CIRRELT-2019-44

How the Minimum Number of Periods Between Regeneration Harvests Induces Modeling Mistakes in the Well-Known Model II Forest Management

Siamak Mushakhian
Mustapha Ouhimmou
Mikael Rönnqvist
Jean-Martin Lussier

October 2019

Bureau de Montréal
Université de Montréal
C.P. 6128, succ. Centre-Ville
Montréal (Québec) H3C 3J7
Tél. : 1-514-343-7575
Télécopie : 1-514-343-7121

Bureau de Québec
Université Laval,
2325, rue de la Terrasse
Pavillon Palasis-Prince, local 2415
Québec (Québec) G1V 0A6
Tél. : 1-418-656-2073
Télécopie : 1-418-656-2624

How the Minimum Number of Periods Between Regeneration Harvests Induces Modeling Mistakes in the Well-Known Model II Forest Management[ł]

Siamak Mushakhian[1,2] *, Mustapha Ouhimmou[1,2], Mikael Rönnqvist[2,3], Jean-Martin Lussier[4]

[1] System Engineering Department, École de technologie supérieure (ÉTS), Suite A-3586, 1100 Notre-Dame Street West, Montreal, Canada H3C 1K3

[2] Interuniversity Research Centre on Enterprise Networks, Logistics and Transportation (CIRRELT)

[3] Department of Mechanical Engineering, Université Laval, Suite 1504, Pavillon Adrien-Pouliot, 1065, avenue de la Médecine, Québec, Canada G1V 0A

[4] Canadian Wood Fiber Center 1055 du P.E.P.S. Street, P.O. Box 10380, Québec, Canada G1V 4C7

Abstract. The Model II is widely used in forest management models and different variants of it are implemented in various software. Overlooking the minimum number of periods between regeneration harvests can cause some modeling mistakes. We have observed two mistakes in the original formulation. The first is a mistake in the area constraints and the second in calculating one important parameter of the model representing discounted net revenue per hectare between periods. In this paper, we provide a slightly revised model together with comments on the computations of a parameter used in the model formulation. Then, in order to validate the problem identified, we solve the Model II with realistic data to address the modeling mistakes and explain how our revised formulation works with the same data. We also describe situations where the mistakes may have a larger impact and explain why they have not been identified earlier.

Keywords: Forest management planning, timber harvest, harvest scheduling, linear programming, Model II.

Acknowledgments. We gratefully thank the FORAC Research Consortium for its financial support.

[ł] Revised version of the CIRRELT-2018-41

---

* Corresponding author: Siamak.Mushakhian@cirrelt.ca

Dépôt légal – Bibliothèque et Archives nationales du Québec
Bibliothèque et Archives Canada, 2019

1. Introduction

One of the most important models for forest management is the so-called Model II (Johnson and Scheurman 1977). Many variants of this model have been implemented in numerous planning systems within companies and government organizations. One of the main uses of the model is to evaluate the net present value (NPV) of a given forest under different silviculture treatment scenarios and discount rates over a set of time periods representing long-term planning. This model is straightforward and has been referenced in articles since its introduction without any revisions. It has also been implemented in various planning systems and it can be assumed that it is a standard tool to make evaluations of discounted forest values. However, a direct implementation of the original formulation may provide erroneous solutions under certain conditions on parameter values. The main reason is that a number of variables are created incorrectly when a minimum number of periods between regeneration harvests are used. Another mistake is in how the objective function coefficients are calculated. These mistakes may or may not have an impact depending on the silviculture options available and used. Also, as the model is often used for NPV values and not for operational planning, there is no actual need to analyze the actual harvest decision in detail. Hence, there has been little reason to verify or find that some variables have erroneous values. We describe the mistakes and propose a new formulation, which is tested on some illustrative examples.

Simple decision support systems cannot be created and applied universally because strategic planning of the forest value chain includes many different players in many different business contexts. Decision support for strategic planning helps decision makers assess the potential consequences of strategic business choices (Anthony 1965, Drucker 1995). For strategic planning, we should take decision makers' values, objectives, and their future business anticipations into account because these decisions will change the future by changing the flow of the resources and opportunities available to the company (Gunn 2005). At the strategic planning level, decisions, goals, and other constraints of decisions makers must be considered because they have a long-term impact on the company and its resources.

The goal of forest management strategy is to answer the following questions: what to supply, from where to supply to, to which market, and for what use to create value and jobs for local communities. It also has impacts on sustainability, carbon sequestration, wildlife habitat, and ecology, controlling invasive species, and social values (like employment). Sustainable long-term supply is often depicted in terms of level flow or nondeclining yield. D’Amours et al. (2016) have stated that renewability is the key feature of the forest as a supplier of raw materials.

There are three specific parts in the forest management linear models: the process of forest growth and management, the sustainability of forest products, and the requirement to provide certain types of forest cover. Four distinctive modeling approaches are found for forest growth and management, including the well-known Model I and Model II (Johnson and Scheurman 1977) and Model III, which is less common (Garcia 1990); however, it forms the base of popular packages like FOLPI (Gunn 2007), and John and Tóth (2015) proposed a new model for spatial forest harvest scheduling called Model IV.

Woodstock software is capable of generating linear programming matrices by the use of a generalized Model II formulation which is markedly more powerful than other harvest scheduling models based on Model II, like MUSYC (Multiple-Use/Sustained Yield Calculation). FORPLAN (FORest PLANning model) version 2 proposes the capabilities of the generalized Model II (Remsoft 1994).

A combination of Model I and Model II has been used as an optimization model to explore how different management regimes would affect the ability of forests to sequester carbon (Backéus et al. 2005). Martin et al. (2017) compared the efficiency of the spatial Model I and Model II and pointed out that Model I outperformed the Model II.

An optimization approach has been applied through a timber supply model which is an extension of the Model II formulation to estimate the cost of overlapping tenure constraint on forest management agreement areas in Northern Alberta (Nanang and Hauer 2006). A novel approach has been represented to simultaneously maximize carbon sequestration in both forest and wood products and abated emissions from product substitution using Remsoft Spatial Planning System (Hennigar et al. 2008). Note that Woodstock is on the basis of an optimized forest treatment scheduling using a model II LP formulation (Hennigar et al. 2008). Nanang and Hauer (2008) examined the long-term impacts of access road development, which is an important factor in determining harvesting and hunter preferences and non-timber benefits, and they used an extension of the Model II formulation. Model II was used for optimal harvest scheduling in a case study in Spain (Diaz-Balteiro et al. 2009). Model II has been utilized in the forestry portion of the FASOM-GHG model which has been modified to simulate the effects of optional and mandatory participation in carbon offset sales programs (Latta et al. 2011). Model II has been applied in the forest sector model of a linked land-use and forest sector models which have been proposed to find how carbon offset sales can affect private forest owners’ land-use and forest management decisions in Western Oregon (USA) (Latta et al. 2016). In order to analyze the impact of operational-level flexibility on long-term wood supply, a hierarchical planning, i.e. strategic, tactical, and operational, has been developed. The authors used a software called SilviLab to formulate the strategic-level model as a Model II linear program (Gautam et al. 2017). Model II has been used in a goal programming to analyze the long-term impact of policy and industry changes at the landscape level (Corrigan and Nieuwenhuis 2017).

The contribution of the paper is important as the Model II is used in many systems. It is difficult to know if any implementation has found and revised the modeling errors or not. However, we have not found any published article that addresses this and it is important for other researchers and users of the system to understand how they are impacted by the mistakes or how to identify if the implementation may provide erroneous results. This paper identifies and proposes a few modeling mistakes in the Model II formulation given in Johnson and Scheurman (1977). Model II is one of the most well-known forest management models, but the original formulation has two mistakes which may overestimate the objective function and mislead the forest manager or researchers over optimal harvest decisions in a specific context. The first mistake occurs in the first set of the area constraints, wherein some additional decision variables are created. These decision variables may take nonzero values and provide wrong information about the objective function and harvest decisions. The second mistake can be found in the way to calculate one of the key parameters of the model. This parameter will be explained in detail in the following sections.

The rest of this paper is organized as follows. Section 2 describes the forest management models, especially Model II in details with the mathematical formulation. In Section 3, we pose questions to Model II and propose a new mathematical formulation. In order to validate our new formulation, a problem would be represented with practical data and the results would be analyzed in Section 4. Finally, conclusions are drawn in Section 5.

2. Forest management models

In literature, four modeling approaches can be found for forest management planning, including the well-known Model I and Model II (Johnson and Scheurman 1977), Model III, which is less common (Garcia 1990), and John and Tóth (2015) proposed a new model for spatial forest harvest scheduling which is called Model IV. In Model I (Johnson and Scheurman 1977), the integrity of each age-class in the first period is kept throughout the planning horizon (see Model I in Figure 1). However, in Model II (Johnson and Scheurman 1977), the integrity of each age-class in the first period is kept until it is regeneration harvested and forms a new age-class until they are again regeneration harvested (see Model II in Figure 1). In Model III (Garcia 1990), in each period, the land in an age-class can be harvested or become one age-class older (see Model III in Figure 1). The aggregation of all stands in Model II is similar to the Model III; however, the network contains fewer nodes and arcs. Model I can be used to model either aggregated or individual stands (Gunn 2007). In the previous models, one decision variable is required for every applicable prescription for each forest management unit. The mentioned models are aspatial and also depend on static volume and revenue coefficients that must be calculated before starting optimization. Finally, John and Tóth (2015) introduced a new model which is called Model IV, using different equations and Boolean algebra for spatial forest harvest scheduling.

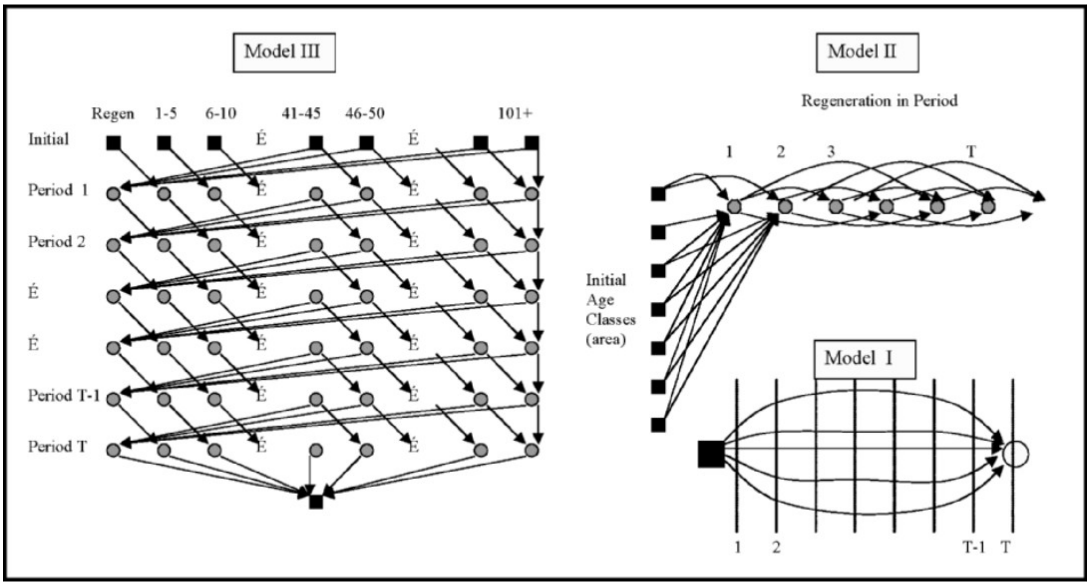


Figure 1: Models I, II, and III (Gunn 2007).

2.1. Model II

Forest management planning aims to schedule timber harvest and investment on an area of timberland under even-aged management. The goal is to maximize the volume or value produced from its timberland while encountering constant or decreasing prices in the volume of timber output (Johnson and Scheurman 1977). The manager may come across land availability limits for harvesting in each time period when the whole area is managed under one cultural treatment regime. A cultural treatment regime is any sequence of silvicultural practices such as planting, pre-commercial thinning, commercial thinning, and fertilization. In addition to area constraints, it may also consider flow constraints (harvest fluctuation and sustainability).

Seven simplifying assumptions have been stated as follows:

1. The forest has one type-site consisting of different age-classes.
2. The area of forestland is fixed during the planning horizon.
3. The number of years representing each time period in the planning horizon is consistent with the years of each age-class.
4. For regeneration harvest, we use clear-cutting.
5. Regeneration occurs in the same period as regeneration harvest.
6. Yield estimates take into account all uncertainties such as fire, insect, and diseases implicitly.
7. The only out-of-pocket costs that should be paid are the cultural treatment costs.

In Model II, each age-class forms a management unit that is harvested. Having regeneration harvested, new age-class is formed till they are again regeneration harvested. Each activity describes a possible management regime for a certain management unit from the time a unit is regenerated until it is regeneration harvested or left as ending inventory at the end of the planning horizon. A management regime includes two parts (Johnson and Scheurman 1977):

1. A regeneration harvest at some time during the planning horizon or an ending inventory at the end of the planning horizon.
2. An associated cultural treatment regime.

We require two sets of area constraints:

- One set on the areas that can be regeneration harvested from, or put aside as ending inventory in, each age-class that exists at the start of planning horizon (See Figure 2 and Constraint 2) (Johnson and Scheurman 1977). Figure 2 indicates that the areas cut from each age-class through different time periods plus the areas left as ending inventory from that age-class are equal to the total number of areas in that age-class at the beginning of planning horizon. For instance, Figure 2b indicates that the total area from age-class one (on the assumption that there is no minimum number of periods between regeneration harvests) at the beginning of the planning horizon can be harvested in different periods starting from one to N, and put aside as ending inventory.

- The second set is on the areas that can be regeneration harvested from, or put aside as ending inventory in, each age-class that is created throughout the planning horizon (See Figure 3 and Constraints 3) (Johnson and Scheurman 1977). Figure 3 illustrates that the areas cut from areas regenerated in period j plus the areas left as ending inventory from areas regenerated in period j are equal to the total number of areas regenerated in period j (j can vary between the first period and the end of the planning horizon). For example, in period j, different age-classes may be harvested, so these areas can be harvested in the following future periods and also put aside as ending inventory.

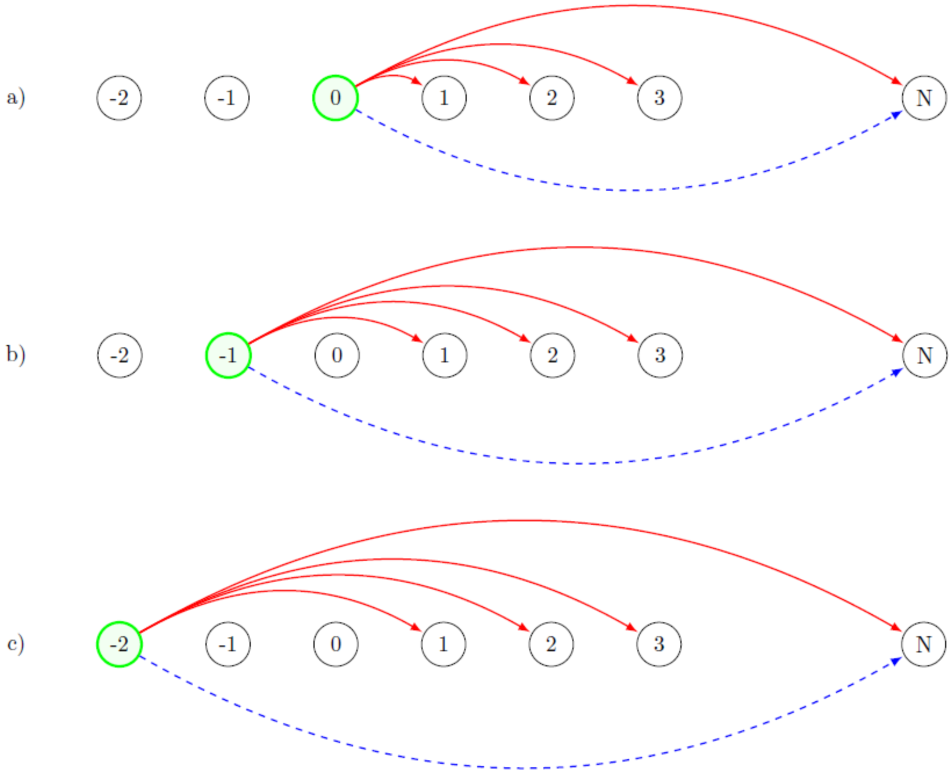


Figure 2: Balance constraint for areas regenerated or put aside as ending inventory at the start of the planning horizon for three different age-classes: 0, 1, and 2 can be seen in a, b, and c, respectively.

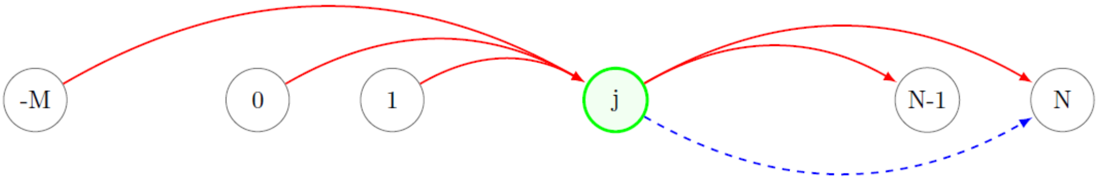


Figure 3: Balance constraint for areas regenerated or put aside as ending inventory throughout the planning horizon.

2.2. Mathematical formulation

The mathematical form of Model II is summarized as follows:

$$\max \sum_{j=1}^{N} \sum_{i=-M}^{j-z} D_{ij} x_{ij} + \sum_{i=-M}^{N} E_{iN} w_{iN} \tag{1}$$

Subject to

$$\sum_{j=1}^{N} x_{ij} + w_{iN} = A_i, \qquad i = -M, \dots, 0 \tag{2}$$

$$\sum_{k=j+z}^{N} x_{jk} + w_{jN} = \sum_{i=-M}^{j-z} x_{ij}\,, \qquad j = 1, 2, \dots, N \tag{3}$$

$$x_{ij} \geq 0, \qquad i = -M, \dots, N, j = 1, 2, \dots, N \tag{4}$$

$$w_{iN} \geq 0, \qquad i = -M, \dots, N \tag{5}$$

Constraint (2) expresses the land availability constraint for the beginning of the planning horizon (see Figure 2). The balance constraint for areas regenerated in period j can be found in Constraint (3) (see Figure 3). Constraint (4) and (5) show the non-negativity.

Authors defined the sets, data, and variables as follows, where:

N	Number of periods in the planning horizon
x_{ij}	Areas regenerated in period i and regeneration harvested again in period j
w_{iN}	Areas regenerated in period i and put aside as ending inventory in period N
A_i	Number of hectares present in period one that were regenerated in period i (i =−M, . . . , 0), with each A_i being a constant at the beginning of the planning horizon (period 1)
M	Number of periods before period zero in which the oldest age-class present in the period one was regenerated
z	Minimum number of periods between regeneration harvests (reasonably it is greater than one, i.e., $z \geq 1$)
D_{ij}	Discounted net revenue per hectare from areas regenerated in period i and

regeneration harvested again in period j. It can be written as shown below:

$$D_{ij} = \sum_{k=\max(i,1)}^{j} \frac{P_{ikj}V_{ikj} - C_{ikj}}{\gamma^{k}}$$

Where

P_{ikj}	Unit price of the volume harvested in period k on areas regenerated in period i and regeneration harvested again in period j
V_{ikj}	Volume per hectare harvested in period k on areas regenerated in period i and regeneration harvested again in period j
C_{ikj}	Cultural treatment costs per hectare in period k on areas regenerated in period i and regeneration harvested again in period j
γ^{j}	Discount rate for period j
E_{iN}	Discounted net revenue per hectare during the planning horizon from areas regenerated in period i and put aside as ending inventory in period N plus discounted net value per hectare of leaving these areas as ending inventory. It can be written as shown below:

$$E_{iN} = \sum_{k=\max(i,1)}^{N} \frac{P_{ikN}V_{ikN} - C_{ikN}}{\gamma^{k}} + \frac{P'_{iN}}{\gamma^{N}}$$

Where

P_{ikN}	Unit price of the volume thinned in period k on areas regenerated in period i and put aside as ending inventory in period N
V_{ikN}	Volume per hectare thinned in period k on areas regenerated in period i and put aside as ending inventory in period N
C_{ikN}	Cultural treatment costs per hectare in period k on areas regenerated in period i and put aside as ending inventory in period N
P'_{iN}	Net value per hectare of leaving areas regenerated in period i as ending inventory in period N

3. Methods

In this section, we first represent the modeling mistakes in Model II and then propose our new model to overcome these mistakes.

3.1. Mistake in the first set of area constraints

In accordance with the definition of z, the minimum number of periods between regeneration harvests, it is not allowed to harvest an area unless at least z periods have been passed since the last regeneration harvest. Unfortunately, Constraint (2) has been mistakenly written in Johnson and Scheurman (1977). In the original formulation, there is a number of extra variables generated that should be forced to be 0 due to the requirement of periods between regeneration harvests. If coefficients of variables in the objective function are negative except for the additional decision variables, it is possible that those extra decision variables take values and if this is the case, we have an erroneous solution. There is a number of situations when this may happen, including partial cutting and thinning operations, where costs exceed revenue from sales.

In order to prove our claim and provide further clarification, consider the following example where we would like to schedule harvests for the next four time periods (N = 4) from a forest that now has three different age-classes aged 0, 1, and 2 (i.e. A_0 = 300, A_{-1} = 200, and A_{-2} = 100) (M = 2). There is a minimum of three time periods between regeneration harvests (z = 3).

Now we want to expand the objective function and first set of area constraints and take a closer look at them. As mentioned above, an area cannot be harvested unless a minimum of z periods have passed since the last regeneration. However, you can find variables in the constraints which are contrary to this law (x_{-11}, x_{01}, and x_{02}).

$$\max \sum_{j=1}^{4} \sum_{i=-2}^{j-3} D_{ij} x_{ij} + \sum_{i=-2}^{4} E_{i4} w_{i4}$$

$$\begin{aligned} = \; & D_{-21}x_{-21} + \\ & D_{-22}x_{-22} + D_{-12}x_{-12} + \\ & D_{-23}x_{-23} + D_{-13}x_{-13} + D_{03}x_{03} + \\ & D_{-24}x_{-24} + D_{-14}x_{-14} + D_{04}x_{04} + D_{14}x_{14} + \end{aligned}$$

$$E_{-24}w_{-24} + E_{-14}w_{-14} + E_{04}w_{04} + E_{14}w_{14} + E_{24}w_{24} + E_{34}w_{34} + E_{44}w_{44}$$

First area constraint

$$\sum_{j=1}^{4} x_{ij} + w_{i4} = A_i, \qquad i = -2, \dots, 0$$

$$x_{-21} + x_{-22} + x_{-23} + x_{-24} + w_{-24} = A_{-2}, \qquad i = -2$$

$$\boldsymbol{x_{-11}} + x_{-12} + x_{-13} + x_{-14} + w_{-14} = A_{-1}, \qquad i = -1$$

$$\boldsymbol{x_{-01}} + \boldsymbol{x_{-02}} + x_{-03} + x_{-04} + w_{-04} = A_{-0}, \qquad i = 0$$

As aforementioned, if the coefficients of variables in the objective function are negative except for the additional decision variables, it is possible that those extra decision variables take values. To clear it up, suppose the following values for parameters D_{ij} and E_{iN} in Tables 1 and 2 for the above-mentioned example.

Table 1: Values for parameters D_{ij}

		Next Harvesting Period (Period j)			
		1	2	3	4
Most Recent Harvesting Period (Period i)	-2	1	1	1	1
	-1	0	**-1**	**-1**	**-1**
	0	0	0	**-1**	**-1**
	1	0	0	0	1
	2	0	0	0	0
	3	0	0	0	0
	4	0	0	0	0

Table 2: Values for parameters E_{i4}

Period i	-2	-1	0	1	2	3	4
E_{i4}	1	**-1**	**-1**	1	1	1	1

We solved the model and the results can be found in Tables 3 and 4. Light gray cells indicate the forbidden periods for the harvesting of each age-class according to the definition of the z parameter. While the rule has been violated by x_{-11} and x_{01}, 200 and 300 are their values, respectively. The objective value is 300.

The repercussion will not be limited to this one. In addition to that, those values would be ignored for future harvest planning. For instance, when a management unit is regeneration harvested in period 1 (x_{-11}= 200), it can be harvested in period 4 or taken into account as an ending inventory while it has been overlooked, likewise for the other unallowable variable (x_{01}= 300).

Table 3: Outcomes for decision variables x_{ij} for original formulation

		Next Harvesting Period (Period *j*)			
		1	2	3	4
Most Recent Harvesting Period (Period *i*)	-2	100	0	0	0
	-1	***200***	0	0	0
	0	***300***	0	0	0
	1	0	0	0	100
	2	0	0	0	0
	3	0	0	0	0
	4	0	0	0	0

Table 4: Outcomes for decision variables w_{i4} for original formulation

Period i	-2	-1	0	1	2	3	4
w_{i4}	0	0	0	0	0	0	100

In order to correct the mistakes, a revised formulation for the first set of area constraints are:

$$\sum_{j=1}^{N} x_{ij} + w_{iN} = A_i, \qquad i = -M, \dots, 1 - z$$

$$\sum_{j=z+i}^{N} x_{ij} + w_{iN} = A_i, \qquad i = 2 - z, \dots, 0$$

We solved the example again by considering the new formulation. The outcomes can be found in Tables 5 and 6. As it can be seen, there is no breach of rule for harvesting. The objective value is -4200.

Table 5: Outcomes for decision variables x_{ij} for revised formulation

		Next Harvesting Period (Period *j*)			
		1	2	3	4
Most Recent Harvesting Period (Period *i*)	-2	100	0	0	0
	-1	0	0	0	200
	0	0	0	300	0
	1	0	0	0	100
	2	0	0	0	0
	3	0	0	0	0
	4	0	0	0	0

Table 6: Outcomes for decision variables w_{i4} for revised formulation

Period i	-2	-1	0	1	2	3	4
w_{i4}	0	0	0	0	0	300	300

3.2. Mistake in calculation of D_{ij}

The model uses the objective function coefficients D_{ij} and E_{iN}. To find the miscalculation of D_{ij} parameter tangibly, consider the following example with given parameters: $N = 7, M = 1, z = 3$.

The objective function is as below:

$$\begin{aligned}
&\max \sum_{j=1}^{7} \sum_{i=-1}^{j-3} D_{ij}x_{ij} + \sum_{i=-1}^{7} E_{i7}w_{i7} \\
&= D_{-12}x_{-12} + \\
&\quad D_{-132}x_{-13} + D_{03}x_{03} + \\
&\quad D_{-14}x_{-14} + D_{04}x_{04} + D_{14}x_{14} + \\
&\quad D_{-15}x_{-15} + D_{05}x_{05} + D_{15}x_{15} + D_{25}x_{25} + \\
&\quad D_{-16}x_{-16} + D_{06}x_{06} + D_{16}x_{16} + D_{26}x_{26} + D_{36}x_{36} + \\
&\quad D_{-17}x_{-17} + D_{07}x_{07} + D_{17}x_{17} + D_{27}x_{27} + D_{37}x_{37} + D_{47}x_{47} + \\
&\quad E_{-17}w_{-17} + E_{07}w_{07} + E_{17}w_{17} + E_{27}w_{27} + E_{37}w_{37} + E_{47}w_{47} + E_{57}w_{57} + E_{67}w_{67} + E_{77}w_{77}
\end{aligned}$$

At each time period, two sets of timber flows are needed, including input (areas regenerated in previous time periods and going to be regeneration harvested again in this period) and output (areas may be regenerated in future or put aside as an ending inventory) flows. Figure 4 represents input and output flows in the aforementioned example ($N = 7, M = 1, z = 3$). For example, in Figure 4c, there are two timber inflows from areas regenerated harvested three and four periods ago (period -1 and 0, respectively) and three timber outflows, two of which will be regenerated again in periods 6 and 7, and the third outflow is related to areas left as ending inventory in period 7. Note that thinned volume obtained from stand thinnings of regeneration harvested areas are not shown in Figure 4.

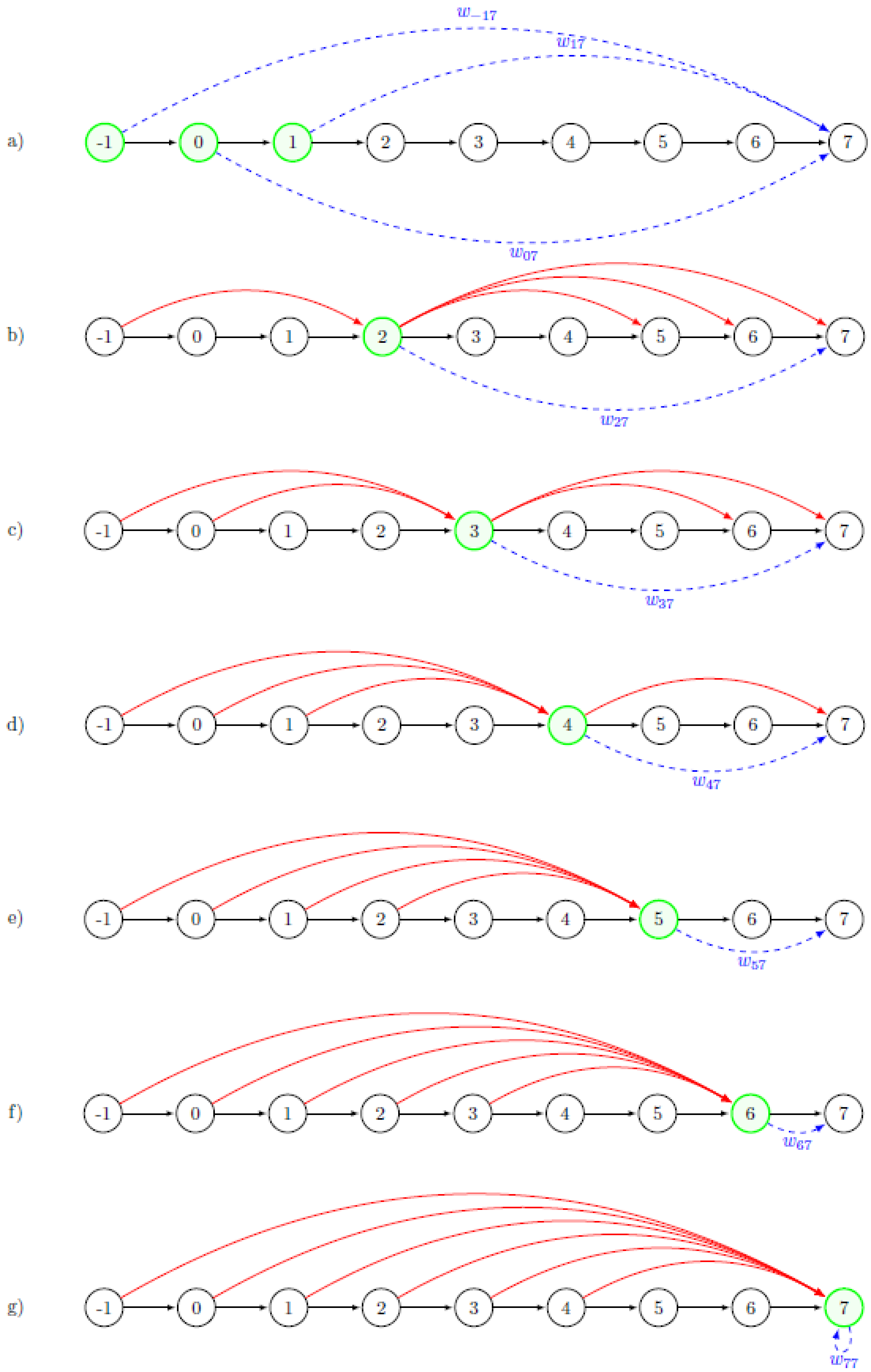


Figure 4: Timber flows for different time periods in an example with N=7, M=1, and z=3. Solid lines show the regenerated areas and dotted lines indicate the areas put aside as an ending inventory.

To represent the mistake which we will come across while we are calculating the coefficients with the use of original formulation, consider the following equations and figures:

$$\begin{aligned} D_{-12} &= \sum_{k=\max(-1,1)}^{2} \frac{P_{-1k2}V_{-1k2} - C_{-1k2}}{\gamma^{k}} \\ &= \boldsymbol{\frac{P_{-112}V_{-112} - C_{-112}}{\gamma}} + \frac{P_{-122}V_{-122} - C_{-122}}{\gamma^{2}} \end{aligned} \tag{6}$$

$$\begin{aligned} D_{-13} &= \sum_{k=\max(-1,1)}^{3} \frac{P_{-1k3}V_{-1k3} - C_{-1k3}}{\gamma^{k}} \\ &= \boldsymbol{\frac{P_{-113}V_{-113} - C_{-113}}{\gamma}} + \boldsymbol{\frac{P_{-123}V_{-123} - C_{-123}}{\gamma^{2}}} + \frac{P_{-133}V_{-133} - C_{-133}}{\gamma^{3}} \end{aligned} \tag{7}$$

$$\begin{aligned} D_{-14} &= \sum_{k=\max(-1,1)}^{4} \frac{P_{-1k4}V_{-1k4} - C_{-1k4}}{\gamma^{k}} \\ &= \boldsymbol{\frac{P_{-114}V_{-114} - C_{-114}}{\gamma}} + \boldsymbol{\frac{P_{-124}V_{-124} - C_{-124}}{\gamma^{2}}} + \boldsymbol{\frac{P_{-134}V_{-134} - C_{-134}}{\gamma^{3}}} \\ &+ \frac{P_{-144}V_{-144} - C_{-144}}{\gamma^{4}} \end{aligned} \tag{8}$$

$$\begin{aligned} D_{-15} &= \sum_{k=\max(-1,1)}^{5} \frac{P_{-1k5}V_{-1k5} - C_{-1k5}}{\gamma^{k}} \\ &= \boldsymbol{\frac{P_{-115}V_{-115} - C_{-115}}{\gamma}} + \frac{P_{-125}V_{-125} - C_{-125}}{\gamma^{2}} + \boldsymbol{\frac{P_{-135}V_{-135} - C_{-135}}{\gamma^{3}}} \\ &+ \boldsymbol{\frac{P_{-145}V_{-145} - C_{-145}}{\gamma^{4}}} + \frac{P_{-155}V_{-155} - C_{-155}}{\gamma^{5}} \end{aligned} \tag{9}$$

$$\begin{aligned} D_{-16} &= \sum_{k=\max(-1,1)}^{6} \frac{P_{-1k6}V_{-1k6} - C_{-1k6}}{\gamma^{k}} \\ &= \boldsymbol{\frac{P_{-116}V_{-116} - C_{-116}}{\gamma}} + \frac{P_{-126}V_{-126} - C_{-126}}{\gamma^{2}} + \frac{P_{-136}V_{-136} - C_{-136}}{\gamma^{3}} \\ &+ \boldsymbol{\frac{P_{-146}V_{-146} - C_{-146}}{\gamma^{4}}} + \boldsymbol{\frac{P_{-156}V_{-156} - C_{-156}}{\gamma^{5}}} + \frac{P_{-166}V_{-166} - C_{-166}}{\gamma^{6}} \end{aligned} \tag{10}$$

Note that the number of the first and last regeneration periods is constant; however, the middle harvested period (k) varies in the formulation. According to the definition of z, some timber flows (only the harvested volume and not the thinned volume of stand thinnings) are impossible; bold segments of the formulas refer to this point. Figure 5 illuminates the possible and impossible timber flows. For instance, as discovered in Figure 5d, D_{-15} is consisted of five timber flows such as $V_{-115}, V_{-125}, V_{-135}, V_{-145}$, and V_{-155}; however, in accordance with the definition of the z parameters, some timber flows are impossible, like V_{-115}, V_{-135}, and V_{-145}. You should be aware that the mistake is not limited to impractical timber flows. In addition, there is an overlap between one fragment of the D_{-12} and D_{-15}. The fragments are as below:

$$\frac{P_{-122}V_{-122} - C_{-122}}{\gamma^2} \quad \& \quad \frac{P_{-125}V_{-125} - C_{-125}}{\gamma^2}$$

These two segments calculate the same timber flow and discount it for two periods. In other words, there is a timber flow in D_{-15} which has been computed in D_{-12}. Furthermore, two overlaps can be found between D_{-16}, D_{-12}, and D_{-13}.

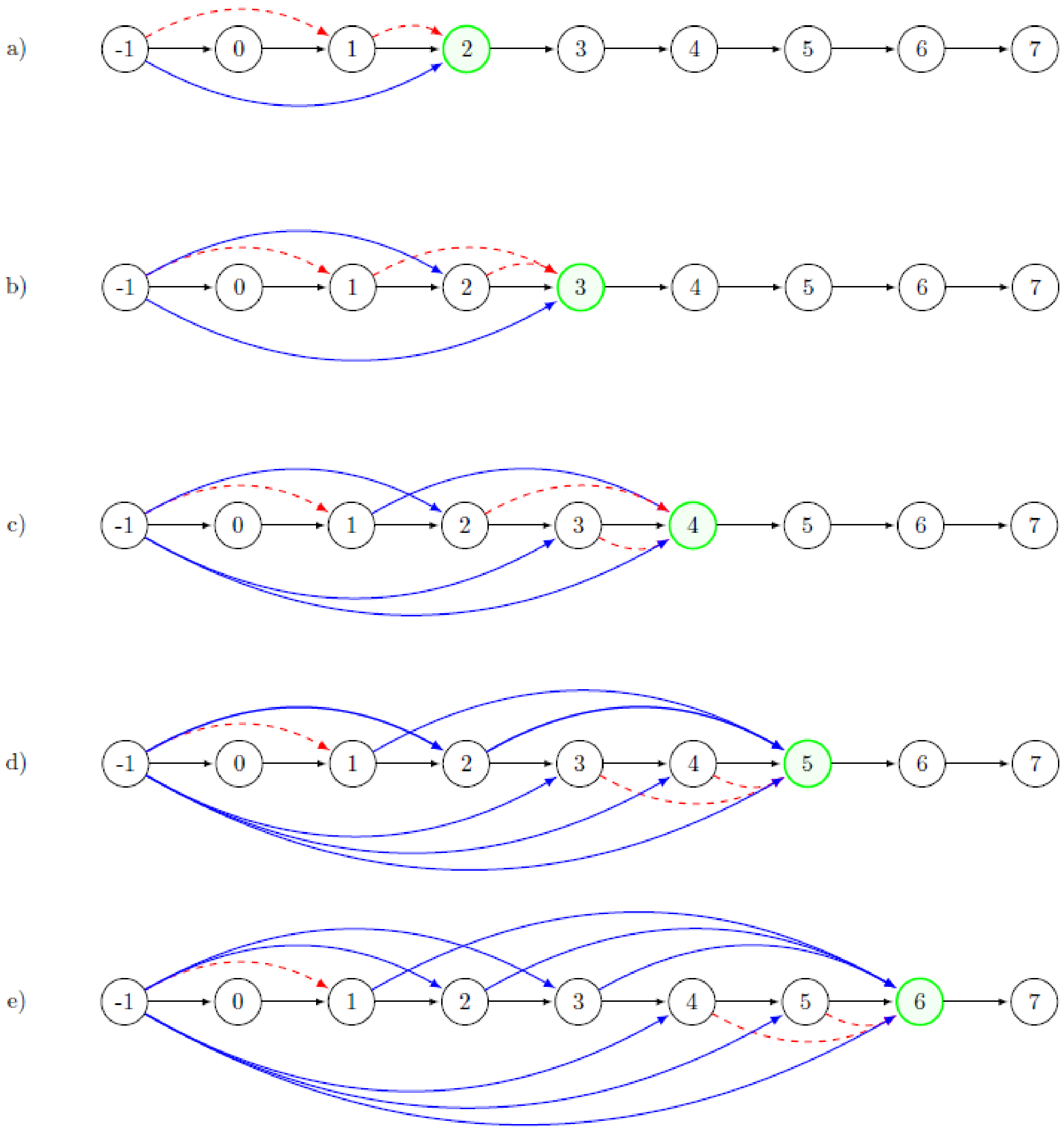


Figure 5: Timber flows used in calculation of discounted net revenue. Solid and dotted lines show the possible and impossible timber flows, respectively

To analyze the profitability of investment, NPV, which is the difference between the present value of cash inflows and outflows discounted by the discount rate, is used in capital budgeting. The coefficient D_{ij} is the discounted net revenue per hectare from areas regenerated in period i and regeneration harvested again in period j. However, in accordance with the definition of z, some timber flows (V_{ikj}) are impossible as was identified in the model constraint. Moreover, there is an overlap between different D_{ij} in the objective function, i.e., the V_{ikj} is counted multiple times in the D_{ij} which results in an overestimation in the calculation of the net present value (NPV). Therefore, in order to correct the formula, D_{ij} should be broken into two segments to consider both revenues from harvested volume (D'_{ij}) and thinned volume (D''_{ij}) as below:

$$D'_{ij} = \frac{P'_{ij}V'_{ij} - C'_{ij}}{(1+\gamma)^{j}} \qquad j = 1, 2, \dots, N \;\&\; i = -M, \dots, j - z \tag{11}$$

$$D''_{ij} = \sum_{k=max(i,1)}^{j} \frac{P''_{ikj}V''_{ikj} - C''_{ikj}}{\gamma^{k}} \tag{12}$$

$$D_{ij} = D'_{ij} + D''_{ij} \tag{13}$$

Where

P'_{ij}	Unit price of the volume harvested in period i and regeneration harvested in period j
V'_{ij}	Volume per hectare harvested in period i and regeneration harvested in period j
C'_{ij}	Cultural treatment costs per hectare in period i and regeneration harvested in period j
P''_{ikj}	Unit price of the volume thinned in period k on areas regenerated in period i and regeneration harvested again in period j
V''_{ikj}	Volume per hectare thinned in period k on areas regenerated in period i and regeneration harvested again in period j
C''_{ikj}	Cultural treatment costs per hectare in period k on areas regenerated in period i and regeneration harvested again in period j

3.3. Full-revised Model II

Mathematically, the fully revised Model II would be as illustrated below:

$$\max \sum_{j=1}^{N} \sum_{i=-M}^{j-z} D_{ij} x_{ij} + \sum_{i=-M}^{N} E_{iN} w_{iN} \tag{14}$$

Subject to

$$\sum_{j=1}^{N} x_{ij} + w_{iN} = A_i, \qquad i = -M, \dots, 1-z \tag{15}$$

$$\sum_{j=z+i}^{N} x_{ij} + w_{iN} = A_i, \qquad i = 2-z, \dots, 0 \tag{16}$$

$$\sum_{k=j+z}^{N} x_{jk} + w_{jN} = \sum_{i=-M}^{j-z} x_{ij}\,, \qquad j = 1, 2, \dots, N \tag{17}$$

$$x_{ij} \geq 0, \qquad i = -M, \dots, N, j = 1, 2, \dots, N \tag{18}$$

$$w_{iN} \geq 0, \qquad i = -M, \dots, N \tag{19}$$

$$D'_{ij} = \frac{P'_{ij} V'_{ij} - C'_{ij}}{(1+\gamma)^j} \qquad j = 1, 2, \dots, N \;\&\; i = -M, \dots, j-z \tag{20}$$

$$D''_{ij} = \sum_{k=max(i,1)}^{j} \frac{P''_{ikj} V''_{ikj} - C''_{ikj}}{\gamma^k} \tag{21}$$

$$D_{ij} = D'_{ij} + D''_{ij} \tag{22}$$

Where

P'_{ij}	Unit price of volume harvested in period i and regeneration harvested in period j
V'_{ij}	Volume per hectare harvested in period i and regeneration harvested in period j
C'_{ij}	Cultural treatment costs per hectare in period i and regeneration harvested in period j

P''_{ikj}	Unit price of volume thinned in period k on areas regenerated in period i and regeneration harvested again in period j
V''_{ikj}	Volume per hectare thinned in period k on areas regenerated in period i and regeneration harvested again in period j
C''_{ikj}	Cultural treatment costs per hectare in period k on areas regenerated in period i and regeneration harvested again in period j

4. Results

As case study, we use a forest located close to Causapsal, in the Gaspesia Region. The stand type is a stand dominated by balsam fir with a small component (< 15%) of White Spruce and White birch on an average site quality (Site Index = 17 for balsam fir). We will consider that the forest stand within the management unit is managed following an even-aged regime based on the shelterwood system. This is common for balsam-fir located in the Gaspesia and the Bas-St-Laurent Regions of Quebec. This regime consists of a partial cutting done to support the establishment and growth of regeneration under the canopy of the residual stand and a few years later the rest of the dominant canopy are removed by a final cut while protecting the advanced regeneration. Balsam fir is very adaptable to this regeneration system since these species are grown by regular seed rain and seedlings are highly shade tolerant. For regenerating this stand type with this regeneration method, only a partial opening of the canopy is required with no soil preparation and no plantation. consequently, regeneration costs are much lower than for the plantation regeneration system. If logging is done properly at the final harvest, the young balsam fir stands would be very dense requiring a precommercial thinning to avoid growth stagnation over time.

4.1. Precommercial thinning

Silvicultural treatment is an operational plan (a sequence of actions, including precommercial thinning (PCT), commercial thinning (CT), shelterwood, selection, buffer, clear-cut, and do nothing) which explains the forest management goals for an area.

In general, stands naturally regenerated are needed to be pre-commercially thinned. In Canada, there are no marketable wood materials during the pre-commercial thinning; it is a cost generator with no intermediate income for the landowner. In order to minimize the cost, PCT should be performed within the first four years of the stand (Forest and Range 2004). Pre-commercial thinning is only conducted in even-aged forests around 15 years old. The trees are too small to be used in the mills and they are always left on site because their decomposition enriches the soil (Forêts, Faune et Parcs Gouvernement du Québec 2003). Precommercial thinning is assumed to be applied at the age of 10. The treatment reduces the canopy of competing hardwoods and regulates the spacing of the softwoods. We assume that the treatment is a prerequisite for obtaining the yields in this study.

Estimating the actual costs of pre-commercial thinning, labor, and equipment costs which vary depending on different issues should be known (De Franceschi and Boylen 1987). Hedin (1982) took into account the PCT costs of $19.80 per hour based on brushsaw ownership, operating costs, and labor union wage. He also supposed that 15 hours should be spent to thin a hectare, i.e. $297 per hectare. In this research, the cost estimate is based on the rates applied by the Ministry of Forests, Wildlife and Parks of Quebec (Gouvernement du Québec 2019). It is calculated with the following function:

$$Precommercial\ thinning = 156.33 + (630.28 * \log(DI) - 5095) * TP$$

Here,

DI: Initial density per hectare, the count of all stems with a stump height diameter (15 cm) greater than 1.5 cm

TP:

- Target composition of the poplar: 0.7705
- Target composition of the softwood: 1.0000
- Target composition of mixed forest with softwood tendency: 1.1004

In our study, we assume an average tree density (DI) of 19 000 $\frac{\text{trees}}{\text{ha}}$ (equals to the average density observed for the fir-dominated ecoregion before PCT treatment in Laflèche and Tremblay (2008)), and a target stand composition dominated by softwoods (TP = 1), then the PCT cost is estimated to be 1272.97 $\frac{\$}{\text{ha}}$.

4.2. Commercial thinning, shelterwood, and growth curve

The growth rate is influenced by numerous variables, such as soil, local climate, light, fertility, and the care you provide. Each tree has its own growth rate curve concluding three phases. In the beginning, the tree is growing and the growth rate is increasing. Gradually, the growth rate decreases until the tree stops growing. Finally, the phase of decay starts and the growth rate reduces further to negative levels.

The goal of the commercial thinning is to cut some trees to make more space for the remaining trees and increase their growth and favour the development of advanced regeneration while providing an intermediate supply of timber before the final harvest. The treatment, however, increases the average tree volume by 24% again in comparison to untreated stands (Pelletier and Pitt 2008) (see Figure 6).

Based on the results from Pelletier and Pitt (2008) commercial thinning has no effect on the cumulative merchantable volume production (thinning + standing volumes) in comparison to untreated stands. Table 7 represents the empirical yield tables for balsam fir stand with and without commercial thinning (Pothier and Auger 2011).

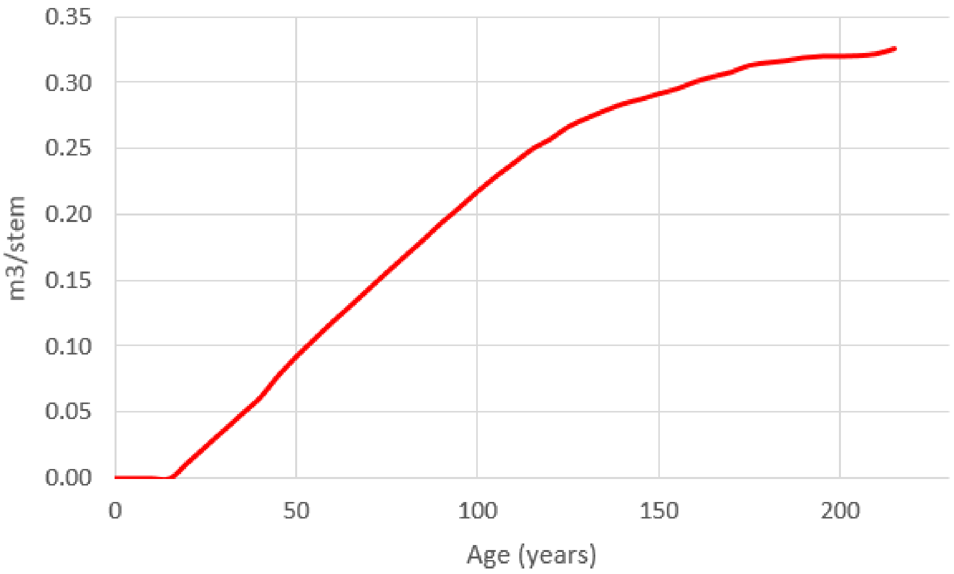


Figure 6: The stem volume growth for balsam fir trees.

Table 7: Balsam fir stand volume with and without commercial thinning

Age Class	Age	Mean tree Volume (m3/tree)	Volume (m3/ha)	Volume after CT (m3/ha)
1	5	0	0	0
2	10	0	0	0
3	15	0	0	0
4	20	0.01	21.1	21.1
5	25	0.02	42.2	42.2
6	30	0.04	63.3	31.7
7	35	0.05	84.6	53.0
8	40	0.08	106.0	74.4
9	45	0.10	139.6	108.0
10	50	0.11	166.5	134.9
11	55	0.13	187.3	155.7
12	60	0.15	202.7	171.1
13	65	0.16	213.1	181.5
14	70	0.18	219.6	188.0
15	75	0.19	222.5	190.9
16	80	0.21	222.4	190.8
17	85	0.22	220.0	188.4
18	90	0.24	215.7	184.1
19	95	0.26	210.2	178.6
20	100	0.27	203.7	172.1
21	105	0.28	196.6	165.0
22	110	0.30	189.3	157.7
23	115	0.31	181.9	150.3
24	120	0.32	174.7	143.1
25	125	0.33	168.1	136.5
26	130	0.34	162.0	130.4
27	135	0.35	156.2	124.6
28	140	0.35	151.4	119.8
29	145	0.36	147.0	115.4
30	150	0.36	143.0	111.4

Commercial thinning is usually done in stands between 30 and 80 years old, with no regeneration objective (Forest Practices Branch, Ministry of Forests, British Columbia, Canada 1999). In this study, commercial thinning is implemented at age 30 and a 50% crown thinning is prescribed, including the effect of skidding trails. Partial harvest is done with a harvester and a forwarder. After commercial thinning the standing volume is assumed to be equal to the volume of an unthinned stand, minus the volume remove at the moment of the thinning (see Table 7 and Figure 7). This follows the results from Pelletier and Pitt (2008).

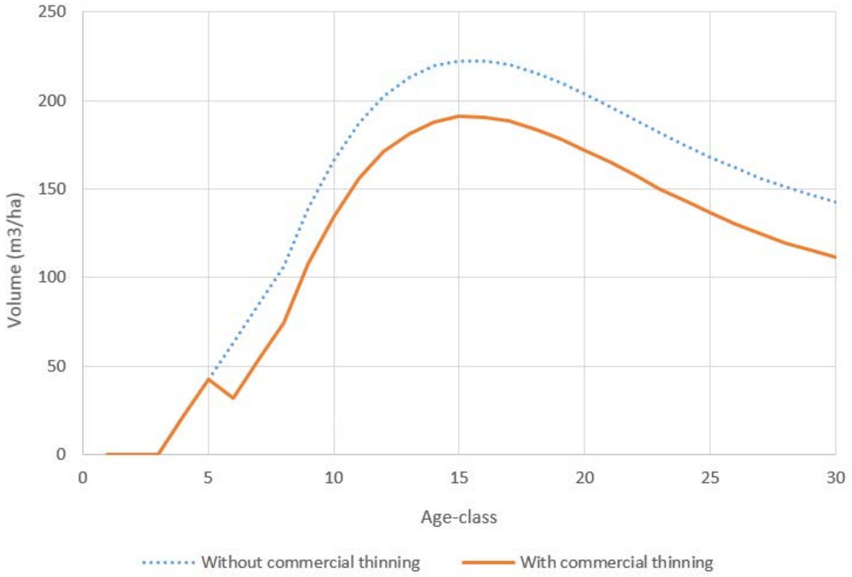


Figure 7: The stand growth curve for balsam fir with and without commercial thinning.

Figure 8 illustrates how the road-side harvest costs for commercial thinning and final harvest for softwood stands change by mean volume of the tree (m^3). The commercial thinning is assumed to be done with a cut-to-lenght system, with a harvester and a forwarder. The cost functions are based on the average productivity observed in Eastern Canada by FPInnovations (Meek 2016, personal communication).

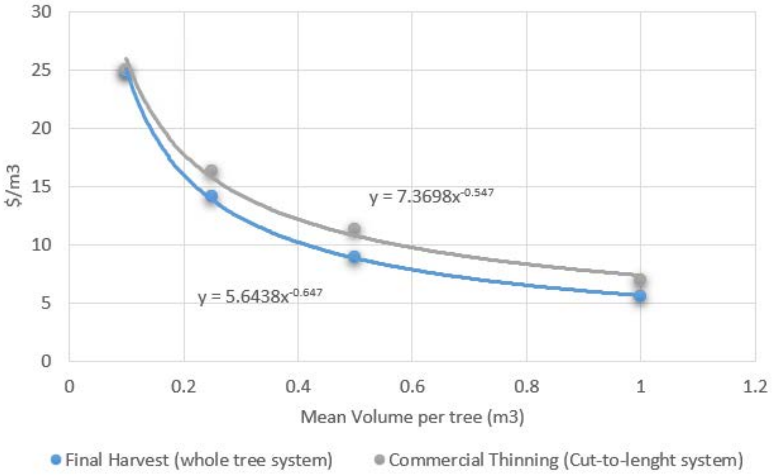


Figure 8: The road-side harvest (commercial thinning and final harvest) costs for softwood stands (Meek 2016, personal communication).

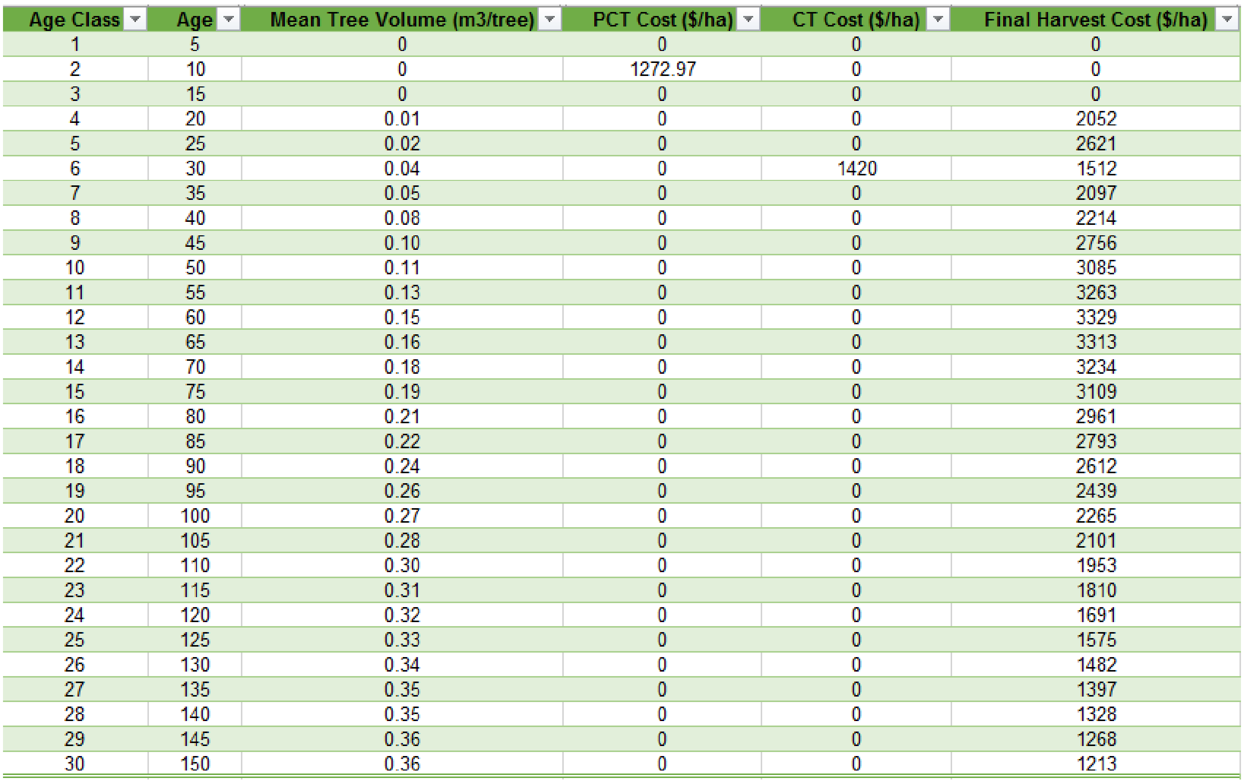
4.3. Final harvest

All merchantable trees in the stand are harvested in the final harvest. Moreover, non-merchantable trees are protected using careful logging techniques. The final harvest is done with a feller-buncher and a skidder. Harvest scheduling is determined by optimization and the harvested volume is equal to the volume of the remaining trees in the stand after the commercial thinning (see Table 7 and Figure 7). Stands are eligible to final harvest when the average tree volume is higher than 0.2 m^3 to ensure that the production of chips for pulpwood does not exceed 50 $\frac{kg}{m^3}$. Considering the mean tree volume estimation in Table 7, the minimum number of periods between two final harvests is 25 years for a stand treated with the above silviculture regime.

In order to estimate the stand yield after commercial thinning in relation with age-class, we used the empirical yield model from Natura-2014 (Pothier and Auger 2011, Auger 2017) which has been estimated by the Chief forester of Quebec by using forest inventory plots to initiate the model and modified with the assumptions presented in section 4.2. The results can be found in Table 7 and Figure 7.

Table 8 represents the treatment costs incurred between two consecutive final harvests. For example, if a stand is going to be harvested at age 60, there would be a PCT cost ($1272.97) at age 10, a CT cost ($1420) at age 30, and finally a final harvest cost at age 60 ($3329).

Table 8: Cultural treatment costs between two consecutive regeneration harvests



Age Class	Age	Mean Tree Volume (m3/tree)	PCT Cost ($/ha)	CT Cost ($/ha)	Final Harvest Cost ($/ha)
1	5	0	0	0	0
2	10	0	1272.97	0	0
3	15	0	0	0	0
4	20	0.01	0	0	2052
5	25	0.02	0	0	2621
6	30	0.04	0	1420	1512
7	35	0.05	0	0	2097
8	40	0.08	0	0	2214
9	45	0.10	0	0	2756
10	50	0.11	0	0	3085
11	55	0.13	0	0	3263
12	60	0.15	0	0	3329
13	65	0.16	0	0	3313
14	70	0.18	0	0	3234
15	75	0.19	0	0	3109
16	80	0.21	0	0	2961
17	85	0.22	0	0	2793
18	90	0.24	0	0	2612
19	95	0.26	0	0	2439
20	100	0.27	0	0	2265
21	105	0.28	0	0	2101
22	110	0.30	0	0	1953
23	115	0.31	0	0	1810
24	120	0.32	0	0	1691
25	125	0.33	0	0	1575
26	130	0.34	0	0	1482
27	135	0.35	0	0	1397
28	140	0.35	0	0	1328
29	145	0.36	0	0	1268
30	150	0.36	0	0	1213

4.4. Harvest revenue

The price of timber depends on different agents such as kind of tree, length, diameter, and quality. Quality is one of the chief agents of price change. In this research, a single price table was estimated for both woods from commercial thinnings and final harvests. Table 9 shows the average price ($) in relation to the average tree volume in the stand (m^3). The estimation has been done in two steps:

4.4.1. Market Search

We had a search for mill prices at www.prixbois.ca which is a wood marketing tool from the Fédération des producteurs forestiers du Québec. The tool provides road-site prices for logs while considering the trucking cost from the forest to the mill. In our analysis, we supposed that the logs are cut in the area of Causapcal (Quebec). Prixbois calculates the road distance and hauling cost from the forest to the mills. The analysis was initially done for 8, 12, 14, and 16-foot logs. However, according to the tool, there is a regional market only for 8- and 12-foot logs. Therefore, if only 8-foot logs are produced, the best price is from JDIrving (Kedgwick, NB) while if a combination of 8- and 12-foot logs are produced, the only mill accepting this assortment is Damabois (Cap Chat).

4.4.2. Bucking simulation

The unit price per unit volume ($\frac{\$}{m^3}$) depends on the number of logs of each sort that can be obtained in the bucking operation. This, in turn, depends on the tree size and taper. A bucking simulator was developed based on the taper equation from Ung et al. (2013) which calculates the number and size of each log based on the average taper profile, given species, and the DBH.

Selling prices ($\frac{\$}{m^3}$) of merchantable volume (from a stump height of 30 cm and a top diameter of 9cm, based on the volume equation from (Perron 2003)) were calculated for a range of DBH. Two scenarios were compared:

1) only producing 8-foot logs and selling them to JDIrving.
2) the production of a combinaison of 8- and 12-foot logs sold to Damabois.

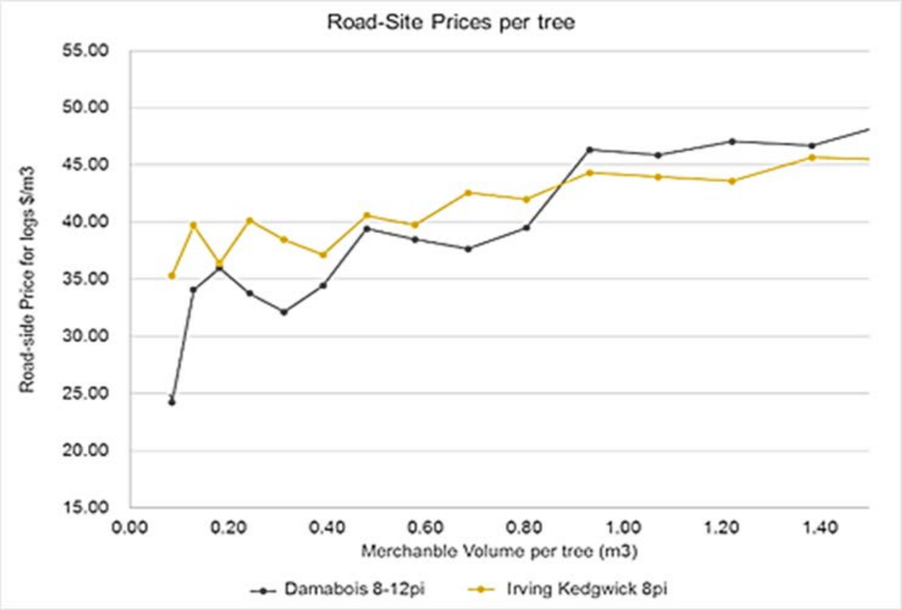


Figure 9: Comparison of selling price of merchantable volume with different length

Figure 9 illustrates that for trees with merchantable volume lower than 1 m^3 selling 8-foot logs to JDIrving is the most profitable option. So, in order to to obtain a good estimation for the price of wood in each age-class, a linear regression has been done to find the best-fitting line as it be found in Figure

10. Table 9 shows the income received from final harvest in relation to the age-class.

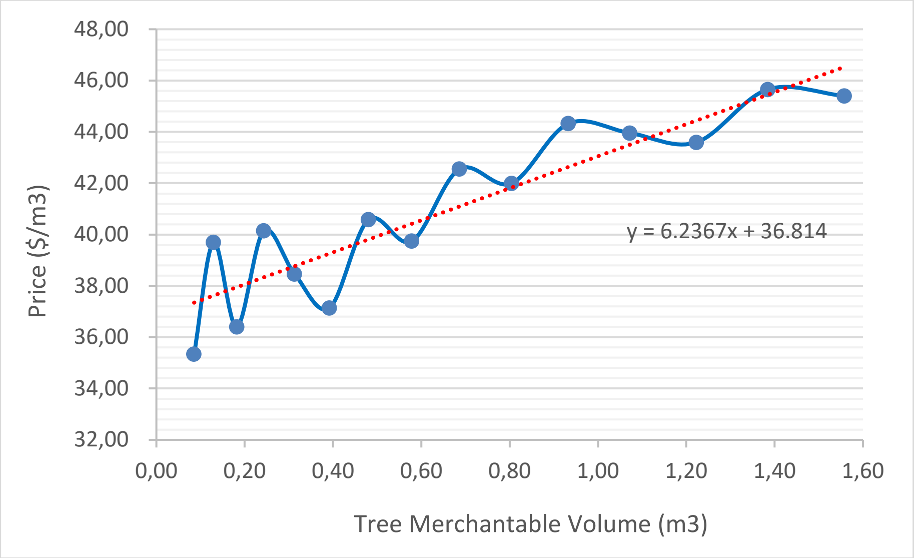
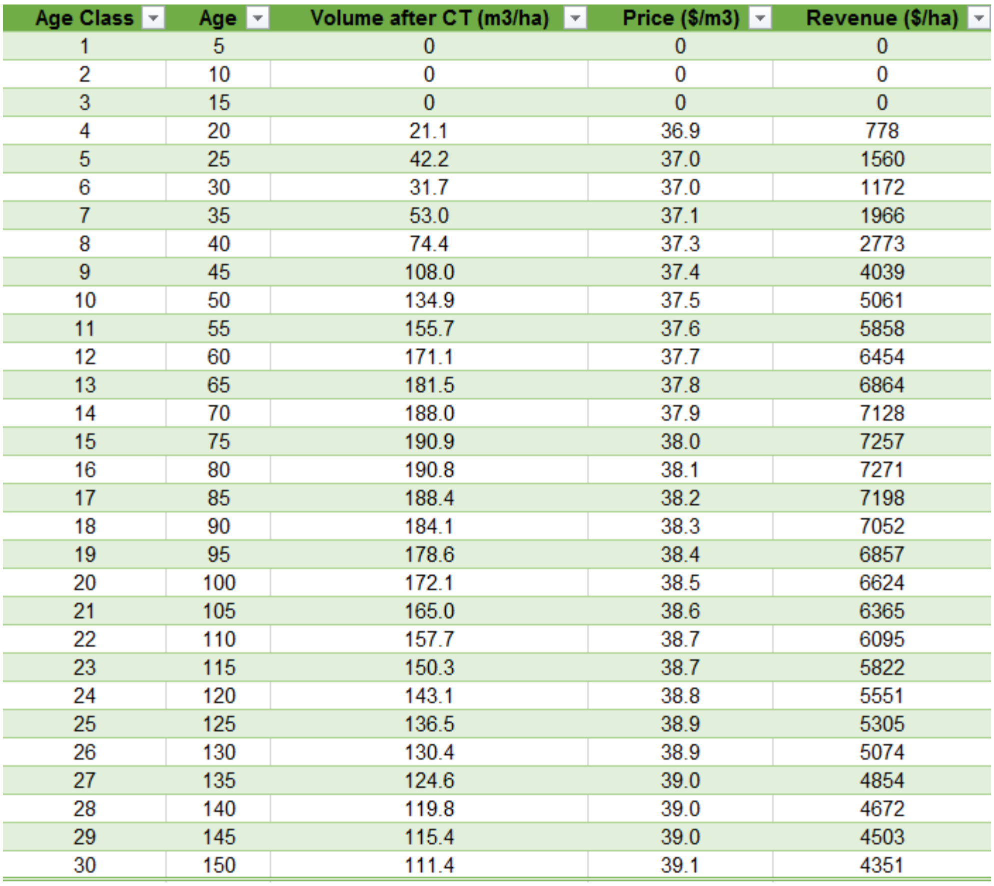


Figure 10: Price table for balsam fir in the Causapscal region

Table 9: Final harvest income at each age class



Age Class	Age	Volume after CT (m3/ha)	Price ($/m3)	Revenue ($/ha)
1	5	0	0	0
2	10	0	0	0
3	15	0	0	0
4	20	21.1	36.9	778
5	25	42.2	37.0	1560
6	30	31.7	37.0	1172
7	35	53.0	37.1	1966
8	40	74.4	37.3	2773
9	45	108.0	37.4	4039
10	50	134.9	37.5	5061
11	55	155.7	37.6	5858
12	60	171.1	37.7	6454
13	65	181.5	37.8	6864
14	70	188.0	37.9	7128
15	75	190.9	38.0	7257
16	80	190.8	38.1	7271
17	85	188.4	38.2	7198
18	90	184.1	38.3	7052
19	95	178.6	38.4	6857
20	100	172.1	38.5	6624
21	105	165.0	38.6	6365
22	110	157.7	38.7	6095
23	115	150.3	38.7	5822
24	120	143.1	38.8	5551
25	125	136.5	38.9	5305
26	130	130.4	38.9	5074
27	135	124.6	39.0	4854
28	140	119.8	39.0	4672
29	145	115.4	39.0	4503
30	150	111.4	39.1	4351

4.5. Forest age class distribution

In this study, we use a typical age class distribution from the Gaspesia Region; it is compiled from the estate model used by the Chief Forester of Quebec for the Forest Unit No. 11161 (Forestier en chef, Woodstock File from the Chief Forester of Quebec for Forest Unit 11161:). Figure 11 shows the percentage of total area of the Forest Unit No. 11161 in different age classes.

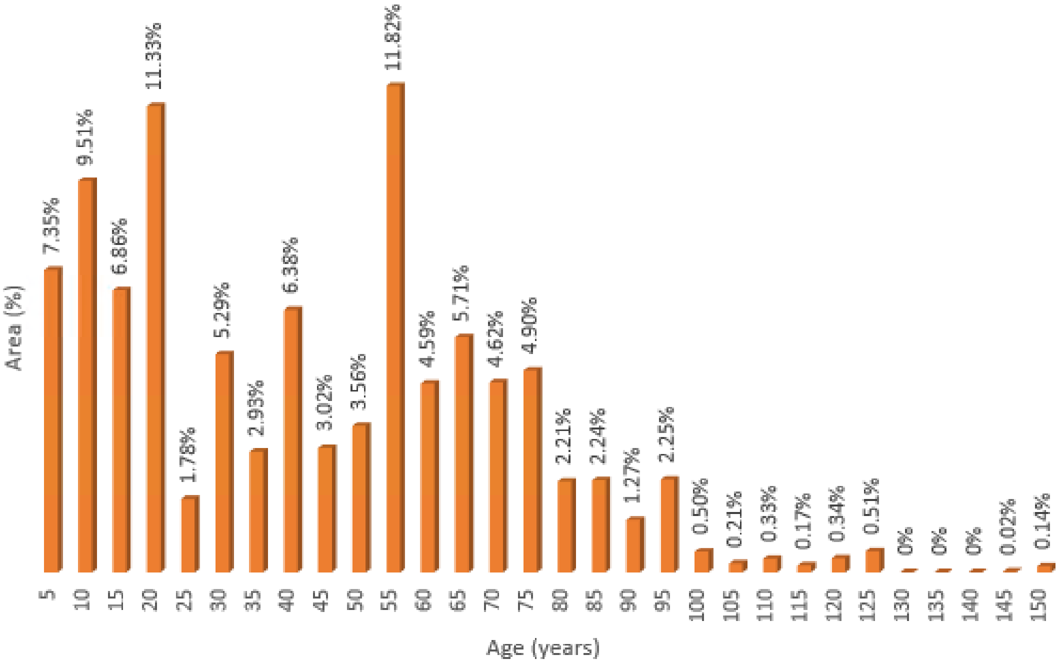
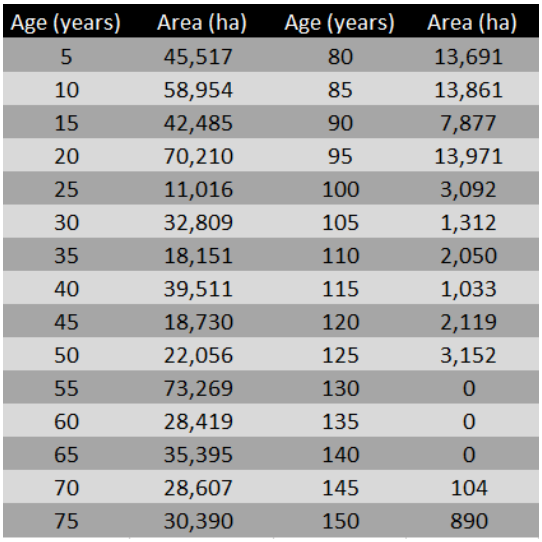


Figure 11: The age class distribution for Forest Unit No. 11161

The total area for the Forest Unit No. 11161 is 619,683 ha. In this study, it is assumed that there is a forest with 30 different age-classes. Therefore, we would have the area for each age class as given in Table 10.

Table 10: The age class distribution taken from Woodstock file from the Chief Forester of Quebec for Forest Unit 11161



Age (years)	Area (ha)	Age (years)	Area (ha)
5	45,517	80	13,691
10	58,954	85	13,861
15	42,485	90	7,877
20	70,210	95	13,971
25	11,016	100	3,092
30	32,809	105	1,312
35	18,151	110	2,050
40	39,511	115	1,033
45	18,730	120	2,119
50	22,056	125	3,152
55	73,269	130	0
60	28,419	135	0
65	35,395	140	0
70	28,607	145	104
75	30,390	150	890

4.6. Comparison of the models

In order to compare the results of the original and proposed models, the following assumptions are used:

- The length of each planning period is 5 years.
- The age-class distribution at the beginning of the planning horizon (A_i) can be found in Table 10.
- The minimum number of periods between regeneration harvests is 5 (z = 5) in other words when the stand is 25 years old.
- The planning horizon is 10 periods (50 years).
- PCT is implemented at age 10, in other words, two planning periods after the regeneration harvest and it would cost $1272.97.
- CT is implemented at age 30 (6 periods after the regeneration harvest) and a 50% crown thinning is prescribed.
- The cultural treatment (CT and final harvest) costs can be found in Table 8.
- The estimation of the unit price of the wood are presented in Table 9.
- Trees older than 150 years old are considered as dead with no value.
- The annual interest rate is assumed to be 1.5% and constant throughout the planning horizon.

- To calculate the net value per hectare of leaving areas regenerated in period i as ending inventory in period N (P'_{iN}), we calculate the value of standing trees which is assumed to be equal to the potential income from wood volume available in the stand. We used the following formula to calculate P'_{iN}:

$$P'_{iN} = \frac{(P_{iN} - C_{iN})V_{iN}}{\gamma^N}$$

P_{iN}	Unit price of volume harvested in period i and regeneration harvested in period N (put aside as ending inventory)
V_{iN}	Volume per hectare harvested in period i and regeneration harvested in period N (put aside as ending inventory)
C_{iN}	Cultural treatment costs per hectare in period i and regeneration harvested in period N (put aside as ending inventory)

According to the aforementioned assumptions and data, D_{ij} and E_{i10} values can be found in Table 11 and Table 12, respectively. Gray cells in Table 11 indicate the impossible values for D_{ij} based on the definition of z parameter.

Table 11: Values for parameters D_{ij} obtained from revised formulation

		Next Harvesting Period (Period *j*)									
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Most Recent Harvesting Period (Period *i*)	**-30**	0	0	0	0	0	0	0	0	0	0
	-29	2,919	0	0	0	0	0	0	0	0	0
	-28	3,009	2,715	0	0	0	0	0	0	0	0
	-27	3,111	2,799	2,526	0	0	0	0	0	0	0
	-26	3,216	2,894	2,604	2,349	0	0	0	0	0	0
	-25	3,341	2,991	2,692	2,422	2,185	0	0	0	0	0
	-24	3,469	3,108	2,783	2,504	2,253	2,033	0	0	0	0
	-23	3,591	3,227	2,891	2,588	2,329	2,096	1,891	0	0	0
	-22	3,732	3,340	3,002	2,690	2,408	2,167	1,950	1,759	0	0
	-21	3,854	3,472	3,107	2,793	2,502	2,240	2,016	1,814	1,636	0
	-20	3,966	3,585	3,229	2,890	2,598	2,328	2,084	1,875	1,687	1,522
	-19	4,055	3,689	3,335	3,004	2,689	2,417	2,165	1,938	1,744	1,569
	-18	4,110	3,772	3,432	3,102	2,794	2,501	2,248	2,014	1,803	1,622
	-17	4,130	3,823	3,509	3,192	2,886	2,600	2,327	2,091	1,874	1,677
	-16	4,098	3,842	3,556	3,264	2,970	2,684	2,418	2,164	1,945	1,743
	-15	4,010	3,812	3,574	3,308	3,036	2,763	2,497	2,249	2,013	1,810
	-14	3,859	3,730	3,546	3,325	3,077	2,825	2,570	2,323	2,093	1,873
	-13	3,623	3,590	3,470	3,299	3,093	2,863	2,628	2,391	2,161	1,947
	-12	3,304	3,370	3,339	3,228	3,068	2,877	2,663	2,444	2,224	2,010
	-11	2,907	3,073	3,135	3,106	3,003	2,854	2,676	2,477	2,274	2,069
	-10	2,414	2,705	2,859	2,916	2,889	2,793	2,655	2,490	2,304	2,115
	-9	1,838	2,246	2,516	2,659	2,713	2,688	2,598	2,470	2,316	2,144
	-8	1,194	1,710	2,089	2,340	2,474	2,524	2,500	2,417	2,298	2,154

	-7	519	1,111	1,591	1,943	2,177	2,301	2,348	2,326	2,248	2,137
	-6	-123	483	1,033	1,480	1,808	2,025	2,141	2,184	2,164	2,092
	-5	-546	-344	219	731	1,147	1,452	1,654	1,761	1,802	1,783
	-4	-987	-508	-320	204	680	1,067	1,350	1,539	1,639	1,676
	-3	0	-918	-473	-298	190	633	992	1,256	1,431	1,524
	-2	0	0	-854	-440	-277	177	589	923	1,168	1,331
	-1	0	0	0	-1,979	-1,593	-1,442	-1,020	-637	-326	-97
	0	0	0	0	0	-1,841	-1,482	-1,341	-949	-592	-303
	1	0	0	0	0	0	-1,712	-1,379	-1,248	-883	-551
	2	0	0	0	0	0	0	-1,593	-1,282	-1,160	-821
	3	0	0	0	0	0	0	0	-1,482	-1,193	-1,080
	4	0	0	0	0	0	0	0	0	-1,378	-1,110
	5	0	0	0	0	0	0	0	0	0	-1,282
	6	0	0	0	0	0	0	0	0	0	0
	7	0	0	0	0	0	0	0	0	0	0
	8	0	0	0	0	0	0	0	0	0	0
	9	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0

Table 12: Values for parameters E_{i10}

Period i	**-30**	**-29**	**-28**	**-27**	**-26**	**-25**	**-24**	**-23**	**-22**	**-21**
E_{i10}	0	0	0	0	0	0	0	0	0	0
Period i	**-20**	**-19**	**-18**	**-17**	**-16**	**-15**	**-14**	**-13**	**-12**	**-11**
E_{i10}	1522	1569	1622	1677	1743	1810	1873	1947	2010	2069
Period i	**-10**	**-9**	**-8**	**-7**	**-6**	**-5**	**-4**	**-3**	**-2**	**-1**
E_{i10}	2115	2144	2154	2137	2092	1783	1676	1524	1331	-97
Period i	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
E_{i10}	-303	-551	-821	-1080	-1110	-1282	-1332	-664	-618	0
Period i	**10**									
E_{i10}	0									

If we solve the problem with two different formulations, the original one and our suggested formulation, there would be a gap between the objective values. The objective value for original and proposed formulations are $1,351{,}867{,}304.26$ and $1{,}347{,}442{,}815.31$ respectively. The new NPV is 0.33% less than the original formulation—the original formulation overestimate the objective function, that is:

$$\frac{(1{,}347{,}442{,}815.31 - 1{,}351{,}867{,}304.26\,)}{1{,}351{,}867{,}304.26} = \mathbf{-0.33}\%$$

Although the gap between two models is small, the value is 4.42 million dollars. Clearly, the original model provides a erroneous solution.

Tables 13 and 14 indicate the outcomes of decision variables if we solve the problem with original formulation. Tables 15 and 16 demonstrate the new values for the decision variables if we use the proposed formulation.

According to the definition of the z parameter, some cells must not take a value (gray cells in Tables 13 and 15); however, as it can be found in the Table 13, the value of x_{-11} is equal to 45,517 which is impossible. The proposed formulation has modified this error and the value of x_{-11} is equal to 0 instead of 45,517. Therefore, in addition to the difference between the objective values, there is a difference between the decision variables of the compared models.

Note that it is hard to estimate the impact on general problems as the type of harvest operations included plays a role.

Table 13: Original formulation - Outcomes for decision variables x_{ij}

		Next Harvesting Period (Period j)									
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Most Recent Harvesting Period (Period i)	**-30**	0	0	0	0	0	0	0	0	890	0
	-29	104	0	0	0	0	0	0	0	0	0
	-28	0	0	0	0	0	0	0	0	0	0
	-27	0	0	0	0	0	0	0	0	0	0
	-26	0	0	0	0	0	0	0	0	0	0
	-25	3,152	0	0	0	0	0	0	0	0	0
	-24	2,119	0	0	0	0	0	0	0	0	0
	-23	1,033	0	0	0	0	0	0	0	0	0
	-22	2,050	0	0	0	0	0	0	0	0	0
	-21	1,312	0	0	0	0	0	0	0	0	0
	-20	3,092	0	0	0	0	0	0	0	0	0
	-19	13,971	0	0	0	0	0	0	0	0	0
	-18	7,877	0	0	0	0	0	0	0	0	0
	-17	13,861	0	0	0	0	0	0	0	0	0
	-16	13,691	0	0	0	0	0	0	0	0	0
	-15	30,390	0	0	0	0	0	0	0	0	0
	-14	28,607	0	0	0	0	0	0	0	0	0

	-13	35,395	0	0	0	0	0	0	0	0	0
	-12	28,419	0	0	0	0	0	0	0	0	0
	-11	73,269	0	0	0	0	0	0	0	0	0
	-10	0	0	0	0	0	0	0	0	22,056	0
	-9	0	0	0	0	0	0	0	0	18,730	0
	-8	0	0	0	0	0	0	0	0	39,511	0
	-7	0	0	0	0	0	0	0	0	18,151	0
	-6	0	0	0	0	0	0	0	0	32,809	0
	-5	0	0	0	0	0	0	0	0	11,016	0
	-4	0	0	0	0	0	0	0	0	0	0
	-3	0	0	0	0	0	0	0	0	0	0
	-2	0	0	0	0	0	0	0	0	0	0
	-1	45,517	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
	1	0	0	0	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	0	0	0
	4	0	0	0	0	0	0	0	0	0	0
	5	0	0	0	0	0	0	0	0	0	0
	6	0	0	0	0	0	0	0	0	0	0
	7	0	0	0	0	0	0	0	0	0	0
	8	0	0	0	0	0	0	0	0	0	0
	9	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0

Table 14: Original formulation - Outcomes for decision variables w_{i10}

Period i	**-30**	**-29**	**-28**	**-27**	**-26**	**-25**	**-24**	**-23**	**-22**	**-21**
w_{i10}	0	0	0	0	0	0	0	0	0	0
Period i	**-20**	**-19**	**-18**	**-17**	**-16**	**-15**	**-14**	**-13**	**-12**	**-11**
w_{i10}	0	0	0	0	0	0	0	0	0	0
Period i	**-10**	**-9**	**-8**	**-7**	**-6**	**-5**	**-4**	**-3**	**-2**	**-1**
w_{i10}	0	0	0	0	0	0	70,210	42,485	58,954	0
Period i	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
w_{i10}	0	258,344	0	0	0	0	0	0	0	143,162
Period i	**10**									
w_{i10}	0									

Table 15: Outcomes for decision variables x_{ij} for revised formulation

		Next Harvesting Period (Period *j*)									
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Most Recent Harvesting Period (Period *i*)	**-30**	0	0	0	0	0	0	0	0	890	0
	-29	104	0	0	0	0	0	0	0	0	0
	-28	0	0	0	0	0	0	0	0	0	0
	-27	0	0	0	0	0	0	0	0	0	0
	-26	0	0	0	0	0	0	0	0	0	0
	-25	3,152	0	0	0	0	0	0	0	0	0
	-24	2,119	0	0	0	0	0	0	0	0	0
	-23	1,033	0	0	0	0	0	0	0	0	0
	-22	2,050	0	0	0	0	0	0	0	0	0
	-21	1,312	0	0	0	0	0	0	0	0	0
	-20	3,092	0	0	0	0	0	0	0	0	0
	-19	13,971	0	0	0	0	0	0	0	0	0
	-18	7,877	0	0	0	0	0	0	0	0	0
	-17	13,861	0	0	0	0	0	0	0	0	0
	-16	13,691	0	0	0	0	0	0	0	0	0
	-15	30,390	0	0	0	0	0	0	0	0	0
	-14	28,607	0	0	0	0	0	0	0	0	0
	-13	35,395	0	0	0	0	0	0	0	0	0
	-12	28,419	0	0	0	0	0	0	0	0	0
	-11	73,269	0	0	0	0	0	0	0	0	0
	-10	0	0	0	0	0	0	0	0	22,056	0
	-9	0	0	0	0	0	0	0	0	18,730	0
	-8	0	0	0	0	0	0	0	0	39,511	0

	-7	0	0	0	0	0	0	0	0	18,151	0
	-6	0	0	0	0	0	0	0	0	32,809	0
	-5	0	0	0	0	0	0	0	0	11,016	0
	-4	0	0	0	0	0	0	0	0	0	70,210
	-3	0	0	0	0	0	0	0	0	0	42,485
	-2	0	0	0	0	0	0	0	0	0	58,954
	-1	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
	1	0	0	0	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	0	0	0
	4	0	0	0	0	0	0	0	0	0	0
	5	0	0	0	0	0	0	0	0	0	0
	6	0	0	0	0	0	0	0	0	0	0
	7	0	0	0	0	0	0	0	0	0	0
	8	0	0	0	0	0	0	0	0	0	0
	9	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0

Table 16: Outcomes for decision variables w_{i10} for revised formulation

Period i	**-30**	**-29**	**-28**	**-27**	**-26**	**-25**	**-24**	**-23**	**-22**	**-21**
w_{i10}	0	0	0	0	0	0	0	0	0	0
Period i	**-20**	**-19**	**-18**	**-17**	**-16**	**-15**	**-14**	**-13**	**-12**	**-11**
w_{i10}	0	0	0	0	0	0	0	0	0	0
Period i	**-10**	**-9**	**-8**	**-7**	**-6**	**-5**	**-4**	**-3**	**-2**	**-1**
w_{i10}	0	0	0	0	0	0	0	0	0	45,517
Period i	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
w_{i10}	0	258,344	0	0	0	0	0	0	0	143,162
Period i	**10**									
w_{i10}	171,649									

5. Concluding remarks

Several models are considered as the basic tools of strategic forest planning by most foresters because they examine the long-term consequences of forest-management inputs (Gunn 2007). In this paper, we focused on Model II and how the minimum number of periods between regeneration harvests, i.e, z parameter, leads to modeling mistakes. The first mistake appears in the first set of area constraints where additional decision variables are included. These variables have no contribution to the objective function; however, in specific contexts, they could take nonzero values. The second mistake is when computing the D_{ij} parameter where overlaps and impossible timber flows could be found in the formulation.

As far as we know in the literature, these mistakes have not been identified and presented by any researcher, since Model II was suggested by Johnson and Scheurman (1977). An illustrative example is given with realistic parameters to verify the modeling errors. data case study from a real forest also supports the findings. Some well-known software, such as Woodstock, FORPLAN, TigerMoth, and SilviLab are based on variants of Model II formulation. We have not verified that these applications use the formulation that was published in the original article by Jonhson and Scheurman (1977).

Furthermore, we have not verified that the models referenced to Jonhson and Scheurman (1977) included the mistakes or that the mistakes were a publication error. It is, however, important to provide information to avoid that the mistakes are implemented in new models and can be corrected, if necessary, in old models. It can be very difficult to identify these errors in particular if the models are only used for computing forest NPV values and it is not necessary to study the detailed harvest plan where the additional variables can be identified.

References

Anthony, R.N., 1965. Planning and control systems; a framework for analysis. Harvard Business School Press, Boston, MA.

Backéus, S., Wikström, P., Lämås, T., 2005. A model for regional analysis of carbon sequestration and timber production. Forest Ecology and Management 216, 28–40.

Auger, I., 2017. Guide d'utilisation du simulateur de croissance forestière Natura-2014 sur Capsis Version 1.0. Gouvernement du Québec, Ministère des Forêts, de la Faune et des Parcs, Direction de la recherche forestière, p. 38.

Forestier en chef. Woodstock File from the Chief Forester of Quebec for Forest Unit 11161: PC_4859_U11161_4FF_V07.are. Available online at https://forestierenchef.gouv.qc.ca/; last accessed Nov. 5, 2019.

Corrigan, E., Nieuwenhuis, M., 2017. Using goal-programming to model the effect of stakeholder determined policy and industry changes on the future management of and ecosystem services provision by Irelands Western Peatland forests. Sustainability 9, 1–18.

D'Amours, S., Ouhimmou, M., Audy, J., Feng, Y., 2016. Forest Value Chain Optimization and Sustainability. CRC Press, Canada.

De Franceschi, J.P., Boylen, D.M., 1987. Costs and benefits of precommercial thinning: an overview and case study, in: Proceedings of the IUFRO International Interdivisional Conference on Thinning, Part II, Moscow and Riga, pp. 252–268.

Diaz-Balteiro, L., Bertomeu, M., Bertomeu, M., 2009. Optimal harvest scheduling in eucalyptus plantations: A case study in Galicia (Spain). Forest Policy and Economics 11, 548–554.

Drucker, P.F., 1995. The information executives truly need. Harvard Business Review 73, 54–63.

Forest and Range, 2004. National learning center for private forest and range landowners. pre commercial thinning. Available online at http://forestandrange.org/southernpine/magement/thinning/precomm.html; last accessed Aug. 23, 2017.

Forest Practices Branch, Ministry of Forests, British Columbia, Canada, 1999. Guidelines for Commercial Thinning. Ministry of Forests, Forest Practices Branch, Victoria, British Columbia Canada.

Forêts, Faune et Parcs Gouvernement du Québec, 2003. Pre-commercial thinning. Available online at https://mffp.gouv.qc.ca/english/forest/understanding/understanding-thinning.jsp; last accessed Aug. 23, 2017.

Garcia, O., 1990. Linear programming and related approaches in forest planning. New Zealand Journal of Forestry Science 20, 307–331.

Gautam, S., LeBel, L., Beaudoin, D., 2017. A hierarchical planning system to assess the impact of operational-level flexibility on long-term wood supply. Canadian Journal of Forest Research 47, 424–432.

Gouvernement du Québec. 2019 Valeur des traitements sylvicoles non commerciaux pour l'année financière 2019-2020 - Forêt publique. Ministère des Forêts de la Faune et des Parcs, Bureau de mise en marché des bois , Direction des évaluations économiques et financières.

Gunn, E.A., 2005. Sustainable forest management: Control, adaptive management, hierarchical planning. Soil and Water 2, 8.

Gunn, E.A., 2007. Models for Strategic Forest Management. Springer, Boston, US. Hedin, I.B., 1982. Five case studies of precommercial thinning in British Columbia and Alberta. Technical Report 62. 29 p. Forest Engineering Research Institute of Canada (FERIC).

Hennigar, C.R., MacLean, D.A., Amos-Binks, L.J., 2008. A novel approach to optimize management strategies for carbon stored in both forests and wood products. Forest Ecology and Management 256, 786–797.

John, R., Tóth, S., 2015. Spatially explicit forest harvest scheduling with difference equations. Annals of Operations Research 232, 235–257.

Johnson, K.N., Scheurman, H.L., 1977. Techniques for prescribing optimal timber harvest and investment under different objectives:discussion and synthesis. Society of American Foresters, Washington, US.

Laflèche, V. and Tremblay, S., 2008. Résultats de cinq ans de la mesure des effets réels du traitement d'éclaircie précommerciale de peuplements à dominance résineuse. Mémoire de recherche forestière. Gouvernement du Québec, Ministère des Ressources naturelles et de la Faune, Direction de la recherche forestière, Québec.

Latta, G.S., Adams, D., Alig, R., White, E., 2011. Simulated effects of mandatory versus voluntary participation in private forest carbon offset markets in the United States. Journal of Forest Economics 17, 127–141.

Latta, G.S., Adams, D., Bell, K.P., Kline, J.D., 2016. Evaluating land-use and private forest management responses to a potential forest carbon offset sales program in Western Oregon (USA). Forest Policy and Economics 65, 1–8.

Martin, A.B., Richards, E., Gunn, E., 2017. Comparing the efficacy of linear programming models I and II for spatial strategic forest management. Canadian Journal of Forest Research 47, 16–27.

Meek, P., 2016, Personal communication.

Nanang, D.M., Hauer, G., 2006. Estimating the costs of overlapping tenure constraints: a case study in Northern Alberta, Canada. Forest Policy and Economics 8, 610–624.

Nanang, D.M., Hauer, G., 2008. Integrating a random utility model for non-timber forest users into a strategic forest planning model. Journal of Forest Economics 14, 133–153.

Pelletier, G., and Pitt, D.G., 2008. Silvicultural responses of two spruce plantations to midrotation commercial thinning in New Brunswick. Canadian Journal of Forest Research 38 (4), 851-867.

Perron, J.-Y., 2003. Tarif de cubage général - Volume marchand brut. Ministère des Ressources naturelles, de la Faune et des Parcs, Forêt Québec, Direction des inventaires forestiers, Québec.

Pothier, D., and Auger, I., 2011. NATURA-2009: un modèle de prévision de la croissance à l'échelle du peuplement pour les forêts du Québec. Gouvernement du Québec, Ministère des Ressources naturelles et de la Faune, Direction de la recherche forestière. Québec, p. 56.

Remsoft, 1994. Linear programming extensions to the woodstock forest modeling system. Available online at http://www.remsoft.com/docs/library/wk_lp.pdf; last accessed Aug. 23, 2017.

Ung, C., Guo, X., and Fortin, M., 2013 Canadian national taper models. Forestry Chronicle 89 (2), 211-224.