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Michel Gendreau

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Bureaux de Montréal :
Université de Montréal
Pavillon André-Aisenstadt
C.P. 6128, succursale Centre-ville
Montréal (Québec)
Canada H3C 3J7
Téléphone : 514 343-7575
Télécopie : 514 343-7121

Bureaux de Québec :
Université Laval
Pavillon Palasis-Prince
2325, de la Terrasse, bureau 2642
Québec (Québec)
Canada G1V 0A6
Téléphone : 418 656-2073
Télécopie : 418 656-2624

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Population-Based Risk Equilibration for the Multi-Mode Hazmat Transport Network Design Problem†

Pirmin Fontaine[1,2,3], Teodor Gabriel Crainic[1,2,*], Stefan Minner[3], Michel Gendreau[1,4]

[1] Interuniversity Research Centre on Enterprise Networks, Logistics and Transportation (CIRRELT)

[2] Department of Management and Technology, Université du Québec à Montréal, P.O. Box 8888, Station Centre-Ville, Montréal, Canada H3C 3P8

[3] School of Management, Technical University of Munich, Arcisstraβe 21, 80333 Munich, Germany

[4] Department of Mathematics and Industrial Engineering, Polytechnique Montréal, P.O. Box 6079, Station Centre-Ville, Montréal, Canada H3C 3A7

Abstract. The shipment of hazardous materials is necessary for most countries and many of these products are flammable, explosive or even radioactive. Despite high security standards, accidents still happen and the transportation of hazmat causes fear among the population who faces the risk of those accidents. Therefore, the society requests a fair distribution of risk by the authorities. To support such a fair distribution, we propose a new population-based risk definition that evaluates the risk for the population in any given area with respect to its multimodal transportation network. Moreover, we propose different objective functions for equilibrating the risk and extend the bilevel Hazmat Transport Network Design Problem by considering several transportation modes. In this problem, the government wants to equilibrate the risk among the population centers by restricting links to the shipment of hazardous goods. When taking that decision, the government has to anticipate the carriers' reaction who want to minimize the transportation costs. This bilevel problem is transformed into a single-level mixed-integer linear program and solved with Xpress. In the numerical results, we show that both objectives have a positive convex correlation and therefore a significant improvement in risk distribution can be achieved at the cost of just a small increase in total risk. The cities with high risk benefit from the risk redistribution in the beginning. However, strong equilibrations just penalize cities with low risk. Moreover, compared to classical approaches in the literature, we achieve a better risk distribution among the population without increasing the total risk.

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Keywords: Hazardous materials transportation, risk equilibration, network design.

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* Corresponding author: TeodorGabriel.Crainic@cirrelt.net

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1 Introduction

Hazardous material accidents can have tremendous consequences for the population. One of the worst accidents of this kind in recent transportation related history happened in July 2013 in Lac-Mégantic, Quebec, Canada. A driverless train with 72 tank cars of petroleum crude oil derailed in the city center, burning it to the ground, causing the death of at least 42 persons, and destroying at least 30 buildings and 115 businesses. It took almost 2 days to control the fire. Yet, the transport of hazardous materials is essential not only for industrial countries like Canada, Germany and the United States, but also for developing countries. The four most frequently shipped hazardous materials are - with 80% of the transported volume in Canada - crude petroleum, gasoline, fuel oils, and non-metallic minerals (Searag et al., 2015). According to the Bureau of Transportation Statistics and U.S. Census Bureau (2015), 2,580 million tons of hazardous materials where shipped throughout the United States in 2012. 59.4% of them where transported by truck, 4.3% by rail, 11% by water and 24.3% by pipeline in single mode transportation. Only 1% was shipped via intermodal transportation. In Canada, railways have a much higher relevance. In 2012, 26.1 million tons were transported by rail and 107.4 million tons by truck. A different structure of the network in Germany, which, compared to North America, is very dense, is reflected in the share of transportation modes used: In 2010, 56 million tons were transported by maritime transport, 48 million tons on inland waterways, 63 million tons by rail, and 140 million tons by trucks (Statistisches Bundesamt Wiesbaden, 2012).

The consideration of different transportation modes is thus essential to adequately represent the hazardous material transportation system, in particular for risk calculation whenever one contemplates regulating the industry. The review of the literature reveals, however, that the different streams of research investigate the transportation of hazardous materials from a single-mode point of view, considering it either on roads (e.g., Kara and Verter, 2004) or on rail (e.g., Verma et al., 2011). We want to fill this gap by integrating different transportation modes in the *Hazmat Transport Network Design Problem* (*HTNDP*).

This raises the double question of, first, how to compute risk and, second, how to equilibrate this risk over the considered population. With respect to the former question, risk in the literature is associated with the individual arcs of the network and the population zones arrayed along them (e.g., Kara and Verter, 2004). This definition neglects the fact that the risk faced by people within any given area is influenced by the combined risk of all the transportation arcs, of all modes, crossing or bordering the area. We refer to such an area as a *population center* and introduce a new population-based risk definition to evaluate the risk in population centers.

Turning now to the latter question, one observes that, in the definitions proposed in the literature, either the total risk of the network is minimized (e.g., Kara and Verter, 2004) or the maximum arc risk is minimized for equilibration (e.g., Bianco et al., 2009). These definitions do not yield, in most cases, a fair distribution of risk. Thus, e.g., even though the total risk over all arcs is minimized, a population center with a high number of arcs will face a higher risk that one with few arcs. Similarly, when one arc supports the transport of a high amount of hazardous material, the maximum risk in the network will be defined by that arc and the distribution of all others arcs and population centers becomes unimportant with respect to the maximum risk function. We therefore introduce and numerically compare several risk-equilibration functions aiming for a fair distribution of risk among the population. Our numerical results show that traditional risk measures are wanting with respect to the equilibration of risk for a given population, and that the new population-based measure, and the introduction of multi-modality, yield a more effective model of risk and equity within a population. They also show that simply equilibrating risk may lead to a significant increase of the total risk in the network, and that all population centers may end up worse than before. Selecting the appropriate measure is thus important, and we investigate the trade-off between risk equilibration and risk minimization, showing a convex correlation between these two objectives. One can therefore achieve a much better distribution of risk among population centers with a small increase of the total risk in the network. To perform meaningful comparisons, we select and generalize, by introducing multiple transportation modes, the bilevel approach, a method well-known and well-represented in the literature. The resulting multimode multicommodity bilevel formulation is transformed into a mixed-integer linear program, and it is used in our numerical experiments to show the benefits of the new population-based risk definition over classical ones.

The contributions of this paper are: (1) A new population-based definition of risk and equity risk measures for hazardous material shipments; (2) an extension of the HTNDP to multimode transportation and risk equilibration; (3) a comparison of different risk equilibration measures; (4) insights on the trade-off between risk equilibration and risk minimization; (5) a comparison to existing models from the literature (single-mode and maximum arc risk equilibration).

This paper is structured as follows. The problem and notation are introduced, and the related literature is summarized, in Section 2. The new population-based risk definition is presented in Section 3, together with a number of risk-equilibration measures. Section 4 introduces the Multimode Hazmat Transport Network Design Problem and its transformation to a mixed-integer linear program. The results of the numerical study are presented and analyzed in Section 5, before ending with the conclusion and an outlook on future research.

2 Problem Definition and Related Literature

According to Erkut et al. (2007), the literature of hazardous material transport can be classified into four categories: risk assessment, routing, combined facility location and routing, and network design. They give a summary on all of these topics. This paper is located in the area of risk assessment and network design, which is so far the least investigated topic in this area.

The transportation network is represented by a graph $G = (N, A)$ with a set of nodes N and a set of arcs A. In a countrywide network, the nodes can be cities, facilities or important points in the network. In a city network, the level of detail needs to be much higher and the nodes represent junctions and entry and exit points of the city. Compared to the existing literature, we consider different transportation modes $m \in M$. Depending on the detail of the model, these modes are the classical modes train, road, rail, air, water and pipeline; however, we also define different vehicle types as a transportation mode. To keep the notation simple, we neglect the fact that in practice not every arc can be used by every transportation mode. Each arc could further have a capacity limit for each mode. Especially pipelines have limitations on the possible amount of shipments. We, however, stay in line with the literature and neglect the fact of capacity restrictions.

K is the set of commodities shipped through the network. Each commodity $k \in K$ is defined by an origin $o_k \in N$, a destination $d_k \in N$ and the transport volume ϕ_k. The transportation costs for shipping one unit of commodity $k \in K$ on arc $(i, j) \in A$ with transportation mode $m \in M$ are c_{ij}^{km}. Each commodity can be shipped partly via different transportation modes. However, we do not allow inter-modal transportation, as this is also not often the case in practice (e.g., Bureau of Transportation Statistics and U.S. Census Bureau, 2015). The probability of an incident on arc $(i, j) \in A$ with mode $m \in M$ is given by σ_{ij}^{km}. Similar to the literature, it is assumed that there is no correlation between accidents and therefore the probability distributions are independent.

To equilibrate the risk among the population, we define a set of population centers C with a population P_c. In a global optimization setting, a population center represents a city; if the risk is equilibrated inside a city, these population centers need to represent districts or parts of the city.

Finally, l_{ij}^{mkc} defines the influence of an accident on arc (i, j) of commodity k using mode m on the population c. This influence factor depends on the distance between the population center and the arc, as well as the hazardous material type: The shorter the distance and the more dangerous the material is, the higher is the influence factor. The literature introduces different methods for calculating the influence of an accident on an arc: Batta and Chiu (1988) use a fixed bandwidth around the route segment, Erkut and Verter (1998) define a danger circle and Patel and Horowitz (1994) use a Gaussian plume model to define the impact of airborne hazmat accidents. We assume that these influence factors are given. Using this notation, we associate the risk with the population center. This is in contrast to the network design literature (e.g., Alp, 1995; Kara and Verter, 2004; Bianco et al., 2009), where risk calculation is associated with arcs. With P_{ij} being the accumulated affected population in the area of arc (i, j), the classical literature defines the risk of an arc by $\sum_{k \in K} \sigma_{ij}^{km} P_{ij} \phi_k x_{ij}^{km}$ (Erkut and Verter, 1998). This definition is used in the models and solution methods for network design problems proposed in the literature.

To evaluate our new risk definition, we model the problem as linear bilevel problem. In this well-studied formulation, the leader represents the government or an authority. The leader can decide if the mode of an arc of the network is allowed for the transportation of hazardous materials or not and the decision is modeled by the binary decision variable y_{ij}^{m}. For simplification, the differentiation between specific hazardous material types is ignored, but the model could easily be extended to include this more realistic setting. The leader decision is subject to the follower optimization problem: The carriers minimize their transportation costs subject to demand satisfaction by deciding over the transportation percentage x_{ij}^{km} of commodity $k \in K$ shipped over arc $(i, j) \in A$ on transportation mode $m \in M$.

Kara and Verter (2004) reformulate the bilevel problem into a single-level formulation using the Karush-Kuhn-Tucker (KKT) conditions. In Verter and Kara (2008), a path-based formulation of the HTNDP is proposed. For each carrier, all possible paths are generated and ordered according to the carrier priority. Bianco et al. (2009) also transform the bilevel problem into a single-level formulation and show that these solutions might not be stable. If several follower solutions exist, the KKT conditions use an optimistic bilevel formulation which assumes that the carriers choose the path with the lowest risk among all shortest paths. In this case, the total risk can increase if a carrier chooses a path with higher risk. They provide a method to evaluate the stability and present a heuristic which always finds stable solutions. Amaldi et al. (2011) proposed a mixed-integer linear program to solve the HTNDP. This global optimization method further guarantees finding stable solutions. Recently, Fontaine and Minner (2018) proposed a Benders decomposition approach to solve larger instances. While most models consider a network design problem, a different approach for a reducing the risk is to introduce tolls for the transportation of hazardous materials (Marcotte et al., 2009).

As linear bilevel problems are already $\mathcal{NP}$-hard (Ben-Ayed and Blair, 1990), different heuristics were proposed as well: Erkut and Alp (2007) present a solution method that starts with a tree structured subset of the network and gradually adds new arcs. In contrast, Erkut and Gzara (2008) keep the bilevel formulation and propose a heuristic algorithm to solve the problem. The objective function of the follower problem is integrated into the objective function of the leader problem in a bi-objective bilevel model. Recently, Bianco et al. (2016) use a game-theoretic approach for regulating the transportation of hazardous materials via tolls. To calculate a Nash-equilibrium, they use a local search heuristic not only to minimize the total risk, but also to equilibrate the risk on the arcs by looking at the maximum risk on an arc. This approach is restricted to one hazmat type and the authors point out that an extension makes the problem much harder as the Nash game is not convex anymore. Sun et al. (2015) include risk uncertainty into the network design problem and introduce a heuristic to find a robust solution.

Besides the solution method, the evaluation of risk for specific paths was investigated as well. Alp

(1995) minimizes the sum of the risk of all used arcs and Erkut and Ingolfsson (2000) propose more advanced objective functions like the mean-variance. In contrast to that, ReVelle et al. (1991) minimize the exposed population on a path and Saccomanno and Chan (1985) the incident probability on a path. Since all these measures assume independent risk probabilities, Kara et al. (2003) propose a method to evaluate the risk on a path accurately by including conditional risk distributions. However, Erkut and Verter (1998) show that the approximation error of the independent risk assumption is small. To include the fact that the population might not be risk-neutral and favor a higher probability of a low-consequences accident over a lower probability with high consequences, Abkowitz et al. (1992) introduce the perceived risk. The exposed population is exponentiated with a risk preference. If this risk preference is greater than 1, a risk averse population is assumed, if it is 1, the population is risk neutral and if it is smaller than 1, the population is risk prone. Bianco et al. (2009) and Bianco et al. (2016) equilibrate the risk over all arcs by minimizing the maximum risk on an arc. We will show that our approach can achieve a better distribution of risk for a population by comparing it to these approaches. Moreover, Taslimi et al. (2017) propose a framework for equity in hazmat network design and hazmat response team location. A summary of different risk measures can also be found in Erkut and Ingolfsson (2005).

Even though risk equilibration is a fairly new topic to hazmat network design, it has been studied in the area of hazardous material routing. Gopalan et al. (1990) propose a shortest-path problem for routing trucks that minimizes the total risk and ensures risk equity between zones by a constraint. This constraint limits the risk deviation between these zones. However, this model ignores the fact that the carrier's main goal is to minimize the costs. Lindner-Dutton et al. (1991) extend this model and also include the sequencing of trucks, to have a fair distribution in every period and not over the whole planning horizon. Carotenuto et al. (2007) define a mixed-interger linear program to find minimal and equitable risk routes for hazardous material shipments. This fair risk distribution is done by distributing it equally among the arcs. The problem is solved by a modified k-shortest path algorithm. For hazmat facility location, Romero et al. (2016) used the Gini coefficient to determine the location of storage facilities under the aspect of a fair risk distribution. Moreover, Sun et al. (2016) include cost equity for the carriers. Since network design decisions influence the costs of each carrier differently, the model includes this unfair aspect and equilibrates the cost increase of all carriers.

3 Population-based Risk Definition and Evaluation

In this section, we first introduce the population-based risk definition. Then we define different possible risk equilibration measures and give an example for the differences between the classical risk definition and ours.

3.1 Risk Definition

In contrast to the classical network design definition of risk on arcs, we define the risk for each population center $c \in C$. Following the classical definition, we assume that only one accident can happen at the same time on an arc. Therefore, the accidents on the different arcs through a center are independent and the expected risk is defined as follows:

$$R_c(x) := P_c \sum_{m \in M} \sum_{k \in K} \sum_{(i,j) \in A} l_{ij}^{mkc} \sigma_{ij}^{km} \phi_k x_{ij}^{km} \tag{1}$$

Thus, the risk of a population center is the sum of the transported volume on all arcs via all modes in the influence area of the center weighted with the accident risk and the potential influence factor.

As long as the overall risk in the network is minimized, this risk definition is fully equivalent to the traditional risk definition, only the order of summation is changed. However, the differences can be huge for equilibrating the risk, i.e., by minimizing the maximum risk.

The network of Figure 1 gives an example of how the risk measures differ when we minimize the maximum risk. We assume two OD-pairs: 10 units from 1 to 4 via road and 10 units from 1 to 4 via rail. In this example, an optimal solution will always ship the 10 rail units via 2, as no other solution exists.

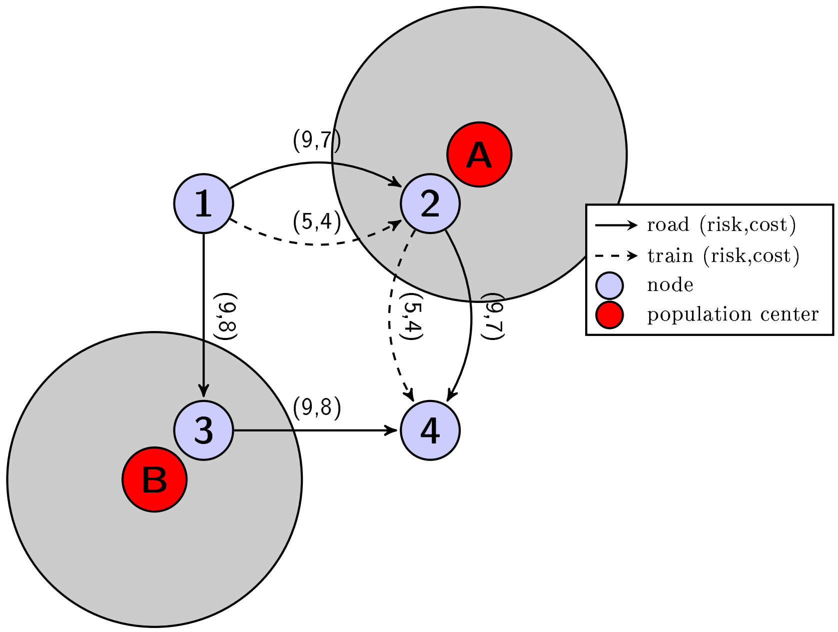

Figure 1: Example of the Population-based Risk Definition

However, the road commodity has two options, shipping either via 2 or via 3. If the maximum risk on each arc is minimized, both paths are the same from a risk perspective. Both will cause a total risk of 180 and no arc will be forbidden. The carrier will choose the cheapest path via 2 and population A will face a total risk of 280 and population B one of zero. If the maximum population risk is minimized, the government will close the road between 1 and 2 and the carrier will have to ship via 3. The risk for population A would be 100 and for population B 180. In both solutions, the maximum risk on an arc is 90 and the classical risk in the network is 280. The only difference is a fair distribution of the risk among the population.

3.2 Risk Equilibration Measures

Erkut and Ingolfsson (2005) summarized different measures the for evaluation of risk on a path, Bianco et al. (2009) equilibrated the risk by minimizing the maximum risk on an arc and Marsh and Schilling (1994) reviewed different equity measures in location theory. Following these risk evaluation ideas, we introduce several possible risk measures for the population-based risk definition of the whole network.

$$\sum_{c \in C} R_c(x) \quad \text{Traditional/Overall risk (Trad)} \tag{2}$$

$$\max_{c \in C} R_c(x) \quad \text{Maximum risk (Max)} \tag{3}$$

$$\frac{1}{|C|}\sum_{c\in C}\left|R_c(x)-\frac{1}{|C|}\sum_{c'\in C}R_{c'}(x)\right| \quad \text{Average deviation to mean (AdM)} \tag{4}$$

$$\max_{c\in C}\left|R_c(x)-\frac{1}{|C|}\sum_{c'\in C}R_{c'}(x)\right| \quad \text{Maximum deviation to mean (MdM)} \tag{5}$$

$$\frac{1}{|C|(|C|-1)}\sum_{c,c'\in C|c<>c'}|R_c(x)-R_{c'}(x)| \quad \text{Average deviation among all (AdA)} \tag{6}$$

$$\max_{c,c'\in C|c<>c'}|R_c(x)-R_{c'}(x)| \quad \text{Maximum deviation among all (MdA)} \tag{7}$$

The traditional risk measure (2) sums the risk of all population centers and is equivalent to the arc definition of the risk. The maximum risk (3) minimizes the maximal risk in a population center. If each population center is defined by one arc, this definition is equivalent to the maximum arc risk definition by Bianco et al. (2009). The risk measures (5) - (7) are different deviation measures which are all zero when the risk is perfectly equilibrated and the risk in every population center is the same. While the first two calculate the average and maximum deviation to the mean, the last two give the average difference between all population centers and the maximum difference between two population centers.

Since in pure equilibration of risk, the total risk of the network is neglected, it can lead to a very high increase of risk in some population centers. To avoid this effect, we introduce social bounds for the risk. Let L be the set of social bounds. Then, each population center c has the bounds b_c^l for each $l \in L$. If the risk is higher than a bound, a penalty p_c^l for all risk above this bound is added to the objective value. The new risk is calculated by adding the following term to the objective function:

$$+\sum_{l\in L}p_c^l P_c \max\left\{0, \sum_{m\in M}\sum_{k\in K}\sum_{(i,j)\in A} l_{ij}^{mkc}\sigma_{ij}^{km}x_{ij}^{km}-b_c^l\right\}. \tag{8}$$

This gives a piecewise linear increasing objective function. Such functions are also often used to equilibrate the user's travel time in traffic assignment problems (Sheffi, 1985). Moreover, this idea is similar to the idea of conditional value at risk and perceived risk with a risk-averse population. For example, Abkowitz et al. (1992) and Erkut and Ingolfsson (2000) used a non-linear function $f(x) = x^\alpha$ with $\alpha > 1$, to take into account that accidents with high probability and low consequences are less undesirable than low probability-high consequence accidents.

4 Multi-Mode Hazmat Transport Network Design Problem

In this section, we first introduce the bilevel formulation for the Multi-Mode Hazmat Transport Network Design Problem (mHTNDP) and explain how the general definition can be adapted to specific network types. Then we transform the model into a mixed-integer linear program.

4.1 Bilevel Formulation

Besides the already introduced decision variables x_{ij}^{km} and y_{ij}^m, let z^{km} be the percentage of commodity $k \in K$ shipped with mode m.

All introduced objective functions can be used in the leader problem, and all transformations shown in this section can be applied as well. Considering the maximum risk objective function, the leader problem can be defined as follows:

$$\min r_{max} \tag{9}$$

$$\text{s.t. } P_c \sum_{m\in M}\sum_{k\in K}\sum_{(i,j)\in A} l_{ij}^{mkc}\sigma_{ij}^{km}\phi_k x_{ij}^{km} \leq r_{max} \qquad \forall c \in C \tag{10}$$

$$r_{max} \geq 0 \tag{11}$$

$$y_{ij}^m \in \{0,1\} \qquad \forall (i,j)\in A, m\in M \tag{12}$$

The follower problem is a multi-mode shortest path problem. The carriers decide how many percent z^{km} of commodity k are shipped via transportation mode m. Equation (14) is the flow conservation constraint and constraint (15) ensures that the full demand is divided into the different transportation modes. Constraint (16) ensures that only arcs which are allowed by the leader can be used. In the objective function, the carriers' user-optimum - the overall transportation costs - is minimized.

$$\min \sum_{k\in K}\sum_{m\in M}\sum_{(i,j)\in A} c_{ij}^{km}x_{ij}^{km} \tag{13}$$

$$\text{s.t. } \sum_{(i,j)\in A} x_{ij}^{km} - \sum_{(j,l)\in A} x_{jl}^{km} = \begin{cases} 0, & \text{if } j \neq o_k, d_k \\ -z^{km}, & \text{if } j = o_k \\ z^{km}, & \text{if } j = d_k \end{cases} \qquad \forall j\in N, k\in K, m\in M \tag{14}$$

$$\sum_{m\in M} z^{km} = 1 \qquad \forall k \in K \tag{15}$$

$$x_{ij}^{km} \leq y_{ij}^m \qquad \forall (i,j)\in A, k\in K, m\in M \tag{16}$$

$$x_{ij}^{km} \geq 0 \qquad \forall (i,j)\in A, k\in K, m\in M \tag{17}$$

$$z^{km} \geq 0 \qquad \forall k\in K, m\in M \tag{18}$$

This model is a generalization of the classical HTNDP. Depending on the network and setting, several special cases are possible: By using only one transportation mode the follower problem is equivalent to the classical shortest path problem by Kara and Verter (2004). In an urban area setting, the only used transportation mode is the road with several vehicle types. Other transportation modes like rail or pipeline exist, but the decision might be taken in a global network design problem. However, for the fair equilibration, it is important to include the risk of these modes as constant into the population center where it appears.

By optimizing the risk distribution in a global setting like a province or a country, the modes can describe not only different vehicle types, but also rail, pipeline and water transport. Because of the transport via rail, pipeline or water, a capacity restriction might become necessary. However, with a capacity restriction, the follower problem becomes a multi-commodity transportation problem. Therefore, no longer a user-optimum but a system-optimum is calculated which is not considered in this paper.

4.2 Transformation to a Mixed-Integer Linear Program

To transform the linear bilevel problem into a non-linear mixed-integer program, we assume the partial cooperation assumption and the follower problem can be replaced by the Karush-Kuhn-Tucker conditions

(Bard, 1998). The assumption of an optimistic bilevel problem can lead unstable solutions. As introduced by Amaldi et al. (2011), an additional term can be added to the follower objective function to assume the pessimistic case. However, we used the optimistic case, since we believe also carriers are not interested in choosing a high risk path and by cost data perturbation also unique follower solutions are possible.

$$\min \; r_{max} \tag{19}$$

$$\text{s.t.} \tag{20}$$

$$P_c \sum_{m\in M} \sum_{k\in K} \sum_{(i,j)\in A} l_{ij}^{mkc} \sigma_{ij}^{km} \phi_k x_{ij}^{km} \leq r_{max} \qquad \forall c \in C \tag{21}$$

$$\sum_{(i,j)\in A} x_{ij}^{km} - \sum_{(j,l)\in A} x_{jl}^{km} = \begin{cases} 0, & \text{if } j \neq o_k, d_k \\ -z^{km}, & \text{if } j = o_k \\ z^{km}, & \text{if } j = d_k \end{cases} \qquad \forall j \in N, k \in K, m \in M \tag{22}$$

$$\sum_{m\in M} z^{km} = 1 \qquad \forall k \in K \tag{23}$$

$$x_{ij}^{km} \leq y_{ij}^{m} \qquad \forall (i,j) \in A, k \in K, m \in M \tag{24}$$

$$\sum_{k\in K} \sum_{m\in M} \sum_{(i,j)\in A} c_{ij}^{km} x_{ij}^{km} \leq \sum_{k\in K} v^k + \sum_{m\in M} \sum_{(i,j)\in A} \sum_{k\in K} t_{ij}^{km} y_{ij}^{m} \tag{25}$$

$$u_j^{km} - u_i^{km} + t_{ij}^{km} \leq c_{ij}^{km} \qquad \forall (i,j) \in A, k \in K, m \in M \tag{26}$$

$$u_{o_k}^{km} - u_{d_k}^{km} + v^k \leq 0 \qquad \forall k \in K, m \in M \tag{27}$$

$$v^k \in \mathbb{R} \qquad \forall k \in K \tag{28}$$

$$u_j^{km} \in \mathbb{R} \qquad \forall j \in N, k \in K, m \in M \tag{29}$$

$$t_{ij}^{km} \leq 0 \qquad \forall (i,j) \in A, k \in K, m \in M \tag{30}$$

$$x_{ij}^{km} \geq 0 \qquad \forall (i,j) \in A, k \in K, m \in M \tag{31}$$

$$z^{km} \geq 0 \qquad \forall k \in K, m \in M \tag{32}$$

$$r_{max} \geq 0 \tag{33}$$

$$y_{ij}^{m} \in \{0,1\} \qquad \forall (i,j) \in A, m \in M \tag{34}$$

To replace the follower problem (13) - (18) with the optimality conditions, we define u_j^{km}, v^k and t_{ij}^{km} as the dual variables of the follower constraints (14) - (16). While (22) - (24) are the primal constraint, equation (26) is the dual constraint associated with the primal variable x_{ij}^{k} and equation (27) is the dual constraint associated withthe primal variable z^{km}. Equation (25) is the optimality condition which equals the primal and dual follower objectives.

As in Cao and Chen (2006), the optimality condition can be linearized by introducing the auxiliary variables w_{ij}^{km} and a Big $\hat{M}$ and by replacing (25) with the following terms:

$$\sum_{k\in K}\sum_{m\in M}\sum_{(i,j)\in A} c_{ij}^{km}x_{ij}^{km} \leq \sum_{k\in K} v^k + \sum_{m\in M}\sum_{(i,j)\in A}\sum_{k\in K} w_{ij}^{km} \tag{35}$$

$$w_{ij}^{km} \leq t_{ij}^{km} + \hat{M}(1-y_{ij}^{m}) \qquad \forall (i,j)\in A, m\in M, k\in K \tag{36}$$

$$w_{ij}^{km} \geq t_{ij}^{km} \qquad \forall (i,j)\in A, m\in M, k\in K \tag{37}$$

$$w_{ij}^{km} \geq -\hat{M}y_{ij}^{m} \qquad \forall (i,j)\in A, m\in M, k\in K \tag{38}$$

$$w_{ij}^{km} \leq 0 \qquad \forall (i,j)\in A, m\in M, k\in K \tag{39}$$

5 Numerical Study

In this section, we show the computational results of the model proposed in the previous section. We used Xpress 7.9 on an Intel Core i7 with 4 cores and 32GB RAM. In the results, we used the risk measure abbreviations of Section 3.2.

First, we used the Sioux Falls network from the literature (Bar-Gera, 2013) to show the convergence properties and the trade-off between risk minimization and risk equilibration. Then, we compared the model to risk formulations from the literature: a one-mode decision model and the maximum arc risk formulation. Finally, we applied our model to a larger US-Canada instance (Orlowski et al., 2010) to confirm the results of the small example.

5.1 Sioux Falls Instances

The Sioux Falls network consists of 24 nodes and 76 arcs. Each arc is defined by a length (in km), however other necessary data was generated as follows: The network is divided into six population centers. The population density of Sioux Falls is 814.4 inhabitants per square kilometer. The population distribution is shown with the definition of the population centers in Figure 2. The influence of an arc on a population center was set to 1 if the arc is inside the center, 0 if not. If an arc is contained in more than one center, the influence was proportionally split into two parts (e.g., arc (11,14)).

Two vehicle types are used with transportation costs of 1.1 per transported unit per km for the smaller vehicle and 0.9 per km for the larger vehicle. The accident probability of the larger vehicle was set 3% higher than the risk of the smaller one. The accident rate on an arc was generated randomly between 9.56×10^{-9} and 1.08×10^{-7} (Erkut and Gzara, 2008) and σ_{ij}^{km} is the product of the accident rate, the length of the arc and the factor for the vehicle type. We assume only one commodity type. Consequently, the risk for all shipped commodities are the same.

Four instances with different demand scenarios were generated. Nodes $1, 2, 13, 20$ were defined as the entry and exit nodes of the network. 43 commodities were shipped through the network (13 out-flows and 30 in-flows). The demand into the city (in-flows) was set between 100 and 1,000 and out of the city (out-flows) between 50 and 150. For each scenario the origin, the destination, and the demand were generated randomly.

For the non-linear function with l social bounds (NLl), we used a simple penalty function, which is defined as follows: The risk interval was divided into $l-1$ equidistant segments and from each point p, a further penalty of p^2 was added to the objective function.

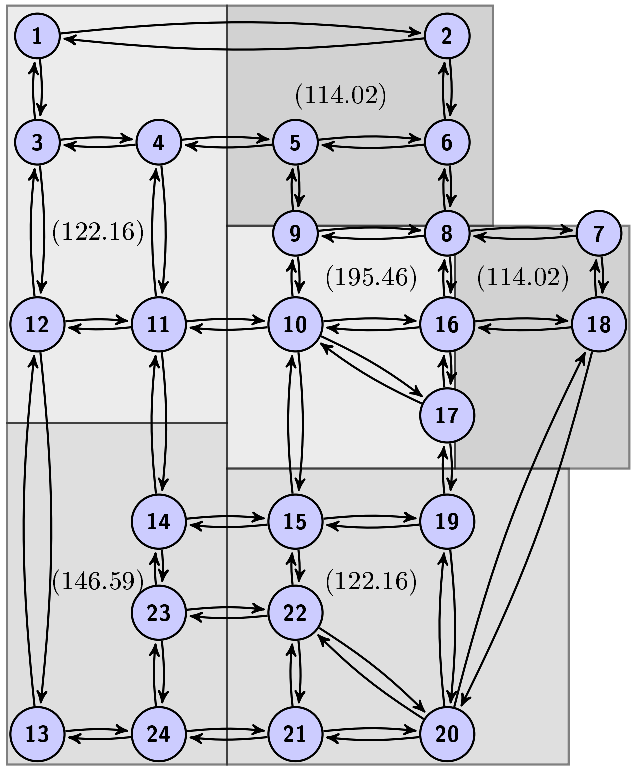

Figure 2: Sioux Falls Network with Population Centers and Population Density

In a first test, we evaluated the convergence of the model. Figure 3 shows that the lower bound stays zero and therefore the GAP cannot be calculated. This is the case for all measures that calculate the deviation: MdM, AdM, MdA, AdA. As mentioned in the definition of the measures, all population centers have the same risk in an optimal equilibration: zero. Most of the improvements happen within the first 20 minutes and there are still some improvements within two hours. Therefore, the time limit of the numerical study was set to 7,200 seconds. For the other objective functions and trade-off calculations, the convergence was significantly better and the average gaps are below 1%.

We first show the effect of risk equilibration and the differences between risk equilibration and total risk minimization and discuss possible approaches to combine them. Then, we compare the effect of using different transportation modes in one model before discussing the difference between our model and the equilibration idea from the literature.

5.1.1 Risk Equilibration

Table 1 shows the results for the different objective functions for demand scenario 1 and the risk of all population centers is reported. The optimized objective function is shown in the first column. Moreover, the optimized risk measure is highlighted in bold.

The results show that just minimizing the deviation or the maximum leads to an extreme increase in the overall risk in the network. The results are quite obvious as an equilibrium is only possible on a high level. It shows that, for an equal distribution of risk, almost every population center comes out worse and the total risk increases by more than 100%. Only the non-linear function, which does not try

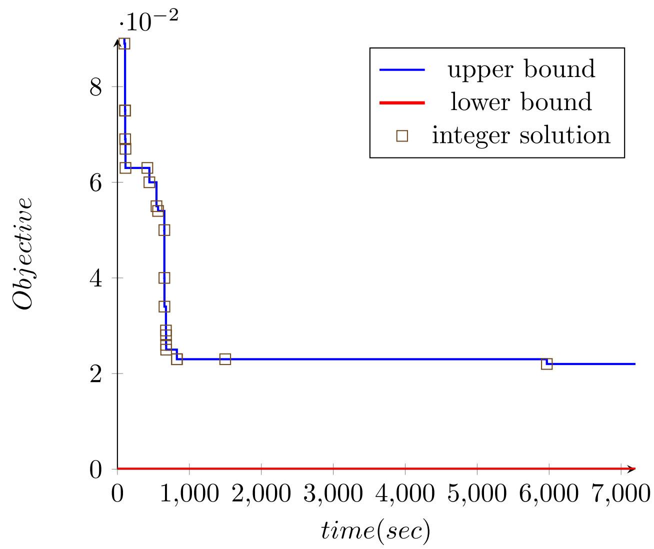

Figure 3: Run Time Analysis for AdA.

Table 1: Risk Evalutation for Demand Scenario 1

Obj	Risk measures						Risk of population centers					
	Trad	Max	AdM	MdM	AdA	MdA	Pop1	Pop2	Pop3	Pop4	Pop5	Pop6
Trad	**1.574**	0.525	0.155	0.262	0.207	0.419	0.525	0.179	0.464	0.105	0.169	0.132
Max	2.144	**0.370**	0.018	0.055	0.023	0.067	0.370	0.367	0.366	0.303	0.370	0.370
AdM	2.680	0.488	**0.014**	0.041	0.024	0.063	0.441	0.425	0.488	0.447	0.447	0.433
MdM	2.251	0.441	0.042	**0.066**	0.065	0.128	0.314	0.371	0.441	0.314	0.421	0.390
AdA	3.194	0.541	0.004	0.012	**0.007**	0.020	0.520	0.534	0.541	0.532	0.533	0.532
MdA	2.482	0.444	0.028	0.042	0.040	**0.072**	0.414	0.372	0.444	0.443	0.436	0.372
NL7	1.574	0.525	0.155	0.262	0.207	0.419	0.525	0.179	0.464	0.105	0.169	0.132

to equalize all populations, distributes the risk better without a dramatic risk increase. These effects are similar for the other 3 instances as well and as the equilibration measures AdM, MdM, AdA, MdM perform very similar and the maximum risk is only effective if there is no population center with a very high risk, we will use the AdA measure for the following analysis.

5.1.2 Trade-off between Risk Equilibration and Risk Minimization

Since the pure risk equilibration increases the total risk significantly, we analyze the trade-off between minimizing the total risk in the network and equilibrating the risk. In the literature (e.g., Gopalan et al., 1990; Lindner-Dutton et al., 1991), this is achieved by adding the level of equilibration as a constraint into the model. This approach is not applicable in bilevel programming, as this constraint would be a so-called coupling constraint (e.g., Dempe, 2002) and then the KKT-transformation is not feasible. Therefore, we used a bi-objective function that combines the AdA equilibration measure with the overall risk function Trad, which is weighted with $\alpha \in \{0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5\}$. As mentioned earlier, the optimality gaps are significantly smaller in this study. The bi-objective function has over all instances an average gap of less than 1%.

Table 2 shows the results for the 4 randomly generated demand scenarios (DS). The minimal risk solutions are 1.574 (DS 1), 1.388 (DS 2), 1.399 (DS 3) and 1.335 (DS 4) and shown in the first line of each scenario. One can see that the risk is better distributed among the population centers without increasing the risk as much as in the pure equilibration measures. Also, a higher weight on the risk minimization (Trad) leads, as expected, to a lower risk with worse equilibration.

The distribution is already much better for an increase of 5 - 15% of the total risk. However, to distribute the risk as fairly as possible, an increase of more than 35% is necessary. Moreover, the equilibration of the first α steps is mainly achieved by a significant reduction of the risk in the population centers with the highest risk and a shift to low risk population centers. But especially in the last steps, the equilibration is achieved by increasing the risk in low risk population centers without reducing the risk in high risk population centers. For example in demand scenario 1, the risk of population center 1 drops from 0.525 to 0.381 in the first step ($\alpha = 0.5\%$). Population center 3 improves in the first step from 0.464 to 0.430. Even though the equilibration improves further for $\alpha \leq 0.5$, this is mostly due to a risk increase in population centers 2, 4, 5 and 6.

A similar effect can be seen in the other scenarios. DS2 shows a significant improvements of population centers 1 and 3 until $\alpha = 0.4$. After that the equilibration is again mostly achieved by increasing the risk in other population centers.

Figure 4 shows the Pareto-optimal curves of the trade-off between an equilibrated network and a network with a low total risk. The results indicate a convex substitution relation on the efficient frontier between both objective functions, which is consistent with our previous findings: With a small increase of total risk, the risk can be much better equilibrated. However, there comes a point from which on the price of total risk in the network for further equilibration is very high.

Besides the bi-objective approach, the non-linear objective function of the previous section showed similar effects and can be a good alternative, especially when risk is perceived differently in different population centers.

Table 2: Trade-off between Trad and AdA for 4 Demand Scenarios

DS	Objective function	Risk measure		Risk of population centers					
		Trad	AdA	Pop 1	Pop 2	Pop 3	Pop 4	Pop 5	Pop 6
1	Trad	1.574	0.207	0.525	0.179	0.464	0.105	0.169	0.132
	AdA + 0.50 Trad	1.637	0.136	0.381	0.272	0.430	0.163	0.227	0.164
	AdA + 0.45 Trad	1.643	0.133	0.402	0.261	0.430	0.180	0.190	0.179
	AdA + 0.40 Trad	1.643	0.133	0.402	0.261	0.430	0.180	0.190	0.179
	AdA + 0.35 Trad	1.660	0.126	0.357	0.283	0.442	0.186	0.210	0.181
	AdA + 0.30 Trad	1.726	0.104	0.348	0.279	0.423	0.202	0.274	0.200
	AdA + 0.25 Trad	1.862	0.068	0.346	0.284	0.423	0.269	0.272	0.269
	AdA + 0.20 Trad	2.024	0.031	0.329	0.328	0.406	0.321	0.320	0.320
	AdA + 0.15 Trad	2.074	0.029	0.351	0.337	0.400	0.327	0.331	0.328
	AdA + 0.10 Trad	2.179	0.015	0.370	0.362	0.378	0.357	0.370	0.341
2	Trad	1.388	0.177	0.393	0.175	0.448	0.164	0.134	0.074
	AdA + 0.50 Trad	1.425	0.151	0.383	0.208	0.417	0.129	0.161	0.127
	AdA + 0.45 Trad	1.425	0.151	0.383	0.208	0.417	0.129	0.161	0.127
	AdA + 0.40 Trad	1.480	0.127	0.372	0.243	0.383	0.147	0.187	0.148
	AdA + 0.35 Trad	1.480	0.127	0.372	0.243	0.383	0.147	0.187	0.148
	AdA + 0.30 Trad	1.494	0.122	0.374	0.247	0.375	0.152	0.194	0.152
	AdA + 0.25 Trad	1.584	0.098	0.368	0.263	0.368	0.193	0.199	0.193
	AdA + 0.20 Trad	1.983	0.014	0.340	0.335	0.347	0.320	0.322	0.320
	AdA + 0.15 Trad	2.063	0.002	0.343	0.343	0.346	0.344	0.342	0.345
	AdA + 0.10 Trad	2.043	0.003	0.341	0.340	0.346	0.340	0.337	0.339
3	Trad	1.399	0.159	0.377	0.199	0.412	0.185	0.154	0.072
	AdA + 0.50 Trad	1.447	0.125	0.358	0.250	0.370	0.145	0.189	0.136
	AdA + 0.45 Trad	1.447	0.125	0.358	0.250	0.370	0.145	0.189	0.136
	AdA + 0.40 Trad	1.540	0.084	0.323	0.306	0.323	0.180	0.237	0.170
	AdA + 0.35 Trad	1.546	0.082	0.322	0.308	0.323	0.178	0.238	0.177
	AdA + 0.30 Trad	1.546	0.082	0.322	0.308	0.323	0.178	0.238	0.177
	AdA + 0.25 Trad	1.654	0.051	0.319	0.303	0.325	0.235	0.237	0.235
	AdA + 0.20 Trad	1.764	0.027	0.314	0.304	0.325	0.275	0.273	0.274
	AdA + 0.15 Trad	1.843	0.011	0.310	0.302	0.327	0.303	0.301	0.301
	AdA + 0.10 Trad	1.875	0.007	0.322	0.308	0.318	0.307	0.309	0.310
4	Trad	1.335	0.165	0.347	0.217	0.419	0.099	0.171	0.081
	AdA + 0.50 Trad	1.402	0.115	0.310	0.280	0.339	0.129	0.233	0.112
	AdA + 0.45 Trad	1.432	0.100	0.301	0.306	0.312	0.125	0.258	0.131
	AdA + 0.40 Trad	1.443	0.096	0.301	0.306	0.309	0.136	0.258	0.133
	AdA + 0.35 Trad	1.443	0.095	0.301	0.305	0.309	0.136	0.259	0.133
	AdA + 0.30 Trad	1.458	0.091	0.301	0.304	0.310	0.143	0.257	0.143
	AdA + 0.25 Trad	1.641	0.042	0.301	0.304	0.310	0.237	0.255	0.234
	AdA + 0.20 Trad	1.768	0.015	0.306	0.304	0.308	0.289	0.284	0.278
	AdA + 0.15 Trad	1.794	0.010	0.298	0.310	0.308	0.294	0.294	0.289
	AdA + 0.10 Trad	1.799	0.009	0.303	0.308	0.307	0.298	0.290	0.293

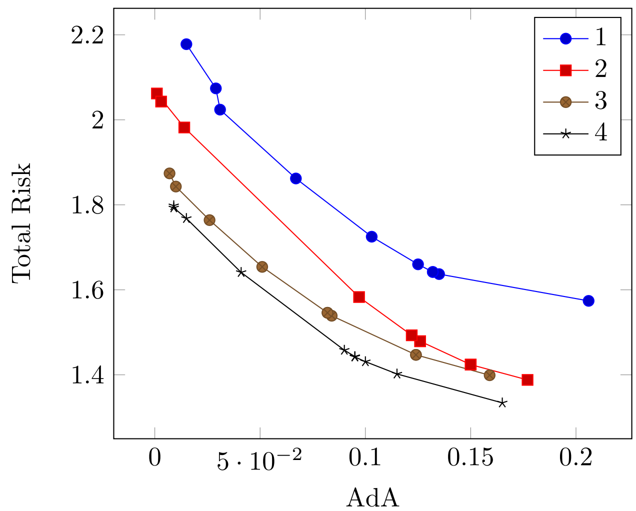

Figure 4: Trade-off between Trad and AdA for 4 Demand Scenarios

5.1.3 Comparison to One-Mode Decision Model

In this subsection, we show the necessity of using a multi-mode decision model by comparing our model to classical single mode models. In the multi-mode decision model the mode is part of the decision process. Therefore, we split the demands according to the modal split of our multi-mode network design model. The resulting demand patterns are used to solve the hazmat network design problem for mode 1 and 2 separately and evaluate the sum of both ("sum" in Table 3). The multi-mode result is compared with the sum of the single mode models. Moreover, the network design of the two single mode models is used in the multi-mode model to see the reaction of the followers on the single mode decisions ("reaction"). For all models, we used the non-linear objective function with 7 approximation points (NL7) as the non-linear function combined risk equilibration and risk minimization in one function. The detailed results are shown in Table 3.

Table 3: Comparison of the multi-mode model with single-level decisions

DS	Model	Risk measure		Risk of population centers					
		Trad	AdA	Pop 1	Pop 2	Pop 3	Pop 4	Pop 5	Pop 6
1	multi mode	1.696	0.119	0.357	0.286	0.413	0.147	0.284	0.209
	sum	1.687	0.127	0.381	0.298	0.402	0.130	0.267	0.209
	reaction	1.681	0.130	0.378	0.294	0.410	0.128	0.264	0.207
2	multi mode	1.524	0.118	0.356	0.288	0.365	0.147	0.211	0.157
	sum	1.522	0.118	0.356	0.289	0.365	0.147	0.208	0.157
	reaction	1.522	0.118	0.356	0.289	0.365	0.147	0.208	0.157
3	multi mode	1.538	0.089	0.326	0.293	0.329	0.208	0.239	0.142
	sum	1.547	0.099	0.362	0.267	0.329	0.215	0.233	0.140
	reaction	1.460	0.142	0.386	0.210	0.396	0.188	0.183	0.097
4	multi mode	1.419	0.110	0.303	0.286	0.336	0.143	0.240	0.112
	sum	1.423	0.118	0.308	0.329	0.303	0.090	0.262	0.131
	reaction	1.424	0.118	0.308	0.329	0.303	0.090	0.262	0.131

In scenario 2, there is no difference between the three models. In the other three scenarios, the distribution got worse. In scenario 1, the total risk is reduced by 5%, but the equilibration is worse by 60%. Population centers 1 and 3, which are the ones with the highest risk, increase their risk. This shift towards risk minimization is mostly caused by the reaction of the followers if they are again allowed to change their mode. In scenario 4, the single mode models result even in a higher total risk and a worse equilibration. This worse equilibration is due to the fact that two equilibrated modes do not need to be equilibrated in the same way as when considering several modes.

This shows that considering different modes in the models is important. The effect of solving the different modes separately is dependent upon the instance and can lead to different risk distributions as with single modes.

5.1.4 Comparison to Maximum Arc Risk Equilibration

To equilibrate risk, the literature so far proposed to minimize the maximum arc risk (Bianco et al., 2009). In Table 4, we compare the solution of a maximum arc risk model to solutions of the Pareto curve of the previous section for the four different demand scenarios.

Table 4: Comparison to Maximum Arc Risk Model

DS	Objective function	Risk measure		Risk of population centers					
		Trad	AdA	Pop 1	Pop 2	Pop 3	Pop 4	Pop 5	Pop 6
1	max arc	1.890	0.217	0.591	0.211	0.534	0.210	0.209	0.135
	AdA + 0.25 Trad	1.862	0.068	0.346	0.284	0.423	0.269	0.272	0.269
	Trad	1.574	0.207	0.525	0.179	0.464	0.105	0.169	0.132
2	max arc	1.789	0.173	0.458	0.264	0.503	0.209	0.211	0.144
	AdA + 0.25 Trad	1.584	0.098	0.368	0.263	0.368	0.193	0.199	0.193
	AdA + 0.20 Trad	1.983	0.014	0.340	0.335	0.347	0.320	0.322	0.320
	Trad	1.388	0.177	0.393	0.175	0.448	0.164	0.134	0.074
3	max arc	1.975	0.170	0.447	0.254	0.508	0.378	0.247	0.142
	AdA + 0.10 Trad	1.875	0.007	0.322	0.308	0.318	0.307	0.309	0.310
	Trad	1.399	0.159	0.377	0.199	0.412	0.185	0.154	0.072
4	max arc	1.760	0.176	0.450	0.350	0.426	0.178	0.271	0.086
	AdA + 0.20 Trad	1.768	0.015	0.306	0.304	0.308	0.289	0.284	0.278
	Trad	1.335	0.165	0.347	0.217	0.419	0.099	0.171	0.081

The results show for all demand scenarios that there exists a solution with a similar total risk in the network but a better distribution within the population centers and a solution with a similar distribution within the population centers but significantly smaller total risk. Using maximum arc risk increased the total risk by more than 35% without distributing the risk better. In all scenarios, every population center has a higher risk, than the risk minimal solution. However, compared to the equilibration measures for population centers, this does not lead to a better distribution of risk. The risk distribution remains more or less the same as in the minimal risk solution. Therefore, a similar risk distribution is always possible with the minimal overall risk solution.

This shows that the maximum arc formulation leads to a very unbalanced risk distribution among the population.

5.2 US-Canada Instance

As a second instance, we used a network of the United States and Canada (Orlowski et al., 2010), which is shown in Figure 5. The network consists of 39 nodes (cities in Canada and the United States) and

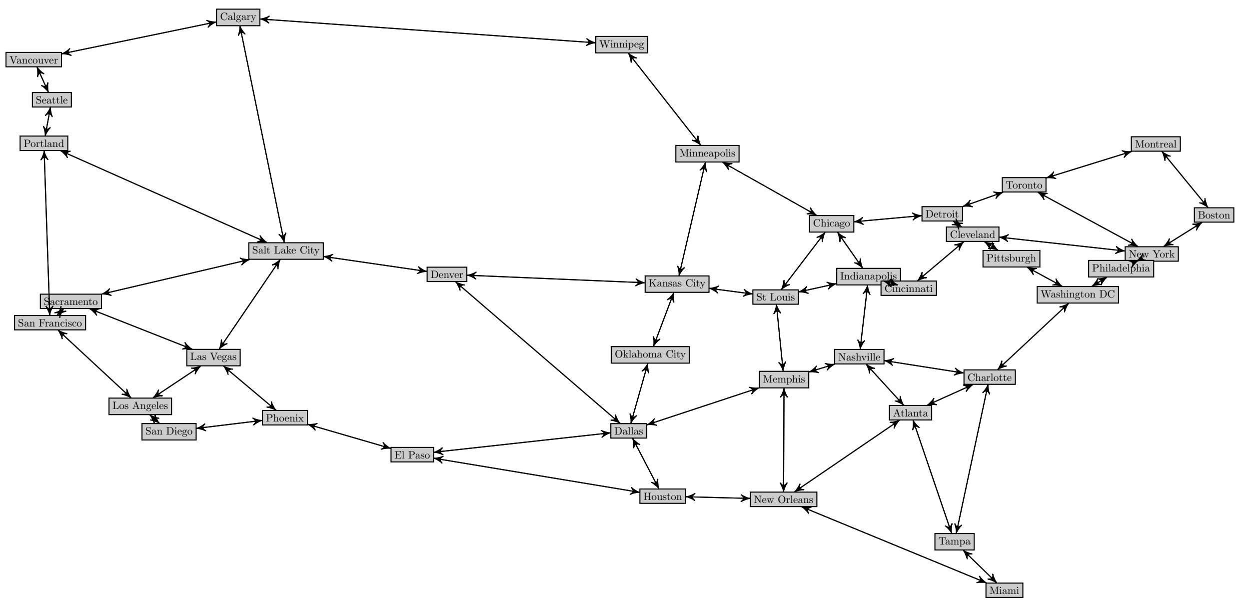

Figure 5: United States and Canada Network

122 directed arcs. The transportation costs are given, and the length of an arc was calculated by using the Euclidean distance between the two nodes. Each city defines one population center. The population of each center is the population of the city and the influence factor was set to 0.5 for the origin and the destination city of an arc. The accident rates on an arc and σ_{ij}^{km} are generated in the same way as for the Sioux Falls network. Two vehicle types are used, the second vehicle costs 5% more while the first vehicle's accident probability is 3% higher . A set of 100 commodities with randomly chosen origin and destination pairs was generated. The demand was defined randomly between 300 and 600.

We used the AdA as equilibration measure and compared the unregulated network with the risk minimal solution, the maximum arc risk solution and several bi-objective functions. The time limit was set to 10 hours and the average gap was below 5% (excluding the maximum arc risk).

The computational results in Table 5 show that the unregulated network has a very high overall risk, a very bad distribution and some cities can have a very high risk. Using the classical approach, the total risk and the distribution can be improved by almost 50%. The maximum risk for a population is reduced from 9.075 to 4.146, which is more than 50%. Using the maximum arc risk formulation from the literature, the maximum risk for a population is similar to that of the Trad risk function. However, the total risk in the network is significantly higher and the distribution is worse. For the bi-objective function, one can see again that a small increase in total risk leads to a significantly better distribution. 7% higher total risk improves the distribution by 20% and the maximum risk of a population center is reduced by over 40%. However, further risk increases only lead to small improvements in the distribution. In fact, for the last case, even though the distribution improves, the maximum risk of a population increases to 2.744.

Table 5: Risk Comparison for the US-Canada Instance

Objective	Trad	AdA	max Pop
free network	60.694	1.684	9.075
max arc	48.429	1.195	4.144
Trad	31.368	0.802	4.146
AdA + 0.03 Trad	33.654	0.634	2.417
AdA + 0.02 Trad	36.237	0.625	2.428
AdA + 0.015 Trad	37.750	0.613	2.417
AdA + 0.01 Trad	39.798	0.597	2.744

Moreover, we analyzed the changes within the population centers. The risk of each city for the different risk measures are shown in Table 6. Comparing the Trad solution with the $AdA + 0.03Trad$ solution, we find that 8 out of 13 cities among the first third of those with the highest risk reduce their risk by 24% on average while the risk of the cities in the last third, 12 suffer an increase of 120% on average. Whereas in total numbers, this equals a reduction of 4.338 in the first third and an increase by 2.507.

This effect, however, changes when comparing $AdA + 0.03Trad$ with $AdA + 0.02Trad$. Only 7 cities in the first third reduce their risk by a total of 0.880 while the risk in 10 cities in the last third increases by 3.121. From $AdA + 0.02Trad$ to $AdA + 0.015Trad$, the risk of 9 out of 13 cities of the first third reduces again, while the risk of 8 out of 13 of the last third increases. However, the reduction is again only 0.778 while the increase is 3.507. This again supports the assumption of the aggregated numbers: In the beginning, the cities with a high risk can benefit from the risk redistribution, but too strong equilibration just penalizes cities with a low risk.

6 Conclusions

We introduced a new population-based risk definition and extended the HTNDP to a multi-mode problem in order to address the problem of risk equity. In the numerical study, we showed the superiority of the new definition over the arc risk definition and the necessity to consider multiple modes in the model. We also showed that the pure equilibration of risk increases the total risk significantly and that one has to find a trade-off between equilibration and risk minimization. But because of the convex correlation between these two measures, a small increase in the total risk can lead to a much better equilibration. As the problem is still very difficult to solve, enhancements for solving it should be considered in further research. Moreover, the differentiation between different hazardous material classes seems to be interesting. Using different classes, could lead to even better distributions of risk.

Table 6: Risk of Population Center for the US-Canada Instance

	free network	max arc	Trad	AdA + 0.03 Trad	AdA + 0.02 Trad	AdA + 0.015 Trad	AdA + 0.01 Trad
Atlanta	0.996	0.829	0.618	0.697	0.596	0.811	0.626
Boston	0.236	0.206	0.205	0.204	0.204	0.204	0.249
Calgary	2.845	1.610	0.513	1.738	1.279	1.589	1.392
Charlotte	1.392	0.920	0.429	0.509	0.729	0.673	0.852
Chicago	7.182	3.090	1.054	2.065	1.926	2.006	1.865
Cincinnati	0.267	0.254	0.254	0.285	0.411	0.444	0.442
Cleveland	0.561	0.545	0.425	0.499	0.729	0.784	0.794
Dallas	4.037	3.187	0.932	1.143	0.992	0.885	1.664
Denver	4.115	4.144	4.146	2.086	2.428	2.221	2.123
Detroit	1.267	0.996	0.501	0.499	0.547	0.592	0.626
El Paso	1.262	0.722	0.331	0.766	0.752	0.849	1.025
Houston	1.533	1.399	0.558	0.868	1.862	1.600	1.153
Indianapolis	1.723	1.336	1.361	1.520	1.472	1.291	1.346
Kansas City	2.717	2.911	3.335	2.274	2.263	2.258	2.292
Las Vegas	0.597	0.625	0.379	0.547	0.633	0.778	0.819
Los Angeles	1.167	2.058	0.630	0.505	0.607	1.099	0.882
Memphis	2.455	2.178	1.167	0.822	0.902	0.960	1.087
Miami	0.143	0.104	0.076	0.432	0.637	0.515	0.708
Minneapolis	0.723	0.498	0.274	0.550	0.491	0.673	0.555
Montreal	0.690	1.393	0.819	0.814	0.814	0.814	1.129
Nashville	2.276	1.710	1.047	0.976	0.729	1.070	0.981
New Orleans	0.597	0.618	0.482	0.575	0.827	0.834	1.049
New York	9.075	3.798	2.414	2.417	2.425	2.417	2.021
Oklahoma City	0.094	0.096	0.152	0.216	0.205	0.172	0.264
Philadelphia	0.658	0.634	0.650	0.650	0.654	0.716	0.794
Phoenix	1.742	0.594	0.445	0.763	1.022	0.885	1.344
Pittsburgh	0.172	0.280	0.355	0.401	0.415	0.398	0.414
Portland	0.878	1.074	0.660	0.613	0.858	0.781	0.913
Sacramento	0.418	0.915	0.363	0.509	0.587	0.784	0.961
Salt Lake City	0.889	1.266	0.871	0.494	0.637	0.461	0.451
San Diego	0.455	0.117	0.189	0.275	0.452	0.217	0.766
San Francisco	0.290	0.219	0.160	0.408	0.359	0.942	0.739
Seattle	0.242	0.380	0.238	0.487	0.589	0.778	0.739
St. Louis	1.205	1.410	1.470	1.062	1.081	0.992	1.081
Tampa	0.320	0.207	0.169	0.491	0.547	0.388	0.445
Toronto	2.403	3.871	2.341	2.331	2.331	2.331	2.744
Vancouver	0.620	0.229	0.184	0.432	0.587	0.781	0.739
Washington DC	0.679	0.831	0.782	0.801	0.849	0.784	0.810
Winnipeg	1.771	1.175	0.388	0.930	0.812	0.975	0.913

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