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October 2016

CIRRELT-2016-49

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Bike Route Choice Modeling Using GPS Data without Choice Sets of Paths

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Abstract. Concerned by the nuisances of motorized travel on urban life, policy makers are faced with the challenge of making cycling a more attractive alternative for everyday transportation. Route choice models can help achieve this objective by gaining insights into the trade-offs cyclists make when choosing their routes and by allowing the effect of infrastructure improvements to be analyzed. We estimate a link-based bike route choice model from a sample of GPS observations in the city of Eugene on a network comprising over 40,000 links. The so-called recursive logit (RL) model (Fosgerau et al., 2013), does not require to sample any choice set of paths. We show the advantages of this approach in the context of prediction by focusing on two applications of the model: link flows and accessibility measures. Compared to the path-based approach which requires to generate choice sets, the RL model proves to make significant gains in computational time and to avoid paradoxical accessibility measure results discussed in previous works, e.g. Nassir et al. (2014).

Keywords: Bike route choice, recursive logit, infinite choice set, accessibility, link flows.

Acknowledgement: This work was partly supported by Mitacs and INRO Software. We are grateful to Josh Roll from the Lane Council of Government for sharing the data with us. We also thank Mike Florian from INRO for the helpful discussions.

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Dépôt légal – Bibliothèque et Archives nationales du Québec
Bibliothèque et Archives Canada, 2016

1 Introduction

The increasing concern of policy makers for the nuisances generated by motorized travel, including air pollution, urban congestion and energy waste, has triggered the need for research into sustainable means of transportation, such as cycling. Cycling is not a popular option for US households, 92% of which owned a car in 2001 (Pucher and Renne, 2003) and used it as their usual commute mode (Polzin and Chu, 2005). In some European countries however, cycling levels have increased sharply since 1975, when efforts were first made to accommodate cyclists on the road network, providing evidence of the powerful impact of policy on travel behaviour (Pucher and Buehler, 2008). The challenge policy makers face nowadays is providing a safe and convenient cycling environment that will encourage a greater shift to this mode.

The high travel demand and the size constraints on the street network make it difficult for urban planners to create a system adapted to cyclists. In order to determine exactly what facilities are worth investing in, urban planners need to understand the behavior of bike users and gain insight into the trade-offs they make when choosing their route. Indeed, cyclists do not always choose the shortest distance path to go from origin to destination, and in fact many other factors play a part. For example, would a cyclist be willing to go far out of their way to avoid a hill, or to use a bike lane?

One way to answer these questions is route choice analysis. Route choice models in a real network deal with identifying the route a traveler would take to go from one location to another. The discrete choice framework and revealed preference (RP) GPS data is used to define a choice probability distribution over paths in the network. This framework is advantageous on multiple levels. Firstly, the interpretation of model parameters quantifies the trade-offs made by cyclists, which provides helpful guidance for improving network infrastructure. Secondly, link flows predicted from the model are useful to target the network areas most in need of improvement. Thirdly, route choice model output provides bike accessibility prediction to higher-level models, e.g. mode choice.

In the literature on route choice models based on RP data in a real network, there are two main modeling approaches. The most common approach is path-based, in the sense that the model describes a discrete choice among paths. A well-known issue associated to this framework is that in a real network the universal set of all paths is intractable. The other approach, put forward by Fosgerau et al. (2013), is link-based. In this model, called recursive logit (RL), the choice of itinerary is modeled as a sequence of link choices.

There is little literature on bike route choice modeling, and all current models are based on the first approach (e.g. Broach et al., 2012, Hood et al., 2011, Menghini et al., 2010). A shortcoming of these models is that due to the exponential number of paths in the network one has to make assumptions about which paths to consider (i.e. sample a restricted choice set). This sampling process may introduce variability in estimation results, as pointed out by Frejinger et al. (2009). Moreover, it is unknown how to use these models to obtain correct predictions, as further detailed in Section 2. On the other hand, link-based models have the advantage of not requiring any sampling of paths.

In fact, it was proven that the RL model is equivalent to a path-based model with unrestricted choice set.

In this work, we propose a link-based bike route choice model which overcomes these challenges. We adapt to the bike route choice problem the RL model formulated by Fosgerau et al. (2013), based on the assumption of an unrestricted choice set and not requiring any sampling of paths. Unlike previous studies, this work addresses both the issues of estimation and prediction. More precisely, we make the following empirical and theoretical contributions. First, we show how non link-additive attributes, such as slope, can be incorporated into the link utilities of the RL model. Second, we provide estimation results based on GPS observations in the network of Eugene, Oregon, which reveal cyclists' preferences and quantifies trade-offs between different network attributes. Third, we provide numerical results which illustrate the advantages of the RL model over path-based models in the context of prediction, in particular regarding gains in computational time. Fourth, we study properties of the RL model and specifically discuss accessibility measures. The analysis illustrates that the paradoxical results reported e.g. by Nassir et al. (2014) obtained when path-based models predict accessibility are due to the necessity to sample paths but can be avoided by the RL model.

The remainder of this paper is structured as follows. In Section 2, we start by describing the state of the art in bike route choice modeling and we highlight gaps in previous research. In Section 3, we review the RL model and in Section 4 we describe the data used for this application. We provide estimation results and discuss their implications in terms of travel behavior in Section 5. Then Section 6 focuses on prediction of link flows and accessibility. Finally, we conclude in Section 7.

2 Literature review

In this section, we review the path-based modeling approach for the route choice problem and highlight differences with the link-based approach. We then focus specifically on bike route choice modeling and describe previous studies.

2.1 Path-based approach to route choice modeling

Route choice analysis based on the multinomial logit framework and RP data can be modeled in two ways. The first and most common approach is path-based and models a discrete choice among paths. A well-known issue associated with this framework is that the set of all feasible paths is intractable and the actual choice sets of paths are unknown to the analyst. In fact, in a real-sized network, there is an unlimited number of paths connecting each origin-destination pair if loops are permitted. In order to estimate such a model, a restricted choice set has to be defined for each path observation. They can be generated with some sort of path-generation algorithm, such as link elimination (e.g. Menghini et al., 2010), or route labeling (e.g. Ben-Akiva et al., 1984). This process can lead to two different hypotheses on the choice set. The classic approach hypothesizes that the generated choice sets contain all the paths considered as alternatives by travelers. As argued by Frejinger et al. (2009), the issue with this approach is that parameter estimates may vary significantly with the definition of choice sets. This led Frejinger et al. (2009) to propose a sampling approach. In this approach, all feasible paths connecting an origin-destination pair are assumed to belong to the choice set, denoted as the universal choice set, and the parameter estimates are corrected for the bias induced by sampling a restricted set.

The issue of choice set generation has been mainly discussed in the context of model estimation. However, the intractability of choice sets is also an issue for prediction. Indeed, having access to the estimated path choice probabilities requires to explicitly enumerate the choice set. In the literature most route choice models follow the classic approach, which counters the problem by assuming that only a subset of alternatives are actually considered as relevant by travelers. The constructed choice set is assumed to contain all of them. However, as argued by Prato (2009), an objective definition of relevant routes is currently missing. Therefore, the correctness of path choice sets for prediction purposes cannot be ascertained. This is an important issue since predictions vary depending on which paths are assumed to be part of the choice set. On the other hand, when a route choice model is estimated based on the hypothesis of an unrestricted choice set, any feasible path is associated to a non zero choice probability. In this setting it is difficult to predict from the estimated choice distribution. There is only one known method to sample paths according to a given distribution without enumerating the choice set, which is Metropolis-Hastings sampling of paths (Flötteröd and Bierlaire, 2013). The method only requires to know the distribution up to a multiplicative constant, which obviates the computation of the denominator in the logit function and avoids path enumeration. However, Metropolis-Hastings sampling is relatively time-consuming. Furthermore, it is arguable that in some cases, it could be useful to not only be able to sample from the estimated distribution, but to also evaluate specific path choice probabilities.

2.2 Bike route choice modeling literature

Until recent years, the literature on bike route choice was exclusively based on stated preference (SP) data. In the simplest case, individuals take part in a survey in which they are asked to evaluate routes based on their main characteristic (e.g. Winters et al., 2011). In other studies like that of Sener et al. (2009), surveys are designed in a way that forces the respondent to make trade-offs between combinations of attributes. Some studies based on SP methods are limited to performing a descriptive analysis without estimating a formal model, while others use multinomial logit or regression analysis methods, including Tilahun et al. (2007), Sener et al. (2009), Hunt and Abraham (2007), and Stinson and Bhat (2003).

Although SP studies can be relatively inexpensively implemented and are able to evaluate alternatives that are not yet available (e.g. nonexistent facilities), they also have a number of well-known shortcomings. The limitations of SP studies arise mostly from the difference between claimed and observed behavior, as described in numerous works, for example by Sener et al. (2009). Indeed, it is difficult for SP studies to put respondents in a setting where they can best reproduce the behavior they exhibit in reality.

RP studies were enabled by the emergence of geographic information systems (GIS) which gave access to new types of data. Data was then still collected through surveys, but instead of being put in hypothetical choice situations, participants had to recall their actual commuting routes, which were subsequently analyzed with GIS. While providing valid insights, these first attempts to analyze bike route choice based on RP data never resulted in the estimation of a full route choice model, as observed by Broach et al. (2012). In particular, the models lack a comprehensive choice set of paths since the recalled route is compared mostly only to the shortest path. In addition, the models focus on predicting specific aspects of route choice, such as distance deviation to shortest path or presence of bike facility, but cannot be applied to predict path probabilities for a large set of routes. In other words, they are certainly useful for behavior analysis, but not for trip distribution in a network.

The first RP study that overcame these various limitations was the work of Menghini et al. (2010). Its main innovation was to exploit automatically processed GPS-based observations. Car route choice models had already been estimated on this kind of data (e.g. Ramming, 2001), since this area of research benefited from a few year's lead in data collecting efforts. However Menghini et al. (2010) were the first to obtain a large-scale GPS sample of cyclists trajectories matched to a suitable network and to estimate a complete bike route choice model.

Some other noteworthy studies followed the steps of Menghini et al. (2010), but overall the literature on bike route choice based on RP is still in its early stages compared to its car counterpart. Notably, Hood et al. (2011) extended the Zürich results of Menghini et al. (2010) to the US context, in a study based in San Francisco. Broach et al. (2012) contributed as well to the state of the art by estimating a model comprising a richer set of attributes.

The previously cited works are all based on the hypothesis that choice sets contain the actual paths considered by cyclists. Part of the focus of their study was then on the development of realistic choice set generation methods. A common measure of the adequacy of choice sets is the coverage of observed routes (Ramming, 2001). In other words, path generation algorithms should be able to reproduce the observed routes for a high proportion of origin-destination pairs. However, the network density and the variety of attributes influencing cyclist's choices make this especially difficult for bike networks. As noted by Broach et al. (2012), common algorithms for car routes based on shortest paths are often not directly applicable. Menghini et al. (2010) developed a choice set generation algorithm for high resolution data (Rieser-Schüssler et al., 2013), deemed suitable for bike networks, and Hood et al. (2011) and Broach et al. (2012) experimented with methods to account for the diversity of attributes. Despite this progress, these studies highlight the challenges raised by the restricted choice set hypothesis, especially for bike route choice. Considered choice sets are rarely observed, thus even the quality measures proposed in the literature have limitations (Frejinger, 2008). Moreover, even based on these criteria the most recent algorithms fail to include all observed alternatives.

As pointed out by Horowitz and Louviere (1995), when there exists no observation on choice sets, it is better to rely solely on the utility function to predict choices, which is the assumption of the RL model.

3 Methodology

In this section, we present the link-based modeling framework. We recall the formulation of the RL model and we review subsequent works which relax its IIA property.

3.1 The recursive logit model

The framework for this bike route choice modeling study is the recursive logit model formulated by Fosgerau et al. (2013). The RL corresponds to a dynamic discrete choice model and the path choice is formulated as a sequence of link choices. At each node in the network, the individual chooses the utility-maximizing link, where the utility is the sum of the instantaneous link cost, the maximum expected utility to the destination and i.i.d. extreme value type I error terms. Therefore, attributes of the RL model are attributes of the links in the network and they are specified to be link-additive, such that the utility of a path is the sum of the utility of each link in the path.

Formally, the model can be described as follows (Fosgerau et al., 2013). The road network is a directed connected graph $G = (\mathcal{A}, \mathcal{V})$, where $\mathcal{A}$ is the set of links and $\mathcal{V}$ is the set of nodes. More precisely, a set of absorbing links without successors, corresponding to the observed destinations, is added to $\mathcal{A}$. We denote links $a, k \in \mathcal{A}$, and the set of outgoing links from k, $A(k)$. Each link pair (k, a) where $a \in A(k)$ then has a deterministic utility component $v(a|k)$, based on the attributes $x(a|k)$ of the link pair. In the terminology of dynamic programming, k is a state and a is an action given k, although in this context choosing an action translates simply to choosing the next link in the path.

Consider now an individual n traveling in this network. The instantaneous random utility for the individual n of a link a conditionally on being in state k can then be defined as:

$$u_n(a|k) = v_n(a|k) + \mu\epsilon_n(a) \tag{1}$$

where $\epsilon_n(a)$ are i.i.d extreme value type 1 error terms with zero mean and μ is a fixed scale parameter. The full utility of link a conditionally on being in state k is obtained by adding to the instantaneous utility $u_n(a|k)$ the maximum expected utility to destination d, denoted the value function $V_n^d(a)$ and defined by the Bellman equation as follows

$$V_n^d(k) = E\left[\max_{a \in A(k)} \left\{v_n(a|k) + V_n^d(a) + \mu\epsilon_n(a)\right\}\right]. \tag{2}$$

Therefore, upon observing the random term $\epsilon_n(a)$, the individual chooses in $A(k)$ the link a which maximizes $u_n(a|k) + V_n^d(a)$.

The probability of choosing a link a given state k conditionally on going to destination d is then given by the multinomial logit model

$$P_n^d(a|k) = \frac{e^{\frac{1}{\mu}v_n(a|k)+V_n^d(a)}}{\sum_{a'\in A(k)} e^{\frac{1}{\mu}v_n(a'|k)+V_n^d(a')}} \tag{3}$$

In this case the value function is the logsum

$$V_n^d(k) = \mu \ln \sum_{a\in A(k)} e^{\frac{1}{\mu}v_n(a|k)+V_n^d(a)}. \tag{4}$$

We note that the denominator in (3) simplifies to $e^{\frac{1}{\mu}V_n^d(k)}$. As a result, the probability of choosing a path $\sigma = \{k_i\}_{i=0}^l$ where k_0 is the origin and $k_l = d$, given by the product of the link choice probabilities, also has a simple expression:

$$P_n^d(\sigma) = \prod_{i=0}^{l-1} e^{\frac{1}{\mu}(v_n(k_{i+1}|k_i)+V_n^d(k_{i+1})-V_n^d(k_i))} \tag{5}$$

$$= \frac{e^{\frac{1}{\mu}\sum_{i=0}^{l-1} v_n(k_{i+1}|k_i)}}{e^{\frac{1}{\mu}V_n^d(k_0)}} \tag{6}$$

Denoting $\sum_{i=0}^{l-1} v_n(k_{i+1}|k_i)$ as $v_n(\sigma)$, Equation (6) can be rewritten as:

$$P_n^d(\sigma) = \frac{e^{\frac{1}{\mu}v_n(\sigma)}}{\sum_{\sigma'\in\mathcal{U}} e^{\frac{1}{\mu}v_n(\sigma')}} \tag{7}$$

where $\mathcal{U}$ is the universal set of all possible paths. Therefore, the RL model is equivalent to a static model of multinomial logit form with an infinite choice set (Fosgerau et al., 2013).

3.2 Modeling correlated utilities

When discrete choice models are used to analyze path choice in a network, it is well known that the IIA property does not hold due to overlapping paths in the network (Ben-Akiva and Bierlaire, 2003). Paths sharing links in the network also share unobserved attributes and route choice models should account for this correlation. Several solutions have been proposed in the literature to model correlated path utilities, as reported by Frejinger and Bierlaire (2007).

Some of these approaches have been adapted so as to be compatible with the recursive logit. A deterministic correction called the Link Size attribute (LS) was proposed by Fosgerau et al. (2013). It resembles the Path Size attribute (Ben-Akiva and Bierlaire, 1999) of path-based route choice models, which heuristically corrects the utility of overlapping paths. The LS attribute is however link-additive and can be used with the RL model.

Even including the LS attribute, the RL model retains the structure of a logit model. Mai et al. (2015a) was the first to relax the IIA property in the RL model by allowing scale parameters of random terms to be link-specific. The model contains scale parameters μ_k for each link $k \in A$ and the utility function becomes

$$u_n(a|k) = v_n(a|k) + \mu_k \epsilon_n(a). \tag{8}$$

The resulting model is called the nested recursive logit (NRL) and it allows path utilities to be correlated in a fashion similar to the nested logit (McFadden, 1978). The path probabilities are in this case defined by

$$P_n^d(\sigma) = \prod_{i=0}^{l-1} e^{\frac{1}{\mu_{k_i}}(v_n(k_{i+1}|k_i)+V_n^d(k_{i+1})-V_n^d(k_i))}. \tag{9}$$

The scales μ_k are parameters of the model to be estimated, similarly to the parameters β associated with the attributes of the instantaneous utilities. Due to the impossibility to estimate a scale parameter for each link in a real network, it is assumed that scale parameters are a function $\mu_k(\beta_{\text{scale}})$ of parameters β_{scale} to be estimated.

There is a trade-off between modeling suitably the correlation structure of path utilities and being able to estimate the models in a reasonable amount of time. The RL model requires to solve systems of linear equations in order to compute the value functions for each value of the parameters evaluated by the estimation algorithm. Computational time can be reduced by using the decomposition method (DeC) proposed by Mai et al. (2015b), which greatly diminishes the number of systems to solve, however the DeC is not compatible with either the LS attribute or the NRL model. In addition, the corresponding systems of equations in the NRL model are non-linear and more time consuming to solve than in the case of the RL model.

4 Data

This study is based on GPS observations of cyclists trajectories in the city of Eugene, Oregon. The data was collected and processed by the Central Lane Metropolitan Planning Organization as part of their ongoing research on bicycle travel behavior in the area. Their goal was to collect the data in an inexpensive manner in terms of time and money, which pointed towards the use of a smartphone application instead of a bicycle-mounted GPS device. This led to the development of the CycleLane smartphone application. CycleLane builds on code provided by the San Francisco County Transportation Authority, who has previously developed a similar application called CycleTracks (see Hood et al., 2011).

Upon downloading the CycleLane application, users are first asked about demographics and cycling frequency. They may then voluntarily record any bike trip they undertake by switching on the application. At the end of a trip, the user fills in the purpose of the trip and the data is automatically sent to the CLMPO.

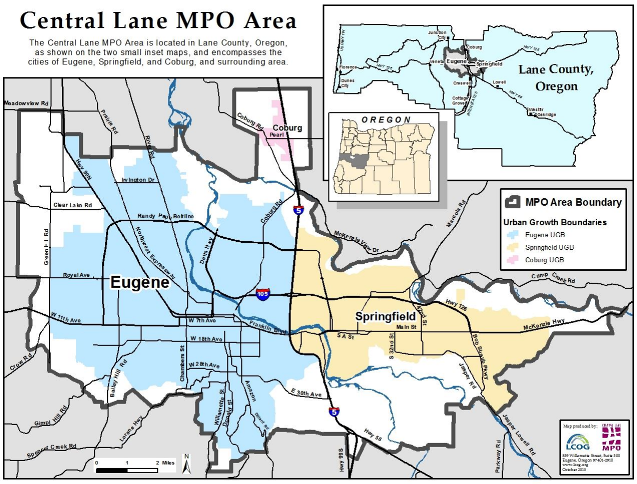


Figure 1: Map of the region of study. Source: www.thempo.org

In total, 648 observations of bike trips were collected from 103 users, after the CLMPO screened observations in order to remove trips not within the region, trips not fully recorded, and duplicate trips. The users were on average frequent cyclists (with 55% of the sample riding several times per week or daily). There is also a bias towards males, who represent 74% of participants, and surprisingly towards people older than 26, who amount to 81%, despite the high number of university students in the region (Roll, 2014).

The observations were matched to the route network of the Eugene Springfield Metropolitan area (Figure 1). The network contains 16,352 nodes and 42,384 links. It was enlarged to include not only traditional car routes but also the many minor alleys and multi-use paths bikes may take. The area comprises some 80 miles of off-street bicycle and pedestrian paths and over 140 miles of bike lanes and bike boulevards, according to the CLMPO. As a result, we can analyze preferences towards different types of bike facilities.

Several network characteristics are available to describe the network's links, such as length, average slope and upslope, estimated car traffic volume, one-way restrictions, speed limit, presence of various types of bike facilities, traffic signals, and stop signs. In contrast with previous path-based studies, the data does not need to be processed in order to compute attribute levels of each generated path. However, link characteristics need to be link-additive in order to be incorporated into the utility function of the RL model. In the following section, we describe how to exploit this data in order to meet the link-based model's assumptions.

5 Recursive bike route choice models

In this section, we apply the recursive modeling framework of Section 3 to the data presented in Section 4 in order to specify and estimate recursive bike route choice models.

5.1 Link utilities

We specify four different models within the recursive framework: the RL model with and without the LS attribute, and the NRL model, also with and without LS attribute.

The RL model formulation requires attributes to be link-additive. As a result, link characteristics such as slope need to be carefully incorporated in the utility function. For example, it is not possible to include slope as a continuous variable, since the average slope of a path consisting of two links is not equal to the added average slopes of each link. In our case, these inherently non link-additive attributes are slope, traffic volume and presence of bike facilities. The solution we adopt is to specify a dummy variable δ_a for each of these attributes and let the dummy variables interact with the link length attribute. On each link a, the variable δ_a takes the value 1 if the attribute is present or greater than a chosen threshold in case of continuous attributes, and 0 else. The interaction term is simply the product of the two attribute values. Not only does this specification allow us to include important characteristics in a way that respects link-additivity, but the interpretation is also simple and intuitive.

As an illustration, let us assume links are characterized by three attributes, link length, slope, and the presence of a bike lane. Let us also assume a certain threshold above which slope affects utility has been chosen. If we denote L_a the length of link a, δ_a^S and δ_a^B the previously introduced dummy variables corresponding to slope and presence of a bike lane respectively, β_L the length parameter, and $\beta_{L,S}$, $\beta_{L,B}$ the parameters corresponding to the interaction terms, then the deterministic utility component of a link a given a state k would be:

$$\begin{aligned} &\beta_L \cdot L_a + \beta_{L,S} \cdot L_a \cdot \delta_a^S + \beta_{L,B} \cdot L_a \cdot \delta_a^B \\ =&\underbrace{\left(\beta_L + \beta_{L,S} \cdot \delta_a^S + \beta_{L,B} \cdot \delta_a^B\right)}_{\beta_{TL}} L_a. \end{aligned}$$

Implied is that length is associated to a total length parameter, referred in this example as β_{TL}, which may take different values across links. For example, for a link a with a slope greater than the chosen threshold and with a bike lane, the variables δ_a^S and δ_a^B would take the value 1. In this case, the parameters of the interaction terms would add to β_L, and β_{TL} would be equal to $\beta_L + \beta_{L,S} + \beta_{L,B}$.

Intuitively, the way individuals perceive length is influenced by other characteristics of the link. In the illustrative example, if $\beta_{L,B}$ is positive, the fact that there is a bike lane will increase the value of the total parameter β_{TL}, making traveling a unit of distance on this link less unpleasant for the individual. Similarly, if $\beta_{L,S}$ is negative, a link with a slope higher than the threshold will cost more per unit of length, making it

Attribute	Description
Length	Link length (1/1000 feet)
Link Constant	A constant equal to one for each link intended to penalize paths with many crossings.
Length · Upslope	Interaction between link length and average upslope > 4%.
Length · Medium Traffic	Interaction between link length and medium traffic volume (between 8000 and 20000 vehicles/day).
Length · Heavy Traffic	Interaction between link length and heavy traffic volume (greater than 20000 vehicles/day).
Length · RMUP	Interaction between link length and regional multi-use path.
Length · Bike Boulevard	Interaction between link length and bike boulevard.
Length · Bike Lane	Interaction between link length and bike lane.
Bridge	Presence of bridge
Bridge · Bike Fac	Interaction between presence of bridge and bike facilities.
No Turn	Straight direction of travel (no turn $\pm 5°$)
No Turn · Crossroad	Straight direction of travel at a crossroad
Left Turn · Crossroad · Medium Traffic	Left turn through medium traffic at crossroad without traffic signal (at an angle between 60° and 179°)
Left Turn · Crossroad · Heavy Traffic	Left turn through heavy traffic at crossroad without traffic signal (at an angle between 60° and 179°).

Table 1: Description of attribute variables

less attractive. The implied behavior is plausible, as travelers might be willing to cope with negative attributes, but more so for relatively short distances.

We summarize in Table 1 the network attributes $x(a|k)$ of each link pair (k, a) included in the deterministic utility specification of all four models. Non link-additive attributes which are included through the specification of one or several dummy variables are traffic volume, average upslope, and three types of bike facilities. Turn attributes are computed based on link orientation at each node. Obtaining these link pair attributes from the network data is straightforward and does not require extensive computations. This makes the model practical to estimate compared to path-based models which require to compute path attributes for each path in the choice set.

In order to account for correlation due to overlap between paths, we follow the methodology detailed in Section 3.2. In addition to the RL model, we specify a RL model with LS attribute, a NRL model, and a NRL model with LS. The LS attribute is specific to each pair of origin-destination (OD). It represents the expected link flow between each OD and is generated from the RL model with chosen parameter values. The two NRL models include link-specific scale parameters μ_k which are a function of a single parameter β_{scale}.

Model	RL			RL-LS		
Attribute	$\hat{\beta}$	$\hat{\sigma}$	t-test	$\hat{\beta}$	$\hat{\sigma}$	t-test
Length	-2.25	0.13	-17.31	-2.28	0.14	-16.28
Link Constant	-1.61	0.02	-80.50	-1.60	0.02	-80.00
Length · Upslope	-3.24	0.55	-5.89	-3.15	0.50	-6.30
Length · Medium Traffic	-0.81	0.08	-10.13	-0.82	0.08	-10.25
Length · Heavy Traffic	-1.01	0.10	-10.10	-1.02	0.08	-12.75
Length · Bike Boulevard	0.74	0.08	9.25	0.76	0.07	10.86
Length · RMUP	1.80	0.07	25.71	1.81	0.07	25.86
Length · Bike Lane	0.92	0.06	15.33	0.87	0.06	14.50
Bridge	-5.41	0.97	-5.58	-4.56	1.00	-4.56
Bridge · Bike Fac.	2.83	0.52	5.44	1.99	0.56	3.56
No Turn	1.37	0.03	45.67	1.33	0.03	44.33
No Turn · Crossroad	-0.28	0.03	-9.33	-0.29	0.03	-9.67
Left Turn · Crossroad · Medium Traffic	-0.28	0.09	3.11	-0.33	0.09	3.67
Left Turn · Crossroad · Heavy Traffic	-1.84	0.33	-5.58	-1.86	0.34	-5.47
Link Size	–	–	–	-0.24	0.03	-8.00
Log likelihood at $\hat{\beta}$	12383			12202		

Table 2: Estimation results: RL model

5.2 Estimation results

We first make some remarks regarding the estimation algorithm and computational times. As described in Fosgerau et al. (2013) and Mai et al. (2015a), the optimization algorithm is a basic trust-region algorithm which uses the BFGS Hessian approximation for the RL model, and the BHHH approximation for the NRL. The models are estimated with MATLAB 2016 [1] following the implementation of Mai et al. (2015a). We have used an Intel(R) Xeon(R) X5675 @ 3.07GHz machine. The machine has a multi-core processor but we only used one processor to estimate the models. As expected, the computational time required to estimate the NRL models (about 15 days) is much greater than that of the RL models (1h without LS using the decomposition method, and 43h with LS).

Tables 2 and 3 display the estimation results for all four model structures and for the chosen utility specification. All parameter estimates are significantly different from zero and have their expected sign. The models with LS attribute have a significantly better in-sample fit than those without, and the NRL model has a significantly better in-sample fit than the RL model. With the LS attribute, the NRL model is the best of all four, but without it is outperformed by the RL model with LS. The ratio between parameter estimates remain similar for the RL and NRL models. Therefore, the interpretation of parameters is consistent with all models considered. In the following discussion, we focus on the estimates of the RL model without LS.

Consistently with the expectation that cyclists are highly put off by long distances,

[1] Code distributed on github

Model	NRL			NRL-LS		
Attribute	$\hat{\beta}$	$\hat{\sigma}$	t-test	$\hat{\beta}$	$\hat{\sigma}$	t-test
Length	-1.48	0.14	-10.57	-1.54	0.17	-9.06
Link Constant	-1.07	0.05	-21.40	-1.09	0.06	-18.17
Length · Upslope	-2.97	0.55	-5.40	-3.05	0.55	-5.55
Length · Medium Traffic	-0.53	0.07	-7.57	-0.59	0.08	-7.38
Length · Heavy Traffic	-0.66	0.08	-8.25	-0.70	0.08	-8.75
Length · Bike Boulevard	0.51	0.06	8.50	0.46	0.06	7.67
Length · RMUP	1.15	0.09	12.78	1.18	0.10	11.80
Length · Bike Lane	0.62	0.06	10.33	0.60	0.06	10.00
Bridge	-2.83	0.56	-5.05	-2.08	0.47	-4.43
Bridge · Bike Fac.	0.86	0.29	2.97	0.19	0.33	-0.58
No Turn	0.89	0.05	17.80	0.88	0.06	14.67
No Turn · Crossroad	-0.14	0.02	-7.00	-0.15	0.02	-7.50
Left Turn · Crossroad · Medium Traffic	-0.05	0.05	1.00	0.03	0.05	0.60
Left Turn · Crossroad · Heavy Traffic	-1.56	0.40	-3.90	-1.28	0.27	-4.74
Scale	-0.11	0.01	-11.00	-0.11	0.02	-5.50
Link Size	–	–	–	-0.16	0.02	-8.00
Log likelihood at $\hat{\beta}$	12325			12143		

Table 3: Estimation results: NRL model

the link length parameter has a negative value. This was found to be the attribute dominating the choices of cyclists by Menghini et al. (2010) and an important factor in virtually all bike route choice research. However, as described in Section 5.1, this parameter represents only part of a total length parameter, the magnitude of which varies across links depending on other relevant characteristics influencing length perception.

Characteristics related to slope were included in the form of a dummy variable interacting with link length. We chose a threshold of an average link upslope higher than 4%. The negative value of the slope parameter, about 1.5 times that of the length parameter, shows that a large upslope considerably increases the magnitude of the total length parameter. We tested a higher threshold of 6% in addition to a 4-6% threshold, but the difference between the estimates was not significant. A 2-4% threshold was also investigated but the estimate was not significantly different from 0. The most similar findings are those of Broach et al. (2012) who included as an attribute the proportion of route length within three categories of average slope (2-4%, 4-6%, 6% and more). They found this specification to perform better than the most common alternatives, such as maximum or average slope of the path, found in Hood et al. (2011) and Menghini et al. (2010).

Traffic volumes also affect the way cyclists perceive distances, but less so than slope. Medium (between 8000 and 20000 vehicles/day) and heavy (more than 20000 vehicles/day) traffic are both associated with negative parameters, however not significantly different. While the value of the total length parameter is -2.25 on a segment with low traffic, assuming no other link characteristics contribute to its value, it becomes -3.06 on a segment with medium traffic volume (and similarly -3.26 with heavy traffic volume). Thus the model did not identify a significant difference between medium and heavy traffic. The ratio between both values indicates that cycling 1 mile surrounded by heavy traffic would be perceived equivalent to cycling 1.45 miles on a low traffic road.

Bike facilities are all associated with a positively signed parameter, indicating that cyclists are willing to travel greater distances to use them. The regional multi-use path is the bike facility with the largest parameter value. Bike lanes and bike boulevards both have a significantly smaller parameter estimate, consistently with the results of Broach et al. (2012). On a segment with a bike facility, the value of the total length parameter increases and is equal to -1.51 if the facility is a bike boulevard, -1.33 for a bike lane and -0.45 for a regional multi-use path. Thus, traveling on a street with a bike boulevard is equivalent to a reduction in distance of 33%. This becomes a reduction of 41% for the bike lane and of 80% for the regional multi-use path. The value placed on separate multi-use paths is surprisingly high, and suggests that cyclists are willing to travel on roads more than 4 times longer to use them. This result may be due to the relatively small number of observations available, many of which use regional multi-use paths. We also note that the bike lane parameter is of a similar magnitude as the ones for traffic volume, which have an opposite sign. This suggests that the presence of a bike lane counterbalances the negative impact of heavy traffic on the utility of a road, however it has no residual value. This last observation supports the conclusions of Broach et al. (2012), who also stated that bike lanes are no more or less attractive than a basic low traffic street.

A bridge is in general an unattractive feature of a path for a cyclist, as the negative value of the estimate shows. However, if the bridge has a separated bike facility, the positive value associated to the bike facility in that case outweighs the negative one, and the sum of both parameters is not significantly different from zero, meaning that in this case bridges are not penalized compared to other links.

The link constant parameter has a negative sign, meaning that paths with many crossings are less attractive to cyclists.

The coefficient associated to a straight direction of travel is significantly positive, probably because many turns may cause detours or result in an intricate path. Cyclists thus have a preference for simple routes. However, the model suggests that at a crossroad (instead of another type of intersection with fewer outgoing links) the incentive for going straight is slightly lowered. In this specification, left and right turns at crossroads do not contribute to the utility, while being still less attractive relative to a straight route. We expect in contrast difficult left turns which cause delays to be especially inconvenient to cyclists. The model shows indeed that left turns through heavy traffic at crossroads without signals are greatly penalized. Cyclists are also sensitive to left turns through medium traffic, but less so.

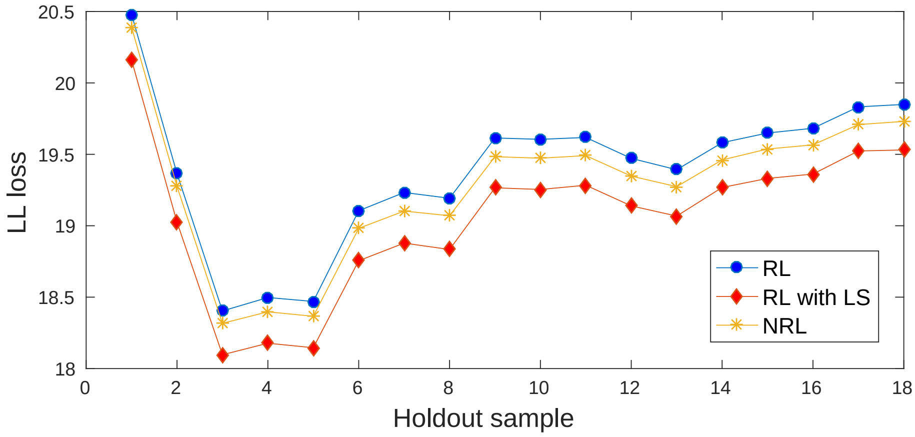


Figure 2: Moving average of err_i across samples $i = 1, ..., 18$

5.3 Cross-validation

In this section, we compare the out-of-sample fit of the four models with a cross-validation approach, in order to check for overfitting. The observations are repeatedly and randomly split into a training set (80% of all observations) and a test set (20%), until 20 different training sets and matching test sets have been generated. The performance of the models is evaluated by computing the log-likelihood loss on the test sets, after having estimated the models on the training sets. The log-likelihood loss of sample i is defined as:

$$err_i = -\frac{1}{|S_i|} \sum_{\sigma \in S_i} \ln P(\sigma, \hat{\beta}_i)$$

where S_i denotes test set i, and $\hat{\beta}_i$ the vector of estimated parameters on training set i. Thus, the lowest the loss is, the best a model performs.

We performed the cross-validation on all four models. However, there were too few observations in the training set for the estimation algorithm of the NRL model with LS to converge and it was excluded from the comparison. The estimation algorithm for the NRL model also did not converge for two training sets, therefore we compare the RL, the RL with LS and the NRL on the 18 remaining sample sets. Figure 2 plots the moving average of err_i across sample sets $i = 1, ..., 18$. The cross-validation is in line with in-sample fit and confirms that the RL model with LS performs best of the three models, followed by the NRL model, and that the RL model has the highest log-likelihood loss.

6 Prediction

In this section, we extend the analysis beyond the interpretation of model parameters. We address the general issue of applying bike route choice models for prediction. In a policy analysis perspective, important applications of the model are i) predicting link-level bike volume and ii) measuring cyclist specific accessibility.

We aim with this section to contrast the path-based approach to prediction with that of the link-based RL model, underlining the advantages of the latter. In both cases, we review the prediction methods provided by all models. We enlighten the methodological issues associated with path-based models, then explain how the RL model overcomes them. For link flows, we provide in addition numerical results which highlight the potential gains of time associated to the RL model.

6.1 Link flows

We start by stating that in the following, the methods we discuss are based on the assumption of an uncongested network. This means that route choice probabilities are independent of the amount of flow on each link, which is reasonable in the case of many bike networks, in particular in North America. In general link-level traffic volume is predicted from route choice models by distributing a given travel demand between each OD pair on the network. We assume that an OD matrix characterizing this demand exists. The recursive models offer two ways to distribute demand in the network according to an estimated model: by simulation or by computing link flows as solutions to systems of linear equations. Both ways make use of destination specific link choice probabilities $P^d(a|k;\widehat{\beta})$ given by (3) but with the parameter estimates $\widehat{\beta}$. We denote $\mathbf{P}^d$ the matrix with elements $P^d(a|k;\widehat{\beta})$.

The first way of distributing demand consists of simulating path choices for each origin destination pair by sampling from $\mathbf{P}^d$. There are different simulation methods available with different computational cost. For the sake of illustration, we use a simple approach in this paper that consists of drawing the same number r of paths for each OD pair. The path choice probabilities are known for each of these paths (6) and we normalize them so that the sum over the r simulated paths for each OD equals one. We then distribute the demand given by the OD matrix according to the path probabilities. While this simulation approach may at a first glance seem similar to the classic way of distributing demand according to a path-based model, there is an important difference. Path-based models make use of choice sets that are arbitrarily generated while the recursive model allows to simulate according to the estimated model $\mathbf{P}^d$ without generating any choice sets.

The second way to distribute demand in the network is grounded in the link-based structure of the RL model and was proposed by Baillon and Cominetti (2008). It allows to compute link flows without resorting to simulation. The method consists in solving a system of linear equations for each destination d in the network, and to sum the resulting link flows over all destinations. Let us denote the demand originating from each link a to destination link d as the vector $\mathbf{G}^d$, the vector of destination-specific link flows as $\mathbf{F}^d$. Then, the vector of expected link flows $\mathbf{F}^d$ is obtained by solving

$$(\mathbf{I} - \mathbf{P}^{d^T})\mathbf{F}^d = \mathbf{G}^d, \tag{10}$$

and the vector of link flows $\mathbf{F}$ resulting from demand with multiple destinations is equal to $\mathbf{F} = \sum_d \mathbf{F}^d$.

To the best of our knowledge, this second method has not been used with an estimated model and a real network before. In the following we compare predictions generated with both methods. The objective is to assess the potential gain in computational time of avoiding simulation.

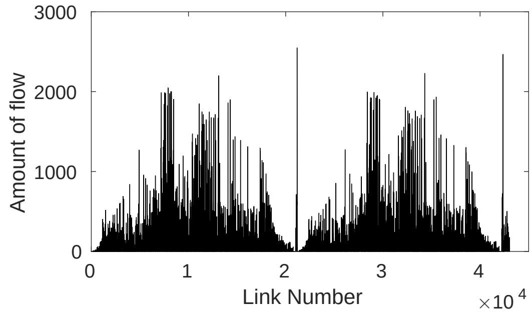
We applied both method to predict link flows in the Eugene bike network with the RL model. Since we assumed an uncongested network, we did not iterate to find a traffic equilibrium condition. The flows were predicted for a given demand matrix consisting of 666 origins and destinations in the Eugene bike network which was obtained from a mode choice model. Figure 3 plots the amount of flow on each link according to each prediction method. The figure indicates that both methods yield very similar results, even with a relatively small number of paths sampled in the choice set. The average flow on each link amounts to 55.36 according to the solution of (10). The average difference of flow on each link when comparing these results with simulated link flows is 3.03 when 10 draws are used, and 2.95 when 20 draws are used. We conclude that it would take a very large number of draws for the simulated flows to converge to the solution of the system of equations, nevertheless the difference is very small. Furthermore, we note that the average difference is inflated by a few links with a very large amount of flow, while for the great majority of links this difference is comprised in the $[-4; 4]$ interval and close to 0, as seen in Figure 4.

A difference between both methods is that solving the linear system of equations assumes that there is a non zero probability of flow on each link. As a result, the amount of flow on each link is strictly positive (although negligible for many links). On the contrary, when link flows are simulated, there is only flow on links of paths that were sampled. When 10 draws were used, we found that 30,205 links out of 43,050 had non zero flow, and this became 31,039 when 20 draws were used. On the other hand, by solving the system of equations, we obtain slightly higher flows: 27,077 links have a flow higher than 1, while this amounts to 25,583 (25,760) links for simulation with 10 (20) draws.

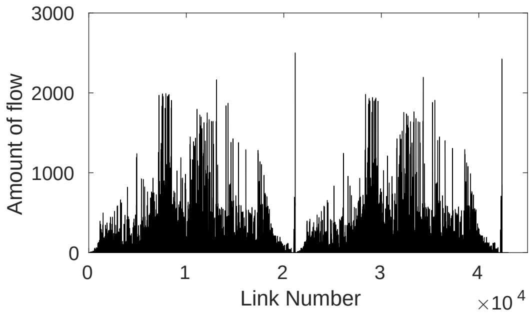
In terms of computational time, solving the system of linear equations for all destinations requires 6 minutes, while simulating link flows via sampling took about 25 hours for $r = 10$, and about 70 hours for $r = 20$ (non-parallelized MATLAB code). Even though the code has not been optimized for simulation, the results illustrate the potential gain associated with solving systems of linear equations as opposed to simulation. Moreover, this approach has the advantage of producing deterministic link flows and hence overcomes the issues associated with simulation bias.

6.2 Accessibility measure

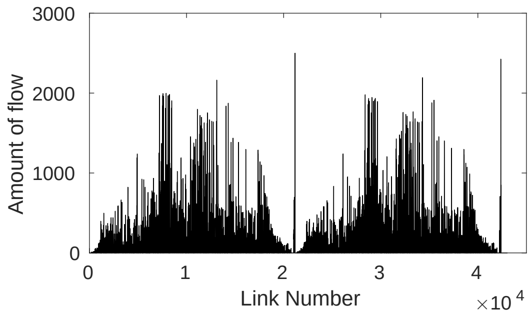
Accessibility is a widely studied notion in transportation, and in this context it can be defined as information evaluating the attractiveness of a network (regardless of activity participation, which is encompassed in a more general definition, e.g. Bhat et al., 2000). Accessibility measures are very helpful in travel demand modeling, as they provide input



(a) System of linear equations

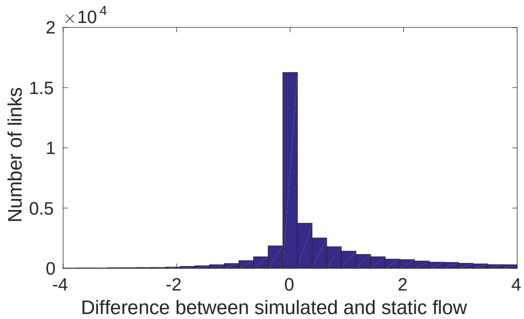


(b) Simulation, $r = 10$

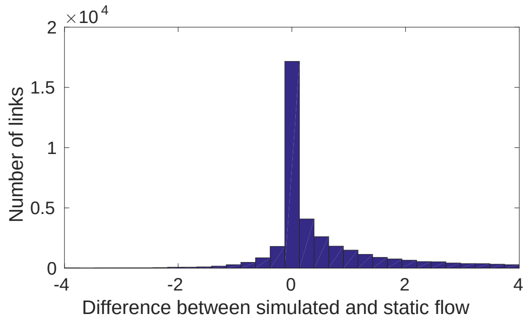


(c) Simulation, $r = 20$

Figure 3: Load profile of links



(a) Simulation, $r = 10$



(b) Simulation, $r = 20$

Figure 4: Histogram of link flow difference

to higher level models, such as mode choice, household location choice or car ownership models. These measures are often OD-specific, in which case they characterize the level of service of a network when traveling from an origin O to a destination D.

In particular, bike accessibility encapsulates information regarding the suitability of the network for cycling, and has been also denoted bikeability in other works (Lowry et al., 2012). According to Hood et al. (2011), current bike accessibility measures used in higher level models are more predictive of automobile travel than cycling, while Mesbah and Nassir (2014) asserts that traditional measures are only based on shortest path computations between OD pairs and thus unsuitable for bike accessibility. As a result, recent works now recognize the importance to improve two aspects of bike accessibility measurement, first to incorporate route choice preferences of cyclists, and secondly to capture the diversity of sub-optimal available routes instead of the utility of the single best path.

Deriving an accessibility measure from a bike route choice model appropriately fits these two purposes and was recently investigated by Nassir et al. (2014). This idea is not new and originates from the general concept of deriving an accessibility measure from a random utility model, first introduced by Ben-Akiva and Lerman (1979). They defined accessibility as the logsum

$$E(\max_{i\in C_n} u_i) = \mu \log \sum_{i\in C_n} e^{\frac{1}{\mu}v_i}. \tag{11}$$

This measure guarantees that accessibility does not decrease if the systematic utility of any alternative in the choice set increases, as proven by Ben-Akiva and Lerman (1979). In other words, if an alternative becomes more attractive, for example as a result of infrastructure enhancements, the accessibility measure mirrors this improvement.

However, with a route choice random utility model based on paths, we argue that this important property no longer holds due to the intractable nature of the choice set C_n in (11). Whether it is assumed that the true choice set consists of all feasible paths or that only a subset of alternatives are in fact considered does not affect the prediction method. In each case, it becomes necessary to define a restricted set of paths in order to evaluate Equation (11). Similarly to the link flow problem, in the absence of a clear methodology any choice set could be selected and the ensuing accessibility measures vary.

Why this implies that the property of monotonicity with respect to systematic utility no longer holds can be straightforwardly explained. Indeed, the sampled choice set $\mathcal{C}_n$ in (11) needs to be updated after network changes in order to account for potential newly attractive paths that were not previously generated. Paths that were sampled in the first choice set may not appear in the second one. However if accessibility after network changes is computed based on a different choice set $\tilde{\mathcal{C}}_n$, there is no basis for comparison. As such there can be no guarantee of monotonicity. This has given rise to what Nassir et al. (2014) denote the Valencia paradox. This paradox was observed when the predicted accessibility counter-intuitively decreased for some origin-destination pairs after network improvements and is tangible proof of the problematic consequences of this limitation.

In essence, we argue that this paradox is an artifact inherent to path-based models and arises from the necessity to explicitly generate a restricted choice set for prediction. On the other hand, the RL model allows to predict accessibility according to the true model with the hypothesis of an unrestricted choice set. The ensuing measure prevents paradoxical predictions. In order to illustrate this assertion, we first derive the accessibility measure resulting from the RL model.

In the RL model, the accessibility of an origin-destination pair as defined previously by the logsum formula is simply equivalent to the value function to destination d at the origin link k (Fosgerau et al., 2013):

$$V^d(k) = \mathrm{E}\left[\max_{a \in A(k)} \Big(v(a|k) + V^d(a) + \mu\epsilon(a)\Big)\right] = \mu \ln \sum_{a \in A(k)} e^{\frac{1}{\mu}(v(a|k)+V^d(a))}. \tag{12}$$

The value function from an origin to a destination encompasses the expected maximum utility of all paths connecting them. This becomes clear when recalling that the RL model is equivalent to a path-based multinomial logit model over the set of all possible paths (see Section 3). This property is what allows the value function to be rewritten in an equivalent non recursive form:

$$V^d(k) = \mu \ln \sum_{\sigma \in \mathcal{U}} e^{\frac{1}{\mu} v(\sigma)} \tag{13}$$

where $\mathcal{U}$ is the set of all paths between origin k and destination d, and $v(\sigma)$ is the deterministic utility component of path σ. It is then apparent that the value functions of the RL model are of the form in (11), and consequently they retain the property of monotonicity with respect to the deterministic part of utilities. The fundamental point here is that, to the difference of path-based models, the value functions of the RL model can be conveniently computed by solving systems of linear equations and do not rely on enumerating the set $\mathcal{U}$.

Naturally, accessibility in (13) could also be approximated with Monte Carlo techniques by generating a subset of paths $\mathcal{C}_n$ from $\mathcal{U}$, just as link flows may be predicted by sampling from the true model. Intuitively, as more paths are sampled and added to $\mathcal{C}_n$, the value obtained converges towards an asymptotic value which is given by the value function. Finally, this means that path-based models can only provide an approximation of accessibility based on the entire network. Whether it is judicious to behaviorally assume that any feasible path should enter the choice set is yet another much debated question (Horowitz and Louviere, 1995). Nevertheless, this work provides evidence that for mathematical reasons, it is very pragmatic to do so.

7 Conclusion

We outlined the development of several versions of a bike route choice model based on the recursive logit framework of Fosgerau et al. (2013), with and without relaxing the IIA property through nesting, as proposed by Mai et al. (2015a). We estimated the models on 648 GPS-based observations of paths collected in Eugene, Oregon, and matched to a network of 16'352 nodes and 42'384 links. Our utility specification successfully incorporates all fundamental attributes impacting cyclists' route choice while respecting the requirement of link-additivity. To do so, we let inherently non link additives such as slope interact with link length.

Estimation results emphasize the sensibility of cyclists to distance, traffic volume, slope, crossings and presence of bike facilities. The preferred facilities are separate multi-use paths, followed by bike lanes and then by bike boulevards. Our results confirm the findings of previous studies, in particular the strong preference for separate paths and the small residual value of bike lanes after compensating the negative effect of high traffic volumes, as highlighted by Broach et al. (2012). On the other hand, our model did not identify as many distinct categories of slope or traffic volume as the one of Broach et al. (2012), distinguishing only between average slope above or below 4%, and traffic volume above or below 8000 vehicles per day.

The RL model is fast to estimate when applying the decomposition method of Mai et al. (2015b). However this is no longer possible when including a link size attribute or when relaxing the IIA property via nesting. Since models accounting for correlated utilities performed better than the simple RL model, a trade-off has to be made between faster estimation or a better likelihood.

In addition to analyzing cyclists' route choice preferences, this paper makes valuable contributions, both theoretical and empirical, in the field of prediction. We experimented two methods to predict traffic flows, simulation and solving a system of linear equations (Baillon and Cominetti, 2008). We find that solving the system requires a significantly lower amount of computational time than sampling paths while resulting in very similar link flows. We also highlighted a theoretical property of the RL model, namely that its value function corresponds to the accessibility measure obtained asymptotically from a path-based model, if the sampled choice set grows towards including all paths. The implication of this result is that the RL model yields an accessibility measure which is monotonous with respect to deterministic utilities, and could be consistently incorporated in higher level models, such as mode choice models. Thus, the result discussed at length in Nassir et al. (2014) and dubbed a paradox is an artifact of the hypothesis of a restricted choice set.

Acknowledgements

This work was partly supported by Mitacs and INRO Software. We are grateful to Josh Roll from the Lane Council of Government for sharing the data with us. We also thank Mike Florian from INRO for the helpful discussions.

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