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Product Quality Improvement Induced by Cost-Sharing Mechanism and Competition

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Abstract. Due to the growing competition in the modern sophisticated business environment, there is an increasing trend to launch new products or to improve the quality of the end products in order to attract more consumers. But the rising costs or uncertainties for this innovation require firms to collaborate with each other. In this paper, we explore supply chain coordination mechanisms arising out of improvement in the quality of the end products and analyze the impact of the cost-sharing contract on the optimal decisions of the supply chain members undertaking quality improvement initiatives. Through a game theoretic approach, we develop and analyze two models of cost-sharing: (1) Simple cost-sharing and (2) Cost-sharing through bargaining. We confirm that cost-sharing contract is able to coordinate the whole supply chain. Further, Upstream competition is desirable since it is profitable from both profit and product quality improvement perspective. Finally, we perform a comprehensive numerical analysis.

Keywords: Quality improvement, competition, supply chain coordination, non-cooperative game, bargaining.

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1. INTRODUCTION

Since the 1980's, quality has become a major influence in the business world. In the recent years, after the price of the product, quality has become the second most important factor influencing consumers' purchasing decision (J.D. Power and Associates 2004). The product quality may be defined as the fulfillment of consumers' expectation levels. A failure to meet this expectation may be a key reason behind the loss of goodwill of a company. So, product quality has a significant role in our modern competitive business environment. For instance, in 2011, the brand's ranking of Ford fell 10 spots in Consumer Report's annual auto reliability survey due to deterioration of quality and subsequently, 'quality' became 'job 1' at Ford (Associated Press 2011). The product quality is important to those consumers who are ready to pay a high price to get the high quality product in return. Again, due to the globalization of modern business, very good quality international products capturing our local markets and to survive in these competitive markets, domestics companies must improve their product quality. Therefore, quality improvement has an important impact on the life and performance of a supply chain.

Nowadays, quality improvement is an important issue of a supply chain quality management. It has a significant impact not only on players' different strategies of a supply chain but also on consumers' satisfaction levels (Lin et al. 2005). Again, higher quality of product sometimes requires high product price, it may also be a reason for higher costs. Moreover, at the same time, demand and profits are influenced by this quality improvement practices (Banker et al., 1998; Baiman et al. 2000). Therefore, quality improvement and pricing decisions are significantly correlated with each other and become important for players in a supply chain.

Our present model of price competition with price and quality sensitive consumer demand is related to three recent streams of papers in operations management literature. The first group looks at quality maintenance and coordination mechanisms in supply chain management. The second group analyzes competition among channel members within a supply chain and competition among supply chains, with quality improvement strategies. The third group discusses cooperative bargaining framework as applied to operations management stream. In a supply chain, coordination among channel members plays an important role in quality-related decisions.

Quality maintenance and channel coordination: Since last two decades, there is a growing literature on quality management and coordinating contracts in the supply chain. In a game theoretic set-up, Reyniers and Tapiero (1995) investigated the effect of a contract on a supplier's quality level and a buyer's inspection policy in both noncooperative and cooperative settings. They further emphasized the significant impacts of strategic and contractual issues on supply chain quality management. Then, Forker (1997) linked quality management with process optimization to address both effectiveness and efficiency concerns. The study suggested that system performance was affected by transaction-specific investments (asset specificity) in the buyer/supplier relationship that lead to poorer component quality and higher transaction costs. Baiman et al. (2000) analyzed the relationship among product quality, the cost of quality, and the information that can be contracted on. Considering a risk neutral setting, they further investigated how information available for contracting affects the efficiency of a supply chain, in which the supplier is responsible for the quality enhancement and the buyer for quality appraisal. Singer et al.(2003) intended to explain the strategic behavior regarding quality within a supplier-retailer partnership in a disposable product industry. They further derived the conditions under which the supplier and the retailer might devise a mutually beneficial transfer contract that simultaneously increases profit and improves quality. Zhu et al.(2007) considered a buyer who designed a product and owned the brand, yet outsourced production to a supplier. Both the buyer and the supplier incurred quality related costs.They explored the roles of different parties in a supply chain in quality improvement and showed that the buyer's involvement could have a significant impact on profits of both parties,and of the supply chain as a whole. They further investigated how quality-improvement decisions interact with operational decisions such as the buyer's order quantity and the supplier's production lot size. Chao et al. [2009] proposed two contractual agreements by which product recall costs can be shared between a manufacturer and a supplier to induce quality improvement effort. Rong et al. (2011) integrated food quality with production and distribution planning in a food supply chain. Through illustrative case studies they showed (i) how the generic model can be implemented in a specific situation, (2) how the product quality can be modeled on a discrete scale, and (3) what kind of results are obtained from the model. Xie et al. (2011a) investigated quality investment and price decision of a make-to-order (MTO) supply chain with uncertain demand in international trade. They considered the risk-averse behavior of the players in three different supply chain strategies and found that both supply chain strategy and risk-averse behavior have significant impacts on quality investment and pricing. Giovanni (2011) distinguished advertising, pricing and quality improvement strategies in a dynamic setting where demand depends on price as well as the goodwill of the product. Tse and Tan (2012) proposed a multi-tier supply chain in quality risk management framework. They considered the situation of asymmetric information between a manufacturer and a supplier. They argued that better visibility of risk in the supply chain could minimize the threat of product harm.

Quality maintenance and Competition: In all the above studies, the effect of quality maintenance and coordination mechanisms are investigated for the single manufacturer and single retailer supply chain. But depending on different choices of consumer, retailer can buy different qualities product with different prices. Since last two decades, there is an increasing trend in many industries where competition is shifting from price and product quantity to product quality and service in the specific market segment. Early research which included attributes like product quality and service can be found in economics literature such as Spence (1975) and Dixit (1979). Jeuland and Shugan (1983) considered non-price variable such as quality and services in their model with the profit function as a linear function of service amount. Moorthy (1988) examined a competition in duopoly market through both price and quality. Banker et al.(1998) then considered supply chain models of oligopolistic competition to investigate whether equilibrium levels of quality increased in competition intensity. In a similar way of Moorthy (1988), Chamber (2006) considered the impact of variable production costs on competitive behavior in a duopoly where manufacturers compete on both quality and price in a two-stage game. Xie et al. (2011b) considered quality improvement in a given segment of the market, shared by two supplier-manufacturer supply chains which offer a given product at the same price but compete on quality. Recently, Giri et al. (2015) developed a supply chain model where multiple oligopolistic manufacturers compete for both quality and selling price of a product with deterministic demand pattern.

cooperative bargaining: It has been recognized that game theory is an effective tool for the analysis of supply chains. Our Modeling approach in studying the coordinating contracts leads to the use of non-cooperative as well as cooperative bargaining as modeling tools. In recent years, there has been a wide variety of research papers that apply non-cooperative game theory to the field of supply chain management. In noncooperative games, the players choose strategies simultaneously and are thereafter committed to their chosen strategies. For a detailed survey of the existing literature on the applications of non-cooperative games to supply chain management, readers are referred to Cachon and Netessine (2004). On the other hand, the application of cooperative game in supply chain management literature is less prevalent. In the recent past, operations management literature has resorted to using the tool of cooperative bargaining as proposed by Nash(1950,1953) to study the impact of negotiations between players in a supply chain context. For a detailed review of operations management work which applies cooperative bargaining framework, we would like readers to refer to Nagarajan and Sosic (2008). In an earlier work, Kohli and Park(1989) considered a bargaining problem in which a buyer and seller negotiate over the order quantity and average unit price. Among recent literature, Bhaskaran and Krishnan (2009) use the bargaining framework to evaluate its impact on product development under investment sharing and innovation sharing.

In all the papers mentioned above, quality management has considered either in the direction of coordinating the supply chain or in the direction of competing market segment. Recently, Wang and Shin (2015) developed and analyzed supply chain contracts in a supply chain with endogenous upstream innovation. Here they showed that inviting upstream competition among upstream suppliers is desirable since it increases the downstream manufacturer's profit under wholesale as well as quality dependent wholesale price contract. But to the best of our knowledge, the existing literature has not considered product quality improvement with the help of coordination mechanisms in competing market segment with both non-cooperative as well as cooperative game framework. Hence, the purpose of this study is to develop a supply chain model by integrating these three research directions and analyze the model with the desire of product quality improvement in the supply chain. The papers closest in spirit to ours are Choi (1991) and Chakraborty et al. (2015) with price-sensitive demands, where quality and cooperative bargaining issues are not considered. In our present study, we consider a market scenario where two manufacturers produce different but substitutable products (brands) and sell their products to the end consumer through a common retail channel. We especially assume that there is only one retailer in the area. In other words, this market scenario can be interpreted as follows: there is no competition among retailers due to the long distance among them. This may be a strong assumption for some market. However, in our present study, this assumption allows us to focus on competition between two manufacturers. We also assume that the demand faced by the retailer is sensitive to price as well as quality improvement of the product. In our paper, we assume the competition between the players with respect to retail price, only.

In our modeling setup, only the manufacturers incur the cost of quality improvement of products while both the retailer and the manufacturers benefit out of this quality sensitive market demand. Since, the single agent takes decisions for an integrated supply chain (Cachon 2003), for better understanding, we develop integrated channel as a benchmark. We then extend our analysis for the decentralized channel. In our paper, we demonstrate Stackelberg leader-follower game theory where the common retailer acts as the leader and each of the manufacturers as the follower. Our basic model is developed on wholesale price contract. To coordinate the supply chain, we consider a cost-sharing mechanism between the common retailer and each of the manufacturers and, explore the impact of this coordinating mechanism on the optimal decisions of our supply chain players, undertaking the quality improvement issue (Chao et al. 2009, Bhaskaran and Krishnan 2009). Here we consider this cost sharing mechanism under two different scenarios - one where the retailer offers the cost-sharing contract and second, where the players negotiate over the cost-sharing parameters.

Our present study thus aims to answer the following questions for the market scenario where two manufacturers compete with respect to retail price: (1) what is the impact of price as well as quality sensitive consumer demand on the key decisions and profitability

of the supply chain players?, (2) What is the significance of the word "collaboration" in product quality improvement initiatives?, (3) What is the impact of cost sharing contract on the optimal decisions of the supply chain players?, (4) How does the negotiation on the cost sharing parameter between players affect their optimal decisions? The practical application of our model can be found in the greening initiatives by different industries. Here by "quality improvement" it means "product greening improvement level". It is found that over the last several years different global firms are actively pursuing the environmentally friendly activities within their supply chain. Greening initiative, one of those important factors, is undertaking by different firms to capture the green sensitive consumer demand. Large firms often require their suppliers to undertake greening initiative but due to extra cost burden, it becomes challenges to the suppliers to implement those greening initiatives. To overcome this difficulty as well as to make spontaneous participation of the suppliers into greening initiatives, the retail giant, like Wal-Mart has adopted some innovative strategies. For example, in the case of procurement of organic cotton, Wal-mart committed to its Tier-1 suppliers of procuring organic cotton over a five-year period reflecting a long-term sourcing commitment (Plambeck and Denend (2011)). The impact of quality sensitive consumer demand on the optimal decisions of a supply chain players and collaboration among them in quality improvement initiatives are important considerations of our model.
The rest of the paper is organized as follows. The next section describes the notation and assumptions adopted in this paper. Section 3 describes model formulation and analysis. In this section, we address coordinating mechanisms through coordinating contracts to coordinate the proposed supply chain. A comprehensive computational analysis will be carried out in Section 4. Section 5 draws conclusions with managerial explanations and suggests some directions for future investigations.

2. Model description

In our supply chain structure, we consider the business environment where two competing manufacturers $M_i,\ i = 1, 2$ sell their products through a common retailer, denoted by R (refer to Figure 1). The common retailer sells the two competing brands with varying degree of product substitutability. Especially, we consider quality investment and price decision of a supply chain which produces goods against specific orders placed by the retailer. Bellow we will discuss the underlying notations and assumptions of our proposed model.

The following notations are used in developing the proposed model:

i	:	index for product $i = 1, 2$
d_i	:	demand faced by the retailer for ith product
Q_i	:	order quantity for the ith product
α_i	:	initial market size of the ith product
β_i	:	product i's own price sensitivity coefficient
γ_i	:	product i's cross price sensitivity coefficient
δ_i	:	quality sensitivity parameter for ith product
η_i	:	quality investment parameter for ith product
c_i	:	cost of manufacturing of ith product per unit

The decision variables are:

P_i	:	retail price per unit of ith product
w_i	:	wholesale price per unit for the ith product
m_i	:	retailer's margin for ith product
θ_i	:	product quality improvement level, a continuous variable

In our study, the consumer is assumed to be sensitive to both retail price and product quality. Similar to the definition of quality in Banker et al. (1998), in our study, the term "quality" means to refer to both design and conformance quality characteristics of interest to the consumer. Here, two manufacturers compete in the same market segment with respect to retail price, only. Since our goal is to investigate the impact of price discrimination between two products (brands) in a competitive market environment, consumer demand function must adequately reflect the substitutability of the two products. Thus, our basic model uses the following duopoly static demand function that captures the product substitution (Jeuland and Shugan, 1983, Choi (1991,1996), Ingene and Parry, 1995, Xie et al. 2011 b etc):

$$\begin{aligned} d_i &= d_i(P_i, P_j, \theta_i) = \alpha_i - \beta_i P_i + \gamma_i P_j + \delta_i \theta_i, \\ & \quad i = 1, 2 \text{ and } j = 3 - i, \end{aligned} \tag{1}$$

where d_i is the price and quality dependent deterministic demand function of the ith product at retail price P_i given that the price of the competitor brand is P_j. $\alpha_i, \beta_i, \gamma_i, \delta_i$ are defined in notations, earlier. We assume that the initial market size α_i of ith product is large enough so that order quantity Q_i will always be non-negative. It is to be perceived that demand function reflects a 'quality' sensitive consumer market where the demand of ith brand's product is linearly decreasing in its own retail price and, increasing in competitor's retail price and quality improvement level of ith product(savaskan and Van Wassenhove (2006)). The parameters are restricted to satisfy $\alpha_i > 0,\ \beta_i > \gamma_i > 0,\ \delta_i > 0$ for $i = 1, 2$. The restriction $\beta_i > \gamma_i$ is necessary for the demand to be well behaved. The difference $(\beta_i - \gamma_i)$ are inversely proportional to the degree of product substitutability between the two products. This means the closer γ_i is to β_i, the greater is the substitution between two products (i.e., less differentiated), therefore, there is more potential price competition (Choi 1991). Also, each manufacturer has a complete knowledge of the demand conditions of its product.

We have already mentioned that in our basic model both manufacturers are assumed to incur the cost of quality improvement whereas the retailer has no handout in quality improvement of the products though both the retailer and the manufacturers benefit out of this quality sensitive market demand. Further, we assume that quality improvement initiative does not increase the marginal cost of the product produced by each firm. Quality improvement investment is an upfront investment and is a function of quality investment level θ_i of the form $\eta_i \theta_i^\rho$, where η_i is the quality improvement investment parameter for ith product (Banker et al. 1998). In order to model the cost of the quality improvement, we consider $\rho = 2$, which implies that the cost of the quality improvement is increasing and convex with respect to θ_i. This quadratic and increasing cost structure suggests diminishing returns. Under the assumptions, any product quality increasing from, say, 90 to 95 % typically requires a greater additive investment than does increase from 80 to 85 % [Tsay and Agrawal 2000].

It is assumed that the manufacturers are always able to produce the required order quantity $Q_i,\ i = 1, 2$ in time for the start of the selling season. In this case, $Q_i = d_i$, since consumer demand is assumed to be deterministic in our model. The lead times of both products are assumed to be zero. In addition, we assume that $P_i > w_i > c_i$ for $i = 1, 2$. These inequalities assure that the chain will not produce infinite quantities of the product and each firm has a positive profit. Further, P_i, w_i and m_i are related as $P_i = w_i + m_i$.

The basic model is developed with duopoly deterministic demand depending on price and quality improvement level of the products. We first formulate and obtain results for integrated as well as decentralized channels to inspire cost sharing and cooperative contracts frameworks and analyses. All the proofs of the analytical results are given in Appendices.

3. Model formulation and analysis

In this section, we first consider an integrated system in which all the decisions are optimized to maximize the performance of the entire supply chain. The integrated system solution serves as a benchmark for the setting of the decentralized system. Then we consider a decentralized supply chain under wholesale price (WP) contract, where the manufacturers incur the complete cost of quality improvement of the products. Here, we formulate and analyze the equilibrium solutions under game theoretic settings. In our present market segment, two competing manufacturers play horizontal Nash game among themselves and Stackelberg game is played among each manufacturer and the common retailer. Here, we derive the sufficient conditions for the existence of the equilibrium solutions. Then we propose a coordinating mechanism to enhance individual profits as well as total supply chain efficiency. In order to encourage the manufacturers to engage in quality improvement of the products, cost-sharing contract plays an important role. Here we consider cost sharing mechanism under two different scenarios: one where the retailer determines the cost sharing parameter as a Stackelberg leader (Simple cost sharing contract), the second where each manufacturer and the common retailer bargain on the cost-sharing parameter (Cost sharing contract through bargaining). Since, given the prices and quality improvement level, demand is known with certainty, the common retailer chooses order quantities just equal to consumer demand. Again, $\eta_i\theta_i^2$ is the cost of quality improvement of the ith product. Thus, based on the above model assumptions the profits of the retailer and the manufacturers are given as

$$\Pi_R = \sum_{i=1}^{2}(P_i - w_i)d_i \text{ for } j = 3-i, \tag{2}$$

$$\Pi_{M_i} = (w_i - c_i)Q_i - \eta_i\theta_i^2, \text{ for } i = 1,\ 2. \tag{3}$$

3.1. Integrated system

Let us consider an integrated system where a single manufacturer produces two different (brand) products and sells them through its own retail channel. The decision variables of the integrated system are the retail prices (P_i, P_j) and the quality improvement level (θ_i, θ_j) of the two products. If $\Pi_I(P_1, P_2, \theta_1, \theta_2)$ denotes the profit of the integrated system then it can be given as

$$\Pi_I(P_1, P_2, \theta_1, \theta_2) = \sum_{i=1}^{2}\Big[(P_i - c_i)d_i - \eta_i\theta_i^2\Big] \text{ for } j = 3-i, \tag{4}$$

Here the objective is to find the optimal values of the decision variables so that profit of the integrated system is maximized. The following proposition assures the existence of the optimal solutions under some parametric restrictions.

Prposition 1

The objective function is a strictly concave function with respect to its decision variables $(P_1, P_2, \theta_1, \theta_2)$ *under the parametric restrictions* **A1** $4\beta_i\beta_j - (\gamma_i + \gamma_j)^2 \geq (\frac{\beta_i\delta_i\delta_j}{\eta_j} + \frac{\beta_j\delta_i^2}{\eta_i}) > 0$ *for* $j = 3-i,\ i = 1, 2.$

The above proposition implies that there exist unique optimal values of the decision variables for which cost of the integrated system will be maximized. From the first order optimality conditions, the optimal values of the decision variables are derived as $P_i^{I*},\ \theta_i^{I*}$ for $i = 1,\ 2$ (See Appendix B). For briefity of the manuscript those calculations are not included into the manuscript. Now if both products are symmetric in production costs (*i.e.*, $c_1 = c_2 = c$) and if the retailer is symmetric in initial market size of each product (α_i), product's own price sensitivity parameter (β_i), product's competitor price sensetivity coefficient (γ_i) and quality improvement investment parameter (η_i) (*i.e.*, $\alpha_1 = \alpha_2 = \alpha,\ \beta_1 = \beta_2 = \beta,\ \gamma_1 = \gamma_2 = \gamma,\ \eta_1 = \eta_2 = \eta$), then the problem becomes symmetric and it becomes quite tractable to solve (Choi 1991). Table 1 summarizes the optimal solutions under symmetric assumtion.

Table 1: Equlibrium solutions under symmetric assumptions

Vars./Profit	Integrated channel	Decentralized channel
θ^*	$\frac{\delta\left(\alpha-c(\beta-\gamma)\right)}{2\eta\left(2\beta-\frac{\delta^2}{2\eta}-2\gamma\right)}$	$\frac{\delta\left\{\alpha-c(\beta-\gamma)\right\}}{2(4\beta\eta-2\eta\gamma-\delta^2)}$
P^*	$c+\left(\frac{\alpha-c(\beta-\gamma)}{2\beta-\frac{\delta^2}{2\eta}-2\gamma}\right)$	$\frac{\alpha(6\beta\eta-4\eta\gamma-\delta^2)+c(2\beta\eta-\delta^2)(\beta-\gamma)}{2(\beta-\gamma)(4\beta\eta-2\eta\gamma-\delta^2)}$
w^*	-	$\frac{\alpha\eta+c(3\beta\eta-\eta\gamma-\delta^2)}{(4\beta\eta-2\eta\gamma-\delta^2)}$
m^*	-	$\frac{\alpha-c(\beta-\gamma)}{2(\beta-\gamma)}$
d^*	$\frac{(\beta-\gamma)\left(\alpha-c(\beta-\gamma)\right)}{\left(2\beta-\frac{\delta^2}{2\eta}-2\gamma\right)}$	$\frac{\beta\eta\left\{\alpha-c(\beta-\gamma)\right\}}{(4\beta\eta-2\eta\gamma-\delta^2)}$
Π_M^*		$\frac{\eta\left(4\beta\eta-\delta^2\right)\left\{\alpha-c(\beta-\gamma)\right\}^2}{4(4\beta\eta-2\eta\gamma-\delta^2)^2}$
Π_R^*		$\frac{\beta\eta\left\{\alpha-c(\beta-\gamma)\right\}^2}{(\beta-\gamma)(4\beta\eta-2\eta\gamma-\delta^2)}$
Π_{SC}^*	$\frac{2(\beta-\gamma)\left(\alpha-c(\beta-\gamma)\right)^2}{\left(2\beta-\frac{\delta^2}{2\eta}-2\gamma\right)^2}$	$\frac{\eta\left[4\beta\eta(3\beta-2\gamma)-(3\beta-\gamma)\delta^2\right]\left\{\alpha-c(\beta-\gamma)\right\}^2}{2(\beta-\gamma)(4\beta\eta-2\eta\gamma-\delta^2)^2}$

3.2. Decentralized channel

In decentralized system, wholesale price (w_i) and quality improvement level (θ_i) are the decision variables of ith manufacturer and retail prices ($P_1,\ P_2$) are the decision variables of the common retailer. The basic model is developed under wholesale price (WP) contract. We assume the following game structure among the channel members (See Figure 1):

1. Two manufacturers will play horizontal Nash game among themselves whereas the common retailer will play Stackelberg game with each of the manufacturers.

2. As a monopoly retailer in the market, there is no loss of generality if the retailer is assumed to act as a Stackelberg leader. In that case, each manufacturer acts as a Stackelberg follower.

Insert **Figure 1** here

3.2.1. Retailer-Stackelberg game

Under the Retailer-Stackelberg game, the common retailer acts as the leader and the manufacturers the followers. In this market scenario, as a dominant retailer, the common retailer moves first to announce the retail margin ($m_i,\ i = 1, 2$) for both productsive to their respective manufacturers. Then both manufacturers respond by choosing their wholesale prices and quality improvement levels. Again, when manufacturers are the followers then the reaction functions of the manufactuers are obtained from the first order optimality conditions conditional on the retail margin of his own product and retail price of competing product. We thus obtain the ith manufacturer's reaction fuction as functions of retail prices as

$$\begin{aligned}
\theta_i^{WRS}(P_1, P_2) &= \frac{\delta_i(\alpha_i - \beta_i P_i + \gamma_i P_j)}{2\beta_i\eta_i - \delta_i^2}, \\
w_i^{WRS}(P_1, P_2) &= c_i + \frac{\alpha_i - \beta_i P_i + \gamma_i P_j}{(\beta_i - \frac{\delta_i^2}{2\eta_i})}, \\
& \text{for } i = 1, 2,\ j = 3 - i. \qquad (5)
\end{aligned}$$

The nature of the above solution is determined through the folloing Proposition.

Proposition 2

The solution given by (5) is a Nash equilibrium between two manufacturers.
Substituting these reaction functions into retailer's objective function we get the profit of the retailer for the Retailer-Stackelberg game. The following proposition assures the existance of the Retailer-Stackelberg game.

Proposition 3

If the condition **A2:** $\eta_i > \frac{\delta_i^2}{2(\beta_i - \gamma_i)}$, *for* $i = 1, 2$ *then there exists a Stackelberg game between each manufacturer and the common retailer under WP contract with retailer as the Stackelberg leader. Moreover, the Stackelberg equilibrium solutions are* $P_i^{WRS*},\ w_i^{WRS*},\ i = 1, 2$

Following the above game sequence , we can determine the equilibrium solutions for Decentralized channel for Retailer-Stackelberg game under WP contract. Under the symmetric assumption, in this case also we get the similar results as of Integrated system for the equilibrium solutions and those are depicted in Table 1. Choi (1991) established the similar results for the integrated system and the decentralized channel and we derive those further for the quality sensitive product to motivate the following coordination mechanism. From the tabulated values, it can be noted that the optimal wholesale price and retail price will be always positive since $\beta > \gamma > 0$ and under assumption **A2**. It can be checked that under the symmetric assumption if **A2** is satisfied then **A1** is satisfied automatically. Now, under this symmetric assumption, the contribution margin of the manufacturers are

$$w_i^{RS*} - c_i = w^* - c = \frac{\eta\Big\{\alpha - c(\beta - \gamma)\Big\}}{(4\beta\eta - 2\eta\gamma - \delta^2)} \text{ for } i = 1, 2. \qquad (6)$$

Hence, for Retailer-Stackelberg game manufacturers will be profitable always for the non-negative values of this margin. Thus, we get the following condition as an upper bound of the production cost under the assumption **A2**:

$$c \leq \frac{\alpha}{\beta - \gamma}. \tag{7}$$

It can be checked that under symmetric assumption If the assumption **A2** is satisfied then the assumption **A1** is satisfied automatically. Now, for Retailer-Stackelberg game, the retail margin is derived as
$m_i^{RS*} = m^* = P_i^{RS*} - w_i^{RS*} = \frac{\alpha - c(\beta-\gamma)}{2(\beta-\gamma)}$ for $i = 1, 2$.
Now, this retail margin will be non-negative always if production cost satisfies the upper limit as given by (7). Again, under the same restriction demand of the consumer will be always non-negative. Furthermore, the quality improvement level $\theta_i^{RS*} = \theta^* = \frac{\delta\left\{\alpha - c(\beta-\gamma)\right\}}{2(4\beta\eta - 2\eta\gamma - \delta^2)} \geq 0$ always under this production cost restriction and under assumption **A2**. This fact ensures that for non-negative quality improvement level, retailer's and manufacturers' profits will always be positive for Retailer-Stackelberg game. When $\delta_1 = \delta_2 = \delta = 0$, *i.e.,* if consumer sensitivity parameter to quality is assumed to be zero then all the tabulated results for Decentralized channel are converted to Choi's (1991) results for Retailer-Stackelberg game.

3.3. Coordination mechanisms: Cost sharing contract

In this section, we will consider a cost sharing contract between each manufacturer and the common retailer. The main focus is how to design cost sharing contract between each manufacturer and the retailer so that retailer's and manufacturers' profits are maximized in decentralized supply chain. From the retailer's profit maximization perspective, we seek to investigate whether cost-sharing contract is beneficial for the common retailer. The framework of our model is same as discussed earlier for wholesale price contract.
Here, our basic model considers the market scenario where Only manufacturers do the developmental work to improve the quality of the product and quality improvement level (θ_i) is also determined by the corresponding manufacturer. Under this cost-sharing contract, in order to encourage the manufacturers in quality improvement initiatives, the common retailer offers to ith manufacturer to share ϕ_i ($0 \leq \phi_i < 1$) proportion of the total investment due to quality improvement of the ith product. Manufacturers can accept or reject that offer. If the ith manufacturer accepts the offer then retailer shares ϕ_i proportion of the total investment due to quality improvement and the ith manufacturer incurs only $(1 - \phi_i)$ proportion of that investment. Here we will follow the following two mechanisms to determine the cost sharing fraction ϕ_i for $i = 1,\ 2$:(i) Simple cost sharing contract (SCS) and (ii) Cost sharing contract through bargaining (CSB).

3.3.1. Simple cost sharing contract (SCS)

Under this contract, as a Stackelberg leader, the common retailer decides the cost sharing fraction $\phi_i,\ i = 1, 2$. Hence, the sequence of the game is

1 The retailer offers to ith manufacturer to share ϕ_i proportion of the total investment in order to improve the quality.

2 In this market segment, before the start of the selling season, as a Stackelberg leader, the common retailer moves first to announce the retail margin $m_i = P_i - w_i$ for both products to their respective manufacturers. Then the manufacturers respond by choosing the their wholesale prices and the quality improvement levels. Finally, for these respond functions, the retailer decides his retail prices taking cost sharing proportion ϕ_i into consideration.

3 For given retail prices, each manufacturer then decides his wholesale price w_i and quality improvement level θ_i taking cost sharing proportion ϕ_i into consideration.

4 Finally, the retailer decides the cost sharing fraction ϕ_i for given retail prices, wholesale prices and quality improvement levels.

Thus, for this game structure, the profits of the retailer and the ith manufacturer are given by

$$\Pi_R^{SCS} = \sum_{i=1}^{2}\Big[(P_i - w_i)d_i - \phi_i\eta_i\theta_i^2\Big] \tag{8}$$

$$\Pi_{M_i}^{SCS} = (w_i - c_i)d_i - \eta_i(1-\phi_i)\theta_i^2. \tag{9}$$

Since, our main interest is to investigate the case where both manufacturers participate in the cost sharing mechanism, in the following we consider that market scenario where both manufacturers accept the cost-sharing contract. We later show why both manufacturers accept this contract.

3.4.1.1 Retailer-Stackelberg game under SCS contract:

Following the above game sequence, reaction functions of ith manufacturer is obtained as

$$\theta_i^{SCS}(P_1, P_2, \phi_1, \phi_2) = \frac{\delta_i(\alpha_i - \beta_i P_i + \gamma_i P_j)}{\Big(2\beta_i\eta_i(1-\phi_i) - \delta_i^2\Big)}, \tag{10}$$

$$w_i^{SCS}(P_1, P_2, \phi_1, \phi_2) = c_i + \frac{\alpha_i - \beta_i P_i + \gamma_i P_j}{\Big(\beta_i - \frac{\delta_i^2}{2\eta_i(1-\phi_i)}\Big)}, \tag{11}$$

for $i = 1,\ 2$ and $j = 3 - i$. Substituting these reaction functions into retailer profit functions we get retailer objective function as a function of retail prices and cost sharing fractions and get the following proposition.

Proposition 4

For a given cost sharing fractions ϕ_1 and ϕ_2, there exists a Stackelberg game between each manufacturer and the common retailer under SCS contract with retailer as the Stackelberg leader. Moreover, the Retailer-Stackelberg equilibrium solutions are $P_i^{SCS^*}$, $w_i^{SCS^*}$ for $i = 1,\ 2$.

Substituting the above values into retailer's profit function, we get retailer's profit as function of cost sharing fractions ϕ_1 and ϕ_2 only. Next objective of the retailer as a Stackelberg leader is to obtain the optimal values of ϕ_1, ϕ_2 for which his profit Π_R^{SCS} is maximized. But due to the complexity of the above solutions, let us consider our simple cost-sharing contract under symmetric assumption. The corresponding Retailer-Stackelberg solutions for given $\phi_1 = \phi_2 = \phi$ is tabulated in Table 2. When $\phi = 0$, then simple cost-sharing contract converted into wholesale price contract. That is why, when $\phi = 0$ all results of Table 2 coincide with all results of Table 1 under Decentralized channel. Hence, under homogeneous assumption, the retailer's optimization problem takes the form

$$\max_{\phi} \Pi_R^{SCS^*}(\phi) \text{ where}$$

$$\Pi_R^{SCS*} = \frac{2(1-\phi)^2\beta^2\eta\Big\{\alpha - c(\beta-\gamma)\Big\}^2}{(\beta-\gamma)\zeta}, \tag{12}$$

where $\zeta = \Big[4\beta\eta(2\beta-\gamma)(1-\phi)^2 - 2\beta(1-\phi)\delta^2 + (\beta-\gamma)\phi\delta^2\Big]$.

Table 2: For given ϕ equilibrium solutions with homogeneous assumptions under cost sharing contract

Vars./Profits	Decentralized channel
θ^{SCS^*}	$\frac{\delta\beta(1-\phi)\Big\{\alpha-c(\beta-\gamma)\Big\}}{\Big[4\beta\eta(2\beta-\gamma)(1-\phi)^2-2\beta(1-\phi)\delta^2+(\beta-\gamma)\phi\delta^2\Big]}$
P^{SCS^*}	$\frac{\alpha\Big\{2\beta\eta(3\beta-2\gamma)(1-\phi)^2-\beta\delta^2(1-\phi)+(\beta-\gamma)\phi\delta^2\Big\}+\beta c(\beta-\gamma)(1-\phi)\Big(2\beta\eta(1-\phi)-\delta^2\Big)}{(\beta-\gamma)\Big[4\beta\eta(2\beta-\gamma)(1-\phi)^2-2\beta(1-\phi)\delta^2+(\beta-\gamma)\phi\delta^2\Big]}$
w^{SCS^*}	$\frac{2\alpha\beta\eta(1-\phi)^2+c\Big\{2\beta\eta(1-\phi)^2(3\beta-\gamma)-2\beta(1-\phi)\delta^2+(\beta-\gamma)\phi\delta^2\Big\}}{\Big[4\beta\eta(2\beta-\gamma)(1-\phi)^2-2\beta(1-\phi)\delta^2+(\beta-\gamma)\phi\delta^2\Big]}$
m^{SCS^*}	$\frac{\Big\{\alpha-c(\beta-\gamma)\Big\}\Big\{2\beta\eta(1-\phi)^2(2\beta-\gamma)-\beta(1-\phi)\delta^2+(\beta-\gamma)\phi\delta^2\Big\}}{(\beta-\gamma)\Big[4\beta\eta(2\beta-\gamma)(1-\phi)^2-2\beta(1-\phi)\delta^2+(\beta-\gamma)\phi\delta^2\Big]}$
d^{SCS^*}	$\frac{2\beta^2\eta(1-\phi)^2\Big\{\alpha-c(\beta-\gamma)\Big\}}{\Big[4\beta\eta(2\beta-\gamma)(1-\phi)^2-2\beta(1-\phi)\delta^2+(\beta-\gamma)\phi\delta^2\Big]}$
$\Pi_M^{SCS^*}$	$\frac{\beta^2\eta(1-\phi)^3\Big\{\alpha-c(\beta-\gamma)\Big\}^2\Big(4\beta\eta(1-\phi)-\delta^2\Big)}{\Big[4\beta\eta(2\beta-\gamma)(1-\phi)^2-2\beta(1-\phi)\delta^2+(\beta-\gamma)\phi\delta^2\Big]^2}$
$\Pi_R^{SCS^*}$	$\frac{2(1-\phi)^2\beta^2\eta\Big\{\alpha-c(\beta-\gamma)\Big\}^2}{(\beta-\gamma)\Big[4\beta\eta(2\beta-\gamma)(1-\phi)^2-2\beta(1-\phi)\delta^2+(\beta-\gamma)\phi\delta^2\Big]}$
$\Pi_d^{SCS^*}$	$\frac{2\Big\{\alpha-c(\beta-\gamma)\Big\}^2\beta^2\eta(1-\phi)^2\Big\{4\beta\eta(1-\phi)^2(3\beta-2\gamma)-(1-\phi)(3\beta-\gamma)\delta^2+(\beta-\gamma)\phi\delta^2\Big\}}{(\beta-\gamma)\Big[4\beta\eta(2\beta-\gamma)(1-\phi)^2-2\beta(1-\phi)\delta^2+(\beta-\gamma)\phi\delta^2\Big]^2}$

Proposition 5

Under symmetric assumption, there exists an optimal solution of ϕ for which retailer's profit under SCS contract is maximized and the optimal value of ϕ is given by $\phi_{Opt}^{SCS^*} = \frac{\beta+\gamma}{(3\beta-\gamma)}$.

Substituting the optimal value of ϕ by $\frac{\beta+\gamma}{(3\beta-\gamma)}$ into the results of Table 2, we get equlibrium solutions under SCS contract and those are given in Table 3. From this tabulated values, it can be verified that under assmption **A3:** $\eta > \frac{\delta^2(3\beta-\gamma)^2}{8\beta(\beta-\gamma)(2\beta-\gamma)}$ and production cost restriction (7), equilibrium wholesale price $w_{Opt}^{SCS^*}$ and retail price $P_{Opt}^{SCS^*}$ are always positive under SCS contract. It is easy to verify that under sysmmetric assumption, the parametric restioction **A3** dominates both restrictions **A1** and **A2**. Now the contribution margin of each manufacturer is given by

$$\begin{aligned} w_{Opt}^{SCS^*} - c &= \frac{8\beta\eta(\beta-\gamma)\Big\{\alpha - c(\beta-\gamma)\Big\}}{\Big[16\beta\eta(\beta-\gamma)(2\beta-\gamma) - (3\beta-\gamma)^2\delta^2} \\ &\geq 0, \end{aligned}$$

under A3 and the production cost resriction (7). Hence, for Retailer-Stackelberg game under SCS contract, each manufacturer will be profitable always due to this contribution margin of each manufacturer. Again, under the same restrictions, retailer margin $m_{Opt}^{SCS^*}$ is always non-negative (see Table 3). Further, for these restrictions, quality improvement level $\theta_{Opt}^{SCS^*}$ is always non-negative. *Hence, for non-negative quality improvement level, the retailer's and the manufacturers' profits are always positive under SCS contract for the Retailer-Stackelberg game.*

Table 3: Equilibrium solutions with homogeneous assumptions under SCS contract

Vars./Profits	Decentralized channel
$\phi_{Opt}^{SCS^*}$	$\frac{\beta+\gamma}{3\beta-\gamma}$
$\theta_{Opt}^{SCS^*}$	$\frac{2\delta\beta(3\beta-\gamma)\Big\{\alpha-c(\beta-\gamma)\Big\}}{\Big[16\beta\eta(\beta-\gamma)(2\beta-\gamma)-(3\beta-\gamma)^2\delta^2\Big]}$
$P_{Opt}^{SCS^*}$	$\frac{\alpha\Big\{8\beta\eta(3\beta-2\gamma)-(3\beta-\gamma)\delta^2\Big\}+2\beta c\Big\{4\beta\eta(\beta-\gamma)-(3\beta-\gamma)\delta^2\Big\}}{\Big[16\beta\eta(\beta-\gamma)(2\beta-\gamma)-(3\beta-\gamma)^2\delta^2\Big]}$
$w_{Opt}^{SCS^*}$	$\frac{8\alpha\beta\eta(\beta-\gamma)+c(3\beta-\gamma)\Big\{8\beta\eta(\beta-\gamma)-(3\beta-\gamma)\delta^2\Big\}}{\Big[16\beta\eta(\beta-\gamma)(2\beta-\gamma)-(3\beta-\gamma)^2\delta^2\Big]}$
$m_{Opt}^{SCS^*}$	$\frac{\Big\{\alpha-c(\beta-\gamma)\Big\}\Big\{8\beta\eta(2\beta-\gamma)-(3\beta-\gamma)\delta^2\Big\}}{\Big[16\beta\eta(\beta-\gamma)(2\beta-\gamma)-(3\beta-\gamma)^2\delta^2\Big]}$
$d_{Opt}^{SCS^*}$	$\frac{8\beta^2\eta(\beta-\gamma)\Big\{\alpha-c(\beta-\gamma)\Big\}}{\Big[16\beta\eta(\beta-\gamma)(2\beta-\gamma)-(3\beta-\gamma)^2\delta^2\Big]}$
$\Pi_{M_{Opt}}^{SCS^*}$	$\frac{8\beta^2\eta(\beta-\gamma)\Big\{\alpha-c(\beta-\gamma)\Big\}^2\Big\{8\beta\eta(\beta-\gamma)-(3\beta-\gamma)\delta^2\Big\}}{\Big[16\beta\eta(\beta-\gamma)(2\beta-\gamma)-(3\beta-\gamma)^2\delta^2\Big]^2}$
$\Pi_{R_{Opt}}^{SCS^*}$	$\frac{8\beta^2\eta\Big\{\alpha-c(\beta-\gamma)\Big\}^2}{\Big[16\beta\eta(\beta-\gamma)(2\beta-\gamma)-(3\beta-\gamma)^2\delta^2\Big]}$
$\Pi_{d_{Opt}}^{SCS^*}$	$\frac{8\beta^2\eta\Big\{\alpha-c(\beta-\gamma)\Big\}^2\Big\{16\beta\eta(\beta-\gamma)(3\beta-2\gamma)-(3\beta-\gamma)(5\beta-3\gamma)\delta^2\Big\}}{\Big[16\beta\eta(\beta-\gamma)(2\beta-\gamma)-(3\beta-\gamma)^2\delta^2\Big]^2}$

Proposition 6

The SCS parameter $\phi_{Opt}^{SCS^*}$ increases with the increase of competition between two products. The above proposition indicates that when manufacturers are more competitive, the retailer offers a higher proportion of cost share. Thus, the cost sharing decision of the retailer is influenced by the degree of competition between the manufacturers.

Proposition 7

The equilibrium values of unit prices and quality improvement level in SCS contract are in the order as mentioned bellow in comparison to the decentralized supply chain values in WP contract under assumptions **A2***,* **A3** *and production cost restriction as given in equation (7):* $\theta_{Opt}^{SCS^*} > \theta^{WRS^*}$, $w_{Opt}^{SCS^*} \geq w^{WRS^*}$, $P_{Opt}^{SCS^*} \geq P^{WRS^*}$.

The above relations are obtained through algebraic comparison of the equilibrium values as tabulated in Tables 1 and 3. Proposition 6 is interesting in that the results show that under SCS contract, product quality improvement level is higher than that of decentralized supply chain under WP contract. So, SCS contract is advantageous from a product quality improvement prospect. However, this higher quality improvement level also results in higher wholesale price and higher retail price of the product. Thus, from consumer's point of view, higher quality product results in the higher price to purchase the product. Thus, each of the manufacturers and the retailer would be interested in participating in SCS contract only when this contract results in more profits than that of Dencentralized channel under WP contract. The impact of the cost-sharing contract on profitability is depicted in the following proposition.

Proposition 8

The equilibrium values of profits in SCS contract are in the order as mentioned bellow in comparison to Decentralized channel under WP contracts with assumptions **A2***,* **A3***:* $\Pi_{M_{Opt}}^{SCS^*} > \Pi_{M}^{WRS^*}$, $\Pi_{R_{Opt}}^{SCS^*} \geq \Pi_{R}^{WRS^*}$ *and* $\Pi_{d_{Opt}}^{SCS^*} \geq \Pi_{d}^{WRS^*}$

The above relations are obtained through algebraic comparison of the equilibrium values as tabulated in Tables 1 and 3. Proposition 7 is significant in the sense that the results show that both the retailer and the manufacturers earn higher profits in SCS contract than that of Decentralized channel under WP contract. Thus, cost sharing with the retailer, helps each of the manufacturers to earn a higher profit. Due to this sharing the quality improvement cost with the retailer, quality improvement cost are lowered for the manufacturers, the manufacturers will also be able to provide a higher quality improvement in the products. Further, though the retailer shares a portion of the quality improvement costs with the manufacturers, the retailer incurs higher profits in this contract than that of Decentralized channel under WP contract. This is the underlying reason of why does the retailer prefer to offer SCS contract rather than WP contract. In practice, product quality improvement is applicable to those market segments where consumers are quality sensitive and are ready to pay a higher price to get the higher quality product in return.

3.3.2. Cost sharing contract through bargaining (CSB)

In this section, let us consider a cost-sharing contract where the cost sharing parameter is determined through bargaining between each manufacturer and the retailer. Our proposed bargaining game is based on the Nash bargaining process as proposed by John Nash (1950, 1953). The first three steps of CSB case are identical with those of SCS contract. The only difference between two contracts lies in step 4. In step 4, given retail prices, wholesale prices and quality improvement levels, the cost sharing fraction is determined from the Nash bargaining optimization problem.

Under the given assumptions, the profit functions of the retailer and the *ith* manufacturer is given by

$$\Pi_{R_P}^{CSB}(\phi_i^{CSB}) = \sum_{i=1}^{2}\Big[(P_i - w_i)d_i - \phi_i^{CSB}\eta_i\theta_i^2\Big] \tag{13}$$
$$\Pi_{M_{i_P}}^{CSB}(\phi_i^{CSB}) = (w_i - c_i)d_i - \eta_i(1-\phi_i^{CSB})\theta_i^2. \tag{14}$$

Now, we have Π_R^{WRS*} and $\Pi_{M_i}^{WRS*}$ as the optimal profits of the retailer and the ith manufacturer for Retailer-Stackelberg game of Decentralized channel under WP contract, respectively. Let $\Pi_{R_P}^{CSB}$ and $\Pi_{M_{i_P}}^{CSB}$ be the Pareto improved profits of the retailer and the ith manufacturer, respectively. We have from Decentralized channel under WP contract, $\Pi_d^{WRS*} = \Pi_R^{WRS*} + \sum_{i=1}^{2}\Pi_{M_i}^{WRS*}$. Now, let $\Pi_{d_P}^{CSB*} = \Pi_{R_P}^{CSB*} + \sum_{i=1}^{2}\Pi_{M_{i_P}}^{CSB*}$. Then Π_d^{WRS*} and $\Pi_{d_P}^{CSB*}$ are the optimal channel profit under WP contract and under cost sharing through bargaining (CSB) contract, respectively. If Π_I^* denotes the optimal profit of the integrated system, then from the definition of the Pareto improvement, we have $\Pi_{R_P}^{CSB*} \geq \Pi_R^{WRS*}$, $\Pi_{M_{i_P}}^{CSB*} \geq \Pi_{M_i}^{WRS*}$ for $i = 1, 2$. In addition we further have $\Pi_d^{WRS*} \leq \Pi_{d_P}^{CSB*} \leq \Pi_I^*$.
It is to be noted that Nash bargaining game is considered at the final step (i.e., at Step 4). Hence, we can define the decision set κ of Pareto improvement as

$$\begin{aligned}\kappa \;=\; & \Big\{\Big(\phi_1^{CSB},\ \phi_2^{CSB}\Big) : \Pi_{R_P}^{CSB*} \geq \Pi_R^{WRS*}, \\ & \Pi_{M_{i_P}}^{CSB*} \geq \Pi_{M_i}^{WRS*} \text{ for } i = 1, 2\Big\}\end{aligned} \tag{15}$$

Then the optimization problem of the Nash (1950, 1953) bargaining game takes the form

$$\max_{\left(\phi_1^{CSB},\ \phi_2^{CSB}\right)\in\kappa} \sum_{i=1}^{2}\Pi_{B_i}\ where$$
$$\Pi_{B_i} = \Big(\Pi_{R_P}^{CSB*} - \Pi_R^{WRS*}\Big)\Big(\Pi_{M_{i_P}}^{CSB*} - \Pi_{M_i}^{WRS*}\Big). \tag{16}$$

Due to the complicated form of the objective function, it is troublesome to derive a closed-form equilibrium solutions of ϕ_1^{CSB*}, ϕ_2^{CSB*} under CSB contract. Hence, as an alternative, to obtain the equilibrium solutions, we carry out a comprehensive numerical study for the non-linear constraint optimization problem.

4. Computational analysis

In the previous section. we derived some analytical results that provide some significant features of our proposed model under Retailer-Stackelberg game. In this section, we perform an extensive numerical study to explain some of those analytical results. Moreover, through this computational approach, we investigate the impacts of quality investment parameter (η), quality sensitivity parameter δ and competition level between the manufacturers ($\beta-\gamma$) on the decision variables and compare and contract optimal solutions under different scenarios.

Insert **Figures 2 to 10** here

For this experimental study, the following parametric values are assumed: $c_1 = 1$; $c_2 = 1$; $\alpha_1 = 1000$; $\alpha_2 = 1000$; $\beta_1 = 45$; $\beta_2 = 45$; $\gamma_1 = 30$; $\gamma_2 = 30$; $\delta_1 = 10$; $\delta_2 = 10$; $\eta_1 = 9$; $\eta_2 = 9$. Production costs always satisfy the restrictions $c_i \leq \alpha_i/(\beta_i - \gamma_i),\ i = 1, 2$ as given by equation (7). Since, under symmetric assumption, the parametric restriction **A3** dominates both **A1, A2**, lower limit of quality improvement investment parameter (η) is obtained from $\eta > \frac{\delta^2(3\beta-\gamma)^2}{8\beta(\beta-\gamma)(2\beta-\gamma)}$ and quality sensitivity parameter must satisfy the range $\delta^2 > \frac{8\beta\eta(\beta-\gamma)(2\beta-\gamma)}{(3\beta-\gamma)^2}$. Through experiment it is found that cost sharing contract can coordinate the supply chain if $15 \leq \beta - \gamma \leq 27$ for our chosen other parametic values. The obtained result is difficult to interprate if $(\beta - \gamma) \leq 15$. Moreover, it is also found that cost sharing contract can coordinate the whole supply chain if the difference between the quality sensitivity parameters of the two products satisfy the range $0 \leq |\delta_1 - \delta_2| \leq 0.71$. All these parametric restrictions ensure us that we work within the feasible region.

4.1. Impact of quality improvement investment:

The variations of the optimal solutions with respect to the quality improment investment (η) are depicted in figures 2 to 7. For our chosen parameters, feasible solutions exist for $\eta > 3.40278$ which is obtained from the assumption **A3** under symmetric assumtion. From the figures, it can be seen that quality improvement investment has negative impact on all the decision variables as well as on optimal profits of the supply chain. Especially, quality improvement levels of the products decrease with the increase of quality improvement investment. That means, for higher quality imprevement cost, optimal quality imprevement level decerases. This is the reason why manufacturers struggle to improve the quality of the products for higher quality improvement cost (Figure 2). Further, the optimal quality improvement level is highest for Integrated system and lowest for Decentralized channel under WP contract. Analytically, we have already shown that the optimal quality improveent level in Simple cost sharing (SCS) contract is higher than that of Decentralized channel under WP contract (Proposition 7). Further, from figures we see that the retailer can get better quality product for Cost sharing through bargaining contract (CSB) case rather than SCS contract case where the common retailer as the Stackelberg leader optimizes the cost sharing fraction. Thus, we get $\theta^{I^*} > \theta^{CSB^*} > \theta^{SCS^*} > \theta^{WRS^*}$. This result indicates that from better quality perspective, negotiation during cost sharing contract is beneficial than that of SCS contract. However, the retailer earns highest profit in SCS case than the cases of CSB and Decentralized system under WP contract, due to the higher retail price in SCS contract where being Stackelberg leader, the retailer optimizes the cost sharing fractions (Figures 3 and 4). On the other hand, in CSB contract, the manufacturers able to produce higher quality product, the manufactures charge higher wholesale price for those products (Figure 5). Subsequently, the manufacturers earns highest profit in CSB case (Figure 6). Further, from the channel profits purspective. channel profit is highest in integrated system, followed by CSB case, followed by SCS case, followed by Dentralized case under WP contract.

Analytically, we have already established that SCS contract can coordinate the proposed supply chain (Proposition 8). Moreover, from figures (Figures 4, 6, 7) we observe that both types of cost sharing contract (SCS and CSB) can coordinate our proposed supply chain. But, as a Stackelberg leader, the retailer would prefer that cost sharing contract which is most profitable to him. Thus, ideally, the retailer would like to participate in SCS contract where he individually can decide the cost sharing fractions.

4.2. Impact of quality sensitive parameter:

In the following, we investigate the impact of the quality sensitivity parameter (δ) to the quality improvement level. For our above chosen parameters, feasible solutions exist for $0 \leq \delta < 15$. Variation of the quality improvement level with respect to quality sensitivity parameter is depicted in Figure 8. From the figure, it can be noticed that quality sensitivity parameter has a positive impact on the quality improvement level. Hence, the manufacturers would like to take the quality improvement initiation for those market segments where consumer are quality sensitive. Moreover, quality improvement level is highest in the integrated system, followed by CSB case followed by SCS case, followed by Decentralized case under WP contract.Thus for the quality conscious consumer market, the highest quality product can be obtained in the integrated system and the lowest quality product is obtained in the case of Decentralised channel under WP contract. The impact of the quality sensitive parameter on the channel profits as well as the individual profits is also positive which can be verified numerically as shown in the previous case. Hence, the quality conscious consumer can encourage the manufacturers to undertake quality improvement initiatives.

4.3. Impact of competition factor:

The degree of competition ($\beta - \gamma$) between two manufacturers has a significant impact on the quality improvement level of the product(Figures 9, 10). The better quality product is obtained when manufacturers are more competitive. Thus, with the increase of ($\beta - \gamma$), i.e., with the decrease of competition between manufacturers, quality improvement level decreases (Figure 9). This implies that the product quality improvement level increases with the increase competition between the manufacturers. Moreover, it can be easily verified that the competition between the manufacturers has positive impacts on the supply chain profits. Further, Figure 10 indicates that the retailer's cost sharing fraction increases with the increase of competition between manufacturers(Figure 10) which supports our analytical result as stated in Proposition 6.

Thus, our computational analysis acknowledges that cost sharing contract (whether it is the case where the retailer decides the cost sharing fraction or the case where cost sharing fraction is obtained through negotiation) is enabled to produce a better quality product in the supply chain. Moreover, the cost-sharing contract is able to produce a higher surplus for all the channel members as well as for the whole supply chain in compare with Decentralized system under WP contract. Hence, the cost-sharing contract is beneficial for both profit and products' quality improvement perspective. Further, this analysis reveals that competition between the manufacturers has positive impacts on the product quality as well as on the profits of the supply chain. Hence, competition among manufacturers can induce the better quality of the products.

5. Conclusions

In the recent years, quality of the product becomes a significant factor on consumers purchasing decisions and hence product quality and quality improvement efforts are being increasingly studies globally. Hence, our primary objective is to study the impact of quality improvement efforts and pricing strategies, simultaneously. Using game theoretic approach, our analytical, as well as computational study, find a number of insights into economic behavior of the channel members of the proposed supply chain.

Our study reveals that competition among the manufacturers is profitable for both profit and products' quality improvement perspective. Further, the cost-sharing mechanism is

beneficial to all the channel members as well as to the whole supply chain. This contract is able to increase not only the quality improvement level of the products but also higher surplus for all the channel members as well as for the whole supply chain. Moreover, higher quality of the product, as well as higher profits, are obtained in that cost-sharing contract where the cost sharing fraction is obtained through negotiation (CSB case) than that the contract case where cost sharing parameter is determined by the common retailer (SCS case). But the common retailer would not be encouraged to participate in bargaining on the cost sharing fraction since bargaining leads to lower profit of the retailer than that the case where he individually determines the cost sharing fraction.
We now explore the limitations of our present study and possible extensions for future research in this research area. In our article, we consider the proposed model under deterministic demand. An obvious extension would be to consider the stochastic demand with an additive as well as with a multiplicative demand shock. A dual channel at the retailer place or e-market selling may be considered for future research. Further, throughout our study, we consider supply chain with monopoly retail channel. Although single retail market segment is common in literature, horizontal competition among multiple retailers would definitely affect the supply chain dynamics in addition to horizontal competition among manufacturers.

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Appendix A

Proof of Proposition 1

Since, profit functions are continuous and twice differentiable, to establish the strictly concavity of the objective function as given by (4) with respect to its decision variables $P_1, P_2, \theta_1, \theta_2$, it is sufficient to show that objective function is negative definite. Thus, we have to show that principal minors of the hesiisan matrix H of the objective function are alternatively, (-)ve, (+)ve and (-)ve, in order i.e., $D_1^I(P_1, P_2, \theta_1, \theta_2) < 0$, $D_2^I(P_1, P_2, \theta_1, \theta_2) > 0$, $D_3^I(P_1, P_2, \theta_1, \theta_2) < 0$ and $D_4^I(P_1, P_2, \theta_1, \theta_2) > 0$, respectively where $D_r^I(P_1, P_2, \theta_1, \theta_2)$ denotes the principal minor of the hessian H of rth order, $r = 1, 2, 3, 4$.Now

$$\begin{aligned}
D_1^I &= -2\beta_1 < 0, \\
D_2^I &= 4\beta_1\beta_2 - (\gamma_1 + \gamma_2)^2 > 0, \text{ under } \mathbf{A1} \\
D_3^I &= -2\eta_1\Big\{4\beta_1\beta_2 - (\gamma_1 + \gamma_2)^2\Big\} + 2\beta_2\delta_1\delta_2 \\
&\leq -\frac{2\eta_1\beta_1\delta_2^2}{\eta_2}, \text{ under assumption } \mathbf{A1} \\
&< 0 \text{ and} \\
D_4^I &= \det H = -4\beta_1\eta_1\delta_1\delta_2 + \delta_1^2\delta_2^2 - 4\delta_1^2\beta_2\eta_2 + 4\eta_1\eta_2\Big\{4\beta_1\beta_2 - (\gamma_1 + \gamma_2)^2\Big\} \\
&\geq 4\eta_1\eta_2\Big\{\frac{\beta_1\delta_1\delta_2}{\eta_2} + \frac{\beta_2\delta_1^2}{\eta_1}\Big\} - 4\beta_1\eta_1\delta_1\delta_2 + \delta_1^2\delta_2^2 - 4\delta_1^2\beta_2\eta_2 \text{ under } \mathbf{A1} \\
&= \delta_1^2\delta_2^2 \\
&> 0.
\end{aligned}$$

This completes the proof of the proposition.

Appendix B

Optimal solutions of the retail prices and the quality improvement levels of the products

$$
\begin{aligned}
P_i^{I*} &= \frac{1}{\Delta_1}\Big[(\gamma_i+\gamma_j)\Big\{\alpha_j-\gamma_i c_i+\Big(\beta_j-\frac{\delta_j^2}{2\eta_j}\Big)c_j\Big\}+\Big(2\beta_j-\frac{\delta_j^2}{2\eta_j}\Big)\Big\{\alpha_i-\gamma_j c_j+\Big(\beta_i-\frac{\delta_i^2}{2\eta_i}\Big)c_i\Big\}\Big],\\
\theta_i^{I*} &= \frac{\Big(P_i^{I*}-c_i\Big)\delta_i}{2\eta_i},\ \text{where}\Delta_1=\Big(2\beta_i-\frac{\delta_i^2}{2\eta_i}\Big)\Big(2\beta_j-\frac{\delta_j^2}{2\eta_j}\Big)-(\gamma_i+\gamma_j)^2\text{for } i=1,2,\ j=3-i.
\end{aligned}
$$

However, uder assumption **A1**, the value of Δ_1 is always positive. This can be verified easily.

Appendix C

Proof of Proposition 2

As profit functions are continuous and twice differentiable, in order to prove that the solutions given by (5) are Nash equilibrium between two manufacturers, we first show that profit function of each manufacturer is concave with respect to its decision variables. Then, each manufacturer's two dimensional strategy space can be converted into one dimensional strartegy space, only. Differentiating equation (3) partially with respect to w_i and θ_i for $i=1,2$, we get $\frac{\partial^2\Pi_{M_i}}{\partial\theta_i^2}=-2\eta_i<0$, $\frac{\partial^2\Pi_{M_i}}{\partial w_i^2}=-2\beta_i<0$ and $\frac{\partial^2\Pi_{M_i}}{\partial\theta_i\partial w_i}=\frac{\partial^2\Pi_{M_i}}{\partial w_i\partial\theta_i}=\delta_i$. These give $\frac{\partial^2\Pi_{M_i}}{\partial w_i^2}\frac{\partial^2\Pi_{M_i}}{\partial\theta_i^2}-(\frac{\partial^2\Pi_{M_i}}{\partial w_i\partial\theta_i})^2=4\beta_i\eta_i-\delta_i^2>0$ under assumption **A2**. Thus, hessian matrix of each manufacturer is negetive definite. Hence, each manufacturer's profit function is jointly concave with respect to $w_i,\ \theta_i$ for $i=1,\ 2$. Now, from the first order condition we get

$$\theta_i(w_i)=\frac{\delta_i(w_i-c_i)}{2\eta_i}.$$

Replacing θ_i by the above expression, we get the profit function of ith manufacturer as

$$\Pi_{M_i} \;=\; (w_i-c_i)\Big(\alpha_i-\beta_i P_i+\gamma_i P_j+\frac{\delta_i^2(w_i-c_i)}{2\eta_i}\Big)-\frac{\eta_i\delta_i^2(w_i-c_i)^2}{(2\eta_i)^2},\ \text{for } i=1,2.$$

From the above functions, we get second order jacobian matrix of two manufacturers as

$$J=\begin{bmatrix}\frac{\partial^2\Pi_{M_1}}{\partial w_1^2} & \frac{\partial^2\Pi_{M_1}}{\partial w_1\partial w_2}\\ \frac{\partial^2\Pi_{M_2}}{\partial w_2\partial w_1} & \frac{\partial^2\Pi_{M_2}}{\partial w_2^2}\end{bmatrix}=\begin{bmatrix}-2\beta_1 & \gamma_1\\ \gamma_2 & -2\beta_2\end{bmatrix}$$

which is negative definite under assumtion **A1** (Choi, 1991). This assures that the solutions given by (5) are Nash equlibrium between two manufacturers. This completes the proof.

Appendix D

Proof of Proposition 3

Substituting the reaction functions of the manufacturers as given by equation (5) into retailer's objective function we get retailer's profit function in Retailer-Stackelberg game as

$$\Pi_R^{RS} = \sum_{i=1}^{2}\Big(P_i - w_i^{RS}(P_1,P_2)\Big)\Big\{\alpha_i - \beta_i P_i + \gamma_i P_j + \delta_i\theta_i^{RS}(P_1,P_2)\Big\} \text{ for } j = 3-i.$$

Now, in order to prove the existance of the Retailer-Stackelberg game, it is sufficient to show that $\Pi_R^{RS}(P_1,P_2)$ is quasi-concave with respect to its decision variables $P_1,\ P_2$. If $D_r^{RS}(P_1,P_2)$ denotes the determinants of the boardered hessian matrix of rth order as before, then the necessary and sufficient condition for the quasi-concavity are $D_1^{RS}(P_1,P_2)\le 0,\ D_2^{RS}(P_1,P_2)\ge 0$ and $D_1^{RS}(P_1,P_2)<0,\ D_2^{RS}(P_1,P_2)>0$, respectively. Now

$$\begin{aligned}
D_1^{RS} &= -\Big(\frac{\partial\Pi_R^{RS}(P_1,\ P_2)}{\partial P_1}\Big)^2 < 0, \text{ always and}\\
D_2^{RS} &= \frac{\partial\Pi_R^{RS}}{\partial P_1}\Big(\frac{\partial^2\Pi_R^{RS}}{\partial P_1\partial P_2}\frac{\partial\Pi_R^{RS}}{\partial P_2} - \frac{\partial\Pi_R^{RS}}{\partial P_1}\frac{\partial^2\Pi_R^{RS}}{\partial P_1\partial P_2}\Big)\\
&+ \frac{\partial\Pi_R^{RS}}{\partial P_2}\Big(\frac{\partial^2\Pi_R^{RS}}{\partial P_1\partial P_2}\frac{\partial\Pi_R^{RS}}{\partial P_1} - \frac{\partial\Pi_R^{RS}}{\partial P_2}\frac{\partial^2\Pi_R^{RS}}{\partial P_1^2}\Big).
\end{aligned}$$

Due to the symmetric nature of the consumer's demand function, $\frac{\partial\Pi_R^{RS}}{\partial P_1}$ and $\frac{\partial\Pi_R^{RS}}{\partial P_2}$ must be of same sign. So, $\frac{\partial\Pi_R^{RS}}{\partial P_1}\frac{\partial\Pi_R^{RS}}{\partial P_2}$ must be greater than zero always. Hence, in order to show that $D_2^{RS}(P_1,P_2)>0$, it is sufficient to show that $\frac{\partial^2\Pi_R^{RS}}{\partial P_1\partial P_2}>0$, $\frac{\partial^2\Pi_R^{RS}}{\partial P_1^2}<0$ and $\frac{\partial^2\Pi_R^{RS}}{\partial P_2^2}<0$. Now,

$$\begin{aligned}
\frac{\partial^2\Pi_R^{RS}(P_1,P_2)}{\partial P_1^2} &< 0,\\
\frac{\partial^2\Pi_R^{RS}(P_1,\ P_2)}{\partial P_2^2} &= -\frac{4\beta_2^2\eta_2(4\beta_2\eta_2-\delta_2^2)}{(2\beta_2\eta_2-\delta_2^2)^2} - \frac{8\beta_1\eta_1^2\gamma_1^2}{(2\beta_1\eta_1-\delta_1^2)^2}\\
&< 0, \text{ under assumption } \mathbf{A2} \text{ and}\\
\frac{\partial^2\Pi_R^{RS}(P_1,P_2)}{\partial P_1 P_2} &= \sum_{i=1}^{2}\Big[\frac{2\beta_i\eta_i\gamma_i(6\beta_i\eta_i-\delta_i^2)}{(2\beta_i\eta_i-\delta_i^2)^2}\Big]\\
&> 0 \quad \text{under assumption } \mathbf{A2}.
\end{aligned}$$

Thus, $\Pi_R^{RS}(P_1,P_2)$ is a quasi-concave function with respect to P_1 and P_2. Hence, there exists a Retailer-Stackelberg game. To derive the equilibrium soutions, we follow the same approach as described for wholesale price contract. The equilibrium solutions of the Retailer-Stackelberg game are given below.

$P_i^{WRS*} = \frac{1}{\Delta_2}\Big[\Big(\frac{2\eta_i\beta_i\gamma_i K_i}{N_i^2} + \frac{2\eta_j\beta_j\gamma_j K_j}{N_j^2}\Big)\Big\{\frac{2\beta_j\eta_j(\alpha_j K_j+\beta_j c_j N_j)}{N_j^2} - \frac{2\beta_i\eta_i\gamma_i(4\alpha_i\eta_i+c_iN_i))}{N_i^2}\Big\}$
$+\Big(\frac{2\eta_j\beta_j^2(K_j+N_j)}{N_j^2} + \frac{8\beta_i\eta_i^2\gamma_i^2}{N_i^2}\Big)\Big\{\frac{2\beta_i\eta_i(\alpha_iK_i+\beta_ic_iN_i)}{N_i^2} - \frac{2\beta_j\eta_j\gamma_j(4\alpha_j\eta_j+c_jN_j))}{N_j^2}\Big\}\Big]$,
$w_i^{WRS*} = c_i + \frac{\alpha_i-\beta_iP_i^{RS*}+\gamma_iP_j^{RS*}}{(\beta_i-\frac{\delta_i^2}{2\eta_i})}$ and $\theta_i^{RS*} = \frac{\delta_i(\alpha_i-\beta_iP_i^{RS*}+\gamma_iP_j^{RS*})}{2\eta_i\beta_i-\delta_i^2}$, where $N_i = 2\beta_i\eta_i-\delta_i^2$,
$K_i = 6\beta_i\eta_i-\delta_i^2$ for $i=1,2,\ j=3-i$ and
$\Delta_2 = \Big(\frac{2\beta_1^2\eta_1(K_1+N_1)}{N_1^2} + \frac{8\beta_2\eta_2^2\gamma_2^2}{N_2^2}\Big)\Big(\frac{2\beta_2^2\eta_2(K_2+N_2)}{N_2^2} + \frac{8\beta_1\eta_1^2\gamma_1^2}{N_1^2}\Big) - \Big(\frac{2\beta_1\eta_1\gamma_1K_1}{N_1^2} + \frac{2\beta_2\eta_2\gamma_2K_2}{N_2^2}\Big)^2$.

This completes the proof of the proposition.

Appendix E

Proof of Proposition 4

If $D_r^{SCS}(P_1, P_2)$ denotes the determinants of the boardered hessian matrix of rth order under Simple cost sharing (SCS) contract, then

$$\begin{aligned} D_1^{SCS} &= -\Big(\frac{\partial \Pi_R^{SCS}(P_1,\ P_2)}{\partial P_1}\Big)^2 < 0,\ \text{always and} \\ D_2^{SCS} &= \sum_{i=1}^{2}\Big[\frac{2\beta_i\eta_i\gamma_i\Big\{(1-\phi_i)\Big(6\beta_i\eta_i(1-\phi_i)-\delta_i^2\Big)+\phi_i\delta_i^2\Big\}}{\Big(2\beta_i\eta_i(1-\phi_i)-\delta_i^2\Big)^2}\Big] \\ &> 0,\ \text{always.} \end{aligned}$$

Then following the same argument as given in **Appendix D** we can prove the existance of the Retailer-Stackelberg game and obtain the equilibrium solutions for Retailer-Stackelberg game as

$$P_i^{SCS^*} = \frac{1}{\Delta_3}\Big[\Big(\frac{2\beta_i\eta_i\gamma_i K_i^{CS}}{(N_i^{CS})^2}+\frac{2\beta_j\eta_j\gamma_j K_j^{CS}}{(N_j^{CS})^2}\Big)\Big\{\frac{2\beta_j\eta_j\Big\{\alpha_j K_j^{CS}+\beta_j c_j(1-\phi_j)N_j^{CS}\Big\}}{(N_j^{CS})^2}-\frac{2\eta_i\gamma_i\Big\{\alpha_i L_i^{CS}+\beta_i c_i(1-\phi_i)N_i^{CS}\Big\}}{(N_i^{CS})^2}\Big\}$$
$$+\Big(\frac{2\beta_j^2\eta_j\Big\{K_j^{CS}+(1-\phi_j)N_j^{CS}\Big\}}{(N_j^{CS})^2}+\frac{2\eta_i\gamma_i^2 L_i^{CS}}{(N_i^{CS})^2}\Big)\Big\{\frac{2\beta_i\eta_i\Big\{\alpha_i K_i^{CS}+\beta_i c_i(1-\phi_i)N_i^{CS}\Big\}}{(N_i^{CS})^2}-\frac{2\eta_j\gamma_j\Big\{\alpha_j L_j^{CS}+\beta_j c_j(1-\phi_j)N_j^{CS}\Big\}}{(N_j^{CS})^2}\Big\},$$

$w_i^{SCS^*} = c_i + \frac{(\alpha_i-\beta_i P_i^{CRS^*}+\gamma_i P_j^{CRS^*})}{\Big(\beta_i-\frac{\delta_i^2}{2\eta_i(1-\phi_i)}\Big)}$ and $\theta_i^{CRS^*} = \frac{\delta_i(\alpha_i-\beta_i P_i^{CRS^*}+\gamma_i P_j^{CRS^*})}{\Big(2\beta_i\eta_i(1-\phi_i)-\delta_i^2\Big)}$ where

$N_i^{CS} = 2\beta_i\eta_i(1-\phi_i)-\delta_i^2,\ K_i^{CS} = (1-\phi_i)\Big(6\beta_i\eta_i(1-\phi_i)-\delta_i^2\Big)+\phi_i\delta_i^2,\ L_i^{CS} = 4\beta_i\eta_i(1-\phi_i)^2+\phi_i\delta_i^2$ for $i=1,2$ $j=3-i$ and

$$\Delta_3 = \Big[\frac{2\beta_1^2\eta_1\Big\{K_1^{CS}+(1-\phi_1)N_1^{CS}\Big\}}{(N_1^{CS})^2}+\frac{2\eta_2\gamma_2^2 L_2^{CS}}{(N_2^{CS})^2}\Big]\Big[\frac{2\beta_2^2\eta_2\Big\{K_2^{CS}+(1-\phi_2)N_2^{CS}\Big\}}{(N_2^{CS})^2}+\frac{2\eta_1\gamma_1^2 L_1^{CS}}{(N_1^{CS})^2}\Big]-\Big[\frac{2\beta_1\eta_1\gamma_1 K_1^{CS}}{(N_1^{CS})^2}+\frac{2\beta_2\eta_2\gamma_2 k_2^{CS}}{(N_2^{CS})^2}\Big]^2.$$

Appendix F

Proof of Proposition 5

From equation (12) we get the retailer's objective function as a function of ϕ only and is $\Pi_R^{SCS^*} = \frac{2\beta^2\eta(1-\phi)^2\Big\{\alpha-c(\beta-\gamma)\Big\}^2}{(\beta-\gamma)\zeta}$, where $\zeta = \Big[4\beta\eta(2\beta-\gamma)(1-\phi)^2-2\beta(1-\phi)\delta^2+(\beta-\gamma)\phi\delta^2\Big]$. Now, second order optimality condition gives

$$\begin{aligned} \Big[\frac{d^2\Pi_R^{SCS^*}(\phi)}{d\phi^2}\Big]_{\phi=\phi_{Opt}^{CRS}} &= -\frac{4\beta^2\delta^2\eta\Big\{\alpha-c(\beta-\gamma)\Big\}^2(3\beta-\gamma)^4}{(\beta-\gamma)^2\Big[16\beta\eta(2\beta-\gamma)(\beta-\gamma)-(3\beta-\gamma)^2\delta^2\Big]^2} \\ &< 0 \text{ under assumption } \mathbf{A2}. \end{aligned}$$

This implies that at $\phi = \phi_{Opt}^{SCS^*}$, $\Pi_R^{SCS^*}$ is strictly concave. Hence, from the first order optimality condition we get the optimal value of ϕ as $\phi_{Opt}^{SCS^*} = \frac{\beta+\gamma}{3\beta-\gamma}$. This completes the proof.

Appendix G

Proof of Proposition 6

As we know that the difference $(\beta-\gamma)$ is inversely proportional to the degree of product substitution, i.e., the degree of price competition between the two products (brands). Thus, the difference $(\beta-\gamma)$ increases means products are more differentiated (i.e., less substitutable). Hence, to prove the proposition it is sufficient to proove that $\frac{\partial \phi_{Opt}^{CRS^*}}{\partial(\beta-\gamma)} < 0$ has negative impact with respect to $(\beta-\gamma)$. This implies that for more differentiated products the retailer would share lower proportion of quality improvement cost with the manufacturers. In other words, for less differentiated (i.e., greater substitutable or greater competitive) products, the retailer would like to share higher proportion of quality improvement costs. Hence, retailer simple cost sharing parameter increases for more competitive products.

Appendix H

Proof of Proposition 7

$\theta_{Opt}^{SCS^*} > \theta^{WRS^*}$: To establish this relation it is sufficient to show that $\theta_{Opt}^{SCS^*} - \theta^{WRS^*} > 0$. After some simplification $\theta_{Opt}^{SCS^*} - \theta^{WRS^*}$ can be written as

$$\begin{aligned}
\theta_{Opt}^{SCS^*} - \theta^{WRS^*} &= \frac{\delta(\beta+\gamma)\Big\{\alpha - c(\beta-\gamma)\Big\}\Big[8\beta\eta(2\beta-\gamma) - (3\beta-\gamma)\delta^2\Big]}{2(4\beta\eta - 2\eta\gamma - \delta^2)\Big[16\beta\eta(\beta-\gamma)(2\beta-\gamma) - (3\beta-\gamma)^2\delta^2\Big]} \\
&> \frac{\delta^3(\beta+\gamma)(5\beta+\gamma)\Big\{\alpha - c(\beta-\gamma)\Big\}}{2(4\beta\eta - 2\eta\gamma - \delta^2)\Big[16\beta\eta(\beta-\gamma)(2\beta-\gamma) - (3\beta-\gamma)^2\delta^2\Big]}, \\
&\geq 0, \text{ under } \textbf{A2, A3 and the restriction (7)}.
\end{aligned}$$

$w_{Opt}^{SCS^*} \geq w^{WRS^*}$: After some simplification we get

$$\begin{aligned}
w_{Opt}^{SCS^*} - w^{WRS^*} &= \frac{\eta\delta^2(\beta+\gamma)^2\Big\{\alpha - c(\beta-\gamma)\Big\}}{(4\beta\eta - 2\eta\gamma - \delta^2)\Big[16\beta\eta(\beta-\gamma)(2\beta-\gamma) - (3\beta-\gamma)^2\delta^2\Big]} \\
&\geq 0 \text{ under } \textbf{A2, A3 and the restriction (7)}.
\end{aligned}$$

$P_{Opt}^{SCS^*} \geq P^{WRS^*}$: After some simplification we get

$$\begin{aligned}
P_{Opt}^{SCS^*} - P^{WRS^*} &= \frac{\delta^2(\beta+\gamma)\xi\Big\{2\beta\eta(7\beta-5\gamma) - (3\beta-\gamma)\delta^2\Big\}}{2(\beta-\gamma)(4\beta\eta - 2\eta\gamma - \delta^2)\tau} \\
&\geq 0 \text{ under } \textbf{A2, A3 and the restriction (7)}, \text{ where} \\
&\quad \xi = \Big\{\alpha - c(\beta-\gamma)\Big\} \\
&\quad \tau = \Big[16\beta\eta(\beta-\gamma)(2\beta-\gamma) - (3\beta-\gamma)^2\delta^2\Big].
\end{aligned}$$

These complete the proof.

Appendix I

Proof of Proposition 8

$\Pi_{M_{Opt}}^{SCS*} > \Pi_{M}^{WRS*}$: We can write the optimal profit of the manufacturer under WP contract as

$$\begin{aligned}\Pi_{M}^{WRS*} &= \frac{(\beta-\gamma)^2}{(\beta-\gamma)^2}\Pi_{M}^{WRS*}\\ &= \frac{\eta(\beta-\gamma)^2(4\beta\eta-\delta^2)\Big\{\alpha-c(\beta-\gamma)\Big\}^2}{4(\beta-\gamma)^2(4\beta\eta-2\eta\gamma-\delta^2)^2}\\ &= \frac{\eta(\beta-\gamma)^2(4\beta\eta-\delta^2)\Big\{\alpha-c(\beta-\gamma)\Big\}^2}{\Big[4\beta\eta(\beta-\gamma)(2\beta-\gamma)-2(\beta-\gamma)\delta^2\Big]^2}.\end{aligned}$$

It can ve easily shown that $\Big[16\beta\eta(\beta-\gamma)(2\beta-\gamma)-(3\beta-\gamma)^2\delta^2\Big] > \Big[4\beta\eta(\beta-\gamma)(2\beta-\gamma)-2(\beta-\gamma)\delta^2\Big]$. Now using the result $A > B \Rightarrow -\frac{1}{A} > -\frac{1}{B}$, we get

$$\begin{aligned}&\Pi_{M_{Opt}}^{SCS*}-\Pi_{M}^{WRS*}\\ >\ & \frac{\eta\Big\{\alpha-c(\beta-\gamma)\Big\}^2\Big[2\beta(\beta-\gamma)\Big\{2\eta(\beta-\gamma)-\delta^2\Big\}\Big]}{\Big[16\beta\eta(\beta-\gamma)(2\beta-\gamma)-(3\beta-\gamma)^2\delta^2\Big]^2}\\ >\ & 0,\end{aligned}$$

under assumptions **A2**, **A3** and the production cost restriction as given by equation (7).

$\Pi_{R_{Opt}}^{SCS*} \geq \Pi_{R}^{WRS*}$: After some simplification we get

$$\begin{aligned}&\Pi_{R_{Opt}}^{SCS*}-\Pi_{R}^{WRS*}\\ =\ & \frac{\beta\eta\delta^2(\beta+\gamma)^2\xi}{(\beta-\gamma)(4\beta\eta-2\eta\gamma-\delta^2)\tau}\\ \geq\ & 0 \text{ under } \mathbf{A2,\ A3} \text{ and the restriction (7), where}\\ & \xi=\Big\{\alpha-c(\beta-\gamma)\Big\}^2 \text{ and}\\ & \tau=\Big[16\beta\eta(\beta-\gamma)(2\beta-\gamma)-(3\beta-\gamma)^2\delta^2\Big].\end{aligned}$$

$\Pi_{d_{Opt}}^{SCS*} \geq \Pi_{d}^{WRS*}$: This part of the proposition follows directly from the above results, since, all the members of the proposed supply chain are benefitted from SCS contract. These complete the proof.