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The Combinatorial Bid Construction Problem with Stochastic Prices for Transportation Services Procurement

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Abstract. In combinatorial auctions for the procurement of transportation services, each carrier has to determine the set of profitable contracts to bid on and the associated ask prices. This is known as the Bid Construction Problem (BCP). Our paper adresses a BCP with stochastic clearing prices taking into account uncertainty on other competing carriers' offers. Contracts' selection and pricing decisions are integrated to generate multiple combinatorial bids. Both exact and heuristic methods are proposed to solve the problem. Our computational results demonstrate that exact solutions can be obtained on instances with up to 50 contracts. Our heuristic obtains most of these optimal solutions in a fraction of the run time, and provides competitive solutions for the remaining instances.

Keywords. Combinatorial auctions, uncertainty, transportation procurement, bid construction problem, stochastic programming, non-linear model.

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1 Introduction

To remain competitive, carriers need to optimize their transport network and obtain new profitable contracts by offering attractive rates for their customers. There are two settings available for carriers to optimize their network (Gansterer and Hartl, 2018). The first one is known as collaborative transportation planning. It involves small- and medium-sized carriers forming a coalition. They exchange, often, less-than-truckload (LTL) contracts to collect shipments by sharing transportation capacities (Lafkihi et al., 2019; Lai et al., 2019). The objective is to maximize the coalition global profit and reduce transportation costs (Ergun et al., 2007; Özener et al., 2011; Wang and Kopfer, 2014). The second setting, which is considered in this paper, is more common and non-collaborative: carriers compete in auction mechanisms by bidding on transportation contracts requested by shippers. Auction mechanisms are generally used for the procurement of full truckload (TL) services resulting in long-term contracts between carriers and shippers. According to Nandiraju and Regan (2005) and Sheffi (2004), shippers prefer, in most cases, mid- to long-term transportation contracts in order to avoid future prices variation and to secure transportation capacity availability.

Combinatorial auctions (CAs), in particular, are commonly used for TL transportation services procurement. In these auctions, bids on combinations of items are allowed so that either all the items in the combination are allocated – if the bid is won – or nothing at all (Caplice and Sheffi, 2006; Abrache et al., 2007; Nisan et al., 2007). Shippers act as auctioneers seeking transportation services between several origin and destination locations, and carriers, acting as bidders, compete by submitting bids on shippers' transportation contracts. A number of decisional problems need be addressed during the CA process. Our paper focuses on the so-called Bid Construction Problem (BCP) also known as the bid generation problem.

A BCP must be solved by each carrier participating in the auction. When combinatorial bidding is allowed, it consists in determining the subsets of auctioned contracts to bid on and the price to ask for serving all the contracts in each bid. The majority of published papers on the BCP for the procurement of transportation services do not explicitly address the bids' pricing problem. They rather focus on determining the set of profitable contracts to incorporate in the carrier's pre-existing network. The pricing part is either ignored or simplistically handled a posteriori once the contracts to bid on are determined (Song and Regan (2003, 2005); Wang and Xia (2005); Lee et al. (2007); Ben Othmane et al. (2019); Hammami et al. (2019)). Moreover, almost all research on the BCP for combinatorial TL transportation procurement auctions consider a deterministic context (Hammami et al., 2019; Ben Othmane et al., 2019; Rekik et al., 2017; Lee et al., 2007; Chen et al., 2009; Chang, 2009; Song and Regan, 2005; Wang and Xia, 2005).

To the best of our knowledge, only Triki et al. (2014) consider a stochastic BCP where bids' clearing prices are assumed uncertain. The authors propose a mathematical model in which all the package bids are enumerated and the corresponding clearing prices are defined as random variables with known probability distributions. The proposed formulation generates only a single combinatorial bid and uses chance constraints to model clearing price uncertainty. The authors report that their model was not tractable and propose two heuristics to solve the problem.

To the best of our knowledge, no exact method for the BCP with stochastic clearing prices exists. Integrating contracts selection and pricing problems to simultaneously generate multiple bids under prices uncertainty has never been addressed in the literature neither. Our paper aims to fill these gaps. It proposes a novel non-enumerative solution approach that enables generating both single and multiple bids depending on auction rules and the carrier's preferences. The proposed solution approach can either act as an exact or a heuristic method depending on parameters tuning. The heuristic form includes two phases. The first phase, referred to as the Contract Selection Problem (CSP), aims to reduce the number of contracts to consider when solving the integrated selection and pricing problem of the second phase by preselecting promising contracts based on their uncertain clearing prices. Uncertainty in CSP is modeled by considering the equivalent deterministic problem of a scenario-based two-stage stochastic model. In the second phase, combinatorial bids composition and associated ask prices are optimally and simultaneously determined using a non-linear model. This problem is referred to as the Contract Selection and Pricing Problem (CSPP). Uncertainty in CSPP is modeled using chance constraints that, contrarily to Triki et al. (2014), cannot be linearized given that the contracts covered by the bids are not known in advance (not enumerated). The first phase, which makes the approach a heuristic one, is not mandatory and can be ignored when the number of auctioned contracts or the contracts in which the carrier is interested is relatively small. In such case, the problem can be solved using only the exact algorithm proposed for the CSPP.

Our experimental study reports the results obtained for the proposed solution approach in both its exact (CSPP only) and heuristic (CSP + CSPP) forms. The heuristic required on average less than one hour to reach the best solutions. The exact algorithm proved optimality for several instances and the average time required to find either the optimal or the best solutions is less than four hours on average. For 72% of the instances, the two-phase heuristic identified either the same or better solutions than the exact approach. For the remaining instances, the loss in solution quality yielded by the heuristic does not exceed 0.68% on average while it offered large savings in computing times.

The remainder of this paper is organized as follows. Section 2 defines the problem and how the uncertainty is modeled. In Section 3, we describe our overall solution algorithm. Section 4 describes the two-stage stochastic model and the two solution methods proposed for the CSP. In Section 5, we present the mathematical model and the exact solution algorithm introduced for the CSPP. Extensive computational results are reported and discussed in Section 6. Section 7 concludes the paper.

2 Problem definition

We consider a BCP with stochastic clearing prices where the carrier seeks to maximize its expected net profit by bidding on new contracts while taking into account its existing commitments, its fleet capacity, and a number of operational constraints. The carrier has then to decide on the number of bids to submit, the contracts covered by each bid, and the associated ask price knowing that other competing carriers are participating in the auction and may propose better bids. On one hand, the carrier has to increase its chances of winning the bids with attractive prices for the shipper compared to its competitors. On the other hand, it aims to achieve large profits. The problem is even more complex because of the uncertainty characterizing clearing prices: no carrier knows in advance the prices at which contracts will be allocated when the auction is cleared.

The stochastic BCP addressed in this paper considers two sets of contracts: a set $K_e = \{1, ..., |K_e|\}$ containing the carrier's existing contracts (contracts it must serve due to previous commitments), and a set $K_n = \{|K_e|+1, ..., |K_e|+|K_n|\}$ composed of the newly auctioned contracts. The set of all contracts is denoted by $K = K_e \cup K_n$. Each contract $k \in K$ is defined by an origin and a destination, denoted respectively by o_k and d_k. Given the TL context, all the volumes picked up at an origin location of a given contract must be driven directly to its destination.

The BCP described above can be represented using a directed graph $G = (V, A)$ in which V denotes the set of vertices and A represents the set of arcs. $V = O \cup D \cup \{0, N\}$ where 0 and N are respectively the start and end point of each route and represent the depot. Set $O = \{o_k; k \in K\}$ ($D = \{d_k; k \in K\}$) denotes the origins (destinations) associated with contracts in K. The set of arcs is defined as: $A = A_e \cup \{(o_k, d_k), k \in K\}$, where A_e represents the empty traveling arcs as follows: $A_e = \{(0, i) : i \in O\} \cup \{(i, j) : i = d_k \in D, j = o_{k'} \in O : k, k' \in K, k \neq k'\} \cup \{(j, N), j \in D\}$. To each arc $(i, j) \in A$ are associated a traveling cost c_{ij} and a traveling time t_{ij}.

A first-price combinatorial auction is considered implying that if a bid is won, then the carrier must be paid exactly the price asked in this bid. The carrier's expected profit is thus computed as the difference between the revenue for ensuring existing and newly selected contracts and the total cost for serving them. The revenue corresponds to the prices already guaranteed and known for serving the existing contracts plus the prices asked in the generated bids. The latter are determined taking into account uncertainty on auctioned contracts' clearing prices. Formally, a price p_k is associated with each existing contract $k \in K_e$ and is assumed to be known with certainty. For newly auctioned contracts, clearing prices are random variables $\tilde{p}_k, k \in K_n$ and, as in Triki et al. (2014), are assumed to follow a normal distribution $N(\overline{p_k}, \sigma_k^2)$.

We assume that the carrier has a homogeneous fleet and each contract can be serviced by any vehicle $l \in L = \{1, ..., |L|\}$. Each vehicle route must start and end at the depot. Each vehicle has a maximum route duration T_{max} and a fixed cost f.

When solving the BCP, the carrier is offered the possibility to generate a single bid or multiple OR bids. OR bids imply that there is no guarantee that all the submitted bids will be allocated to the carrier (Nisan, 2010). However, if all these bids are won, the carrier must have the internal capacity to serve them. OR bids are assumed to not overlap avoiding the case where the same contract is present in two OR bids. We also consider the case where the number of generated OR bids is limited. The maximum number of generated bids is denoted by γ. Similarly, the number of contracts submitted in a bid could be restricted to a given value denoted by η. As reported in Rekik et al. (2017), such constraints may be imposed by the shipper or considered by the carrier to increase its chances to win when multiple OR bids are submitted.

3 Solution methods: overview

As already mentioned, the proposed solution method can either act as an exact method or a heuristic depending on whether the CSP phase is performed or not. In what follows, we describe the more general two-phase heuristic approach. One should keep in mind that the exact approach corresponds to the CSPP (phase 2) with the whole set of auctioned contracts. In the first phase, we address the CSP to preselect a subset of profitable contracts $K_n^* \subseteq K_n$ to bid on without tackling the pricing problem. A restricted CSPP is then solved at the second phase considering only the contracts in K_n^* to generate the final bids. In what follows, we briefly explain the modeling and solution approaches proposed for each phase. Details and formal descriptions are provided in Section 4 for the CSP and in Section 5 for the CSPP.

A scenario-based two-stage stochastic model is proposed to formulate the CSP with stochastic prices. The objective is to maximize the carrier's expected profit taking into account routing constraints. It is assumed that the probability distributions of the uncertain clearing prices are available and can thus be used to generate plausible future scenarios. A representative sample Ω of plausible scenarios is then determined to obtain the equivalent deterministic model, referred to in the following as M_{CSP}.
Two solution methods are proposed to solve M_{CSP}. The first method consists in running the branch-and-cut procedure of a commercial solver on M_{CSP}. The second approach decomposes the problem into a series of deterministic sub-problems, one sub-problem for each scenario. Based on the output of each sub-problem, a winning ratio is computed for each auctioned contract $k \in K_n$ to measure the percentage of times it was selected in the different scenarios. A deterministic CSP is then considered in which contracts with larger winning ratios are prioritized. The resulting deterministic model is solved by branch-and-cut to determine the set of preselected contracts K_n^*.

In the second phase, package bids and associated ask prices are simultaneously determined by solving the CSPP. A non-linear mathematical formulation is proposed where uncertainty on clearing prices is modeled with probabilistic constraints. The latter are defined for the generated bids to ensure that the probability of the bid ask price (a decision variable) being lower than the bid clearing price is greater than a given value (to be specified by the carrier depending on its risk tolerance). As will be explained in Section 5, probabilistic constraints cannot be linearized as in Triki et al. (2014) given that bids composition are not known in advance. However, if the contracts covered by the bid are known, linearizing such constraints is simple. Based on this observation, we propose an iterative exact solution algorithm to solve the CSPP. More details are given in Section 5.

4 Contract selection problem

Section 4.1 presents the two-stage stochastic model proposed for the CSP. In Section 4.2, we describe the heuristic proposed to solve it.

4.1 Two-stage stochastic model

We first present a deterministic model for the CSP in which contracts clearing prices p_k are assumed to be known with certainty. We consider the following decision variables:

- $y_k = 1$ if contract k is selected, $y_k = 0$, otherwise; $\forall k \in K_n$.
- $x_{ij}^l = 1$ if arc $(i,j) \in A$ is traversed using vehicle $l \in L$, $x_{ij}^l = 0$, otherwise; $\forall (i,j) \in A, l \in L$;
- $B_i \geq 0$ and integer indicating the order of visit of node $i \in V$.

The deterministic CSP is formulated as follows:

$$\max \sum_{k \in K_e} p_k + \sum_{k \in K_n} p_k y_k - \sum_{l \in L} \sum_{(i,j) \in A} c_{ij} x_{ij}^l - \sum_{l \in L} \sum_{j \in O} f x_{0j}^l \tag{1}$$

$$\text{s.t.} \quad \sum_{l \in L} x^l_{o_k d_k} = y_k \qquad \forall k \in K_n \tag{2}$$

$$\sum_{l \in L} x^l_{o_k d_k} = 1 \qquad \forall k \in K_e \tag{3}$$

$$x^l_{o_k d_k} \leq \sum_{j \in O} x^l_{0j} \leq 1 \qquad \forall l \in L, k \in K \tag{4}$$

$$x^l_{o_k d_k} \leq \sum_{i \in D} x^l_{i,N} \leq 1 \qquad \forall l \in L, k \in K \tag{5}$$

$$x^l_{o_k d_k} \leq \sum_{k' \in K, k' \leq k} x^{(l-1)}_{o_{k'} d_{k'}} \qquad \forall k \in K, l \in L \setminus \{1\} \tag{6}$$

$$\sum_{j \in O} x^l_{0j} \leq \sum_{j \in O} x^{(l-1)s}_{0j} \qquad \forall l \in L \setminus \{1\} \tag{7}$$

$$x^1_{o_1 d_1} = 1 \tag{8}$$

$$\sum_{j:(j,i) \in A} x^l_{ji} = \sum_{j:(i,j) \in A} x^l_{ij} \qquad \forall l \in L, i \in V \setminus \{0, N\} \tag{9}$$

$$\sum_{(i,j) \in A} t_{ij} x^l_{ij} \leq T_{max} \qquad \forall l \in L, \tag{10}$$

$$B_i + \sum_{l \in L} x^l_{ij} - |V| \left(1 - \sum_{l \in L} x^l_{ij}\right) \leq B_j \qquad \forall (i,j) \in A \tag{11}$$

$$B_0 = 0 \tag{12}$$

$$x^l_{ij} \in \{0, 1\} \qquad \forall (i,j) \in A, l \in L \tag{13}$$

$$0 \leq B_i \leq |V| \qquad \forall i \in V \tag{14}$$

$$y_k \in \{0, 1\} \qquad \forall k \in K_n. \tag{15}$$

The objective function (1) maximizes the carrier's net profit defined as the difference between the revenues collected from serving the contracts, the fixed costs associated with the use of the vehicles, and the traveling costs. Constraints (2) ensure that if selected, a new contract must be served by a single vehicle. These constraints also ensure the link between y and x^l_{ij} variables. Constraints (3) ensure that all pre-existing contracts are served exactly once. Constraints (4) and (5) imply that each route starts and ends at the depot. Constraints (6)–(8) are symmetry breaking constraints. Constraints (6) impose an order for serving contracts. Constraints (7) impose that vehicle l is used if and only if vehicle $l-1$ is already used. Constraint (8) arbitrarily and without loss of generality forces the first pre-existing contract to be assigned to the lowest indexed vehicle. Network connectivity is ensured via constraints (9). Constraints (10) impose maximum tour length. Constraints (11) impose an order for visiting nodes. Constraints (12) ensure that each route starts from the depot. Constraints (13)–(15) define the domain of the decision variables.

Contracts clearing prices $\tilde{p}_k, k \in K_n$ are not known. We propose hereafter a two-stage stochastic model in which the first-stage variables are the contract selection variables (y_k) and the recourse variables are the routing decisions (x^l_{ij} and B_i). As mentioned in Section 2, it is assumed that the probability distributions of the clearing prices are available and can thus be used to generate plausible future scenarios. More precisely, $\tilde{p}_k$ follows a normal distribution $N(\overline{p_k}, \sigma_k^2)$. A scenario is defined as a compound event which is the result of the juxtaposition of random processes related to the different contracts. Given these stochastic process and using a Monte Carlo procedure, we define a scenario by generating independent pseudo-random numbers uniformly distributed on the interval $[0,1]$, and use them to compute the inverse of the normal distributions of all the random variables. All the generated scenarios are equiprobable.

Due to the fact that all random variables have continuous probability distributions, the number of plausible scenarios is infinite. Thus, the Monte Carlo procedure is employed to generate samples of future scenarios, denoted Ω. For a scenario $\omega \in \Omega$, the instance of the contracts clearing prices is denoted $p^\omega_k, \forall k \in K_n$. The equivalent deterministic problem associated with the two-stage stochastic model for a sample Ω is modeled as:

$$M_{CSP} : \max_{y,x} \sum_{k \in K_e} p_k + \frac{1}{|\Omega|} \sum_{\omega \in \Omega} \left(\sum_{k \in K_n} p^\omega_k y_k - \sum_{l \in L} \sum_{(i,j) \in A} c_{ij} x^{l\omega}_{ij} - \sum_{l \in L} \sum_{j \in O} f x^{l\omega}_{0j} \right) \tag{16}$$

$$
\begin{aligned}
\text{s.t.}\ & \sum_{l \in L} x^{l\omega}_{o_k d_k} = y_k && \forall k \in K_n, \omega \in \Omega && (17)\\
& \sum_{l \in L} x^{l\omega}_{o_k d_k} = 1 && \forall k \in K_e, \omega \in \Omega && (18)\\
& x^{l\omega}_{o_k d_k} \le \sum_{j \in O} x^{l\omega}_{0j} \le 1 && \forall l \in L, k \in K, \omega \in \Omega && (19)\\
& x^{l\omega}_{o_k d_k} \le \sum_{i \in D} x^{l\omega}_{i,N} \le 1 && \forall l \in L, k \in K, \omega \in \Omega && (20)\\
& x^{l\omega}_{o_k d_k} \le \sum_{k' \in K, k' \le k} x^{(l-1)\omega}_{o_{k'} d_{k'}} && \forall k \in K, l \in L \setminus \{1\}, \omega \in \Omega && (21)\\
& \sum_{j \in O} x^{l\omega}_{0j} \le \sum_{j \in O} x^{(l-1)s}_{0j} && \forall l \in L \setminus \{1\}, \omega \in \Omega && (22)\\
& x^{1\omega}_{o_1 d_1} = 1 && \forall \omega \in \Omega && (23)\\
& \sum_{j:(j,i) \in A} x^{l\omega}_{ji} = \sum_{j:(i,j) \in A} x^{l\omega}_{ij} && \forall l \in L, i \in V \setminus \{0, N\}, \omega \in \Omega && (24)\\
& \sum_{(i,j) \in A} t_{ij} x^{l\omega}_{ij} \le T_{max} && \forall l \in L, \omega \in \Omega && (25)\\
& B^{\omega}_i + \sum_{l \in L} x^{l\omega}_{ij} - |V| \left(1 - \sum_{l \in L} x^{l\omega}_{ij}\right) \le B^{\omega}_j && \forall (i,j) \in A, \omega \in \Omega && (26)\\
& B^{\omega}_0 = 0 && \forall \omega \in \Omega && (27)\\
& x^{l\omega}_{ij} \in \{0,1\} && \forall (i,j) \in A, l \in L, \omega \in \Omega && (28)\\
& 0 \le B^{\omega}_i \le |V| && \forall i \in V, \omega \in \Omega && (29)\\
& y_k \in \{0,1\} && \forall k \in K_n. && (30)
\end{aligned}
$$

By solving M_{CSP}, we obtain the set of profitable contracts to bid on as: $K^*_n = \{k \in K_n : \overline{y_k} = 1\}$ where the vector $(\overline{y_k})_{k \in K_n}$ represents the values of the first-stage variable y in the final solution.

Observe that generated samples should be large enough to be representative of prices' uncertainty without impacting the solvability of the associated model. When the number of scenarios in Ω is relatively small, the equivalent deterministic problem formulated with M_{CSP} can be solved to optimality by a branch-and-cut procedure available in commercial solvers. However when the sample size gets large, M_{CSP} becomes intractable. In the following, we propose a heuristic approach to tackle it.

4.2 Heuristic solution approach

In the first step, $|\Omega|$ CSP deterministic models (1)–(15), denoted $M_1^\omega, \omega \in \Omega$, are solved in parallel by branch-and-cut. When solving model $M_1^\omega, \omega \in \Omega$, the price p_k considered for a contract $k \in K_n$ is the one generated in scenario ω. The objective here is to determine for each scenario $\omega \in \Omega$ the set of profitable contracts to bid on. Based on these solutions, we determine for each contract $k \in K_n$, a winning ratio $\theta_k \in [0, 1]$ as follows:

$$\theta_k = \frac{\sum_{\omega \in \Omega} y_k^*(\omega)}{|\Omega|},$$

where $y^*(\omega)$ is the optimal solution value for the contract selection variables of model $M_1^\omega, \omega \in \Omega$. The winning ratio gives the percentage of times a contract $k \in K_n$ was selected for all the scenarios in Ω.

The second step of the proposed heuristic considers a deterministic model, denoted M_2, including the same constraints as the deterministic CSP (2)–(15) but with an objective function that prioritizes the selection of auctioned contracts with higher winning ratios:

$$\max \sum_{k \in K_e} p_k + \sum_{l \in L} \sum_{k \in K_n} (1 + \theta_k)\overline{p_k} x^l_{o_k d_k} - \sum_{l \in L} \sum_{(i,j) \in A} c_{ij} x^l_{ij} - \sum_{l \in L} \sum_{j \in O} f x^l_{0j}. \tag{31}$$

By solving this model, we obtain the subset K_n^* of preselected contracts that are likely to be profitable to bid on. Observe that a contract $k \in K_n$ with a null winning ratio still has a chance to be considered in the second step of our approach. Such a contract could be selected, for example, if not all the contracts with a non-null winning ratio could be served by the carrier because of fleet size or other operational constraints.

5 Contract selection and pricing problem

Given the set of preselected contracts K_n^* obtained from the first phase, the objective of CSPP is to determine the number of bids to submit, the contracts covered by each bid and the associated ask price given the uncertainty on contracts' clearing prices. As mentioned before, CSPP is formulated so that multiple non-overlapping OR bids could be generated and all the contracts they cover served by the carrier (in case it wins all of them). The proposed modeling and solution approaches could however be easily adapted to the case where a unique bid is requested through the parameter γ that limits the number of generated bids.

Let $\mathfrak{B}$ represent the set of bids that could be generated. Given our assumptions, $|\mathfrak{B}| = \min\{\gamma, |K_n^*|\}$. A bid $b \in \mathfrak{B}$ is defined by a pair (K_b, p_b) where $K_b \subset K_n^*$ includes the set of contracts covered by b and p_b is the price asked for serving all the contracts in K_b. As will be detailed in the following, K_b and p_b are determined by solving a chance-constrained mathematical model. The proposed model is presented in Section 5.1. An exact approach to solve it is described in Section 5.2.

5.1 Chance-constrained model

We consider the following parameters and decision variables. Let w_b equal to 1 if bid b is generated, $w_b = 0$, otherwise; $z_{kb} = 1$ if contract k is covered by bid b, $z_{kb} = 0$, otherwise; and $p_b \geq 0$ represents the ask price for bid b, $\forall b \in \mathfrak{B}$. Let also $\tilde{C}_b$ represent the clearing price of bid b, $\forall b \in \mathfrak{B}$. The parameters of our model are $\alpha \in [0, 1]$ indicating the risk accepted by the carrier to loose a bid, γ indicating the maximum number of generated bids and η indicating the maximum number of contracts covered by a bid.

The chance-constrained model is given by:

$$\max \quad \sum_{k \in K_e} p_k + \sum_{b \in \mathfrak{B}} p_b w_b - \sum_{l \in L} \sum_{(i,j) \in A} c_{ij} x_{ij}^l - \sum_{l \in L} \sum_{j \in O} f x_{0j}^l \tag{32}$$

s.t. (3)–(14), and to

$$P(p_b w_b \leq \tilde{C}_b) \geq 1 - \alpha \qquad \forall b \in \mathfrak{B} \tag{33}$$

$$w_b \leq \sum_{k \in K_n^*} z_{kb} \qquad \forall b \in \mathfrak{B} \tag{34}$$

$$z_{kb} \leq w_b \qquad \forall k \in K_n^*, b \in \mathfrak{B} \tag{35}$$

$$\sum_{b \in \mathfrak{B}} z_{kb} \leq 1 \qquad \forall k \in K_n^* \tag{36}$$

$$\sum_{k \in K_n^*} z_{kb} \leq \eta \qquad \forall b \in \mathfrak{B} \tag{37}$$

$$w_b \leq w_{b-1} \qquad \forall b \in \mathfrak{B} \setminus \{1\} \tag{38}$$

$$p_b \leq p_{b-1} \qquad \forall b \in \mathfrak{B} \setminus \{1\} \tag{39}$$

$$\sum_{l \in L} x_{o_k d_k}^l = \sum_{b \in \mathfrak{B}} z_{kb} \qquad \forall k \in K_n^* \tag{40}$$

$$z_{kb} \in \{0,1\} \qquad \forall b \in \mathfrak{B}, k \in K_n^* \tag{41}$$

$$w_b \in \{0,1\} \qquad \forall b \in \mathfrak{B} \tag{42}$$

$$p_b \geq 0 \qquad \forall b \in \mathfrak{B}. \tag{43}$$

The objective function (32) maximizes the carrier profit defined as the difference between the pre-existing contracts revenues, the bidding prices and transportation costs. Constraints (3)–(14) guarantee that if a contract k is covered by the generated bids, a feasible routing solution is ensured for this contract in the carrier's network. Chance constraints (33) express a winning probability of $(1-\alpha)$ for each generated bid b (P refers to the probability function). Constraints (34) and (35) link w_b and z_{kb} variables so that a bid is generated if and only if it covers at least one contract. Constraints (36) ensure that each auctioned contract is allocated to at most one bid. Constraints (37) are inspired from the business side constraints considered in Rekik et al. (2017) and impose an upper bound on the number of auctioned contracts per bid. Constraints (38) and (39) are symmetry breaking constraints. Constraints (40) link routing variables x_{ij}^l to z_{kb} variables. Finally, constraints (41)–(43) define the domain of the decision variables.

Although objective function (32) is non-linear (because of the product $p_b w_b$), it can be easily linearized by adding a continuous variable $\zeta_b \geq 0, \forall b \in \mathfrak{B}$ (to replace the product $p_b w_b$) and appropriate linking constraints as follows:

$$\zeta_b \leq M w_b \qquad \forall b \in \mathfrak{B} \tag{44}$$

$$\zeta_b \leq p_b \qquad \forall b \in \mathfrak{B} \tag{45}$$

$$p_b + M(w_b - 1) \leq \zeta_b \qquad \forall b \in \mathfrak{B}, \tag{46}$$

where M is a big-M parameter.
Then the CSPP can be formulated as:

$$M_p : \max \sum_{k \in K_e} p_k + \sum_{b \in \mathfrak{B}} \zeta_b - \sum_{l \in L} \sum_{(i,j) \in A} c_{ij} x_{ij}^l - \sum_{l \in L} \sum_{j \in O} f x_{0j}^l \tag{47}$$

$$\text{s.t. } (3)-(14), (34)-(43), (44)-(46), \text{ and to}$$

$$P(\zeta_b \leq \tilde{C}_b) \geq 1 - \alpha \qquad \forall b \in \mathfrak{B} \tag{48}$$

$$\zeta_b \geq 0 \qquad \forall b \in \mathfrak{B}. \tag{49}$$

Model M_p is still non-linear because of chance constraints (48). In Triki et al. (2014), similar probabilistic constraints were proposed in which all possible $2^{|K_n|} - 1$ bid packages were enumerated. This becomes intractable when the number of contracts $|K_n|$ increases. Assuming that contracts prices follow a normal distribution and given that the package of contracts covered by each enumerated bid is known, linearizing such constraints was possible in Triki et al. (2014). In our case, similar linearization is not possible since one must determine the auction clearing price $\tilde{C}_b$ for bid b without knowing the contracts it covers. To handle this, we first express the bid clearing price $\tilde{C}_b, b \in \mathfrak{B}$ as follows:

$$\tilde{C}_b = \sum_{k \in K_n^*} z_{kb} \tilde{p}_k.$$

Since auctioned contracts' prices are independent and normally distributed ($\tilde{p}_k = N(\overline{p_k}, \sigma_k^2)$, $k \in K_n$), their sum is also normally distributed and we have:

$$\tilde{C}_b \equiv N\left(\sum_{k \in K_n^*} z_{kb}\overline{p_k}, \sum_{k \in K_n^*} z_{kb}^2 \sigma_k^2\right) \rightarrow Z_b = \frac{\tilde{C}_b - \sum_{k \in K_n^*} z_{kb}\overline{p_k}}{\sqrt{\sum_{k \in K_n^*} z_{kb}^2 \sigma_k^2}} \qquad \forall b \in \mathfrak{B},$$

where Z_b is the standardized variable associated with $\tilde{C}_b$.
This implies that chance constraints (48) can be written as:

$$P\left(\frac{\zeta_b - \sum_{k \in K_n^*} z_{kb}\overline{p_k}}{\sqrt{\sum_{k \in K_n^*} z_{kb}^2 \sigma_k^2}} \leq Z_b\right) \geq 1 - \alpha \leftrightarrow P\left(Z_b \leq \frac{\zeta_b - \sum_{k \in K_n^*} z_{kb}\overline{p_k}}{\sqrt{\sum_{k \in K_n^*} z_{kb}^2 \sigma_k^2}}\right) \leq \alpha \quad \forall b \in \mathfrak{B}.$$

Using the inverse cumulative distribution function for a standard normal distribution, denoted Φ^{-1}, we have:

$$\zeta_b - \sum_{k \in K_n^*} \overline{p_k} z_{kb} \leq \Phi^{-1}(\alpha) \sqrt{\sum_{k \in K_n^*} z_{kb}^2 \sigma_k^2}, \qquad \forall b \in \mathfrak{B}. \tag{50}$$

As constraints (50) cannot be handled by common commercial solvers, an exact solution approach is proposed in the following section to solve the chance-constrained model.

5.2 Exact solution approach

Our solution approach is based on two main observations. First, the difficulty in linearizing chance constraints (50) is essentially due to the fact that the contracts covered by a bid are not known in advance. If the associated package of covered contracts K_b (i.e., z_{kb} values) was known, then the corresponding chance constraint (50) reduces to a bounding constraint on the ζ_b variable. So, one way to obtain relatively good quality solutions is to generate interesting contracts packages and to restrict M_p to bids composed of these packages so that M_p becomes linear and could be solved by branch-and-cut. Solving this restricted model results in a lower bound for the CSPP.

Second, non-linearity in chance constraints (50) is due to the right-hand side expression $\Phi^{-1}(\alpha)\sqrt{\sum_{k\in K_n^*} z_{kb}^2\sigma_k^2}$. So, if one is able to bound this non-linear expression, the resulting model is linear and is a relaxation of M_p. Solving it gives a valid upper bound to M_p but not necessarily a feasible solution. Infeasibility, if any, would be due only to the bid ask prices that may violate original chance constraints (50). Although infeasible, such solutions are interesting: they respect all the other constraints of M_p and one knows the contracts covered by each generated bid. Thus, one could keep only the information on the contracts and solve a restricted model as explained before. Furthermore, the upper bound of the relaxed model could be improved by adding to the relaxed model new bid variables for which the covered contracts are known (but not the price) and incorporate the corresponding chance constraints which have the property to be linear. The restricted and the relaxed problems are explained in details next.

5.2.1 Relaxed problem

In what follows, it is assumed that $\alpha \leq \frac{1}{2}$ implying that the chance of winning a bid is equal to or larger than 50% which is more likely to be considered by a carrier.
Let $\sigma_{min} = \min\{\sigma_k, k \in K_n^*\}$. Then, $\forall b \in \mathfrak{B}$, we have $\sqrt{\sum_{k\in K_n^*} z_{kb}^2\sigma_{min}^2} \leq \sqrt{\sum_{k\in K_n^*} z_{kb}^2\sigma_k^2}$.
Since $\sqrt{\sum_{k\in K_n^*} z_{kb}^2\sigma_{min}^2} = \sigma_{min}\sqrt{\sum_{k\in K_n^*} z_{kb}^2}$ and $\sqrt{\sum_{k\in K_n^*} z_{kb}^2} = \sqrt{\sum_{k\in K_n^*} z_{kb}}$ $(z_{kb} \in \{0,1\})$, then $\sigma_{min}\sqrt{\sum_{k\in K_n^*} z_{kb}} \leq \sqrt{\sum_{k\in K_n^*} z_{kb}^2\sigma_k^2}$.

Given constraints (34), it follows $\sigma_{min} w_b \leq \sqrt{\sum_{k\in K_n^*} z_{kb}^2\sigma_k^2}$. Since $\alpha \leq \frac{1}{2} \rightarrow \Phi^{-1}(\alpha) \leq 0$, then, $\Phi^{-1}(\alpha)\sigma_{min} w_b \geq \Phi^{-1}(\alpha)\sqrt{\sum_{k\in K_n^*} z_{kb}^2\sigma_k^2}$.

Replacing chance constraints (50) with the following constraints:

$$\zeta_b - \sum_{k\in K_n^*} \overline{p_k} z_{kb} \leq \Phi^{-1}(\alpha)\sigma_{min} w_b, \qquad \forall b \in \mathfrak{B} \tag{51}$$

results in a Mixed Integer Linear Programming model that is a relaxation of model M_p.

We refer to it as $\overline{M_p}$.

5.2.2 Restricted problem

A restricted problem is obtained by considering a particular set of partial bids, denoted by $\overline{\mathfrak{B}}$, for which the set of contracts covered by each bid is known but not the associated price.
Let δ_{kb} be a binary parameter defined for each contract k in K_n^* and each bid $b \in \overline{\mathfrak{B}}$ such that $\delta_{kb} = 1$, if bid b covers contract k, and $\delta_{kb} = 0$, otherwise. Assume also that bids in $\overline{\mathfrak{B}}$ are indexed in an ascending order and that the bid with the lowest index is denoted by $\bar{b}_{min}$. The restricted model, denoted by $\underline{M_p}$, can be formulated as follows:

$$\underline{M_p} : \max \sum_{k \in K_e} p_k + \sum_{b \in \overline{\mathfrak{B}}} \zeta_b - \sum_{l \in L} \sum_{(i,j) \in A} c_{ij} x_{ij}^l - \sum_{l \in L} \sum_{j \in O} f x_{0j}^l \tag{52}$$

$$\text{s.t. } (3)-(14), (44)-(46), \text{ and to}$$

$$\zeta_b \leq \sum_{k \in K_n^*} \overline{p_k} \delta_{kb} + \Phi^{-1}(\alpha) \sqrt{\sum_{k \in K_n^*} \delta_{kb} \sigma_k^2} \qquad \forall b \in \overline{\mathfrak{B}} \tag{53}$$

$$\sum_{b \in \overline{\mathfrak{B}}} \delta_{kb} w_b \leq 1 \qquad \forall k \in K_n^* \tag{54}$$

$$w_b \leq w_{b-1} \qquad \forall b \in \overline{\mathfrak{B}} \setminus \{\bar{b}_{min}\} \tag{55}$$

$$p_b \leq p_{b-1} \qquad \forall b \in \overline{\mathfrak{B}} \setminus \{\bar{b}_{min}\} \tag{56}$$

$$\sum_{l \in L} x_{o_k d_k}^l = \sum_{b \in \overline{\mathfrak{B}}} \delta_{kb} w_b \qquad \forall k \in K_n^* \tag{57}$$

$$w_b \in \{0, 1\} \qquad \forall b \in \overline{\mathfrak{B}} \tag{58}$$

$$p_b, \zeta_b \geq 0 \qquad \forall b \in \overline{\mathfrak{B}}. \tag{59}$$

As will be detailed in Section 5.2.3, the proposed solution approach is iterative. At each iteration, both a restricted and a relaxed problem are solved by considering a different set $\overline{\mathfrak{B}}$ that is iteratively updated. In what follows, the restricted (relaxed) model defined with respect to a set $\overline{\mathfrak{B}}$ is referred to as $\underline{M_p}(\overline{\mathfrak{B}})$ ($\overline{M_p}(\overline{\mathfrak{B}})$).

5.2.3 Solution algorithm

The general structure of the proposed approach is illustrated in Algorithm 1. At each iteration, valid lower and upper bounds (denoted respectively LB and UB) are updated by solving appropriate restricted and relaxed problems. The process iterates until $LB = UB$ or a time limit is met.

Algorithm 1 General structure of the exact solution approach for CSPP

$\overline{\mathfrak{B}} \leftarrow \emptyset$, $\bar{b} \leftarrow |\mathfrak{B}|$, $LB \leftarrow 0$, $UB \leftarrow +\infty$.
while a time limit is not met and $LB < UB$ **do**
  $UB \leftarrow$ Solve $\overline{M_p}(\overline{\mathfrak{B}})$ by branch-and-cut
  for each optimal solution of $\overline{M_p}(\overline{\mathfrak{B}})$ **do**
    for each generated bid $b \in \mathfrak{B}$, **do**
      if $\left(\overline{\zeta_b} - \sum_{k \in K_n^*} \overline{p_k}\delta_{kb} > \Phi^{-1}(\alpha)\sqrt{\sum_{k \in K_n^*} \delta_{kb}\sigma_k^2}\right)$ **then**
        $\bar{b} \leftarrow \bar{b} + 1$
        for each $k \in K_n^*$ **do**
          if $(\delta_{kb} = 1)$ **then**
            $\delta_{k\bar{b}} \leftarrow 1$
          else
            $\delta_{k\bar{b}} \leftarrow 0$
          end if
        end for
        $\overline{\mathfrak{B}} \leftarrow \overline{\mathfrak{B}} \cup \{\bar{b}\}$
        Add to $M_p^r(\mathfrak{B} \cup \overline{\mathfrak{B}})$ no-good cuts to forbid the bid b
        Add to $M_p^r(\mathfrak{B} \cup \overline{\mathfrak{B}})$ the chance constraint corresponding to bid $\bar{b} \in \overline{\mathfrak{B}}$
      end if
    end for
  end for
  $LB \leftarrow$ Solve $\underline{M_p}(\overline{\mathfrak{B}})$ by branch-and-cut
end while

The relaxed model $\overline{M_p}(\overline{\mathfrak{B}})$ – at a given iteration – is similar to model M_p except that: (i) in addition to set $\mathfrak{B}$, it incorporates a set $\overline{\mathfrak{B}}$ of partially defined bids b for which the set of covered contracts K_b is known but not the ask price, (ii) chance constraints (50) for bids in $\overline{\mathfrak{B}}$ are linear and formulated as in (53), (iii) chance constraints (50) for bids in $\mathfrak{B}$ are replaced by relaxed constraints (51), (iv) it includes the so-called no-good cuts (defined later in (66)) to forbid generating bids that are already in $\overline{\mathfrak{B}}$.

Solving the relaxed model at a given iteration with branch-and-cut results in a solution (w^0, p^0, z^0, ζ^0) that is not necessarily feasible for M_p (because of the chance constraints relaxation). For each generated bid b deduced from (w^0, p^0, z^0, ζ^0) (i.e., $b : w_b^0 = 1$), we know the contracts it covers ($k : z_{kb}^0 = 1$) and the associated price (p_b^0). If bid b violates the chance constraint (50), a bid $\overline{b}$ covering the same contracts as b and for which no price is fixed is added to the set of partially defined bids $\overline{\mathfrak{B}}$. The relaxed model considered at the next iteration includes chance constraints (50) for the bids newly added to $\overline{\mathfrak{B}}$ as well as no-goods cuts to forbid generating a bid that covers the same contracts as those covered in $\overline{\mathfrak{B}}$.

The relaxed model $\overline{M_p}(\overline{\mathfrak{B}})$ is formulated as:

$$\overline{M_p}(\overline{\mathfrak{B}}) : \max \sum_{k \in K_e} p_k + \sum_{b \in \mathfrak{B} \cup \overline{\mathfrak{B}}} \zeta_b - \sum_{l \in L} \sum_{(i,j) \in A} c_{ij} x_{ij}^l - \sum_{l \in L} \sum_{j \in O} f x_{0j}^l \tag{60}$$

$$\text{s.t. } (3)-(14), (34)-(35), (37)-(39), (41), (48), \text{ and to}$$

$$\zeta_b \leq \sum_{k \in K_n^*} \overline{p_k} \delta_{kb} + \Phi^{-1}(\alpha) \sqrt{\sum_{k \in K_n^*} \delta_{kb} \sigma_k^2} \qquad \forall b \in \overline{\mathfrak{B}} \tag{61}$$

$$\zeta_b - \sum_{k \in K_n^*} \overline{p_k} z_{kb} \leq \Phi^{-1}(\alpha) \sigma_{min} w_b \qquad \forall b \in \mathfrak{B} \tag{62}$$

$$\sum_{b \in \mathfrak{B} \cup \overline{\mathfrak{B}}} w_b \leq \gamma \tag{63}$$

$$\sum_{b \in \mathfrak{B}} z_{kb} + \sum_{b \in \overline{\mathfrak{B}}} \delta_{kb} w_b \leq 1 \qquad \forall k \in K_n^* \tag{64}$$

$$\sum_{l \in L} x_{o_k d_k}^l = \sum_{b \in \mathfrak{B}} z_{kb} + \sum_{b \in \overline{\mathfrak{B}}} \delta_{kb} w_b \qquad \forall k \in K_n^* \tag{65}$$

$$\sum_{\substack{k \in K_n^*: \\ \delta_{kb'} = 0}} z_{kb} + \sum_{\substack{k \in K_n^*: \\ \delta_{kb'} = 1}} (1 - z_{kb}) \geq 1 \qquad \forall b \in \mathfrak{B}, \forall b' \in \overline{\mathfrak{B}} \tag{66}$$

$$\zeta_b \leq M w_b \qquad \forall b \in \mathfrak{B} \cup \overline{\mathfrak{B}} \tag{67}$$

$$\zeta_b \leq p_b \qquad \forall b \in \mathfrak{B} \cup \overline{\mathfrak{B}} \tag{68}$$

$$p_b + M(w_b - 1) \leq \zeta_b \qquad \forall b \in \mathfrak{B} \cup \overline{\mathfrak{B}} \tag{69}$$

$$w_b \in \{0,1\}, p_b \in \{0,1\}, \zeta_b \geq 0 \qquad \forall b \in \mathfrak{B} \cup \overline{\mathfrak{B}}. \tag{70}$$

6 Computational results

The proposed solution methods are coded in Java and OPL and the mathematical models solved using the branch-and-cut of CPLEX 12.9.0. The experiments are conducted on computers mounted in parallel and equipped with Intel(R) Xeon(TM) Gold 6148 processors clocked at 2.40 GHz with up to 32 threads and 186 Gigabyte of RAM.

We start by describing the instances and parameters setting in Section 6.1. Detailed computational results assessing the solution quality and the computational time of our two-phase solution approach are presented in Section 6.2 for the CSP phase and in Section 6.3 for the CSPP phase. In Section 6.4, we compare the performance of the two-phase heuristic approach (CSP+CSPP) to the exact approach (CSPP only).

6.1 Problem instances and parameters setting

Several tests were conducted to assess the performance of the proposed solution methods. We consider 50 instances inspired from the ones proposed by Hammami et al. (2019). These instances are obtained by varying the number of pre-existing and auctioned contracts, the number of vehicles and their fixed costs. Contracts' origin-destination pairs are randomly selected from real locations in USA and Canada. Traveling costs (c_{ij}) and times (t_{ij}) associated with arcs $(i, j) \in A$ are computed using Google Maps. The maximum tour duration T_{max} is set to 1500 minutes for each instance. The mean $\overline{p}_k$ of the normal distribution function associated with contract $k \in K_n$ is uniformly generated within the interval $[2 \times c_{o_k d_k}, 4 \times c_{o_k d_k}]$. The standard deviation $\sigma_k, k \in K_n$ is set to $15\%\overline{p_k}$. Table 1 reports for each instance, the fleet size ($|L|$), the vehicles fixed cost (f), the number of existing contracts ($|K_e|$), and the number of auctioned contracts ($|K_n|$). The maximum number of bids γ is set to 5 and the maximum number of contracts per bid η is set to $\left\lceil \frac{|K_n^*|}{|\mathfrak{B}|} \right\rceil$. The risk α accepted by the carrier to loose a bid is fixed to 5%.

A time limit of 7,200 seconds is fixed for CPLEX to solve the equivalent deterministic

Table 1: Characteristics of the instances

Instance	$\lvert L \rvert$	f	$\lvert K_e \rvert$	$\lvert K_n \rvert$
1	2	500	10	20
2	2	1000	10	20
3	3	500	10	20
4	3	1000	10	20
5	4	500	10	20
6	4	1000	10	20
7	3	500	15	15
8	3	1000	15	15
9	4	500	15	15
10	4	1000	15	15
11	5	500	15	15
12	5	1000	15	15
13	4	500	15	20
14	4	1000	15	20
15	5	500	15	20
16	5	1000	15	20
17	5	500	20	15
18	5	1000	20	15
19	6	500	20	15
20	6	1000	20	15
21	4	500	15	25
22	4	1000	15	25
23	5	500	15	25
24	5	1000	15	25
25	6	500	15	25
26	6	1000	15	25
27	5	500	20	20
28	5	1000	20	20
29	6	500	20	20
30	6	1000	20	20
31	7	500	20	20
32	7	1000	20	20
33	5	500	20	25
34	5	1000	20	25
35	6	500	20	25
36	6	1000	20	25
37	7	500	20	25
38	7	1000	20	25
39	6	500	25	20
40	6	1000	25	20
41	7	500	25	20
42	7	1000	25	20
43	8	500	25	20
44	8	1000	25	20
45	6	500	20	30
46	6	1000	20	30
47	7	500	25	25
48	7	1000	25	25
49	9	500	25	25
50	9	1000	25	25

problem M_{CSP}. Besides, a time limit of 3,600 seconds is considered to solve each of the M_1^{ω} models, $\omega \in \Omega$ and model M_2 when considering the heuristic proposed for CSP. A time limit of 43,200 seconds is fixed for the exact solution method proposed for the CSPP.

For each instance, 25 independent and equiprobable scenarios are generated to solve CSP using the Monte Carlo procedure. A size of 25 was validated through a preprocessing procedure where we have first fixed the sample size so that at least half of the instances could be solved to optimality with the branch-and-cut of CPLEX within 7,200 seconds. With $|\Omega| = 25$, 19 instances were solved to optimality. Then, for each of these instances, we have generated 10 samples of size 25 each and have solved the corresponding equivalent deterministic problem M_{CSP} to optimality. We have then calculated, for each instance, the average and standard deviation of the optimal objective values over the 10 samples. We observed that the standard deviations were small. Detailed results obtained for each instance are reported in the online supplement.

6.2 Results of the first-phase contract selection problem (CSP)

In order to evaluate the performance of the heuristic proposed for the CSP, we compare the expected profit yielded by the first-stage solution values output by our heuristic (contracts in K_n^*) to that obtained when applying a branch-and-cut procedure to solve model M_{CSP}. To this end, we solve for each scenario $\omega \in \Omega$, the following deterministic model:

$$
\begin{aligned}
M_3^\omega: \quad & \max \ z^\omega = \sum_{k \in K_e} p_k + \sum_{k \in K_n^*} p_k^\omega - \sum_{l \in L} \sum_{(i,j) \in A} c_{ij} x_{ij}^l - \sum_{l \in L} \sum_{j \in O} f x_{0j}^l \\
& \text{s.t.} \quad (3)-(15), \text{ and to} \\
& \sum_{l \in L} x_{o_k d_k}^l = 1 \qquad\qquad \forall k \in K_n^*.
\end{aligned}
$$

The expected profit yielded by our heuristic, denoted by LB^h, is given by: $LB^h = \frac{\sum_{\omega \in \Omega} z^\omega}{|\Omega|}$. The expected profit yielded by the branch-and-cut procedure of CPLEX is the optimal solution of model M_{CSP}.

Table 2 reports for each instance, the lower bound (LB^c) and the upper bound (UB^c) obtained with CPLEX, the gap between LB^c and UB^c in percentage ($Gap = \frac{UB^c - LB^c}{UB^c}$), and the CPU time (in seconds) required by CPLEX to identify an optimal solution within a time limit of 7,200 seconds. Table 2 also reports, for each instance, the expected profit LB^h obtained with our heuristic, the required computing time (in seconds), the gaps in percentage between our expected profit and the upper bound of CPLEX ($Gap_{UB}^h = \frac{UB^c - LB^h}{UB^c}$) and its lower bound ($Gap_{LB}^h = \frac{LB^h - LB^c}{LB^c}$).

The results of Table 2 show that our heuristic outperforms CPLEX on different levels. Indeed, CPLEX was able to solve to optimality only 19 of the 50 instances with an average computing time of $1,287$ seconds. It was unable to provide a feasible solution for 7 instances within the time limit of $7,200$ seconds. For the remaining 24 instances for which only feasible solutions were obtained, the average gap to the upper bound exceeds 21%. Our heuristic solves to optimality all the 19 instances solved by CPLEX in only 448.93 seconds, on average. Either an optimal or a feasible solution was identified for all the instances with an average gap equal to 5.49% with respect to the best upper bound CPLEX obtained within 7,200 seconds. For the 24 instances for which CPLEX identifies only feasible solutions, our heuristic generally yields a better expected profit

Table 2: Computational results for the CSP

Instance	**CPLEX**				Our heuristic			
	LB^c	UB^c	CPU	Gap	LB^h	CPU	Gap^h_{UB}	Gap^h_{LB}
1	5057	5057	36.24	0.00	5057	1.29	0.00	0.00
2	4120	4120	41.30	0.00	4120	1.31	0.00	0.00
3	6749	6749	191.96	0.00	6749	9.77	0.00	0.00
4	5555	5555	191.60	0.00	5555	37.33	0.00	0.00
5	6506	6506	181.85	0.00	6506	25.74	0.00	0.00
6	5925	5925	323.62	0.00	5925	33.71	0.00	0.00
7	5517	5517	59.47	0.00	5517	2.86	0.00	0.00
8	4813	4813	121.92	0.00	4813	1.99	0.00	0.00
9	7330	7330	238.62	0.00	7330	161.77	0.00	0.00
10	8183	8183	143.45	0.00	8183	67.32	0.00	0.00
11	11747	12037	7200.00	2.41	11747	3946.57	2.41	0.00
12	9833	9833	5143.62	0.00	9833	2375.32	0.00	0.00
13	8497	8497	590.49	0.00	8497	249.67	0.00	0.00
14	9068	9068	480.96	0.00	9068	254.53	0.00	0.00
15	11781	12199	7200.00	3.43	11725	4253.66	3.89	-0.48
16	8683	8832	7200.00	1.69	8718	2837.15	1.29	0.40
17	10633	10633	5080.28	0.00	10633	2008.19	0.00	0.00
18	9199	9199	5805.61	0.00	9199	1764.84	0.00	0.00
19	-	13135	7200.00	-	12485	6653.41	4.95	-
20	6333	10945	7200.00	42.14	9951	6852.30	9.08	57.13
21	-	9440	7200.00	-	8897	6.00	5.75	-
22	-	7167	7200.00	-	5883	6.61	17.92	-
23	5386	14239	7200.00	62.17	11454	168.42	19.56	112.65
24	3372	13249	7200.00	74.55	9194	115.39	30.60	172.66
25	14423	14883	7200.00	3.09	14423	1143.97	3.09	0.00
26	11114	11643	7200.00	4.54	11114	1776.99	4.54	0.00
27	8818	12237	7200.00	27.94	10371	25.83	15.25	17.61
28	7772	7772	505.80	0.00	7772	77.09	0.00	0.00
29	12271	13133	7200.00	6.56	12406	2124.90	5.54	1.10
30	11648	13035	7200.00	10.64	11648	2895.32	10.64	0.00
31	17940	18328	7200.00	2.12	17940	5935.96	2.12	0.00
32	13175	13767	7200.00	4.30	13209	7200.00	4.06	0.25
33	11201	11201	742.90	0.00	11201	139.04	0.00	0.00
34	7626	7626	819.54	0.00	7626	121.82	0.00	0.00
35	12298	13938	7200.00	11.77	13291	1317.58	4.64	8.08
36	9594	10874	7200.00	11.77	10428	2500.01	4.10	8.69
37	16380	16928	7200.00	3.24	16380	2527.57	3.24	0.00
38	10891	11390	7200.00	4.38	10891	6862.75	4.38	0.00
39	-	12801	7200.00	-	12208	970.21	4.63	-
40	-	10650	7200.00	-	10369	782.57	2.63	-
41	-	15123	7200.00	-	14497	1754.45	4.14	-
42	-	13414	7200.00	-	12966	6847.10	3.34	-
43	5381	20860	7200.00	74.20	16307	6670.56	21.83	203.05
44	12174	12842	7200.00	5.20	12261	7064.75	4.52	0.72
45	13861	14307	7200.00	3.12	14043	2797.19	1.85	1.31
46	9322	9322	3594.60	0.00	9322	1196.23	0.00	0.00
47	14712	14742	7200.00	0.20	14712	2956.28	0.20	0.00
48	11866	13062	7200.00	9.16	12196	6672.02	6.63	2.78
49	7852	23581	7200.00	66.70	17660	7200.00	25.11	124.91
50	3914	21596	7200.00	81.88	12378	7200.00	42.68	216.25
Average	**9268**	**11346**	**4949.88**	**12.03**	**10493**	**2371.91**	**5.49**	**21.56**

(Gap^h_{LB} varies between -0.48% and 216.25% for these instances) in shorter computing times ($3,876$ seconds on average).

6.3 Results for the second-phase contract selection and pricing problem (CSPP)

Table 3 reports, for each instance, the number of auctioned contracts considered in the CSPP ($|K_n^*|$) as output by the first phase, the LB, the UB, the relative gap (in percentage) $Gap = \frac{UB-LB}{UB}$, the CPU time (in seconds) required to identify the best feasible solution (CPU^*), the total CPU time (in seconds) required to run the exact method described in Section 5.2.3 (CPU), and the number of cuts generated during the solution process.

The results of Table 3 show that, for the second phase, optimal solutions were identified for 46 over the 50 instances. The average gap is equal to 0.42% and the average computational time reaches $8,847.31$ seconds. Optimality was proven for the 46 instances with an average computational time of $5,860.12$ seconds. For the four instances not solved to optimality, the gap varies between 2.08% and 8.96% with an average value of 5.29%. Although these gaps are relatively large for instances with $|K_n^*| \geq 16$, we observed during our experiments that the LB value tends to stabilize quickly while the UB value keeps slowly decreasing as the size of K_n^* increases. This is further confirmed when comparing the CPU time required to find an optimal solution (CPU^* for the 46 instances where $Gap = 0\%$) and the time required to prove its optimality (CPU). For the 46 instances solved to optimality, only 361.70 seconds were required in average to find the optimal solutions compared to $5,860.12$ seconds to prove it.

The performance of the exact method proposed for the CSPP is indeed closely dependent on the number of pre-selected contracts $|K_n^*|$. When this number increases, the number of generated cuts gets large and $\overline{M_p}$ becomes more difficult to solve. For example, instances with fewer than 10 pre-selected contracts are solved to optimality in less than 950 seconds on average and the average number of generated cuts is equal to 137. Instances solved

Table 3: Computational results for the CSPP

Instance	$\lvert K_n^* \rvert$	LB	UB	Gap	CPU^*	CPU	Cuts
1	9	4383	4383	0.00	0.34	13.84	70
2	9	3501	3501	0.00	0.21	44.41	51
3	17	5884	6109	3.68	13.69	43200.00	1410
4	15	4569	4569	0.00	121.25	6532.51	2852
5	18	5711	6104	6.43	34.62	43200.00	1831
6	16	4969	5075	2.08	33.61	43200.00	3650
7	2	5048	5048	0.00	0.35	0.81	1
8	2	4324	4324	0.00	0.36	0.82	1
9	7	6238	6238	0.00	110.25	221.72	205
10	6	6916	6916	0.00	7.11	35.45	35
11	12	9992	9992	0.00	58.00	10532.65	425
12	12	8311	8311	0.00	1106.52	21314.06	327
13	8	7540	7540	0.00	77.23	220.71	39
14	9	8102	8102	0.00	5.55	34.85	285
15	12	10226	10226	0.00	302.02	1324.03	329
16	13	7183	7183	0.00	137.10	1914.69	830
17	7	9566	9566	0.00	189.42	477.44	114
18	7	8262	8262	0.00	53.88	433.39	83
19	12	11182	11182	0.00	3006.30	12601.71	327
20	12	8747	8747	0.00	1506.61	7030.67	326
21	4	8073	8073	0.00	0.51	1.46	6
22	4	5176	5176	0.00	0.33	0.88	5
23	8	10101	10101	0.00	11.49	63.68	339
24	9	7917	7917	0.00	15.88	87.12	134
25	12	12575	12575	0.00	208.78	13441.67	344
26	13	9368	9368	0.00	57.56	599.90	4116
27	3	9909	9909	0.00	15.70	35.62	2
28	3	7324	7324	0.00	15.02	33.39	2
29	8	11115	11115	0.00	174.72	4291.02	128
30	8	10192	10192	0.00	1.53	262.53	42
31	12	16222	16222	0.00	93.17	10701.13	391
32	12	11586	11586	0.00	215.29	7817.58	366
33	8	9625	9625	0.00	50.11	213.06	174
34	8	6100	6100	0.00	1.59	1451.06	268
35	12	11488	11488	0.00	38.50	4721.88	289
36	14	8588	8588	0.00	934.17	14576.88	512
37	16	13911	13911	0.00	239.57	16737.05	655
38	16	9243	9243	0.00	4.76	16007.13	643
39	7	10951	10951	0.00	24.84	7138.29	58
40	7	9021	9021	0.00	46.12	277.32	81
41	9	12651	12651	0.00	66.20	871.73	226
42	11	11417	11417	0.00	548.66	18448.22	261
43	14	14331	14331	0.00	1069.44	7050.13	504
44	15	10485	10485	0.00	1645.39	6391.92	451
45	14	12043	12043	0.00	143.52	10088.40	494
46	15	7594	7594	0.00	248.41	13089.84	515
47	15	13272	13272	0.00	214.41	13688.07	501
48	12	10801	10801	0.00	917.19	14835.39	332
49	20	15621	17160	8.96	2851.01	43200.00	5523
50	16	10718	10718	0.00	2952.65	23909.59	726
Average		**9161**	**9206**	**0.42**	**391.42**	**8847.31**	**626**

to optimality and for which the number of pre-selected contracts is between 11 and 16 required between 599.90 and $23,909.59$ seconds. The three instances for which $|K_n^*| \geq 17$ were not solved to optimality, or at least optimality was not proven for the obtained solutions, within $43,200$ seconds. However, as previously observed, our approach quickly located optimal solutions but required more time to prove their optimality.

The results reported in Table 3 are for the 50 instances described in Section 6.1 in which a number of problem parameters were fixed to pre-specified values. A sensitivity analysis was conducted to evaluate the impact of certain parameters on the computational performance of the proposed exact method. Three parameters were tested: the risk accepted by the carrier to lose a bid: three values are considered $\alpha = 1\%$, 5% and 10%; the maximum number of generated bids: three values are considered $\gamma = 3$, 4 and 5; and the maximum number of contracts covered by a bid: three values are considered $\eta = 3$, 4 and 5. When a parameter value is varied, all the other parameters are kept fixed. Table 4 summarizes our results. It reports for each parameters' combination the percentage of instances for which optimality was proven, the average gap over the 50 instances, and the average CPU time (in seconds). Detailed results for each instance are presented in Table 6 in the appendix. Our results first prove that varying the value of the α parameter has almost no impact on computing times. The exact method identifies optimal solutions for 46 instances over the 50 considered independently of the value of α. Second, as the value of η increases, the number of instances for which optimality is proven decreases and average CPU times increase. Finally, increasing the value of γ allows to prove optimality for more instances and reduces the average computational time of the algorithm.

Table 4: Summary of the sensivity analysis for the exact method

Parameters	$\left(\gamma = 5, \eta = \left\lceil \frac{\lvert K_n^* \rvert}{\lvert \mathfrak{B} \rvert} \right\rceil\right)$			$(\gamma = 5, \alpha = 5\%)$			$(\eta = 5, \alpha = 5\%)$		
	$\alpha = 1\%$	$\alpha = 5\%$	$\alpha = 10\%$	$\eta = 3$	$\eta = 4$	$\eta = 5$	$\gamma = 3$	$\gamma = 4$	$\gamma = 5$
Optimality (%)	92	92	92	86	80	72	27	66	72
Average gap	0.54	0.42	0.33	0.75	0.82	1.34	1.66	1.36	1.34
Average CPU	9,134.52	8,847.31	9,027.54	11,577.08	17,335.69	21,936.36	25,350.59	23,484.59	21,936.36

6.4 Heuristic versus exact approach

As already mentioned, by dropping off the first-phase and considering all auctioned contracts (K^n) in the CSPP, our solution approach becomes exact. The object of this section is to measure the relevance of considering the CSP phase on computing times and solution quality. To do this, we compare the performance of the heuristic and exact approaches over all the instances described in Section 6.1 in which we fix $\gamma = 5, \eta = 3$, and $\alpha = 5\%$. Recall that for the heuristic approach a time limit of two, respectively, 12 hours was fixed for the first, respectively, the second, phase. Then, we fix a time limit of 14 (=2+12) hours for the exact approach.

Table 5 reports the results obtained for each instance and each solution approach with regard to: the best solution found (LB^e and LB^h, for the exact and heuristic approaches, respectively), the time required to reach the best solution (CPU^{*e} and CPU^{*h}) and, the total CPU time (CPU^e and CPU^h). For the exact approach, we additionally report the best upper bound (UB^e), and the relative gap between the lower and upper bounds ($Gap^e = \frac{UB^e - LB^e}{UB^e}$). The last two columns display: the relative difference (in percentage) between the best solutions found by the heuristic versus the exact approach ($Gap_{LB} = \frac{LB^h - LB^e}{LB^e}$), and the absolute difference in seconds in the CPU times required to identify the best solutions ($Gap_{CPU^*} = CPU^{*h} - CPU^{*e}$). Observe that a value in bold under the columns LB^e and LB^h indicates that it corresponds to the best solution found. A positive value for Gap_{LB} indicates that the heuristic identifies a better solution than the exact approach. A negative value for Gap_{CPU^*} implies that the heuristic was faster to identify its best solution than the exact method to identify its own.

The results of Table 5 prove that the proposed heuristic offers the best trade-off between solution quality and computing times. Both approaches identify the same solutions for 26 instances over the 50 considered. However, the exact approach requires much more time than the heuristic to find these solutions: CPU times vary between 142 and 49,188 seconds with an average of 10,223 seconds for the exact approach. They range between 3 and 8,358 seconds for the heuristic with an average of 1,839 seconds. Moreover, four

Table 5: Heuristic vs exact approach

Ins	Exact approach					Heuristic			Saving/loss	
	LB^e	UB^e	Gap^e	CPU^{*e}	CPU^e	LB^h	CPU^{*h}	CPU^h	Gap_{LB}	Gap_{CPU^*}
1	**4511**	4692	3.86	120.30	50400.00	4489	37.30	57.22	-0.49	-83.00
2	**3610**	3781	4.52	326.72	50400.00	**3610**	75.22	244.08	0.00	-251.50
3	**5748**	6325	9.12	267.58	50400.00	5670	129.58	43209.77	-1.38	-138.00
4	**4570**	5316	14.03	388.91	50400.00	**4570**	158.58	6569.84	0.00	-230.33
5	**5467**	6025	9.26	240.86	50400.00	5425	123.78	43225.74	-0.77	-117.08
6	**4782**	5274	9.33	357.91	50400.00	**4782**	192.78	43233.71	0.00	-165.13
7	**5178**	5178	0.00	208.52	208.52	**5178**	3.86	3.93	0.00	-204.66
8	**4459**	4459	0.00	189.19	189.19	**4459**	3.20	3.20	0.00	-185.99
9	**6407**	6848	6.44	1795.95	50400.00	**6407**	262.93	262.93	0.00	-1533.02
10	**7140**	7507	4.89	152.28	50400.00	**7140**	84.01	258.59	0.00	-68.27
11	9814	10870	9.71	26564.72	50400.00	**9992**	4004.57	14479.22	1.78	-22560.15
12	8181	8453	3.22	10273.58	50400.00	**8311**	3481.84	23689.38	1.56	-6791.74
13	**7712**	8201	5.96	10844.08	50400.00	7673	275.78	646.70	-0.51	-10568.30
14	**8299**	8810	5.80	10875.40	50400.00	**8299**	293.96	6370.84	0.00	-10581.44
15	**10226**	10325	0.96	13132.05	50400.00	**10226**	4555.68	5577.69	0.00	-8576.37
16	7176	7253	1.06	20789.75	50400.00	**7183**	2974.25	4751.84	0.10	-17815.50
17	9734	10317	5.65	11956.07	50400.00	**9740**	6595.70	6595.70	0.06	-5360.37
18	8377	8874	5.60	11178.59	50400.00	**8411**	7213.27	7213.27	0.40	-3965.32
19	11077	12253	9.60	8976.96	50400.00	**11182**	8659.71	19255.12	0.94	-317.25
20	**8747**	9649	9.35	37925.51	50400.00	**8747**	8358.91	13882.97	0.00	-29566.60
21	**8361**	8361	0.00	142.49	142.49	**8361**	9.20	9.20	0.00	-133.29
22	**5415**	5415	0.00	755.76	755.76	**5415**	11.40	11.40	0.00	-744.36
23	**10305**	10522	2.06	368.35	50400.00	**10305**	291.47	701.43	0.00	-76.88
24	**8157**	8955	8.91	488.39	50400.00	**8157**	205.18	325.04	0.00	-283.21
25	**12575**	13935	9.76	1743.82	50400.00	**12575**	1352.75	14585.64	0.00	-391.07
26	**9379**	10663	12.04	21684.91	50400.00	9368	1834.55	2376.89	-0.12	-19850.36
27	**10087**	10087	0.00	1736.59	1599.61	**10087**	45.08	45.08	0.00	-1691.51
28	**7492**	7492	0.00	801.26	1755.99	**7492**	99.62	156.98	0.00	-701.64
29	**11305**	11919	5.15	4062.01	50400.00	**11305**	2474.96	11337.06	0.00	-1587.05
30	**10405**	11092	6.19	4689.07	50400.00	**10405**	3197.14	33588.80	0.00	-1491.93
31	**16223**	16397	1.06	25036.16	50400.00	**16223**	6029.13	16637.09	0.00	-19007.03
32	**11586**	11662	0.65	49188.41	50400.00	**11586**	7415.29	15017.58	0.00	-41773.12
33	**9988**	10873	8.14	384.69	50400.00	9897	197.30	3225.47	-0.92	-187.39
34	**6364**	6887	7.59	298.98	50400.00	**6364**	216.06	2562.86	0.00	-82.92
35	**11488**	12889	10.87	7871.52	50400.00	**11488**	1356.08	6039.46	0.00	-6515.44
36	8534	8792	2.93	40259.17	50400.00	**8588**	3434.18	17076.89	0.63	-36824.99
37	13480	15780	14.58	28762.95	50400.00	**13488**	2705.48	45727.57	0.06	-26057.47
38	**8814**	10062	12.40	20393.56	50400.00	8763	10383.46	50062.75	-0.58	-10010.10
39	11112	11866	6.35	10959.46	50400.00	**11129**	1226.36	1226.36	0.15	-9733.10
40	**9219**	9817	6.09	35489.24	50400.00	**9219**	927.51	1595.67	0.00	-34561.73
41	**12971**	13975	7.18	31059.57	50400.00	**12971**	2470.71	17012.62	0.00	-28588.86
42	11392	12624	9.76	42395.44	50400.00	**11417**	7395.76	25295.32	0.22	-34999.68
43	**14331**	15890	9.81	36719.21	50400.00	**14331**	7740.00	13720.69	0.00	-28979.21
44	10452	11876	11.99	27971.49	50400.00	**10485**	8710.15	13456.68	0.31	-19261.35
45	12007	13600	11.71	25296.46	50400.00	**12043**	2940.71	12885.59	0.30	-22355.75
46	7526	8397	10.37	3333.63	50400.00	**7594**	1444.65	14286.08	0.90	-1888.98
47	13155	14201	7.37	4001.59	50400.00	**13272**	3170.69	16644.35	0.88	-830.90
48	10393	11364	8.54	12463.82	50400.00	**10801**	7589.21	21507.41	3.78	-4874.61
49	14520	16092	9.77	19958.25	50400.00	**14524**	14769.07	50400.00	0.03	-5189.18
50	10061	11650	13.64	45357.96	50400.00	**10376**	15861.83	50400.00	3.04	-29496.13
Average	**9166**	**9870**	**6.67**	**13404.78**	**44445.03**	**9190**	**3259.80**	**13948.99**	**0.21**	**-10144.99**

solutions were proved to be optimal ($Gap^e = 0\%$) by the exact method. All these solutions were obtained with the heuristic in much less time. For 17 instances, the heuristic found better solutions than the exact method. The improvement in solution value varies between 0.06% and 3.78%. For the seven instances where the solution obtained with the exact method was better, the variation in solution values ranges between 0.12% and 1.36% at the expense of an average increase in computing times of $5,851$ seconds. Finally, the average results obtained for the 50 instances show that the heuristic slightly decreases solution value (a loss of 0.21% on average) but considerably reduces computing times with an absolute gain of 101,44 seconds on average.

7 Conclusion

In this paper, we have addressed the BCP with stochastic prices for combinatorial transportation procurement auctions. A two-phase solution approach was proposed in which two problems were sequentially solved. The first problem is a contracts' selection problem (CSP) with stochastic prices that helps the carrier pre-select a set of interesting contracts to bid on. The second problem is a stochastic contracts' selection and pricing problem (CSPP) which allows the carrier to simultaneously generate contracts packages and associated ask-prices. The first phase, although not mandatory, allows reducing the size of the CSPP so that it could be tackled by the exact solution approach we proposed.

To the best of our knowledge, our paper is the first to address and solve a BCP with stochastic prices with an exact solution method that cleverly handles non-linearity in chance constraints without exhaustively enumerating all contracts combinations. It is also the first to address a stochastic BCP in which multiple OR bids are generated. A future research avenue would be to develop acceleration strategies to faster improve upper-bounds and reduce the computing time required to solve relaxed problems through the algorithm iterations.

Table 6: Sensivity analysis for the exact method

Ins	$\left(\gamma = 5, \eta = \left\lceil \frac{\lvert K_n^* \rvert}{\lvert \mathfrak{B} \rvert} \right\rceil\right)$						$(\gamma = 5, \alpha = 5\%)$						$(\eta = 5, \alpha = 5\%)$						$\lvert K_n^* \rvert$
	$\alpha = 1\%$		$\alpha = 5\%$		$\alpha = 10\%$		$\eta = 3$		$\eta = 4$		$\eta = 5$		$\gamma = 3$		$\gamma = 4$		$\gamma = 5$		
	Gap	**CPU**	**Gap**	**CPU**	**Gap**	**CPU**	**Gap**	**CPU**	**Gap**	**CPU**	**Gap**	**CPU**	**Gap**	**CPU**	**Gap**	**CPU**	**Gap**	**CPU**	
1	0.00	7.24	0.00	13.84	0.00	6.72	0.00	55.93	0.00	155.15	0.00	120.89	0.00	807.15	0.00	297.82	0.00	120.89	9
2	0.00	123.03	0.00	44.41	0.00	41.69	0.00	242.77	0.00	208.88	0.00	589.92	0.00	1516.26	0.00	432.84	0.00	589.92	9
3	3.90	43200.00	3.68	43200.00	4.72	43200.00	6.09	43200.00	3.68	43200.00	7.91	43200.00	0.47	43200.00	3.49	43200.00	7.91	43200.00	17
4	0.00	7239.25	0.00	6532.51	0.00	6778.92	0.00	6532.51	5.04	43200.00	2.54	43200.00	3.15	43200.00	3.11	43200.00	2.54	43200.00	15
5	4.94	43200.00	6.44	43200.00	4.32	43200.00	6.80	43200.00	6.44	43200.00	9.03	43200.00	2.35	43200.00	2.75	43200.00	9.03	43200.00	18
6	6.34	43200.00	2.09	43200.00	2.95	43200.00	3.94	43200.00	2.09	43200.00	2.99	43200.00	5.66	43200.00	3.17	43200.00	2.99	43200.00	18
7	0.00	1.26	0.00	0.81	0.00	1.32	0.00	1.07	0.00	1.07	0.00	0.99	0.00	0.49	0.00	0.56	0.00	0.99	2
8	0.00	1.07	0.00	0.82	0.00	1.32	0.00	1.21	0.00	1.42	0.00	1.05	0.00	0.88	0.00	0.97	0.00	1.05	2
9	0.00	39.25	0.00	221.72	0.00	65.98	0.00	101.16	0.00	249.57	0.00	568.56	0.00	1241.59	0.00	805.72	0.00	568.56	7
10	0.00	32.87	0.00	35.45	0.00	78.49	0.00	191.27	0.00	359.43	0.00	54.07	0.00	81.72	0.00	35.18	0.00	54.07	11
11	0.00	11036.51	0.00	10532.65	0.00	10506.82	0.00	10532.65	0.00	35892.04	0.00	41588.02	3.13	43200.00	0.00	42892.71	0.00	41588.02	12
12	0.00	20894.20	0.00	21314.06	0.00	19569.27	0.00	21314.06	0.00	27869.01	0.00	30336.71	0.00	41966.71	0.00	40009.48	0.00	30336.71	12
13	0.00	301.84	0.00	220.71	0.00	301.47	0.00	397.03	0.00	990.02	0.00	1488.64	0.00	4892.07	0.00	3167.27	0.00	1488.64	8
14	0.00	861.93	0.00	34.85	0.00	486.05	0.00	6116.31	0.00	1729.08	0.00	2584.12	0.00	7836.84	0.00	5416.13	0.00	2584.12	9
15	0.00	1059.20	0.00	1324.03	0.00	1914.78	0.00	1324.03	0.00	12207.95	0.00	35697.05	3.56	43200.00	5.02	43200.00	0.00	35697.05	12
16	0.00	3702.51	0.00	1914.69	0.00	1898.61	0.00	1914.69	0.00	37504.77	0.00	39571.09	3.71	43200.00	0.00	42099.42	0.00	39571.09	13
17	0.00	1060.90	0.00	477.44	0.00	1428.39	0.00	4587.51	0.00	10831.68	0.00	1685.25	0.00	3727.25	0.00	1112.29	0.00	1685.25	13
18	0.00	744.26	0.00	433.39	0.00	308.22	0.00	5448.43	0.00	8165.60	0.00	11555.42	0.00	8111.20	0.00	8092.03	0.00	11555.42	7
19	0.00	12883.69	0.00	12601.71	0.00	13003.87	0.00	12601.71	0.00	18504.94	0.00	22408.24	0.00	34589.08	0.00	33157.05	0.00	22408.24	12
20	0.00	5933.24	0.00	7030.67	0.00	8521.24	0.00	7030.67	0.00	15479.22	0.00	34995.07	2.46	43200.00	0.00	37004.82	0.00	34995.07	12
21	0.00	1.84	0.00	1.46	0.00	2.17	0.00	3.20	0.00	12.04	0.00	7.41	0.00	6.98	0.00	5.87	0.00	7.41	4
22	0.00	1.64	0.00	0.88	0.00	1.99	0.00	4.79	0.00	103.37	0.00	7.06	0.00	4.95	0.00	5.14	0.00	7.06	4
23	0.00	251.51	0.00	63.68	0.00	159.20	0.00	533.01	0.00	131.05	0.00	485.00	0.00	1063.71	0.00	579.54	0.00	485.00	8
24	0.00	27.37	0.00	87.12	0.00	33.46	0.00	209.65	0.00	71.43	0.00	1028.74	0.00	2689.04	0.00	1895.50	0.00	1028.74	9
25	0.00	13548.17	0.00	13441.67	0.00	13265.05	0.00	13441.67	0.00	19983.71	0.00	20489.10	2.89	43200.00	0.00	32377.61	0.00	20489.10	12
26	0.00	925.21	0.00	599.90	0.00	537.29	0.00	599.90	1.56	43200.00	3.50	43200.00	3.24	43200.00	4.23	43200.00	3.50	43200.00	13
27	0.00	4.05	0.00	35.62	0.00	4.15	0.00	19.25	0.00	49.45	0.00	18.97	0.00	21.85	0.00	109.08	0.00	18.97	3
28	0.00	20.55	0.00	33.39	0.00	17.09	0.00	79.89	0.00	79.80	0.00	79.51	0.00	25.15	0.00	76.93	0.00	79.51	5
29	0.00	4857.29	0.00	4291.02	0.00	4897.11	0.00	9212.16	0.00	13047.87	0.00	10060.75	0.00	18210.18	0.00	5594.66	0.00	10060.75	8
30	0.00	3139.81	0.00	262.53	0.00	301.91	0.00	30693.48	0.00	10384.15	0.00	8425.77	0.00	24891.27	0.00	1465.91	0.00	8425.77	8
31	0.00	11463.10	0.00	10701.13	0.00	10253.69	0.00	10701.13	0.00	25192.25	0.00	34801.53	2.21	43200.00	0.00	41662.81	0.00	34801.53	12
32	0.00	8036.51	0.00	7817.58	0.00	6924.61	0.00	7817.58	0.00	21342.56	0.00	39523.07	1.84	43200.00	0.00	42930.01	0.00	39523.07	12
33	0.00	422.08	0.00	213.06	0.00	645.52	0.00	3086.43	0.00	615.14	0.00	1936.06	0.00	3049.21	0.00	2433.71	0.00	1936.06	8
34	0.00	845.25	0.00	1451.06	0.00	518.06	0.00	2441.04	0.00	2145.47	0.00	1173.49	0.00	2934.28	0.00	1193.28	0.00	1173.49	8
35	0.00	4788.95	0.00	4721.88	0.00	4503.80	0.00	4721.88	0.00	7056.44	0.00	31489.92	5.09	43200.00	5.44	43200.00	0.00	31489.92	12
36	0.00	15026.41	0.00	14576.88	0.00	13982.05	0.00	14576.88	0.00	18028.17	0.00	29740.47	0.00	38995.42	0.00	36091.51	0.00	29740.47	14
37	0.00	16259.08	0.00	16737.05	0.00	16251.02	2.40	43200.00	0.00	16737.05	1.45	43200.00	0.59	43200.00	0.06	43200.00	1.45	43200.00	16
38	0.00	17056.27	0.00	16007.13	0.00	16852.61	3.97	43200.00	0.00	16007.13	7.90	43200.00	6.25	43200.00	6.29	43200.00	7.90	43200.00	16
39	0.00	149.73	0.00	7138.29	0.00	155.51	0.00	256.15	0.00	6169.82	0.00	1241.58	0.00	2884.84	0.00	2589.47	0.00	1241.58	7
40	0.00	185.89	0.00	277.32	0.00	311.61	0.00	813.10	0.00	1753.80	0.00	1898.89	0.00	5493.13	0.00	3483.20	0.00	1898.89	7
41	0.00	300.52	0.00	871.73	0.00	217.62	0.00	15258.17	0.00	3720.50	0.00	12253.74	0.00	27634.44	0.00	13555.07	0.00	12253.74	9
42	0.00	17537.61	0.00	18448.22	0.00	16357.18	0.00	18448.22	0.00	17593.05	0.14	43200.00	3.84	43200.00	1.35	43200.00	0.14	43200.00	11
43	0.00	9334.42	0.00	7050.13	0.00	10064.27	0.00	7050.13	2.86	43200.00	3.68	43200.00	0.61	43200.00	5.55	43200.00	3.68	43200.00	14
44	0.00	8025.69	0.00	6391.92	0.00	9001.05	0.00	6391.92	8.25	43200.00	3.62	43200.00	5.48	43200.00	5.32	43200.00	3.62	43200.00	15
45	0.00	9572.68	0.00	10088.40	0.00	10362.25	0.00	10088.40	1.34	43200.00	5.96	43200.00	6.15	43200.00	5.99	43200.00	5.96	43200.00	14
46	0.00	14486.04	0.00	13089.84	0.00	16009.82	0.00	13089.84	0.00	33610.04	0.00	35710.66	0.00	41253.73	0.00	39256.07	0.00	35710.66	15
47	0.00	20536.29	0.00	13688.07	0.00	20728.21	0.00	13688.07	0.00	26891.06	0.00	38400.94	5.52	43200.00	5.32	43200.00	0.00	38400.94	15
48	0.00	15204.11	0.00	14835.39	0.00	16258.14	0.00	14835.39	1.08	43200.00	3.68	43200.00	3.64	43200.00	3.43	43200.00	3.68	43200.00	12
49	11.60	43200.00	8.97	43200.00	4.31	43200.00	7.99	43200.00	8.97	43200.00	8.40	43200.00	6.56	43200.00	1.56	43200.00	8.40	43200.00	20
50	0.00	25995.79	0.00	23909.59	0.00	25047.29	6.57	43200.00	0.00	23909.59	6.09	43200.00	4.78	43200.00	6.05	43200.00	6.09	43200.00	19
Average	**0.54**	**9134.52**	**0.42**	**8847.31**	**0.33**	**9027.54**	**0.75**	**11577.08**	**0.82**	**17335.69**	**1.34**	**21936.36**	**1.66**	**25350.59**	**1.36**	**23484.59**	**1.34**	**21936.36**	-

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