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A Stochastic Programming Approach for the Capacitated Supplier Selection Problem with Total Quantity Discount and Activation Costs

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Abstract. We study the Capacitated Supplier Selection problem with Total Quantity Discount policy and Activation Costs, a procurement problem where a company needs a certain quantity of different products from a set of potential suppliers, and introduce its variant under uncertainty. In its deterministic form, the problem aims at selecting a subset of the suppliers and the relative purchasing plan satisfying the demands at minimum cost, taking into account that the suppliers offer discounts based on the total quantity of products purchased and that the activation of a business activity with a supplier has a fixed cost. However, due to the long-term nature of the problem, several parameters may be affected by uncertainty. Thus, we propose a two-stage stochastic programming formulation with recourse, highlighting the strategic and the operational decisions involved, as well as the effect of the different sources of uncertainty. In particular, we focus on the cases in which only the products price or only the products demand are stochastic. The general model and the recourse actions are adapted for these special cases, and the resulting modeling approaches are validated on a large set of instances. The experiments show the convenience of having in place models considering uncertainty explicitly with respect to using expected values for approximating it, and give rise to interesting managerial insights. Due to the computational burden of solving the resulting stochastic models (for a sufficiently large number of scenarios), we also propose a simple solution framework based on valid inequalities and other accelerating mechanisms.

Keywords. Procurement logistics, total quantity discount, stochastic prices, stochastic demands, stochastic programming.

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1. Introduction

Procurement logistics problems mainly concern the operational decisions of a company that needs to buy products/raw materials from several potential suppliers. Since a large portion of a firm total cost is represented by procurement expenditure, regardless of the type of purchased goods, it is of crucial importance to optimize this aspect through the use of sophisticated mathematical models and efficient solution algorithms. That is why, despite of its long history in the specialized literature, procurement problems still foster new research contributions (see, e.g., Manerba, 2015).

In general, decisions underlying the procurement processes aim to elaborate a purchasing plan that adequately satisfies an internal demand while minimizing the procurement costs depending on product prices. Let M be a set of suppliers, indexed by i, and let K be a set of products, indexed by k. Each product $k \in K$, for which a positive integer demand d_k is required, can be purchased in a subset $M_k \subseteq M$ of suppliers at a positive basic price f_{ik}, potentially different for each supplier $i \in M_k$. The so-called Supplier Selection (SS) problem consists in deciding which suppliers have to be visited and which amount of each product has to be purchased in each visited supplier (Aissaoui et al., 2007). However, real procurement settings may be complicated by several factors making the basic SS problems inadequate to provide a solution to the actual decision process. Common complicating factors are:

- **restricted product availabilities:** when a supplier cannot guarantee a priori to satisfy completely a product demand, then the purchase of each product has to be split over different suppliers thus complicating the creation of a purchasing plan. In this work we assume that, for each product $k \in K$, a quantity q_{ik} is available at each supplier $i \in M_k$ (*capacitated* SS);
- **discount policies:** to be more competitive, suppliers often try to push-up their sales by offering discounts.) In this paper we assume that all the suppliers propose the so-called *total quantity discount* (TQD), a policy in which the cumulative quantity purchased (i.e., the number of units bought regardless of the type of products involved) determines the discount rate applied by the supplier to the total purchase cost. More precisely, each supplier $i \in M$ defines a set $R_i = \{1, \dots, r_i\}$, indexed by r, of r_i consecutive and non-overlapping intervals represented by $[l_{ir}, u_{ir}]$ and associated with a discount rate $\delta_{ir} \in [0, 1)$ such that $\delta_{i,r+1} \geq \delta_{ir}$ $r = 1, ..., r_i - 1$ (i.e., the higher the interval, the greater the discount). Then, for each supplier $i \in M$, the discount rate δ_{ir} is applied to the total purchase cost if the total quantity purchased lies in the interval $r \in R_i$, i.e., is greater than or equal to l_{ir} and less than or equal to u_{ir}. An example of the cost function for a specific supplier i offering a TQD policy is plotted in Figure 1, where only the lower bound of each interval is indicated;
- **contract activation costs:** in general, clients can benefit from the discounts only by previously activating a contract with the selected suppliers. In this paper we consider a cost a_i for each supplier $i \in M$, corresponding to the fee that the company has to paid in order to undertake a business activity with that supplier. Trade-offs between these activation costs and the possible saving due to the discounted purchase further complicate the supplier selection;

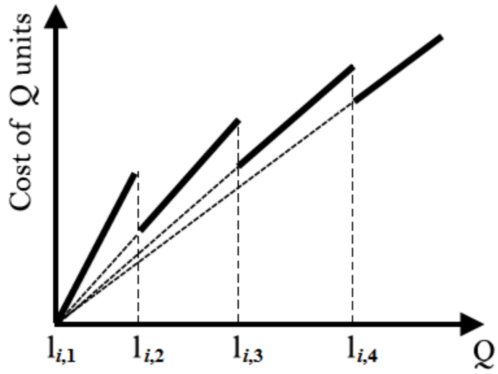


Figure 1: Piece-wise linear function representing the cost of buying Q units from a supplier i offering a TQD policy.

- **data uncertainty:** the high competition on the markets and the globalization, with the consequent larger possibility of products availability but also with a longer time for obtaining them (due to the delocalization of the factories), push the companies to sign long-term purchasing contracts in order to sustain their offer. This increases the volatility of some parameters particularly affected by uncertainty. For example, a precise forecast of the future product demand is hard to obtain since it depends from several unknown a-priori internal and external factors. Again, product prices and availabilities at the suppliers are also subject to fluctuations due to market and environmental conditions. Since using approximated/estimated data may results in non-convenient or even infeasible purchasing plans, when the actual information reveals, in this work we propose an explicit modeling of the possible sources of uncertainty for the problem.

Under this setting, some research questions arise. First, how the data uncertainty can affect the supplier selection when involving total quantity discounts? Second, can we incorporate this uncertainty in a model able to ensure a competitive advantage to the company?

To face the aforementioned questions, we consider the Capacitated Supplier Selection problem with Total Quantity Discount policy and Activation Costs (CTQD-AC) and introduce its variant under uncertainty (CTQD-AC$_u$). The problem aims at selecting a subset of the suppliers and the relative purchasing plan that satisfies the demands at minimum cost, taking into account that the suppliers offer discounts based on the total quantity of products purchased and that the activation of a business activity with a supplier has a fixed cost. In order to tackle different sources of uncertainty we propose two-stage Stochastic Programming (SP) formulations with recourse, highlighting the strategical and the operational decisions involved. To the best of our knowledge, this represents the first study concerning the CTQD-AC problem under uncertainty.

The rest of the paper is organized as follows. In Section 2, we review the literature concerning SS problems with quantity discount policies and stochastic data. In Section 3, we propose a mixed-integer linear programming (MILP) formulation for the deterministic CTQD-AC. In Section 4, we study the possible sources of uncertainty of the problem and propose a two-stage SP model to explicitly cope with them. In particular, we focus on the cases in which only the product prices

(Sections 4.2) or only the product demands (Section 4.3) are stochastic variables. A solution framework is presented in Section 5, whereas the generation of the deterministic benchmark instances and that of the scenario trees are presented in Section 6. Section 7 is devoted to the experimental validation of our modeling and solution approaches and to the discussion of some economic/managerial insights. Finally, conclusions and possible future extensions are drawn in Section 8.

2. Literature review

Quantity discounts, i.e. price discounts provided by suppliers with respect to large orders of products (Munson and Rosenblatt, 1998), have existed as ubiquitous tools of commerce incentive for hundreds of years within any type of application context. Evidently, they have a great impact on purchasing, transportation, and inventory costs as well as on marketing and supply-chain coordination goals. In a recent monograph, Munson and Jackson (2015) review the most relevant quantity discount scenarios from both the buyer's and seller's perspectives. Concerning the buyer's side, it emerges that the most common issues to face are about a) how many units should be ordered when suppliers offer quantity discount schedule, and b) in which conditions should a buyer attempt to negotiate a discount schedule from its suppliers. In particular, the order sizing issue, complicated by the selection of suppliers among a predefined set, represents the SS problem already discussed in the previous section and has a long story in the specialized literature (see, e.g., Benton, 1991).

Since the last two decades, the SS problem and lots of its variants involving quantity discounts have been studied under a quantitative perspective and the assumption of deterministic data. For these problems, mathematical programs (mostly MILPs) have been developed and both exact and heuristic solution algorithms have been proposed. For example, Mirmohammadi et al. (2009) consider a single-item multi-period material requirement planning problem and propose a branch-and-bound algorithm exploiting some properties as fathoming rules. Munson and Hu (2010) analyze the inventory impact of incorporating quantity discounts into centralized purchasing scenarios for a multi-site organization, whereas Krichen et al. (2011) study the convenience of a retailers coalition in the presence of a single-supplier and permissible delay in payments through cooperative game theory. Recently, Jolai et al. (2013) propose a multi-objective mixed integer nonlinear programming model for solving a multi-item multi-period and multi-supplier problem taking into account linear discount pricing scheme, the limits on the suppliers availability, the delivery rate and the quality of the items, the minimum order quantities, and the budgetary limitations. In the present paper we study a multi-supplier and multi-product SS problem considering *total quantity discounts* and fixed costs for contract activations.

Different discount policies have been studied in the literature, including *incremental discount*, *fixed fees*, *truckload discount*, but the total quantity (also called *all-unit*) discount represents the most popular form applied so far (Munson and Jackson, 2015). A motivation for this preference depends on the fact that TQD fits well many multi-product procurement settings where the purchase is completed at a single point in time, i.e. without auctions or other rebate mechanisms. The existence of a single point in time for completing the purchase under a TQD policy is forced by the fact that further purchases achieving an higher discount interval may results in a smaller total cost (look at point discontinuities of the function plotted in Figure 1), thus in the necessity for suppliers to give back money. Application contexts in which total quantity discounts have been used are incredibly various, from dairy (McConnel and Galligan, 2004) to chemical industry (Crama et al., 2004), from project's resource investment (Shahsavar et al., 2016) to telecommunication systems (van de Klundert et al., 2005). Goossens et al. (2007) study the basic SS procurement problem, in which suppliers are assumed to have unlimited availability for offered products, under the presence of total quantity discounts and show its $\mathcal{NP}$-completeness by reduction from the 3-Dimensional Matching Problem, also demonstrating that it can not be solved by a polynomial-time approximation algorithm with a constant ratio (unless $\mathcal{P} = \mathcal{NP}$). They propose a branch-and-bound algorithm based on a min-cost flow formulation of the problem and also study four variants taking into account market share constraints, the possibility of buying more than products demand (*more-for-less*), a limited number of winning suppliers, and multi-period scenarios, respectively. The capacitated version of this problem (Capacitated Total Quantity Discount Problem, CTQDP), in which quantities of product available at suppliers are limited, has been studied in Manerba and Mansini (2012b, 2014) where the authors propose efficient branch-and-cut and Variable Neighborhood Decomposition Search (VNDS) matheuristic solution approaches. The CTQDP has been extended to include transportation costs based on truckload shipping rates in Mansini et al. (2012). The authors propose iterative rounding schemes based on the linear programming relaxation to heuristically solve the problem and demonstrate their efficiency on a large set of randomly generated instances.

There also exist some works considering more than one type of discounts simultaneously. For example, Stadtler (2007) creates an intricate MIP to handle, in the same model, both all-units and incremental discounts for a multi-period SS problem. More recently, Ebrahim et al. (2009) propose a multi-objective mathematical model and a scatter search algorithm for the solution of a single-item purchasing problem considering all-unit, incremental, and total business volume discounts.

Contrarily to the conspicuous literature on supplier selection problems under data uncertainty (Anupindi and Akella, 1993, Awasthi et al., 2009), only few contributions can be found about stochasticity in the presence of quantity discount schedules. Sen et al. (2013) consider a multi-item, multi-period, and multi-supplier problem involving quantity discounts and formulate a scenario-based multi-stage stochastic optimization model. They also propose certainty-equivalent heuristics and evaluate them for three bidding events (involving random events such as a drop in price, a price change in the spot market or a new discount offer) of a large manufacturing company. More recently, Hammami et al. (2014) develop a two-stage stochastic programming model for a SS problem (faced by a multi-site buyer in the context of automotive manufacturing) integrating the exchange rate fluctuation uncertainties with price discounts while explicitly considering transportation and inventory costs. Finally, even if not explicitly including discounts, Zhang and Zhang (2011) address a SS problem with a similar combinatorial structure, given by the presence of fixed selection costs and limitation on minimum and maximum order sizes. Since the demand is stochastic, in their model a penalty cost or an holding cost is incurred if the ordered quantity is less or more than the realized demand, respectively.

We precise that the procurement setting studied in this paper assumes that transporting the products from the suppliers to the company's depot is an outsourced operation, and it does not take part of the optimization. Instead, if a company uses its own fleet of vehicles to achieve the procurement process, at an operating level, each involved vehicle has also to be routed through its purchasing trip so to minimize the traveling costs as well. The single and multi-vehicle Traveling Purchaser Problem (TPP) are well-known routing problems combining purchasing costs, traveling costs, and supplier selection (Manerba et al., 2017) that find many applications in procurement logistics contexts (Beraldi et al., 2017, Gendreau et al., 2016). Interesting enough, a TPP variant involving total quantity discounts is studied in Manerba and Mansini (2012a) where a branch-and-cut approach, exploiting valid inequalities and matheuristic strategies for the TPP and CTQDP subproblems, is proposed. As far as we know, the only other work integrating quantity discount and routing decisions is proposed by Nguyen et al. (2014). In particular, they study an Open Vehicle Routing Problem where the trucks utilization is optimized by building multi-stop routes and by increasing order sizes through purchase incentives. In this case, however, the problem is applied in the outbound transportation context, where the decision maker is a seller.

3. The deterministic CTQD-AC

Let x_i be a binary variable taking value 1 if a purchasing contract is activated with supplier $i \in M$ and the corresponding activation cost is paid, and 0 otherwise. Let z_{ikr} be a variable representing the units of product $k \in K$ purchased from supplier $i \in M_k$ in interval $r \in R_i$ and yielding a discount rate δ_{ir}. Finally, let y_{ir} be a binary variable taking value 1 if supplier $i \in M$ applies the discount interval $r \in R_i$ (i.e., if $\sum_{k \in K} z_{ikr} \in [l_{ir}, u_{ir}]$), and 0 otherwise. Then, a MIP formulation for the CTQD-AC is as follows:

$$\min \quad \sum_{i \in M} a_i x_i + \sum_{k \in K} \sum_{i \in M_k} \sum_{r \in R_i} (1 - \delta_{ir}) f_{ik} z_{ikr} \tag{1}$$

subject to

$$\sum_{i \in M_k} \sum_{r \in R_i} z_{ikr} \geq d_k \qquad k \in K \tag{2}$$

$$\sum_{r \in R_i} z_{ikr} \leq q_{ik} \qquad k \in K, i \in M_k \tag{3}$$

$$l_{ir} y_{ir} \leq \sum_{k \in K} z_{ikr} \leq u_{ir} y_{ir} \qquad i \in M, r \in R_i \tag{4}$$

$$\sum_{r \in R_i} y_{ir} \leq x_i \qquad i \in M \tag{5}$$

$$z_{ikr} \geq 0 \qquad k \in K, i \in M_k, r \in R_i \tag{6}$$

$$x_i \in \{0, 1\} \qquad i \in M \tag{7}$$

$$y_{ir} \in \{0, 1\} \qquad i \in M, r \in R_i. \tag{8}$$

Objective function (1) establishes the minimization of the sum of activation and purchasing costs. Constraints (2) ensure that the demand d_k for each product $k \in K$ is satisfied, whereas constraints (3) state that it is not possible to purchase from supplier i an amount of product k larger than the quantity available. Constraints (4) define interval bounds for each supplier. If interval r for supplier i is selected ($y_{ir} = 1$), then the total amount purchased has to lie between the lower bound l_{ir} and the upper bound u_{ir}. On the contrary, if interval r is not selected ($y_{ir} = 0$), then $\sum_{k \in K} z_{ikr} = 0$. Constraints (5) guarantee that at most one interval for each supplier is selected if supplier i is visited ($x_i = 1$), and that no interval is selected if the supplier is not visited ($x_i = 0$). Finally, constraints (6)–(8) are non-negativity and binary conditions on variables. Note that, the integrality of z variables is not explicitly declared in the formulation since a solution with this property always exists if demands, products availabilities, and lower/upper bounds are integral values.

In order to avoid pathologies, we make some assumptions. First, note that a feasible plan can exist only if the total availability of each product over the suppliers is greater or equal to its demand, hence we will assume $\sum_{i \in M_k} q_{ik} \geq d_k, \forall k \in K$. Second, to guarantee the correct application of the discount policy to the entire purchase, we assume $l_{i1} = 0, \forall i \in M$ and $\sum_{k \in K} q_{ik} \leq u_{i,r_i}, \forall i \in M$.

The CTQD-AC generalizes the *more-for-less* TQD problem variant described in Goossens et al. (2007) where suppliers have unlimited availability for offered products and activation costs are not considered. In this variant, as stated in constraints (2) of our model, it is allowed to buy more than the required demand d_k, for each product k, in order to achieve higher discounts. In turn, this implies that it is possible to buy more quantities from a supplier paying less because of the discount applies to the entire purchase (look at the points of discontinuity in Figure 1). This anomaly, along with the behavioral cooperation possibilities in order to take advantage from that, is studied for example in Jucker and Rosenblatt (1985). Even if, in the original TQD problem, the product demand has to be satisfied exactly, we consider the *more-for-less* variant because it better models a real long-term procurement process as the one we want to study.

4. The CTQD-AC under uncertainty

In this section, we study all the possible sources of uncertainty for the just presented CTQD-AC. Even if unpredictability could theoretically affect each type of data, in this problem it makes sense to consider it only on the product prices f_{ik}, on the product demand d_k, and on the product availability q_{ik}. These data represent, in fact, information strongly related to operational decisions (as the daily construction of a purchasing plan) that are the most affected by variability. We are confident that all the remaining problem data, as the ones related to the discount policies or the activation costs proposed by the suppliers, can be considered deterministic over the optimization horizon, and known a priori (actually, they are part of the contract clauses stipulated with the suppliers). We denote the resulting stochastic problem as CTQD-AC$_u$.

4.1. A general two-stage SP formulation for CTQD-AC$_u$

In the following, we propose a two-stage stochastic programming formulation for the CTQD-AC in which the product prices, the demands, and the availabilities are stochastic variables. We assume that each uncertain parameter consists of two components, an estimated deterministic component and a stochastic oscillation. More precisely, given a product $k \in K$, let d_k and $d_k^{\Delta}(\xi)$ be the estimated deterministic component and the stochastic oscillation of each product demand, respectively. Similarly, given a product $k \in K$ and a supplier $i \in M_k$, let f_{ik} and $f_{ik}^{\Delta}(\xi)$ be the estimated deterministic component and the stochastic oscillation of each product price, respectively, whereas let q_{ik} and $q_{ik}^{\Delta}(\xi)$ be the estimated deterministic component and the stochastic oscillation of each product availability, respectively. To ensure the feasibility of the stochastic problem, we must assume that the condition $\sum_{i \in M_k}(q_{ik} + q_{ik}^{\Delta}) \geq d_k + d_k^{\Delta}$ holds for each $k \in K$, too.

The first-stage decision is about which suppliers are involved in the purchasing, how much we expect to purchase from each supplier and, consequently, in which discount interval we expect the total quantity of products purchased lies. The second-stage recourse decision deals instead with adjusting the quantity purchased of each product in each supplier, thus possibly impacting also on the chosen discount interval for each supplier (that may be changed with respect to the first-stage decisions). Note that this last recourse action is necessary to obtain a stochastic programming formulation with complete recourse, given that product demands and availabilities are stochastic. Instead, we do not allow the possibility to activate new contracts or to exclude any contracts with respect to those already decided in the first stage. Then, the CTQD-AC$_u$ problem can be stated as follows:

$$\min \sum_{i \in M} a_i x_i + \sum_{k \in K} \sum_{i \in M_k} \sum_{r \in R_i} (1 - \delta_{ir}) f_{ik} z_{ikr} + \mathbb{E}[Q(\mathbf{x}, \mathbf{y}, \mathbf{z}, \xi)] \tag{9}$$

subject to

$$\sum_{i \in M_k} \sum_{r \in R_i} z_{ikr} \geq d_k \qquad k \in K \tag{10}$$

$$\sum_{r \in R_i} z_{ikr} \leq q_{ik} \qquad k \in K, i \in M_k \tag{11}$$

$$l_{ir} y_{ir} \leq \sum_{k \in K} z_{ikr} \leq u_{ir} y_{ir} \qquad i \in M, r \in R_i \tag{12}$$

$$\sum_{r \in R_i} y_{ir} \leq x_i \qquad i \in M \tag{13}$$

$$z_{ikr} \geq 0 \qquad k \in K, i \in M_k, r \in R_i \tag{14}$$

$$x_i \in \{0, 1\} \qquad i \in M \tag{15}$$

$$y_{ir} \in \{0, 1\} \qquad i \in M, r \in R_i. \tag{16}$$

Here, the constraints have been already explained, whereas the objective function is further guided through the expected value of a function $Q(\mathbf{x}, \mathbf{y}, \mathbf{z}, \xi)$ corresponding to the second-stage optimization problem. In order to formulate this problem, we introduce the following second-stage variables:

- z_{ikr}^{2+} := extra units of product k purchased from supplier i in interval r with respect to the first stage decision, $k \in K$, $i \in M_k$, $r \in R_i$;
- z_{ikr}^{2-} := reduction in units of purchased product k from supplier i in interval r with respect to the first stage decision, $k \in K$, $i \in M_k$, $r \in R_i$;

- $y^2_{ir} := \begin{cases} 1 & \text{if } \sum_{k\in K}(z_{ikr} + z^{2+}_{ikr} - z^{2-}_{ikr}) \in [l_{ir}, u_{ir}] \\ 0 & \text{otherwise} \end{cases}, \quad i \in M,\ r \in R_i;$
- $\lambda^{2+}_{ikr} := 1$ if an extra purchase occurs for product k from supplier i in interval r, 0 otherwise, $k \in K$, $i \in M_k$, $r \in R_i$;
- $\lambda^{2-}_{ikr} := 1$ if a reduction of the purchase occurs for product k from supplier i in interval r, 0 otherwise, $k \in K$, $i \in M_k$, $r \in R_i$.

Then, the second-stage optimization problem is as follows:

$$Q(\mathbf{x}, \mathbf{y}, \mathbf{z}, \xi) := \min \sum_{k\in K}\sum_{i\in M_k}\sum_{r\in R_i}(1-\delta_{ir})[f^{\Delta}_{ik}(z_{ikr} + z^{2+}_{ikr} - z^{2-}_{ikr}) + f_{ik}(z^{2+}_{ikr} - z^{2-}_{ikr})] \tag{17}$$

subject to

$$\sum_{i\in M_k}\sum_{r\in R_i}(z_{ikr} + z^{2+}_{ikr} - z^{2-}_{ikr}) \geq d_k + d^{\Delta}_k \qquad k \in K \tag{18}$$

$$\sum_{r\in R_i}(z_{ikr} + z^{2+}_{ikr} - z^{2-}_{ikr}) \leq q_{ik} + q^{\Delta}_{ik} \qquad k \in K, i \in M_k \tag{19}$$

$$l_{ir}y^2_{ik} \leq \sum_{k\in K}(z_{ikr} + z^{2+}_{ikr} - z^{2-}_{ikr}) \leq u_{ir}y^2_{ik} \qquad i \in M, r \in R_i \tag{20}$$

$$\sum_{r\in R_i} y^2_{ir} \leq x_i \qquad i \in M \tag{21}$$

$$z^{2-}_{ikr} \leq z_{ikr} \qquad k \in K, i \in M_k, r \in R_i \tag{22}$$

$$z^{2+}_{ikr} \leq (q_{ik} + q^{\Delta}_{ik})\lambda^{2+}_{ikr} \qquad k \in K, i \in M_k, r \in R_i \tag{23}$$

$$z^{2-}_{ikr} \leq (q_{ik} + q^{\Delta}_{ik})\lambda^{2-}_{ikr} \qquad k \in K, i \in M_k, r \in R_i \tag{24}$$

$$\lambda^{2+}_{ikr} + \lambda^{2-}_{ikr} \leq 1 \qquad k \in K, i \in M_k, r \in R_i \tag{25}$$

$$z^{2+}_{ikr}, z^{2-}_{ikr} \geq 0 \qquad k \in K, i \in M_k, r \in R_i \tag{26}$$

$$\lambda^{2+}_{ikr}, \lambda^{2-}_{ikr} \in \{0, 1\} \qquad k \in K, i \in M_k, r \in R_i \tag{27}$$

Objective function (17) establishes the minimization of the purchasing costs derived from the variation of the purchased quantities and from the price oscillation. Constraints (18) ensure that the real demand (i.e., the expected one plus its oscillation) for each product is satisfied. Constraints (19) state that it is not possible to purchase from supplier i an amount of product k larger than the real availability (i.e., the expected one plus its oscillation). Constraints (20) define interval bounds for each supplier with respect to the second-stage binary variables y^2. If interval r for supplier i is selected ($y^2_{ir} = 1$), then the total amount purchased has to lie between the lower bound l_{ir} and the upper bound u_{ir}. On the contrary, if interval r is not selected ($y^2_{ir} = 0$), then $\sum_{k\in K}(z_{ikr} + z^{2+}_{ikr} - z^{2-}_{ikr}) = 0$. Constraints (21) guarantee that at most one interval r for each supplier is selected if supplier i is visited ($x_i = 1$), and that no interval is selected if the supplier is not visited ($x_i = 0$). Constraints (22) state that a product purchase can not be reduced more than the quantity decided in the first stage. Constraints (23)–(25) together ensure the orthogonality between z_{ikr}^{2+} and z_{ikr}^{2-} through the use of binary variables λ_{ikr}^{2+} and λ_{ikr}^{2-}. This way, the purchase of a product cannot be increased and reduced by a positive quantity at the same time. Finally, constraints (26)–(27) state non-negativity and binary conditions on variables.

The above presented model is very general and exploits the separation of strategic from operative decisions of the problem guaranteeing a complete recourse. However, it has two main drawbacks:

1. all the interdependences among the different stochastic variables in the problem constraints make hard to point out any useful consideration about the uncertainty without having previously considered the various stochastic aspects by themselves;
2. the proposed recourse action, i.e. the possibility to modify the discount interval for each supplier on the base of the current realization of the uncertainties, is not very realistic in a long-term procurement setting. In general, in fact, the purchaser and the supplier decide the interval in the contract clauses, i.e. the established discount is granted only if minimum and maximum quantities are respected.

In order to overcome this situation, in the following, we will focus on the particular cases in which only the products price or only the products demand is stochastic. We denote these problems as CTQD-AC$_{up}$ and CTQD-AC$_{ud}$, respectively. For them we will propose two-stage SP formulations in which the decisions about the discount intervals are locked by the first stage and the recourse actions consist in modifying the purchased quantities within a predefined interval, or, if necessary, to purchase outside from the selected suppliers to satisfy the demand.

Despite of the importance of all the sources of uncertainty discussed, product prices and demands represent the stronger uncertain factors in these type of procurement settings. Instead, minimum available quantities for products may be guaranteed by the suppliers within the contract activation clauses.

4.2. The CTQD-AC with stochastic prices (CTQD-AC$_{up}$)

As already said, the only considered recourse action in this case is adjusting the purchased quantities, given that discount intervals are locked by the first-stage decision for each supplier. Hence, the two-stage stochastic programming formulation for CTQD-AC$_{up}$ reduces to the same first stage (9)–(16) in which the recourse function is as follows:

$$Q(\mathbf{x}, \mathbf{y}, \mathbf{z}, \xi) := \min \sum_{k \in K} \sum_{i \in M_k} \sum_{r \in R_i} (1 - \delta_{ir})[f_{ik}^{\Delta}(z_{ikr} + z_{ikr}^{2+} - z_{ikr}^{2-}) + f_{ik}(z_{ikr}^{2+} - z_{ikr}^{2-})] \tag{28}$$

subject to

$$\sum_{i \in M_k} \sum_{r \in R_i} (z_{ikr} + z_{ikr}^{2+} - z_{ikr}^{2-}) \geq d_k \qquad k \in K \tag{29}$$

$$\sum_{r \in R_i} (z_{ikr} + z_{ikr}^{2+} - z_{ikr}^{2-}) \leq q_{ik} \qquad k \in K, i \in M_k \tag{30}$$

$$l_{ir} y_{ik} \leq \sum_{k \in K} (z_{ikr} + z_{ikr}^{2+} - z_{ikr}^{2-}) \leq u_{ir} y_{ik} \qquad i \in M, r \in R_i \tag{31}$$

$$z^{2-}_{ikr} \leq z_{ikr} \qquad k \in K, i \in M_k, r \in R_i \tag{32}$$
$$z^{2+}_{ikr} \leq q_{ik}\lambda^{2+}_{ikr} \qquad k \in K, i \in M_k, r \in R_i \tag{33}$$
$$z^{2-}_{ikr} \leq q_{ik}\lambda^{2-}_{ikr} \qquad k \in K, i \in M_k, r \in R_i \tag{34}$$
$$\lambda^{2+}_{ikr} + \lambda^{2-}_{ikr} \leq y_{ir} \qquad k \in K, i \in M_k, r \in R_i \tag{35}$$
$$z^{2+}_{ikr}, z^{2-}_{ikr} \geq 0 \qquad k \in K, i \in M_k, r \in R_i \tag{36}$$
$$\lambda^{2+}_{ikr}, \lambda^{2-}_{ikr} \in \{0,1\} \qquad k \in K, i \in M_k, r \in R_i. \tag{37}$$

The two-stage model proposed for the problem with stochastic prices cannot be solved in the present form because of the difficult evaluation of the second-stage objective function that involves the calculation of a multidimensional integral. In the practical applications (see, e.g., Wallace and Ziemba, 2005), it is common to assume that the random vector ξ follows a known discrete distribution involving a finite number of possible scenarios and to solve the so-called Deterministic Equivalent Problem (DEP) considering all those scenarios. More precisely, we explicitly consider a set S of potential scenarios. Each scenario $s \in S$ is associated with a realization of the price oscillation f^{Δ}_{iks} occurring with probability p_s, such that the standard axiom $\sum_{s\in S} p_s = 1$ is satisfied. Then, the DEP of the two-stage stochastic model (9)–(16) and (28)–(37) is as follows:

$$\min \sum_{i\in M} a_i x_i + \sum_{s\in S} p_s \sum_{k\in K}\sum_{i\in M_k}\sum_{r\in R_i}(1-\delta_{ir})(f_{ik}+f^{\Delta}_{iks})(z_{ikr}+z^{2+}_{ikrs}-z^{2-}_{ikrs}) \tag{38}$$

subject to

$$\sum_{i\in M_k}\sum_{r\in R_i} z_{ikr} \geq d_k \qquad k \in K \tag{39}$$
$$\sum_{r\in R_i} z_{ikr} \leq q_{ik} \qquad k \in K, i \in M_k \tag{40}$$
$$l_{ir}y_{ir} \leq \sum_{k\in K} z_{ikr} \leq u_{ir}y_{ir} \qquad i \in M, r \in R_i \tag{41}$$
$$\sum_{r\in R_i} y_{ir} \leq x_i \qquad i \in M \tag{42}$$
$$\sum_{i\in M_k}\sum_{r\in R_i}(z_{ikr}+z^{+}_{ikrs}-z^{-}_{ikrs}) \geq d_k \qquad k \in K, s \in S \tag{43}$$
$$\sum_{r\in R_i}(z_{ikr}+z^{+}_{ikrs}-z^{-}_{ikrs}) \leq q_{ik} \qquad k \in K, i \in M_k, s \in S \tag{44}$$
$$l_{ir}y_{ir} \leq \sum_{k\in K}(z_{ikr}+z^{+}_{ikrs}-z^{-}_{ikrs}) \leq u_{ir}y_{ir} \qquad i \in M, r \in R_i, s \in S \tag{45}$$
$$z^{-}_{ikrs} \leq z_{ikr} \qquad k \in K, i \in M_k, r \in R_i, s \in S \tag{46}$$
$$z^{+}_{ikrs} \leq q_{ik}\lambda^{+}_{ikrs} \qquad k \in K, i \in M_k, r \in R_i, s \in S \tag{47}$$
$$z^{-}_{ikrs} \leq q_{ik}\lambda^{-}_{ikrs} \qquad k \in K, i \in M_k, r \in R_i, s \in S \tag{48}$$
$$\lambda^{+}_{ikrs} + \lambda^{-}_{ikrs} \leq y_{ir} \qquad k \in K, i \in M_k, r \in R_i, s \in S \tag{49}$$

$$x_i \in \{0,1\} \qquad i \in M \tag{50}$$
$$y_{ir} \in \{0,1\} \qquad i \in M, r \in R_i \tag{51}$$
$$z_{ikr} \geq 0 \qquad k \in K, i \in M_k, r \in R_i \tag{52}$$
$$z^{+}_{ikrs}, z^{-}_{ikrs} \geq 0 \qquad k \in K, i \in M_k, r \in R_i, s \in S \tag{53}$$
$$\lambda^{+}_{ikrs}, \lambda^{-}_{ikrs} \in \{0,1\} \qquad k \in K, i \in M_k, r \in R_i, s \in S \tag{54}$$

The just proposed model has been already explained in each part, considering that the new variables z^{+}_{ikrs}, z^{-}_{ikrs}, λ^{+}_{ikrs}, and λ^{+}_{ikrs} have, for a given scenario $s \in S$, the same meaning of z^{2+}_{ikr}, z^{2-}_{ikr}, λ^{2+}_{ikr}, and λ^{2+}_{ikr}, respectively.

4.3. The CTQD-AC with stochastic demands (CTQD-AC$_{ud}$)

A similar approach can be used for modeling the CTQD-AC$_{ud}$. However, it is easy to understand that some second-stage feasibility issues may appear when the stochastic oscillation $d^{\Delta}_k(\xi)$ is positive. In order to prevent infeasibility, we complete the recourse by introducing a slack variable $w_k \geq 0$ for each product k representing the units of product k that has to be purchased outside from the selected suppliers to satisfy the demand. The two-stage stochastic programming formulation of CTQD-AC$_{ud}$ is then composed by the first stage (9)–(16) and by the following recourse function:

$$Q(\mathbf{x}, \mathbf{y}, \mathbf{z}, \xi) := \min \sum_{k \in K} \sum_{i \in M_k} \sum_{r \in R_i} (1 - \delta_{ir}) f_{ik} (z^{2+}_{ikr} - z^{2-}_{ikr}) + \sum_{k \in K} F_k w_k \tag{55}$$

subject to

$$\sum_{i \in M_k} \sum_{r \in R_i} (z_{ikr} + z^{2+}_{ikr} - z^{2-}_{ikr}) + w_k \geq d_k + d^{\Delta}_k \qquad k \in K \tag{56}$$
$$\sum_{r \in R_i} (z_{ikr} + z^{2+}_{ikr} - z^{2-}_{ikr}) \leq q_{ik} \qquad k \in K, i \in M_k \tag{57}$$
$$l_{ir} y_{ik} \leq \sum_{k \in K} (z_{ikr} + z^{2+}_{ikr} - z^{2-}_{ikr}) \leq u_{ir} y_{ik} \qquad i \in M, r \in R_i \tag{58}$$
$$z^{2-}_{ikr} \leq z_{ikr} \qquad k \in K, i \in M_k, r \in R_i \tag{59}$$
$$z^{2+}_{ikr} \leq q_{ik} \lambda^{2+}_{ikr} \qquad k \in K, i \in M_k, r \in R_i \tag{60}$$
$$z^{2-}_{ikr} \leq q_{ik} \lambda^{2-}_{ikr} \qquad k \in K, i \in M_k, r \in R_i \tag{61}$$
$$\lambda^{2+}_{ikr} + \lambda^{2-}_{ikr} \leq y_{ir} \qquad k \in K, i \in M_k, r \in R_i \tag{62}$$
$$z^{2+}_{ikr}, z^{2-}_{ikr} \geq 0 \qquad k \in K, i \in M_k, r \in R_i \tag{63}$$
$$\lambda^{2+}_{ikr}, \lambda^{2-}_{ikr} \in \{0,1\} \qquad k \in K, i \in M_k, r \in R_i \tag{64}$$
$$w_k \geq 0 \qquad k \in K. \tag{65}$$

Since the use of external supplies, i.e., buy in the spot market, has to be discouraged unless for the strict need of satisfying the demand, these w_k variables are penalized in the objective function (55) by using a (possibly disadvantageous) external price F_k for each product k.

Again, by considering a set S of scenarios in which each scenario $s \in S$ is associated with a realization of the demand oscillation d^{Δ}_{ks} occurring with probability p_s, we present in the following the DEP formulation of the stochastic model (9)–(16) and (55)–(65):

$$\min \sum_{i \in M} a_i x_i + \sum_{s \in S} p_s \left[\sum_{k \in K} \sum_{i \in M_k} \sum_{r \in R_i} (1 - \delta_{ir}) f_{ik} (z_{ikr} + z^{2+}_{ikrs} - z^{2-}_{ikrs}) + \sum_{k \in K} F_k w_{ks} \right] \quad (66)$$

subject to

$$\sum_{i \in M_k} \sum_{r \in R_i} z_{ikr} \geq d_k \qquad k \in K \quad (67)$$

$$\sum_{r \in R_i} z_{ikr} \leq q_{ik} \qquad k \in K, i \in M_k \quad (68)$$

$$l_{ir} y_{ir} \leq \sum_{k \in K} z_{ikr} \leq u_{ir} y_{ir} \qquad i \in M, r \in R_i \quad (69)$$

$$\sum_{r \in R_i} y_{ir} \leq x_i \qquad i \in M \quad (70)$$

$$\sum_{i \in M_k} \sum_{r \in R_i} (z_{ikr} + z^{+}_{ikrs} - z^{-}_{ikrs}) + w_{ks} \geq d_k + d^{\Delta}_{ks} \qquad k \in K, s \in S \quad (71)$$

$$\sum_{r \in R_i} (z_{ikr} + z^{+}_{ikrs} - z^{-}_{ikrs}) \leq q_{ik} \qquad k \in K, i \in M_k, s \in S \quad (72)$$

$$l_{ir} y_{ir} \leq \sum_{k \in K} (z_{ikr} + z^{+}_{ikrs} - z^{-}_{ikrs}) \leq u_{ir} y_{ir} \qquad i \in M, r \in R_i, s \in S \quad (73)$$

$$z^{-}_{ikrs} \leq z_{ikr} \qquad k \in K, i \in M_k, r \in R_i, s \in S \quad (74)$$

$$z^{+}_{ikrs} \leq q_{ik} \lambda^{+}_{ikrs} \qquad k \in K, i \in M_k, r \in R_i, s \in S \quad (75)$$

$$z^{-}_{ikrs} \leq q_{ik} \lambda^{-}_{ikrs} \qquad k \in K, i \in M_k, r \in R_i, s \in S \quad (76)$$

$$\lambda^{+}_{ikrs} + \lambda^{-}_{ikrs} \leq y_{ir} \qquad k \in K, i \in M_k, r \in R_i, s \in S \quad (77)$$

$$x_i \in \{0, 1\} \qquad i \in M \quad (78)$$

$$y_{ir} \in \{0, 1\} \qquad i \in M, r \in R_i \quad (79)$$

$$z_{ikr} \geq 0 \qquad k \in K, i \in M_k, r \in R_i \quad (80)$$

$$z^{+}_{ikrs}, z^{-}_{ikrs} \geq 0 \qquad k \in K, i \in M_k, r \in R_i, s \in S \quad (81)$$

$$\lambda^{+}_{ikrs}, \lambda^{-}_{ikrs} \in \{0, 1\} \qquad k \in K, i \in M_k, r \in R_i, s \in S \quad (82)$$

$$w_{ks} \geq 0 \qquad k \in K, s \in S. \quad (83)$$

For each product $k \in K$, variable w_{ks} has the same meaning of w_k in a given scenario $s \in S$.

5. The solution framework

Thanks to the discretization, the DEP models of CTQD-AC$_{up}$ and of CTQD-AC$_{ud}$ can be easily casted into an off-the-shelf MIP solver such as Cplex or Gurobi. However, due to the enormous number of scenario-dependent constraints and variables, its plain exact solution procedure results computationally too expensive even for relatively small instances and number of scenarios. Hence, we propose a tailored exact solution framework based on valid inequalities for the problem and several accelerating techniques. The procedure has been partially inherited from the branch-and-cut approach proposed for the deterministic CTQDP in Manerba and Mansini (2012b) and adapted to the multi-scenario context. The purpose of this method is, on the one hand, to speed up the collection of all the data useful for our analysis and, on the other hand, to put a first consistent base point for developing and comparing other future ad-hoc solution approaches.

In our solution method we adopt IBM Ilog Cplex 12.6.1 as a general branch-and-cut framework, implementing and incorporating some improvements to the model (preprocessing routines, the creation of a MIP start solution, and the separation of valid inequalities) through the *C++ Concert Technology*'s callbacks provided by the solver. We precise that other possible mathematical programming techniques can be applied apart from the ones proposed in the following. For example, Stadtler (2007) suggests to use a Special Ordered Sets of Type 1 (SOS1) for modeling each set of variables $\{y_{ir} | r \in R_i\}$, since at most one interval can be selected for any supplier $i \in M$. However, we found this adjustment ineffective in some preliminary experiments.

5.1. Preprocessing

Some simple routines can be applied to reduce the number of variables or to strengthen the coefficients of the problem. First, note that the binary condition on the variables corresponding to the selection of a supplier's first interval is not strictly necessary since $l_{i1} = 0, \forall i \in M$. Hence, we can substitute (51) in CTQD-AC$_{up}$ and (79) in CTQD-AC$_{ud}$ with

$$y_{ir} \in \{0, 1\}, \quad i \in M, r \in R_i \setminus \{1\} \qquad \text{and} \qquad y_{i1} \geq 0, \quad i \in M.$$

Note that the model constraints already guarantee the unitary upper bound on y variables.

Second, let $R_i^* := \{r \in R_i : \sum_{k \in K} q_{ik} < l_{ir}\}$ be the set of discount intervals that can never be reached even buying all quantities available at supplier $i \in M$. Then, in both our models, we can set to zero variables $y_i^r, i \in S, r \in R_i^*$. In turn, this implies $z_{ik}^r = 0, i \in S, k \in K, r \in R_i^*$, and, for each scenario $s \in S$, $z_{ikrs}^+, z_{ikrs}^-, \lambda_{ikrs}^+, \lambda_{ikrs}^- = 0, k \in K, i \in M_k, r \in R_i^*$.

Finally, let $M^* := \left\{ i \in M : \exists k \in K \text{ such that } \sum_{j \in M_k \setminus \{i\}} q_{jk} < d_k \right\}$ be the set of suppliers that has to be necessarily selected in any feasible solution of the problem. Then, we can add to CTQD-AC$_{up}$ the following constraints

$$\sum_{r \in R_i} z_{ikr} \geq \max \left\{ 0, d_k - \sum_{j \in M_k \setminus \{i\}} q_{ik} \right\} \qquad k \in K, i \in M^*,$$

and set to 1 variables x_i for each $i \in M^*$. Note that this does not hold for CTQD-AC$_{ud}$ because of the presence of w variables.

5.2. MIP start solution

In general, having a feasible initial solution helps in pruning the branch-and-cut decision tree and speeds up the solution. We take advantage from the fact that in CTQD-AC$_{up}$ the uncertainty affects only the product prices and not the required or the available product quantities. This implies that each feasible solution of a CTQDP-AC$_{up}$ deterministic approximation (in which each stochastic price is substituted by any realization) is feasible as well for its DEP considering several scenarios, possibly generating a different objective function value. For this reason, we solve the DEP to optimality considering only a single scenario, i.e. the one in which stochastic variables are replaced by their expected values, and provide the resulting solution as a MIP starting solution for the multi-scenario complete problem.

The same property does not hold for CTQD-AC$_{ud}$, where there is no guarantees that a feasible solution for a given scenario remains feasible for another one due to the demand oscillation. However, any infeasible CTQD-AC$_{ud}$ solution $(x, y, z, z^+, z^-, \lambda^+, \lambda^-, w)$ can be always converted to a feasible one by duly adjusting the w slack variables. Hence, in order to get a feasible solution quickly, we optimally solve (as above) a single-scenario CTQD-AC$_{ud}$ and provide the non-complete solution $(x, y, z, z^+, z^-, \lambda^+, \lambda^-)$ to the solver so it can easily adjust the remaining variables for reaching the feasibility.

5.3. Valid inequalities

The linear relaxation of our DEP models can be strengthened introducing some classes of valid inequalities. For example, the following cuts have been shown to be effective in solving problem with a TQD structure (see e.g. Manerba and Mansini, 2012b, 2014):

$$z_{ikr} \leq \min\{q_{ik}, u_{ir}\} y_{ir}, \quad i \in M_k, r \in R_i, k \in K. \tag{84}$$

Constraints (84) bounds the quantity that can be purchased for each product in a given interval by the minimum between quantity available for that product and the upper bound of the interval. This excludes the systematic selection of the last intervals of each supplier in the LP relaxation. Moreover, we directly add the following cuts

$$\sum_{r \in R_i} z_{ikr} \leq q_{ik} x_i \qquad k \in K, i \in M_k, \tag{85}$$

that trivially improve inequalities (40) and (68) in CTQD-AC$_{up}$ and CTQD-AC$_{ud}$, respectively.

6. Instances generation

The CTQD-AC$_{up}$ has never been studied before in the literature, hence no benchmark instances exist for the problem (neither for its deterministic counterpart). For this reason, we generate a new collection of hard-to-solve benchmark instances for providing a realistic testbed for our stochastic approaches and fostering other contributions on the topic. In this section, we present the detailed method used to generate the deterministic CTQD-AC instances and to produce the scenario trees for the stochastic data in CTQD-AC$_{up}$ and CTQD-AC$_{ud}$.

First, each product $k \in K$ is assigned to a random subset M_k of suppliers, and to a basic price f_k, randomly chosen in $[10, 200]$. Then, for each product k sold by supplier i, the available quantity q_{ik} is generated uniformly in [1, 15] whereas the product price f_{ik} is randomly generated in $[0.9f_k, 1.1f_k]$ (i.e., the prices of the same product cannot vary more than the 20% from a supplier to another). Moreover, the penalty price F_k is generated for each product $k \in K$ such that $F_k := 1.2 \max_{i \in M_k} f_{ik}$ (i.e., it is the 20% more costly than the maximum price among the considered suppliers), in order to discourage external supplies.

The demand for each product k is generated as follows:

$$d_k := \left\lceil \overline{d}_k - \left((\overline{d}_k - 1) \quad \frac{f_k}{\max_{k \in K}\{f_k\}} \right) \right\rceil ,$$

where

$$\overline{d}_k := \left\lceil \lambda \max_{i \in M_k}\{q_{ik}\} + (1-\lambda) \sum_{i \in M_k} q_{ik} \right\rceil \text{ with } \lambda \in (0,1).$$

The term $\overline{d}_k$ allows to control, through parameter λ varying in $(0, 1)$, the number of suppliers included in a feasible solution. More precisely, the lower the value of λ, the higher the number of suppliers required to satisfy the entire demand. Finally, the real demand is adjusted with respect to the average expensiveness of a product (i.e., the more expensive the product, the lower the corresponding demand). Two values of λ will be considered.

Data concerning the discounts offered by the suppliers are inspired on the generation method proposed in Manerba and Mansini (2012b) for the CTQDP. More precisely, for each supplier $i \in M$ and each interval $r \in R_i$, lower bounds are generated as an a priori percentage $0 \leq \alpha_{ir} < 1$ of the total amount of products available from a specific supplier, i.e. $l_{ir} = \lfloor \alpha_{ir} \sum_{k \in K} q_{ik} \rfloor$. Upper bounds u_{ir} for interval $r \in R \setminus \{r_i\}$ of supplier $i \in M$ are set to the value corresponding to the lower bound of the following interval minus 1, whereas $u_{i,r_i} = \sum_{k \in K} q_{ik}, i \in M$. We consider two different discount policies (DP):

- *DP1*: given a supplier $i \in M$, the number of discount intervals ranges randomly between 3 and 5, with a random interval width (even if, in order to generate reasonable instances, we set $\alpha_{i,1} \geq 0.6$). Discount rates applied in all intervals are randomly generated such that $\delta_{i,r_i} \leq 0.05$ and $\delta_{ir} \leq \delta_{i,r+1}$, $r \in R \setminus \{r_i\}$;

- *DP2*: the number of intervals is equal to 3 for all suppliers, with $\alpha_{i,1} = 0.7$ and $\alpha_{i,2} = 0.9$, $i \in M$. Moreover, discount rates are fixed for all the suppliers and equal to 1%, 2%, and 3%, respectively. We precise that, even when all the suppliers use common discount rates, the resulting problem is still $\mathcal{NP}-$hard (see Goossens et al., 2007).

Thus, while *DP1* is a more random generation, *DP2* creates very similar discount conditions for all the suppliers thus highlighting the role of the activation costs and of the product availabilities.

Finally, two types of activation costs (AC) have been considered. In the first type, *AC1*, the activation cost for a business activity is generated as a defined percentage γ of the total quantity of products available in the supplier multiplied by a global average product price, i.e.

$$a_i := \overline{f} \sum_{k \in K} \gamma q_{ik}, \quad \text{where} \quad \overline{f} := \frac{\sum_{i \in M} \overline{f}_i}{|S|} \quad \text{and} \quad \overline{f}_i := \frac{\sum_{k \in K} f_{ik}}{|K|}.$$

On the contrary, in *AC2* instances, activation costs are inversely proportional to the supplier's average product price, i.e.

$$a_i := \left(\overline{f} + \frac{\overline{f}}{\overline{f}_i} \right) \sum_{k \in K} \gamma q_{ik}.$$

Parameter γ has been set equal to 0.1, to create activation costs proportional to the average purchasing ones.

In the end, we generate an instance for each combination of $|M| = \{5, 10, 20\}$ (the number of suppliers), $|K| = \{10, 20, 30\}$ (the number of products), $DP = \{DP1, DP2\}$ (the type of discount policy), $AC = \{AC1, AC2\}$ (the type of activation cost), and $\lambda = \{0.1, 0.8\}$. This means that our complete dataset is composed by 72 deterministic instances.

6.1. Scenario tree generation for stochastic prices

In the following we describe how the scenario tree is built for CTQD-AC$_{up}$, for any given deterministic instance. For each scenario $s \in S$, we generate the stochastic price f_{iks} of each product $k \in K$ and supplier $i \in M_k$ and then calculate its oscillation $f^{\Delta}_{iks} := f_{iks} - f_{ik}$. The value of f_{iks} is drawn according to a probability distribution with *location* parameter $\mu = f_{ik}$ and truncated between $[f_{ik} - \beta f_{ik}, f_{ik} + \beta f_{ik}]$, with $\beta = \{0.1, 0.3\}$ (i.e., the oscillation of each price f_{ik} may vary no more than βf_{ik}). In our tests, in order to reasonably model the uncertainty of a product price around an estimated mean value, we used the *Normal* and the *Gumbel* distributions. Since our truncated distribution support varies proportionally to the mean value, we also need to modify the distribution *scale* parameter accordingly. In particular, we use a standard deviation $\sigma = 0.25\mu$ for the Normal distribution and a scale parameter $\sigma = 0.20\mu$ for the Gumbel distribution. These coefficients have been empirically calculated such that the probability of discarding a generated value never exceeds 0.2 for both the distributions, i.e.,

$$P[f_{iks} \leq f_{ik} - \beta f_{ik}] + P[f_{iks} \geq f_{ik} + \beta f_{ik}] \leq 0.2.$$

6.2. Scenario tree generation for stochastic demands

The scenario tree for CTQD-AC$_{ud}$ is built for any given deterministic instance in a similar way. For each scenario $s \in S$, we generate the stochastic price d_{ks} for each $k \in K$ and calculate its oscillation $d^{\Delta}_{ks} := d_{ks} - d_k$. The value of d_{ks} is drawn according to a *Uniform* or a *Gumbel* probability distribution in $[0.5d_k, 2d_k]$ (i.e., the demand d_k may be halved or doubled at most). In the same spirit of the scenario tree generation for the stochastic prices, for the *Gumbel* distribution we decided to use a location parameter $\mu = d_k$ and a proportional scale parameter $\sigma = 20\mu$.

7. Computational experiments

The benchmark instances presented in Section 6 are used to empirically validate our modeling approaches through stability and economical analysis of the solutions, and to show the efficiency of our solution framework. The computational resources for the preliminary stability analysis have been provided by HPC@POLITO cluster (`http://hpc.polito.it`). All the other experiments have been done on a *Intel(R) Core(TM) i7-5930K* CPU@3.50 GHz machine with 64 GB RAM and running *Windows 7* 64-bit operating system.

7.1. Stability analysis

Being part of the stochastic models we proposed for CTQD-AC$_{up}$ and for CTQD-AC$_{ud}$, the scenario tree generation procedures described above must be validated in terms of stability. We evaluate their *in-sample stability* by considering a subset of instances corresponding to several combinations of $|M| = \{5, 10, 20\}$, $|K| = \{10, 20, 30\}$, and $\lambda = \{0.1, 0.8\}$. We generate 10 different scenario trees for each instance and solve the corresponding stochastic problem varying the number of considered scenarios. The percentage ratios between the standard deviation and the mean of the optimal solution values are computed over the 10 runs and plotted in Figures 2 and 3.

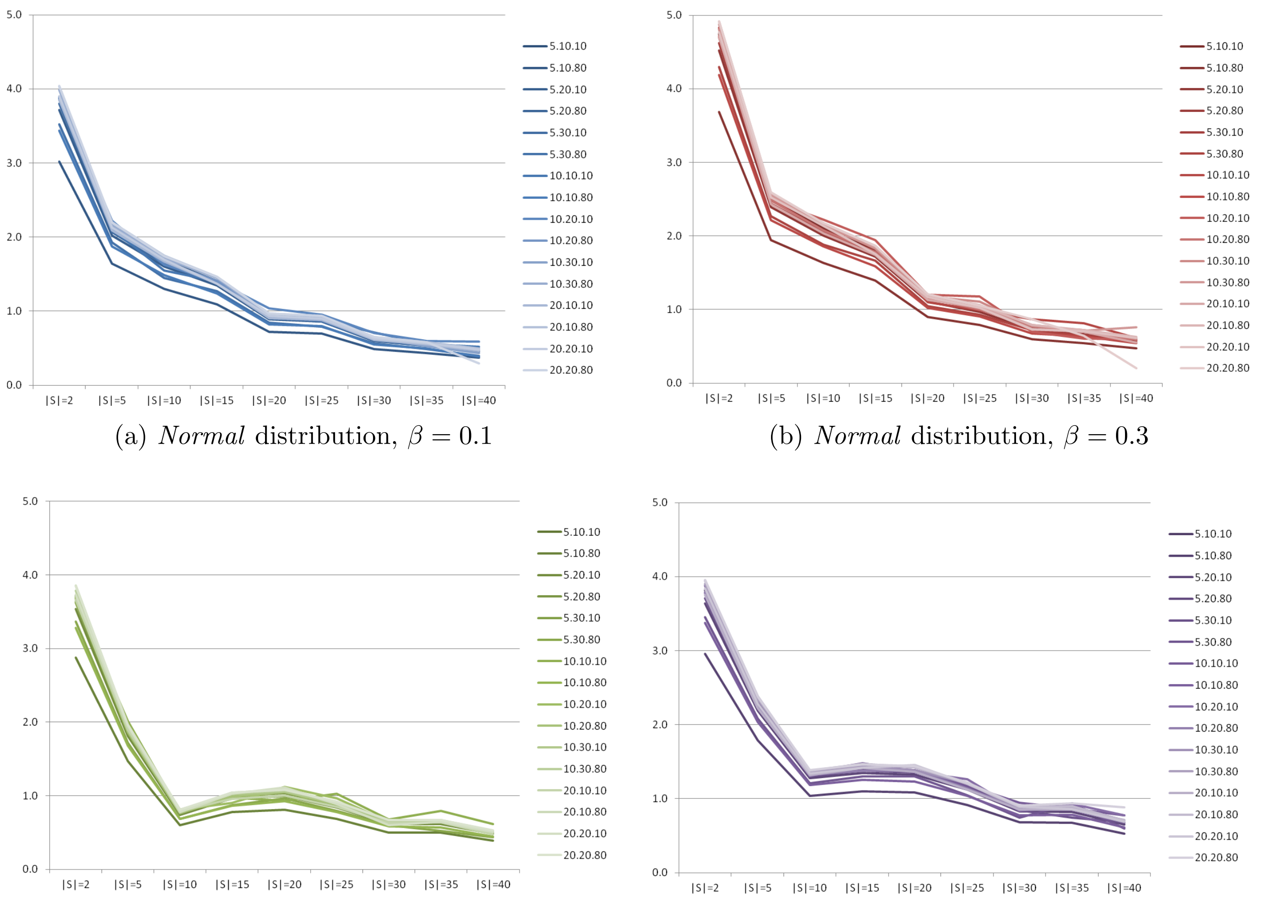
In particular, Figure 2 shows the results concerning CTQD-AC$_{up}$ for any combination of distributions (*Normal* and *Gumbel*) and value of β used ($\beta = \{0.1, 0.3\}$). The number of scenarios considered varies from 2 up to 40. As expected, the stability of the solutions tends to increase rapidly with the number of scenarios and it is a little bit harder to achieve for the distributions with a larger domain (i.e., with $\beta = 0.3$). More important, the stability is independent from the number of suppliers considered, the number of products involved, and the demand generation type (the λ value). Note also that, by using the *Gumbel* distribution, the standard deviation decreases with more oscillations. However, in all the cases, 30 scenarios seem sufficient to maintain the percentage ratio between the standard deviation and the mean of the optimal solution values decisively under the 1% threshold, which is a precision suitable with the problem setting (Perboli et al., 2014).

Figure 3 shows instead the results concerning CTQD-AC$_{ud}$ for any distributions considered (*Uniform* and *Gumbel* in $[0.5d_k, 2d_k]$). Here, the number of scenarios considered varies from 10 up to 100. We can see that, for both the distributions, now there are more different converging trends and a wider span of values for the same number of scenarios but for different instances. This is reasonable, given the wider domains of the distributions for the products demand. The convergence for the *Uniform* distribution is harder to obtain, in particular when the number of products is lower. However, for both the distributions, 100 scenarios are sufficient to maintain the percentage ratio between the standard deviation and the mean of the optimal solution values under, or very near to, the 1% threshold.

According to the above stability analysis, we will consider 30 scenarios for CTQD-AC$_{up}$ instances and 100 scenarios for the CTQD-AC$_{ud}$ ones in all the following experiments.

7.2. Evaluation of the solution framework

In order to settle the benefits from using the solution framework proposed in Section 5 (referred in the following as `B&C`), we have compared its performance with the resolution by Cplex of the plain



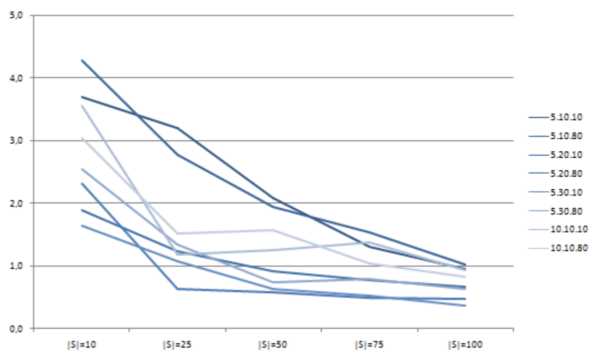
(a) *Normal* distribution, $\beta = 0.1$ (b) *Normal* distribution, $\beta = 0.3$

(c) *Gumbel* distribution, $\beta = 0.1$ (d) *Gumbel* distribution, $\beta = 0.3$

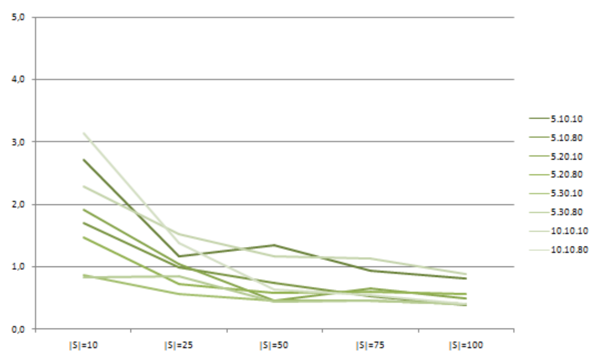
Figure 2: Percentage ratio between the standard deviation and the mean of the optimal solution values of 16 instances identified by $|M|.|K|.\lambda$ over 10 different scenario tree generations for stochastic prices.

DEP formulations of the two problem variants (`Cplex`). The results of this comparison, on a subset of instances, are shown in Tables 1 and 2 for CTQD-AC$_{up}$ and CTQD-AC$_{ud}$, respectively. For each instance (identified by $|M|$, $|K|$, and λ parameters) and for both the solution methods, we present the CPU time in seconds *t(s)* needed to prove the optimality, the time-to-best *ttb(s)*, i.e. the time needed to achieve the best solution, and the number of nodes explored by the branch-and-bound algorithm on which both the methods rely (*BBn*).

It appears evident that `B&C` outperforms the MIP solver on average and also on each single simulation, despite of the characteristics or the dimension of the instances. Computational times for `B&C` are, on average, ten times lower than the `Cplex`'s ones on CTQD-AC$_{up}$ instances, and the time-to-best values follow the same proportion. The effort of `B&C` on strengthening the bounds of the solutions (through the introduction of cuts and good initial feasible solutions) clearly results in much smaller branch-and-bound trees, thus speeding up the convergence toward optimality. The difference in performance is exacerbated on CTQD-AC$_{ud}$ instances, where a greater number of scenarios is considered. In fact, while the `B&C` times remain good and reasonable, `Cplex` ceases to be a suitable solution alternative (look for example at instances involving 10 supplier and 10



(a) Uniform distribution in $[0.5d_k, 2d_k]$



(b) Gumbel distribution truncated in $[0.5d_k, 2d_k]$

Figure 3: Percentage ratio between the standard deviation and the mean of the optimal solution values of 8 instances (identified by $|M|.|K|.\lambda$) over 10 different scenario tree generations for stochastic demands.

			`Cplex`			`B&C`		
$\lvert M\rvert$	$\lvert K\rvert$	λ	t(s)	ttb(s)	BBn	t(s)	ttb(s)	BBn
5	10	0.1	58	58	35	6	2	8
5	10	0.8	112	97	119	13	9	27
5	20	0.1	370	339	200	35	30	23
5	20	0.8	567	293	4837	21	14	17
5	30	0.1	681	516	524	58	31	21
5	30	0.8	527	518	785	58	56	20
10	10	0.1	3520	2664	2219	166	117	317
10	10	0.8	3180	2649	1918	126	62	249
10	20	0.1	7035	2745	3347	666	488	416
10	20	0.8	3471	2667	717	306	302	128
		avg:	1952.0	1254.5	1470.1	145.6	111.0	122.6

Table 1: `Cplex` vs `B&C` for CTQD-AC$_{up}$

products for which about two days are needed to reach optimality).

7.3. Analysis of the stochastic solution

In the following, we evaluate the economic advantage of considering uncertainty through the use of stochastic models with recourse for CTQD-AC$_{up}$ and CTQD-AC$_{ud}$ with respect of using expected values for approximating the stochastic variables. To this aim, we compute two well-known stochastic programming measures (see, e.g., Birge, 1982 or Birge and Louveaux, 1997), i.e. the *Value of the Stochastic Solution* (VSS) and the *Expected Value of Perfect Information* (EVPI), for both the problems on the complete set of generated instances and considering all the above described scenario tree generations. More precisely, $VSS := EEV - RP$ and $EVPI := RP - WS$, where RP is the objective value of the stochastic solution (*recourse problem solution*), EEV is the solution value of the stochastic model with the first-stage decision fixed by solving the deterministic problem using expected values for approximating the random parameters (*expected value solution*), and WS is the solution value of a problem in which it is assumed to know at the first-stage the realizations of all the stochastic variables (*wait-and-see solution*).

Tables 3-5 show, for each deterministic instance (uniquely identified by $|M|$, $|K|$, λ, DP, and AC parameters) and for each scenario tree generation concerning CTQD-AC$_{up}$, the percentage values of VSS and EVPI with respect to the objective value of the stochastic solution. More precisely,

			Cplex			B&C		
$\|M\|$	$\|K\|$	λ	t(s)	ttb(s)	BBn	t(s)	ttb(s)	BBn
5	10	0.1	1193	1029	934	63	23	15
5	10	0.8	5092	4756	1210	81	81	25
5	20	0.1	72348	64168	7380	738	542	62
5	20	0.8	4546	4012	1171	202	182	27
5	30	0.1	32455	24530	3750	1049	945	33
5	30	0.8	19113	18081	3250	844	819	27
10	10	0.1	154458	54805	15076	4183	4140	678
10	10	0.8	154295	149507	8595	4160	3761	800
		avg:	55437.5	40111.0	5170.8	1415.1	1311.7	208.4

Table 2: Cplex vs B&C for CTQD-AC$_{ud}$

VSS% and EVPI% are computed as VSS/RP*100 and EVPI/RP*100, respectively. For each set of instances involving the same number of suppliers, average and maximum values of VSS% and EVPI% are also shown.

					Normal distribution				Gumbel distribution			
Instance					$\beta = 0.1$		$\beta = 0.3$		$\beta = 0.1$		$\beta = 0.3$	
$\|M\|$	$\|K\|$	λ	DP	AC	VSS%	EVPI%	VSS%	EVPI%	VSS%	EVPI%	VSS%	EVPI%
5	10	0.1	1	1	0.0	0.0	0.0	1.6	0.0	0.0	0.0	0.5
5	10	0.1	1	2	0.0	0.2	1.1	0.5	0.0	0.2	0.1	0.6
5	10	0.1	2	1	0.0	0.2	0.0	0.3	0.4	0.1	0.0	0.1
5	10	0.1	2	2	0.5	0.3	2.7	0.4	0.4	0.3	0.5	0.3
5	10	0.8	1	1	0.0	0.4	0.8	1.7	0.0	0.4	0.4	0.9
5	10	0.8	1	2	0.2	0.1	0.0	0.2	0.2	0.1	0.0	0.1
5	10	0.8	2	1	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0
5	10	0.8	2	2	0.1	0.2	0.5	1.0	0.0	0.2	0.0	1.8
5	20	0.1	1	1	0.2	0.2	2.0	0.3	0.1	0.2	0.6	0.2
5	20	0.1	1	2	0.2	0.2	1.8	0.2	0.0	0.1	1.4	0.2
5	20	0.1	2	1	0.0	0.1	0.3	0.1	0.0	0.1	0.3	0.1
5	20	0.1	2	2	0.4	0.2	1.5	0.3	0.3	0.2	0.3	0.3
5	20	0.8	1	1	0.0	0.2	0.0	0.5	0.0	0.1	0.2	0.5
5	20	0.8	1	2	0.2	0.1	1.0	0.3	0.1	0.1	0.6	0.5
5	20	0.8	2	1	0.0	0.0	0.0	0.5	0.0	0.0	0.0	0.2
5	20	0.8	2	2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	30	0.1	1	1	0.1	0.1	0.0	0.0	0.1	0.1	0.4	0.1
5	30	0.1	1	2	0.1	0.1	0.9	0.0	0.1	0.1	0.6	0.0
5	30	0.1	2	1	0.3	0.1	1.6	0.1	0.0	0.1	0.1	0.1
5	30	0.1	2	2	0.3	0.1	0.3	0.1	0.3	0.1	0.2	0.1
5	30	0.8	1	1	0.3	0.0	0.3	0.1	0.0	0.1	0.6	0.1
5	30	0.8	1	2	0.2	0.1	1.4	0.3	0.0	0.1	0.7	0.1
5	30	0.8	2	1	0.0	0.3	0.0	0.7	0.0	0.3	0.3	0.9
5	30	0.8	2	2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
				avg:	0.1	0.1	0.7	0.4	0.1	0.1	0.3	0.3
				max:	0.5	0.4	2.7	1.7	0.4	0.4	1.4	1.8

Table 3: VSS and EVPI for CTQD-AC$_{up}$ instances with $|M| = 5$.

We can clearly see that the VSS% values are quite low, on average and also for the most part of the instances, independently from their dimension and the discounts or activation costs characteristics. More precisely, the percentage VSS for the distributions with the smaller domain (i.e., with $\beta = 0.1$) is always lower than 0.7% whereas, for bigger variations of the prices (i.e., with $\beta = 0.3$) the average values goes up to 1.5% with some picks around the 3% (see in particular the $|K| = 10$ product instances in the *Normal distribution* columns). Thus, the explicit consideration of uncertainty through a SP approach does not give a particular conservativeness to the solutions with respect to use of deterministic expected values for the unknown parameters, unless the procurement involves only few products subject to very strong price fluctuations.

Similarly to Tables 3-5, Tables 6 and 7 report the VSS% and EVPI% values for the CTQD-AC$_{ud}$ concerning the two types of scenario tree generation. Note that, in Table 7, the results for the 16

Instance					Normal distribution				Gumbel distribution			
					$\beta = 0.1$		$\beta = 0.3$		$\beta = 0.1$		$\beta = 0.3$	
$\|M\|$	$\|K\|$	λ	DP	AC	VSS%	EVPI%	VSS%	EVPI%	VSS%	EVPI%	VSS%	EVPI%
10	10	0.1	1	1	0.1	0.8	1.5	2.4	0.1	0.8	0.4	2.3
10	10	0.1	1	2	0.0	1.1	3.1	3.0	0.3	1.1	1.5	2.4
10	10	0.1	2	1	0.1	0.9	2.2	2.7	0.1	0.8	1.3	2.8
10	10	0.1	2	2	0.0	0.5	2.3	1.7	0.0	0.7	1.0	1.5
10	10	0.8	1	1	0.1	1.9	2.3	4.2	0.1	1.8	1.4	3.9
10	10	0.8	1	2	0.2	1.3	2.0	4.3	0.6	1.2	0.3	3.9
10	10	0.8	2	1	0.0	1.2	1.8	3.2	0.0	1.0	1.1	2.5
10	10	0.8	2	2	0.2	0.9	3.0	3.7	0.3	0.8	2.0	3.2
10	20	0.1	1	1	0.4	0.5	1.3	1.0	0.2	0.5	1.9	0.9
10	20	0.1	1	2	0.4	0.5	0.8	1.0	0.2	0.6	1.1	0.8
10	20	0.1	2	1	0.5	0.6	1.4	1.5	0.4	0.6	1.9	1.2
10	20	0.1	2	2	0.0	0.4	2.4	0.9	0.5	0.4	1.5	0.8
10	20	0.8	1	1	0.5	0.5	2.2	2.7	0.5	0.3	1.1	1.4
10	20	0.8	1	2	0.4	0.6	1.9	1.8	0.1	0.5	1.1	0.8
10	20	0.8	2	1	0.4	0.0	0.0	1.4	0.0	0.1	0.0	1.3
10	20	0.8	2	2	0.2	0.3	1.3	1.4	0.2	0.3	1.0	1.5
10	30	0.1	1	1	0.3	0.4	1.9	0.5	0.3	0.4	1.4	0.6
10	30	0.1	1	2	0.4	0.2	1.8	0.9	0.3	0.3	1.1	0.7
10	30	0.1	2	1	0.5	0.2	2.5	0.9	0.2	0.2	0.7	1.0
10	30	0.1	2	2	0.3	0.2	0.6	0.5	0.5	0.3	0.6	0.4
10	30	0.8	1	1	0.1	0.3	2.0	1.9	0.2	0.2	0.8	1.6
10	30	0.8	1	2	0.0	0.0	0.0	1.2	0.0	0.0	0.0	1.0
10	30	0.8	2	1	0.2	0.6	0.4	1.7	0.0	0.6	0.6	1.4
10	30	0.8	2	2	0.0	0.3	0.0	1.5	0.0	0.5	0.4	1.2
				avg:	0.2	0.6	1.6	1.9	0.2	0.6	1.0	1.6
				max:	0.5	1.9	3.1	4.3	0.6	1.8	2.0	3.9

Table 4: VSS and EVPI for CTQD-AC$_{up}$ instances with $|M| = 10$.

Instance					Normal distribution				Gumbel distribution			
					$\beta = 0.1$		$\beta = 0.3$		$\beta = 0.1$		$\beta = 0.3$	
$\|M\|$	$\|K\|$	λ	DP	AC	VSS%	EVPI%	VSS%	EVPI%	VSS%	EVPI%	VSS%	EVPI%
20	10	0.1	1	1	0.1	1.0	2.3	3.2	0.2	1.0	0.8	3.1
20	10	0.1	1	2	0.0	1.3	1.7	4.8	0.0	1.3	0.8	3.9
20	10	0.1	2	1	0.7	1.4	2.8	4.4	0.4	1.4	1.2	4.4
20	10	0.1	2	2	0.4	1.3	2.2	3.3	0.3	1.0	1.6	2.9
20	10	0.8	1	1	0.3	2.9	2.2	6.9	0.2	2.9	2.5	6.5
20	10	0.8	1	2	0.2	2.5	0.7	8.2	0.1	2.0	0.6	6.4
20	10	0.8	2	1	0.0	0.9	0.4	4.7	0.0	0.9	0.2	4.1
20	10	0.8	2	2	0.1	1.5	0.9	4.7	0.2	1.6	1.0	4.4
20	20	0.1	1	1	0.4	0.5	2.1	1.0	0.2	0.4	1.9	0.9
20	20	0.1	1	2	0.6	0.7	1.8	2.5	0.6	0.6	1.7	2.1
20	20	0.1	2	1	0.2	1.0	2.9	2.0	0.3	1.0	2.2	1.8
20	20	0.1	2	2	0.5	0.9	2.4	2.0	0.3	0.8	2.6	1.9
20	20	0.8	1	1	0.5	1.6	1.5	4.6	0.5	1.5	0.7	4.1
20	20	0.8	1	2	0.3	0.9	0.8	3.4	0.3	1.2	1.3	3.2
20	20	0.8	2	1	0.4	1.3	1.2	3.9	0.4	1.5	0.4	3.3
20	20	0.8	2	2	0.5	0.7	2.4	2.8	0.3	0.8	0.5	3.2
20	30	0.1	1	1	0.6	0.4	2.0	0.9	0.4	0.5	1.5	1.0
20	30	0.1	1	2	0.1	0.6	2.5	1.6	0.2	0.4	1.4	1.5
20	30	0.1	2	1	0.3	0.5	1.7	1.3	0.3	0.6	1.4	1.2
20	30	0.1	2	2	0.2	0.6	2.9	1.3	0.4	0.5	1.1	1.2
20	30	0.8	1	1	0.2	0.8	2.0	3.1	0.2	0.7	0.4	2.5
20	30	0.8	1	2	0.2	1.0	1.6	3.6	0.2	0.8	1.8	2.6
20	30	0.8	2	1	0.4	0.5	0.0	2.5	0.3	0.6	0.5	2.3
20	30	0.8	2	2	0.0	0.6	0.8	3.0	0.0	0.6	0.5	2.8
				avg:	0.3	1.1	1.7	3.3	0.3	1.0	1.2	3.0
				max:	0.7	2.9	2.9	8.2	0.6	2.9	2.6	6.5

Table 5: VSS and EVPI for CTQD-AC$_{up}$ instances with $|M| = 20$.

biggest instances (i.e., instances involving 20 suppliers and 20 or 30 products) are missing. Due to the great number of considered scenarios, in fact, we have not be able to optimally solve those instances in a reasonable amount of time even through our solution framework.

The results considering demand uncertainty show a completely different impact of the SP approach on the solution quality with respect to the case under price uncertainty. In this case, the VSS% values are considerable high, in particular in presence of a lower number of suppliers, and

Instance				$\lvert M\rvert = 5$				$\lvert M\rvert = 10$			
				Uniform distribution		Gumbel distribution		Uniform distribution		Gumbel distribution	
$\lvert K\rvert$	λ	DP	AC	VSS%	EVPI%	VSS%	EVPI%	VSS%	EVPI%	VSS%	EVPI%
10	0.1	1	1	9.5	3.3	5.1	3.7	6.1	1.7	4.9	1.1
10	0.1	1	2	27.9	2.7	21.0	3.2	8.5	1.8	5.1	1.7
10	0.1	2	1	4.9	5.4	28.6	6.3	11.9	2.0	8.9	2.2
10	0.1	2	2	25.2	7.0	11.9	9.6	8.3	2.1	9.5	1.9
10	0.8	1	1	43.2	10.4	31.1	11.8	15.0	5.8	7.9	6.2
10	0.8	1	2	8.3	4.8	1.4	6.1	4.7	3.6	19.6	3.7
10	0.8	2	1	43.1	6.2	40.0	8.7	31.3	2.3	20.2	2.4
10	0.8	2	2	34.9	9.7	29.2	10.6	21.0	4.7	8.7	4.5
20	0.1	1	1	13.6	3.9	20.1	5.7	11.0	1.4	8.3	2.2
20	0.1	1	2	14.0	3.8	6.0	5.3	14.9	1.8	3.5	2.5
20	0.1	2	1	34.3	4.1	20.8	6.1	13.3	1.8	11.5	2.0
20	0.1	2	2	21.2	3.2	14.5	4.4	16.7	2.1	6.3	2.8
20	0.8	1	1	13.4	11.9	7.1	15.5	36.6	1.8	25.8	1.6
20	0.8	1	2	25.8	5.5	18.6	7.4	18.4	4.0	31.7	4.8
20	0.8	2	1	8.0	4.0	32.9	5.4	10.3	2.4	6.1	3.1
20	0.8	2	2	41.7	4.2	31.7	5.9	9.9	4.2	5.6	4.7
30	0.1	1	1	30.3	3.0	5.4	4.7	11.1	1.6	8.6	2.3
30	0.1	1	2	12.7	3.2	5.2	4.8	12.2	1.2	5.0	1.8
30	0.1	2	1	36.8	2.6	28.5	3.6	18.0	1.6	10.6	2.5
30	0.1	2	2	17.9	2.7	10.9	4.3	18.1	1.3	7.8	2.1
30	0.8	1	1	44.6	6.5	30.9	8.8	5.6	2.2	5.9	2.4
30	0.8	1	2	35.8	3.8	24.8	6.0	10.4	2.9	14.7	3.8
30	0.8	2	1	11.5	7.8	38.8	10.2	22.3	2.6	17.0	3.3
30	0.8	2	2	2.9	10.7	4.6	13.9	7.0	5.8	2.1	7.8
			avg:	23.4	5.4	19.6	7.2	14.3	2.6	10.6	3.1
			max:	44.6	11.9	40.0	15.5	36.6	5.8	31.7	7.8

Table 6: VSS and EVPI for CTQD-AC$_{ud}$ instances with $|M| = 5$ and $|M| = 10$

Instance					Uniform distribution		Gumbel distribution	
$\lvert M\rvert$	$\lvert K\rvert$	λ	DP	AC	VSS%	EVPI%	VSS%	EVPI%
20	10	0.1	1	1	17.5	2.1	14.5	1.5
20	10	0.1	1	2	10.3	2.3	6.8	1.7
20	10	0.1	2	1	8.0	4.5	11.0	4.9
20	10	0.1	2	2	25.2	2.2	15.2	2.3
20	10	0.8	1	1	10.2	3.1	6.2	2.3
20	10	0.8	1	2	9.1	3.7	10.4	3.0
20	10	0.8	2	1	16.2	4.1	12.5	3.9
20	10	0.8	2	2	21.2	5.5	7.4	7.0
				avg:	14.7	3.4	10.5	3.3
				max:	25.2	5.5	15.2	7.0

Table 7: VSS and EVPI for CTQD-AC$_{ud}$ instances with $|M| = 20$

is a little bit higher for the *Uniform* distribution than for the *Gumbel*. More precisely, the average VSS% considering the *Uniform* distribution is equal to 23.4% for $|M| = 5$ instances, while goes down to about 14% for $|M| = \{10, 20\}$ ones. The average values for the *Gumbel* are about 4% lower than the relative values for the *Uniform*. Remarkably, 18 times out of the total instances, the VSS% exceeds the 30%. In general, this means that the demand uncertainty is the very critical factor to consider in procurement settings and that solutions coming up from an explicit study of the demand fluctuations (as we did through our SP approach) can be very useful on the long-term minimization of the costs.

7.4. Economic and managerial considerations

The previous analysis gives some insightful results. Clearly, it emerges how the more relevant source of uncertainty in this setting is represented by the demand, presenting quite large VSS.

In particular, a potential error due to the usage of the mean values can bring to a gap between the stochastic and the expected value solutions of more than 15% for medium-large procurements, and of more than 20% when few suppliers are involved. From a managerial point of view, this means that an integration of our models and algorithms for the CTQD-AC$_{ud}$ in a Decision Support System (DSS) might effectively give a strategic advantage to a company facing such a type of procurement. Since the discounts are granted if a certain amount of product is purchased, this DSS may also foster some collaborative interactions with other companies in order to further aggregate the demand (Perboli et al., 2016).

Due to the great number of scenarios to consider (see Section 7.1), the size of the problems that we can manage is still limited. However, it can be compatible with lots of different markets as, for example, the *Automotive* one where the number of big suppliers for a single part (body, chassis, electrical components, interiors, powertrain, and so on) are globally less than 100 with the 53% of the suppliers with revenues over 5 billions dollars (Jetli, 2014; Berret et al., 2016). Since in the last years the Automotive market had a profit around 7.5%, a competitive company needs a conservativeness for its long-term procurements as the one prospected by our modeling approach.

On the contrary, the volatility of the products price seems affecting the problem in a negligible way. First, it has to be said that even such as small percentages of saving on procurement costs could be of interest in those markets where the profits are very marginal (e.g., the market of computer/hi-tech hardware components where the profit is in the order of 4-5%). Second, and more important, it seems that stipulating total quantity discount contracts with the suppliers (in which discount clauses are known in advance) naturally prevents undesirable expenditures due to the product prices uncertainty. The price fluctuations are in fact absorbed by the discount applied to the total amount of products purchased at a supplier. Note that the product aggregation given by the model, needed to calculate this type of discounts, leads to a built-in control of the uncertainty on each single product parameters. The quite low EVPI% values for the CTQD-AC$_{up}$ also confirm that having precise information about the product prices does not impact valuably on the solution quality in this type of procurement setting.

Another critical issue, from a managerial point of view, is how much CTQD-AC$_{ud}$ optimal solutions make use of external supplies in the spot market (corresponding to the use of w variables) to satisfy the demand due to a lack of supply. In Figure 4, for both the distributions, we plot the percentage of product quantity purchased out of the selected suppliers with respect to the total amount purchased. These percentages, calculated on all the solved instances, have been averaged by different values of λ (first chart), by number of suppliers (second chart), and by number of products (third chart). Spot market is, on average, more consistent for instances with more suppliers and when the average available quantities tend to be closer to the product demands (i.e., when λ=0.1). Instead, a higher number of products leads to less spot market supplies. Again, the diversification of the products and the aggregation process to calculate the discounts helps in controlling the undesirable use of too many spot-suppliers. The highlighted trends are similar for both the distributions, but the values are halved (or more than halved) when considering a Gumbel distribution for the demands. More precisely, the percentages for the Uniform distribution can

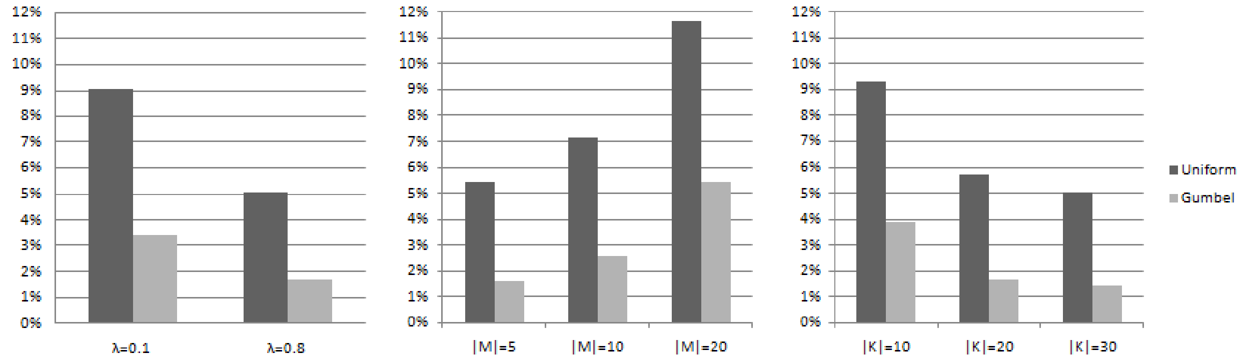


Figure 4: Average percentage of quantities purchased at external suppliers with respect to the total amount purchased.

reach the 10-12% of the total purchase and never go under the 5%. Instead, for the Gumbel, the higher value is about 5% with many cases where the external supplies are irrelevant (1-2% or less).

8. Conclusions and future research

In this paper, we have studied a long-term multi-product multi-supplier procurement problem called Capacitated Supplier Selection problem with Total Quantity Discount policy and Activation Costs, introducing its variant under uncertainty. In its deterministic form, the problem aims at selecting a subset of the suppliers and the relative purchasing plan such that the product demands are satisfied at minimum cost, also taking into account the discount policy offered by the suppliers and the cost to activate a business activity with them. Discounts are based on the total quantity of products purchased, regardless of the type of product. Since the assumption of having deterministic data is quite unrealistic in a long-term perspective, we explicitly consider all the reasonable sources of uncertainty and propose a general two-stage Stochastic Programming formulation with recourse to cope with them. Then, we adapt the model and the recourse actions for two particular cases, i.e. the ones in which only the product prices or only the product demands are stochastic variables. The modeling approaches are evaluated on a large set of instances and by considering different probability distributions and different parameterizations. In particular, we show how using total quantity discount contracts to select suppliers represents itself a good way in mitigating the effects of products price fluctuations. On the contrary, it is evident, how a more conservative approach (as the SP one) is crucial to cope with demand uncertainty in terms of solution quality and percentage of quantities purchased at external suppliers.

Some future extensions can be sketched. First, the results in this work are obtained through a branch-and-cut solution framework that has resulted more efficient than state-of-the-art MIP solvers. However, the computational times needed to reach the optimality for medium-large instances (with a reasonable number of scenarios) are still too high. We believe that scenario-based decomposition methods, like a heuristic *Progressive Hedging* (Perboli et al., 2017), or a Logit-based approximation (Tadei et al., 2012) might be more suitable. These methods could also allow to tackle instances with a larger number of scenarios or a more complex scenario tree structure necessary to model specific market cases, as economic crises or changes in market shares (Crainic et al., 2016). Second, once studied the basic supplier selection problem with quantity discounts under uncertainty, it could be interesting to consider additional complicating features (in order to address more realistic applications). Some of them, as highlighted for example in Goossens et al. (2007) or Sadrian and Yoon (1994), are the introduction of inventory costs for the products, *market share* constraints, and limitation on the number of selected suppliers.

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