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Abstract. Fairness has recently become a key concern for crisis managers. In the aftermath of a disaster, when needs overcome response's capacity, decision makers are expected to distribute the available relief efficiently, but also in such a way that nobody might perceive any injustice in the access to relief. This paper presents three multi-period models to support relief distribution decisions. The models consider fairness but also tackle the demand's and offer's changes over time. In addition, demand can be backordered as it is the case in realistic situations. The paper discusses the notion of fairness and proposes several proxies to measure it. Numerical tests are run on a set of academic instances to analyze the behaviour of the considered models and assess their performance.

Keywords: Emergency logistics, network design problem, relief distribution, fairness.

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1. Introduction

In the past decade, the number of scientific contributions on relief distribution has grown significantly and nowadays *humanitarian logistics* has become an independent and important body of research. In particular, *relief distribution logistic*s, have many characteristics that differentiates them from business logistics (Holguín-Veras et al. 2012; Kovács & Spens 2007). Among them, three major features inspire and motivate this work. Firstly, demand is uncertain and it may evolve very quickly as the population might tend to mobilize after a disaster. Thus multi-period models are needed to tackle the demand's dynamics. Secondly, relief distribution logistics focuses on demand satisfaction rather than profit maximization or costs minimization, because if the relief demand is not satisfied in terms of both quantities and time, the safety and well-being of the affected people are jeopardized. Thirdly, relief must be distributed in a "fair" manner among the people in need (Anaya-Arenas et al. 2014; Holguín-Veras et al. 2012; Holguín-Veras et al. 2013). Our concern on justice in distribution was also promoted by Fujitsu Consulting (Canada) Inc., one of our previous partners in emergency logistics developments. Past results on design and planning of relief distribution (Rekik et al. 2013) showed that when the response's capacity is smaller than demand, and if fairness is not seek explicitly in the optimization models, some PoDs can be completely neglected while others are fully covered. This behavior was rejected by our partner, understanding then the importance of the principals of justice and impartiality for humanitarian decision makers.

This paper proposes two main contributions. Firstly, we propose a multi-period formulation for the design of a relief distribution network where the opening decisions of humanitarian aid distribution centers (HADC), and the allocation of demand to the HACDs, are reconsidered at every period. In addition, because of the limited amounts of relief available, we allow the demand to be backordered, although only for a limited time or number of periods. A limited backorder of demand allows us to provide a more flexible and realistic logistic plan. Secondly, since this network must be planned in such a way that it maximizes fairness in the distribution, three different objective functions will be proposed and their performance assessed using a set of metrics.

The rest of this article is organized as follows. Section 2 reports the most relevant works in the literature. Section 3 presents a discussion on the notion of fairness, and what we expect from a fair relief distribution plan. Section 4 states the addressed problem. Section 5 presents three different mathematical formulations for fair distribution network design and operations. Section 6 presents numerical experiments, while Section 7 concludes this work and suggests research perspectives.

2. Fair relief distribution in humanitarian logistics

Relief distribution logistics (also called post disaster humanitarian logistics) have received increasing attention in recent years. For instance, in their review, Anaya-Arenas et al. (2014) reported 83 articles, from which 76 were published after 2004. However, the most recent works claim that despite of the important contributions made up to date, additional efforts must be made in order to truly understand the complexity of relief distribution and capturing the challenges of disaster response in optimization models.

One of these challenges is to capture the variety of managers' objectives. According to Anaya-Arenas et al. (2014)'s review of the existing literature, 50% of the relief distribution literature focuses on cost minimization. However, the acknowledgment of saving lives as the ultimate purpose of relief distribution motivates the proposition of different objectives as satisfaction of demand (20% of the literature) or rapidity of the response (20%). To the best of our knowledge, only 10% of the literature on relief distribution networks has considered the fairness concept, and it is almost exclusively done from a routing problems' perspective. For instance, Tzeng et al. (2007) proposed a multi-objective optimization problem for a transportation problem. Fairness of distribution is the third objective, after cost and rapidity, and it is sought by maximizing the minimum satisfaction level among clients. Suzuki (2012) used a similar approach. Lin et al. (2011) also proposed a transportation problem for relief distribution with three different objectives, and it is pursued by the minimization of the maximum gap of the unsatisfied demand. Vitoriano et al. (2010; 2009) proposed a multi-criteria optimization model as well, where fairness is sought by minimizing the deviation of normalized unsatisfied demands. The model considered both costs and routes' reliability. Finally, Huang et al. (2012) presented three different ways to measure what they call equity. The first approach computed a deviation measure like the one used in Lin et al. (2011) and Tzeng et al. (2007). The second one measured the standard deviation of the demand satisfaction using a non-linear formulation. The third approach used a piecewise-linear function to penalize inequity. They tested the three approaches on a set of routing instances and compared their efficiency. Only two contributions on the relief distribution network design literature included the fairness objective. Lin et al. (2012) included a penalty cost for unfairness in service in its cost minimization objective function to design temporal depots in the affected-area, and Yushimito et al. (2012) used an economic function including social cost. As we can see, the use of fairness objectives is still spare, and the most common approach for it is a deviation minimization. However, we will show how other cost functions can indeed be more efficient, as it was presented by Huang et al. (2012) and Holguín-Veras et al. (2013).

Holguín-Veras et al. (2013) is one of the few works to underline the need for objective functions to represent the real challenge of post-disaster humanitarian logistic models. They discussed how social costs must be included in the objective function, in addition to the logistics cost. To do it, a monotonic, non-linear and convex cost function, in respect to the deprivation time, is proposed to estimate the human suffering caused by the supplies' deficit of goods or services to a community in the aftermath of a disaster. Their contribution was later extended in Pérez-Rodríguez and Holguín-Veras (2015) were this proposition was applied for a routing and inventory allocation problem. Through their analysis, they underlined the opportunity cost linked to the satisfaction of clients' demand and the need for multi-period models that account for the temporal effect of demand's dissatisfaction. These two aspects are highly important for the development of a fair distribution chain and are therefore captured in our proposition.

Although several objectives are pursued when designing and operating a relief distribution network, the fairness objective is the major focus of our work. This is due to its significance and rareness, which also makes this principle the hardest one to define and measure. The next section discusses fairness and how to apply and measure it in a relief distribution network design.

3. What should be expected and how to measure fairness in relief distribution?

Relief distribution decisions need to be anchored in the principle of fairness and justice. The principle of justice is based on how *each person has an inviolability founded on justice that even the welfare of society as a whole cannot override* (Rawls 2009). In the context of goods or relief distribution, several aspects need to be observed. First, in terms of demand satisfaction, equity should be pursued. The term equity implies that everyone's demand will be satisfied to the same proportion, according to their needs. Since each point of demand (PoD) may require a different amount of help, equity should be measured as a percentage of satisfied demand or, alternatively, as a percentage of the demand's shortage. Furthermore, this idea of equity in demand satisfaction (or dissatisfaction) needs to be refined in a dynamic, multiperiod context. What we mean by this is that, even if all the PoDs have the same percentage of demand satisfaction at the end of the planning horizon, the way in which they receive relief during the planning horizon is of paramount importance. Clearly, a fair relief distribution should ensure that every PoD maintains a supply level as close as possible, not only to its total needs, but to its needs at each period. It follows that eventual shortages should be "distributed" in a fair manner among the PoD and, whenever there is a shortage at a given PoD, it should be "compensated" as soon as possible. Finally, response time should be, in the best possible way, the same for every PoD, and this interest is beyond a distance minimization objective. Of course, geography and population distribution make this goal very difficult to attain, but a fairness's perception can be reinforced by ensuring that every PoD can be supplied in less than a given time by at least one open HADC. The next subsection discusses how to assess the fairness of a given distribution plan.

3.1. Fairness metrics

Beamon and Balcik (2008) developed a complete framework for performance measurement inspired by commercial supply chains. They suggested that a good relief distribution network needs to achieve a high performance in efficiency, effectiveness and flexibility. They recognised the need of fairness, but they did not specify any indicator or proxy related to it.

We believe that in a multiperiod context, fairness needs to be achieved within the same period, and across periods. With *fairness within periods* we aim at minimizing the differences on the percentage of demand satisfaction between PoDs in a given period. Consequently, if there is a shortness at a given period t, it is preferred to deliver to each customer an equal fraction of their demand rather than fully satisfy some PoDs while leaving others suffering important shortages. On the other hand, *fairness across periods* refers to how the distribution plan balances eventual shortages by "rationing" the available supplies among the PoDs during the planning horizon. In other words, it might be preferable to deliver PoDs with a portion of their demand, rather than to fully satisfy demand in a period and fully unsatisfying it in another. In the following, we name the fairness within periods as "equity" and fairness across periods as "stability". Aiming at quantifying the fairness of a given distribution plan, we define four measures on the differences in shortage among the deserved PoDs. These measures are based on the range and the dispersion and specifically concern equity and stability.

Equity and stability as ranges in proportion of demand shortage

Let u_{zt} be the shortage (in percentage) of PoD z at period t. We define the two following range-based measures:

$$\overline{R_1} = \frac{\sum_{t \in T}(\max_z\{u_{zt}\} - \min_z\{u_{zt}\})}{|T|} \qquad \text{and} \qquad \overline{R_2} = \frac{\sum_{z \in Z}(\max_t\{u_{zt}\} - \min_t\{u_{zt}\})}{|Z|}$$

where T refers to the set of periods in the planning horizon and Z to the set of PoDs. Range $\overline{R_1}$ computes the average, over all periods, of the demand shortage ranges of PoDs for each period. Alternatively, range $\overline{R_2}$ computes the average, over all PoDs, of the range of each PoD shortage over all periods. Therefore, whereas a small value in $\overline{R_1}$ shows that all the PoDs are similarly satisfied at each period, a small value of $\overline{R_2}$ testifies that, on average, PoDs have received a relatively stable satisfaction of demand.

Equity and stability in terms of global dispersion

Variance and standard deviation are measures used to quantify the dispersion of a set of data around its average value. Let us define $u_{..}$ as the global average of the demand shortage over all PoDs and all periods (in percentage), and ${\sigma^2}_{global}$ be the shortage's global variance over all PoDs and periods computed as:

$${\sigma^2}_{global} = \frac{\sum_{t\, \in T} \sum_{z\, \in Z} (u_{zt} - u_{..})^2}{|T| \times |Z| - 1}$$

The following paragraphs show how a classic analysis of variance allows us to identify the components of equity (within a period) and stability (across periods) in the relief distribution decisions. The numerator of ${\sigma^2}_{global}$ is a Total Sum of Squares (TSS) of the deviations of the shortages' values from their average value. Taking periods as a main factor, TSS can be decomposed in two independent terms: Sum of Squares Within periods ($SSWT$) and Between periods ($SSBT$). This decomposition let us quantify how much of the global dispersion is due to the variability inside (within) the periods and how much is due to the variability of distribution decisions between the periods. More precisely,

$$\sum_{t\, \in T} \sum_{z\, \in Z} (u_{zt} - u_{..})^2 = TSS = SSWT + SSBT$$

with:

$$SSWT = \sum_{t\, \in T} \sum_{z\, \in Z} (u_{zt} - u_{\cdot t})^2 \qquad \text{and} \qquad SSBT = |Z| \sum_{t\, \in T} (u_{\cdot t} - u_{..})^2 ,$$

where $u_{\cdot t}$ is the average over all PoD's of the shortage percentage for period t (i.e. $u_{\cdot t} = \sum_{z\, \in Z} u_{zt} \,/\, |Z|$). $SSWT$ measures in the planning horizon if, period by period, the PoDs are all similarly satisfied; therefore, it is the basic component of *equity* among PoDs. On his side, $SSBT$ shows the dispersion of the average demand shortage per period around the global mean value ($u_{..}$). Therefore, $SSBT$ is related to the *stability* of the distribution decisions in time (all PoDs combined). We believe that $SSBT$ is a good measurement of how the distribution decisions are able to "smooth" the supply variations in the planning horizon.

Since global variance (${\sigma^2}_{global}$) is computed by dividing TSS by its degrees of freedom ($|T| \times |Z| - 1$), one can find the mean value of each component by dividing it by its respective degree of freedom. Therefore, and based in the previous decomposition analysis, it is possible to define two dispersion measures, named $\overline{WT}$, $\overline{BT}$, as follows:

$$\overline{WT} = \frac{SSWT}{|T| \times (|Z| - 1)} \quad \text{and} \quad \overline{BT} = \frac{SSBT}{|T| - 1}$$

It is worth mentioning that, although the variance decomposition presented in the previous paragraphs uses periods as main factor, a similar decomposition could have been done using the PoDs as the main factor. Indeed, doing so, the decomposition exercise would have led to two alternative dispersion measures on stability ($\overline{WZ}$) and equity ($\overline{BZ}$) :

$$\overline{WZ} = \frac{\sum_{t \in T} \sum_{z \in Z} (u_{zt} - u_{z\cdot})^2}{|Z| \times (|T| - 1)} \quad \text{and} \quad \overline{BZ} = \frac{|T| \sum_{t \in T} (u_{z\cdot} - u_{..})^2}{|Z| - 1}$$

4. Problem definition and formulations

This section defines the considered distribution context and proposes three different mathematical formulations that aimed tackling explicitly the notion of fairness in the satisfaction of the PoDs' demand. We consider a multi period planning horizon composed of $t \in T$ periods. The network includes three types of nodes: the outside suppliers $s \in S$, the potential HADCs $l \in L$, and the PoDs $z \in Z$. The exact location of all the nodes is known, and the transportation time between each two nodes i and j is denoted c_{ij}. We consider a set of different products' families or relief's kits, named in the following humanitarian functions $f \in F$ such as *survival* (e.g., meals, water), *safety*, *medical*, *technical*, etc. (Rekik et al, 2013). We assume that each PoD z has a given demand d_{zft} for each particular function f and period t, expressed in number of pallets or any other standard measure.

If a PoD does not receive its complete demand for a given period, we assume that it can be backordered and fulfilled within the next period. If the backordered demand is not delivered during the next period, this demand is considered lost. We have fixed the limit of backordered demand to one period, considering that compensating demand after more than one period can be too late and cause as much damage as if demand is never delivered. Evidently, this can be adjusted accordingly to the time's discretization used and the needs of the crisis managers. Please notice that the formulation can be easily extended to consider two periods of backorder or more. On the other side, allowing backorders gives flexibility to managers, but it must be avoided whenever possible. To reflect this, we establish a penalty cost β_{1f} if the demand for a function f is delayed by one period, and β_{2f} if demand is lost, with $\beta_{2f} \gg \beta_{1f}$.

Each HADC l has a specific global capacity limit by period ($Glob_{lt}$) and a capacity limit by function ($CapD_{lft}$), also by period. This capacity is expressed in number of pallets. On the other hand, suppliers' capacity is also limited to a number of pallets for each function at each period ($CapS_{sft}$). Finally, each HADC needs a specific number of professionals (n_l) to operate at its full capacity. However, there is a restriction on the total number of personnel N_t available at each period t, which in fact limits the total number of HADC to open. Also, the opening of an HADC requires some setting-up activities, decreasing in practice the center's available operation time for the period. We therefore assume that, during the period when a center is open, its capacity of incoming and outgoing flow is reduced by a factor $\alpha < 1$. Contrariwise, a center can be closed at any period without any additional cost.

The transportation of relief (from suppliers to HADCs and finally from HADCs to PoDs) is assumed to be done by truckloads of capacity P (same vehicle type). A transportation capacity limit is established for both the number of trips between a supplier s and a center l (Vs_{sl}), as well as the number of trips between a pair of HADC l and PoD z (Vd_{lz}) at any period.

The proposed optimization models seek to define a relief distribution network focusing in three primary aspects. First, we seek to minimize the demand shortage while maximizing fairness. The secondary objective (efficiency) is achieved by minimizing the total travel time, affecting directly the allocation decisions. Lastly, rapidity in distribution is not included in the objective function, but assured using maximum access time constraints for the supply (τ_1) and the distribution (τ_2). In the following, we introduce additional notation and then we present common constraints for the three models. Finally, each of the alternative objective functions are proposed. Notice that all the quantities as well as capacities are expressed in pallets, but they can be expressed using any other standard measure.

Sets

S_l Set of suppliers within the maximum distance of HADC l ($s \in S_l : c_{sl} \leq \tau_1$);

L_s Set of HADCs that are within the maximum distance of supplier s ($l \in L_s : c_{sl} \leq \tau_1$);

L_z Set of HADCs that are within the maximum distance of PoD z ($l \in L_z : c_{lz} \leq \tau_2$);

Z_l Set of PoDs that are within the maximum distance of HADC l ($z \in Z_l : c_{lz} \leq \tau_2$);

Decisions variables

x_{tl} binary variable equal to 1 if the HADC l is open at period t, zero otherwise;

y_{tl} binary variable equal to 1 if the HADC l is operating at period t, zero otherwise;

Qs_{slft} quantity of function f sent from supplier s to HADC l at the beginning of period t;

Qd_{lzft} quantity of function f sent from HADC $l \in L_z$ to PoD $z \in Z$ during period t;

$S^{-}_{zft,t+1}$ quantity of function f not delivered at PoD z during period t, scheduled to be delivered at period $\mathrm{t}+1$;

$S^{-}_{zft,t+2}$ quantity of function f not delivered at PoD z during period t, and that will not be delivered at the end of period $t + 1$, so it is counted as lost demand;

I_{lft} inventory at HADC l of function f at the end of period t;

4.1. Common constraints

The following set of constraints defines the general framework of the described system (as the distribution and flow dynamics) and the basic restrictions of the resources (capacity limits). These constraints are common to the three proposed models.

$$S^{-}_{zf1,2} + S^{-}_{zf1,3} + \sum_{l \in L_z} Qd_{lzf1} = d_{zf1} \qquad \forall\, z \in Z, f \in F \qquad (1)$$

$$S^{-}_{zft,t+1} + S^{-}_{zft,t+2} + \sum_{l \in L_z} Qd_{lzft} = d_{zft} + S^{-}_{zft-1,t} \qquad \forall\, z \in Z, f \in F, t = 2, \dots T \qquad (2)$$

$$S^{-}_{zft,t+1} + S^{-}_{zft,t+2} \leq d_{zft} \qquad \forall\, z \in Z, f \in F, t \in T \qquad (3)$$

$$\sum_{l \in L_z} (x_{lt} + y_{lt}) \geq 1 \qquad \forall\, z \in Z, t \in T \qquad (4)$$

$$\sum_{l \in L_s} Qs_{slft} \leq CapS_{sft} \qquad \forall\, s \in S, f \in F, t \in T \qquad (5)$$

$$I_{lf0} = 0 \qquad \forall\, l \in L, f \in F \qquad (6)$$

$$I_{lft} = I_{lft-1} + \sum_{s \in S_l} Qs_{slft} - \sum_{z \in Z_l} Qd_{lzft} \qquad \forall\, l \in L, f \in F, t \in T \qquad (7)$$

$$\sum_f^F I_{lft} \leq Glob_{lt}(x_{lt} + y_{lt}) \qquad \forall\, l \in L, t \in T \qquad (8)$$

$$I_{lft} \leq CapD_{lft} \qquad \forall\, l \in L, f \in F, t \in T \qquad (9)$$

$$\frac{\sum_f^F Qs_{slft}}{P} \leq Vs_{slt}(\alpha x_{lt} + y_{lt}) \qquad \forall\, s \in S_l, l \in L, t \in T \qquad (10)$$

$$\frac{\sum_f^F Qd_{lzft}}{P} \leq Vd_{lzt}(\alpha x_{lt} + y_{lt}) \qquad \forall\, z \in Z_l, l \in L, t \in T \qquad (11)$$

$$\sum_l^L n_l(x_{lt} + y_{lt}) \leq N_t \qquad \forall\, t \in T \qquad (12)$$

$$x_{lt} + y_{lt} \leq 1 \qquad \forall\, l \in L, t \in T \qquad (13)$$

$$y_{lt} \leq x_{lt-1} + y_{lt-1} \qquad \forall\, l \in L, t = 2 \dots T \qquad (14)$$

$$y_{lt-1} + x_{lt} \leq 1 \qquad \forall\, l \in L, t = 2 \dots T \qquad (15)$$

$$x_{lt-1} + x_{lt} \leq 1 \qquad \forall\, l \in L, t = 2 \dots T \qquad (16)$$

$$x_{lt}, y_{lt} = \{0,1\} \qquad \forall\, l \in L, t \in T \qquad (17)$$

$$S^-_{zft,t+1}, S^-_{zft,t+2}, Q_{slft}, Q_{lzft}, I_{lft}, \geq 0 \qquad \forall\, s \in S_l\; z \in Z_l, l \in L, f \in F, t \in T \qquad (18)$$

Constraints set (1) defines the quantity of the humanitarian function f not delivered to PoD z at period 1, which are scheduled to be delivered at period 2 or will be considered as lost. This equation is generalized in constraints (2) for the other periods. Constraints (3) limit the quantity that can be backordered (or lost) for a given period to the demand of the period. These constraints also assure that backordered demand is delivered during the period where it is expected. Constraints (4) require that an active HADC (opened or already in operation) within the covering distance of each PoD must be open at every period. Constraints (5) state that the total flow of a given function sent to the HADCs from a given supplier s at period t must respect the supplier's capacity. Constraints (6) and (7) stablish the balance of flow to define the inventory levels at each center l, at each period t and for each function f. This inventory level has to respect the total capacity limit of every HADC (8), as well as the capacity limit by function (9), at every period. Constraints (10) and (11) define the number of loads that will traverse an arc (s, l) and (l, z), respectively, at period t considering that each trip can carry P pallets, and state that the total number of trips, from suppliers to HADCs and from HADCs to PoDs, needs to respect the imposed limits. They also consider that the capacity of a center l is reduced by α at its opening period. Finally, constraints (12) establish the available staff's limit at every period. Constraints (13) to (16) link the opening and the operation variables for every HADC at every period. They ensure that a HADC cannot be operating and opened at the same period (constraint 13) and if it is operating a certain period t, it is because it was already opened or operating (constraint 14). Constraints 15 states that in order to open a HADC in a period t, it has to be closed (not operating) in the previous period. Finally, constraints (16) state that a HADC cannot be opened for two periods in a row. These last two constraints are only useful in the specific case where the global capacity is greater than the demand; otherwise they are redundant with constraints (10) and (11).

4.2. Fair distribution modeling approaches

As mentioned before, among the variety of objectives that can be pursued in the design of relief distribution networks, this paper seeks to minimize the percentage of unsatisfied demand with a major focus on fairness. We therefore present three objective functions to seek a fair relief distribution. In addition we include an efficiency objective by minimizing the total travel time seeking to guide the model to make smart decisions in the use of resources. Thus, we propose three different multi-criteria objective functions to be minimized using *weighted-sum* optimization method. The details of each objective function and the additional constraints required for each model are presented in the following.

4.2.1. M1: Minimization of the penalty associated to the total unsatisfied demand

The first model is one of the most popular in relief distribution. It concentrates in minimizing the penalty due to the unsatisfied demand. However, this approach has been adapted to account for backordered and lost demand. Finally, it includes the efficiency objective of minimizing total travel time. To present this three objectives in a single objective function, each term i is affected by a penalty factor δ_i. Let u_{zft} be the percentage of unsatisfied demand of humanitarian function f at PoD z in period t, if any. We define Obj_1 (19) as the penalty cost for the percentage of unsatisfied demand penalized by factor δ_1; Obj_2 (20) is the penalty cost of backordered and lost demand penalized by factor δ_2; and Obj_3 (21) is the cost associated with the number of trips multiplied by their distance and penalized by factor δ_3. Each objective is formulated as follows:

$$Obj_1 = \delta_1\left(\sum_{t\in T}\sum_{z\in Z}\sum_{f\in F} u_{zft}\right) \tag{19}$$

$$Obj_2 = \delta_2\left(\sum_{t\in T}\sum_{z\in Z}\sum_{f\in F}\beta_{1f}\frac{S^-_{zft,t+1}}{d_{zft}} + \sum_{t\in T}\sum_{z\in Z}\sum_{f\in F}\beta_{2f}\frac{S^-_{zft,t+2}}{d_{zft}}\right) \tag{20}$$

$$Obj_3 = \delta_3\left(\sum_{t\in T}\sum_{s\in S}\sum_{l\in L} c_{sl}\frac{\sum_f^F Qs_{slft}}{P} + \sum_{t\in T}\sum_{l\in L}\sum_{z\in Z} c_{lz}\frac{\sum_f^F Qd_{lzft}}{P}\right) \tag{21}$$

Model 1 (M1) is then formulated as:

$$\text{Min } Obj_1 + Obj_2 + Obj_3 \tag{22}$$

subject to:

$$u_{zft} \geq 1 - \frac{\sum_{l\in L_z} Qd_{lzft}}{d_{zft}} \qquad \forall\, z\in Z, f\in F, t = 1,\ldots T \tag{23}$$

$$u_{zft} \in [0,1] \qquad \forall\, z\in Z, f\in F, t\in T \tag{24}$$

in addition to constraints (1) to (18) .

4.2.2. M2: Minimization of the maximum gap

The second approach to maximize distribution fairness is similar to the one in Tzeng (2007) and Lin et al. (2011). It consists in minimizing the largest gap among the unsatisfied demand (in percentage) for all pairs of zones. We have adapted and extended this approach to take backorders into account. Let $\gamma_{ft,t+1}$ be the maximum gap between PoDs of the percentage of demand of humanitarian function f that is backordered at period t to be payed at period $t+1$. Likewise, we define $\gamma_{ft,t+2}$ as the maximum gap among the PoDs of the percentage of demand of humanitarian function f that is lost at period t. We thus define Obj_4 (25) as the penalty cost for unsatisfied demand's range, penalized by factor δ_4 and we add constraints (27) and (28) to the model.

$$Obj_4 = \delta_4\left(\sum_{t \in T} \sum_{f \in F} \beta_{1f}\gamma_{ft,t+1} + \beta_{2f}\gamma_{ft,t+2}\right) \qquad (25)$$

In this model we include also the minimization of backorders and lost demands and total travel time as presented in M1 (Obj_2 and Obj_3)

The second model (M2) can be stated as follows:

$$\text{Min } Obj_4 + Obj_2 + Obj_3 \qquad (26)$$

Subject to:

$$\frac{S^{-}_{ift,t+1}}{d_{ift}} - \frac{S^{-}_{jft,t+1}}{d_{jft}} \le \gamma_{ft,t+1} \qquad \forall\, i, j \in Z\ (i \neq j), f \in F, t \in T \qquad (27)$$

$$\frac{S^{-}_{ift,t+2}}{d_{ift}} - \frac{S^{-}_{jft,t+2}}{d_{jft}} \le \gamma_{ft,t+2} \qquad \forall\, i, j \in Z\ (i \neq j), f \in F, t \in T \qquad (28)$$

in addition to constraints (1) to (18).

M3: Minimum dissatisfaction cost with a piecewise penalty function

We were inspired by Holguín-Veras et al. (2013) and Huang et al. (2012), which suggested the use of a monotonic, non-linear and convex function to express the cost associated to human suffering caused by the deficit of supplies or services in the aftermath of a disaster. Indeed, the perception of the people in need is clearly not linear, and higher values of dissatisfaction of demand must be penalized more strongly than lower values. In a similar manner, delays of two periods in demand's satisfaction has a larger cost (penalty) than twice the penalty for one period delay. We propose to model penalties related to the percentage of unsatisfied demand as an exponential function. Using non-linear penalties in the objective function is a way to seek fairness. In order to introduce such effect in our model while keeping its linearity, we approximated the penalty curve for a given unsatisfied demand percentage of PoD z, humanitarian function f at period t (i.e. u_{zft}) by a piecewise linear function as depicted in Figure 1, where it can be observed that the penalty increases significantly from one piece k to the next one. We refer the reader interested in the mathematical aspects of this linearization to Padberg (2000).

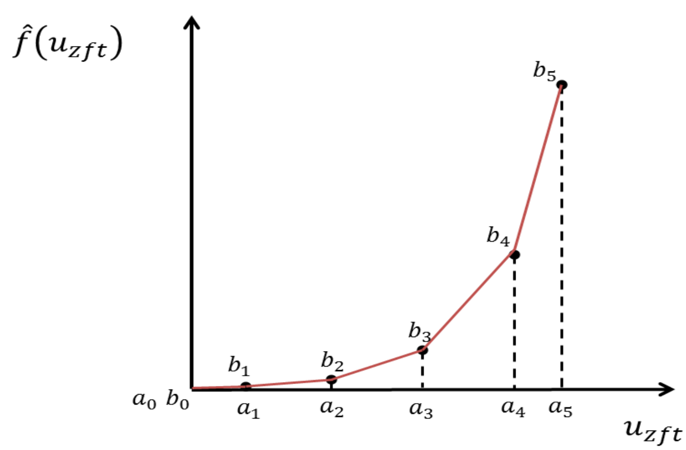

Figure 1 – Example of a piecewise cost function for $\boldsymbol{u_{zft}}$.

In addition, we need to adapt this approach to take backorders and lost demands into account. We thus define *Obj*$_5$ (29) as the piecewise penalty cost for the percentage of total dissatisfaction percentage with a penalty weight of δ_5 and *Obj*$_6$ (30) as the piecewise penalty cost for backordered and lost demand percentage with a penalty weight of δ_6:

$$Obj_5 = \delta_5 \sum_{t \in T} \sum_{z \in Z} \sum_{f \in F} \ \left[\hat{f}\left(u_{zftk}\right)\right] \tag{29}$$

$$Obj_6 = \delta_6\left[\beta_{1f}\hat{f}\left(u_{zft,t+1,k}\right) + \beta_{2f}\hat{f}\left(u_{zft,t+2,k}\right)\right] \tag{30}$$

Then, we include the efficiency objective (Obj_3) as is done in M1 and M2. M3 can be stated as follows:

$$Min\ Obj_5 + Obj_6 + \ Obj_3 \tag{31}$$

The first two terms of (31) accounts for the penalty associated to unsatisfied demand (as well as the backorder and lost demand) for each product, each PoD and each period. This is given by the piecewise linear function defined by the following functions:

$$\hat{f}\left(u_{zftk}\right) = \sum_{k=1}^{K} c_k \ u_{zftk} \tag{32}$$

$$\hat{f}\left(u_{zft,t+1,k}\right) = \sum_{k=1}^{K} c_k \ u_{z,f,t,t+1,k} \tag{33}$$

$$\hat{f}\left(u_{zft,t+2,k}\right) = \sum_{k=1}^{K} c_k \ u_{z,f,t,t+2,k} \tag{34}$$

where $u_{zftk} \ \in [0, a_k - a_{k-1}]$ is the percentage, of the demand of function f not delivered to PoD z at period t, that is inside the piece k, c_k is the slope of piece k ($c_k = \frac{b_k - b_{k-1}}{a_k - a_{k-1}}$) and (a_k, b_k) is the breaking point of the piecewise function related to piece k (same for backorder and lost demand). The last term computes the penalty associated to distribution time as in the previous models. Model M3 requires the following constraints (in addition to constraints (1) to (18)):

$$\sum_{k=1}^{K} u_{zftk} \geq 1 - \frac{\sum_{l \in L_z} Qd_{lzft}}{d_{zft}} \qquad \forall\, z \in Z, f \in F, t = 1, \dots T \qquad (35)$$

$$\sum_{k=1}^{K} u_{z,f,t,t+1,k} = \frac{S^{-}_{zft,t+1}}{d_{zft}} \qquad \forall\, z \in Z, f \in F, t = 1, \dots T \qquad (36)$$

$$\sum_{k=1}^{K} u_{z,f,t,t+2,k} = \frac{S^{-}_{zft,t+2}}{d_{zft}} \qquad \forall\, z \in Z, f \in F, t = 1, \dots T \qquad (37)$$

$$u_{zftk} \leq (a_k - a_{k-1}) \qquad \forall\, z \in Z, f \in F, t = 1, \dots T, k \in K \qquad (38)$$

$$u_{zft,t+1,k} \leq (a_k - a_{k-1}) \qquad \forall\, z \in Z, f \in F, t = 1, \dots T, k \in K \qquad (39)$$

$$u_{zft,t+2,k} \leq (a_k - a_{k-1}) \qquad \forall\, z \in Z, f \in F, t = 1, \dots T, k \in K \qquad (40)$$

$$u_{zftk}, u_{zft,t+1,k}, u_{zft,t+2,k} \in [0,1] \qquad \forall\, z \in Z, f \in F, t \in T \qquad (41)$$

Constraints (35) to (37) link the piecewise variables and the demand shortage quantities, computing the total percentage of unsatisfied demand, the demand backorder at periods $\mathrm{t}+1$ and $\mathrm{t}+2$ (lost demand) respectively, divided by demand of period t. Notice that in the case of a compensation (i.e. if backordered demand is paid in a given period t), the total delivery might be higher than the demand of t. In this case, the dissatisfaction percentage is computed as null. Constraints (38) to (40) ensure that variables $u_{ztf,t+1,k}$ and $u_{ztf,t+2,k}$ cannot be greater that the length of interval k. These constraints, together with the objective function, force the sequential use of each piece of the piecewise function for variables $u_{zftk}, u_{zft,t+1,k}, u_{zft,t+2,k}$ respectively. Constraint set (41) define the domain for the piecewise variables. Needless to say, the quality of the solutions produced by model M3 depends on the number and the bounds of the pieces used in the piecewise function. Therefore, a heuristic procedure is proposed to find a good compromise in this matter. This method is presented in the next paragraphs.

4.3. Iterative approach to construct the piecewise linear function

The number of pieces in the piecewise linear function and their breakpoints have a strong influence on the quality of the solution as well as on its solvability. Three considerations must be kept in mind when designing the piecewise function. First, a better approximation of the exponential function may be obtained by using more pieces, but by doing so the model becomes more difficult to solve. Second, since the piecewise cost function is intended to enforce equity by trying to group all PoDs in the same dissatisfaction level (the same piece) pieces should be small enough. Third, the fairest value of unsatisfied demand depends on the offer/demand ratio of a period and its evolution in time. In other words, the proper number and value of each piece can differ according to the specific instance. We therefore propose a heuristic procedure to fix the number of pieces, the bounds of each one as well as the slope of each piece by an iterative approach.

In the following, we illustrate the algorithm used to set the piecewise function of a particular humanitarian function f (i.e. considering u_{zft} as u_{zt}). The heuristic is initialized with only two pieces per variable ($|K| = 2$). We define A as the set of breakpoints a_k. A is initialized with the minimum value of the ratio offer/demand in the horizon (named $min_t\, \rho_t$), seeking to fix an upper bound of dissatisfaction i.e. $A = \{0; \min_t \rho_t\,;\ 1\}$. Then, M3 is solved to optimality and the solution produced is analyzed in order to decide if new pieces should be added to the piecewise function and the model solved again (next iteration).

At a given iteration i, the average unsatisfied demand's percentage ($u_{..}^{i}$) and the global standard deviation (σ_{global}^{i} as defined in Section 3.1) of the present solution are computed. If σ_{global}^{i} is greater than the standard deviation goal (σ_{wanted} set arbitrary to zero), three new pieces are added around $u_{..}^{i}$, with three new breakpoints added to set A as $\left\{u_{..}^{i} - \frac{\sigma_{global}^{i}}{2};\ u_{..}^{i};\ u_{..}^{i} + \frac{\sigma_{global}^{i}}{2}\right\}$. Slopes for all the pieces are recalculated. To this end, we set a base penalty value for the first piece in the function, and then the slope for each piece is increased by 1.1 times the rate between the highest and the lowest demand (i.e. $c_k = c_{k-1} \times 1.1\ \frac{D_{max}}{D_{min}}$). After the recalculating the cost piecewise function, the model is redefined and solved again. If the new solution results in a reduction in σ_{global}^{i} a new iteration $i+1$ begins. If no improvement is achieved, the same procedure is applied over the backorder and lost demand variables. The procedure is repeated until a given stop criterion is met (e.g. maximal number of iterations, or until the improvement obtained with current iteration is not significant or null). At the end, the last solution is retained and reported as the solution of M3. The *Algorithm 1* allows us to adapt the shape of the piecewise function dynamically.

Algorithm 1 – Procedure to construct our piecewise function.

1. Initialize: **Set** $|\mathrm{K}| = 2$ with $A = \{0; \min_t \rho_t;\ 1\}$, $\sigma_{global}^{0} = \infty$; $i = 0$ and maxIter = 5.
2. **Set** $i = i + 1$, $s \leftarrow$ MIP Solution to optimality using A as bounds, estimate σ_{global}^{i} and $u_{..i}$
3. **If** $\sigma_{global}^{i} > \sigma_{wanted}$ and $\sigma_{global}^{i} < \sigma_{global}^{i-1}$ **then**

    Go to step 4

   else

    Go to step 6.
4. Estimate breakpoints of three new pieces:

   4.1. **Fix** breakpoint = $u_{..i} - \sigma_{global}^{i}/2$

   4.2. **Fix** breakpoint = $u_{..i}$

   4.3. **Fix** breakpoint = $u_{..i} + \sigma_{global}^{i}/2$
5. **If** the new bounds defined do not exist in A and $i \leq maxIter$, **then**

    5.1 add the bounds to A

    5.2 go to step 2;

else

Go to step 6.

6. **Return** s

5. Numerical experiments

This section seeks to examine and to analyse, through numerical experiments, the behaviour of the three different modeling approaches for a fair relief distribution over different scenarios.

5.1. Problem generation and demand scenarios

In order to test the models, a flexible instance generator was designed to define and create a large variety of test scenarios. All the parameters specified in the following paragraphs can be adapted to the needs of a particular problem. The size of an instance is defined by the cardinality of the following sets: PoDs ($|Z|$), HADCs ($|L|$), suppliers ($|S|$), humanitarian functions ($|F|$), and the number of periods in the planning horizon ($|T|$). A problem is defined over a total area (TA) of $[1000 \times 900]$, inside of which we define an affected area (AA) of $[600 \times 500]$. The PoDs' and HADCs' location is randomly generated inside the AA, and the set of suppliers inside the TA, but outside the AA. The demand for each PoD at the first period is randomly generated in the range of [20;70] for every humanitarian function. The capacity of any HADC l is set at 60% of the total demand ($CapD_{lft} = 0.6 \sum_{z \in Z} d_{zft}$). In all our numerical experiments we seek to represent our main interest in minimizing the unsatisfied demand percentage and the fairness objective, which has been overlooked in the past. Therefore, in M1 we applied $\delta_1 \gg \delta_2 \gg \delta_3$, with $\delta_1 \cong 100\delta_2 \cong 1000\delta_3$; for M2 we applied $\delta_4 \gg \delta_2 \gg \delta_3$, with $\delta_4 \cong 100\delta_2 \cong 1000\delta_3$ and for M3 we applied $\delta_5 \gg \delta_6 \gg \delta_3$, with $\delta_5 \cong 100\delta_6 \cong 1000\delta_3$.

Depending on the nature and the gravity of the event (demand) and the availability of resources (number and capacity of responders), different supply scenarios can be considered. Following that, we defined two basic theoretical scenarios.

Scenario 1 - Temporary shortness of resources

In the first periods in the aftermath of a disaster, the available resources are limited and vary from one period to another on the planning horizon. In other words, periods of shortness alternate with others showing reasonable offer levels corresponding to the arrival of help from national and international organizations. In this case, the backordering of the unsatisfied demand becomes an interesting solution available to crisis managers.

Scenario 2: Extreme shortness of resources

In this scenario, the available supplies, in addition to the foreseen arrivals of relief, will be systematically under the requirements. Crisis managers cannot make a commitment towards future deliveries to compensate for the shortness. In this case, crisis managers would try to distribute the available relief in the most fair manner.

We model both of our instances' scenarios for a particular humanitarian function on an offer/demand ratio ρ_t for each time periods. In temporary shortness, suppliers have the capacity to respond to the demand during the first periods. Then, ρ_t drops under one when local supplies are finished, and finally external supplies start to arrive ($\rho_t > 1$). In extreme shortness, we consider that, during the first periods, local capacity is limited ($\rho_t \approx 0.8$), and then it decreases until a deep strong ($\rho_t \approx 0.2$). In the following, numerical results produced for temporary shortness are presented, followed by those produced for extreme shortness.

5.2. Models' performance in a temporary shortness scenario

In order to characterize the behavior of the solutions produced by the three models with respect to fairness, we will use in this section a set of 10 small instances (two suppliers, three HADCs, six

PoDs, one humanitarian function, and eight periods). Instances were solved to optimality with Gurobi v.6.0 for M1, M2, and M3. This later is solved several times according to the iterative heuristic described in section 4.3.

We have thoroughly analyzed the solutions produced to the first instance I1 where ρ_t, the offer/demand ratio at each period, is set to {1.0; 1.0; 0.7; 0.5; 0.9; 1.2; 1.2;1.2}. In other words, after two periods in which the demand can be satisfied, there is a three periods shortness where the offer falls to only 70% and then to 50% of the demand. From period five, offer rises to 90% of the demand and, during the last three periods, it exceeds the requirements. Figure 2 shows the dissatisfaction percentage at each PoD and period in the solutions produced by M1 (leftmost chart), M2 (central chart) and M3 (rightmost chart) and how they behave in very different manners.

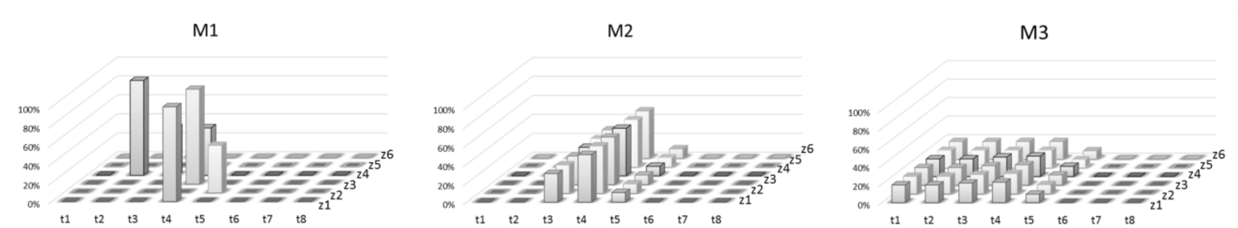

Figure 2 – Dissatisfaction percentage for instance I1

If we look at how shortage is shared between the PoDs for a given period, M2 and M3 split the shortness in a rather homogeneous manner: all the PoDs suffer similar shortages. However, M1 concentrates shortages only on a few PoDs, and those will experience very high values of dissatisfaction. For instance, PoD four’s demand is 100%, 50% and 50% unsatisfied in periods two to four. If we now look at how the global shortage is handled in time, we observe that M1 and M2 simply distribute the available quantities at each period. However, M3 shows a more elaborated behavior, which translates in a smoother distribution. In fact, M3 reserves some quantities during periods one and two in order to minimize the impact of the shortage in periods three to five. Doing so, the maximum dissatisfaction percentage suffered by any PoD and at any period is under 20%, while in M1's solution some PoDs experience up to 100% of unsatisfied demand and in M2's solution PoDs suffer up to 60%. The piecewise approximation achieves a rationalization of resources, resulting in an equitable distribution among PoDs (the same or almost the same dissatisfaction level) in a period, and this in a stable matter across time in the shortness periods. To sum up, both M2 and M3's solutions achieve a good "equity" between the PoDs, but M3 is also able to achieve an excellent "stability".

Let us now see how these behaviors are captured by the proposed numerical indicators. *Table 1* reports the numerical results produced by models M1, M2 and M3. To measure the quality of the distribution plan obtained by each model, we report two global measures: the global average dissatisfaction percentage ($u_{..}$), and the global standard deviation (σ_{global}), which concerns to the dispersion in distribution. Then, we also compute mean sum of squares within time ($\overline{WT}$) and between time ($\overline{BT}$) and the two range indicators ($\overline{R_1}$ and $\overline{R_2}$). Finally, we calculate the total traveled distance (D) and record the total computation time to solve each model in seconds.

Let us consider first the results produced for I1. We observe that M1 achieves a lower value for $u_{..}$ (i.e. a better global satisfaction). The reason is that, although all the three models distribute the same quantity of help, M1 prefers to give slightly higher quantities to PoDs 5 and 6 because the marginal impact of a single additional help unit is higher for PoDs with small demand. On the other hand, doing so deteriorates the equity and stability objectives. In fact, M1 is clearly outperformed by both M2 and M3 for almost all the others indicators (excluding $\overline{BT}$ in which M2 shows the poorest performance). As expected, concerning σ_{global} (global dispersion over all PoDs and periods), M2 achieves 18% and is clearly dominated by M3, which produces only 9%. As per range indicators, $\overline{R_1}$ and $\overline{R_2}$ show the poor performance of M1. M2 has a perfect score in terms of equity ($\overline{R_1}$) and M3 shows an almost equal performance, but M3 offers a better performance with respect to stability ($\overline{R_2}$). This particular behaviour is confirmed by the dispersion indicators. Indeed, M2 and M3 achieve equal "perfect" scores for equity ($\overline{WT}$), but M3 offers better results for stability ($\overline{BT}$). This result is easily explained by the cost function structure of M3. The fact that the domain of u_{zft} is discretized in different pieces, with a higher cost function (slope) for each successive piece, makes it possible to seek the same (or almost the same) dissatisfaction percentage for each period, PoD and humanitarian function.

We now analyze the rest of the results in Table 1. Lines *Avg.* show the average values for each column over the 10 instances and lines *# best* counts the number of instances in which the model achieved the best value of the indicator.

Table 1 – Results produced for 10 small instances (temporary shortness).

		$\boldsymbol{u_{..}}$	$\boldsymbol{\sigma_{global}}$	$\boldsymbol{\overline{WT}}$	$\boldsymbol{\overline{BT}}$	$\boldsymbol{\overline{R_1}}$	$\boldsymbol{\overline{R_2}}$	**D**	**Sec.**
I1	M1	9%	26%	6%	12%	37%	50%	364	0.2
	M2	11%	18%	0%	21%	0%	50%	367	0.2
	M3	11%	9%	0%	6%	1%	21%	371	0.6
Avg.	M1	9%	24%	5%	11%	35%	45%	1338	0.2
	M2	11%	17%	0%	19%	0%	50%	1329	0.1
	M3	11%	9%	0%	6%	3%	20%	1380	0.6
# best	M1	10	0	0	0	0	0	3	4
	M2	0	0	10	0	10	0	7	7
	M3	0	10	10	10	0	10	0	0

Globally speaking, results are quite similar to the ones produced for instance I1. M1 systematically achieves the best average percentage of unsatisfied demand but at the cost of a very poor equity and stability performances. M2 offers the best performance with regards to $\overline{R_1}$, but M3's performance is always very close to M2's. As expected, M3's stability performance is the best over the three models. To sum up, M2 and M3 achieve equal (perfect) scores for $\overline{WT}$ (equity indicator), but M3 offers better results when measuring stability ($\overline{BT}$).

Before moving on to the experiments on extreme shortness scenario, let us say few words about the efficiency of the produced solutions. Both M1 and M2 produce the shortest total distance for three and seven instances. However, M3's average total distance is only 3.9% higher than M2. We can therefore conclude that an equitable and smooth distribution does not cost much in terms of transportation efficiency.

5.3. Models' performance in an extreme shortness scenario

In an extreme shortness scenario, when resources are really scarce, assuring equity in a period and across the horizon is of the highest importance, because rationalization is the only way to minimize suffering and reduce disparity in the relief given to affected people.

To validate the behavior of the solutions produced by the three proposed models, we use the same set of instances of section 5.2., but this time the offer/demand ratio ρ_t per period was set to $\{0.8, 0.8, 0.8, 0.6, 0.6, 0.2, 0.2, 0.6\}$. In the same way as it was done in previous section, we'll use a single instance (I11) as an illustrative example to carefully explain each model's behavior. Figure 3 shows the dissatisfaction percentage at each PoD and period for this instance.

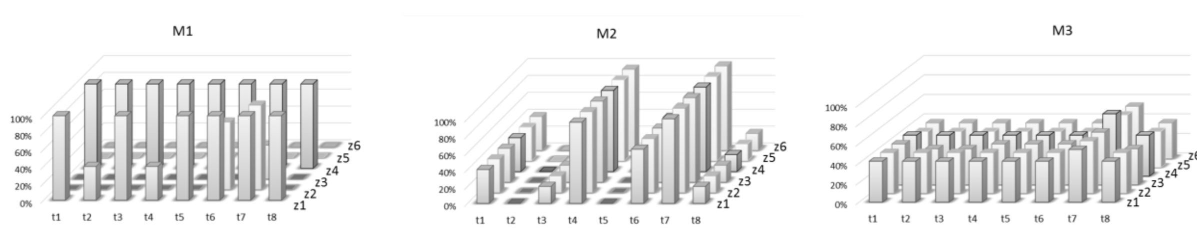

Figure 3 – Dissatisfaction percentage for instance I11.

First, it can be observed that M1 never visits PoD4, which is the POD with the highest demand. On the other hand, PoDs 5 and 6, the ones having the lowest demand, are always visited. PoD1 is also strongly penalized and is not visited in six out of eight periods. We believe that this behaviour should not be tolerated in practice. On its side, M2 shares the amount of relief available, assuring equity at every period, but again, it is not able to balance deliveries between periods. Hence, all PoDs suffer equivalent penuries, but theirs demand is fully met in some periods and totally unsatisfied in others (periods 4 and 7). We consider this as a questionable decision because the lowest offer/demand ratio on the horizon is 20%. Indeed, M3 is the only formulation that allows for equity among PoDs and stability throughout time, thus reducing the maximum non-satisfaction level from 80% to 54% (in period seven) and 42% in the other periods. Table 2 reports the performance values achieved by the solutions produced by models M1 to M3.

Table 2 – Results produced for 10 small instances (extreme shortness).

		$\boldsymbol{u}_{..}$	$\boldsymbol{\sigma_{global}}$	$\overline{\boldsymbol{WT}}$	$\overline{\boldsymbol{BT}}$	$\overline{\boldsymbol{R_1}}$	$\overline{\boldsymbol{R_2}}$	**D**	**Sec.**
I11	M1	35%	46%	24%	8%	100%	33%	204	0.1
	M2	42%	35%	0%	86%	0%	97%	219	0.0
	M3	42%	5%	0%	2%	4%	16%	219	0.4
Avg.	M1	33%	46%	24%	4%	100%	22%	824	0.1
	M2	42%	31%	0%	68%	0%	89%	813	0.1
	M3	42%	5%	0%	2%	4%	14%	819	0.6
# best	M1	10	0	0	7	0	2	3	4
	M2	0	0	10	0	10	0	6	6
	M3	0	10	10	8	0	8	1	0

As expected, M1 achieves again the lowest total dissatisfaction value. M3 shows a total deviation of only 5% while M1 and M2 produce values of up to 46% and 35%, respectively. Again, M2 shows a perfect balance for all the PoDs within the same period, and M3's results are not far. Indeed, M3 also achieves a perfect score of 0% for $\overline{WT}$ and only 4% for $\overline{R_1}$. On the other hand, M3 clearly outperforms both M1 and M2 in terms of distribution stability. We extend our analysis to nine more random generated instances. The numerical results are reported in Table 2.

As in the temporary shortness case, M3 minimizes global deviation and offers the best possible equity and stability performances at a negligible increase in the distribution distance.

5.4. Models' performance in larger-sized instances

We will dedicate the last part of this section to show the models' performance over a set of 20 instances with a more realistic size. The objective is to test the models' capacity to ensure a fair distribution over a much larger set of PoDs, and to test, at the same time, the computational effort of each model. Following the pattern described in section 5.2 and 5.3, we solve 10 instances for temporary shortness and 10 instances for extreme shortness cases. For each scenario we test five medium-size instances and five large-size instances. We define as "medium-size" instances with a total of 20 PoDs, 10 potential HADCs, six suppliers, one humanitarian function and eight periods. Large-size instances have 50 PoDs, 20 HADCs, six suppliers, one function and eight periods. Table 3 and Table 4 summarize the results for the temporary and extreme shortness scenarios respectively. In the following, we will concentrate our analysis to models M2 and M3, because M1 shows still the poorest performance in most of the indicators. Table 3 and Table 4 confirm that the models' behaviors follow the same line observed in the smaller instances. M2 and M3 achieved almost the same dissatisfaction percentage, but distributed the limited resources in a very different way. M3 achieves the best score in average global dispersion of only 9% and around 3% for the temporary and extreme shortness cases respectively. Therefore, we can observe that increasing the number of PoDs did not have an impact in the quality of the solution. M3 can still ensure an equitable distribution among PoDs as M2 does and a much stronger stability in time. For the temporary shortness case, M3 has an $\overline{R_2}$ of only 19% vs. 49% for M2, while for the extreme shortness case M3 has an $\overline{R_2}$ of only 4% vs. 93% and 76% for M2.

Table 3 – Results produced for larger-sized instances in temporary shortness.

		$\boldsymbol{u_{..}}$	$\boldsymbol{\sigma_{global}}$	$\overline{\boldsymbol{WT}}$	$\overline{\boldsymbol{BT}}$	$\overline{\boldsymbol{R_1}}$	$\overline{\boldsymbol{R_2}}$	**D**	**Sec.**
Avg. over 5 medium instances	M1	9%	26%	6%	35%	62%	45%	3590	0.1
	M2	11%	17%	0%	63%	0%	49%	3475	2.3
	M3	11%	9%	0%	18%	3%	19%	4005	1.6
Avg. over 5 large instances	M1	9%	27%	6%	89%	62%	49%	8003	0.3
	M2	11%	16%	0%	143%	0%	49%	7782	35.3
	M3	10%	9%	0%	45%	4%	19%	8840	3.4

Table 4 – Results produced for larger-sized instances in extreme shortness.

		$\boldsymbol{u_{..}}$	$\boldsymbol{\sigma_{global}}$	$\overline{\boldsymbol{WT}}$	$\overline{\boldsymbol{BT}}$	$\overline{\boldsymbol{R_1}}$	$\overline{\boldsymbol{R_2}}$	**D**	**Sec.**
Avg. over 5 medium instances	M1	33%	47%	23%	6%	100%	12%	2110	0.1
	M2	43%	30%	0%	205%	0%	93%	2062	2.9
	M3	42%	2%	0%	0%	4%	4%	2069	1.4
Avg. over 5 large instances	M1	33%	47%	22%	1%	100%	4%	4751	0.2
	M2	42%	26%	0%	413%	0%	76%	4657	10.2
	M3	42%	3%	0%	1%	5%	4%	4723	0.9

The numerical values previously reported show their sensibility with respect to the specific type of the scenario considered. In the extreme shortness scenario the obtained values tend to be higher than in the temporary shortness one. This is related to the variability of the offer/demand ratio defining each type of scenario. Finally, let us take a second to analyze the computational effort needed to solve larger instances. All the models can be solved to optimality in short time (less than a minute in average), even for large instances. However, M2 shows a high variability in CPU time, having a lot of trouble to close the optimality gap. For instance, in the temporary shortness case (for large instances) M2 takes 35 seconds on average with a standard deviation of 27 seconds due to extremes values in three out of five instances, while M3 takes 3,4 seconds on average with a standard deviation of only 0,4 seconds. We can conclude that M3 is the modeling approach that best suits the different objectives set for the complex problem of relief distribution.

6. Conclusions and future research

In this paper we proposed and discussed three different approaches for the design and the operation of a relief distribution network. This work is mainly centered in two important components that had been overlooked in the past: the fairness principle and the multi-period nature of relief distribution. We strongly believe that these major aspects need to be covered in response logistics, and they need to be addressed from the beginning of the response plan (the network design phase) in order to improve the other logistic tasks (procurement, delivery plans and transportation problems). Three important contributions were made in the fair relief distribution problem. First of all, a discussion on what can be defined as a fair distribution was presented, concluding that in order to obtain fairness, crisis managers should warrant equity in distribution within periods, but also, stability in delivery in the best possible way. In addition, we considered and modeled shortness by including the possibility of backordered demand. This allows crisis managers to gain flexibility in the distribution and seek compensation of unsatisfied PoDs on the planning horizon. Secondly, we proposed and adapted five performance indicators to measure the two components of fairness. Finally, we proposed and tested three different formulations to handle the complex context of relief distribution. These formulations seek mainly minimization of unsatisfied demand and effectiveness in distribution, but also two of them explicitly include the fairness objective. We compared them in some numerical examples and concluded that, M2 achieves a perfect score in equity in distribution in a single period, but is unable to maintain a stable distribution on the planning horizon. We proved, on its side, that M3 accounts for both equity and stability.

Several promising research paths are currently being considered. For instance, the extension of our proposition to include routing planning, supplying an integral planning tool to CMs in response and preparedness of relief distribution.

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