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Location-Routing Models for Designing a Two-Echelon Freight Distribution System

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Abstract. In this paper, we address the decision problem of designing a two-echelon freight distribution system for an urban area. The scope of the paper is twofold. At first it describes a two-level distribution system focusing on its structure, components and related decision problems. Then the problem arising in this context is formulated as a two-echelon location-routing problem. Three mixed integer programming models will be proposed, aimed at defining location and number of two kinds of capacitated facilities, size of two different vehicle fleets and related routes. An instance generator has been developed and results of proposed models on a wide set of small and medium instances are reported, comparing their performances in terms of quality of solution and computation times.

Keywords. Two-echelon freight distribution system, two-echelon location-routing.

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1 Introduction

Logistic strategic decisions consist in defining the location, number and type of facilities of a distribution system, customer assignments to these facilities, transportation modes and vehicle routes. These decisions significantly affect the total logistic cost and customer service level. Most of literature on logistic problems tackles freight distribution within the context of supply chain optimization. However, in the last years, researchers and practitioners linked logistics, freight distribution and traffic management so generating the idea of *City logistics* (*CL*), which is defined as strategical, tactical and operational planning of freight flows within city limits. Main target of *CL* is minimizing negative effects due to freight distribution (as congestion, environmental nuisances, etc.) in order to improve sustainability, mobility, and quality of life of urban areas. This may be obtained through better fleet management practices, rationalization of distribution activities, traffic control, freight consolidation/coordination, and deployment of *intermediate facilities*. This last measure plays a fundamental role in most of *CL* projects based on distribution system, where transportation to and from an urban area is performed through *platforms*, also called *City Distribution Centers(CDC)*, located far from city limits. Freights, directed to a city and arriving by different transportation modes and vehicles, are consolidated at platforms on trucks in charge of final distribution. Platforms had a great impact on effectiveness of freight distribution in urban areas, but their use is showing some deficiencies because of two main reasons: their position (often far from final customers) and the constrained structure of urban areas. To overcome the limits of such *single-echelon* system, distribution systems have been proposed, where another intermediate level of facilities is added between platforms and final customers. These facilities, referred as *satellites* or *transit points*, perform no storage activities and are devoted to transfer and consolidate freights coming from platforms on trucks into *smaller* vehicles, more suitable for distribution in city centers ([7] and [9]). This *two-echelon* system could determine an increase of costs due to additional operations at satellites. Anyway these costs should be compensated by freight consolidation, decrease of empty trips, economy of scale, reduction of traffic congestion and environmental safety.

In this distribution system a two-echelon location-routing problem (*2E-LRP*) can be defined. In literature no exact or heuristic methods have been proposed for this new class of location-routing problems. The objective of this modeling paper is twofold. At first it provides a description of the described system focusing on its structure, components and related decision problems. Then a basic version of the *2E-LRP* is defined and three mixed-integer programming models are proposed in order to explore modeling issues and compare them.

The paper is structured as follows. In Section *2* the two-echelon freight distribution system is described with the assumptions required for the basic *2E-LRP*. In Section *3* we provide a literature review about main modeling approaches for two-echelon location and routing problems. Section *4* is devoted to presentation of three mixed integer programming models derived from previous *VRP* formulations, comparing them and highlighting weakness and strength points. Proposed models tackle the basic *2E-LRP*, i.e. single-commodity, single sourcing, static, deterministic problem with capacity constraints and without time constraints. They are aimed at defining location and number of two different kinds of facilities, platforms and satellites, size of two different vehicle fleets and related routing on each echelon. Finally, Section *5* describes an ad-hoc developed instance generator and presents results obtained experiencing and comparing proposed models on a wide set of small and medium size instances, differing for spatial facility distribution, by XPRESS-MP solver.

2 Problem Definition

In the following we describe the general structure of proposed two-echelon freight distribution system for an urban area and then we present the assumptions required to define the basic problem addressed in this paper.

2.1 System components and interactions

In a two-echelon freight-distribution system platforms, satellites and customers interact, linked by two or more vehicle fleets performing distribution:

1. *Primary facilities or platforms*: high capacitated facilities generally located far from final customers. At this locations, freight is broken and charged on trucks (referred as *first echelon trucks* or *urban trucks*) of different sizes and characteristics, which perform distribution to successive levels. Each urban truck serves one or more satellites or customers and then travels back to platforms to start a new route.

2. *Secondary facilities or satellites*: low capacitated facilities devoted to transshipment operations. Freights arriving from platforms on urban trucks are transferred on smaller vehicles (referred as *second echelon vehicles* or *city freighters*), more suitable for distribution in congested urban areas. Each city freighter performs a route to serve several customers and in general travels back to satellites to start a new route. Each satellite is served by at least one platform and one urban truck.

3. *Customers*: end points of distribution (shops, retailers, etc.), mostly located in city center. Each customer is served by at least one truck or vehicle coming from upper levels.

Freight movements among the two echelons, capacity constraints and different vehicle fleets give a first idea of decision problem complexity. Moreover the problem of designing a two-echelon freight distribution system is made even harder by all other parameters, constraints, relations and interactions among system components which should be taken into account to make the system effectively work. These additional features can be classified according to:

- ***Freight typology.*** Freight flows directed to an urban area could be very heterogeneous (multi-commodity). This imply several difficulties concerning complete knowledge of origin-destination demand for each commodity, freight consolidation operations, usage of vehicles and sourcing strategies (*single or multi sourcing*).
- ***Time constraints.*** Two main kinds of constraints should be considered: *time-windows* constraints for satellites and final customers, and *synchronization* constraints, requiring fleets of the two-echelons to be well coordinated to avoid long waiting time at satellites. These aspects are well treated in [9].
- ***Time dependency.*** A freight distribution system could be designed considering a single period (*static problem*) or a multi-period horizon (*dynamic problem*), to take into account variability of some parameters.
- ***Data uncertainty.*** Several input data for the problem as customer demands, traveling time, transportation costs, etc., so as problem variables could be not always deterministic, especially over multi-period horizon (*stochastic problem*).

Hence, we can say that the definition of a two-echelon freight distribution system is a very hard problem, where many strategic decisions have to be taken into account simultaneously. In its more complex form, it could be a *multi-commodity, multi-sourcing, dynamic, stochastic problem with time-windows and synchronization constraints.*

2.2 Assumptions for basic two-echelon freight distribution problem

According to the goal of the paper, in this section we start to explore the proposed new class of location-routing problems by tackling the basic *2E-LRP*, for which the following assumptions are required:

- A single substitute product is considered (*single-commodity*). Demand of each customer is known in advance (*deterministic*) and referred to a *single planning period.*

- All freight starts at platforms. Distribution cannot be managed by direct shipping from platforms to customers, but freight must be first consolidated at satellites. In particular: 1^{st} echelon routes start from a platform, serve one or more satellites and ends to the same platform; 2^{nd} echelon routes start from a satellite, serve one or more customers and ends to the same satellite.
- Platforms and satellites are capacitated. Platform capacity is much higher than satellite capacity. Facilities belonging to the same echelon can have different capacity.
- Each satellite has to be served by a single platform and by a single urban truck. Each customer has to be served by a single satellite and by a single vehicle (*single sourcing at both echelons*).
- Vehicles operating in the same echelon have the same capacity value. Capacity of each urban truck is higher than capacity of each city freighter and is higher or equal to capacity of each satellite. Capacity of each city freighter is much higher than demand of each customers.
- No time windows and synchronization constraints are considered.

The problem thus consists in:

- *Location decisions*: define number and locations of platforms and satellites;
- *Allocation decisions*: assign customers to each open satellite and open satellites to open platforms;
- *Fleet size and routing decisions*: number of vehicles to use for distribution in each echelon and related routes.

The defined problem is a *basic two-echelon location-routing problem* (*2E-LRP*). It is *single-commodity, single sourcing, static, deterministic problem with capacity constraints and without time constraints.* A schematic representation of such *2E-LRP* is provided in Figure 1.

3 Literature Review

2E-LRP is easily seen to be *NP-hard* via a reduction to facility location and vehicle routing problems (*FLP* or *VRP*). Indeed if we require customers and satellites to be directly linked to a facility, becomes a standard multi-level facility location problem.

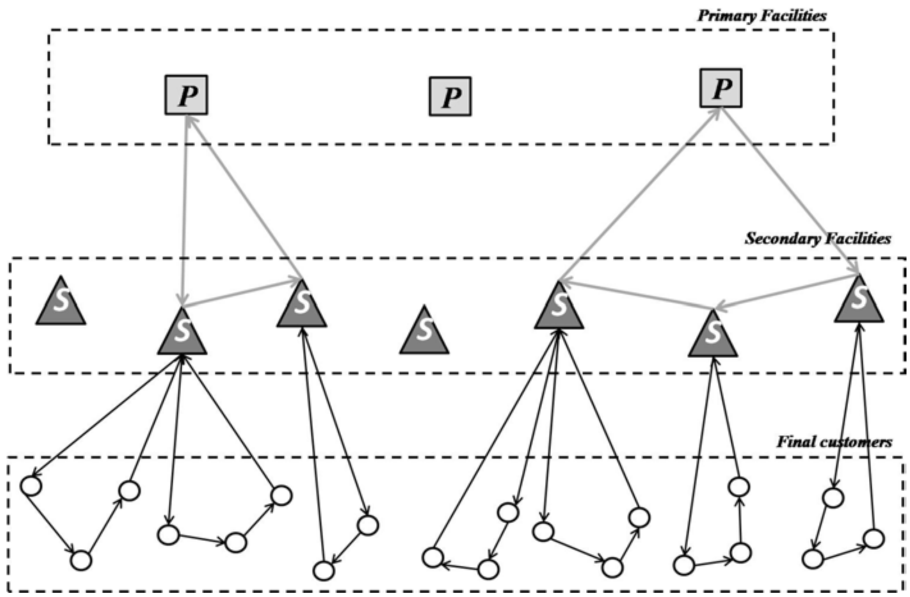

Figure 1: Two-echelon location routing problem representation.

If, on the other hand, we fix facility locations, *2E-LRP* reduces to a multi-level vehicle routing problem.

As previously said no contributions are available in literature for *2E-LRP*. For this reason, in the following we review literature about close problems as single and multi-echelon problems classified in three main research streams: two-echelon facility location (*2E-FLP*), two-echelon vehicle routing (*2E-VRP*) and two-echelon location-routing (*2E-LRP*).

Many variants of single and multi-level location-allocation problems have been proposed in literature within context of supply chain network. *FLP* in its more general form consists in determining number and location of one or more kind of facilities, minimizing location and transportation costs and satisfying capacity constraints. In these models transportation costs are approximated with direct distance between nodes of successive levels, given the assumption that each level is served by near full load trucks, which perform a dedicated route. A complete overview of *FLP* literature, exact and heuristic methods, can be obtained in many survey papers and among them we cite [30], [14], [25], [26] and [19].

In the last years *2E-VRP* models have been proposed aimed at defining first and second echelon routes minimizing transportation costs and satisfying demand of customers without violating vehicle capacity constraints. This problem arises to overcome limits of previous literature which explicitly considered routing problem only at the last level of a distribution system, while a simplified routing problem was considered at higher levels, so underestimating routing costs. Literature about *2E-VRP*, specifically referred to systems composed of platforms and satellites, is recent and started with [22], where a flow based model for the *2E-VRP*, several valid inequalities and two-math heuristics are proposed. Recent developments for this problem are in [8], [9] and [23]. In particular in [8] a clustering heuristic able to solve large instances is proposed. This method is based on the decomposition of the *2E-VRP* in two routing sub-problems, one for each echelon. In [9] the day-before planning problem arising in the two-echelon freight distribution system proposed in [7] is tackled. The aim is the determination of the fleet size performing the distribution on the two echelons, satisfying time windows and synchronization constraints between the two echelons. In [23] new classes of traveling salesman problem *TSP* and capacitated vehicle routing problem *CVRP* based valid inequalities are proposed for the flow based formulation proposed in [22].

In many cases distribution costs are significantly affected by facility locations. Indeed location and routing decisions are strongly interrelated and have to be modeled and optimized simultaneously. Inappropriateness of approaching them through pure location models has been pointed out in several papers ([31], [6], [11], [32], [24], [27], [5]). The idea of combining two decisional levels, strategical and tactical, for a transportation system dates back to 1960 [18], but a greatest number of papers on generalized *LRP* started to appear just from the '80s. A complete overview of *LRP* literature, exact and heuristic approaches, can be obtained in some survey papers and among them we cite [2], [15], [16], [20] and [21]. Laporte [15] provided a definition of location-routing problems and a classification based on number of interacting levels and kind of routes connecting them. Laporte introduced the following expression to synthetically represent a location-routing problem: $\lambda/M_1/.../M_{\lambda-1}$, where λ is the number of layers and $M_1/.../M_{\lambda-1}$ are kind of tours among two consecutive layers. *R routes* are dedicated routes, whereas *T routes* are multiple node routes. An *LRP* is characterized by location decisions and *T routes* at least for one level. We propose a simple enhancement of this notation to clearly indicate the number of levels where location decisions have to be taken into account. We mark a route identification letter (R or T) with an *overline* if location decisions have to be taken on the same echelon. By default location decisions concern always the starting point of the routes. For example with the expression $3/\overline{R}/\overline{T}$, we refer to a problem with three layers, location decisions for primary and secondary facilities, R routes between first and second level (first echelon) and T routes between second and third level (second echelon). With expression $3/R/\overline{T}$ location decisions involve just secondary facilities.

The literature on multi-level location-routing problems is very limited. To the best of our knowledge the only contributions on this topic are in [13], [17], [4], [28], [1] and [3]. In [13], [17] and [4] the $3/T/\overline{T}$ problem is tackled by respectively two sequential heuristics and a clustering heuristic. In [28] a $3/T/\overline{T}$ problem is tackled by exact and heuristic approaches. A mathematical model is proposed to determine the location and the size of satellites, referred as *public logistic terminals*. Satellites are considered as small platforms and not just as transshipment points. In [1] a complex $4/R/\overline{T}/\overline{T}$ is tackled by mixed-integer programming models. different formulations are proposed for a distribution network design problem with location decisions for primary and secondary facilities in static and dynamic scenarios, extending three-index arc formulations proposed in [24] and three index flow formulation proposed in [12]. Finally in [3] a $3/\overline{T}/\overline{T}$ problem is tackled by a tabu search metaheuristic where the *2E-LRP* is decomposed in four subproblems, one *FLP* and one *VRP* for each echelon. Then the four sub-problems are sequentially and iteratively solved and their solutions are opportunely combined in order to determine a good global solution.

4 Two-echelon Location-Routing Formulations

In this section three formulations for *2E-LRP* are proposed, differing for kind and number of routing variables. First and third model derive directly from classical *VRP* formulations proposed in literature [29]. Second model, instead, is based on a multi-depot vehicle-routing formulation (*MDVRP*) proposed in [10].

4.1 Problem setting

Based on assumptions of section *2* and on Laporte's notation, the addressed problem of designing a two-echelon freight distribution system can be referred as a $3/\overline{T}/\overline{T}$ problem and is described by a multi-level network $\mathcal{G} = \{\mathcal{N}, \mathcal{A}\}$, where:

$\mathcal{N} = \{\mathcal{P} \cup \mathcal{S} \cup \mathcal{Z}\}$
$\mathcal{P} = \{p\}$: set of the possible platform locations;
$\mathcal{S} = \{s\}$: set of the possible satellite locations,
$\mathcal{Z} = \{z\}$: set of customers whose positions are fixed and known in advance.

$\mathcal{A} = \{\mathcal{A}^1 \cup \mathcal{A}^2\}$
$\mathcal{A}^1$: set of first echelon arcs from node i to node j , $i \in \mathcal{P} \cup \mathcal{S}, j \in \mathcal{S}$;
$\mathcal{A}^2$: set of second echelon arcs from node i to node j , $i, j \in \mathcal{S} \cup \mathcal{Z}$.

Two sets of trucks and vehicles are considered:
$\mathcal{T} = \{t\}$: set of *urban trucks*;
$\mathcal{V} = \{v\}$: set of *city freighters.*

The input data are the following:
$K_i, i \in \mathcal{P} \cup \mathcal{S}$: facility capacity value;
$k^i, i \in \mathcal{T} \cup \mathcal{V}$: capacity of urban trucks and city freighters;
$H_i, i \in \mathcal{P} \cup \mathcal{S}$: facility location cost, depending on related capacity;
$h^i, i \in \mathcal{T} \cup \mathcal{V}$: cost for using a truck;

$D_z, z \in \mathcal{Z}$: demand of each client.
C_{ij} : transportation costs for moving a vehicle from node i to node j on an arc. For first echelon $i \in \mathcal{P} \cup \mathcal{S}, j \in \mathcal{S}$. For second echelon i to node j , $i, j \in \mathcal{S} \cup \mathcal{Z}$.

Given this parameter setting, the only variables common to all proposed formulations are location ones:
$y_i = \{0, 1\}, i \in \mathcal{P} \cup \mathcal{S}$: *1*, if a facility is open at node i, *0* otherwise.

Additional variables and, in particular, routing and vehicle variables will be specifically defined for each formulation.

4.2 Three index formulation: *3i-2E-LRP*

This model has been inspired by multi-echelon *LRP* formulation proposed in [1]. Differences derive from problem setting and used subtour elimination constraints. The additional following sets of variables will be used:
- $r^t_{ij} = \{0, 1\}, i, j \in \mathcal{P} \cup \mathcal{S}, t \in \mathcal{T}$: *1*, if i precedes j in first echelon route, performed by a urban truck t, *0* otherwise;
- $x^v_{ij} = \{0, 1\}, i, j \in \mathcal{S} \cup \mathcal{Z}, v \in \mathcal{V}$: *1*, if i precedes j in second echelon route, performed by a city freighter v, *0* otherwise;
- $w_{sz} = \{0, 1\}, s \in \mathcal{S}, z \in \mathcal{Z}$: *1*, if customer z is assigned to satellite s, *0* otherwise;
- $u_i = \{0, 1\}, i \in \mathcal{T} \cup \mathcal{V}$: *1*, if a truck i is used on an echelon, *0* otherwise.
- $f^t_{ps} \geq 0, p \in \mathcal{P}, s \in \mathcal{S}, t \in \mathcal{T}$: flow from platform p to satellite s on urban truck t.

This formulation will be referred as three-index formulation since routing variables (r^t_{ij} and x^v_{ig}) are defined using three indices. A representation of parameters and variables is shown in Figure 2 (variables are indicated in lower case and bold letters).

The problem can be formulated as follows:

$$\text{Minimize} \quad \sum_{p\in\mathcal{P}} H_p\, y_p + \sum_{s\in\mathcal{S}} H_s\, y_s + \sum_{t\in\mathcal{T}} h^t\, u_t + \sum_{v\in\mathcal{V}} h^v\, u_v + \\ + \sum_{v\in\mathcal{V}} \sum_{i\in\mathcal{S}\cup\mathcal{Z}} \sum_{j\in\mathcal{S}\cup\mathcal{Z}} C_{ij}\, x^v_{ij} + \sum_{t\in T} \sum_{i\in\mathcal{P}\cup\mathcal{S}} \sum_{j\in\mathcal{P}\cup\mathcal{S}} C_{ij}\, r^t_{ij} \tag{1}$$

subject to

$$\sum_{v\in\mathcal{V}} \sum_{j\in\mathcal{S}\cup\mathcal{Z}} x^v_{zj} = 1 \qquad \forall z \in \mathcal{Z} \tag{2}$$

$$\sum_{l\in\mathcal{S}\cup\mathcal{Z}} x^v_{lj} - \sum_{l\in\mathcal{S}\cup\mathcal{Z}} x^v_{jl} = 0 \qquad \forall j \in \mathcal{Z} \cup \mathcal{S}, \forall v \in \mathcal{V} \tag{3}$$

$$L_i - L_j + (|S| + |Z|) \sum_{v\in\mathcal{V}} x^v_{ij} \leq (|S| + |Z| - 1) \qquad \forall i, j \in \mathcal{Z} \cup \mathcal{S}, i \neq j \tag{4}$$

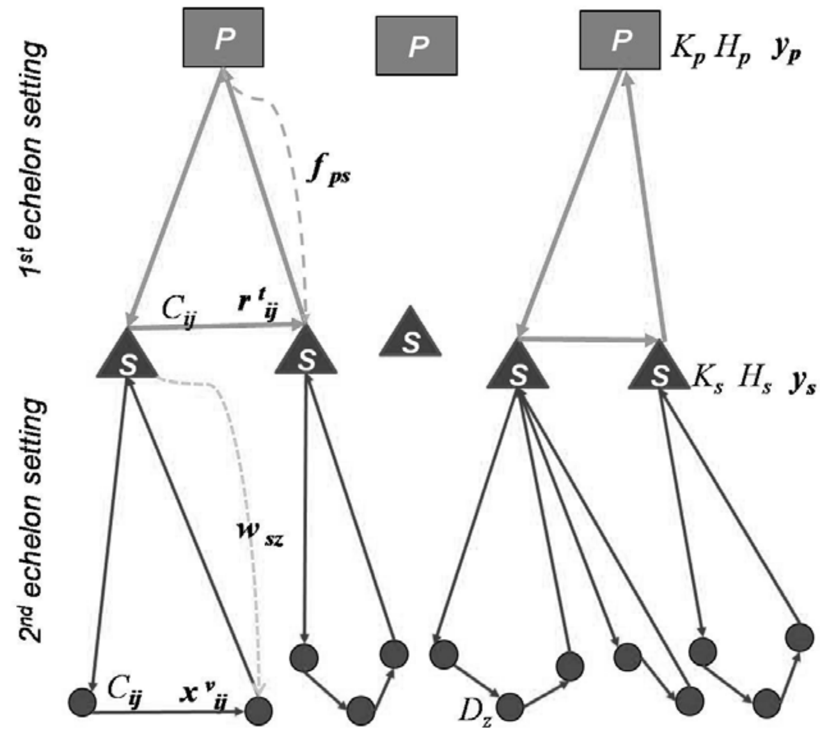

Figure 2: Variables and parameters of *2E-LRP* three-index formulation.

$$\sum_{l \in \mathcal{S} \cup \mathcal{Z}} \sum_{j \in \mathcal{S}} x^v_{lj} \leq 1 \qquad \forall v \in \mathcal{V} \tag{5}$$

$$\sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{P} \cup \mathcal{S}} r^t_{lj} = y_l \qquad \forall l \in \mathcal{S} \tag{6}$$

$$\sum_{l \in \mathcal{P} \cup \mathcal{S}} r^t_{lh} - \sum_{l \in \mathcal{P} \cup \mathcal{S}} r^t_{hl} = 0 \qquad \forall h \in \mathcal{P} \cup \mathcal{S}, \forall t \in \mathcal{T} \tag{7}$$

$$L_i - L_j + (|P| + |S|) \sum_{t \in \mathcal{T}} r^t_{ij} \leq (|P| + |S| - 1) \qquad \forall i, j \in \mathcal{S} \cup \mathcal{P}, i \neq j \tag{8}$$

$$\sum_{l \in \mathcal{P} \cup \mathcal{S}} \sum_{j \in \mathcal{P}} r^t_{lj} \leq 1 \qquad \forall t \in \mathcal{T} \tag{9}$$

$$\sum_{h \in \mathcal{S} \cup \mathcal{Z}} x^v_{zh} + \sum_{h \in \mathcal{S} \cup \mathcal{Z}} x^v_{sh} - w_{zs} \leq 1 \qquad \forall z \in \mathcal{Z}, \forall v \in \mathcal{V}, \forall s \in \mathcal{S} \tag{10}$$

$$\sum_{s \in \mathcal{S}} w_{zs} = 1 \qquad \forall z \in \mathcal{Z} \tag{11}$$

$$\sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} f^t_{ps} - \sum_{z \in \mathcal{Z}} D_z \, w_{zs} = 0 \qquad \forall s \in \mathcal{S} \tag{12}$$

$$\sum_{s \in \mathcal{S}} f_{ps} - K_p \, y_p \leq 0 \qquad \forall p \in \mathcal{P} \tag{13}$$

$$\sum_{p\in\mathcal{P}}\sum_{t\in\mathcal{T}} f^t_{ps} - K_s\ y_s \leq 0 \qquad \forall s \in \mathcal{S} \tag{14}$$

$$k^t \sum_{h\in\mathcal{S}\cup\mathcal{P}} r^t_{sh} - f^t_{ps} \geq 0 \qquad \forall t \in \mathcal{T}, \forall s \in \mathcal{S}, \forall p \in \mathcal{P} \tag{15}$$

$$k^v \sum_{h\in\mathcal{S}\cup\mathcal{P}} r^t_{ph} - f^t_{ps} \geq 0 \qquad \forall t \in \mathcal{T}, \forall s \in \mathcal{S}, \forall p \in \mathcal{P} \tag{16}$$

$$\sum_{i\in\mathcal{Z}} D_z \sum_{j\in\mathcal{S}\cup\mathcal{Z}} x^v_{zj} \leq k^v\ u_v \qquad \forall v \in \mathcal{V} \tag{17}$$

$$\sum_{p\in\mathcal{P}}\sum_{s\in\mathcal{S}} f^t_{ps} \leq k_t\ u^t \qquad \forall t \in \mathcal{T} \tag{18}$$

$$\begin{aligned}
r^t_{ij} &= \{0,1\} && \forall i,j \in \mathcal{P}\cup\mathcal{S}, t \in \mathcal{T}\\
x^v_{ij} &= \{0,1\} && \forall i,j \in \mathcal{S}\cup\mathcal{Z}, v \in \mathcal{V}\\
w_{zs} &= \{0,1\} && \forall z \in \mathcal{Z}, s \in \mathcal{S}\\
y_p &= \{0,1\} && \forall p \in \mathcal{P}\\
y_s &= \{0,1\} && \forall s \in \mathcal{S}\\
u_t &= \{0,1\} && \forall t \in \mathcal{T}\\
u_v &= \{0,1\} && \forall v \in \mathcal{V}\\
f^t_{ps} &\geq 0 && \forall p \in \mathcal{P}, s \in \mathcal{S}, t \in \mathcal{T}
\end{aligned} \tag{19}$$

The objective function 1 is the sum of six cost components: location cost for platforms and satellites, fixed cost for usage of trucks, transportation cost on the two echelons. For what concerns model constraints, they can be classified in function of: routing of first and second echelon; flow conservation; facility and truck capacity; consistency constraints. Indeed constraints (2) impose that each customer z, $z \in \mathcal{Z}$, is served by exactly one city freighter v, $v \in \mathcal{V}$. Constraints (3) impose that for each vehicle v, $v \in \mathcal{V}$, number of arcs entering in a node i, $i \in \mathcal{Z}\cup\mathcal{S}$, is equal to number of arcs leaving the node. Constraints (4) are subtour elimination constraints, where L_i and L_j are continuous non-negative variables, imposing presence of at least a satellite in each route performed by a city freighter. Constraints (5) impose that each city freighter v, $v \in \mathcal{V}$, has to be assigned unambiguously to one satellite s, $s \in \mathcal{S}$, i.e. each vehicle can perform just one route. Constraints (6), (7), (8), (9) are routing constraints imposing on first echelon the same conditions that constraints (2), (3), (4) and (5) impose on second echelon. Constraints (12) are flow conservation constraints at satellites. Constraints (13) impose that flow leaving a platform p, $p \in \mathcal{P}$, has to be less than its own capacity if the facility is open. Constraints (14) impose that flow entering in a satellite $s \in \mathcal{S}$ has to be less than its own capacity if the facility is open. Constraints (17) impose that demand satisfied by city freighter v, $v \in \mathcal{V}$ has to be less than its own capacity if the vehicle is used. Constraints (18) impose that amount of flow transferred by an urban truck t, $t \in \mathcal{T}$, has to be less than its own capacity if the vehicle is used. Constraints (10) link allocation and routing variables. In fact by constraint 2 each customer is assigned to exactly one truck v. This constraint together with constraints 3, 4 and 5, imply that exactly one satellite is on the route of truck v. Therefore if we consider any given customer z^*, assigned to a route v^*, which contains also a satellite s^*, then $\sum_{h \in \mathcal{S} \cup \mathcal{Z}} x^{v^*}_{z^*h} = 1$ and $\sum_{h \in \mathcal{S} \cup \mathcal{Z}} x^{v^*}_{s^*h} = 1$, and consequently $w_{z^*s^*} = 1$. If customer is not on a route starting from satellite s^*, then constraints (10) are satisfied for both $w_{zs} = 0$ and $w_{zs} = 1$, but since each customer has to be assigned to just one satellite, then it will be assigned to the one satisfying its demand. Constraints (11) imposes that each customer z has to be assigned to a satellite s. These constraints are redundant, but allow to slightly improve the bounds of linear relaxations. Constraints (15) and (16) guarantee that amount of flow on a vehicle t, $t \in \mathcal{T}$, from a platform p, $p \in \mathcal{P}$, to a satellite s, $s \in \mathcal{S}$, is positive if and only if both satellite and platform are visited by the same vehicle t. In end constraints (19) express integrality constraints and non-negativity constraints.

Three index model presents a very flexible formulation and can tackle location-routing problems with both symmetric and asymmetric cost matrices. It can be extended to keep into account other features of the problem, as multi-commodity flows, introduction of time windows for customers, maximum length constraints, heterogeneous fleets, etc.. On the other side it is hard to solve, since it requires the definition of a large number of variables and constraints.

4.3 Two-index formulation: *2i-2E-LRP*

The two-index formulation for *2E-LRP* is an extension to the two-echelon case, integrated with location variables, of *MDVRP* formulation proposed in [10]. In this formulation routing constraints are defined with two-index variables, more precisely assignment variables and sequencing variables.

The additional integer and non negative variables used in this model are the following:
- $a_{zv} = \{0, 1\}, z \in \mathcal{Z}, v \in \mathcal{V}$: 1 if a customer z is assigned to a city freighter v, 0 otherwise;
- $b_{sv} = \{0, 1\}, s \in \mathcal{S}, v \in \mathcal{V}$: 1 if a city freighter v is assigned to a satellite s, 0 otherwise;
- $x_{ij} = \{0, 1\}, i, j \in \mathcal{Z} \mid i < j$: 1 if customer i is visited before customer j, 0 otherwise.

A single variable is defined for each couple of node, on the base of relative ordering of nodes, i.e. x_{ij} exists if $ord(i) < ord(j)$ (where $ord(i)$ indicates relative position of element i in customer set $\mathcal{Z}$). In this way number of sequencing variable is cut by half;
- $m_{st} = \{0, 1\}, s \in \mathcal{S}, t \in \mathcal{T}$: 1 if open satellite s, is assigned to urban truck t, 0 otherwise;
- $n_{pt} = \{0, 1\}, p \in \mathcal{P}, t \in \mathcal{T}$: 1 if a urban truck t is assigned to an open platform p, 0 otherwise;
- $r_{ij} = \{0, 1\}, i, j \in \mathcal{S} \mid i < j$: 1 if satellite i is visited before satellite j, 0 otherwise.

The above consideration on number of variables x_{ij} is valid also for r_{ij} variables;
- $w_{ij} = \{0,1\}$: *1*, if node i is assigned to node j, *0* otherwise. For first echelon $i \in \mathcal{S}, j \in \mathcal{P}$; for second echelon $i \in \mathcal{S}, j \in \mathcal{P}$
- $C^1(s) \geq 0, s \in \mathcal{S}$: routing cost on first echelon up to a satellite s starting from an open platform;
- $C^2(z) \geq 0, z \in \mathcal{Z}$: routing cost on second echelon up to a customer z starting from an open satellite;
- $C^V(v) \geq 0, v \in \mathcal{V}$: total routing cost for a city freighter v;
- $C^T(t) \geq 0, t \in \mathcal{T}$: total routing cost for an urban truck t.

A representation of involved parameters and variables of two-index formulation is shown in Figure 3

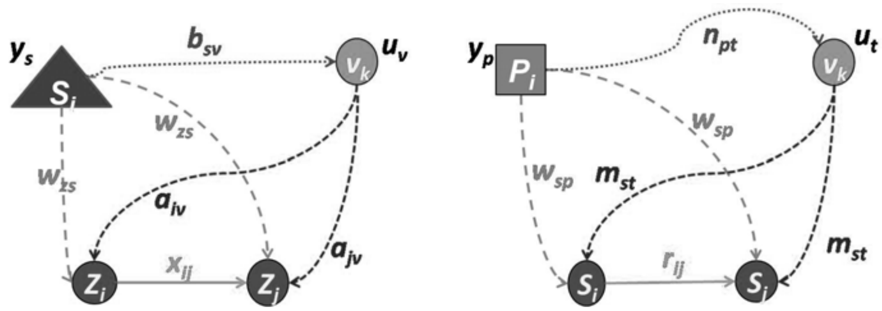

Figure 3: Variables of *2E-LRP* two-index formulation.

The two-index formulation for *2E-LRP* is the following:

$$\text{Minimize} \quad \sum_{p\in\mathcal{P}} H_p\ y_p + \sum_{s\in\mathcal{S}} H_s\ y_s + h^t \sum_{p\in\mathcal{P}} \sum_{t\in\mathcal{T}} n_{pt} +$$

$$+h^v \sum_{s\in\mathcal{S}} \sum_{v\in\mathcal{V}} b_{sv} + \sum_{v\in\mathcal{V}} C^V(v) + \sum_{t\in\mathcal{T}} C^T(t) \tag{20}$$

Subject to

$$\sum_{v\in\mathcal{V}} a_{zv} = 1 \qquad \forall z \in \mathcal{Z} \tag{21}$$

$$\sum_{s\in\mathcal{S}} b_{sv} \leq 1 \qquad \forall v \in \mathcal{V} \tag{22}$$

$$C^2(i) \geq C_{si}\,(b_{jv} + a_{iv} - 1) \qquad \forall i \in \mathcal{Z}, s \in \mathcal{S}, v \in \mathcal{V} \tag{23}$$

$$C^2(j) \geq C^2(i) + C_{ji} - M\,(1 - x_{ij}) - M\,(2 - a_{jv} + a_{iv}) \qquad \forall i,j \in \mathcal{Z}_{i<j}, v \in \mathcal{V} \tag{24}$$

$$C^2(i) \geq C^2(j) + C_{ij} - M\,(x_{ij}) - M\,(2 - a_{jv} + a_{iv}) \qquad \forall i,j \in \mathcal{Z}_{i<j}, v \in \mathcal{V} \tag{25}$$

$$C^V v \geq C^2(i) + C_{is} - M\,(2 - b_{sv} - a_{iv}) \qquad \forall i \in \mathcal{Z}, s \in \mathcal{S}, v \in \mathcal{V} \tag{26}$$

$$\sum_{t\in\mathcal{T}} m_{st} = y_s \qquad \forall s \in \mathcal{S} \tag{27}$$

$$\sum_{p \in \mathcal{P}} n_{pt} \leq 1 \qquad \forall t \in \mathcal{T} \tag{28}$$

$$C^1(i) \geq C_{ji}\left(n_{jt} + a_{it} - 1\right) \qquad \forall i \in \mathcal{S}, j \in \mathcal{P}, t \in \mathcal{T} \tag{29}$$

$$C^1(j) \geq C^1(i) + C_{ji} - M\left(1 - r_{ij}\right) - M\left(2 - a_{jt} + a_{it}\right) \qquad \forall i, j \in \mathcal{S}_{i<j}, t \in \mathcal{T} \tag{30}$$

$$C^1(i) \geq C^1(j) + C_{ij} - M\left(r_{ij}\right) - M\left(2 - a_{jt} + a_{it}\right) \qquad \forall i, j \in \mathcal{S}_{i<j}, t \in \mathcal{T} \tag{31}$$

$$C^T t \geq C^1(i) + c_{ip} - M\left(2 - b_{pt} - a_{it}\right) \qquad \forall i \in \mathcal{S}, p \in \mathcal{P}, t \in \mathcal{T} \tag{32}$$

$$\sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} f_{ps}^t - \sum_{z \in \mathcal{Z}} D_z \; w_{zs} = 0 \qquad \forall s \in \mathcal{S} \tag{33}$$

$$\sum_{z \in \mathcal{Z}} D_z \; w_{zs} \leq K_s \; y_s \qquad \forall s \in \mathcal{S} \tag{34}$$

$$\sum_{s \in \mathcal{S}} f_{ps}^t - K_p \; y_p \leq 0 \qquad \forall p \in \mathcal{P} \tag{35}$$

$$\sum_{z \in \mathcal{Z}} D_z \; a_{zv} \leq k^v \sum_{s \in \mathcal{S}} b_{sv} \qquad \forall v \in \mathcal{V} \tag{36}$$

$$a_{zv} + b_{sv} - w_{zs} \leq 1 \qquad \forall i \in \mathcal{Z}, s \in \mathcal{S}, v \in \mathcal{V} \tag{37}$$

$$k^t \; n_{pt} - f_{ps}^t \geq 0 \qquad \forall t \in \mathcal{T}, \forall s \in \mathcal{S}, \forall p \in \mathcal{P} \tag{38}$$

$$k^t \; m_{st} - f_{ps}^t \geq 0 \qquad \forall t \in \mathcal{T}, \forall s \in \mathcal{S}, \forall p \in \mathcal{P} \tag{39}$$

$$\begin{array}{rlrl}
a_{zv} = \{0,1\} & \forall z \in \mathcal{Z}, v \in \mathcal{V} & b_{sv} = \{0,1\} & \forall s \in \mathcal{S}, v \in \mathcal{V} \\
w_{zs} = \{0,1\} & \forall z \in \mathcal{Z}, s \in \mathcal{S} & x_{ij} = \{0,1\} & \forall i, j \in \mathcal{Z}_{(i<j)} \\
m_{st} = \{0,1\} & \forall s \in \mathcal{S}, t \in \mathcal{T} & n_{pt} = \{0,1\} & \forall p \in \mathcal{P}, t \in \mathcal{T} \\
r_{ij} = \{0,1\} & \forall i, j \in \mathcal{S}_{(i<j)} & y_s = \{0,1\} & \forall s \in \mathcal{S} \\
y_p = \{0,1\} & \forall p \in \mathcal{P} & &
\end{array} \tag{40}$$

The objective function (20) is aimed at minimizing the six component of the total cost. Constraints (21) and (22) assign respectively customers and satellites to a single city freighter. Constraints (23) defines the least cost for a city freighter to reach a customer. Constraints (24) and (25) define relationship between traveling costs up to nodes $i, j \in \mathcal{Z}$ on same tour. In fact being C_{ij} the least travel cost from node i to node j on vehicle v, if both nodes are on the same tour, i.e. ($y_{iv} = y_{jv} = 1$ and if node i is visited before, i.e. ($x_{ij} = 1$), then constraints (24) states that routing cost from the satellite to customer j, $C^2(j)$ must always be greater than $C^2(i)$, by at least C_{ij}. On the other side if node j is visited earlier ($x_{ij} = 0$), the reverse statement holds. Constraints from (27) to (31) are the just explained routing constraints but referred to first echelon. Constraints (33) are flow conservation constraints at open satellites. Constraints (34), (35) and (36) are capacity constraints related to respectively satellites, platforms and urban trucks.

Constraints (37) are consistency constraints between assignment variables. Constraints (38) and (39) are consistency constraints between flow and assignment variables. Finally constraints (40) are integrality and non-negativity constraints.

This formulation is suitable to solve symmetric problems. As three index formulation it requires the definition of a wide set of constraints and variables. Anyway the two described models present opposite features. Two-index formulation requires a minor number of variables but a greater number of constraints. To provide a comparison between the two formulations, we can say that for an instance with 2 platforms, 8 satellites, 20 customers, 5 urban trucks and 8 city freighters, the number of variables and constraints are respectively: 7063 variables and 2242 constraints for three-index formulation and 783 variables and 7552 constraints for two-index formulation.

4.4 One-index formulation: *1i-2E-LRP*

The one-index formulation, derived from *VRP*, is a path based formulation. It is an extension of single-echelon location routing formulation present in literature. In this formulation a variable is defined for all feasible routes. In our case two different sets of routes will be defined, one for each echelon $\mathcal{T}^1$ and $\mathcal{T}^2$. Let us also indicate as $\mathcal{T}_p^1$ the subset of $\mathcal{T}^1$ composed of routes starting from platform p. In order to formalize the problem we need to define the following variables:
- $x_i = \{0,1\}, i \in \mathcal{T}^2$: 1 if a second echelon route is selected, 0 otherwise;
- $r_i = \{0,1\}, i \in \mathcal{T}^1$: 1 if a first echelon route is selected, 0 otherwise.
- $f(i) \geq 0, i \in \mathcal{T}^1$: flow traveling on first echelon route i from a platform to a satellite.

Each path on the two echelons is associated to a cost indicated with C_i. In order to take into account which nodes belong to a first or a second echelon path, we will use the following path-node incidence matrix, respectively referred as A and B, whose generic element assumes the following values:
- $\alpha_{is} = \{0,1\}$ *1*, if a satellite s, $s \in \mathcal{S}$ is covered by first echelon route i, $i \in \mathcal{T}^1$, 0 otherwise;
- $\beta_{iz} = \{0,1\}$ *1*, if a customer z, $z \in \mathcal{Z}$ is covered by second echelon route i, $i \in \mathcal{T}^2$, 0 otherwise.

Since not all platform-satellite and satellite customer assignments are possible, then two incidence matrices have to be defined, referred as E and F, where:
- $\epsilon_{ps} = \{0,1\}$ *1*, if a satellite s, $s \in \mathcal{S}$ may be covered by a platform p, $p \in \mathcal{P}$, 0 otherwise;
- $\varphi_{sz} = \{0,1\}$ *1*, if a customer z, $z \in \mathcal{Z}$ may be covered by a satellite s, $s \in \mathcal{S}$, 0 otherwise.

Hence the problem can be formulated as follows:

$$\text{Minimize} \quad \sum_{p \in \mathcal{P}} H_p \; y_p + \sum_{s \in \mathcal{S}} H_s \; y_s + h^t \sum_{i \in \mathcal{T}^1} r_i + h^v \sum_{i \in \mathcal{T}^2} x_i + \sum_{i \in \mathcal{T}^1} C_i \; r_i + \sum_{i \in \mathcal{T}^2} C_i \; x_i \tag{41}$$

subject to

$$\sum_{s \in \mathcal{S}} \varphi_{sz} \; y_s = 1 \qquad \forall z \in \mathcal{Z} \tag{42}$$

$$\sum_{i \in \mathcal{T}^2} \beta_{iz} \; x_i = \sum_{j \in \mathcal{S}} \varphi_{jz} \; y_j \qquad \forall z \in \mathcal{Z} \tag{43}$$

$$\sum_{p \in \mathcal{P}} \epsilon_{ps} \; y_p = y_s \qquad \forall j \in \mathcal{S} \tag{44}$$

$$\sum_{i \in \mathcal{T}^1} \alpha_{is} \; r_i = \sum_{p \in \mathcal{P}} \epsilon_{ps} \; y_p \qquad \forall s \in \mathcal{S} \tag{45}$$

$$\sum_{i \in \mathcal{T}^1} f_i \; \alpha_{is} - \sum_{z \in \mathcal{Z}} D_z \; \varphi_{sz} \; y_s = 0 \qquad \forall p \in \mathcal{P} \tag{46}$$

$$\sum_{z \in \mathcal{Z}} D_z \; \varphi_{sz} \; y_s \le K_s \; y_s \qquad \forall s \in \mathcal{S} \tag{47}$$

$$\sum_{i \in \mathcal{T}^1_p} f_i \le K_p \; y_p \qquad \forall p \in \mathcal{P} \tag{48}$$

$$k^g \; r_i - f_i \ge 0 \qquad \forall i \in \mathcal{T}^1 \tag{49}$$

$$\begin{aligned} r_i &= \{0,1\} && \forall i \in \mathcal{T}^1 \\ x_i &= \{0,1\} && \forall i \in \mathcal{T}^2 \\ y_p &= \{0,1\} && \forall p \in \mathcal{P} \\ y_s &= \{0,1\} && \forall s \in \mathcal{S} \\ f_i &\ge 0 && \forall i \in \mathcal{T}^1 \end{aligned} \tag{50}$$

The objective function (41) minimizes the total costs. Constraints (42) impose that each customer is served by just one open satellite. Constraint (43) imposes that if a customer is served by a satellite, then it has to be served on just one route passing through the same satellite. Constraint (44) and (45) impose same routing conditions described for customers, but referred to first echelon. Constraints (46) are flow balance constraints for satellites. Constraints (47) and (48) are capacity constraints respectively for satellites and platforms. Constraints (49) are consistency constraints between flow and routing variables. Constraints (49) are the binary and non-negativity constraints.

This formulation, referred in literature also as set partitioning formulation is very compact and flexible since it can be easily adapted to take into account other problem settings and constraints. Indeed it has been widely used in solving different variants of *VRP*. On the other side this formulation is based on the definition of a huge number of variables associated with all the possible route. This requires an important pre-processing phase in order to generate feasible routes and its resolution would require the usage of a dynamic column generation approach which is beyond the scope of this paper.

5 Computational Experiments

In this section we present the results of computational experiments with the three and two index formulations proposed for *2E-LRP* on small and medium instances. Results have been obtained by XPRESS-MP solver and compared in terms of computation time, bounds and quality of solutions. One-index formulation has not been tested, since as already explained, an effective usage of such model should require a specific algorithmic approach, out of the scope of this paper. This approach could be a future line search for this problem. As said above the literature on *2E-LRP* is still limited and no instances are available to compare with. For this reason, an original random instance generator, coded in C++, has been developed to experience the proposed models. Used test beds are available on request from the authors.

5.1 Instance generator

The aim of the instance generator is to reproduce a schematic representation of a multi-level urban area. Customers and facilities are located within concentric circle rings of increasing ray (Figure 4).

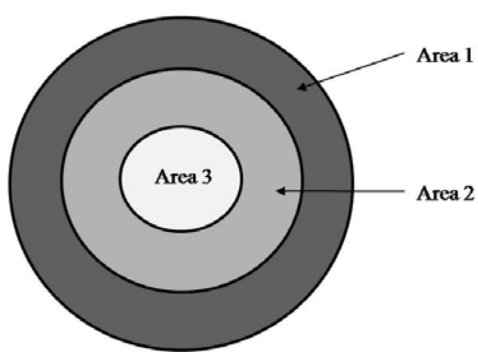

Figure 4: Schematic representation of a multi-level urban area.

The instance generator is based on the definition of some parameter values:

- Urban area size: ray values, ray_1, ray_2, and ray_3, respectively for *Area 1*, *Area 2* and *Area 3* are fixed, satisfying the condition: $ray_1 \geq ray_2 \geq ray_3$.

- Instance size: customer and facility number to locate are fixed. These values will be referred respectively as $|Z|$, $|S|$ and $|P|$.

- Customer and facility distribution: position of customers, satellites and platforms are fixed randomly generating the (X, Y) coordinates in the defined urban area and with the following criteria:

  – *Customer position*: customers are randomly located within *Area 3*.

  – *Satellite distribution*: satellite can be located within *Area 2* and/or *Area 3*. A percentage α is defined and satellites are distributed as follows:
  - α% of total number of satellites is randomly located in *Area 2*;
  - $(1 - \alpha)$% of total number of satellites is randomly located within *Area 3*.

  – *Platform distribution*: platforms are randomly located within *Area 1*

- Euclidean distances among nodes are computed.

- Customer demands (D_z) are randomly generated in a range $[D_{min}, D_{max}]$.

- Facility capacities (K_p and K_s) are randomly generated in range $[K_{min}, K_{max}]$ and subdivided in a prefixed number of classes of equal length.

- Facility location costs are randomly generated in the range $[H_{min}, H_{max}]$ and are subdivided in the same number of classes of equal length as for facility capacity. Moreover they are related to capacity values. For each randomly chosen capacity value, a random location cost is chosen within the correspondent cost class. A representation of this simple process is shown in Figure 5.

- Truck capacity (k^t and k^v) and related costs (h^t and h^v) are fixed.

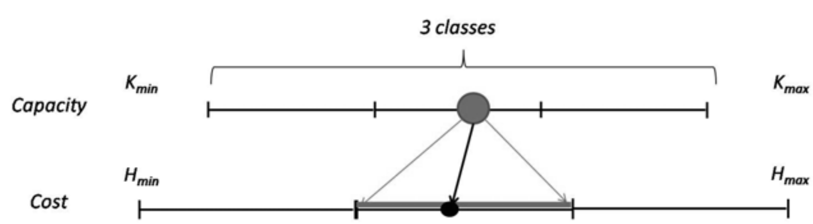

Figure 5: Capacity and location cost values

5.2 *2E-LRP* models computational results

Models have been experienced on three sets of instances differing for satellite distribution

- *Instance set I1*: satellites in *Area 2*;
- *Instance set I2*: satellites equally distributed in *Area 2* and *Area 3*;
- emphInstance set I3: satellites in *Area 3*.

The distribution of satellites within the defined areas in the three set of instances is graphically represented in Figure 6.

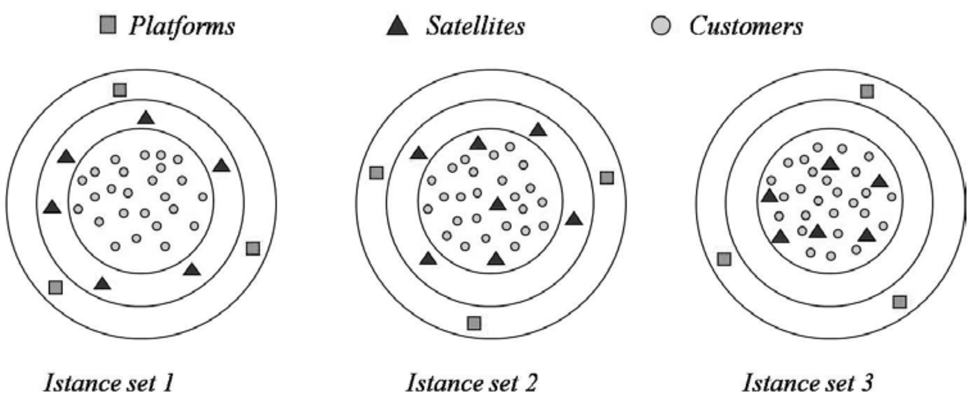

Figure 6: Satellite distribution in the threes set of instances

Different combinations of numbers of customers, satellites, and platforms were used for each instance set

- Customers: $\{8, 10, 15, 20, 25\}$;
- Satellites: $\{3, 5, 8, 10\}$;
- Platforms: $\{2, 3\}$.

The following parameter values were used:

- Customer demands: $D_z \in [1, 100]$;
- Satellite capacities and costs: three classes have been considered with ranges $U_s \in [300, 600]$ and $H_s \in [50, 90]$;
- Platform capacities and costs: three classes have been considered with ranges $U_p \in [900, 1600]$ and $H_p \in [100, 160]$;
- Urban truck capacity and cost: $k_t = 800$ and $h_t = 50$;

- City freighter capacity and cost: $k_v = 200$ and $h_v = 25$.

In the following, instances will be indicated with a notation composed by *test set name* and cardinality of $\boldsymbol{P}$, $\boldsymbol{S}$ and $\boldsymbol{Z}$. For example $I1/2-8-20$ stands for an instance of set *I1* with 2 platforms, 8 satellites and 20 customers. Models have been solved by XPRESS-MP solver, and instances were run on an Intel(R) Pentium(R) 4 (2.40 GHz, RAM 4.00 GB).

In Tables 1, 2, and 3 the results obtained on small and medium instances with proposed models are reported. For each instance and for each model the values of linear relaxation (*LR*), best lower bound (*BL*), best found solution (*BS*), percentage gap between *BS* and related *BL* (*Gap*%) and computation time in seconds (Time) are reported. The evaluation of the gap for a generic instance *I*, is computed with the following expression:

$$GAP(\%) = \frac{BS(I) - BL(I)}{BS(I)} \cdot 100 \tag{51}$$

Concerning computation time, the execution stops when an overload occurs. The best solution for each instance is highlighted in bold and the optimal solutions are marked by asterisk.

From Tables 1, 2 and 3 we can observe that for small instances both formulations are generally able to determine the optimal solution by XPRESS-MP solver within the predefined execution time, but the two-index formulation requires much lower computation times. On the other side, concerning medium instances with more than five satellites, the three-index formulation performs better than two-index formulation in terms of quality of solution and comparison with related best bound. This is basically due to the fact that three-index formulation linear relaxation provides higher lower bound values and moreover the improvement of best determined bound during the running time for three-index formulation is much higher than the improvement of best bound for two-index formulation, which remains very near to the linear relaxation. This is basically due to *big M* parameter used in subtour elimination constraints. A good estimation of M in function of maximum length route could slightly improve performances of the solver.

Finally in Table 4 the average percentage gap values for each formulation and for each instance set are reported. The first two columns (*3i-2E-LRP* and *3i-2E-LRP*) report the average of the already shown gaps of Table 1, 2 and 3. The third and the fourth columns (*3i-2E-LRP** and *3i-2E-LRP**) report the average gap values computed by using the higher best bound obtained by the two models. From this Table we can observe that performances of two-index model significantly improve if compared with best available lower bounds, whereas the ones of three-index formulation are almost equivalent. This confirms the need of proposing adjustments for both formulations in order to obtain better bounds.

Instance	Nodes			3i-2E-LRP					2i-2E-LRP				
	$\|P\|$	$\|S\|$	$\|Z\|$	**LR**	**BL**	**BS**	**Gap (%)**	**Time**	**LR**	**BB**	**BS**	**Gap (%)**	**Time**
I1/2-3-8	2	3	8	415.46	678.28	**678.27***	**0.00**	61	224.03	678.28	**678.27***	**0.00**	8
I1/2-3-10	2	3	10	552.09	877.65	**877.64***	**0.00**	4198	272.42	877.65	**877.64***	**0.00**	671
I1/2-4-10	2	4	10	444.24	838.22	**838.21***	**0.00**	3152	215.69	838.22	**838.21***	**0.00**	363
I1/2-4-15	2	4	15	646.12	999.44	**1155.71**	13.52	21223	411.26	897.00	1274.40	29.61	19678
I1/2-8-20	2	8	20	628.91	821.72	**1104.13**	25.58	24043	520.49	642.67	1057.42	39.22	27131
I1/2-8-25	2	8	25	823.94	1068.98	**1445.43**	26.04	21043	646.05	850.05	1514.24	43.86	29913
I1/2-10-15	2	10	15	415.11	642.37	**760.00**	15.48	19275	332.50	437.38	771.54	43.31	22123
I1/2-10-20	2	10	20	701.57	929.49	**1051.87**	11.63	27336	505.15	719.15	1098.23	34.52	31879
I1/2-10-25	2	10	25	745.20	956.89	**1331.46**	28.13	31087	590.63	773.45	1307.90	40.86	35471
I1/3-5-10	3	5	10	399.11	848.17	**848.16***	**0.00**	4745	226.89	848.17	**848.16***	**0.00**	92
I1/3-5-15	3	5	15	623.11	830.96	**1003.12**	17.16	22153	375.46	830.96	**1003.12**	17.16	21187
I1/3-8-10	3	8	10	338.11	512.91	**512.90***	**0.00**	3300	260.00	447.86	**512.91**	12.68	15200
I1/3-8-15	3	8	15	577.42	719.92	**842.13**	14.51	23215	410.89	603.25	875.34	31.08	28768
I1/3-8-20	3	8	20	603.84	715.29	**898.21**	20.37	24000	441.49	627.55	949.96	33.94	29980
I1/3-8-25	3	8	25	763.77	877.46	**1212.42**	27.63	26467	557.38	798.00	1288.02	38.04	31732
I1/3-10-15	3	10	15	427.71	564.15	**711.12**	20.67	21533	328.19	437.50	754.54	42.02	27533
I1/3-10-20	3	10	20	591.30	790.11	**1030.02**	23.29	19325	436.71	595.55	1099.52	45.84	29815
I1/3-10-25	3	10	25	847.81	1437.24	1696.74	15.29	36000	524.18	871.76	**1527.42**	42.93	39138

Table 1: Results of 3-index and 2-index formulations on instance set I1.

Instance	Nodes			3i-2E-LRP					2i-2E-LRP				
	$\lvert P \rvert$	$\lvert S \rvert$	$\lvert Z \rvert$	**LR**	**BL**	**BS**	**Gap (%)**	**Time**	**LR**	**BB**	**BS**	**Gap (%)**	**Time**
I2/2-3-8	2	3	8	501.19	648.19	**648.18***	**0.00**	179	283.44	648.19	**648.19**	**0.00**	13
I2/2-3-10	2	3	10	693.57	972.55	**972.55***	**0.00**	3416	443.28	972.55	**972.55**	**0.00**	541
I2/2-4-10	2	4	10	723.89	931.82	**931.816***	**0.00**	8221	428.53	931.82	**931.82**	**0.00**	3134
I2/2-4-15	2	4	15	805.11	936.09	1078.07	13.17	21604	387.63	874.65	**987.96**	11.47	18345
I2/2-8-20	2	8	20	676.00	765.22	**945.79**	19.09	28132	468.75	697.22	1133.04	38.46	30254
I2/2-8-25	2	8	25	766.33	899.13	**1280.14**	29.76	29123	553.38	777.22	1331.88	41.64	33398
I2/2-10-15	2	10	15	559.54	741.16	**835.09**	11.25	22737	465.66	579.36	910.37	36.36	26458
I2/2-10-20	2	10	20	666.63	805.76	**1104.33**	27.04	26734	478.57	678.32	1265.49	46.40	31123
I2/2-10-25	2	10	25	845.75	1009.21	**1435.67**	29.70	27452	624.21	861.35	1505.50	42.79	37139
I2/3-5-10	3	5	10	574.64	755.84	**793.21**	4.71	19803	422.18	793.21	**793.21***	**0.00**	3186
I2/3-5-15	3	5	15	680.10	785.13	**957.25**	17.98	19245	470.40	813.76	**957.25**	14.99	17132
I2/3-8-10	3	8	10	449.23	633.22	**654.20**	3.21	22546	344.73	617.28	**654.20**	5.64	25854
I2/3-8-15	3	8	15	501.70	690.37	**822.78**	16.09	24004	376.63	516.78	825.52	37.40	28452
I2/3-8-20	3	8	20	695.41	846.78	**1058.34**	19.99	26887	509.24	708.75	1085.03	34.68	30475
I2/3-8-25	3	8	25	830.34	1063.13	**1462.03**	27.28	27698	558.91	864.50	1596.02	45.83	37484
I2/3-10-15	3	10	15	617.68	872.14	**1016.48**	14.20	25316	442.69	656.09	1186.14	44.69	30211
I2/3-10-20	3	10	20	619.15	800.27	**1064.02**	24.79	36342	444.32	632.20	1076.06	41.25	39876
I2/3-10-25	3	10	25	841.57	989.04	**1463.65**	32.43	39941	623.62	855.16	1594.71	46.38	45124

Table 2: Results of 3-index and 2-index formulations on instance set I2.

Instance	Nodes			3i-2E-LRP					2i-2E-LRP				
	$\|P\|$	$\|S\|$	$\|Z\|$	**LR**	**BL**	**BS**	**Gap (%)**	**Time**	**LR**	**BB**	**BS**	**Gap (%)**	**Time**
I3/2-3-8	2	3	8	712.05	976.20	**976.19***	**0.00**	352	502.04	976.20	**976.19***	**0.00**	35
I3/2-3-10	2	3	10	597.36	807.60	**807.60***	**0.00**	3103	368.19	807.60	**807.60***	**0.00**	57
I3/2-4-10	2	4	10	689.27	984.85	**984.85***	**0.00**	5106	475.38	984.85	**984.85***	**0.00**	1858
I3/2-4-15	2	4	15	849.39	1077.12	**1405.15**	23.34	19345	496.68	1114.28	1431.67	22.17	17456
I3/2-8-20	2	8	20	883.99	999.13	**1424.26**	29.85	27321	620.40	893.75	1537.08	41.85	31477
I3/2-8-25	2	8	25	772.87	967.84	**1375.75**	29.65	28653	528.38	795.52	1418.50	43.92	33223
I3/2-10-15	2	10	15	600.63	677.68	**906.15**	25.21	18765	397.60	609.83	1021.49	40.30	25567
I3/2-10-20	2	10	20	704.11	765.09	**1255.21**	39.05	27335	502.42	716.14	1328.19	46.08	36245
I3/2-10-25	2	10	25	830.79	937.23	**1364.25**	31.30	30000	574.30	870.00	1640.28	46.96	39987
I3/3-5-10	3	5	10	445.86	983.60	**983.59***	**0.00**	15995	243.62	983.60	**983.59***	**0.00**	2369
I3/3-5-15	3	5	15	560.14	843.10	**1045.86**	19.39	21225	297.48	933.19	**1045.86**	10.77	22733
I3/3-8-10	3	8	10	478.27	625.35	**736.49**	15.09	18436	305.87	493.23	741.36	33.47	21298
I3/3-8-15	3	8	15	548.07	701.03	**953.55**	26.48	25598	419.05	599.82	1024.88	41.47	29984
I3/3-8-20	3	8	20	648.58	822.39	**1084.05**	24.14	23987	472.31	665.49	1280.12	48.01	31135
I3/3-8-25	3	8	25	803.14	876.74	**1199.91**	26.93	28766	538.13	825.00	1412.43	41.59	35567
I3/3-10-15	3	10	15	505.44	679.13	**929.60**	26.94	21435	340.67	532.50	986.65	46.03	27745
I3/3-10-20	3	10	20	658.60	784.99	**1101.76**	28.75	27645	480.86	671.16	1322.60	49.26	40567
I3/3-10-25	3	10	25	798.53	1030.87	**1419.73**	27.39	36000	544.86	847.50	1690.61	49.87	42113

Table 3: Results of 3-index and 2-index formulations on instance set I3.

	Average Gap (%) Values			
	3i-2E-LRP	**2i-2E-LRP**	**3i-2E-LRP***	**2i-2E-LRP***
Instance Set 1	16.37	27.50	14.50	15.72
Instance Set 2	17.22	27.48	15.98	17.93
Instance Set 3	20.75	31.21	20.13	25.02

Table 4: Average percentage GAP values on the three set of instances.

6 Conclusions

The problem of designing a two-echelon freight distribution system is an important logistic problem and it has been modeled as a two-echelon location-routing problem (*2E-LRP*). This problem represents a new class of location-routing problems which has never been tackled in literature. In this work we explored modeling issues for the 2E-LRP, proposing three mixed integer programming models obtained extending and/or adapting known *VRP* and *MDVRP* formulations. The proposed models assume as known the position of customers and are able to find the optimal or sub-optimal locations of platforms and satellites and fleet routing on two echelons. Given a real case with a predefined location of platforms they could also be used to determine the optimal location of satellites and optimal vehicle routes, so defining the best structure of a distribution system in a simpler case. Models have been experienced on test instances of varying dimensions by XPRESS-MP solver. Test instances have been generated through an original instance generator. Obtained results in terms of quality of solution and computation time are encouraging to continue the work on this research line to improve the performances of the models by adding ad-hoc effective cuts and valid inequalities already proposed in literature for classical *FLP* and *VRP* in order to improve bounds. On the other side the NP-hardness of the problem pushes towards the development of specific heuristic and metaheuristics approach in order to tackle the problem on large size instances coming from real applications.

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