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The Load Planning Problem for Double-Stack Intermodal Trains

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Abstract. This paper presents a general methodology that addresses the load planning problem for intermodal trains. We propose a general model that can deal with single- or double-stack railcars as well as arbitrary containers-to-cars matching rules. Moreover, we model center-of-gravity constraints, stacking rules and technical loading restrictions associated with specific container types and/or goods. We propose an integer linear programming (ILP) formulation, where the objective is to choose the optimal subset of containers and the optimal way of loading them on outbound railcars such that the resulting loading cost is minimized. An extensive numerical study shows that ignoring center-of-gravity constraints and containers-to-cars matching rules, may lead to an overestimation of the train capacity and to load plans that are not feasible in practice. We also show that we solve realistic size instances to optimality in less than 20 minutes on average using a commercial ILP solver.

Keywords: Freight transportation, intermodal trains, load Planning, double-stack containers, railway terminals.

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1 Introduction

Nowadays an essential ingredient of a competitive economy is a cost effective freight transportation system. Intermodal transportation is an important part of this system where different transport modes are linked in order to move freight from a point of origin to a point of destination. Taking advantages of economies of scale, low volume demands are firstly shipped to an intermediate point, a consolidation terminal or hub, where traffic is sorted (classified) and grouped (consolidated). Then, the consolidated traffic is moved between hubs by efficient transport modes. In this paper we deal with intermodal railway transportation where containers are consolidated and transported by train on the long-haul part of the trip. We focus on the North American market and double-stack trains.

Intermodal transportation relies heavily on containerization, because the latter ensures a faster and safer handling and transfer between modes and decreases transportation costs. Intermodal containers are steel frame boxes designed to move goods across the world using different transport modes without any re-handling of the cargo. Since 2005, the containerized worldwide traffic has increased from 382 to 684 million of TEU (Twenty Foot Equivalent Unit) (CBRE Research, 2015). North American ports have seen container traffic grown by an annual average of 5.3% since 1990 (International Association of Ports and HarborsA, 2015). This growth is placing a heavy burden on the entire consolidation-based transportation system, which must provide efficient, reliable and cost-effective services.

Terminals are major components of any intermodal transportation system and thus they are the backbone of the entire international trade. They are special transshipment nodes which provide equipment and space where containers are processed, loaded, unloaded and stored to ensure a seamless transfer between different modes. Carriers, in our case railroads, face a number of challenging planning issues, which may be examined according to the classical categorization with respect to the planning horizon, i.e. strategic, tactical, operational. In this study, we focus on the *load planning problem* which is an operational problem arising at intermodal railway terminals.

Given a set of containers stored in a terminal and a departing train, the objective of the load planning is to select the optimal set of containers to load and the optimal way of loading them, using as much as possible of the available capacity. We address this problem for double-stack trains. This is a challenging problem because the load plan must respect a number of loading rules that depend on container and railcar characteristics. For example, stacking rules for certain sizes of containers and types of goods, weight capacity and center-of-gravity restrictions. While the methodology in this paper is general, we use the North American market as a case study because it is particularly challenging. Indeed, there are many types of railcars and there are also more container sizes than the standard 20 feet (ft) and 40 ft ones.

As we detail in Section 3, with one exception, previous studies in the literature do not address the load planning problem for double-stack trains. Moreover, they make simplifying assumptions that may lead to load plans that violate important loading rules and hence cannot be used in practice. For example, none of the studies model the center-of-gravity restrictions. The objective of this paper is to propose a general methodology that adresses the load planning problem of double-stack trains taking into account all the different loading rules.

There is a large number of possible ways – so-called loading patterns – to load different container types on a given railcar type. The multitude of railcar types and the cardinality of the associated sets of loading patterns is a key issue. We refer to this problem as container-to-car matching. In this context, we make a number of contributions. First, we propose a general model that can deal with single- and double-stack trains where railcars can be of different types and have different loading rules. Second, the model accounts for loading constraints related to different container types as well as weight and center-of-gravity restrictions. Third, we present extensive numerical results using the North American market as case study.

The numerical results illustrate that we can solve very large instances in reasonable time using a commercial solver. They also show that failing to account for container-to-car matching as well as center-of-gravity and stacking restrictions may lead to an overestimation of the available train capacity and to load plans that cannot be used in practice.

The remainder of the paper is structured as follows. In Section 2 we describe the load planning problem in detail. Section 3 is dedicated to a literature review of the studies on containers assignment on railcars, highlighting the main contributions of this study. We present the ILP formulation in Section 4. Numerical results are presented in the Section 5. We conclude in Section 6 where we discuss possible directions for future research.

2 Double-stack train loading

The way containers can be loaded on a train depends on the characteristics of the containers, the railcars and on the way they can be matched to each other. Intermodal containers exist in several sizes which are standardized for facilitating their handling (e.g. Wikipedia, 2016b). ISO standard containers which are used worldwide are of four types: 20 ft high cube, 40 ft low and high cube, 45 ft high cube. In this paper we focus on the North America market, where some additional sizes are available: 45 ft, 48 ft and 53 ft (all high-cube containers). In addition to size, there are different types of cargo-specific containers, for example, general purpose (so-called dry, which make up about in ninety percent of the global container fleet), temperature controlled, open top and tank containers.

The container size, or more specifically, the location of the load bearing along the length of the container, as well as the type of container determines how they can be stacked. On trains, containers are stacked at the 40 ft load bearing location. This means that 20 ft containers cannot by stacked on top of any other container size. Moreover, it is not possible to stack any container size on top of a single 20 ft. Other container sizes can be double-stacked, for example, a 40 ft container can be stacked on top of a 53 ft container and vice-versa.

Certain types of goods are transported in special container types or have specific loading rules associated with them. We call these *technical loading restrictions*. For the North American market we have identified six such restrictions, we provide a few examples in the following. Tanker and dangerous containers have restrictions with respect to the position in the stack they may occupy. Refrigerated containers, that do not have their own power unit, have to be loaded in proximity of a generator, and thus, their loading depends on the railcar sequence. An open-top container cannot be double stacked. Depending on company policies and on country regulations, there may exist different rules.

In addition to the aforementioned physical characteristics of the containers, we assume that there is a cost associated with a container departing or not departing with a given train. For example, customer penalties for late arrival and storage costs in the terminal.

Intermodal trains are composed of sequences of railcars, that are designed to carry single or double-stacked containers. The railcars differ on attributes such as the number of platforms (also called wells), the length of each platform and weight loading limit. Each platform consists of two slots, the bottom and the top one, which can accommodate up to two containers (two 20 ft only in case of bottom slot). Figure 1 illustrates the slots of a three platforms double-stack railcar. It is expensive to operate a train because of costs associated with locomotives, crew and fuel (see e.g. Bouzaiene-Ayari et al., 2014). Hence, there is a cost associated with leaving a slot on the train unused.

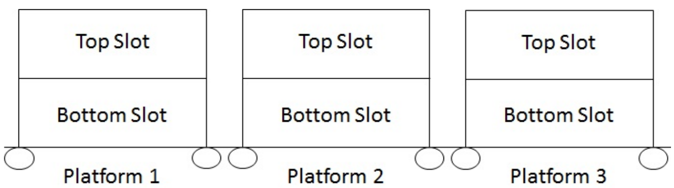

Figure 1: Slots on a three platforms double stacked railcar

The matching of container and railcars depends on the characteristics of both. We start by describing the matching with respect to container size. For the North American market, this is covered by the so-called AAR Guide (Association American Railroads, 2014b), a reference document for inspectors which describes each series of railcars and provides information on which containers sizes that can be loaded in the bottom and top slot of each platform. We provide an example in Table 1 where the first block of rows is an exact copy of the AAR Guide and the following block of rows show four examples of loading that satisfy the guide. The platform units on a given railcar are denoted by letters: front unit is A, rear unit is B, and in-between C,D,E (from front to rear on 5-unit car). We underline that AAR Guide reports the loading capabilities, but the guide does not show all the possible ways to match container sizes to railcars. The loading of certain platforms may depend on the loading of the others and thus the railcar loading problem cannot be decomposed by platform. The example in Table 1 illustrates such a dependency since the 53 ft containers can only be loaded in certain top slots, if 40 ft containers are loaded in the other top slots. The set of loading patterns must account for this dependency and its cardinality may therefore be large, in particular for 5 platforms railcars (see Section 4.1 for more details).

Bottom slot					**Top slot**				
A	*C*	*D*	*E*	*B*	*A*	*C*	*D*	*E*	*B*
AAR Guide									
$2-20'$ $1-40'$	$2-20'$ $1-40'$	$2-20'$ $1-40'$	$2-20'$ $1-40'$	$2-20'$ $1-40'$	$1-40'$ $1-45'$ $1-48'$ $1-53'^{(\mathbf{1})}$	$1-40'$ $1-45'$ $1-48'$	$1-40'$ $1-45'$ $1-48'$ $1-53'^{(\mathbf{1})}$	$1-40'$ $1-45'$ $1-48'$	$1-40'$ $1-45'$ $1-48'$ $1-53'^{(\mathbf{1})}$
Some examples satisfying AAR Guide									
$2-20'$	$2-20'$	$2-20'$	$2-20'$	$2-20'$	$1-48'$		$1-40'$		$1-45'$
$1-40'$	$2-20'$	$1-40'$	$2-20'$	$1-40'$	$1-45'$	$1-40'$	$1-53'$	$1-40'$	$1-53'$
$2-20'$	$1-40'$	$2-20'$	$1-40'$	$1-40'$	$1-48'$	$1-45'$	$1-48'$	$1-45'$	$1-48'$
$1-40'$	$1-40'$	$1-40'$	$1-40'$	$1-40'$	$1-40'$	$1-40'$	$1-40'$		$1-40'$

Table 1: Example of AAR Guide railcars BN 63900 - 63909 type IBC 100 tons **(1)**: 53 ft containers in top slot of platforms A, D and B only when 40 ft containers are loaded in top slot of the platforms C and E.

We refer to the several loading rules, differentiated by car type and containers size as *container to cars matching rules*. The size of the container in the bottom slot cannot exceed the length of the platform. For instance, in a 40 ft platform it is possible to load either a 40 ft container or at most two 20 ft. The rules for the top slot are more complex: the container sizes that can be loaded in the top slot depend on other characteristics of the railcar. In general, the size of the container loaded in the top slot can exceed the length of the one loaded in the bottom slot provided that there is enough space between the platforms. For example, while it may be feasible to load a 53 ft container in the top slot of a platform of length 40 ft, it may not be feasible to load a 53 ft containers in the top slot on two or more contiguous 40 ft platforms.

The matching of railcars and containers does not only depend on size but also on weight. There are two types of restrictions with respect to the weight of the load. First, the total weight of the containers loaded on a given platform cannot exceed its weight holding capacity. This ensures that the total weight of containers loaded on a given railcar does not exceed the car weight holding capacity.

The second type of restriction concerns the vertical center of gravity (we adopt the term from the North American railroad industry but note that it is the same as the center of mass). It is the unique point where the weighted relative position of the distributed mass sums to zero or the point where, if a force is applied, causes it to move in direction of force without rotation (Wikipedia, 2016a). Since there is not restriction on the horizontal center of gravity, it is sufficient to ensure that it does not exceed a certain height above the the top of rail, as stated by the AAR Guide: "*..the Center of Gravity (COG) for a double-stack car and the load in the platform must be less than or equal to 98 inches at top of rail (ATR). Reference Rule 89, Section C.2.e. in the AAR Field Manual*" (Association American Railroads, 2014a). We provide a detailed discussion on the modeling of the center of gravity restrictions in Section 4.2.

In summary, we focus on the load planning problem: given a set of containers stored in a terminal and a departing train, the objective is to select the optimal set of containers to load and the optimal way of loading them, using as much as possible of the available capacity. The principal metric currently used to measure the efficiency of the train loading plan is the *slot utilization*, which measures the percentage of the available slots on intermodal outbound cars that accommodate containers in the load plan (Burriss, 2003). We note that we focus on a deterministic setting, and that we do not model the different handling costs associated with retrieving containers from the terminal. The objective is to develop a general methodology, which can be used as a decision support tool for providing feasible and reliable loading plans. We deal with all the loading rules and restrictions that arise for double-stack trains, by taking into account also the multitude of containers and railcars types that exist in the North American market.

3 Literature review

We present an overview of the studies that focus on the load planning problem. We start by noting that Heggen et al. (2016) provide a recent classification of the literature. For surveys with a broader scope, we refer the reader to Crainic and

Kim (2007) and Carlo et al. (2014). Moreover, Boysen et al. (2013), presents a comprehensive overview of the planning issues that arise specifically in railway yards, including the load planning problem.

Most of the studies in the literature focus on the single stack load planning problems. The first contribution was made by Feo and Gonzalez-Velarde (1995), while later on, Powell and Carvalho (1998) dealt with the problem of balancing the flat cars over a network from a load planning perspective. The problem for single-stack trains is considerably simpler than for double-stack ones and the studies mainly focus on the loading containers such that handling costs in the yard (e.g. Ambrosino et al., 2011, Ambrosino and Siri, 2015, Corry and Kozan, 2006, 2008) or train set-up costs (Bruns et al., 2014, Bruns and Knust, 2012) are minimized. They deal with load planning at different degrees of detail. For example, Corry and Kozan (2008) consider matching different container and railcar types, while Corry and Kozan (2006) do not. Bruns and Knust (2012) extend the work by Corry and Kozan (2008) by considering both the matching problem between containers and railcars and the weight constraints. Heggen et al. (2016) build on this model and integrate a number of practical loading constraints. Ambrosino et al. (2011), and Ambrosino and Siri (2015) minimize rehandles in the yard and unproductive movements of cranes. Finally, Bruns et al. (2014) consider several sources of uncertainty (e.g. weights, lengths and equipment failures) in a robust optimization approach.

The aforementioned studies focus on single-stack trains, where the main challenge is associated with optimizing yard operations rather than the train loading. In our problem, loading double-stack trains is difficult because we consider the matching problem between a multitude of different railcar and container types. Moreover, we enforce the center-of-gravity restrictions and stacking rules. This is a rather complex setting and, thus, we focus load planning problem and handling costs are ignored.

To the best of our knowledge, Lai et al. (2008a) is the only previous study on the double-stack load planning problem. Similarily to this study, they also ignore handling costs. Their focus is on minimizing the aerodynamic drag of double-stack trains that depends on the gaps between containers and the location of these gaps along the train. They present an integer linear programming formulation for the double-stack trains load planning problem but they make a number of simplifying assumptions. First, they address the matching among containers and railcars types deriving the loading patterns without considering platform dependencies. This implies that loading rules can be defined for platforms independently (we show a real example in Table 1 where this assumption is not valid). Second, they ignore the case when loading may be constrained by the railcar sequence, in which case is not possible to decompose the loading by platform (e.g., refrigerated containers require to be loaded in physical proximity of a generator). Third, the problem is studied without accounting for center-of-gravity restrictions and stacking rules. The authors also extend the model to a rolling horizon setting and show that the loading can be improved by considering several trains at a time (Lai et al., 2008b).

4 Mathematical formulation

As we discuss at length in the previous sections, the load plan must respect dimensional and weight capacity constraints as well as stacking rules. In this section, we present an integer linear programming (ILP) formulation where we maximize the slot utilization by minimizing the cost associated with the loading plan. The cost is computed with respect to the used railcars and the containers left on the ground. We describe in the following how we address the dimensional constraints through loading patterns that define feasible matches between different sizes of containers and railcars. We then present the center-of-gravity constraints, followed by the full ILP formulation.

4.1 Modeling containers-to-cars matching

We model the containers-to-cars matching through *loading patterns*. A loading pattern is a feasible assignment of container sizes to railcars. In the construction of loading patterns, the containers are only characterized by their length. Corry and Kozan (2008) and Lai et al. (2008b) also use loading patterns, the main difference here lies in the fact that we account for dependencies between the loading of platforms on a same railcar (see example in Table 1). This leads to an exponential increase in the number of loading patterns as the number of platforms increases.

A specific loading pattern for a given platform is defined by a n-tuple specifying the total number of containers of each length that can be loaded in the slots. When a railcar consists of several platforms these tuples are concatenated from left to right. For example, a particular pattern for a railcar comprising three platforms is described by a 3 n-tuples. A set of loading patterns is a collection of loading patterns complying with the loading capabilities of a specific railcar (as described in Association American Railroads, 2014b).

Let T be the set of railcar types and H be the set of container types, defined by the container dimensions in feet. For instance $H = \{20, 40, 45, 48, 53\}$. For each railcar type $t \in T$, we derive a set of feasible loading patterns $K_t \subset K$, where K is the set of all feasible loading patterns for all railcars. A loading pattern $k \in K_t$ provides the information about the number of containers of each type $h \in H$ that can be loaded on each platform of that car.

k / h	*20*	*40*	*45*	*48*	*53*
1	0	0	0	0	0
2	2	0	0	0	0
3	0	1	0	0	0
4	2	1	0	0	0
5	2	0	1	0	0
6	2	0	0	1	0
7	2	0	0	0	1
8	0	2	0	0	0
9	0	1	1	0	0
10	0	1	0	1	0
11	0	1	0	0	1

Table 2: Example of K_t, where rows are the configuration $k \in K_t$ and columns are the containers types $h \in H$

We provide an example of set K_t in Table 2, where for a railcar type t covering all the double-stack one platform railcars on which two 20 ft or one 40 ft can be loaded in the bottom slot and any size $h \in H$ can be loaded in the top slot. There is a total of 11 loading patterns ($|K_t| = 11$). In this example, the first row corresponds to an empty car and the second one to two 20 ft (loadable necessarily in the bottom slot).

It is easy to see that the number of loading patterns increases exponentially with the number of platforms. Some of them are however redundant. Consider for example loading only one 40 ft container on a three platform railcar. The three possible loading patterns representing the loading in one of the platform can be represented by a single one (loading on any platform is possible). We remove redundancies by defining equivalence classes, where each class is represented by a single representative element. This leads to an important reduction of the sets cardinality.

Table 3 reports the descriptive statistics of the number of loading patterns for railcar types in the North American fleet, after removing redundancies. For each number of platforms, we report the average, maximum and minimum number of loading patterns, as well as the standard deviation, for double and single stack railcars. While the number of loading patterns increases with the number of platforms, we note that the equivalence classes still make the number reasonable. We note that there is a large variation in the number of loading patters. For instance, the minimum number for a 53 ft railcar is 56 and the maximum 53,130. This is due to the fact that some railcars can only take one or two container sizes. Moreover, there are only few railcar types with four platforms and the minimum number is quite high because these few types can take several container dimensions (there are no single stack four platform railcars).

	Double-Stack Railcars				Single Stack Railcars			
Number of platforms	**AVG**	**MAX**	**MIN**	**STD**	**AVG**	**MAX**	**MIN**	**STD**
1	24	792	12	67	6	6	6	0
3	1,704	1,771	348	277	37	80	35	8
4	5,998	7,315	4,845	1,275	-	-	-	-
5	6,269	53,130	56	9,133	353	371	126	62

Table 3: Descriptive statistics on loading patterns for railcar types in the North American fleet

4.2 Modeling weight restrictions and the vertical center of gravity

The weights of containers loaded on any platform must satisfy both weight and center of gravity (COG) constraints. The former ensures that the total weight of the containers does not exceed the weight holding capacity of the platform, which in turn ensures that the weight holding capacity of the railcar is not exceeded.

Based on a standard solid-body formula, it is possible to compute an approximate maximum permissible weight limit that ensures that the COG does not exceed 98 inches above the top of the rail (Association American Railroads, 2014a). The approximation stems from the assumption that the load is uniformly distributed in the container (an exact formula would take the actual load distribution into account). This leads to a simple formula of $C = (BE + D_b F_b + D_t F_t)/(E + F_b + F_t)$ where C is the COG, E is the railcar tare and B the center of gravity of the empty railcar above top of the rail. D_b and D_t center of gravity of the bottom and top load above top of rail and F_b and F_t the weight of the bottom and top loads respectively. Since the COG must not exceed 98 inches above the top of the rail, we can compute the maximum weight limit F_t with respect to the threshold as

$$F_t = \frac{E(maxCOG - B) + F_b(maxCOG - D_b)}{(D_t - maxCOG)}. \tag{1}$$

We note that if the weight in the top slot is heavier than in the bottom slot, the COG constraint is always satisfied. We note that F_t depends on the height of the containers D_b and D_t (in bottom and top slot, respectively). Since there are low and high cube containers, we consider four possible cases that are illustrated in Figure 2. Finally we note that it is necessary to consider the case when two 20 ft containers occupy the bottom slot (all 20 ft containers have the same height). We

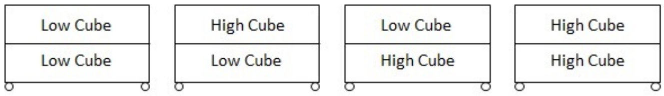

Figure 2: Different stacking loading patterns with respect to center of gravity restrictions

explain how this can be done through linear constraints in the following section where we present the ILP formulation for the load planning problem.

4.3 ILP formulation

A container $i \in N_h$, $N = \bigcup_{h \in H} N_h$, is characterized by its weight g_i, length l_i, size type $h \in H$, cost π_i and by possibly one or several good type(s) having specific loading rules. Let $N_s \subseteq N$ be the set of containers of good type $s \in S$, and because not all the containers have specific technical loading restrictions $\bigcup_{s \in S} N_s \subseteq N$.

A railcar $j \in J$ is characterized by its weight holding capacity G_j and a cost τ_j. We denote P the set of platforms of all railcars and P_j the set of platforms of railcar $j \in J$. Each platform p is characterized by its length L_p, its weight holding capacity G_p, and a sequence number γ_p, numbered from head to tail of the train. We denote Q the set of all slots, and Q_p the set of slots of a given platform p. Moreover, let μ_q be a binary parameter equal to 1 if $q \in Q$ is a bottom slot, 0 otherwise.

Railcars are defined by their type as presented in Section 4.1. For the sake of notational simplicity we denote K_j the set of loading patterns for railcar $j \in J$. Let $n^h_{k(p)}$ be the number of containers of type $h \in H$ in each platform p in loading pattern $k \in K$.

We have two types of *decision variables*. First, v_{iq} that equals one if container $i \in N$ is assigned to slot $q \in Q$ and zero otherwise. Second, w_{jk} that equals one if railcar $j \in J$ is assigned loading pattern $k \in K_j$ and zero otherwise. In addition, we need two types of auxiliary binary variables linking the assignment variables v_{iq} to platforms $p \in P$ and to railcars $j \in J$. More precisely, y_{ip} equals one if $i \in N$ is loaded on $p \in P$ and x_{ij} equals one if $i \in N$ is loaded on $j \in J$, 0 otherwise.

The model then becomes:

$$\min \sum_{i \in N} \pi_i \Big(\sum_{q \in Q} (1 - v_{iq})\Big) + \sum_{j \in J} \tau_j \Big(\sum_{k \in K_j} w_{jk}\Big) \tag{2}$$

$$\text{s.t} \qquad \sum_{q \in Q} v_{iq} \leq 1 \qquad \forall i \in N \tag{3}$$

$$y_{ip} = \sum_{q \in Q_p} v_{iq} \qquad \forall i \in N,\ \forall p \in P \qquad (4)$$

$$x_{ij} = \sum_{p \in P_j} y_{ip} \qquad \forall i \in N,\ \forall j \in J \qquad (5)$$

$$\sum_{k \in K_j} w_{jk} = 1 \qquad \forall j \in J \qquad (6)$$

$$\sum_{k \in K_j} n^h_{k(p)} w_{jk} = \sum_{i \in N_h} y_{ip} \qquad \forall p \in P_j,\ \forall j \in J,\ \forall h \in H \qquad (7)$$

$$\sum_{i \in N} \sum_{q \in Q_p} \mu_q v_{iq} l_i \leq L_p \qquad \forall p \in P \qquad (8)$$

$$\sum_{i \in N} y_{ip} g_i \leq G_p \qquad \forall p \in P \qquad (9)$$

$$\sum_{i \in N_{l_c}} \sum_{q \in Q_p} (1-\mu_q) v_{iq} g_i \leq \sum_{i \in N} \sum_{q \in Q_p} \mu_q v_{iq} c^p_{il_c} \qquad \forall p \in P \qquad (10)$$

$$\sum_{i \in N_{h_c}} \sum_{q \in Q_p} (1-\mu_q) v_{iq} g_i \leq \sum_{i \in N} \sum_{q \in Q_p} \mu_q v_{iq} c^p_{ih_c} \qquad \forall p \in P \qquad (11)$$

$$\sum_{j \in J} x_{ij} \ (G_j - \ D_W) \geq 0 \qquad \forall i \in N_{s_1} \qquad (12)$$

$$\sum_{j \in J} x_{ij} \ F_j = 0 \qquad \forall i \in N_{s_2} \qquad (13)$$

$$\sum_{j \in J} x_{ij} \ (\alpha_j U_j - \ g_i) \geq 0 \qquad \forall i \in N_{s_3} \qquad (14)$$

$$\sum_{q \in Q} v_{iq} \ (1 - \ \mu_q) = 0 \qquad \forall i \in N_{s_4} \qquad (15)$$

$$y_{ip} + \sum_{q \in Q_p} v_{i'q}(1-\mu_p) \leq 1 \qquad \forall i \in N_{s_5},\ \forall i' \in N \setminus i,\ \forall p \in P \qquad (16)$$

$$\sum_{p \in P} \gamma_p y_{ip} - \sum_{p \in P} \gamma_p y_{i'p} \leq R + (|P| - R)(1 - \sum_{p \in P} y_{i'p}) \quad \forall i, i' \in N_{s_6}, i \neq i' \qquad (17)$$

$$v_{iq} \in \{0,1\} \qquad \forall i \in N,\ \forall q \in Q \qquad (18)$$

$$y_{ip} \in \{0,1\} \qquad \forall i \in N,\ \forall p \in P \qquad (19)$$

$$x_{ij} \in \{0,1\} \qquad \forall i \in N,\ \forall j \in J \qquad (20)$$

$$w_{jk} \in \{0,1\} \qquad \forall j \in J,\ \forall k \in K \qquad (21)$$

The objective (2) of the ILP model seeks to minimize the total cost of containers left on the ground and of the used railcars, i.e. that are assigned at least one container. We note that $\sum_{k \in K_j} w_{jk} = 0$ if railcar j is not used by the model. This generalised cost leads to the maximization of the slot utilization on the used railcars in case of excess capacity.

There are five sets of loading constraints that we present in the following. The *assignment constraints* (3) ensure that each container $i \in N$ can be assigned to at most one slot $q \in Q$. For a given container $i \in N$, $\sum_{q \in Q} v_{iq} = 0$ implies that the container is not assigned to any slot and thus is left on the ground. Constraints (5) ensure that, for a given container $i \in N$, the auxiliary variable y_{ip} equals one if i is assigned to a slot q of platform p ($q \in Q_p$), and (6) ensure that the auxiliary variable x_{ij} equals one if i is assigned to one platform $p \in P_j$ of the railcar j.

The *loading pattern constraints* (7) ensure that exactly one loading pattern $k \in K_j$ is assigned to each railcar $j \in J$. We can link the variables w_{jk} to the variables y_{ip} through constraints (8), which enforce that the number of loaded containers of each type $h \in H$ on the platform $p \in P$ equals $n^h_{k(p)}$. Constraints (9) ensure that the length of the container(s) loaded in the bottom slot of a platform $p \in P$ does not exceed the length of the platform.

The *weight capacity constraints* (10) ensure that the maximum allowable weight limit of a platform is not exceeded by the total weights of the loaded containers.

The *center-of-gravity restrictions* are modelled by (11) and (12). Let N_{l_c} and N_{h_c} denote the set of low cube and high cube containers, respectively. c^p_{i1} and c^p_{i2} are respectively the low-cube and high-cube weight limit of the top slot for container $i \in N$ loaded in the bottom slot of platform $p \in P$, calculated using (1). The doubling of 20 ft containers at the bottom is taken care of while calculating c^p_{i1} and c^p_{i2} by replacing using the sum of the weights of the two 20 ft containers. This is a generic formulation to handle any combination of containers in the bottom slot through linear constraints.

In addition to dimensional and weight restrictions, there are also a variety of *technical loading restrictions* imposing or forbidding the loading of certain types of containers on specific railcars or slots. For the sake of illustration, we use six types of technical loading restrictions that have been derived for the North American market, but note that these can be easily extended to a specific company policy or regulations.

Constraints (13) state that containers $i \in N_{s_1} \subseteq N$ can only be loaded on railcars that have a certain minimum weight holding capacity D_W. Containers $i \in N_{s_2} \subseteq N$ have restrictions (14) on which railcar in the sequence of railcars they can loaded on. F_j is a pre-processed parameter that equals one if it is forbidden to load containers $i \in N_{s_2}$ on railcar j. Containers $i \in N_{s_3} \subseteq N$ can be loaded only on railcars with high weight capacity. For a given car $j \in J$, α_j equals one if

j can be used to load containers $i \in N_{s_3}$. Moreover let U_j be the maximum weight limit of a single container $i \in N_{s_3}$ that can be loaded on that car. Constraints (15) ensure that a container $i \in N_{s_3}$ is loaded on a railcar j with $\alpha_j = 1$, and g_i does not exceed U_j.

There are two types of stacking constraints. First, containers $i \in N_{s_4} \subseteq N$ cannot be loaded in the top slot (16). Second, containers $i \in N_{s_5} \subseteq N_{s_4} \subseteq N$ cannot be loaded in a top slot and cannot be double stacked (17).

Constraints (18) concern the storage of refrigerated containers that need a source of electrical power. For each pair of containers belonging to the set N_{s_6} loaded into the train, the distance among the two should not exceed R consecutive platforms on the train. Finally, expressions (19)-(22) define the domain of to the decision variables.

5 Numerical results

We present two numerical studies. The first is designed to assess the importance of containers-to-cars matching and center-of-gravity restrictions. The second one is designed to assess how computational time varies with instances having different characteristics.

Before presenting the numerical studies in more detail, we note that we have implemented the data processing, the loading pattern generation and the solution post processing in JAVA. All tests were run on an Intel(R) Core(TM) i5-5300U, 2.30 GHz CPU processor equipped with 24 GB RAM. The optimization model is solved using a 32-bit version of the IBM ILOG CPLEX 12.6 solver, with a pre-set computational time limit of 10 hours. The reported time only accounts for the CPLEX solver CPU time, the computational time associated with the other operations is negligible and loading patterns have been generated a priori.

5.1 Assessing the importance of containers-to-cars matching and center-of-gravity restrictions

In order to isolate and measure the effect of containers-to-cars matching and center-of-gravity restrictions we design a stylized experiment. It is based on a total of 396 instances, where we focus only on the main container and railcar characteristics, such as length and weight. The goal is to show that the train capacity estimation changes with respect to the instance composition. In the following, we first describe the instance generation, and then we present the results.

In all instances we keep the train length fixed to $5,000$ ft (1.54 km). The capacity in terms of number of slots can still vary since platforms have different lengths. More precisely, we define four railcar scenarios: one or five platform cars with 40 or 53 ft platforms. The resulting train capacity is 250 40 ft slots or 200 53 ft slots (train length divided by platform length times two). For each railcar scenario we choose one railcar type so that the set of loading patterns is the same for all railcars in a given scenario.

We consider 18 different scenarios for the container sets. The number of containers in each set equals the number of slots on the train. They have different characteristics in terms of mix of container sizes and weights. There are five different size mixes: 50 % 40 ft containers and 50 % 53 ft containers, 75 % 40 ft containers and 25 % 53 ft containers and vice versa, 100% 40 ft and 100% 53 ft. The containers are assigned weights in three different ways, two deterministic and one random. The deterministic cases represent favorable weight distributions, i.e. when the maximum capacity can be used because there are no issues related to the center of gravity. This is the case when either all containers have equal weight, or half of the containers are light and half heavy. The random weights are drawn from an uniform distribution in [8,000-62,000] lb, and [11,000-72,000] lb (Wikipedia, 2016b), and we generate 20 instances for each size mix. We define light and heavy to be the first and third quantile, respectively. The 18 different container set scenarios are denoted S1–S18 and each scenario has 22 instances (20 random and 2 deterministic) so we solve in total 396 instances.

Tables 4 and 5 describe the results for instances with one and five platform railcars, respectively. In both tables, the first two columns show the number of loaded containers and the number of used railcars in the optimal solution. The third column shows CPLEX solution time. Optimality gap is not reported because all the instances are solved to optimality. Note that in case of random weights, we report an average over the 20 instances.

The results show that the solution time is less than 200 seconds for one-platform railcars, while it increases to a maximum of 935 seconds for five-platforms railcars. This is due to the increased cardinality of sets of loading patterns K_j. In the case of 40 ft one platform railcars, the maximum number of containers that can be loaded, that is equal to the number of slots, is 250, but 53 ft containers can only be loaded in the top slot because of the platform length. So, as long as there are less than 125 53 ft containers in the instances (S1-S4 in Table 4), all slots can be used under favorable weight settings (deterministic instances). In Table 4, S5 is an example of scenario where the number of 53 ft containers exceeds the number of top slots, and thus, also under favorable weight settings, some of the containers cannot be loaded.

Due to loading patterns restrictions, fewer slots can be used on five-platform railcars. Indeed, as described in Section 4.1, it is not possible to load 53 ft containers in the top slots on two or more contiguous 40 ft length platforms. This can be seen in the results in Table 5 for S13 and S14. Regardless of the weights

250 CONTAINERS AVAILABLE	125 ONE 40ft PLATFORM RAILCARS AVAILABLE		
INSTANCE DESCRIPTION	LOADED CONTAINERS	USED RAILCARS	SOLUTION TIME [sec]
S1 : 250 40ft containers			
1) Containers same weights	250	125	17.8
2) Containers half low and half high weights	250	125	22.07
3) Containers random weights	250	125	46.3
S2: 200 40ft containers and 50 53ft containers			
1) Containers same weights	250	125	14.61
2) Containers half low and half high weights	250	125	22.02
3) Containers random weights	244	122.3	51.54
S3: 150 40ft containers and 100 53ft containers			
1) Containers same weights	250	125	11.21
2) Containers half low and half high weights	250	125	14.49
3) Containers random weights	238	119.5	76.0
S4: 125 40ft containers and 125 53ft containers			
1) Containers same weights	250	125	10.22
2) Containers half low and half high weights	250	125	11.49
3) Containers random weights	234	117.45	200.82
S5: 100 40ft containers and 150 53ft containers			
1) Containers same weights	200	100	9.28
2) Containers half low and half high weights	200	100	12.81
3) Containers random weights	200	100	35.62
200 CONTAINERS AVAILABLE	**100 ONE 53ft PLATFORM RAILCARS AVAILABLE**		
INSTANCE DESCRIPTION	LOADED CONTAINERS	USED RAILCARS	SOLUTION TIME [sec]
S6: 200 40ft containers			
1) Containers same weights	200	100	8.10
2) Containers low and high weights	200	100	16.22
3) Containers random weights	200	100	15.63
S7: 125 40ft containers and 75 53ft containers			
1) Containers same weights	200	100	9.69
2) Containers low and high weights	200	100	13.55
3) Containers random weights	200	100	23.47
S8: 75 40ft containers and 125 53ft containers			
1) Containers same weights	200	100	9.45
2) Containers low and high weights	200	100	15.77
3) Containers random weights	200	100	28.07
S9: 0 40ft containers and 200 53ft containers			
1) Containers same weights	200	100	9.84
2) Containers low and high weights	200	100	8.74
3) Containers random weights	200	100	27.09

Table 4: Importance of the matching problem and the center of gravity: number of loaded containers, number of used railcars and solution time for one platform railcars

250 CONTAINERS AVAILABLE	25 FIVE 40ft PLATFORM RAILCARS AVAILABLE		
INSTANCE DESCRIPTION	LOADED CONTAINERS	USED RAILCARS	SOLUTION TIME [sec]
S10: 250 40ft containers			
1) Containers same weights	250	25	111.69
2) Containers half low and half high weights	250	25	167.99
3) Containers random weights	250	25	178.14
S11: 200 40ft containers and 50 53ft containers			
1) Containers same weights	250	25	126.59
2) Containers half low and half high weights	250	25	132.83
3) Containers random weights	233	23.75	935.88
S12: 150 40ft containers and 100 53ft containers			
1) Containers same weights	250	25	119.32
2) Containers half low and half high weights	250	25	120.38
3) Containers random weights	219	23.6	764.72
S13: 125 40ft containers and 125 53ft containers			
1) Containers same weights	200	25	116.75
2) Containers half low and half high weights	200	25	127.73
3) Containers random weights	200	25	331.79
S14: 100 40ft containers and 150 53ft containers			
1) Containers same weights	175	25	113.67
2) Containers half low and half high weights	175	25	125.89
3) Containers random weights	175	25	329.88
200 CONTAINERS AVAILABLE	**20 FIVE 53ft PLATFORM RAILCARS AVAILABLE**		
INSTANCE DESCRIPTION	LOADED CONTAINERS	USED RAILCARS	SOLUTION TIME [sec]
S15: 200 40ft containers			
1) Containers same weights	200	20	514.81
2) Containers low and high weights	200	20	539.24
3) Containers random weights	200	20	733.83
S16: 125 40ft containers and 75 53ft containers			
1) Containers same weights	200	20	574.13
2) Containers low and high weights	200	20	636.86
3) Containers random weights	200	20	799.33
S17: 75 40ft containers and 125 53ft containers			
1) Containers same weights	200	20	471.43
2) Containers low and high weights	200	20	844.02
3) Containers random weights	200	20	932.13
S18: 0 40ft containers and 200 53ft containers			
1) Containers same weights	200	20	429.88
2) Containers low and high weights	200	20	513.44
3) Containers random weights	200	20	859.54

Table 5: Importance of the matching problem and the center of gravity: number of loaded containers, number of used railcars and solution time for five-platform railcars

(deterministic instances), all available slots cannot be used because of the high share of 53 ft containers. The different container weight settings clearly illustrate that the maximum capacity of 40 ft railcars can only be reached under favorable weight settings. In the case of random container weights when center-of-gravity restrictions play a role, the results clearly indicate a drop in the average number of loaded containers, even when the sizes match well. The 53 ft platform railcars are more flexible because they can take 53 ft containers also in the bottom position. Under all weight settings, it is possible to load all the containers. However, since 53 ft railcars are longer, there are only 200 slots compared to 250 of the 40 ft platform railcars.

In order to load as many containers as possible, there is a trade-off between using 53 ft versus 40 ft platforms and this trade-off depends on both the size of the containers and their weights. For example, the 250 slots on 40 ft platforms can only be used under the most favorable settings and, as the share of 53 ft containers increases (in particular for the random weight setting), the actual capacity is closer to 200 slots or even worse ($S14$).

This stylized numerical study shows that ignoring center-of-gravity restrictions and the containers-to-cars matching may lead to an overestimation of the available capacity of 40 ft railcars, and more in general, of the train.

5.2 Numerical results of computational time

In this section we present numerical results illustrating the computational time required to solve realistic instances of varying size. We generate instances with different characteristics by drawing railcars from the types available in the North American fleet. Similar to the previous section, we also generate container sets of varying size and where the containers have different characteristics. In the following we present the instance generation in detail.

Figure 3 shows an overview of the instance generation process. We consider three train lengths: 2,000 ft (0.6 km), 6,000 ft (1.8 km) and 10,000 ft (3 km). We note that trains in North America may be longer than in other parts of the world and 6,000 ft can already be considered long (Wikipedia, 2016). The length of 10,000 ft is mainly included for the sake of comparison.

For each train length we generate 20 sequences of railcars by sampling from the different railcar types. We use two different sampling protocols: simple random (10 sequences) and stratified random (10 sequences). We classify railcars in the North American fleet according to their flexibility in accommodating different container sizes. We compute the latter for each railcar type as the average number of loading patterns of each platform on the railcar. Figure 4 shows an histogram of the share of railcar types having different values of the flexibility index. We note that majority of the railcar types have a high value, which means that they can

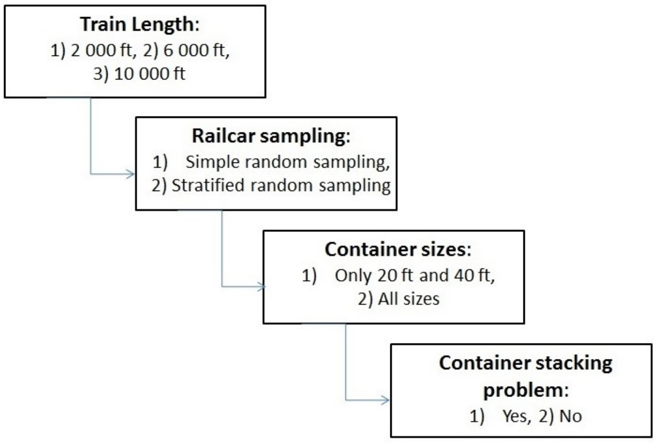

Figure 3: Overview of the instance generation process

load different sizes of containers in the slots. This implies that railcar sequences generated by simple random sampling have a higher share of railcars with high flexibility index value than the sequences generated by stratified sampling.

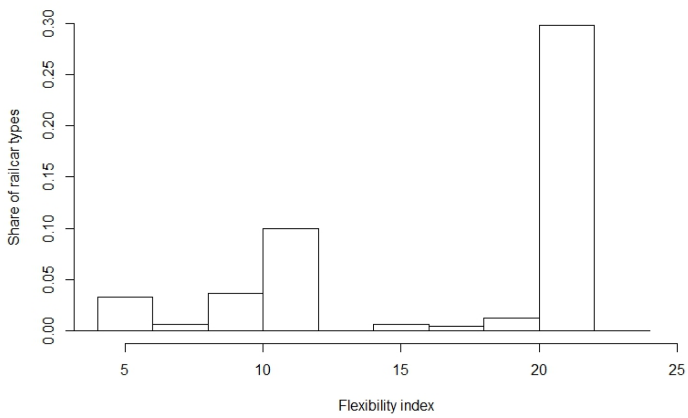

Figure 4: Share of North America railcar types with respect to the flexibility index

We now turn our attention to the sampling of containers. For each train length, and for each railcar sequence, we consider four sets of containers. The size of the sets is 1.5 times the number of slots in the railcar sequence which means that it should be possible to achieve close to a 100% slot utilization. There are two size mixes: one with only 20 ft and 40 ft containers and one with all sizes

(20,40,45,48 and 53 ft). The containers are assigned a weight by drawing from a weight distribution that is conditional on container size (same way as in the previous section). In order to assess the influence of technical loading restrictions (i.e. stacking problems) on the computational time, there are instances with and without containers having such restrictions. In the case of restrictions, 5% of the containers are randomly assigned one of technical loading restrictions defined in Section 4.3.

In total we solve 240 instances (six railcar sequence scenarios with 10 sequences each, and four container sets per railcar sequence). Table 6 presents the average computational times. The slot utilization is not reported since, as expected, it is close to 100 % for all instances.

The results show that we find an optimal solution for instances with short trains (2,000 ft) in less than 24 seconds on average, for all scenarios. The stratified random sampling contains a higher share of railcars with low flexibility index than the other which results in longer computational time. The average computational time is longer for instances having trains of 6,000 ft. Still, all instances can be solved in less than 17 minutes on average. We note that also the very large instances (10,000 ft), which are reported for the sake of comparison, can be solved to optimality. The most complex setting, namely the one with containers having all sizes and stacking restrictions, requires on average 3.5 hours. The cardinality of the loading pattern set has an important impact on computational time. This can clearly be seen by comparing the computational time for 20/40 ft size compared to all container sizes.

	Containers without technical loading restrictions		**Containers with technical loading restrictions**	
Train length / Railcar sampling protocol	**20 and 40 ft**	**All sizes**	**20 and 40 ft**	**All sizes**
2,000 ft				
Random sampling	7.11	13.10	7.95	14.36
Stratified random sampling	11.97	21.20	12.92	24.74
6,000 ft				
Random sampling	184.59	450.96	661.04	636.37
Stratified random sampling	209.15	576.12	377.63	1,077.98
10,000 ft				
Random sampling	967.42	4,010.52	1,963.78	8,266.35
Stratified random sampling	1,653.16	4,217.13	2,755.56	13,254.41

Table 6: Average computational time in seconds

This numerical study shows that we can solve realistic size instances to optimality in reasonable computational time, even for very long trains. In this context it is worth mentioning that that the train loading can be decomposed into smaller load planning problems. Indeed, railcars are typically grouped according to intermediate destinations, so-called blocks. The load planning problem can hence be solved by block and not by train. For more details about the blocking problem we refer to e.g. Barnhart et al. (2000), Bodin et al. (1980), Newton et al. (1998).

6 Conclusions and future research directions

In this paper we studied the load planning problem for double-stack intermodal trains. Given a set of containers stored in a terminal and a departing train, the objective is to select the optimal set of containers to load and the optimal way of loading them, using the maximum of the available capacity. Previous studies in the literature either do not address the load planning problem for double-stack trains or make simplifying assumptions that may lead to load plans that violate important loading rules. The problem related to double-stack trains is challenging because the load plan must respect a number of loading rules that depend on container and railcars characteristics such as containers-to-cars matching and center-of-gravity restrictions.

We formulated an ILP model and made a number of contributions. First, we proposed a general methodology that can deal with double- or single-stack railcars with arbitrary loading patterns. The patterns account for loading dependencies between the platforms on a given railcar. Second, we modelled center-of-gravity restrictions, stacking rules and a number of technical loading restrictions that are associated with certain types of containers and/or goods.

We presented two numerical studies showing, on the one hand, that we can solve realistic size instances in reasonable time using a commercial ILP solver, and on the other hand, that failing to account for containers-to-cars matching as well as center-of-gravity restrictions may lead to an overestimation of the available train capacity. The results showed that the computational time varies with the size and characteristics of the instances. For example, it is more time consuming to solve instances with five platform railcars and several container sizes compared to fewer platforms and only 20 and 40 ft containers. This is due to the cardinality of the sets of loading patterns. It is also more time consuming to solve instances with containers having technical loading restrictions than those without.

The proposed methodology can be used in decision-aid tools for terminal managers in charge of the load planning. It can also be used in a more tactical or strategic planning setting to assess railcar fleet management decisions. We also note that the model can be used to plan several trains ahead under perfect information, similar to Lai et al. (2008b).

There are several possible directions for future research. The model can be extended to consider handling costs in the yard, for example, by selecting containers according to their location in stacks. Furthermore, several aspects of the problem may be subject to uncertainty. For example, the availability and characteristics of containers and railcars. Modelling this uncertainty is another topic of future research.

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