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Joint Design of Product Family and Supply Chain: Between Diversity and Standardization

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Abstract. In this paper, we study the problem of designing a product family which has to satisfy diversified customer requirements. Modular design strategies allow a bill of materials to be generated for various finished products from a limited subset of modules. Simultaneously, a production location must be selected for the manufacture of each module. From a product point of view, the strategies adopted are often extreme, proposing either the fabrication of the total diversity (all possible products) or a strongly standardized range of products (only a few products are proposed). The objective of this paper is to investigate intermediate cases on this continuum, in order to better understand the potential for profit. Our comparison is based on the product family design (selection of the best modules) that minimizes production and transportation costs under time constraints.

Keywords. Product family, supply chain, modular design, optimization.

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1 Introduction

For several decades now, the industry has attempted to propose increasingly diversified products to customers to satisfy most segments of the market [21]. With this marketing approach, the design of unique products that are independent of one another is dropped in favor of unlikely families of products, based on a common platform, with a set of options which makes it possible to achieve the desired diversity [20].

This has resulted in the evolution of product conception, to the point where a finished product is now seen as a base to which options are added which permit customization. From a production point of view, this diversity is difficult to manage. If fabricating the base leads to the implementation of many similar production lines, then finalization of the product becomes very complex. Products can no longer be made for stock, because of the large number of possible finished products. An assemble-to-order production policy becomes the only possible alternative. Furthermore, the product very often cannot be fabricated in as short a time as demanded by the customer, or at what the customer considers a reasonable price [14]. The grouping of functions into modules assembled in advance makes it possible to resolve these difficulties, since these modules, in smaller numbers, can be made for stock in low-cost production facilities, and sent to a final assembly site close to the market.

The objective of this study is to find an acceptable balance between the number of modules to fabricate at distant sites, to choose those sites according to manufacturing and logistical costs, and to determine the bills of materials of finished products to satisfy the time constraints of the final assembly. For the most part, two extreme strategies exist to achieve this. The first is to define a limited number of modules which will serve as a base on which to create the bill of materials for every product (possibly including one or more functions in the product, even if the customer does not ask for them). This is the principle of standardization, which makes it possible to reduce the logistical costs generated by diversity, even if it means losing money on some finished products. The second strategy is to create bills of materials corresponding exactly to the required composition of the finished product (and no extra functions). In that case, the profits on the cost of components are obvious, but the cost of the management of the diversity increases.

In our investigation of intermediate strategies, we accept standardization and the inclusion of functions which are not required (but in a limited number). To our knowledge, there is no work on these strategies in the literature. We first give an outline of the work that does exist in the literature (section 2). The problem is described formally in section 3, and the proposed models are presented in section 4. The experiments are presented in section 5. We conclude and suggest some future research tracks in section 6.

2 State of the art

Mass customization, which is aimed at meeting the needs of individual customers, while ensuring the low costs and high level of responsiveness typically achieved by mass production [24], has received extensive attention since its emergence. Manufacturers must differentiate their products by focusing on individual customer needs without sacrificing efficiency, effectiveness, and the low cost customers expect.

The challenge of designing product families with a common platform in order to achieve product customization, while maintaining the economy of scale of mass production, has been well recognized in academia and industry alike [19].

Integrating modules of components into the design is a strategy that helps customize a large variety of high-demand products. Modularization makes it possible to organize complex designs and process operations more efficiently by decomposing complex systems into simpler portions [20], [23], [29]. A module can be defined as a group of standard and interchangeable components [10]; it is a complex group that allocates a function to the product and which can be changed, replaced, and produced independently [31]. A modular system is made up of independent units which can be easily assembled and which behave in a certain way in a whole system [4]. The term *modularity* is used to designate a common and independent part for the creation of a variety of products [16].

At the same time, the concept of supply chain management is garnering a great deal of interest, since the opportunity for integrated supply chain management can reduce the propagation of undesirable (or unexpected) events through the network, and can decisively affect the profitability of all its members [13]. There have been several articles recently on modeling traditional supply chain management, which can be classified into two major categories: configuration-level issues and coordination-level issues [14]. The configuration-level issues include articles on:

- Product design decisions, which deal with product types, materials to be used, product differentiation, and modularity [8], [12].
- Supply decisions, which are aimed at determining the supply strategy (make or buy decisions, outsourcing, among others), and also at determining which suppliers have to be selected [7], [27], [18].
- Production decisions, which are aimed at determining the number of factories and their location, the capacity of each factory, and the products to be manufactured at each factory [26], [3].
- Distribution decisions, which focus on distribution channels, the number of those channels, location of warehouses, and transportation modes [15], [22], [28].

The coordination-level issues include articles on:

- [17], [25], [30].

- Performance measures, which are aimed at developing suitable performance measures for supply chain management [6], [5].

Some recent work has been carried out on global design modeling for a supply chain that supports product family manufacturing. Agard et al. [1] propose a genetic algorithm to minimize the mean assembly time of finished products for a given demand, and Agard and Penz [2] propose a model for minimizing module production costs and a solution based on simulated annealing. However, these models do not consider the variable costs arising from the number of modules to be manufactured. Lamothe et al. [21] use a generic bill of materials representation to identify the best bill of materials for each product and the optimal structure of the associated supply chain simultaneously, although this approach requires that a predefined generic bill of materials be generated for the product family.

3 Strategic design problems

The problem considered here was introduced by El Hadj Khalaf et al. [9]. Consider the following industrial context (Figure 1). The producer receives customers' orders for finished products containing options and variants. Each individual product is then manufactured from a set of modules that come from various suppliers.

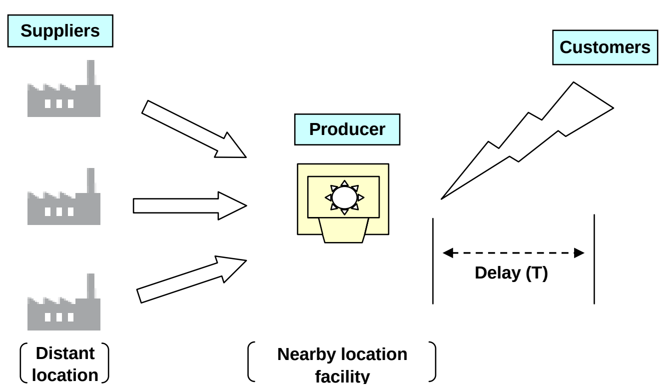

Figure 1: Structure of the supply chain

The producer has only a short time (T) in which to respond to each customer demand. That time is less than the time required to assemble the products from elementary components. In addition to this, the producer has to provide the product exactly according to the customer's requirements (without extra features). This constraint comes from technical considerations or simply to avoid the supplementary cost of providing features that were not requested.

To satisfy customers, the producer brings in pre-assembled components, called modules, from many suppliers located at distant facilities around the world. The suppliers' facilities are characterized by low production costs. The modules are then assembled at the producer's facility, which we assume to be close to the customers and thus characterized by a high level of responsiveness and reduced lead-time.

The strategic problem is then to design the product family *i.e.* to determine the bill of materials for each product. A product will be made up of a set of modules. Simultaneously, for a set of required modules *i.e.* the modules that appear in at least one bill of materials, we determine where those modules must be produced in order to minimize production and transportation costs.

The various elements of the problem, as well as the main costs to be taken into account, are described more formally below. First, we introduce the notions of functions, products, modules, and distant sites:

- $\mathcal{F} = \{F_1, ..., F_q\}$: set of q functions that can appear in both finished products and modules;
- $\mathcal{P} = \{P_1, ..., P_n\}$: set of n possible finished products that may be demanded by at least one customer, with D_i the estimated demand of the product P_i during the life cycle of the product family.
- $\mathcal{M} = \{M_1, ..., M_m\}$: set of m possible modules.
- $\mathcal{S} = \{S_1, ..., S_s\}$: set of s distant production facilities

The problem data are expressed as follows:

- F_j^A : the fixed cost of module M_j at the nearby facility (management costs).
- V_j^A : the variable cost of module M_j at the nearby facility (assembly, storage, transportation, etc.).
- F_{jl}^P : the fixed cost of module M_j at facility S_l (management).
- V_{jl}^P : the variable cost of module M_j at facility S_l (assembly, storage, etc.).
- t_j : the time required to assemble module M_j into a finished product.
- T : maximum assembly time allowed for a finished product.
- W_{jl} : the work load caused by producing module M_j at facility S_l.

Under these assumptions, a product (or a module) is represented by a binary vector of size q. Each element shows whether the corresponding function is required in the product (value $= 1$) or not (value $= 0$). The set $\mathcal{M}$ contains m modules. $\mathcal{M}$ may be all the possible modules from the whole combinatory or a subset of those modules defined by the engineering.

The problem of optimization is now simple to express. It is necessary to determine the subset $\mathcal{M}^{'}$ of modules that has to be manufactured. This subset has to contain all the modules necessary for the elaboration of the bills of materials of all the possible finished products. When all the bills of materials (Figure 2) have been determined, we can easily deduce the demand for each module and assign its production to the various distant production sites. As we have just stated, a natural initiative could be to first determine the subset $\mathcal{M}^{'}$ and define the bills of materials of finished products, and then to assign the production of modules to those distant sites. Our results show that this approach is not successful on problems where there is no standardization [9]. The objective is to solve this optimization problem globally, rather than to undertake a succession of partial optimizations.

The bills of materials shown in (Figure 2) correspond to the assembly strategy of producing a finished product exactly as needed, *i.e.* wwithout extra functions (if function k, for example, is not present in the product, then it must not be present in the modules constituting that product's bill of materials), and without function redundancy (if function k is present in the product, then it is present in only one module among those constituting that product's bill of materials). Other assembly strategies are explored in this paper as well, like the standardization strategy (authorization to include extra functions in the finished product that were not requested) and the redundancy strategy (the same function could be present in more than one module in the product's bill of materials).

The described problem is NP-hard in the strong sense, because it includes the classic set partitioning problem [11].

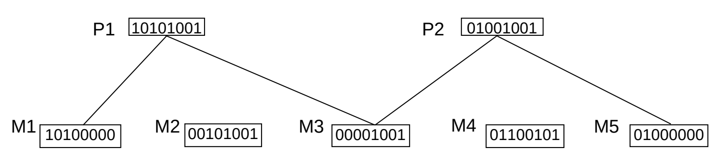

Figure 2: Which modules must be included in the bills of materials?

4 Mathematical models

First, we present the optimization model, which allows us to determine optimal solutions for the problem of total diversity, *i.e.* without standardization or redundancy. Then, we define the model in which we accept a limited number of supplementary functions, but without function redundancy, *i.e.* the same function is not present in more than one module. Finally, we present the model with extra functions and function redundancy.

4.1 Model without standardization or redundancy

In order to solve this model optimally, it will be necessary to precisely determine the bill of materials for each product. For this, we define the binary variable X_{ij}, which takes the value 1 only if the product P_i has the module M_j as a component. If that is the case, the binary variable Y_j, which means that M_j is manufactured at one distant site at least S_l will take the value 1. The binary variable Y_{jl} takes the value 1 only if M_j is manufactured, at least partially, at site S_l. Finally the integer variable Q_{jl} represents the quantity of M_j produced at site S_l.

In order to simplify the writing of the model, we introduce the parameters δ_{ik} and λ_{jk}. The binary parameter δ_{ik} equals 1 if the function F_k is present in the product P_i. Also, the parameter λ_{jk} equals 1 if the function F_k is present in the module M_j. With these notations, we can now write the Mixed Integer Linear Program of that model. The objective function is expressed as the sum of costs:

$$min \left(\sum_{j=1}^{m} F_j^A Y_j + \sum_{j=1}^{m} V_j^A \left(\sum_{i=1}^{n} D_i X_{ij} \right) + \sum_{l=1}^{s} \sum_{j=1}^{m} F_{jl}^P Y_{jl} + \sum_{l=1}^{s} \sum_{j=1}^{m} V_{jl}^P Q_{jl} \right) \tag{1}$$

s.t.

$$\sum_{j=1}^{m} \lambda_{jk} X_{ij} = \delta_{ik} \forall i \in \{1, \cdots, n\} \ \forall k \in \{1, \cdots, q\} \tag{2}$$

$$\sum_{j=1}^{m} t_j X_{ij} \leq T \forall i \in \{1, \cdots, n\} \tag{3}$$

$$X_{ij} \leq Y_j \forall i \in \{1, \cdots, n\} \ \forall j \in \{1, \cdots, m\} \tag{4}$$

$$\sum_{l=1}^{s} Q_{jl} = \sum_{i=1}^{n} D_i X_{ij} \forall j \in \{1, \cdots, m\} \tag{5}$$

$$\sum_{j=1}^{m} W_{jl} Q_{jl} \leq W_l \forall l \in \{1, \cdots, s\} \tag{6}$$

$$Q_{jl} \leq K_{jl} Y_{jl} \forall j \in \{1, \cdots, m\} \ \forall l \in \{1, \cdots, s\} \tag{7}$$

$$Q_{jl} \geq 0 \forall j \in \{1, \cdots, m\} \ \forall l \in \{1, \cdots, s\} \tag{8}$$

$$Y_j, X_{ij}, Y_{jl} \in \{0, 1\} \forall i \in \{1, \cdots, n\} \forall j \in \{1, \cdots, m\} \forall l \in \{1, \cdots, s\} \tag{9}$$

The objective function minimizes the costs (fixed and variable) incurred at the nearby facility, where $(\sum_{i=1}^{n} D_i X_{ij})$ is the total demand for module M_j, and the costs (fixed and variable) incurred at the distant facilities. Constraint (2) shows that a finished product P_i must be assembled exactly as requested by the customer. If the function is not present, then it must not appear in any of the product's components. Constraint (3) indicates that products must be assembled within the time window T, in order to respect the delivery time. According to constraint (4), if module M_j is used in the bill of materials of product P_i, then the module M_j must be produced somewhere. Constraint (5) indicates that the production of a module M_j must satisfy the requirements. Constraint (6) shows that production at facility S_l must not exceed its capacity. Constraint (7) expresses the relation between the variables Q_{jl} and Y_{jl}. A module M_j can be produced in S_l only if M_j is assigned at S_l $(Y_{jl} = 1)$. The parameter K_{jl} is a large constant value representing the maximal quantity that can be manufactured at the distant site S_l. It can be calculated by the following formula:

$$K_{jl} = min\{W_l/W_{jl}, \sum_{i=1}^{n} D_i\} \quad \forall j \in \{1, \cdots, m\} \tag{10}$$

Finally, constraints (8) and (9) guarantee the positivity of the quantities of modules produced and ensure that the decision variables are binary.

4.2 Model with total standardization and without redundancy

We can now transform the previous model in order to formulate the total standardization problem. For this, we substitute constraint (2) by:

$$\sum_{j=1}^{m} \lambda_{jk} X_{ij} = 1 \quad \forall P_i \ and \ k/\delta_{ik} = 1 \tag{11}$$

$$\sum_{j=1}^{m} \lambda_{jk} X_{ij} \leq 1 \quad \forall P_i \ and \ k/\delta_{ik} = 0 \tag{12}$$

Constraint (11) expresses the fact that, if a function F_k is present in a finished product, it must appear in precisely one, and only one, module among those detailed in the product's bill of materials. Constraint (12) indicates that, if a function F_k is not present in the product P_i, then it could appear in its bill of materials (again, only in one component).

This model thus demands that a required function be present in a single copy of the product's bill of materials, and that a function which is not needed be present in at most a single copy of the product's bill of materials. Later, we will investigate the variants in which the standardization is limited.

4.3 Partial standardization without redundancy

Depending on the industrial strategy adopted by the company, standardization can be limited. Limitation may stem from a desire on the part of management to reduce the cost generated by adding unneeded functions to the product, or by other objectives, such as not increasing the weight of the basic product by including those functions. For example, it can be detrimental to install an electronic card in a laptop if the card is not used.

In that case, a constraint must be added to the model in order to limit standardization. This constraint is expressed as follows:

$$\sum_{k=1}^{q} \sum_{j=1}^{m} \lambda_{jk} X_{ij} \leq q_i + \alpha_i \quad \forall P_i \tag{13}$$

Constraint (13) makes it possible to count the total number of functions provided by the product's components (bill of materials). The value q_i gives the number of functions needed in P_i and the parameter α_i is the number of extra functions tolerated for the product. We note here that is possible to limit standardization in a specific way for each finished product.

4.4 Model with function redundancy and without standardization

In certain applications, redundancy is sometimes acceptable. It occurs when a requested function is installed twice (provided by two different modules) in the same finished product. This is common, in the computer industry. Let us suppose that a manufacturer proposes two versions of a computer. The first version contains, among other things, a motherboard and a basic graphics card. The more sophisticated version has the same motherboard, but a more powerful version of the graphics card, requested by only 5% of customers who are interested in video games. The manufacturer can assemble the motherboard and the appropriate video card according to the customer's request, but he can also install a motherboard in this computer which already contains the basic graphics card, and install it in all the computers. He will add to this motherboard, which includes the basic graphics card, the more powerful graphics card when asked to do so by the customer. He will then have only two cards to manage, and for only 5% of the customers will he have an over cost on the components of the basic graphics card. In that case, the same function, F_k, should appear several times in the product, and supplementary constraints should be added to avoid the appearance of that function several times in the product's bill of materials.

The redundancy that we address here is different and concerns the apparition of the same function, F_k, several times. This is the case, for example, with the electric beams, where there can be wires in a beam that are not used. In our modeling, it is sufficient to replace constraints (11) and (12) by the following ones:

$$\sum_{j=1}^{m} \lambda_{jk} X_{ij} \leq 2 \quad \forall P_i \text{ and } k/\delta_{ik} = 1 \tag{14}$$

$$\sum_{j=1}^{m} \lambda_{jk} X_{ij} \geq 1 \quad \forall P_i \text{ and } k/\delta_{ik} = 1 \tag{15}$$

$$\sum_{j=1}^{m} \lambda_{jk} X_{ij} = 0 \quad \forall P_i \text{ and } k/\delta_{ik} = 0 \tag{16}$$

Constraint (14) allows a redundancy only on the requested functions (that must be present in the finished product). By modifying the value 2 by a parameter, we could easily accept that certain functions appear more than twice in a finished product, but this does not seem very realistic from an industrial point of view. We could also impose a parameter that depends on F_k, which means that we apply a redundancy number for each function and for each product. Constraint (15) guarantees that the needed functions have to be present at least once. Finally, constraint (16) prevents standardization.

If we wish in addition to limit the number of redundancies, we must count the total number of functions present in the product's bill of materials and to compare it with the number of the product's requested functions. This constraint is the following one:

$$\sum_{k=1}^{q} \sum_{j=1}^{m} \lambda_{jk} X_{ij} \leq q_i + \beta_i \quad \forall P_i \tag{17}$$

In that case, β_i gives the number of functions generated by the redundancy that will be tolerated.

4.5 Limited standardization with redundancy

In the most general case, it is possible to have a redundancy and extra functions not requested in the finished product at the same time. The model then has to contain constraints (14) and (15) to allow the redundancy, constraint (11) to allow the standardization, and constraint (13) to limit the number of extra functions. In that case, the parameter α_i will represent the number of extra functions, including both those stemming from the redundancy and those stemming from the standardization.

To differentiate the supplementary functions according to their origin, the addition of following two constraints would be necessary:

$$\sum_{k/\delta_{ik}=1} \sum_{j=1}^{m} \lambda_{jk} X_{ij} \leq q_i + \beta_i \quad \forall P_i \tag{18}$$

$$\sum_{k/\delta_{ik}=0} \sum_{j=1}^{m} \lambda_{jk} X_{ij} \leq \gamma_i \qquad \forall P_i \tag{19}$$

The parameter β_i gives the the maximum allowable number of redundant functions and the parameter γ_i gives the maximum allowable number of extra functions.

4.6 Comments on the supply sources

Up to now, we have looked at the impact on the model when redundancy and standardization are introduced as alternative strategies for the determination of the product's bill of materials. Variants can also appear in the logistical part of the model. It is possible to limit the number of sites where the module M_j will be produced. To do this, the following two constraints must be added:

$$\sum_{l=1}^{s} Y_{jl} \leq \eta_j Y_j \quad \forall M_j \tag{20}$$

$$\sum_{l=1}^{s} Y_{jl} \geq \epsilon_j Y_j \quad \forall M_j \tag{21}$$

Constraint (20) demands that the number of sites not exceed η_j for the module M_j, to avoid too wide a distribution of suppliers. Constraint (21) calls for production at least ϵ_j sites. This latter constraint can take the value 1, which guarantees that at least one supplier is required, but also a larger value, which calls for an increase in the number of supply sources to anticipate a stock shortage.

In this last case, it is also possible to force every supplier to produce at least a certain percentage of the total demand for the module M_j. This guarantees that every supplier will mass produce the item, enabling them to reduce production costs. The constraint is then:

$$Q_{jl} \geq \tau_{jl} \sum_{i=1}^{n} D_i X_{ij} - (1 - Y_{jl})M \quad \forall M_j \, and \, S_l \tag{22}$$

The parameter τ_{jl} indicates the minimum percentage of the quantity of the module M_j required that has to be manufactured at the distant site S_l and M is a large number.

5 Computational experiments

5.1 Datasets, experimental conditions, and indicators

The goal of the paper is not to provide a fast solution method, but to compare scenarios in order to better understand the influence of standardization and redundancy in different contexts. So, the experiments were conducted on small examples, and the optimal solution of the models calculated with a standard optimization solver (Cplex).

The objective of the experiments was to compare the various assembly strategies presented in this paper for several cost configurations and for different time windows T. To achieve this, small examples were randomly generated on which the set of possible modules, the finished product set, the distant facility set, the demands D_i, the assembly operating times t_j, and the distant facility capacities are fixed, while the costs vary.

Assuming that the demand D_i for a product P_i is a decreasing function of the number of functions in the product, when a finished product contains more options, the demand for it becomes less than if it had fewer functions. The individual assembly operating times t_j are fixed to 1, so that constraint (2) results in a limitation in the number of modules for each bill of materials.

Fixed and variable costs associated with the bills of materials (F_j^A and V_j^A) are defined using a square root function of q_j (the number of functions in module M_j). The assumption is that assembling a module containing q_j functions is less expensive than assembling two modules containing q_{j1} and q_{j2} functions respectively, such that $q_j = q_{j1} + q_{j2}$.

To explore several cost configurations, three parameters are used:

- X: which represents the ratio between assembly costs and production costs. Three possible values are assigned to this parameter:
  - A : indicating that the assembly costs are much higher than the production costs;
  - B : indicating that the assembly and production costs are almost equivalent;
  - C : indicating that the production costs are higher than the assembly costs.
- Y: which represents the ratio between fixed assembly costs and variable assembly costs. Three possible values are also assigned to this parameter:
  - $+$: indicating that the fixed costs are higher than the variable costs;
  - 1 : indicating that the fixed and variable costs are almost equivalent;
  - $-$: indicating that the variable costs are higher than the fixed costs.
- Z: which represents the ratio between production fixed costs and production variable costs. This parameter takes exactly the same values as Y.

With these three parameters, twenty-seven cost configurations were generated and used in the tests. Table describes the parameter values for each cost file. Each cost file is characterized by a specific ratio between the various problem costs. For more detail on the cost generation procedure, readers can refer to [9].

Costs	C1	C2	C3	C4	C5	C6	C7	C8	C9
X	A	A	A	A	A	A	A	A	A
Y	+	+	+	1	1	1	-	-	-
Z	+	1	-	+	1	-	+	1	-
Costs	C10	C11	C12	C13	C14	C15	C16	C17	C18
X	B	B	B	B	B	B	B	B	B
Y	+	+	+	1	1	1	-	-	-
Z	+	1	-	+	1	-	+	1	-
Costs	C19	C20	C21	C22	C23	C24	C25	C26	C27
X	C	C	C	C	C	C	C	C	C
Y	+	+	+	1	1	1	-	-	-
Z	+	1	-	+	1	-	+	1	-

Table 1: Cost configurations

For each of the 27 configurations, 10 instances were generated. The problem data were fixed as follows: the number of functions $q = 8$, the number of finished products $n = 30$, where each product has at least $q_{min} = 3$ functions and at most $q_{max} = 6$ functions, $m = 255$ (all possible combinations of modules) and the number of production facilities $s = 2$. T varied from 3 to 6. For $T > 6$ (q_{max}) the solution is the same as for $T = 6$. For $T \leq 2$, the final assembly will consider a maximum of 2 assembly operations for each final product, which does not seem reasonable from a practical point of view.

The tests were conducted in C++ with the Ilog Cplex 9.0 library. They were solved on a 1.6 Hz DELL workstation with 512 Go of RAM.

5.2 Results analysis

We call the initial model without standardization and redundancy the basic model. We first analyze the profits accrued by each assembly strategy in comparison with that of the basic model assembly strategy according to the various cost structures. The basic model results will always be used as the reference value.

We use the following notations:

- $|\mathcal{M}'|$: the number of the modules selected in $\mathcal{M}'$ (the solution size);
- Module requirement: the quantity of module M_j required to assemble the finished products required: $Req_j = \sum_{i=1}^{n} D_i X_{ij}$;

- Solution requirement: the sum of the requirements of the solution modules $\sum_{j=1}^{m}\sum_{i=1}^{n} D_i X_{ij}$;
- Red: designates the model with function redundancy without standardization;
- St_n: designates the model with partial standardization and without redundancy where $\alpha_i = n \ \forall P_i$;
- St: designates the model with total standardization and without redundancy;
- StRed: designates the model with total standardization and with redundancy.

Figure 3 shows the gap between the objective function values of the basic model and the model with function redundancy (T is fixed to 4). We see here that the function redundancy strategy is not profitable, but only if production costs are high relative to assembly costs. Indeed, in the B zone, production costs become almost equivalent to assembly costs, and in this case we find a small gap which reaches the maximum value in the C zone, when the production costs are the highest. However, the gap is not great and does not exceed 10%.

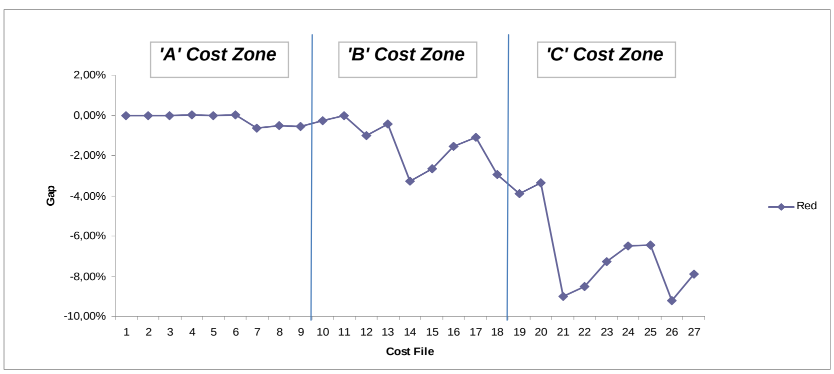

Figure 3: Objective function gap for the function redundancy model (T=4)

The standardization strategy is much more profitable than the function redundancy strategy (see Figures 4 and 5), mainly because the first strategy makes it possible to reduce the solution size since the possibility of finding shared modules is greater than in the basic model strategy (see Figure 6 and 7).

With the standardization strategy, a module may be on the product's bill of materials even though it contains more functions than the product itself. The flexibility of this strategy leads to a reduction in the number of components in

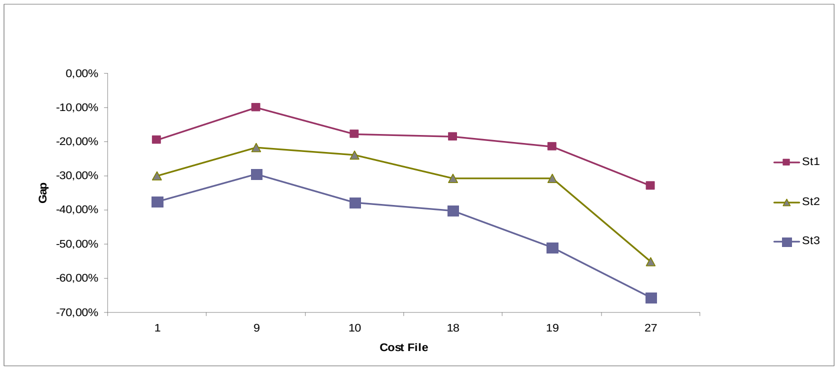

Figure 4: Objective function gap for the standardization model with $1 \leq \alpha \leq 3$ and T=4

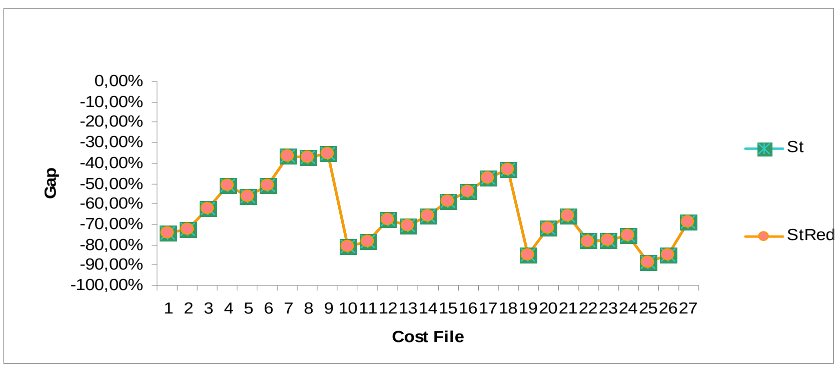

Figure 5: Objective function gap for the total standardization model (T=4)

the bill of materials of the products. The results show that, for $T = 4$, the majority of products could be assembled from two modules in the standardization model optimal solution, and also some products may contain one module in their bills of materials. This leads to a solution where the total number of requirements of modules is lower (see Figure 8), which in turn leads to a reduction in the total variable costs (see Figure 6 and 7).

Of course, when α (which is the number of supplementary functions authorized per product) increases, the gap also increases (see Figure 9) with the larger number of additional functions tolerated, as it becomes easier to reduce the solution size and the solution requirements, which in turn leads to a reduction in both the fixed and variable costs.

However, for a fixed value of α, the gap rate decreases when T increases

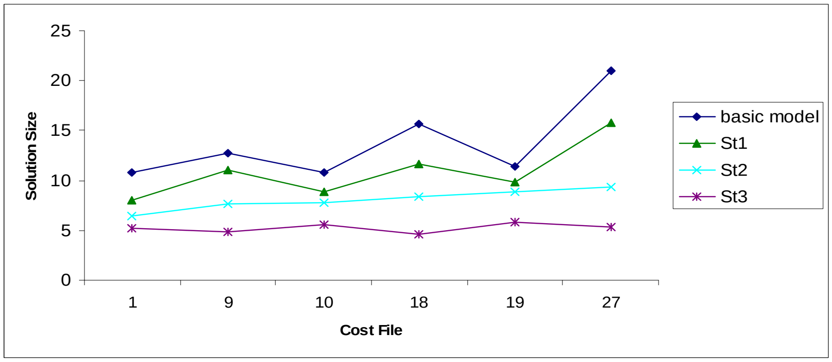

Figure 6: The solution size for the standardization model (T=4)

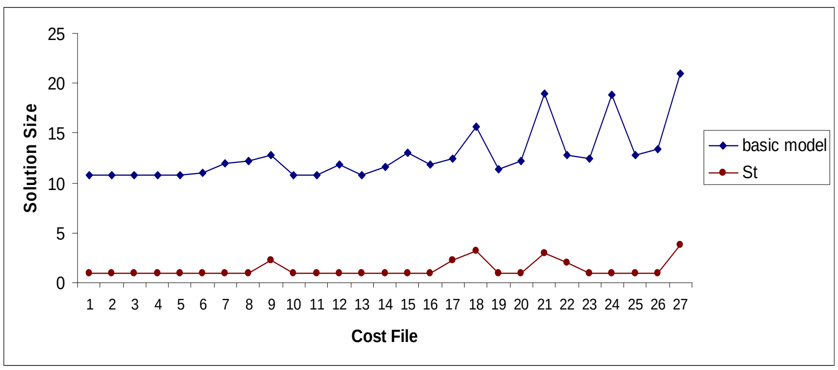

Figure 7: The solution size for the total standardization model (T=4)

(see Figure 10). As explained before, the standardization strategy leads to a reduced number of modules used to assemble a product. Thus, constraint (3) becomes less of an influence on the solution, which is why increasing T does not participate significantly in the improvement of the objective function of the standardization strategy model, especially when α increases (see Figure 11). At the same time, T is highly important for the basic model, and its rise permits a relatively large improvement. So, increasing T causes a significant fall in the objective function for the basic model, and a non significant one for the standardization model. This is why the gap decreases when T increases.

Of course, the maximum profit of the standardization strategy is reached with the total standardization model. It is obvious that with such a strategy the module (11111111) can be included in the bill of materials of any product, because it contains all the functions. However, the optimal solution is not

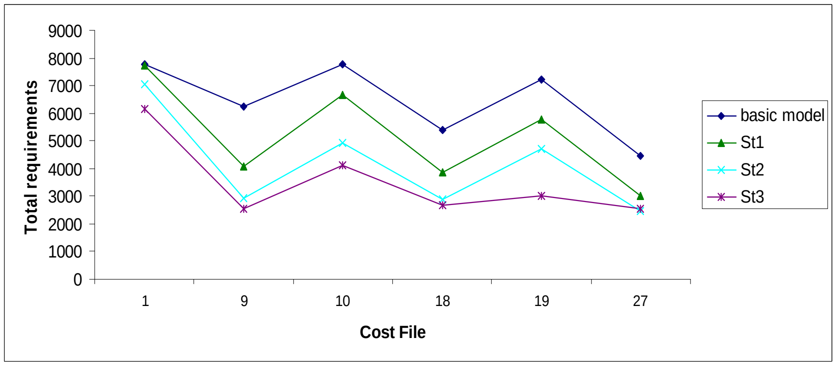

Figure 8: The total requirements solution for the standardization strategy (T=4)

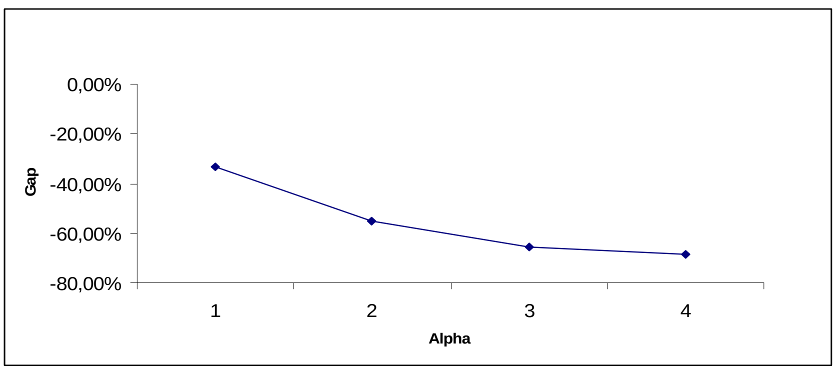

Figure 9: Evolution of the objective function gap according to α for cost 27 (T=4)

always to manufacture this module. For some costs, we have to produce other modules as well (which certainly contain many functions) in order to optimize variable costs. The assembly costs of module (11111111) are very high, because it contains the whole set of functions (based on our assumptions). For some costs, it is of greater interest to produce other modules like (10111111) (which has an assembly cost that is less than that of module (11111111) because it contains fewer functions) and use it to assemble compatible products. Then, if we assemble a product P_i from the module (10111111) instead of (11111111), we gain the following assembly variable costs: $D_i \times (CV^A_{11111111} - CV^A_{10111111})$. This is the case for costs 9, 17, 18, 21, 22, and 27, where the configuration is such that variable costs are greater than fixed costs. For these costs, the optimal

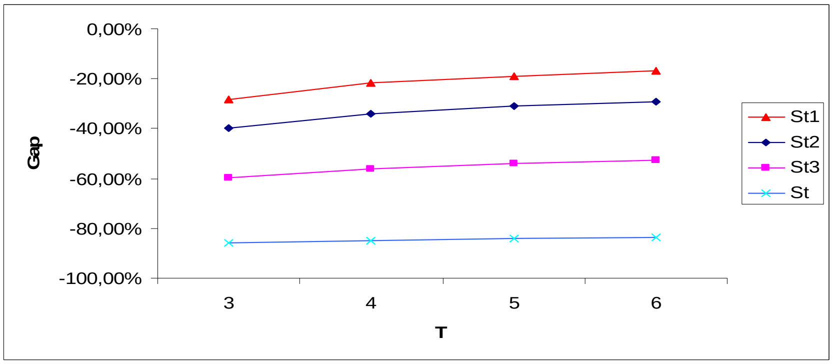

Figure 10: Evolution of the objective function gap according to T for cost 19

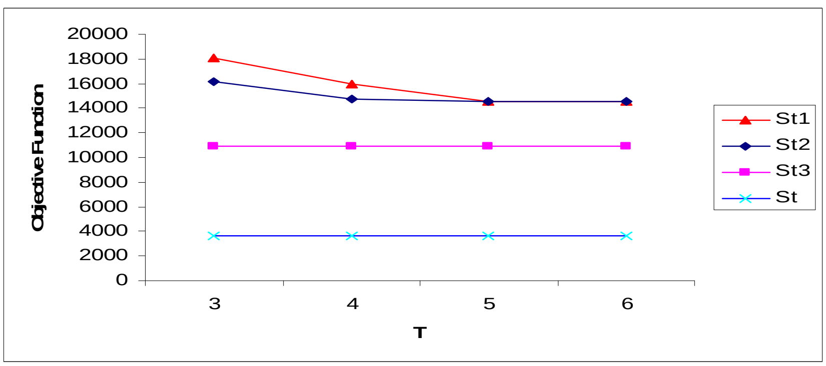

Figure 11: Evolution of the objective function according to T for cost 10

solution size is greater than 1 (see Figure 7).

From the point of view of computational time, we note that resolution of the function redundancy model and of the total standardization model is very fast, generally a few minutes. In return, when we impose a partial standardization, the resolution takes much more computational time, and in fact may require more than four hours of computation time. The standardization with function redundancy model also takes much more resolution time, while the optimal solution is exactly the same as that for the total standardization model.

6 Conclusion

The objective of this article was to propose general models for the resolution of problems associated with the simultaneous design of a product family and its logistical chain. We began by describing an existing model, where every finished product has to contain precisely the functions that are needed, and each function has to be present only once in the product's bill of materials. We then proposed models which allow for controlled standardization and/or redundancy.

But what is the advantage of partial standardization or function redundancy authorization? This is the question that we attempted to answer with the numerical tests presented in this paper. Indeed, the authorization of function redundancy does not seem to be a profitable strategy of interest. The expected gains do not exceed 10% in the best case. The standardization strategy, by contrast, is of much greater interest, with the potential of significantly higher profits, notwithstanding the cost configuration. The advantage of the standardization strategy is that it leads to a reduction in the solution size (thereby reducing the fixed costs) and also the number of modules used in a bill of materials, which reduces the total number of modules needed (thereby reducing the variable costs).

These mathematical models are difficult to solve (in terms of complexity theory), and therefore almost impossible to solve in the case of industry-wide problems. That is why a heuristic approach has to be investigated. A previous method based on a taboo search algorithm has been developed and tested, and has been shown to perform well on the basic model. It would be interesting to extend this method to the resolution of the other models.

The modular approach presented here implicitly considers the bill of materials in one level, that is, a bill of materials where the finished product is assembled directly from a set of independent modules. An interesting track would be to address the problem with bills of materials of depth greater than 1, or, in other words, bills of materials where the modules may themselves be assembled from smaller modules, with the possibility of dedicating some sites to the assembly of small modules and others to the assembly of large modules (even of finished products) from the small ones.

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