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Robert Beauregard

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Bureaux de Montréal :

Université de Montréal
C.P. 6128, succ. Centre-ville
Montréal (Québec)
Canada H3C 3J7
Téléphone : 514 343-7575
Télécopie : 514 343-7121

Bureaux de Québec :

Université Laval
2325, de la Terrasse, bureau 2642
Québec (Québec)
Canada G1V 0A6
Téléphone : 418 656-2073
Télécopie : 418 656-2624

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A Stochastic Programming Approach for Coordinated Contract Decisions in a Make-to-Order Manufacturing Supply Chain

Yan Feng^{1,2,*}, Alain Martel^{1,3}, Sophie D'Amours^{1,2}, Robert Beauregard⁴

¹ Interuniversity Research Centre on Enterprise Networks, Logistics and Transportation (CIRRELT)

² Département de génie mécanique, Université Laval, Pavillon Adrien-Pouliot, 1065, avenue de la médecine, Québec, Canada G1V 0A6

³ Département des opérations et systèmes de décision, Pavillon Palasis-Prince, 2325, rue de la Terrasse, Université Laval, Québec, Canada G1V 0A6

⁴ Département des sciences du bois et de la forêt, Pavillon Abitibi-Price, 2405, rue de la Terrasse, Université Laval, Québec, G1V 0A6

Abstract. This article proposes a stochastic programming approach for coordinated contract design, allocation and selection decisions, from a manufacturer's point of view, in a three-tier manufacturing supply chain. In a capacitated make-to-order manufacturing system, the manufacturer wishes to offer different customer-contracts to satisfy their needs, to accept the contracts that optimize resource capacity allocations, and to select supplier-contracts that guarantee the satisfaction of the demand in order to maximize profits. Using a two-stage stochastic programming model with recourse, these decisions are addressed under a stochastic economic, market, supply, and system environment. The computational results show that the proposed model provides more realistic and robust solutions, with expected 12% performance improvement over the solutions provided by a deterministic mixed integer programming model.

Keywords. Supply chain coordination, contract design, sales and operations planning (S&OP), make-to-order manufacturing, stochastic programming, oriented strand board (OSB) industry.

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* Corresponding author: Yan.Feng@cirrelt.ca

1. Introduction

Effective supply chain management requires collaboration and coordination between independently managed business entities along the supply chain. This function is generally governed by supply chain contracts (or agreements). There is a growing body of research on supply chain contracts defining relationships between supply chain partners. Most of the existing literature focuses on two-tier supplier-buyer contracts, with few exceptions that extend the contract decisions to a general supply chain network context (D'Amours et al. 2000). We consider a three-tier manufacturing supply chain, as illustrated in Figure 1, where a multi-site manufacturer purchases various raw materials from multiple suppliers, and produces different specialty and commodity products for a random demand and price market. Thus, there are contract relationships at both demand and supply ends of the supply chain.

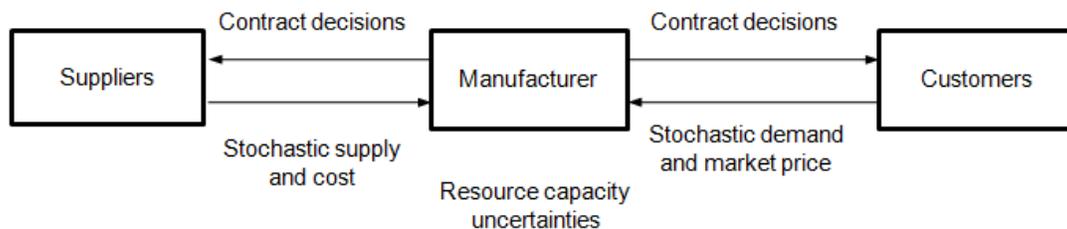


Figure 1. Three-tier Supply Chain in a Stochastic Environment

Generally, contract decisions are made at the beginning of the planning horizon. In a capacitated make-to-order manufacturing system, this decision involves selecting the contract customers so that their demand satisfaction is guaranteed and selecting the contract suppliers so that the raw material supplies are guaranteed while the manufacturer's financial objectives are reached. The manufacturer signs a contract with a customer only if there is enough capacity to satisfy the customer's demand. Hence, the manufacturer would typically allocate a certain proportion of the capacity to contract customers, keeping a capacity buffer for unexpected demand increases and/or to serve spot markets possibly for greater profitability. From a financial point of view, if the market becomes stronger, preserving or increasing contract sales would possibly cause contract demand backlogs and limit the opportunities for greater profitability. However, if the market weakens, reducing contract sales would potentially put the manufacturer at risk of incurring lower profits. Similar scenarios apply to the supply end where the manufacturer has the options to purchase raw materials through contract suppliers or from the open market (spot market) where greater discounts may be possible.

Once the demand contracts are signed with the customers, both contract and capacity allocations are determined, which blocks a proportion of the capacity for the entire contract duration term.

Consequently, sub-optimal contract decisions would have significant impacts on both contract and spot sales, production and logistic performances, as well as raw material supply. Therefore, the demand contract decisions cannot be made in isolation. They must be coordinated both horizontally across different functions of the supply chain and vertically anticipating the impacts on the down stream operational decisions. This is a typical hierarchical planning problem where one has a decision time hierarchy. The objective of this article is to develop an optimization model to help the manufacturer to coordinate contract decisions at both demand and supply ends, and to allocate capacity, in such a way as to maximize the manufacturer's profitability while hedging against uncertainty.

In reality, during the course of the contract period, many uncertain events may happen related to economic conditions, market prices, customer demand, supply availability, and system capacity due to machine failures. This renders the decision-makers under significant risks when making contract decisions. In order to making robust contract decisions that are capable of coping with various uncertainties, a mathematical model that can anticipate the system performances under different plausible futures is required. In this article, we propose a stochastic programming approach to address coordinated contract design, allocation and selection decisions in a three-tier manufacturing supply chain. The research was carried out based on a real case in the Oriented Strand Board (OSB) industry.

OSB is a wood based structural panel widely used in North America as building material for wall, roof, and floor sheathings as well as I-joists. It is made of wood strands mixed with synthetic resins and wax compressed under high temperature and pressure in a hot press. The production is carried out on a highly automated production line, either in batch or in a continuous manner, depending on the type of hot press used. The production line is capable of making a wide range of OSB products including specialty and commodity products with different physical and mechanical properties. The products are mainly sold on contract and non-contract basis, in different markets, to four categories of customers: manufacturers (producing houses or house components), distributors, wholesalers, and retailers. The demand is highly seasonal with strong correlations with the activities in the building construction industry, whereas the supply, particularly for wood logs from forests, is affected by seasonal harvesting operations and long replenishment lead-times.

We address the three-tier supply chain contract design, allocation, and selection problem from the manufacturer's point of view. The manufacturer wishes to offer different contracts to suit the customers' needs and effectively allocate its resource capacities to the right customers, products, and locations. Among different types of contracts found in the literature and practice, we consider

four types of contracts that the manufacturer may offer: i) price-only, ii) periodical minimum quantity commitment, iii) periodical commitment with order band, iv) periodical stationary commitment. The manufacturer also needs to determine which supply contract to accept from which suppliers in order to guarantee the satisfaction of the contract and non-contract demand at lowest procurement cost. In this study, we limit the supply contracts to total minimum quantity commitments with different terms and prices from different suppliers.

We begin the article with a literature review in Section 2 to establish the foundation for this research. In Section 3, the problem definitions are provided with supply chain characteristics, economic, market, and contract descriptions. The two-stage stochastic programming model is presented in Section 4, followed by the solution approach in Section 5. Scenario sampling and model implementation are discussed in Section 6 with computational results being presented in Section 7. Section 8 provides the concluding remarks and future research directions.

2. Literature Review

Since the 1990s, extensive work has been carried out in the general area of supply chain contracts. Tsay et al. (1999) and Cachon (2003) presented detailed reviews of various forms of contracts. Among them, the price-only contract is probably one of the simplest dominant forms of contracts used in practice. In this type of contract, a manufacturer quotes a unit wholesale price to a customer, and the customer has the flexibility to order any quantity in each period during the contract duration term. Lariviere (1999) pointed out that in price only contracts, suppliers tend to sell at a wholesale price above the production marginal cost, which induces the retailer to set a retail price above what an integrated firm would charge (also known as double marginalization), which could result in lower sales and profits than what an integrated channel would achieve. Lariviere and Porteus (2001) studied the price-only contract in a two-echelon distribution channel with a supplier selling to a single retailer facing a single-period newsvendor problem. It was concluded that price-only contracts cannot provide supply chain coordination.

Another widely applied form of contract is quantity discount contract. This type of contract introduces price incentives so as to stimulate sales and maximize supplier's profits. Monahan (1984) studied a single period quantity discount contract between a buyer and a supplier assuming the buyer is likely to react to any supplier's discount proposal. Weng (1995) investigated the effects of a single period quantity discount model on channel coordination and profit maximization. The analysis shows that quantity discount contracts do not guarantee joint profit maximization. However, channel coordination can be reached by employing quantity discounts and franchise fees simultaneously. Munson and Rosenblatt (2001) studied a quantity discount model in a three-echelon supply chain with the middle echelon being the decision maker

offering different discount schemes. Clearly, discounts can be offered in combination with different contract forms where price incentives are necessary.

Under total minimum quantity commitment contracts, while a supplier offers a discounted price, a total minimum quantity commitment is required and, as the total minimum commitment increases, the unit price decreases. The buyer commits to purchase, during the entire contract horizon, at least the minimum quantity at the discounted price. There is no restriction on the maximum amount that can be purchased, nor requirement on the exact amount purchased in each period. Observations found that in a stochastic demand environment, the buyer inclines to purchase exactly its demand requirement, thus passing its demand uncertainties onto the supplier. Nevertheless, total minimum commitment contracts have been widely used as suppliers wish to increase market shares by locking-in buyers to commit to purchase in a longer term. On the other hand, if there is any uncertainty in the supply process, a buyer may wish to enter into such a contract to ensure long term supply (Anupindi and Bassok 1999). Bassok and Anupindi (1997) provided early work on supply contracts with total minimum quantity commitment for a single-product periodical review inventory problem with random demand. By studying a multi-period setting, Anupindi and Bassok (1999) argue that although the total minimum quantity commitment provides buying flexibility at discounted price, it may lead to supplier loss.

One of the remedy to this problem is the periodical commitment contract. Unlike the total minimum commitment contract, the periodical commitment contract imposes restrictions on periodical purchases and, thus, reduces the uncertainty in the order process. This contract may take various forms depending on the nature of periodical commitments and the flexibility offered. Broadly, the commitments could be stationary or dynamic. Stationary commitment contracts were analysed by Moinzadeh and Nahmias (2000) and Anupindi and Akella (1997). With a stationary commitment, a buyer is required to purchase a fixed minimum amount in each period. Discounts are given based on the level of minimum commitment. Additional units can be purchased but at an extra cost and the delivery may be delayed. This contract provides a greater level of demand certainty for the supplier and just-in-time delivery for the customer. With dynamic commitments, the minimum amount can be updated periodically in a rolling horizon manner. The use of rolling horizon procedures in contract based planning was investigated by D'Amours et al. (2000) in a manufacturing supply chain context. More recently, Lian and Deshmukh (2009) studied a rolling horizon planning contract with dynamic commitment and quantity flexibility between a buyer and a supplier for a single product. The flexibility in the contract can be offered in the form of an order band, where all order quantities are required to be within stationary lower and upper limits. Order-band contracts were initially studied by Kumar (1992) and Anupindi (1993) in a game-theoretic setting. Scheller-Wolf and Tayur (1998)

extended the study in a Markovian demand environment. These contracts can also offer quantity flexibility through changing minimum and maximum limits revised in percentages that vary in accordance with the number of periods away from the delivery (Anupindi and Bassok 1999). Earlier studies on quantity flexibility contracts were published by Bassok and Anupindi (1997), Tsay (1999), and Tsay and Lovejoy (1999).

In supply chain contract design, a decision-maker has to determine what types of contract to offer, with what terms and conditions, and what reactions are possible from the customers. To tackle these questions, most of the researchers adopted an agent-based approach focusing on a contract between a buyer and a supplier. The buyer's optimization problem is solved first to determine his optimal order quantity according to the contract offered by the supplier. Then the supplier's optimization problem is solved for the buyer's optimal order quantity to determine the optimal supply contract. A Nash-equilibrium is reached and the costs (profits) of the buyer and supplier are examined to determine the optimal contract setting (Corbett and Tang 1999, Schneeweiss et al. 2004). When a manufacturer serves several customer-product-locations competing for its limited capacity, such as in our case, contract decisions becomes more complex. Unfortunately, such concerns have not yet been considered in most of the literature. One of the difficulties of addressing the coordinated contract design and allocation problem in a single supplier serving multiple customers is the ability to understand the possible reactions of the customers to the contract(s) offered. Consider that, instead of addressing the supplier's contract design problem based on a single-factor customer cost structure, like what has been assumed in most of the contract analysis and design problems, it is possible that the customer's choice of a contract is affected by several factors, the combined attributes of the contract policy, for instance. In this context, whether or not the customer will choose an offered contract policy is a probabilistic discrete choice problem, depending on the economic evaluation of the customer, as well as his perceived product qualities, the services provided, and socio-economic considerations. According Ben-Akiva and Lerman (1994) and Vila et al. (2007), such probability may be determined based on random utility theory using a logit discrete choice model. Vila et al. (2007) applied this method to determine the customer-contract choice probabilities for several customers, where the customers' reactions to the contracts offered are anticipated in a strategic supply chain design model. Similar approaches are adopted in bidding problems for a manufacturer facing multiple customer classes, as shown in Easton and Moodie (1999) and Watanapa and Techanitisawad (2004).

Furthermore, in the contract analysis and design problems, most of the models proposed assume a deterministic structure, with a few exceptions found in van Delft and Vial (2001), Zou et al. (2008), and Xu and Nozick (2008). Van Delft and Vial (2001) presented a stochastic

programming approach for multi-period supply contract analysis between a buyer and a supplier. Zou et al. (2008) proposed a stochastic dynamic programming approach to design a supply contract between an assembler and two suppliers in an assembly system. Xu and Nozick (2008) proposed a two-stage stochastic model for facility location and network design with the possibility of using option contracts to hedge against uncertain events which could cause capacity loss at one or several suppliers in a geographic area.

In this study, the contract design and allocation problem at the demand end is addressed, and the possible customer reactions to a contract offer are anticipated through probabilistic customer-contract choice analysis. The stochastic programming model presented in the following section is based on the deterministic model for multi-site supply chain sales and operations planning (SC-S&OP) developed by Feng et al. (2008) and the case therein.

3. Problem Definition

3.1 Supply Chain Characteristics

In this study, we consider a manufacturing supply chain network, consisting of a manufacturer, and several customers, suppliers and third-party distribution centres (DCs), as shown in Figure 2. The manufacturer has many production sites scattered in different regions. We define $\mathcal{V} = (S, M, D, C)$ as the set of network nodes (vertices), where S , M , D , and C are subsets associated to raw material suppliers, manufacturing sites, DCs, and customers respectively. Let $\mathcal{R} = (S \times M, M \times D, M \times C, D \times C)$ be the set of inbound and outbound arcs, corresponding to ordered pairs of elements of \mathcal{V} . The manufacturer produces both specialty and commodity products. The specialty products ($i \in I_{\text{spe}}$) are sold through contract agreements, and commodity products ($i \in I_{\text{com}}$) can be sold through contract agreements or on the spot market. Both contract and spot market demands are highly seasonal. Customers ordering specialty products prefer a contract relationship in order to secure their supply. If the contract is not awarded, the customer is likely to seek other sources from competitors. A customer ordering commodity products may also choose a contract relationship, however she may purchase from the manufacturer through spot sales when a contract is not signed. The spot market is considered as a recourse which can absorb any production amount.

Each manufacturing site $m \in M$ has a single capacitated production line producing a set $I = I_{\text{spe}} \cup I_{\text{com}}$ of product families¹ on an MTO basis with small on-site inventory capacity. Every manufacturing site can produce all products $i \in I$ and everyone can contribute to satisfy a given

¹ In the remainder of the text, the word “product” should be interpreted as “product family”.

contract subject to capacity constraints. However a contract may be satisfied more economically by one site than others due to its efficiency and location. We assume that production capacity is affected by unexpected machine down time, and hence, for plant m in planning period $t \in T$, it is an independent random variable K_{mt} with cumulative distribution function $F_{K_{mt}}(\cdot)$.

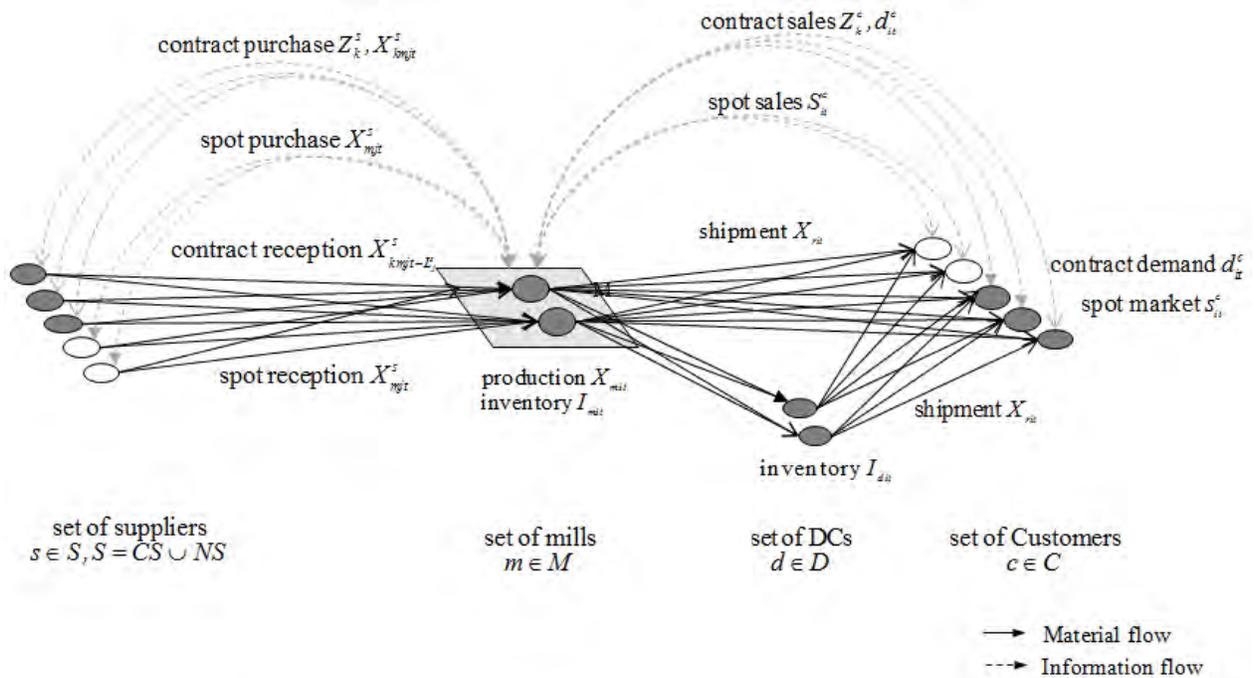


Figure 2. Contract Relationships in a Three-tier Manufacturing Supply Chain Network

The production of each product $i \in I$ consumes a set of J raw materials with different ratios defined by a product recipe. The manufacturer purchases these raw materials from suppliers $s \in S$, including several potential contract suppliers ($CS \subset S$) as well as non-contract spot market suppliers ($NS \subset S$). Suppliers have different procurement lead-times L_j^s for raw material $j \in J$. Raw materials are classified into different categories and stored using different storage technologies. Let G be the set of storage technologies and g , a particular technology with storage capacity KI_{ng} for mill m . Also, let $J_g \subset J$ be the subset of raw materials that can be stored with technology g . The raw material inventory is managed internally complying with safety stock policies. We assume that inbound raw material shipments are carried out by the suppliers, and that their shipping costs are included in the procurement costs.

The outbound shipments of the products from the manufacturing sites to the customers are carried out by third party logistic (3PL) providers, either directly or indirectly via a DC $d \in D$. The manufacturer has an access to several third party DCs which are assumed to have unlimited capacity. We assume a shipment cost is incurred for the flows on each outbound arc with a unit variable rate.

3.2 *Economic Trends*

At the time of making contract decisions, the manufacturer faces uncertainties in market prices, customer demand (both contract and non-contract), customer-contract preferences, raw material prices, and raw material supply availability. These uncertainties are related to the actions of competitors and, in particular, to the state of the economy. In order to take this into account, we assume that the random variables used to model these exogenous factors depend on a finite set Ξ of plausible economic trends over the planning horizon considered. The probability $P(e)$ that economic trend $e \in \Xi$ will prevail over the horizon is estimated subjectively by a panel of industry experts. More specifically, we assume that the probability distribution of the random variables associated to planning period $t \in T$ depends on the prevailing economic trend $e \in \Xi$. The trend is defined by a function of the period index $t \in T$ and it is applied to the value of the original random variables. A typical case would be the consideration of expanding, stable and weakening economic trends defined by multiplying a given random variable by a linearly increasing (decreasing) per-period inflation (deflation) factor.

3.3 *Customer Contract Policies*

As described in Section 1, we examine four potential forms of contracts that the manufacturer may offer to customers: price-only, periodical minimum quantity commitment, periodical commitment with an order band, and periodical stationary commitment. These forms of contracts provide different levels of quantity commitments and flexibilities. For each form of contract, the manufacturer may develop different policies with different contract duration terms and price incentives. Let K^C be the entire set of potential customer contract policies the manufacturer offers. Each contract policy $k \in K^C$ is characterized by a number of distinguishing attributes that influence customer decisions. Without loss of generality, such attributes may include a price discount factor ϕ_k , a fixed contract charge a_k , a quantity flexibility expressed by minimum and maximum quantities lb_k and ub_k , a contract starting period t_k , and a contract duration term N_k (in periods). These attribute values may be determined by the manufacturer's observations of the historical customer ordering behaviours, contract strategies, and pricing experiences. Obviously, the price-only contract provides the greatest quantity flexibility with lb_k being "0" and ub_k being a sufficient large number, while periodical stationary commitment has the least flexibility with $lb_k = ub_k$.

Given the contract commitments and flexibilities, since contract demand may vary randomly, it may be impossible to satisfy the entire contract demand in each period with the finite capacity available. Hence, backlogs are allowed for contract demand. Different backlog penalty costs are

used for different forms of contracts, so that backlog, should it becomes necessary, is more likely to occur for contracts with greater quantity flexibilities (such as price-only contract).

3.4 Customer-Contract Choice Analysis

A manufacturer's decision to offer a contract to a customer does not guarantee that the contract will be signed, but implies that it is feasible and economically advantageous for the manufacturer. Whether or not a customer c will accept a contract k offered, under economic trend e , is modelled using a discrete choice random variable ξ_{ke}^c . In an industrial environment, this choice is affected by many factors such as prices, commitment requirements, customer demand, contract duration terms, product quality, service standards, location, and socio-economic considerations. It is also affected by the competitors' offers available in the market. Let K be the universal contract set offered to some customer population, including all the contract policies offered by the manufacturer, the competitors, as well as the virtual contract ($k = 0$) offered by the spot market, ($K \supset K^c$). Each customer c in the customer population has a preference to a subset of the contracts $\mathcal{K}^c \subset K$. According to Ben-Akiva and Lerman (1994) and Vila et al. (2007), the customer's preferences for one contract over the alternative subset of contracts can be modeled based on economic consumer theory, assuming that the customer has the ability to compare all possible contracts, using discrete choice analysis.

In discrete choice analysis, the attractiveness of each alternative contract can be evaluated by a vector of the attribute values, such as $v_k = (\phi_k, lb_k, ub_k, N_k, id(k))$, where $id(k)$ provides the identity of the manufacturer who is making the offer. Based on random utility theory, the choice preference of customer c for a contract k under economic trend e can be modeled as a linear utility function:

$$U_e^c(k) = \beta_{1e}\phi_k + \beta_{2e}lb_k + \beta_{3e}ub_k + \beta_{4e}N_k + \beta_{5e}id(k) + \varepsilon_{cek}, \quad c \in C, e \in \mathcal{E}, k \in \mathcal{K}^c$$

where $\beta_{1e}, \dots, \beta_{5e}$ are parameters to be estimated, and ε_{cek} is an independent Gumbel distributed random disturbance. This random disturbance is introduced to take into account any unexpected influences.

Customer c will likely choose a contract policy $k \in \mathcal{K}^c$ that has the highest utility value. Thus, the probability that customer c chooses a contract k under economic trend e can be expressed by:

$$P_e^c(k) = P(U_e^c(k) \geq U_e^c(l), \forall l \in \mathcal{K}^c, l \neq k)$$

Note that for a given contract horizon $T_k = \{t_k, \dots, t_k + N_k - I\}$, the manufacturer could only offer a single contract policy $k \in K^c$ to a customer. In order to calculate the probability $P_e^c(k)$, only offer k and offers of the competitors should be considered. Let $\mathcal{K}^c(k) \subset \mathcal{K}^c$ be the set of these

offers. When using a Multinomial Logit discrete choice model, the probability that the contract k would be signed if offered to customer c under economic trend e can be calculated using the expression:

$$P_e^c(k) = \frac{e^{\mu(\beta_{1e}\phi_k + \beta_{2e}lb_k + \beta_{3e}ub_k + \beta_{4e}N_k + \beta_{5e}\text{id}(k))}}{\sum_{l \in \mathcal{K}^c(k)} e^{\mu(\beta_{1e}\phi_l + \beta_{2e}lb_l + \beta_{3e}ub_l + \beta_{4e}N_l + \beta_{5e}\text{id}(l))}}, \quad c \in C, e \in \Xi, k \in \mathcal{K}^c(k)$$

where μ is a positive scale parameter.

In order to calculate these probabilities, it is necessary to estimate the parameter values $\beta_{1e}, \dots, \beta_{5e}$, $e \in \Xi$. This can be done using revealed preference data (Ben-Akiva and Lerman, 1994) or stated preference data (Louviere *et al.*, 2000). The former is derived from the analysis of each customer's behaviour based on the demand observations available. The later is obtained from a questionnaire with hypothetical offers submitted to a sample of customers. With this data, maximum likelihood estimators are used to obtain the parameter values. This can be implemented, for example, with the BIOGEME software developed by Bierlaire and available on the Web at <http://roso.epfl.ch/biogeme>. Alternatively, with insufficient customer preference data, subjective preference probabilities $P_e^c(k)$, $e \in \Xi$, may be assigned by the company sales force to each customer c for each contract k .

3.5 Customer Demand

When a customer c chooses a contract $k \in K^c$, his demand must comply with the contract duration terms and quantity commitments. We assume that the requirements of customer $c \in C$ for product $i \in I$ during period $t \in T$, under economic trend $e \in \Xi$, is an independent random variable d_{ite}^c with cumulative distribution function $F_{d_{ite}^c}(\cdot)$. Taking into account the contract terms, quantity commitments, and customer choices, the contract demand of customer c under contract k for product i in periods t is defined by:

$$d_{kite}^c = \begin{cases} \min(\max(lb_k, d_{ite}^c), ub_k), & \text{if } \xi_{ke}^c = 1 \\ 0, & \text{otherwise} \end{cases} \quad \forall i, e, k \in K^c, t \in T_k$$

Note that the contract demand therefore depends on three random variables: the economic trend e , the discrete choice ξ_{ke}^c , and the customer requirements d_{ite}^c . When no contract is signed with customer c for period t , the potential spot demand is assumed to be equal to the customer requirements d_{ite}^c for commodity products $i \in I_{\text{com}}$, and to "0" for specialty products $i \in I_{\text{spe}}$.

3.6 Contract and Spot Market Pricing

In the OSB industry, the manufacturers' contract and spot sales price are influenced by a market reference price, which depends on the economic trend $e \in \Xi$. In order to win customer contracts, manufacturer may use different pricing strategies. For contract pricing, we assume the

manufacturer uses an n -period backward moving average of the market reference price, adjusted by an appropriate contract discount factor ϕ_k . The contract price of product i for customer c under contract k in period t and economic trend e can thus be defined by $p_{kite}^c = \phi_k \sum_{t'=t-n}^{t-1} p_{it'e}^{c,ref} / n$, $\forall c, k, i, t, e$, where $p_{ite}^{c,ref}$ is the market reference price in the customer's market under economic trend e , which is an independent random variable having cumulative distribution function $F_{p_{ite}^{c,ref}}(\cdot)$. For spot sales pricing, we assume the manufacturer uses the market reference price, i.e. $p_{ite}^c = p_{ite}^{c,ref}$, $\forall c, i, t, e$.

3.7 Supply Contracts and Spot Market Alternatives

At the procurement end, the manufacturer may purchase raw materials from contract suppliers ($s \in CS$) or on the spot market ($s \in NS$). At the beginning of each planning horizon, potential contract suppliers offer several supply contracts. Let K^S be the entire set of potential contract policies offered by the suppliers. We assume that suppliers offer only "total minimum quantity commitment" contracts, where each contract policy $k \in K^S$ is characterized by a unique pair of unit purchase cost c_{kjt}^s and total minimum quantity commitment requirement lb_k^s . Alternatively, the manufacturer may purchase raw materials from the spot market at price c_{jte}^s , subject to the market availability KS_{te}^s . The spot market prices and availabilities are assumed to be independent random variables affected by the plausible economic trends, and with cumulative distribution functions $F_{c_{jte}^s}(\cdot)$ and $F_{KS_{te}^s}(\cdot)$.

4. Stochastic Programming Formulation

The superposition of specific realizations of the random variables defined previously gives rise to a set Ω of plausible future scenarios. This is the set of all the scenarios that may occur over the planning horizon under the different plausible economic trends considered. As explained later (in section 6.2), scenarios can be generated using Monte Carlo methods, and a scenario $\omega \in \Omega$ is associated to the following set of specific random variable realizations

$$\left\{ \xi_k^c(\omega), d_{kit}^c(\omega), d_{it}^c(\omega), p_{kit}^c(\omega), p_{it}^c(\omega), c_{jt}^s(\omega), KS_t^s(\omega), K_m(\omega), \forall c, s, k, i, j, m, t \right\}$$

We assume that all the contract decisions must be taken at the beginning of the planning horizon, which enables us to model the problem as a two-stage stochastic program with fixed recourse. In the model, the contract decisions (for both demand and supply) are first stage decision variables. In the second stage, future operational decisions and performances are anticipated for given first stage contract decisions, under a given scenario $\omega \in \Omega$. The objective of the model is to find efficient and robust solutions, (1) for the selection of customer demand contracts according to perceived customer choice probabilities, in order to best allocate the manufacturer's capacities; and (2) for the selection of supplier contracts in order to guarantee the satisfaction of the demand.

The model maximizes the manufacturer's expected global profitability while hedging against uncertainty.

4.1 Mathematical Notation

The following notations are required to formulate the model:

Indexes and sets

$m \in M$	Set of manufacturing mills
$c \in C$	Set of customers
$s \in S$	Set of contract (CS) and spot (NS) raw material suppliers ($S = CS \cup NS$)
$d \in D$	Set of distribution centres (DCs)
$i \in I$	Set of specialty (I_{spe}) and commodity (I_{com}) products ($I = I_{spe} \cup I_{com}$)
$j \in J$	Set of raw materials
$g \in G$	Set of raw material storage technologies
J_g	Set of raw materials requiring storage technology g ($J_g \subset J$)
$r \in \mathcal{R}^{MC}$	Set of outbound arcs from mills to customers ($\mathcal{R}^{MC} = M \times C$)
$r \in \mathcal{R}^{MD}$	Set of outbound arcs from mills to DCs ($\mathcal{R}^{MD} = M \times D$)
$r \in \mathcal{R}^{DC}$	Set of outbound arcs from DCs to customers ($\mathcal{R}^{DC} = D \times C$)
$r \in \mathcal{R}^O$	Set of all outbound arcs ($\mathcal{R}^O = \mathcal{R}^{MC} \cup \mathcal{R}^{MD} \cup \mathcal{R}^{DC}$)
$k \in K^C$	Set of contract policies the manufacturer offers to customers
$k \in K^S$	Set of contract policies offered by the raw material suppliers
$e \in \Xi$	Set of plausible economic trend over the planning horizon
$t \in T$	Set of planning periods
T_k	Set of planning periods covered by contract k ($T_k \subseteq T$)

Parameters

Sales

$\xi_k^c(\omega)$	Binary choice parameter of customer c for contract policy $k \in K^C$ under scenario ω
a_k	Fixed charge of a demand contract policy $k \in K^C$
a_k^s	Fixed cost of a supply contract with supplier s under contract policy $k \in K^S$
$p_{kit}^c(\omega)$	Sales price of product i for customer c with contract policy k in period t for scenario ω
$p_{it}^c(\omega)$	Spot sales price of product i for customer c in period t for scenario ω
$d_{kit}^c(\omega)$	Contract demand of product i from customer c choosing contract policy k in period t for scenario ω
$d_{it}^c(\omega)$	Spot demand of product i from customer c in period t for scenario ω
π_k	Multiplicative penalty factor for contract $k \in K^C$ demand backlogs

Production

c_{mi}	Unit production cost for product i at mill m
h_{mi}	Inventory holding per unit for product i at mill m
α_{mi}	Capacity consumption coefficient for product i at mill m
$K_{mt}(\omega)$	Production capacity of mill m in period t for scenario ω
u_{mji}	Quantity of raw material j required to produce one unit of product i at mill m
h_{mj}	Unit inventory holding cost of raw material j at mill m
ss_{mj}	Safety stock of raw material j at mill m
KI_m	Finished product storage capacity at mill m (expressed in terms of an upper bound on the inventory level)
KI_{mg}	Raw material storage capacity of technology $g \in G$ at mill m (expressed in terms of an upper bound on the inventory level)

Distribution

c_{ri}	Unit shipping cost for product i on arc r
h_{di}	Inventory holding cost per unit for product i at distribution centre d
tr_{di}	Transshipment cost per unit for product i through distribution centre d

Procurement

c_{kjt}^s	Unit raw material j purchase cost from supplier $s \in CS$ in period t under contract $k \in K^s$
$c_{jt}^s(\omega)$	Unit raw material j spot purchase cost from supplier $s \in NS$ in period t for scenario ω
lb_k^s	Minimum purchase quantity defined by contract policy $k \in K^s$ offered by supplier $s \in CS$
KS_t^s	Supply capacity of contract supplier $s \in CS$ in period t
$KS_t^s(\omega)$	Supply capacity of spot supplier $s \in NS$ in period t for scenario ω
L_j^s	Procurement lead-time of raw material j provided by supplier $s \in S$

Decision variables

First stage variables

Z_k^c	Binary variable equal to “1” if sale contract policy $k \in K^c$ is offered to customer c , and “0” otherwise
Z_k^s	Binary variable equal to “1” if procurement contract $k \in K^s$ is signed with supplier s , and “0” otherwise

Sales recourse variables

$S_{it}^c(\omega)$	Spot sales of product i to customer c in period t for scenario ω
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$z_{kit}^c(\omega)$ Product i backlog for the demand of customer c under contract k at the end of period t for scenario ω

Production recourse variables

$X_{mit}(\omega)$ Production quantity of product i at mill m in period t for scenario ω
 $I_{mit}(\omega)$ Inventory of product i in mill m at the end of period t for scenario ω
 $I_{mjt}(\omega)$ Inventory of raw material j at mill m at the end of period t in scenario ω

Distribution recourse variables

$X_{rit}(\omega)$ Quantity of product i shipped on arc r in period t for scenario ω
 $I_{dit}(\omega)$ Inventory of product i in distribution centre d at the end of period t for scenario ω

Procurement recourse variables

$X_{kmjt}^s(\omega)$ Amount of raw material j purchased by mill m from contract supplier $s \in CS$ under contract $k \in K^s$ in period t for scenario ω
 $X_{mjt}^s(\omega)$ Amount of raw material j purchased by mill m from spot supplier $s \in NS$ in period t for scenario ω
 $z_{ij}^s(\omega)$ Raw material j procurement underage with respect to the minimum commitment quantity imposed by contract $k \in K^s$ with supplier $s \in CS$ for scenario ω

4.2 Scenario Based Stochastic Programming Model

The first stage program is formulated as follows:

$$\max f(\mathbf{Z}) = E_{\Omega} [Q(\mathbf{Z}, \omega)] - \sum_{s \in CS} \sum_{k \in K^s} a_k^s Z_k^s \quad (4.1)$$

subject to

$$\sum_{k \in K^c | t \in T_k} Z_k^c \leq 1 \quad \forall c, t \quad (4.2)$$

$$\sum_{k \in K^s} Z_k^s \leq 1 \quad s \in CS \quad (4.3)$$

$$Z_k^c \in \{0, 1\}, \forall c, k \in K^c; \quad Z_k^s \in \{0, 1\}, \forall s, k \in K^s \quad (4.4)$$

In the objective function (4.1), $E_{\Omega}[\cdot]$ denotes expected value over all scenarios $\omega \in \Omega$, and \mathbf{Z} the vector of all first stage decision variables Z_k^c and Z_k^s . The function $Q(\mathbf{Z}, \omega)$, provides the value of the optimal solution of the second stage program for a given \mathbf{Z} and $\omega \in \Omega$. Constraints (4.2) state that the manufacturer cannot have more than one contract with a customer in any period t . Constraints (4.3) state that the manufacturer cannot have more than one contract with a supplier and (4.4) define the domain for the demand and supply contract decision variables.

The objective function of the second stage program is following:

$$Q(\mathbf{Z}, \omega) = \max_{\mathbf{Y}(\omega) \geq 0} q(\mathbf{Z}, \mathbf{Y}(\omega)) \quad (4.5)$$

$$\begin{aligned} q(\mathbf{Z}, \mathbf{Y}(\omega)) = & \left(\sum_{c \in C} \sum_{k \in K^c} \xi_k^c(\omega) a_k Z_k^c \right) + \left(\sum_{c \in C} \sum_{k \in K^c} \sum_{i \in I} \sum_{t \in T_k} p_{kit}^c(\omega) d_{kit}^c(\omega) Z_k^c + \sum_{c \in C} \sum_{i \in I} \sum_{t \in T} p_{it}^c(\omega) S_{it}^c(\omega) \right) \\ & - \left(\sum_{m \in M} \sum_{i \in I} \sum_{t \in T} (c_{mi} X_{mit}(\omega) + h_{mi} I_{mit}(\omega)) \right) \\ & - \sum_{i \in I} \sum_{t \in T} \left(\sum_{r \in \mathcal{R}^o} c_{rit} X_{rit}(\omega) + \sum_{r \in \mathcal{R}^{MD}} tr_{dit} X_{rit}(\omega) + \sum_{d \in D} h_{dit} I_{dit}(\omega) \right) \\ & - \sum_{m \in M} \sum_{j \in J} \sum_{t \in T} \left(\sum_{s \in CS} \sum_{k \in K^s} (c_{kjt}^s X_{kmjt}^s(\omega) + c_{kjt}^s z_{kj}^s(\omega)) + \sum_{s \in NS} c_{jt}^s(\omega) X_{mjt}^s(\omega) + h_{mj} I_{mjt}(\omega) \right) \\ & - \left(\sum_{c \in C} \sum_{k \in K^c} \sum_{i \in I} \sum_{t \in T_k} \pi_k p_{kit}^c(\omega) z_{kit}^c(\omega) \right) \end{aligned} \quad (4.6)$$

where $\mathbf{Y}(\omega)$ is the vector of all the second stage decision variables for scenario ω . The function $q(\cdot)$ defines the net profit calculated by summing the revenues from the fixed contract charge, as well as contract and spot sales, as shown in the first two sets of brackets, minus the total cost of production, distribution, procurement, and any penalties as expressed by the third, fourth, fifth, and sixth sets of brackets. In the first set of brackets, the fixed contract charge is applied only when the contract is accepted by both parties ($\xi_k^c(\omega) = Z_k^c = I$). In the third set of brackets, the production cost includes the costs of making and inventory holding at the mills. The backlog penalty cost is considered in the last set of brackets. The distribution cost, as shown in the fourth set of brackets, consists of the total cost of shipping, transshipment, and inventory holding at the DCs. The procurement cost, as shown in the fifth set of brackets, includes the costs of both contract and non-contract raw material purchases, the inventory holding, as well as the raw material purchase underage $z_{kj}^s(\omega)$, with respect to the contract minimum quantity commitment. The last set of the brackets provides the penalty cost for the backlogs of the contract demand $z_{kit}^c(\omega)$. The recourse variables, $z_{kit}^c(\omega)$ and $z_{kj}^s(\omega)$, ensure the feasibility of the second stage program for all \mathbf{Z} .

The second stage program includes the following constraints:

Constraints concerning sales:

$$\sum_{r \in (\mathcal{R}^{MC} \cup \mathcal{R}^{DC})} X_{rit}(\omega) = \sum_{k \in K^c} (Z_k^c d_{kit}^c(\omega) + z_{kit-l}^c(\omega) - z_{kit}^c(\omega)) + S_{it}^c(\omega) \quad \forall c, i, t, \omega \quad (4.7)$$

$$z_{kit}^c(\omega) \leq Z_k^c d_{kit}^c(\omega) \quad \forall c, i, t, \omega, k \in K^c \quad (4.8)$$

$$S_{it}^c(\omega) \leq (1 - Z_k^c) d_{it}^c(\omega) \quad \forall c, t, \omega, k \in K^c, i \in I_{com} \quad (4.9)$$

Constraints (4.7) describe the flow balance at a customer node, stating that the shipments to the customer must be equal to the contract sales quantity (if a contract is provided) plus the backlog in the previous period minus the backlog at the end of the current period, or otherwise, the spot sales quantity. Constraints (4.8) provide the bound for the contract demand backlog. Constraints (4.9) state that, when a customer is not served by a contract, spot sales should not exceed the customer's non-contract demand $d_{it}^c(\omega)$.

Constraints concerning production and distribution:

$$X_{mit}(\omega) + I_{mit-1}(\omega) - I_{mit}(\omega) = \sum_{r \in (\mathcal{R}^{MD} \cup \mathcal{R}^{MC})} X_{rit}(\omega) \quad \forall m, i, t, \omega \quad (4.10)$$

$$\sum_{r \in \mathcal{R}^{MD}} X_{rit}(\omega) + I_{dit-1}(\omega) - I_{dit}(\omega) = \sum_{r \in \mathcal{R}^{DC}} X_{rit}(\omega) \quad \forall d, i, t, \omega \quad (4.11)$$

$$\sum_{i \in I} \alpha_{mi} X_{mit}(\omega) \leq K_m(\omega) \quad \forall m, t, \omega \quad (4.12)$$

$$\sum_{i \in I} I_{mit}(\omega) \leq KI_m \quad \forall m, t, \omega \quad (4.13)$$

Constraints (4.10) and (4.11) are the flow conservation constraints at the mills and the DCs. Constraints (4.12) and (4.13) are capacity constraints for production and inventory, respectively.

Constraints concerning procurement:

$$\sum_{s \in CS} \sum_{k \in K^s} X_{kmjt-L_j^s}^s(\omega) + \sum_{s \in NS} X_{mjt}^s(\omega) + I_{mjt-L}(\omega) - I_{mjt}(\omega) = \sum_{i \in I} u_{mji} X_{mit}(\omega) \quad \forall m, j, t = 1 + L_j^s, \dots, T, \omega \quad (4.14)$$

$$\sum_{j \in J} \left(\sum_{m \in M} \sum_{t \in T} X_{kmjt}^s(\omega) + z_{kj}^s(\omega) \right) \geq Z_k^s lb_k^s \quad \forall s \in CS, k \in K^s, \omega \quad (4.15)$$

$$\sum_{k \in K^s} \sum_{m \in M} \sum_{j \in J} X_{kmjt-L_j^s}^s(\omega) \leq \sum_{k \in K^s} Z_k^s KS_t^s \quad \forall s \in CS, t = 1 + L_j^s, \dots, T, \omega \quad (4.16)$$

$$\sum_{m \in M} \sum_{j \in J} X_{mjt}^s(\omega) \leq KS_t^s(\omega) \quad \forall s \in NS, t, \omega \quad (4.17)$$

$$\sum_{j \in J_t} I_{mjt}(\omega) \leq KI_{mg} \quad \forall m, g, t, \omega \quad (4.18)$$

$$I_{mjt}(\omega) \geq ss_{mj} \quad \forall m, j, t, \omega \quad (4.19)$$

Constraints (4.14) are the flow balance constraints for raw material requirements at mills, taking into account the supplier lead times. Constraints (4.15) impose the total minimum quantity commitment the manufacturer must comply with when a supply contract is signed. Constraints (4.16) and (4.17) are capacity constraints for the contract and spot suppliers, respectively. Constraints (4.18) are raw material inventory capacity constraints. Safety stock requirement constraints are given by (4.19).

Valid cuts:

In order to improve the solution time, the following cuts are added to the model:

$$\sum_{m \in M} (X_{mit}(\omega) + I_{mit-1}(\omega) - I_{mit}(\omega)) + \sum_{d \in D} (I_{dit-1}(\omega) - I_{dit}(\omega)) = \sum_{c \in C} \left(\sum_{k \in K^c} (Z_k^c d_{kit}^c(\omega) + z_{kit-1}^c(\omega) - z_{kit}^c(\omega)) + S_{it}^c(\omega) \right) \quad \forall i, t, \omega \quad (4.20)$$

These cuts define the aggregate flow balance over the manufacturing sites, DCs and customers. They are valid since they are a linear combination of constraints (4.7), (4.10) and (4.11). Our preliminary tests have shown that the cuts reduce computation time by a factor of about 6.

5. Sample Average Approximation

In most applications, the set Ω includes an infinite number of scenarios, which makes the proposed stochastic programming model impossible to solve. The Sample Average Approximation (SAA) method can however be used to obtain near optimal solutions. This method has been theoretically analysed by several authors (Mak et al., 1999; Shapiro, 2003) and applied to solve various stochastic supply chain design problems (Santoso et al., 2005; Vila et al., 2007). It involves solving the problem for samples of scenarios randomly selected from the population Ω . For this purpose, B random samples $\Omega_b^N = \{\omega_b^1, \dots, \omega_b^N\}$, $b = 1, \dots, B$, of N scenarios are generated using Monte Carlo methods. For a sample b , the true problem (4.1) – (4.19), with the expected value function $E_\Omega [Q(\mathbf{Z}, \omega)]$ in (4.1), is replaced by the following SAA program:

$$\max \hat{f}_b^N(\mathbf{Z}) = \frac{1}{N} \sum_{n=1}^N q(\mathbf{Z}, \mathbf{Y}(\omega_b^n)) - \sum_{s \in CS} \sum_{k \in K^s} a_k^s Z_k^s \quad (5.1)$$

subject to constraints (4.2) – (4.4) and (4.7) – (4.19).

Note that in this program, the second stage constraints (4.7) – (4.19) are defined over the scenarios $\omega \in \Omega_b^N$ of the sample considered. Program (5.1) is solved for the B samples generated and the best solution found is selected. The SAA program (5.1) is a large mixed integer program (MIP) but, for a moderate sample size N , it can be solved using commercial solvers such as CPLEX. Even if a moderate sample size is used, we expect that the contract decisions made using this approach are considerably more robust than the solutions provided by a deterministic model. Clearly, as the number of scenarios N increases, the quality of the decisions improves. As shown by Shapiro (2003), under mild regularity conditions, the solution of the SAA model converges with probability one to the optimal solution of the true problem, as sample size N increases. Also, using B independent random samples of size N increases the probability of finding the true optimal solution.

An important issue is how to select the best solution among the B solutions found, and how close this solution is to the optimal solution of the true problem. The quality of a candidate solution can be evaluated by estimating a statistical optimality gap and confidence intervals. In the following

paragraphs, we present the SAA solution algorithm developed to solve our model. Similar procedures are found in Santoso et al. (2005) and Vila et al. (2007).

SAA Algorithm:

Step 1. Generate B independent samples of N scenarios Ω_b^N , $b = 1, \dots, B$. For each sample, solve the SAA model (5.1). Let v_b^N and $\hat{\mathbf{Z}}_b^N$ be the corresponding optimal objective value and optimal solution, respectively.

Step 2: Compute the statistical upper bound and variance estimators.

$$\bar{U}_{N,B} = \frac{1}{B} \sum_{b=1}^B v_b^N \quad (5.2)$$

It can be shown that $\bar{U}_{N,B} \geq v^*$, where v^* denotes the optimal value of the true problem (Mak et al. 1999). Thus $\bar{U}_{N,B}$ provides a statistical upper bound. Since the B samples generated, and hence v_1^N, \dots, v_B^N , are independent, the variance of $\bar{U}_{N,B}$ is given by:

$$\hat{\sigma}_{\bar{U}_{N,B}}^2 = \frac{1}{B(B-1)} \sum_{b=1}^B (v_b^N - \bar{U}_{N,B})^2 \quad (5.3)$$

Step 3: Compute the statistical lower bound and variance estimators.

For each distinct candidate solution $\hat{\mathbf{Z}}_b^N$ obtained in *Step 1*, estimate the true objective function value $f(\hat{\mathbf{Z}}_b^N)$ as follows:

$$\tilde{f}_{N_l}(\hat{\mathbf{Z}}_b^N) = \frac{1}{N_l} \sum_{n=1}^{N_l} Q(\hat{\mathbf{Z}}_b^N, \omega^n) - \sum_{s \in CS} \sum_{k \in K^s} a_k^s (\hat{\mathbf{Z}}_k^s)_b^N \quad (5.4)$$

where $\omega^1, \dots, \omega^{N_l}$ is a sample of size $N_l \square N$ generated independently of the samples used to obtain $\hat{\mathbf{Z}}_b^N$ in *Step 1*. Note that $\tilde{f}_{N_l}(\hat{\mathbf{Z}}_b^N)$ is an unbiased estimator of $f(\hat{\mathbf{Z}}_b^N)$. Since $\hat{\mathbf{Z}}_b^N$ is a feasible solution to the true problem, we have $f(\hat{\mathbf{Z}}_b^N) \leq v^*$. Thus, $\tilde{f}_{N_l}(\hat{\mathbf{Z}}_b^N)$ provides a lower statistical bound on v^* . Since we have an independent sample, the variance of this estimator is given by:

$$\hat{\sigma}_{\tilde{f}_{N_l}(\hat{\mathbf{Z}}_b^N)}^2 = \frac{1}{N_l(N_l-1)} \sum_{n=1}^{N_l} \left(Q(\hat{\mathbf{Z}}_b^N, \omega^n) - \sum_{s \in CS} \sum_{k \in K^s} a_k^s (\hat{\mathbf{Z}}_k^s)_b^N - \tilde{f}_{N_l}(\hat{\mathbf{Z}}_b^N) \right)^2 \quad (5.5)$$

Step 4: Calculate the optimality gap and the confidence interval.

Having determined the statistical upper and lower bounds from *Step 2* and *3*, the optimality gap of solution $\hat{\mathbf{Z}}_b^N$ can be estimated by:

$$Gap_{N,B,N_l}(\hat{\mathbf{Z}}_b^N) = \max \left\{ 0, \bar{U}_{N,B} - \tilde{f}_{N_l}(\hat{\mathbf{Z}}_b^N) \right\} \quad (5.6)$$

The estimated variance of the gap is given by:

$$\hat{\sigma}_{Gap}^2 = \hat{\sigma}_{\bar{U}_{N,B}}^2 + \hat{\sigma}_{\tilde{f}_{N_l}(\hat{\mathbf{Z}}_b^N)}^2 \quad (5.7)$$

An approximate $100(1-\alpha)$ percent confidence interval for the optimality gap at $\hat{\mathbf{Z}}_b^N$ is given by:

$$\left[0, \text{Gap}_{N,B,N_l}(\hat{\mathbf{Z}}_b^N) + \frac{t_{\frac{\alpha}{2}, B-1} \hat{\sigma}_{\bar{U}_{N,B}}}{\sqrt{B}} + \frac{t_{\frac{\alpha}{2}, N_l-1} \hat{\sigma}_{\tilde{f}_{N_l}(\hat{\mathbf{z}}_b^N)}}{\sqrt{N_l}} \right] \quad (5.8)$$

assuming random variables v_b^N and $Q(\hat{\mathbf{Z}}_b^N, \omega^n)$ follow a t-distribution with $B-1$ and N_l-1 degrees of freedom, respectively (Mak et al. 1999).

Step 5: Select the solution $\hat{\mathbf{Z}}_b^N, b = 1, \dots, B$, with the highest estimated true objective function value $\tilde{f}_{N_l}(\hat{\mathbf{Z}}_b^N)$.

Having selected the best contract solution, its quality can be evaluated by examining the gap and confidence interval. If the gap and confidence interval are not acceptable, a larger number of samples B and/or sample size N must be used in order to find better solutions.

6. Application to an OSB Industrial Case

6.1 Case Description

In order to validate the methodology, the two-stage stochastic programming model proposed was applied to the real industrial case context presented in Feng et al. (2008). The numerical tests were based on the field data obtained from a single OSB mill. The mill has a single capacitated production line. Production is carried out in batches using a multi-daylight hot press. The production line produces 11 products, on an MTO basis, and it consumes 8 raw materials supplied by 11 raw material suppliers. The products are sold to 140 customers across 5 different regions in North America. In order to effectively apply the methodology, following a Pareto analysis, 20 customers, accounting for 80% of the sales in the 5 regions, were explicitly considered. The rest of the customers and their demands were aggregated to form the spot market in each of the regions. The shipping unit costs to each of the customers are known, and for the spot markets they were estimated based on the weighted cost to each of the customers within the region. On the raw material procurement side, the lead time varies depending on the suppliers and raw material types, being either 0 or 1 period. For demand contracts, 4 forms of contract were offered to the customers, as described in Section 3.3, with different discount, fixed charges, minimum/maximum quantities, and contract horizons, yielding 28 contract policies. For procurement contracts, we considered 7 supply contract offers from 7 suppliers all with yearly contract duration term. The study was conducted with monthly planning periods and a planning horizon of one year. The scope of the case is outlined in Table 1.

Table 1. The Scope of the OSB Case

Indexes	Sizes
Mills	1
Facilities	1
Distribution centers	2
Products	11
Customers	20
Spot market	5
Raw material suppliers	11
Raw materials	8
Demand contract potential offers	28
Supply contract offers	7
Planning horizon	12 months

In this study, the deterministic parameters were derived from field data as explained in Feng et al. (2008). For the random parameters, probability distributions were estimated respectively, using five years market data for the reference price, and one year’s data for customer demand, production capacity, raw material spot price, and raw material spot capacity. The best fit for the market reference price, demand, raw material spot price and raw material spot capacity was a Normal distribution, and the standard deviations were obtained by multiplying the historic means by an estimated coefficient of variation (0.05, 0.20, 0.05, and 0.20, respectively). The best fit for the manufacturing capacity, based on down time analysis, was a Uniform distribution. Three possible economic trends were considered: stable (S), expanding (E), or weakening (W). The corresponding estimated probabilities were $P(S) = 70\%$, $P(E) = 20\%$, $P(W) = 10\%$ and the trends were defined by linearly increasing (decreasing) annual inflation (deflation) factors $\lambda_{et} = (a_e/T)t + 1$ with a_e , being 0%, 10%, and -10%, respectively, for all $e \in \Xi = \{S, E, W\}$, over the planning horizon of $T = 12$ monthly periods. The distributions for the random variables and corresponding inflation (deflation) factors are shown in Table 2.

Table 2. Random variables, their probability distributions and inflation (deflation) factors

Random Variables	Distributions	inflation/deflation factors
Market reference price $p_{ite}^{c,ref}$	$F_{p_{ite}^{c,ref}}(\cdot) = Normal(\mu(p_{ite}^{c,ref}), \sigma(p_{ite}^{c,ref}))$	λ_{et}
Demand d_{ite}^c	$F_{d_{ite}^c}(\cdot) = Normal(\mu(d_{ite}^c), \sigma(d_{ite}^c))$	λ_{et}
Raw material spot price c_{jte}^s	$F_{c_{jte}^s}(\cdot) = Normal(\mu(c_{jte}^s), \sigma(c_{jte}^s))$	λ_{et}
Raw material spot capacity KS_{ite}^s	$F_{KS_{ite}^s}(\cdot) = Normal(\mu(KS_{ite}^s), \sigma(KS_{ite}^s))$	$\lambda_{et}^{KS} = -\frac{a_e}{T}t + 1$
Production capacity K_{mt}	$F_{K_{mt}}(\cdot) = Uniform(\theta_1^{K_{mt}}, \theta_2^{K_{mt}})$	--

The notations $\mu(\cdot)$ and $\sigma(\cdot)$ are the mean and standard deviation of the Normal variables, $\theta_i^{K_m}$, $\theta_2^{K_m}$ are the lower and upper bounds of the Uniform variables (85% and 98% of the standard production capacity, respectively). Due to the sensitivity of the contract issues on customer-manufacturer-supplier relationships, and a confidentiality agreement, the detailed data is not presented.

The SAA model generator was written using Optimization Programming Language OPL 6.3, with a Microsoft Access database connection to automatically read data inputs and write solution outputs. The MIPs were solved using CPLEX 11.2. The program was run on a Intel Core 2 Duo workstation with CPU 2.00GHz, 4.00GB of RAM, and Windows Vista Home Edition Version 2007.

6.2 Scenario Generation

Plausible scenarios are generated using the following Monte Carlo procedure, which is based on the stochastic processes defined in Section 3. In the procedure, u denotes a uniformly distributed pseudo random number in $[0,1]$. The procedure starts by selecting an economic trend. It then sequentially generates demands and prices for the customers, capacities and prices for the spot raw material suppliers, and manufacturing capacities. In order to obtain a sample of N scenarios, one simply runs the procedure N times.

Scenario ω Generation Procedure:

Step 1. Select an economic trend e randomly using $P(e)$, $e \in \Xi$

Step 2. For all customers $c \in C$, do:

Generate customer-contract choices

$$\xi_k^c(\omega) = \begin{cases} 1 & \text{if } u \in [0, P_e^c(k)] \\ 0 & \text{otherwise} \end{cases}, \quad \forall k \in K^C$$

Generate customer requirements and market reference prices

$$d_{it}^c(\omega) = \lambda_{et} F_{d_{it}^c}^{-1}(u), \quad p_{it}^{c,ref}(\omega) = \lambda_{et} F_{p_{it}^{c,ref}}^{-1}(u), \quad \forall i, t \in T$$

Derive contract and spot demands from customer requirements

$$d_{kit}^c(\omega) = \begin{cases} \min(\max(lb_k, d_{it}^c(\omega)), ub_k), & \text{if } \xi_k^c(\omega) = 1 \\ 0, & \text{otherwise} \end{cases}, \quad \forall i, t \in T_k, k \in K^C$$

$$d_{it}^c(\omega) = \begin{cases} d_{it}^c(\omega) \\ 0 & \text{otherwise} \end{cases}, \quad \forall i \in I_{com}, t \in T$$

Derive contract and spot prices from market reference prices

$$p_{kit}^c(\omega) = \frac{\phi_k}{n} \sum_{t'=t-n}^{t-1} p_{it'}^{c,ref}(\omega), \quad \forall i, t \in T_k, k \in K^C$$

$$p_{it}^c(\omega) = p_{it}^{c,ref}(\omega), \forall i \in I_{com}, t \in T,$$

Step 3. For all spot suppliers $s \in NS$, and all periods $t \in T$, do:

Generate the spot supplier's raw material capacity and prices

$$KS_t^s(\omega) = \lambda_{et}^{KS} F_{KS_{ie}^s}^{-1}(u), \quad c_{jt}^s(\omega) = \lambda_{et} F_{c_{je}^s}^{-1}(u) \quad \forall j \in J$$

Step 4. For all mills $m \in M$, and all periods $t \in T$, do:

Generate the manufacturing capacity

$$K_{mt}(\omega) = F_{K_m}^{-1}(u)$$

7. Computational Results

In order to investigate the solvability of the SAA program (5.1), and the quality of the solutions obtained, experiments were initially carried out using 5 samples of scenarios ($B = 5$), each of size $N = 1, 5, 10, 15, 20$ and 25 . Table 3 shows that as the sample size N increases, the SAA program size and the computational times grow significantly. Figure 3 illustrates the time variance in solving the problem for the 5 different samples of the varying sample sizes. Obviously there is a trade-off between the problem size, computational efforts, and solution quality. To obtain good quality solutions while preserving the solvability of the model, we used $B = 10$ samples of $N = 25$ scenarios in our calculations, yielding 10 candidate solutions.

Table 3. Comparison of Model Complexity with Different Sample Size N .

Sample size (N)	Continuous variables	Binary variables	Constraints	Time (Sec)
1	6952	177	8570	3
5	40687	245	5338	18
10	84417	264	112189	55
15	140246	294	193378	758
20	204442	354	290038	377
25	270500	385	390118	2223

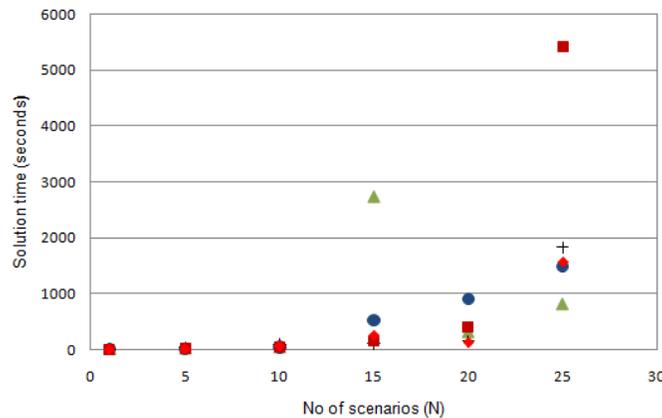


Figure 3. Computation Times Variation.

The statistical validation of the solutions is carried out by evaluating the objective function values with respect to each of the candidate solutions using $N_l = 100, 200, 300$ sampled scenarios. Table 4 presents the statistics for three candidate solutions, denoted $\hat{\mathbf{Z}}_1$, $\hat{\mathbf{Z}}_2$, and $\hat{\mathbf{Z}}_3$. Among the three candidate solutions, the performances are very similar. The objective function values increase and the optimality gaps and confidence intervals become very tight as the sample size N_l increases. This result indicates that the stochastic modeling method can produce robust solutions with good performances in various uncertain environments. The candidate solution $\hat{\mathbf{Z}}_1$ provides slightly superior results among the three solutions. Note that when $N_l = 300$, we observed $\tilde{f}_{N_l}(\hat{\mathbf{Z}}_1^N) > \bar{U}_{N,B}$ resulting in a negative gap. This negative gap is known to be caused by the separate sampling approach used to estimate the statistical upper and lower bounds, and the relatively smaller values of B and N_l . A similar phenomenon was observed by Mak (1999), where a common random number (CRN) sampling approach was proposed. In the CRN sampling approach, instead of developing a confidence interval of the optimality gap by estimating the upper- and lower- bounds separately using independent sample scenarios, the same set of sample scenarios is used. It was reported that using CRN sampling can eliminate the negative gap with improved confidence interval without significantly increasing the sample sizes.

In order to investigate the necessity of applying stochastic programming in solving contract design, allocation, and selection problems, the problem is also solved using MIP deterministic model. The performances of the contract solutions derived using the two models are then compared. In the deterministic model, the random variables such as demand, market price, raw material supplier price and capacity, as well as the manufacturing capacity, are replaced by mean values under a stable economic environment. The customer-contract choice parameters are generated randomly and independently. Ten replicates of customer-contract choice parameters are generated and the MIP model is solved for each replicate yielding ten candidate solutions. These solutions are also evaluated using $N_l = 100, 200, 300$ sampled scenarios. The performances with respect to the deterministic contract solutions are compared in Figure 4 with those obtained from the stochastic contract solutions. The contract solutions obtained from the stochastic programming model perform significantly better than those obtained from the deterministic model with a 12% performance improvement on average equivalent to \$ 7 million dollar increase in profit. The performances from the ten candidate solutions obtained using stochastic programming model are consistent with little variations, while those from the candidate solutions obtained using deterministic model vary considerably, ranging from \$53 to \$64 million dollars.

Table 4. Stochastic programming model results

		Stochastic								
Units: Million \$										
Point estimate: $\bar{U}_{N,B}$		67.100								
Stdev: $\hat{\sigma}_{\bar{U}_{N,B}}$		0.419								
Candidate solutions:		\hat{Z}_1			\hat{Z}_2			\hat{Z}_3		
Sample size: N_l		100	200	300	100	200	300	100	200	300
Point estimate: $\tilde{f}_{N_l}(\hat{Z}_b)$		66.164	67.062	67.371	66.148	67.051	67.368	66.171	67.043	67.352
Standard deviation: $\hat{\sigma}_{\tilde{f}_{N_l}(\hat{Z}_b)}$		1.075	0.708	0.581	1.090	0.718	0.587	1.070	0.704	0.579
Gap $_{N,B,N_l}(\hat{Z}_b)$		0.936	0.038	0.000	0.952	0.050	0.000	0.930	0.057	0.000
Standard deviation: $\hat{\sigma}_{Gap}$		1.494	1.126	1.000	1.509	1.137	1.006	1.489	1.123	0.997
Confidence interval (95%):		[0, 1.447]	[0, 0.436]	[0, 0.365]	[0, 1.467]	[0, 448]	[0, 0.366]	[0, 1.440]	[0, 0.454]	[0, 0.365]
CPU time (minutes):		34	37	41	31	34	38	29	32	36

Table 5. Deterministic model results

		Deterministic								
Units: Million \$										
Candidate solutions:		\hat{Z}_{MIP1}			\hat{Z}_{MIP2}			\hat{Z}_{MIP3}		
Objective function values: $OF_{\hat{Z}_{MIP}}$		68.965			68.685			70.696		
Sample size: N_l		100	200	300	100	200	300	100	200	300
Point estimate: $\tilde{f}_{N_l}(\hat{Z}_b)$		63.054	63.956	64.262	60.632	61.683	61.807	60.590	61.440	61.736
Standard deviation: $\hat{\sigma}_{\tilde{f}_{N_l}(\hat{Z}_b)}$		1.165	0.760	0.608	1.219	0.778	0.643	1.209	0.778	0.629
Gap $_{N,B,N_l}(\hat{Z}_b)$		5.911	5.009	4.703	8.053	7.002	6.879	10.106	9.256	8.960
Standard deviation: $\hat{\sigma}_{Gap}$		1.165	0.760	0.608	1.219	0.778	0.643	1.209	0.778	0.629
Confidence interval (95%):		[0, 6.141]	[0, 5.115]	[0, 4.772]	[0, 8.293]	[0, 7.110]	[0, 6.952]	[0, 10.345]	[0, 9.364]	[0, 9.032]
CPU time (minutes):		7	10	13	8	11	14	8	11	1

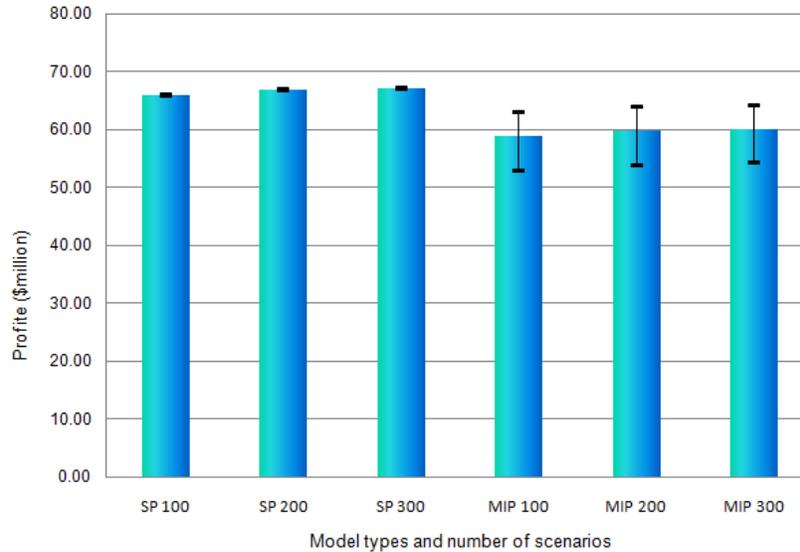


Figure 4. Comparison of objective function values from stochastic and deterministic contract solutions with $N_l = 100, 200, 300$ samples, respectively.

Table 5 present the statistics for three candidate solutions from the deterministic model, denoted $\hat{\mathbf{Z}}_{MIP1}$, $\hat{\mathbf{Z}}_{MIP2}$, and $\hat{\mathbf{Z}}_{MIP3}$. Note that in deterministic case, contract decisions that yield high objective function values do not necessarily perform well in uncertain business situations. As shown in Table 5, particularly candidate solution $\hat{\mathbf{Z}}_{MIP3}$, despite an objective function value higher than the upper bound $\bar{U}_{N,B}$ obtained from the stochastic programming model, has a low $\tilde{f}_{N_l}(\hat{\mathbf{Z}}_{MIP3})$ value and a large gap. Contract decisions are affected by many factors, such as, market price, customer demand and customer-contract choices, among other factors, which are rarely known with certainty. Yet, since in deterministic models, mean parameter values are used and a single customer choice scenario is considered, the decisions made adapt poorly for different plausible scenarios. Thus, decisions provided by deterministic models are less robust, and often inadequate. Stochastic programming is therefore a more appropriate modeling approach for contract decision problems, and the SAA solution approach can be practically applied.

Table 6 presents statistics on the contract decisions provided by the three stochastic and deterministic candidate solutions, respectively. It can be observed that the demand contract decisions vary in terms of contract forms, policies, allocations, and the number of contract customers. This indicates that the decisions are sensitive to the sample scenarios, particularly the customer-contract choices, customer demand, and market prices. This is particularly true for the contract decisions obtained from the deterministic model as shown by the larger variations observed. With the scenarios generated, not all 20 potential high volume customers have been offered a contract. The models have suggested reserving a proportion of the capacity to absorb the contract demand variation and/or serve the spot market. The manufacturer may choose an

alternative contract decisions based on particular contract relationship considerations with full awareness of the financial implications. For the supply contract decisions, the results are rather consistent. Six distinct supply contracts are selected from the 7 contract offers in most of the cases. This indicates that the contract decisions are relatively insensitive for the level of raw material market price and availability changes studied in this case.

Table 6. Candidate solutions.

Candidate solution	No. of contract forms	No of contract policies	Contract demand allocation	No of contract customers	No of supply contract
\hat{Z}_1	2	6	63%	16	6
\hat{Z}_2	2	7	60%	16	6
\hat{Z}_3	3	7	63%	15	6
\hat{Z}_{MIP1}	2	9	67%	17	7
\hat{Z}_{MIP2}	1	7	67%	18	6
\hat{Z}_{MIP3}	4	11	73%	17	6

8. Conclusions and Future Research Opportunities

In this article, we present a two-stage stochastic programming model for coordinated contract design, allocation, and selection decisions from a manufacturer's point of view, in a divergent three-tier manufacturing supply chain, under stochastic economic, market, supply, and system environments. In this capacitated make-to-order manufacturing system, the manufacturer wishes to offer different contracts to satisfy customers' needs, to accept the contract that optimize the resource capacity allocation, and to select the right contracts from the suppliers that guarantee the satisfaction of the contract and non-contract demand at lowest procurement cost. Four forms of contracts are evaluated for the demand contract design including the price-only, periodical minimum quantity commitment, periodical commitment with order band, and periodical stationary commitment contracts, each with different duration terms and price incentives. Stochastic customer-contract choices are incorporated in the scenarios generated in order to provide meaningful solutions for the demand contract decisions. The two-stage stochastic programming model with fixed recourse proposed is solved using the SAA approach. Feasible solutions are obtained in all cases. Computation analysis shows that by using stochastic programming model, more realistic and robust solutions can be obtained, with expected 12% superior financial performances, on average, to those obtained using a MIP deterministic model.

This research has been focused on two-stage stochastic programming to solve a contract decision problem in which all contract decisions are made at the beginning of the planning horizon. In real industrial environments, customer may choose a short term contract, for example, a three month contract, and leave the decisions on future contracts to a later date. A multi-stage stochastic programming approach could thus be investigated to address multiple contract decision points during the planning horizon. Note however that, given the additional complexity introduced by a multi-stage stochastic programming approach, using our model on a rolling horizon basis provides a practical way to reach contract decisions that are near optimal. A comparison of these two approaches would certainly be interesting, despite the fact that the multi-stage models would be much more difficult to solve.

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References

- [1] Anupindi, R., 1993, Supply management under uncertainty. PhD thesis, Graduate School of Industrial Administration, Carnegie Mellon University, Pittsburgh, PA 15213. In: Tayur, S., Ganeshan, R., Magazine, M. (Eds.) Quantitative Models for Supply Chain Management. Kluwer Academic Publishers, Boston, pp. 199-232.
- [2] Anupindi, R., Akella, R., 1997, An inventory model with commitments. Working paper. IN: Tayur, S., Ganeshan, R., Magazine, M. 1999. (Eds.) Quantitative Models for Supply Chain Management. Kluwer Academic Publishers, Boston, ISBN 0792383443, 9780792383444. pp. 199-232
- [3] Anupindi, R., Bassok, Y., 1999, Supply contracts with quantity commitments and stochastic demand. In: Tayur, S., Ganeshan, R., Magazine, M. (Eds.) Quantitative Models for Supply Chain Management. Kluwer Academic Publishers, Boston, pp. 199-232
- [4] Bassok, Y., Anupindi, R., 1997, Analysis of supply contracts with total minimum commitment. IIE Transactions, 29, 373–381.
- [5] Ben-Akiva, M., Lerman, S., 1994. Discrete Choice Analysis: Theory and Application to Travel Demand. MIT, Cambridge, Massachusetts, USA.
- [6] Cachon, G.P., 2003. Supply chain coordination with contracts. In: Graves, S.C., de Kok, A.G. (Eds.), Handbooks in Operations Research and Management Science: Supply Chain Management: Design, Coordination and Operation, Vol. 11. North-Holland, Amsterdam, pp. 227-339.
- [7] Corbett, C.J., Tang, C.S., 1999, Designing supply contracts: Contract type and information asymmetry. In: Tayur, S., Ganeshan, R., Magazine, M. (Eds.) Quantitative Models for Supply Chain Management. Kluwer Academic Publishers, Boston, pp. 271-297.
- [8] D'Amours S., Montreuil, B., Moke, J., Desaulniers, G., 2000. Contract-based tactical planning of supply chain, INFORMS Conference, San Antonio, TX, U.S.A.
- [9] Easton, F.F., Moodie, D.R., 1999. Pricing and lead time decisions for make-to-order firms with contingent orders. European Journal of Operational Research, 116, 305-318.
- [10] Feng, Y., D'Amours, S., and Beauregard, R., 2008a. The value of sales and operations planning in oriented strand board industry with make-to-order manufacturing system: cross functional integration under deterministic demand and spot market recourse. International Journal of Production Economics, 115 (1), 189–209.
- [11] Kumar, A., 1992, Supply contracts and manufacturing decisions. PhD thesis, Graduate School of Industrial Administration, Carnegie Mellon University, Pittsburgh, PA 15213. In: Tayur, S., Ganeshan, R., Magazine, M. (Eds.) Quantitative Models for Supply Chain Management. Kluwer Academic Publishers, Boston, pp. 199-232.
- [12] Lariviere, M.A., 1999, Supply chain contracting and coordination with stochastic demand. In: Tayur, S., Ganeshan, R., Magazine, M. (Eds.) Quantitative Models for Supply Chain Management. Kluwer Academic Publishers, Boston, pp. 235-268.
- [13] Lariviere, M.A., Porteus, E.L., 2001, Selling to the newsvendor: An analysis of price-only contracts. Manufacturing and Service Operations Management, 3(4), 293-305.
- [14] Lian, Z., Deshmukh, A., 2009, Analysis of supply contracts with quantity flexibility. European Journal of Operational Research, 196, 526-533.

- [15] Louviere, J., D. Hensher and J. Swait, 2000, *Stated Choice Methods*, Cambridge University Press.
- [16] Mak, W.K., Morton, D.P., Wood, R.K., 1999. Monte Carlo bounding techniques for determining solution quality in stochastic programs. *Operations Research Letters*, 24, 47-56.
- [17] Monahan, J.P., 1984, A quantity discount pricing model to increase vendor profits. *Management Science*, 30(6), 720-726.
- [18] Moizadeh, K., Nahmias, S., 2000, Adjustment strategies for a fixed delivery contract. *Operations Research*, 48(3), 408-423.
- [19] Munson, C.L., Rosenblatt, M.J., 2001. Coordinating a three-level supply chain with quantity discounts. *IIE Transactions* 33 (5), 371–384.
- [20] Santoso, T., Ahmed, S., Goetschalckx, M, Shapiro, A., 2005, A stochastic programming approach for supplychain network design under uncertainty. *European Journal of Operational Research*, 167, 96-115.
- [21] Scheller-Wolf, A., Tayur, S., 1998, A Markovian Dual-source Production Inventory Model with Order Bands. Working Paper, #1998-E200, Graduate School of Industrial Administration, Carnegie Mellon University, Pittsburgh, PA 15213. In: Tayur, S., Ganeshan, R., Magazine, M. (Eds.) *Quantitative Models for Supply Chain Management*. Kluwer Academic Publishers, Boston, pp. 199-232.
- [22] Schneeweiss, C., Zimmer, K., Zimmermann, M., 2004. The design of contracts to coordinate operational interdependencies within the supply chain. *International Journal of Production Economics*, 92, 43-59.
- [23] Shapiro, A., 2003. Monte Carlo sampling methods. IN: Ruszczyński, A., Shapiro, A., (eds), *Stochastic Programming, Handbooks in Operations Research and Management Sciences*, 10, Elsevier.
- [24] Tsay, A.A., 1999, The quantity flexibility contract and supplier-customer incentives. *Management Science*, 45, 1339–1358.
- [25] Tsay, A.A., Lovejoy, W.S., 1999 Quantity flexibility contracts and supply chain performance. *Manufacturing and Service Operations Management*, 1(2), 89–111.
- [26] Tsay, A.A., Nahmias, S., Agrawal, N., 1999, Modeling supply chain contracts: A review. In: Tayur, S., Ganeshan, R., Magazine, M. (Eds.) *Quantitative Models for Supply Chain Management*. Kluwer Academic Publishers, Boston, pp. 300-336.
- [27] van Delft, Ch., Vial, J.-Ph. (2001). Quantitative analysis of multi-periodic supply chain contracts with options via stochastic programming. Working Paper, Department of management Studies, University of Geneva, 40 Bd du Pont d’Arve, CH-1211 Geneva 4, Switzerland.
- [28] Vila, D., Martel, A., Beauregard, R., 2007. Taking market forces into account in the design of production-distribution networks: A positioning by anticipation approach. *Journal of Industrial and Management Optimization*, 3(1), 29-50.
- [29] Watanapa, B., Techanitisawad, Anulark, 2004. Simultaneous price and due date settings for multiple customer classes. *European Journal of Operational Research*, doi:10.1016/j.ejor. 2004.02.011
- [30] Weng, Z.K., 1995, Channel coordination and quantity discounts. *Management Science*, 41(9), 1509-1522.

- [31] Xu, N., Nozick, L., 2008, Modeling supplier selection and the use of options contracts for global supply chain design. *Computers & Operations Research*, doi:10.1016/j.cor.2008.12.013
- [32] Zou, X., Pokharel, S., Piplani, R., 2008, A two-period supply contract model for a decentralized assembly system. *European Journal of Operational Research*, 187, 257-274.