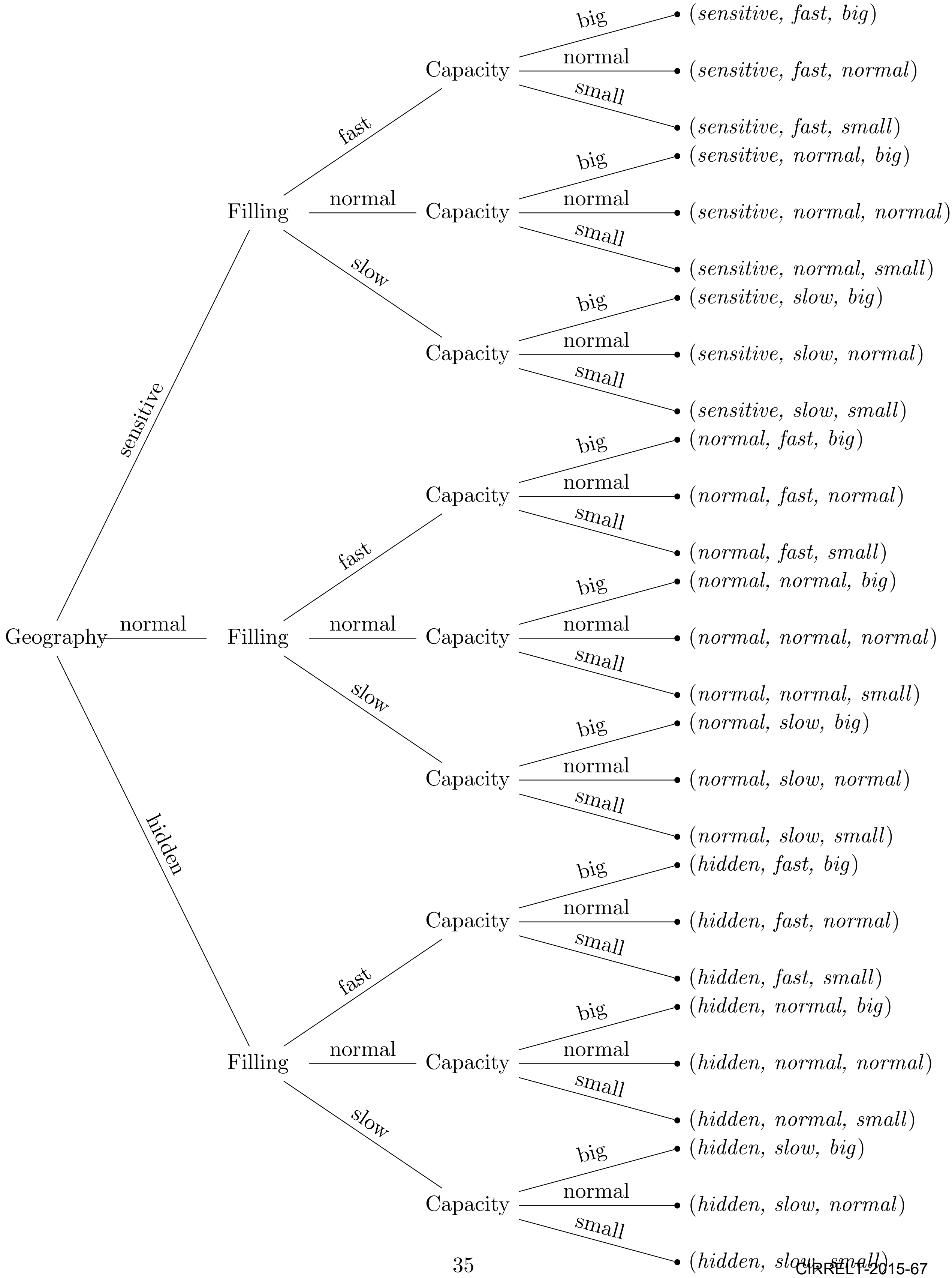

Figure D.5: Complete tree of the subsets of families of cubes