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How the Minimum Number of Periods Between Regeneration Harvests Induces Modeling Mistakes in the Well-Known Model II Forest Management

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Abstract. One of the well-known forest growth and management models is Model II. Overlooking the minimum number of periods between regeneration harvests can cause some modeling mistakes. Two mistakes were found in the original Model II formulation. The first is a mistake in the area constraints and the second in calculating one important parameter of the model representing discounted net revenue per hectare between periods. The second mistake is in the calculation of two important parameters of the model representing discounted net revenue per hectare between periods. In this paper, we provide an illustrative example and describe the mistakes. Next, we propose a newly revised formulation for the Model II. Then, in order to validate the problem identified, we solve the Model II with realistic data to address the modeling mistakes and explain how our revised formulation works with the same data. We also describe situations where the mistakes may have a larger impact and explain why they have not been identified earlier.

Keywords: Forest management planning, timber harvest, harvest scheduling, linear programming, Model II.

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1. Introduction

One of the most important models for forest management is the so-called Model II (Johnson and Scheurman 1977). This model has been implemented in numerous planning systems within companies and government organizations. One of the main uses of the model is to evaluate the net present value (NPV) of a given forest given different silverculture treatment scenarios and discount rates over a set of time periods representing long-term planning. The model is simple to formulate and implement and hence is included in many systems. However, when there is a minimum number of regeneration periods imposed, the model has some mistakes. One is that there are variables allowed that should be omitted and two others are in how to compute some of the parameters in the model. These mistakes may or may not have an impact depending on the silviculture options available and used. Also, as the model is often used for NPV values and not for operational planning, there is no actual need to analyze the actual harvest decision in detail. Hence, there has been little reason to verify or find that some variables have erroneous values. We describe the mistakes and propose a new formulation, which is tested on some illustrative examples.

Simple decision support systems cannot be created and applied universally because strategic planning of the forest value chain includes many different players in many different business contexts. Decision support for strategic planning helps decision makers assess the potential consequences of strategic business choices (Anthony 1965, Drucker 1995). For strategic planning, we should take decision makers' values, objectives, and their future business anticipations into account because these decisions will change the future by changing the flow of the resources and opportunities available to the company (Gunn 2005). At the strategic planning level, decisions, goals, and other constraints of decisions makers must be considered because they have a long-term impact on the company and its resources.

The goal of forest management strategy is to answer the following questions: what to supply, from where to supply to, to which market, and for what use to create value and jobs for local communities. It also has impacts on the sustainability, carbon sequestration, wildlife habitat, and ecology, controlling invasive species, and social values (like employment). Sustainable long-term supply is often depicted in terms of level flow or nondeclining yield. D'Amours et al. (2016) have stated that renewability is the key feature of the forest as a supplier of raw materials.

There are three specific parts in the forest management linear models: the process of forest growth and management, the sustainability of forest products, and the requirement to provide certain types of forest cover. Four distinctive modelling approaches are found for forest growth and management, including the well-known Model I and Model II (Johnson and Scheurman 1977) and Model III, which is less common (Garcia 1990); however, it forms the base of popular packages like FOLPI (Gunn 2007), and John and Tóth (2015) proposed a new model for spatial forest harvest scheduling called Model IV.

Woodstock is capable of generating linear programming matrices by the use of a generalized Model II formulation which is markedly more powerful than other harvest scheduling models based on Model II, like MUSYC (Multiple-Use/Sustained Yield Calculation). FORPLAN (FORest PLANning model) version 2 proposes the capabilities of the generalized Model II (Remsoft 1994).

A combination of Model I and Model II has been used as an optimization model to explore how different management regimes would affect the ability of forests to sequester carbon (Backéus et al. 2005). Martin et al. (2017) compared the efficiency of the spatial Model I and Model II and pointed out that Model I outperformed the Model II.

An optimization approach has been applied through a timber supply model which is an extension of the Model II formulation to estimate the cost of overlapping tenure constraint on forest management agreement areas in Northern Alberta (Nanang and Hauer 2006). A novel approach has been represented to simultaneously maximize carbon sequestration in both forest and wood products and abated emissions from product substitution using Remsoft Spatial Planning System (Hennigar et al. 2008). Note that Woodstock is on the basis of an optimized forest treatment scheduling using a model II LP formulation (Hennigar et al. 2008). Nanang and Hauer (2008) examined the long-term impacts of access road development, which is an important factor in determining harvesting and hunter preferences and non-timber benefits, and they used an extension of the Model II formulation. Model II was used for optimal harvest scheduling in a case study in Spain (Diaz-Balteiro et al. 2009). Model II has been utilized in the forestry portion of the FASOM-GHG model which has been modified to simulate the effects of optional and mandatory participation in carbon offset sales programs (Latta et al. 2011). Model II has been applied in the forest sector model of a linked land-use and forest sector models which have been proposed to find how carbon offset sales can affect private forest owners' land-use and forest management decisions in Western Oregon (USA) (Latta et al. 2016). In order to analyze the impact of operational-level flexibility on long-term wood supply, a hierarchical planning, i.e. strategic, tactical, and operational, has been developed. The authors used a software called SilviLab to formulate the strategic-level model as a Model II linear program (Gautam et al. 2017). Model II has been used in a goal programming to analyze the long-term impact of policy and industry changes at the landscape level (Corrigan and Nieuwenhuis 2017).

The contribution of the paper is important as the Model II is used in many systems. It is difficult to know if any implementation has found and revised the modeling errors or not. However, we have not found any published article that addresses this and it is important for other researchers and users of the system to understand how they are impacted by the mistakes or how to identify if the implementation may provide erroneous results. This paper identifies and proposes a few modeling mistakes in the Model II formulation given in Johnson and Scheurman (1977). Model II is one of the most well-known forest management models, but the original formulation has two mistakes which may overestimate the objective function and mislead the forest manger or researchers over optimal harvest decisions in a specific context. The first mistake occurs in the first set of the area constraints, wherein some extra decision variables are created. These decision variables may take nonzero values and provide wrong information about the objective function and harvest decisions. The second mistake can be found in the way to calculate one of the key parameters of the model. This parameter will be explained in detail in the following sections.

The rest of this paper is organized as follows. Section 2 describes the forest management models, especially Model II in details with mathematical formulation. In Section 3, we pose questions to Model II and propose a new mathematical formulation. In order to validate our new formulation, a problem would be represented with practical data and the results would be analyzed in Section 4. Finally, conclusions are drawn in Section 5.

2. Forest management models

In literature, four modelling approaches can be found for forest management planning, including the well-known Model I and Model II (Johnson and Scheurman 1977), Model III, which is less common (Garcia 1990), and John and Tóth (2015) proposed a new model for spatial forest harvest scheduling which is called Model IV. In Model I (Johnson and Scheurman 1977), the integrity of each age class in the first period is kept throughout the planning horizon (see Model I in Figure 1). However, in Model II (Johnson and Scheurman 1977), the integrity of each age class in the first period is kept until it is regeneration harvested and forms a new age class until they are again regeneration harvested (see Model

II in Figure 1). In Model III (Garcia 1990), in each period, the land in an age class can be harvested or become one age class older (see Model III in Figure 1). The aggregation of all stands in Model II is similar to the Model III; however, the network contains fewer nodes and arcs. Model I can be used to model either aggregated or individual stands (Gunn 2007). In the previous models, one decision variable is required for every applicable prescription for each forest management unit. The mentioned models are aspatial and also depend on static volume and revenue coefficients that must be calculated before starting optimization. Finally, John and Tóth (2015) introduced a new model which is called Model IV, using different equations and Boolean algebra for spatial forest harvest scheduling.

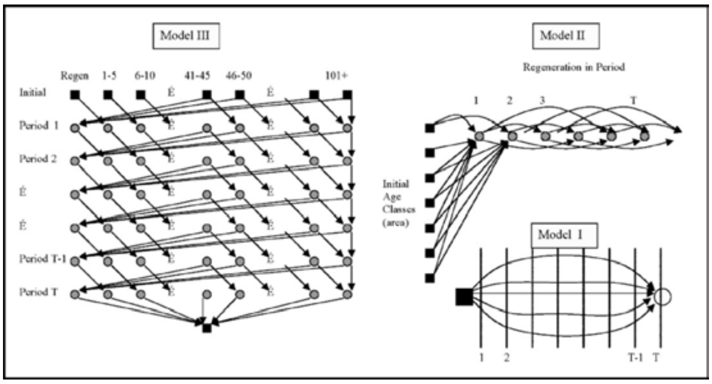


Figure 1: Models I, II, and III (Gunn 2007).

2.1. Model II

Forest management planning aims to schedule timber harvest and investment on an area of timberland under even-aged management. The goal is to maximize the volume or value produced from its timberland, while encountering constant or decreasing prices in the volume of timber output (Johnson and Scheurman 1977). The manager may come across land availability limits for harvesting in each time period, when the whole area is managed under one cultural treatment regime. A cultural treatment regime is any sequence of silvicultural practices such as planting, pre-commercial thinning, commercial thinning, and fertilization. In addition to area constraints, it may also consider flow constraints (harvest fluctuation and sustainability).

Seven simplifying assumptions have been stated as follows:

1. The forest has one type-site consisting of different age classes.

2. The area of forestland is fixed during the planning horizon.
3. The number of years representing each time period in the planning horizon is consistent with the years of each age class.
4. For regeneration harvest, we use clear-cutting.
5. Regeneration occurs in the same period as a regeneration harvest.
6. Yield estimates take into account all uncertainties such as fire, insect, and diseases implicitly.
7. The only out-of-pocket costs that should be paid are cultural treatment costs.

In Model II, each age class forms a management unit that is harvested. Having regeneration harvested, new age class is formed till they are again regeneration harvested. Each activity describes a possible management regime for a certain management unit from the time a unit is regenerated until it is regeneration harvested or left as ending inventory at the end of the planning horizon. A management regime includes two parts (Johnson and Scheurman 1977):

1. A regeneration harvest at some time during the planning horizon or an ending inventory at the end of the planning horizon.
2. An associated cultural treatment regime.

We require two sets of area constraints:

- One set on the areas that can be regeneration harvested from, or put aside as ending inventory in, each age class that exists at the start of planning horizon (See Figure 2 and Equation 2) (Johnson and Scheurman 1977). Figure 2 indicates that the areas cut from each age class through different time periods plus the areas left as ending inventory from that age class are equal to the total number of areas in that age class in the beginning of planning horizon. For instance, Figure 2b indicates that the total area from age class one (on the assumption that there is no minimum number of periods between regeneration harvests) at the beginning of the planning horizon can be harvested in different periods starting from one to N, and put aside as ending inventory.
- The second set is on the areas that can be regeneration harvested from, or put aside as ending inventory in, each age class that is created throughout the planning horizon (See Figure 3 and Equation 3) (Johnson and Scheurman 1977). Figure 3 illustrates that the areas cut from areas regenerated in period j plus the areas left as ending inventory from areas regenerated in period j are equal to the total number of areas regenerated in period j (j can vary between the first period and the end of the planning horizon). For example, in period j, different age classes may be

harvested, so these areas can be harvested in the following future periods and also put aside as ending inventory.

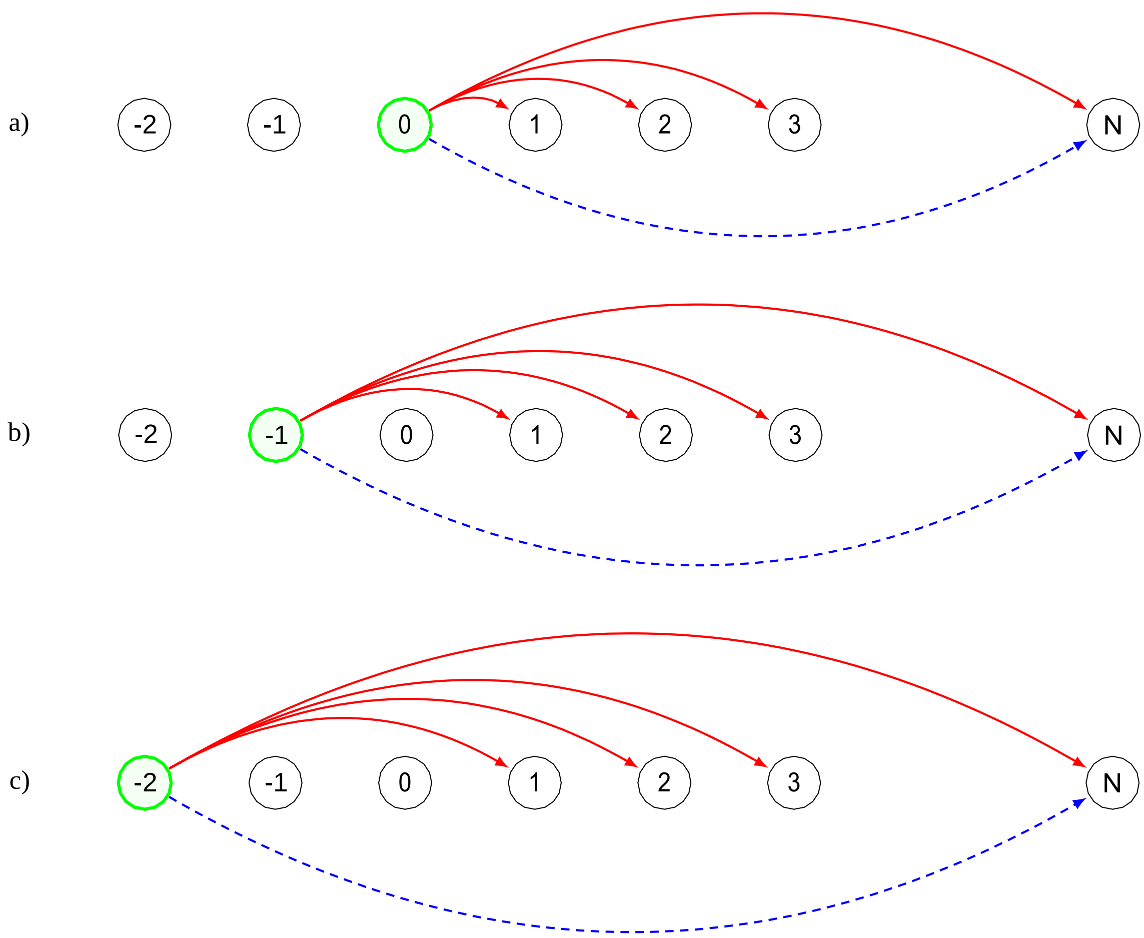


Figure 2: Balance constraint for areas regenerated or put aside as ending inventory at the start of the planning horizon for three different age classes: 0, 1, and 2 can be seen in a, b, and c, respectively.

2.2. Mathematical formulation

The mathematical form of Model II is summarized as follows:

$$\max \sum_{j=1}^{N} \sum_{i=-M}^{j-z} D_{ij} x_{ij} + \sum_{i=-M}^{N} E_{iN} w_{iN} \tag{1}$$

Subject to

$$\sum_{j=1}^{N} x_{ij} + w_{iN} = A_i, \qquad i = -M, \dots, 0 \tag{2}$$

$$\sum_{k=j+z}^{N} x_{jk} + w_{jN} = \sum_{i=-M}^{j-z} x_{ij}\,, \qquad j = 1, 2, \dots, N \tag{3}$$

$$x_{ij} \geq 0, \qquad i = -M, \dots, N, j = 1, 2, \dots, N \tag{4}$$

$$w_{iN} \geq 0, \qquad i = -M, \dots, N \tag{5}$$

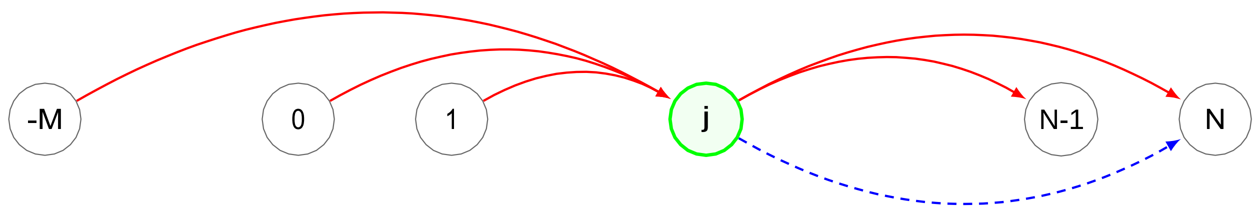


Figure 3: Balance constraint for areas regenerated or put aside as ending inventory throughout the planning horizon.

Equation (2) expresses the land availability constraint for the beginning of planning horizon (see Figure 2). The balance constraint for areas regenerated in period j can be found in Equation (3) (see Figure 3). Equations (4) and (5) show the non-negativity.

Authors defined the sets, data, and variables as follows, where:

- N Number of periods in the planning horizon
- x_{ij} Areas regenerated in period i and regeneration harvested again in period j
- w_{iN} Areas regenerated in period i and put aside as ending inventory in period N
- A_i Number of hectares present in period one that were regenerated in period i ($i = -M, \dots, 0$), with each A_i being a constant at the beginning of the planning horizon (period 1)
- M number of periods before period zero in which the oldest age class present in period one was regenerated
- z Minimum number of periods between regeneration harvests (reasonably it is greater than one, i.e., $z \geq 1$)
- D_{ij} Discounted net revenue per hectare from areas regenerated in period i and regeneration harvested again in period j. It can be written as shown below:

$$D_{ij} = \sum_{k=\max(i,1)}^{j} \frac{P_{ikj}V_{ikj} - C_{ikj}}{\gamma^k}$$

Where

P_{ikj} Unit price of volume harvested in period k on areas regenerated in period i and regeneration harvested again in period j

V_{ikj} Volume per hectare harvested in period k on areas regenerated in period i and regeneration harvested again in period j

C_{ikj} Cultural treatment costs per hectare in period k on areas regenerated in period i and regeneration harvested again in period j

γ^j Discount rate for period j

E_{iN} Discounted net revenue per hectare during the planning horizon from areas regenerated in period i and put aside as ending inventory in period N plus discounted net value per hectare of leaving these areas as ending inventory. It can be written as shown below:

$$E_{iN} = \sum_{k=\max(i,1)}^{N} \frac{P_{ikN}V_{ikN} - C_{ikN}}{\gamma^k} + \frac{P'_{iN}}{\gamma^N}$$

Where

P_{ikN} Unit price of volume thinned in period k on areas regenerated in period i and put aside as ending inventory in period N

V_{ikN} Volume per hectare thinned in period k on areas regenerated in period i and put aside as ending inventory in period N

C_{ikN} Cultural treatment costs per hectare in period k on areas regenerated in period i and put aside as ending inventory in period N

P'_{iN} Net value per hectare of leaving areas regenerated in period i as ending inventory in period N

3. Methods

In this section, we firstly represent the modeling mistakes in Model II and then propose our new model to overcome these mistakes.

3.1. Mistake in the first set of area constraints

At first sight, the mathematical model seems right. However, if you look more closely, you will find a mistake in the first set of area constraints. In accordance with the definition of z, minimum number of periods between regeneration harvests, it is not allowed to harvest an area unless at least z periods have been passed since the last regeneration harvest. Unfortunately, Equation (2) has been mistakenly written in (Johnson and Scheurman 1977). The current constraints contain extra decision variables which are possible to take values; however, we know that their values must be equal to zero because they are not allowed to be harvested.

In the first set of area constraints, if all the coefficients of the variables in the objective function are negative except for the extra decision variables, it is possible that those extra decision variables take values and if this is the case, they will be ignored for future planning; consequently, we lose some values in the objective function.

In order to prove our claim and provide further clarification, consider the following example where we would like to schedule harvests for the next four time periods (N=4) from a forest that now has three different age classes aged 0, 1, and 2 (i.e. A_0=300, A_{-1}=200, and A_{-2}=100)(M=2). There is a minimum of three time periods between regeneration harvests (z=3).

Now we want to expand the objective function and first set of area constraints and take a closer look at them. As mentioned above, an area cannot be harvested unless a minimum of z periods have passed since the last regeneration. However, you can find variables in the constraints which are contrary to this law (x_{-11}, x_{01}, and x_{02}).

$$\max \sum_{j=1}^{4} \sum_{i=-2}^{j-3} D_{ij} x_{ij} + \sum_{i=-2}^{4} E_{i4} w_{i4}$$

$$= D_{-21} x_{-21} +$$

$$D_{-22} x_{-22} + D_{-12} x_{-12} +$$

$$D_{-23}x_{-23} + D_{-13}x_{-13} + D_{03}x_{03} +$$
$$D_{-24}x_{-24} + D_{-14}x_{-14} + D_{04}x_{04} + D_{14}x_{14} +$$
$$E_{-24}w_{-24} + E_{-14}w_{-14} + E_{04}w_{04} + E_{14}w_{14} + E_{24}w_{24} + E_{34}w_{34} + E_{44}w_{44}$$

First area constraint

$$\sum_{j=1}^{4} x_{ij} + w_{i4} = A_i, \qquad i = -2, \dots, 0$$

$$x_{-21} + x_{-22} + x_{-23} + x_{-24} + w_{-24} = A_{-2}, \qquad i = -2$$

$$\boldsymbol{x_{-11}} + x_{-12} + x_{-13} + x_{-14} + w_{-14} = A_{-1}, \qquad i = -1$$

$$\boldsymbol{x_{-01}} + \boldsymbol{x_{-02}} + x_{-03} + x_{-04} + w_{-04} = A_{-0}, \qquad i = 0$$

It might be said that those variables are not involved in the objective function, but note that in specific circumstances, they take value and affect decision variables in the objective function. To clear it up, suppose the following values for parameters D_{ij} and E_{iN} in Tables 1 and 2 for the above-mentioned example.

Table 1: Values for parameters D_{ij}

		Next Harvesting Period (Period *j*)			
		1	2	3	4
Most Recent Harvesting Period (Period *i*)	-2	1	1	1	1
	-1	0	**-1**	**-1**	**-1**
	0	0	0	**-1**	**-1**
	1	0	0	0	1
	2	0	0	0	0
	3	0	0	0	0
	4	0	0	0	0

Table 2: Values for parameters E_{i4}

Period i	-2	-1	0	1	2	3	4
E_{i4}	1	**-1**	**-1**	1	1	1	1

We solved the model and the results can be found in Tables 3 and 4. Light gray cells indicate the forbidden periods for harvesting of each age class according to the definition of the z parameter. While the rule has been violated by x_{-11} and x_{01}, 200 and 300 are their values, respectively. The objective value is 300.

The repercussion will not be limited to this one. In addition to that, those values would be ignored for future harvest planning. For instance, when a management unit is regeneration harvested in period 1 (x_{-11}= 200), it can be harvested in period 4 or taken into account as an ending inventory while it has been overlooked, likewise for the other unallowable variable (x_{01}= 300).

Table 3: Outcomes for decision variables X_{ij}

		Next Harvesting Period (Period *j*)			
		1	2	3	4
Most Recent Harvesting Period (Period *i*)	-2	100	1	1	1
	-1	***200***	0	0	0
	0	***300***	0	0	0
	1	0	0	0	100
	2	0	0	0	0
	3	0	0	0	0
	4	0	0	0	0

Table 4: Outcomes for decision variables w_{i4}

Period i	-2	-1	0	1	2	3	4
w_{i4}	0	0	0	0	0	0	100

In order to overcome this drawback, a new formulation has been proposed for the first set of area constraints as shown below:

$$\sum_{j=1}^{N} x_{ij} + w_{iN} = A_i, \qquad i = -M, \dots, 1-z$$

$$\sum_{j=z+i}^{N} x_{ij} + w_{iN} = A_i, \qquad i = 2-z, \dots, 0$$

We solved the example again by considering the new formulation. The outcomes can be found in Tables 5 and 6. As it can be seen, there is no breach of rule for harvesting. The objective value is -4200.

Table 5: Outcomes for decision variables X_{ij} for revised formulation

Most Recent Harvesting Period (Period i)		Next Harvesting Period (Period j) 1	2	3	4
	-2	100	1	1	1
	-1	0	0	0	200
	0	0	0	0	300
	1	0	0	0	100
	2	0	0	0	0
	3	0	0	0	0
	4	0	0	0	0

Table 6: Outcomes for decision variables w_{i4} for revised formulation

Period i	-2	-1	0	1	2	3	4
w_{i4}	0	0	0	0	0	0	600

3.2. Mistake in calculation of D_{ij}

In order to calculate the objective function we should first find the values of coefficients of decision variables in the objective function, i.e., values for D_{ij} and E_{iN}. Therefore, we should use the following formulas:

If we delve deeply into these formulas, we will perceive mistakes in them. Consider the following example with given parameters: $N = 7, M = 1, z = 3$.

$$D_{ij} = \sum_{k=\max(i,1)}^{j} \frac{P_{ikj}V_{ikj} - C_{ikj}}{\gamma^k}$$

$$E_{iN} = \sum_{k=\max(i,1)}^{N} \frac{P_{ikN}V_{ikN} - C_{ikN}}{\gamma^k} + \frac{P'_{iN}}{\gamma^N}$$

This is its objective function:

$$\max \sum_{j=1}^{7} \sum_{i=-1}^{j-3} D_{ij}x_{ij} + \sum_{i=-1}^{7} E_{i7}w_{i7}$$

$$= D_{-12}x_{-12} +$$

$$D_{-132}x_{-13} + D_{03}x_{03} +$$

$$D_{-14}x_{-14} + D_{04}x_{04} + D_{14}x_{14} +$$

$$D_{-15}x_{-15} + D_{05}x_{05} + D_{15}x_{15} + D_{25}x_{25} +$$

$$D_{-16}x_{-16} + D_{06}x_{06} + D_{16}x_{16} + D_{26}x_{26} + D_{36}x_{36} +$$

$$D_{-17}x_{-17} + D_{07}x_{07} + D_{17}x_{17} + D_{27}x_{27} + D_{37}x_{37} + D_{47}x_{47} +$$

$$E_{-17}w_{-17} + E_{07}w_{07} + E_{17}w_{17} + E_{27}w_{27} + E_{37}w_{37} + E_{47}w_{47} + E_{57}w_{57} + E_{67}w_{67} + E_{77}w_{77}$$

At each time period, two sets of timber flows are needed, including input (areas regenerated in previous time periods and going to be regeneration harvested again in this period) and output (areas may be regenerated in future or put aside as an ending inventory) flows. Figure 4 represents input and output flows in the aforementioned example ($N = 7, M = 1, z = 3$). For example, in Figure 4c, there are two timber inflows from areas regenerated harvested three and four periods ago (period -1 and 0, respectively) and three timber outflows, two of which will be regenerated again in periods 6 and 7, and the third outflow is related to areas left as ending inventory in period 7.

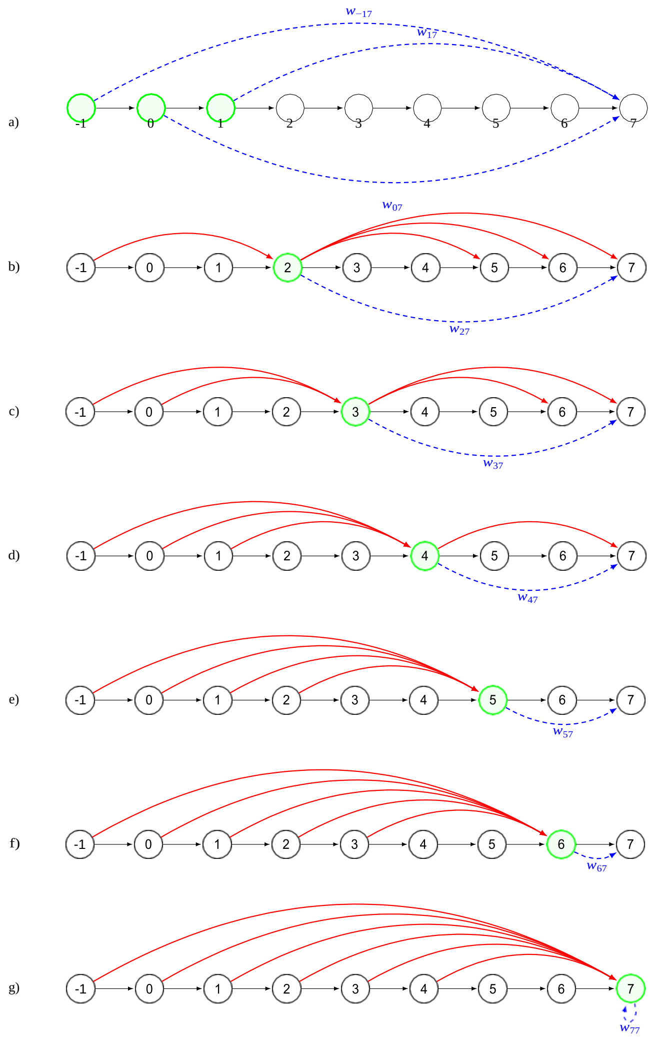


Figure 4: Timber flows for different time periods in an example with N=7, M=1, and z=3. Solid lines show the regenerated areas and dotted lines indicate the areas put aside as an ending inventory.

To represent the mistake which we will come across while we are calculating the coefficients, consider the following equations and figures:

$$D_{-12} = \sum_{k=\max(-1,1)}^{2} \frac{P_{-1k2}V_{-1k2} - C_{-1k2}}{\gamma^k}$$

$$= \frac{\boldsymbol{P_{-112}V_{-112} - C_{-112}}}{\boldsymbol{\gamma}} + \frac{P_{-122}V_{-122} - C_{-122}}{\gamma^2} \tag{6}$$

$$D_{-13} = \sum_{k=\max(-1,1)}^{3} \frac{P_{-1k3}V_{-1k3} - C_{-1k3}}{\gamma^k}$$

$$= \frac{\boldsymbol{P_{-113}V_{-113} - C_{-113}}}{\boldsymbol{\gamma}} + \frac{\boldsymbol{P_{-123}V_{-123} - C_{-123}}}{\boldsymbol{\gamma^2}} + \frac{P_{-133}V_{-133} - C_{-133}}{\gamma^3} \tag{7}$$

$$D_{-14} = \sum_{k=\max(-1,1)}^{4} \frac{P_{-1k4}V_{-1k4} - C_{-1k4}}{\gamma^k}$$

$$= \frac{\boldsymbol{P_{-114}V_{-114} - C_{-114}}}{\boldsymbol{\gamma}} + \frac{\boldsymbol{P_{-124}V_{-124} - C_{-124}}}{\boldsymbol{\gamma^2}} + \frac{\boldsymbol{P_{-134}V_{-134} - C_{-134}}}{\boldsymbol{\gamma^3}} + \frac{P_{-144}V_{-144} - C_{-144}}{\gamma^4} \tag{8}$$

$$D_{-15} = \sum_{k=\max(-1,1)}^{5} \frac{P_{-1k5}V_{-1k5} - C_{-1k5}}{\gamma^k}$$

$$= \frac{\boldsymbol{P_{-115}V_{-115} - C_{-115}}}{\boldsymbol{\gamma}} + \frac{P_{-125}V_{-125} - C_{-125}}{\gamma^2} + \frac{\boldsymbol{P_{-135}V_{-135} - C_{-135}}}{\boldsymbol{\gamma^3}} + \frac{\boldsymbol{P_{-145}V_{-145} - C_{-145}}}{\boldsymbol{\gamma^4}} + \frac{P_{-155}V_{-155} - C_{-155}}{\gamma^5} \tag{9}$$

$$D_{-16} = \sum_{k=\max(-1,1)}^{6} \frac{P_{-1k6}V_{-1k6} - C_{-1k6}}{\gamma^k}$$

$$= \frac{\boldsymbol{P_{-116}V_{-116} - C_{-116}}}{\boldsymbol{\gamma}} + \frac{P_{-126}V_{-126} - C_{-126}}{\gamma^2} + \frac{P_{-136}V_{-136} - C_{-136}}{\gamma^3} + \frac{\boldsymbol{P_{-146}V_{-146} - C_{-146}}}{\boldsymbol{\gamma^4}} + \frac{\boldsymbol{P_{-156}V_{-156} - C_{-156}}}{\boldsymbol{\gamma^5}} + \frac{P_{-166}V_{-166} - C_{-166}}{\gamma^6} \tag{10}$$

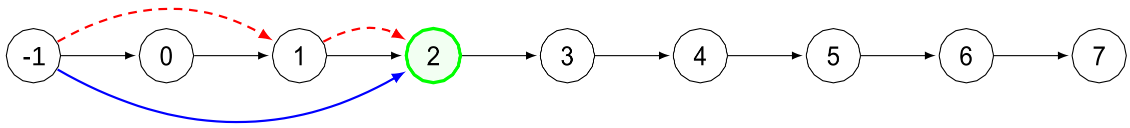
Note that the number of the first and last regeneration periods is constant; however, the middle harvested period (k) varies in the formulation. According to the definition of z, some timber flows are impossible; bold segments of the formulas refer to this point. Figure 5 illuminates the possible and impossible timber flows. For instance, as discovered in Figure 5d, D_{-15} is consisted of five timber flows such as $V_{-115}, V_{-125}, V_{-135}, V_{-145}$, and V_{-155}; however, in accordance with the definition of the z parameters, some timber flows are impossible, like V_{-115}, V_{-135}, and V_{-145}. You should be aware that the mistake is not limited to impractical timber flows. In addition, there is an overlap between one fragment of the D_{-12} and D_{-15}.

The fragments are as below:

$$\frac{P_{-122}V_{-122} - C_{-122}}{\gamma^2} \quad \& \quad \frac{P_{-125}V_{-125} - C_{-125}}{\gamma^2}$$

These two segments calculate the same timber flow and discount it for two periods. In other words, there is a timber flow in D_{-15}which has been computed in D_{-12}. Furthermore, two overlaps can be found between D_{-16}, D_{-12}, and D_{-13}.

a)



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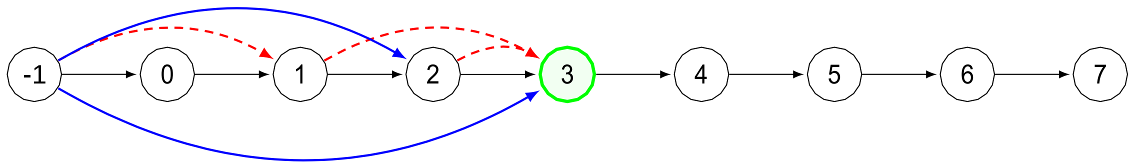
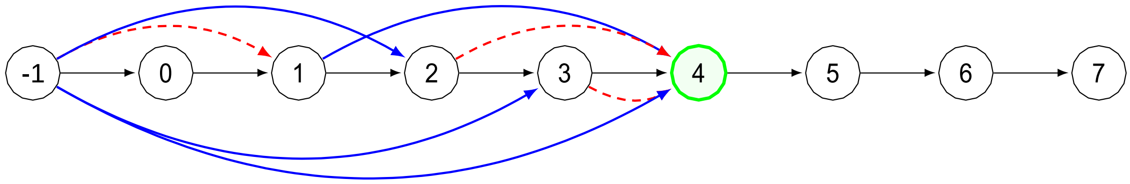


c)



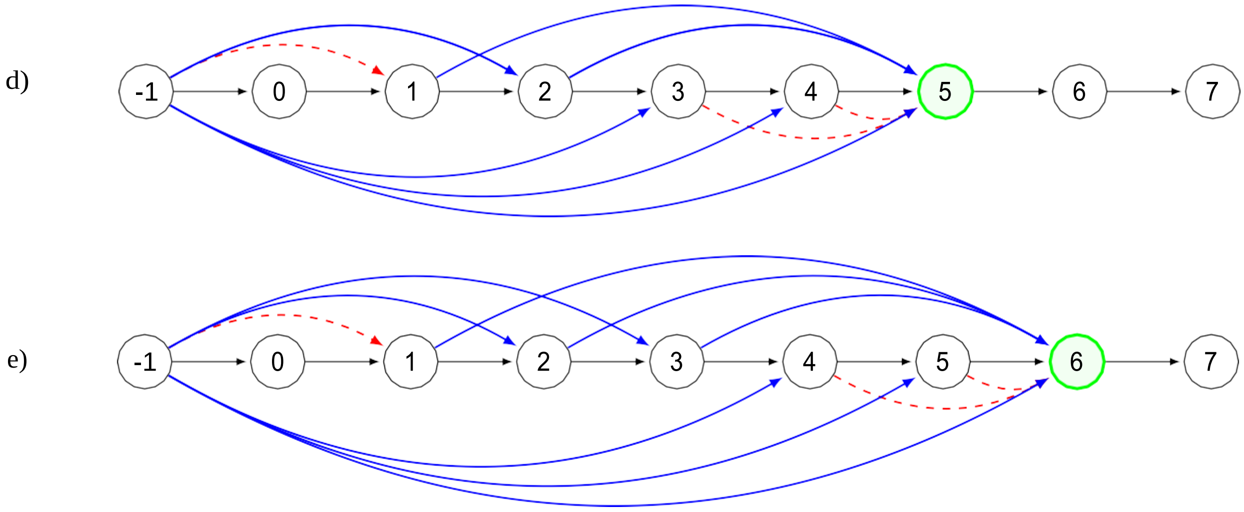


Figure 5: Timber flows used in calculation of discounted net revenue. Solid and dotted lines show the possible and impossible timber flows, respectively

To analyze the profitability of investment, NPV, which is the difference between the present value of cash inflows and outflows discounted by the discount rate, is used in capital budgeting. Therefore, in order to solve the mistake, we have to change the formulation of D_{ij} as below:

$$D_{ij} = \frac{P_{ij}V_{ij} - C_{ij}}{(1+\gamma)^{j}} \qquad j = 1, 2, \ldots, N \quad and \quad i = -M, \ldots, j - z$$

Where

P_{ij} Unit price of volume harvested in period i and regeneration harvested in periodj

V_{ij} Volume per hectare harvested in period i and regeneration harvested in period j

C_{ij} Cultural treatment costs per hectare in period i and regeneration harvested in period j

In order to illustrate the differences between the new and old values of D_{ij}, we assumed the following values in table 7.

Table 7: Values needed to calculate D_{ij}

		Harvesting Period					
		1	2	3	4	5	6
Parameters	Price ($/$m^3$)	17.19	17.19	17.19	17.19	17.19	17.19
	Volume (m^3/ha)	0	0	0	4	32	66
	Income ($/ha)	0	0	0	69	550	1135
	PCT Cost ($/ha)	0	0	300	300	300	300
	Harvesting Cost ($/ha)	2000	2000	2000	2000	2000	2000
	Cumulative Cost ($/ha)	2000	2000	2300	2300	2300	2300
	D_{ij}	-1923	-1849	-1984	-1496	-958	-391

Now we use both formulations to calculate D_{-15}, as an example, to find the differences between them. In accordance with the original formulation, we have:

$$D_{-15} = \sum_{k=\max(-1,1)}^{5} \frac{P_{-1k5}V_{-1k5} - C_{-1k5}}{\gamma^k}$$

$$= \frac{P_{-115}V_{-115} - C_{-115}}{\gamma} + \frac{P_{-125}V_{-125} - C_{-125}}{\gamma^2} + \frac{P_{-135}V_{-135} - C_{-135}}{\gamma^3} + \frac{P_{-145}V_{-145} - C_{-145}}{\gamma^4} + \frac{P_{-155}V_{-155} - C_{-155}}{\gamma^5}$$

$$= (-1496) + (-1984) + (-1849) + (-1923) + (-391) = -7643$$

D_{-15} has been made up of five segments. The first segment represents the net present value of volume harvested from trees in 4 age-class, because V_{-115}means volume per hectare harvested in period 1 on areas regenerated in period -1 (trees in first age class) and regeneration harvested again in period 5, so the trees have been harvested again after four periods. V_{-115} means the volume obtained from harvesting trees in 6 age-class because at the beginning of the planning horizon, we have trees in one age-class and they will be cut five periods later. However, in the new formulation, we only consider -391 for D_{-15}.

3.3. Mistake in calculation of E_{iN}

Now if we carefully look at the formulation of the E_{iN} in detail, you will find a mistake in calculation of this parameter. In order to clarify the mistake we provide an example and illustrate where that mistake occurs. For instance, suppose the planning horizon is 20 years ($N = 4$), there are two age classes of trees in the forest ($M = 2$), and the minimum number of periods between regeneration harvests is 3 ($z = 3$). In the objective function, we have:

$$\sum_{i=-2}^{4} E_{i4}w_{i4} = E_{-24}w_{-24} + E_{-14}w_{-14} + E_{04}w_{04} + E_{14}w_{14} + E_{24}w_{24} + E_{34}w_{34} + E_{44}w_{44}$$

For finding the values for each E_{iN}, we use the formulation below. Therefore, we have:

$$E_{iN} = \sum_{k=\max(i,1)}^{N} \frac{P_{ikN}V_{ikN} - C_{ikN}}{\gamma^k} + \frac{P'_{iN}}{\gamma^N}$$

$$E_{-24} = \sum_{k=\max(-2,1)}^{4} \frac{P_{-2k4}V_{-2k4} - C_{-2k4}}{\gamma^k} + \frac{P'_{-24}}{\gamma^4}$$

$$= \frac{P_{-214}V_{-214} - C_{-214}}{\gamma} + \frac{P_{-224}V_{-224} - C_{-224}}{\gamma^2} + \frac{P_{-234}V_{-234} - C_{-234}}{\gamma^3} + \frac{P_{-244}V_{-244} - C_{-244}}{\gamma^4} + \frac{P'_{-24}}{\gamma^4} \quad (11)$$

$$E_{-14} = \sum_{k=\max(-1,1)}^{4} \frac{P_{-1k4}V_{-1k4} - C_{-1k4}}{\gamma^k} + \frac{P'_{-14}}{\gamma^4}$$

$$= \frac{P_{-114}V_{-114} - C_{-114}}{\gamma} + \frac{P_{-124}V_{-124} - C_{-124}}{\gamma^2} + \frac{P_{-134}V_{-134} - C_{-134}}{\gamma^3} + \frac{P_{-144}V_{-144} - C_{-144}}{\gamma^4} + \frac{P'_{-14}}{\gamma^4} \quad (12)$$

$$E_{04} = \sum_{k=\max(0,1)}^{4} \frac{P_{0k4}V_{0k4} - C_{0k4}}{\gamma^k} + \frac{P'_{04}}{\gamma^4}$$

$$= \frac{P_{014}V_{014} - C_{014}}{\gamma} + \frac{P_{024}V_{024} - C_{024}}{\gamma^2} + \frac{P_{034}V_{034} - C_{034}}{\gamma^3} + \frac{P_{044}V_{044} - C_{044}}{\gamma^4} + \frac{P'_{04}}{\gamma^4} \quad (13)$$

$$E_{14} = \sum_{k=\max(1,1)}^{4} \frac{P_{1k4}V_{1k4} - C_{1k4}}{\gamma^k} + \frac{P'_{14}}{\gamma^4}$$

$$= \frac{\boldsymbol{P_{114}V_{114} - C_{114}}}{\boldsymbol{\gamma}} + \frac{P_{124}V_{124} - C_{124}}{\gamma^2} + \frac{P_{134}V_{134} - C_{134}}{\gamma^3} + \frac{P_{144}V_{144} - C_{144}}{\gamma^4} + \frac{P'_{14}}{\gamma^4} \quad (14)$$

$$E_{24} = \sum_{k=\max(2,1)}^{4} \frac{P_{2k4}V_{2k4} - C_{2k4}}{\gamma^k} + \frac{P'_{24}}{\gamma^4}$$

$$= \frac{\boldsymbol{P_{224}V_{224} - C_{224}}}{\boldsymbol{\gamma^2}} + \frac{P_{234}V_{234} - C_{234}}{\gamma^3} + \frac{P_{244}V_{244} - C_{244}}{\gamma^4} + \frac{P'_{24}}{\gamma^4} \quad (15)$$

$$E_{34} = \sum_{k=\max(3,1)}^{4} \frac{P_{3k4}V_{3k4} - C_{3k4}}{\gamma^k} + \frac{P'_{34}}{\gamma^4}$$

$$= \frac{\boldsymbol{P_{334}V_{334} - C_{334}}}{\boldsymbol{\gamma^3}} + \frac{P_{344}V_{344} - C_{344}}{\gamma^4} + \frac{P'_{34}}{\gamma^4} \quad (16)$$

$$E_{44} = \sum_{k=\max(4,1)}^{4} \frac{P_{4k4}V_{4k4} - C_{4k4}}{\gamma^k} + \frac{P'_{44}}{\gamma^4}$$

$$= \frac{\boldsymbol{P_{444}V_{444} - C_{444}}}{\boldsymbol{\gamma^4}} + \frac{P'_{44}}{\gamma^4} \quad (17)$$

In accordance with the description of V_{ikN}, the values for bold segments in the afore-mentioned statements should be equal to zero; there is no volume to be thinned for hectares which have been recently harvested, in other words. Therefore, when $i = k$, there would be no volume to be thinned and consequently the treatment cost would be zero. Figure 6 represents the feasible and infeasible flows of the thinned volume. To conquer this mistake, we suggest to add a caveat to the original formulation as follows:

$$E_{iN} = \sum_{k=\max(i,1)}^{N} \frac{P_{ikN}V_{ikN} - C_{ikN}}{(1+\gamma)^k} + \frac{P'_{iN}}{(1+\gamma)^N} \qquad and \quad k \neq i$$

Therefore, for E_{14}, E_{24}, E_{34} and E_{44}, we would have the followings:

$$E_{14} = \sum_{k=\max(1,1)}^{4} \frac{P_{1k4}V_{1k4} - C_{1k4}}{(1+\gamma)^k} + \frac{P'_{14}}{(1+\gamma)^4}$$

$$= \frac{P_{124}V_{124} - C_{124}}{(1+\gamma)^2} + \frac{P_{134}V_{134} - C_{134}}{(1+\gamma)^3} + \frac{P_{144}V_{144} - C_{144}}{(1+\gamma)^4} + \frac{P'_{14}}{(1+\gamma)^4} \quad (18)$$

$$E_{24} = \sum_{k=\max(2,1)}^{4} \frac{P_{2k4}V_{2k4} - C_{2k4}}{(1+\gamma)^k} + \frac{P'_{24}}{(1+\gamma)^4}$$

$$= \frac{P_{234}V_{234} - C_{234}}{(1+\gamma)^3} + \frac{P_{244}V_{244} - C_{244}}{(1+\gamma)^4} + \frac{P'_{24}}{(1+\gamma)^4} \tag{19}$$

$$E_{34} = \sum_{k=\max(3,1)}^{4} \frac{P_{3k4}V_{3k4} - C_{3k4}}{(1+\gamma)^k} + \frac{P'_{34}}{(1+\gamma)^4}$$

$$= \frac{P_{344}V_{344} - C_{344}}{(1+\gamma)^4} + \frac{P'_{34}}{(1+\gamma)^4} \tag{20}$$

$$E_{44} = \sum_{k=\max(4,1)}^{4} \frac{P_{4k4}V_{4k4} - C_{4k4}}{(1+\gamma)^k} + \frac{P'_{44}}{(1+\gamma)^4}$$

$$= \frac{P'_{44}}{(1+\gamma)^4} \tag{21}$$

a) b) c) d) e) f) g)

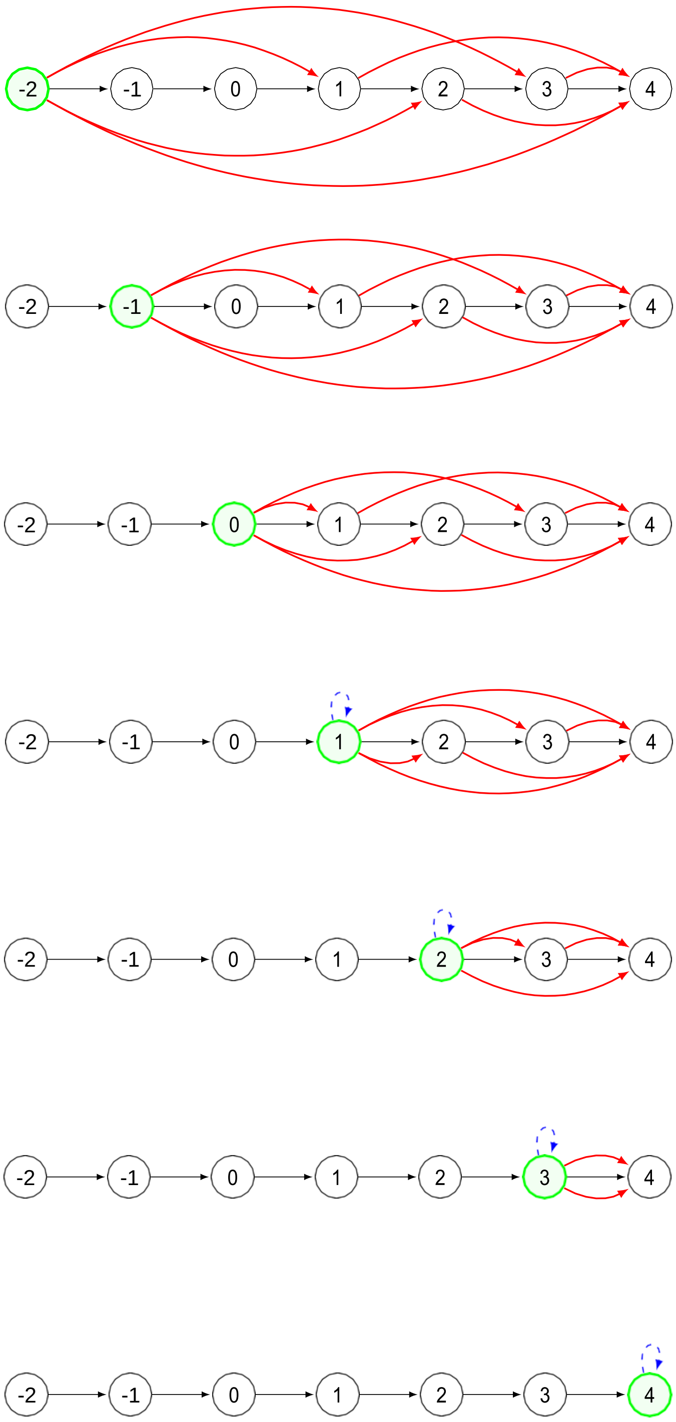


Figure 6: Possible and impossible flows of volume thinned. Solid and dotted lines show the possible and impossible timber flows, respectively.

3.4. Full-revised Model II

Mathematically, the fully revised Model II would be as illustrated below:

$$\max \sum_{j=1}^{N} \sum_{i=-M}^{j-z} D_{ij} x_{ij} + \sum_{i=-M}^{N} E_{iN} w_{iN} \tag{22}$$

Subject to

$$\sum_{j=1}^{N} x_{ij} + w_{iN} = A_i, \qquad i = -M, \dots, 1-z \tag{23}$$

$$\sum_{j=z+i}^{N} x_{ij} + w_{iN} = A_i, \qquad i = 2-z, \dots, 0 \tag{24}$$

$$\sum_{k=j+z}^{N} x_{jk} + w_{jN} = \sum_{i=-M}^{j-z} x_{ij}\,, \qquad j = 1, 2, \dots, N \tag{25}$$

$$x_{ij} \geq 0, \qquad i = -M, \dots, N, j = 1, 2, \dots, N \tag{26}$$

$$w_{iN} \geq 0, \qquad i = -M, \dots, N \tag{27}$$

We use the same definition for the parameters and variables as described by Johnson and Scheurman (1977) in the original paper; however, k index has been taken away from D_{ij} formula and caveat has been added to the E_{iN} formula as follows to overcome the aforementioned mistakes.

$$D_{ij} = \frac{P_{ij} V_{ij} - C_{ij}}{(1+\gamma)^j} \qquad j = 1, 2, \dots, N \quad and \quad i = -M, \dots, j-z$$

$$E_{iN} = \sum_{k=\max(i,1)}^{N} \frac{P_{ikN} V_{ikN} - C_{ikN}}{(1+\gamma)^k} + \frac{P'_{iN}}{(1+\gamma)^N} \quad and \quad k \neq i$$

4. Results

Having encountered problems and proposed a way to deal with them, we want to solve a real problem in order to validate our formulation. In order to solve the problem, first of all, we should compute the values of objective coefficients. Therefore, the price for timbers, volume harvested, and cost of cultural treatments will be discussed in the following paragraphs.

4.1. Case

The price of timber depends on different agents such as kind of tree, length, minimum diameter, and quality. Quality is one of the chief agents of price change. We obtained data on price of spruce trees from the Bureau de mise en marché des bois. There were two sets of data based on different qualities for different zones. The average unit price for spruce lumbers of quality "B" is 17.19 \$/m3 which has been taken into account for P_{ij} and P_{ikN} = 0.75*P_{ij} parameters. In order to calculate the value of E_{iN} , we supposed \$450 for P'_{iN} parameter.

Growth rate is influenced by numerous variables, such as soil, local climate, light, fertility, and care you provide. Each tree has its own growth rate curve concluding three phases. At the beginning, the tree is growing and the growth rate is increasing. Gradually, the growth rate decreases until the tree stops growing. Finally, the phase of decay starts and the growth rate reduces further to negative levels. Figure 7 shows the growth rate curve for spruce trees.

Figure 7: The growth curve for spruce stands used for calculation of discounted net revenue of timber flow.

V_{ikN} indicates the volume thinned per hectare and it is about one-sixth of the marketable timber; in other words, V_{ikN} = V_{ij}/6 (Poulin 2013).

Silvicultural treatment is an operational plan (a sequence of actions, including precommercial thinning (PCT), commercial thinning (CT), shelterwood, selection, buffer, clear-cut, and do nothing) which explains the forest management goals for an area.

In general, stands naturally regenerated are needed to be pre-commercially thinned. There are no marketable wood materials during the thinning; it is a cost generator with no immediate income for the landowner. In order to minimize the cost, PCT should be performed within the first four years of stand

(Forest and Range 2004). Pre-commercial thinning is only conducted in even-aged forests around 15 years old. The trees are too small to be used in the mills and they are always left on site, because their decomposition enriches the soil (Forêts, Faune et Parcs Gouvernement du Québec 2003).

Estimating the actual costs of pre-commercial thinning, labour, and equipment costs which vary depending on different issues should be known (De Franceschi and Boylen 1987). Hedin (1982) took into account the PCT costs of $19.80 per hour based on brushsaw ownership, operating costs, and labour union wage. He also supposed that 15 hours should be spent to thin a hectare, i.e. $297 per hectare. In this paper, we assumed $300 per hectare for pre-commercial thinning.

Commercial thinning is an intermediate harvest process and can provide some intermediate return covering part or all of the cost of harvesting from salable wood. Commercial thinning is usually done in stands between 30 and 80 years old, with no regeneration objective (Forest Practices Branch, Ministry of Forests, British Columbia, Canada 1999).

There is a formulation to calculate the commercial thinning cost for softwood, which has been published by the economic and financial assessments branch of Bureau de mise en marché des bois as below (Direction des dévaluations déconomiques et financières 2017).

$$\textit{Commercial Thinning Cost} = (C1 + (4.797\, vr^{-0.682} - 7.388\, vp^{-0.391})) \times P$$

$$C1 = -0.0385\, P + \frac{325.112}{P} + 4.221$$

Where

P: Sample, m^3/ha

vr: m^3/medium stem to be harvested

vp: m^3/average stand stem, before harvest

The "P" parameter represents the volume which has been cut out of the stand for sampling per hectare. The "vr" indicates the average volume per stem cut for sampling and "vp" points out the average volume per stem for whole trees of the stand. Now, a possible question may spring to your mind: how can the values for "P", "vr", and "vp" be found? In commercial thinning, one sixth of the market volume is thinned. In other words, P = 16.67% (Poulin 2013) and the value for "vr" and "vp" can be considered equal to one (Prégent 2003).

In Canada, the full-tree method is mostly utilized as a harvesting method. The total roadside cost for harvesting under the conventional full tree system is $15.54 per m3. In this paper, the harvesting cost has been supposed $2000 per hectare (Pulkki 1998).

4.2. An illustrative example with realistic data

Suppose we have a forest with six different age classes of trees (See Table 8) and given areas and minimum number of periods between regeneration harvests is 3 ($z = 3$), as below:

Table 8: Values for parameters A_i

Period i	-6	-5	-4	-3	-2	-1	0
A_i	100	200	300	400	500	600	700

We take the following prescription into account as an example for the silvicultural treatment:

- Naturally regenerate
- PCT at age 5
- PCT at age 15
- CT at age 45
- CT at age 70
- Clear-cut at age 85
- Regenerate and repeat

According to the aforementioned cultural treatment costs, D_{ij} and E_{i6} values can be found in Table 9 and Table 10, respectively.

Table 9: Values for parameters D_{ij}

		Next Harvesting Period (Period j)					
		1	2	3	4	5	6
Most Recent Harvesting Period (Period i)	-6	**-188**	376	789	858	1051	1268
	-5	**-832**	**-180**	361	759	825	1010
	-4	**-1394**	**-800**	**-173**	348	730	793
	-3	**-1857**	**-1341**	**-769**	**-167**	334	702
	-2	**-1923**	**-2063**	**-1556**	**-996**	**-407**	84
	-1	**-1923**	**-1849**	**-1984**	**-1496**	**-958**	**-391**
	0	**-1923**	**-2126**	**-2045**	**-2164**	**-1685**	**-1158**
	1	0	**-1923**	**-2126**	**-2045**	**-2164**	**-1685**
	2	0	0	**-1923**	**-2126**	**-2045**	**-2164**
	3	0	0	0	**-1923**	**-2126**	**-2045**

	4	0	0	0	0	**-1923**	**-2126**
	5	0	0	0	0	0	**-1923**
	6	0	0	0	0	0	0

Table 10: Values for parameters E_{i6}

Period i	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
E_{i6}	412	410	408	406	67	78	***-200***	***-178***	***-158***	***-138***	109	119	356

If we solve the problem with two different formulations, the original one and our suggested formulation, there would be a gap between the objective values. The objective value for original and proposed formulations are 1,435,192 and 1,375,344, respectively. The new NPV is 9.5% less than the original formulation—the original formulation overestimate the objective function, that is.

$$\frac{(1,330,345-1,470,009)}{1,470,009} = -9.5\%$$

Tables 11 and 12 indicate the outcomes of decision variables if we solve the problem with original formulation. Tables 13 and 14 demonstrate the new values for the decision variables if we use the proposed formulation. According to the definition of the z parameter, some decision variables must not take a value (gray cells in tables 11 and 13); however, as it can be found in the Table 11, the value of x_{01} is equal to 700. This is the principal reason for the gap between two formulations. In the proposed formulation, the value of x_{01} is equal to 0 instead of 700 and it goes to E_{06} with coefficient of -85 in the objective function.

When discounted costs of cultural treatment are greater than the discounted income obtained from thinned volume plus discounted income from non-timber values, E_{iN} can sometimes have negative value. In accordance with the definition of E_{iN} , we can describe E_{16} as discounted net revenue per hectare during the planning horizon from areas regenerated in period 1 and put aside as ending inventory in period 6 plus discounted net value per hectare of leaving these areas as ending inventory. Thus, the trees have been harvested in period 1 and we put them aside until the end of planning horizon, period 6, when we put them aside for five years. As we supposed in this example, we do precommercial thinning in period 1 and 3, so in accordance with growth rate curve there is no income, but we incur precommercial thinning cost in periods 2 and 4, ($600 per hectare). Moreover, P'_{iN} is supposed to be $450 per hectare. Thus,

$$E_{16} = \sum_{k=\max(1,1)}^{6} \frac{P_{1k6}V_{1k6} - C_{1k6}}{(1+\gamma)^k} + \frac{P'_{16}}{(1+\gamma)^6} \qquad and \quad k \neq 1$$

$$= \frac{-300}{(1+\gamma)^2} + \frac{-300}{(1+\gamma)^4} + \frac{450}{(1+\gamma)^6}$$

$$= -534 + 356$$

5. Concluding remarks

Models are considered as the basic tools of strategic forest planning by most foresters because they examine the long-term consequences of forest-management inputs (Gunn 2007). In this paper, we focused on Model II and how the minimum number of periods between regeneration harvests, i.e, z parameter, could lead to modeling mistakes. The first mistake came about in the first set of area constraints wherein some unnecessary decision variables appeared. They had no coefficient in the objective function; however, in specific contexts they could take nonzero values. The second one took place when we computes the D_{ij} parameter wherein overlaps and impossible timber flows could be found in the formulation.

As far as we know about the literature, these mistakes have not been identified by any researcher, since the Model II was suggested by Johnson and Scheurman (1977). In addition, an example has been given with realistic parameters in order to authenticate our claim. It is well worth mentioning that mistakes had impact on the real data. Some well-known software, such as Woodstock, FORPLAN, TigerMoth and SilviLab are based on variants of Model II formulation. We have not verified that these applications use the formulation that was published in Jonhson and Scheurman. Furthermore, we have not verified that the models referenced by Jonhson and Scheurman included the mistakes or that the mistakes were a publication error.

Table 11: Outcomes for decision variables X_{ij}

Most Recent Harvesting Period (Period *i*)	Next Harvesting Period (Period *j*): 1	2	3	4	5	6
-6	0	0	0	0	0	100
-5	0	0	0	0	0	200
-4	0	0	0	0	0	300
-3	0	0	0	0	0	400
-2	0	0	0	0	0	500
-1	0	0	0	0	0	0
0	***700***	0	0	0	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
4	0	0	0	0	0	0
5	0	0	0	0	0	0
6	0	0	0	0	0	0

Table 12: Outcomes for decision variables w_{i6}

Period i	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
w_{i6}	0	0	0	0	0	600	0	0	0	0	0	0	1500

Table 13: Outcomes for decision variables X_{ij} for revised formulation

		Next Harvesting Period (Period j)					
		1	2	3	4	5	6
Most Recent Harvesting Period (Period i)	-6	0	0	0	0	0	100
	-5	0	0	0	0	0	200
	-4	0	0	0	0	0	300
	-3	0	0	0	0	0	400
	-2	0	0	0	0	0	500
	-1	0	0	0	0	0	0
	0	0	0	0	0	0	0
	1	0	0	0	0	0	0
	2	0	0	0	0	0	0
	3	0	0	0	0	0	0
	4	0	0	0	0	0	0
	5	0	0	0	0	0	0
	6	0	0	0	0	0	0

Table 14: Outcomes for decision variables w_{i6} for revised formulation

Period i	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
w_{i6}	0	0	0	0	0	600	700	0	0	0	0	0	1500

6. Acknowledgments

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