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Detecting, Non-Transitive, Inconsistent Responses in Discrete Choice Experiments

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Abstract. Surveys focusing on choice behaviour, and in particular, Discrete Choice Experiments (DCEs) are widely used in studies across many disciplines, including transportation, marketing research, health economics, labour economics and environmental studies. Investigation of the ‘rationality’ of responses in choice experiments has received a fair bit of attention from researchers. Most of this research has focused on the identification of irrational behaviour as it relates to non-satiation or lexicographic behaviour. At the same time, irrational behaviour indicated by non-transitive choices (often referred to as inconsistent behaviour) has received less attention by researchers. Until now the identification of non-transitive inconsistent behaviour has concentrated on relatively simple choice experiments. This research aims to extend previous work by developing a method to identify non-transitive inconsistent behaviour in more complex experiments. In particular, a systematic test procedure to detect inconsistent behaviour is developed and applied to three DCEs. The consistency test is implying that each respondent has a given preference structure and that her/his choices should be consistent with this structure across their choices, and therefore satisfy the axiom of transitivity. As such, choices that are not consistent with an individual’s observed preference structure are identified as inconsistent with his/her own choices. Our analysis shows that inconsistent choices commonly occur in DCEs with multiple tasks and attributes. Moreover, more inconsistent behaviour is detected in more complex experiments. Also, such behaviour has a significant impact on the valuation of respondent sensitivity to attributes in models estimated from DCE data. Finally, excluding inconsistent responses results in significant improvements in models fit.

Keywords. Discrete Choice Experiments (DCE), rationality, transitivity, inconsistent behaviour, dominance-based approach.

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1. INTRODUCTION

Choice behaviour modelling aims to understand people's behaviour by statistically analysing their choices. Random utility theory is the most commonly used basis on which to model and predict individual choice behaviour. Choice data is typically collected using either Revealed Preference (RP) or Stated Preference (SP) approaches. SP approaches, and Discrete Choice Experiments (DCEs) more specifically, use specialized surveys where respondents are asked to choose between hypothetical alternatives in a series of different scenarios, or choice tasks. An important methodological issue in the use of these surveys is whether preferences elicited via these experiments are consistent with the axioms of preference-based consumer theory. Recently, a fair bit of research has been published focusing on the issue of testing for the violation of normative axioms that hypothesize how 'rational' individuals 'should' make choices (Lancsar & Louviere, 2006). The completeness axiom stipulates that each decision maker has "a well-defined preference between any two possible alternatives" A and B (Lancsar & Louviere, 2006), that is either A>B or A<B. This axiom can be tested by repeated choice sets in experiments (Ryan & San Miguel, 2003). The axiom of transitivity states that for alternatives A, B and C in a choice set, if A>B and B>C, then A>C (Rulleau & Dachary-Bernard, 2012; McIntosh & Ryan, 2002). Choices that violate the transitivity axiom, and are therefore not consistent with previous or subsequent choices are considered as inconsistent choices (Sælensminde, 2002). Research in the transportation literature to have considered the issue of inconsistency in choice behaviour and its effects on models estimated with such data include Sælensminde (2001), Sælensminde (2002), Hess et al. (2010), Rose et al. (2013), and Rezaei & Patterson (2015). The axiom of monotonicity explains that, more is preferred to less and it "implies that the utility function is increasing" (Lancsar & Louviere, 2006). A test for violation of this axiom could be including a dominant alternative in one choice set (Burge & Rohr, 2004). The continuity axiom states that respondents are assumed to consider all attributes of the options in a choice set and to choose the option they prefer

(Rulleau & Dachary-Bernard, 2012). However, some respondents may rank the attributes and make choices based on the attribute with which they associate the highest priority, which is referred as lexicographic behaviour (Rulleau & Dachary-Bernard, 2012). Some exploration of lexicographic behaviour in DCEs has been done by Hess et al. (2010) and by Rose et al. (2013). Lexicographic behaviour is non-compensatory, so that respondents do not consider all attributes but rather adopt an attribute processing strategy to ease their decision-making, such as always choosing the cheapest alternative (Campbell & Lorimer, 2009). Finally, non-trading choice behaviour occurs, especially in the case of labelled choice experiments, when a respondent always chooses the same alternative across choice sets (Hess, et al., 2010).

Despite interest in detecting and analysing the source of irrationality in responses, some researchers have argued against the removal of the irrational choices from SP data. Lancsar and Louviere (Lancsar & Louviere, 2006) believe that deleting responses/individuals that seem to be irrational may result in the removal of valid preferences; impose sample selection bias; and reduce the statistical efficiency and power of the estimated choice models. They state that several factors may make rational behaviour appear irrational, such as shortcomings in the design and implementation of choice experiment. In this case choice may be influenced by attributes that are not included in a choice experiment (Viney, et al., 2002). For example, if quality is not explicitly included in the experiment, respondents could infer that a higher quality is associated with a higher price. Also, a number of alternative approaches to consumer theory have been proposed to account for violations of the standard preference-based axioms (Chorus & Bierlaire, 2013). That is, what may appear irrational using the standard preference-based approach may equally be explained as rational using an alternative approach to consumer theory (Lancsar & Louviere, 2006). The contextual concavity, and random regret models explicitly allow for particular types of reference dependencies and choice set composition effects that are considered irrational under the classical utility-based model. For example, take the compromise effect; this effect can be judged as irrational under the classical model paradigm, but has been found to be robust in other choice contexts (Chorus & Bierlaire, 2013).

Research considering the issue of rationality in DCE responses, have mostly focused on detecting non-trading and, lexicographic behaviour. However, these behaviours don't contradict rationality nor do they cause problems in representing preferences in terms of a utility function (Lancsar & Louviere, 2006). Also, evidence suggests random utility theory (RUT) can cope with such preferences (Lancsar & Louviere, 2006). So, it is suggested that if rationality is of interest and if one intends to employ a preference-based view of consumer theory, then research might be better directed towards the axioms of transitivity and completeness, rather than focusing on non-satiation (dominance) and lexicographic preferences (Lancsar & Louviere, 2006).

The aim of this paper is not to detect and remove responses deemed to be irrational. Instead, it is to propose a systematic approach to test the transitivity of respondent choices as an axiom of rationality and check the effects that the removal of inconsistent responses may have on model estimation results. As noted by (Samuelson, 1938), transitivity is at the centre of the theory of choice and has the greatest empirical content of those axioms responsible for the existence of preferences.. However, very few studies have actually tested for consistency in this way (McIntosh & Ryan, 2002; Lancsar & Louviere, 2006). Previous literature investigating inconsistencies in responses can be split between parametric, and simple inspection approaches for detecting inconsistent behaviour across respondent choices. In parametric approaches, one might allow for different error variances within a single model, such as using the scaling approach (Ben-Akiva & Lerman, 1985; Rose & Black, 2006); or estimate a panel model with respondent-specific scale parameters for the latent random utility distribution, in which each respondent is treated as his/her own individual data set with its own scale factor; or use each respondent's multiple observations to estimate a separate model (Johnson & Desvousges, 1997). Another parametric approach is to include decision strategy selection as an explicit factor in the choice model (Swait & Adamowicz, 1997). In the simple inspection approach, one detects the occurrence of choices that violate the axiom of transitivity resulting in inconsistency across individual respondent choices (Hess, et al., 2010). According to this approach, in the case of only two attributes, if respondents are observed in one choice task to choose an alternative with a substitution ratio benefit (between two attributes) relative to all other alternatives of a given value (e.g. X), but then later reject an alternative with a substitution ratio benefit relative to all other alternatives of a value greater than X in a subsequent choice task (Hess, et al., 2010; Rose, et al., 2013), they are identified as having behaved inconsistently. This consistency test is based on the assumption that each individual respondent has a given preference structure and that her/his choices should be consistent with this structure across her/his own choices, and therefore satisfy the axiom of transitivity (Sælensminde, 2002). While detecting inconsistent choice behaviour can be easily performed in simpler experiments (e.g. experiments using only two attributes such as time and cost) difficulties arise in experiments with more attributes (Hess, et al., 2010).The test proposed in this paper uses a systematic decision rule model that focuses on the axiom of transitivity, which is considered a necessary condition for a preference-based view of consumer theory (Ben-Akiva & Lerman, 1985; Lancsar & Louviere, 2006). Transitivity implies, for example in the case of a binary choice, that if an alternative 'A' is selected in one choice, that same option should, transitively, be chosen in any other choice where it is better in at least one attribute and no worse on the others (McIntosh & Ryan, 2002). Adopting an approach developed by Greco et al. (2001), we find a subset of attributes and their associated thresholds so that if a respondent is faced with a choice task where a given difference in attribute values across alternatives is exceeded, the respondent *should* (according to their preference structure as suggested from other choices) choose the task. The central idea of this approach is the representation (approximation) of upward and downward unions of decisions, by "granules of knowledge" generated by attributes. These granules (or condition profiles) are dominance cones in attribute value space. Each condition profile defines a dominance cone in *n*-dimensional (*n* being the number of attributes) condition space $\mathbb{R}^n$, and each decision profile defines a dominance cone in one-dimensional decision space {select, reject}. In general, this consistency test is based on the assumption that each respondent has a given preference structure and that her/his choices should be consistent with this structure across their own choices, and therefore satisfy the axiom of transitivity (Sælensminde, 2002).

The paper starts with a simple (two alternatives, two attributes) example to explain what inconsistency across individual choices means. Also, an approach to transform a stated choice data set so that inconsistencies can be easily detected is presented. In section three, the dominance approach to find dominance cones and condition profiles, and how they are used to develop individual decision rules, is explained. In section four, we outline three case studies and discuss the results derived from detecting and excluding inconsistent responses in choice model estimation. Finally, section five provides concluding remarks and suggestions on how this might be able to influence future research in the design and analysis of SP studies.

2. MODELLING INCONSISTENT BEHAVIOUR

In this paper we try to identify inconsistent choice behaviour across choice tasks. Inconsistencies are considered to occur when a violation of the axiom of transitivity in the dominance principle is observed. As an example, we may think of a binary choice situation (i.e. two alternatives) with two attributes, *att.1*, *att.2*, each of which includes three levels, L_1, L_2 and L_3 (where $L_1 < L_2 < L_3$). Suppose also that the utility of an alternative increases if the level of any of its attributes increases. The first five columns of Table 1 present an example of attribute levels and decisions made by a respondent in four different tasks. Each of these decisions seems to be rational (taken on its own), yet there might be inconsistency between the different decisions.

Like previous research, our inconsistency test is based on the differences between the attribute levels of alternatives (Sælensminde, 2002; Hess, et al., 2010; Rose, et al., 2013). In the example below, there are five possible attribute level differences for each attribute: two levels better (*2*; e.g. L_3-L_1), one level better (*1*; e.g. L_2-L_1), no difference (*0*; e.g. L_3-L_3), one level worse (-*1*; e.g. L_2-L_3) and two levels worse (-*2*; e.g. L_1-L_3). Also, the decision made can be classified in two different ways: the alternative is selected meaning that the alternative was considered better (*b*) compared to the other alternative; or it can be expressed as the rejection of the other alternative meaning that the alternative was considered worse (*w*). The differences between attributes levels are presented in the last three columns of Table 1. For each choice task, the first line presents the attribute levels of alternative one *minus* the attribute levels of alternative 2 (*Alt.1-Alt.2*). The second line presents (*Alt.2-Alt.1*). Evidently, the components of these two lines are symmetric.

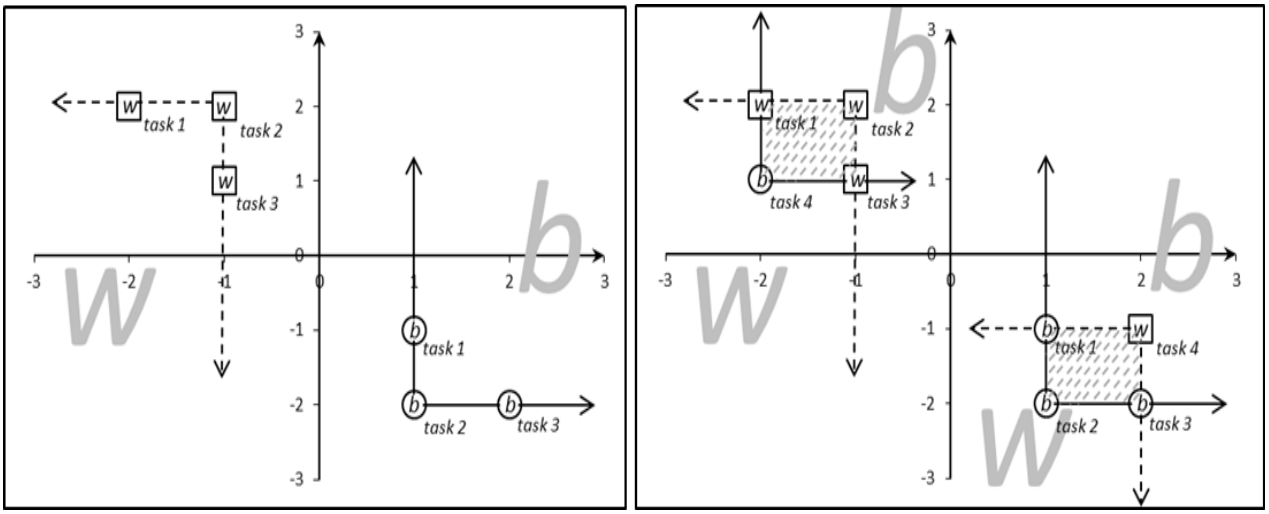
TABLE 1 An Example Of Attribute Levels And Decisions Made

		Alternative levels			Difference between attribute levels		
Task	Alt.	Att.1	Att.2	Decision	Att.1	Att.2	Decision
1	1	L_3	L_1	*Select*	*1*	*-1*	*b*
	2	L_2	L_2	*Reject*	*-1*	*1*	*w*
2	1	L_3	L_1	*Select*	*1*	*-2*	*b*
	2	L_2	L_3	*Reject*	*-1*	*2*	*w*
3	1	L_3	L_1	*Select*	*2*	*-2*	*b*
	2	L_1	L_3	*Reject*	*-2*	*2*	*w*
4	1	L_3	L_2	*Reject*	*2*	*-1*	*w*
	2	L_1	L_3	*Select*	*-2*	*1*	*b*

A close look at the “data” in Table 1, focusing on the first line of each task (i.e. Alt.1-Alt.2), shows the following: in all tasks Alt.1 dominates Alt.2 with respect to att.1; in all tasks Alt.2 dominates Alt.1 with respect to att.2. So, in all cases there is a trade-off between the two alternatives. In the first case, the respondent chose an alternative that is one level better with respect to att.1, but one level worse with respect to att.2. This implies that this respondent values att.1 more than att.2. This choice is not inconsistent with the choice made in task 2. There, the respondent chose the same alternative when att.2 is two levels worse. Indeed, task 2 provides analysts with more information than task 1. That is, the respondent values att.1 much more than att.2. The choice in task 3 is consistent with the preference structure implied by the first two choices tasks.

The situation until now, is represented in Figure 1A. This figure is symmetric since *Alt.1-Alt.2 = - (Alt.2-Alt.1)*. All points representing *Alt.1-Alt.2* fall in the lower right quadrant (4^{th} quadrant), and the points associated with *Alt.2-Alt.1* fall in the upper left (2^{nd}) quadrant. Because results are symmetric in the two quadrants, we concentrate the following explanation on the 4^{th} quadrant, recognizing the results are generalizable to the 2^{nd} as well. The 2D cone drawn by the two continuous lines emanating from the point represented by the choice made in task 2 shows an area inside which all points dominate task 2. Based on the decision made in task 2, *Alt.1* was the implied better alternative (*b*). In all tasks for which the point representing *Alt.1-Alt.2* falls inside the cone, the respondent *should* make the same decision, i.e. choose *Alt.1*.

As will be seen in the following section, the Dominance-based rough set approach, describes a respondent’s choice behaviour structure and finds cones (described by decision rules) in which all alternatives are dominant with respect to chosen alternatives (or dominated with respect to rejected alternatives) – like the cone referred to above. In task 4, however, the first alternative is two levels better and one level worse than the second one with respect to *att.1* and *att.2*, respectively. This is shown in Figure 1B. This task dominates all other tasks with respect to both attributes (i.e. it falls inside the cone). In the other word, in the task 4 *Alt.1* is more superior to *Alt.2*, compared to task 2, with respect to the both attributes. So, based on the axiom of transitivity, and given the knowledge inferred from the first three tasks, the respondent *should* choose *Alt.1*. However, s/he selected *Alt.2*, which is *inconsistent* with the other choices.



(A) **(B)**

FIGURE 1 - Dominance cones and inconsistent behaviour (A – consistent, B – inconsistent)

What has been described above, is another way of representing what others have done in the past when considering the question of inconsistent choice behaviour in stated preference experiments in the field of transportation (i.e. (Hess, et al., 2010; Rose, et al., 2013)). Using their terminology, one can infer from task 2 that the respondent is willing to choose an alternative with an *att.1-att.2* ratio benefit relative to the other alternative of 1/2. In task 4, however, the respondent rejects an alternative with *att.1-att.2* ratio benefit relative to the other alternative of 2/1. As such, they would also identify this respondent as having behaved inconsistently. It is worth noting that in the graphical representation above, for each choice, the ratio benefit value is equal to the negative of the inverse of the slope of the line connecting the origin of the coordinate system to the point associated with that choice.

The reason for representing choices as we have described above, is that it allows us to extend the approach to more complex experiments. In particular, and as will be explained in the following section, we can increase complexity along the following dimensions. First we can include more attributes, which can be treated by mapping the information in a higher level coordinate system (e.g. *m*D coordinate system with *m*D cones for an experiment with *m* attributes). Second, we can include more alternatives per choice task. In this case, imagine there are l alternatives alt_1, alt_2 … alt_l. As noted by Louviere, et al. (2000), any given choice tells us that the respondent prefers one alternative, alt_k, to the other $l-1$ alternatives. That is, $alt_k > alt_1, \dots, alt_k > alt_{k-1}, alt_k > alt_{k+1}, \dots, alt_k > alt_l$. In this case, each *mutual dominance relation* between alt_k and the other alternatives can be treated as a binary choice and mapped on the *m*D coordinate system. It is worth noting that this type of experiment (i.e. a standard stated choice experiment without ranking of alternatives) cannot provide the analyst with information on the mutual dominance relation between other $l-1$ alternatives (Louviere, et al., 2000).

Finally, and at the same time, the approach can be extended to experiments that *do* include the ranking of alternatives in a stated preference setting. In this case, any given response tells us that the respondent prefers the h^{th} alternative, $h=\{1, . . ., l\}$, in the preference ranking, to the $l-h$ alternative that are less preferred. This could be shown by $l-h$ binary choices, and therefore all *mutual dominance relations* between alternatives in a task can be treated as $\frac{(l)\times(l-1)}{2}$ binary choices and mapped on the *m*D coordinate system. It is worth noting that in this case, inconsistencies could occur even among the pieces of information drawn from the same response.

3. METHODOLOGY

This section explains the Dominance-based Rough Set Approach (Greco, et al., 2001), which we propose as a tool to systematically detect inconsistent behaviour in complex experiments.

3.1. Dominance Approach

Generally, Rough Set (RS) is a “mathematical framework that deals with vagueness and uncertainty in the fields of Artificial Intelligence (AI), knowledge discovery in databases and Data Mining (DM)” (Witlox & Tindesman, 2004). The goal of this approach is reasoning from imprecise data, or more specifically, discovering relationships in data. Witlox and Tindemans’ work {, 2004 #15} was the first to employ RS theory for travel choice pattern modelling. Classic RS theory considers attributes without preference ordering of attributes. DRSA is an extension of RS that explicitly takes into account the preference ordering of attributes (Greco, et al., 2001) which has allowed it to be applied in several fields such as the analysis of customer satisfaction (Greco, et al., 2007), Kansei engineering (customer psychological impressions or feelings about product) (Zhai, et al., 2009), and the prediction of airline passenger (Liou, 2009; Nassiri & Rezaei, 2012). The latest applications have benefited from an advanced version of DRSA called the Variable Consistency Dominance-based Rough Set Approach (VC-DRSA). This version allows some inconsistencies in the lower approximations of sets by defining a parameter called the “consistency level”. Its prediction model is in the form of decision rules (Liou, 2009). The basic concepts of DRSA are described as follows (Dembczyński, et al., 2009; Zhai, et al., 2009; Liou, 2009).

3.2. Dominance-Based Rough Set Approach

According to DRSA theory (Greco, et al., 2001), information regarding choice is represented in the form of an information table. The rows of the table refer to distinct objects (actions), while the columns refer to attributes that are considered. Each cell of the table indicates a quantitative or qualitative evaluation of the object attribute placed in the corresponding row and column, respectively.

Formally, an information table is the 4-tuple information system $IS = (U, Q, V, f)$, where U is a finite set of objects (universe), $Q=\{q1, q2, . . .,qm\}$ is a finite set of attributes, $V=\cup_{q\in Q}Vq$ in which Vq is the domain of attribute q, and $f\colon U \times Q \rightarrow V$ is a total function so that $f(x,q) \in Vq$ for each $q \in Q$, $x \in U$, called the information function. The set Q is, in general, divided into set C of condition attributes and set D of decision attributes.

3.2.1. Rough Approximation by Means of the Dominance Relationship

Let $\succeq_q$ be an outranking (also called weak preference) relation on U with reference to criterion $q \in Q$, so that $x \succeq_q y$ means that ''with respect to criterion q, x is at least as good as y''. Suppose that $\succeq_q$ is a complete pre-order, i.e., a strongly complete (which means that for each x, $y \in U$, at least one of $x \succeq_q y$ and $y \succeq_q x$ is verified, and hence with respect to criterion q, x and y are always comparable) and transitive binary relation. Moreover, let $\boldsymbol{Cl} = \{Cl_t,\ t \in T\}$, $T = \{1, \ldots, n\}$, be a set of classes of U, so that each $x \in U$ belongs to one and only one class $Cl_t \in \boldsymbol{Cl}$. We assume that all r, $s \in T$, so that $r > s$, each element of Cl_r is preferred to each element Cl_s. That is, if ≥ is a comprehensive outranking relation on U, then it is supposed that

$$(x \geq Cl_r,\ y \in Cl_s,\ r > s) \Rightarrow x > y, \qquad (1)$$

where $x>y$ means $x \geq y$ and not $y \geq x$.

In the example presented in the previous section $\boldsymbol{Cl}$ included two classes that is $\boldsymbol{Cl} = \{b, w\}$, so that $b > w$. Let's define unions of classes by a specific dominated or dominating class – these unions of classes are called upward and downward unions of classes, respectively. The upward union of classes is defined as:

$$Cl_t^{\geq} = \bigcup_{s \geq t} Cl_s, \qquad t = 1, \ldots, n.; \text{ and} \qquad (2)$$

The downward union of classes is defined as:

$$Cl_t^{\leq} = \bigcup_{s \leq t} Cl_s, \qquad t = 1, \ldots, n. \qquad (3)$$

The statement $x \in Cl_t^{\geq}$ means that "x belongs at least to class Cl_t", while $x \in Cl_t^{\leq}$ means that "x belongs at most to class Cl_t". To clarify, the union $Cl_t^{\geq}$ is the set of objects belonging to class Cl_t or a more desired class, whereas the union $Cl_t^{\leqq}$ is the set of objects belonging to class Cl_t or a less desired class. It should be noted that $Cl_1^{\geqq} = Cl_n^{\leqq} = U$, $Cl_n^{\geqq} = Cl_n$ and also $Cl_1^{\leqq} = Cl_1$. Consequently, for $t = 2,...,n$, we have $Cl_t^{\geq} = U - Cl_{t-1}^{\leq}$, that is, all the objects belonging to class Cl_t or more desirable belong to class U minus Cl_{t-1} or less desirable and similarly $Cl_{t-1}^{\leq} = U - Cl_t^{\geq}$.

In DRSA approaches, where among condition attributes there is at least one criterion, and decision classes are preference-ordered, the knowledge approximated is a collection of upward and downward unions of decision classes and the ‘‘granules of knowledge’’ are sets of objects being defined using a dominance relation instead of the indiscernible relation. This is the main difference between the classical RS approach and DRSA approaches.

It is said that object x P-dominates object y (or, x ***P**-dominates* y) with respect to $P \subseteq C$, denoted as $xD_P\, y$, if $x \succsim_q y$ for all $q \in P$, and $D_P = \bigcap_{q \in P} \succsim_q$, then the dominance relation D_P is a partial preorder. Given $P \subseteq C$ and $x \in U$, the “granules of knowledge” are:

$$D_p^+(x) = \{y \in U : yD_P x\}\,, \tag{4}$$

$$D_p^-(x) = \{y \in U : xD_P y\} \tag{5}$$

called the P-dominating set (a set of knowledge dominating x) and the P-dominated set (a set of knowledge dominated by x), respectively.

For any set of criteria $P \subseteq C$, we say that the inclusion of object $x \in U$ to the upward union of classes $Cl_t^{\geq}$, for $t = 2,...,n$, makes an *inconsistency* if one of the following conditions happens:

(1) x belongs to class Cl_t or better while being P-dominated by an object y belonging to a class worse than Cl_t, in other words, $x \in Cl_t^{\geq}$ but $D_p^+(x) \bigcap Cl_{t-1}^{\leq} \neq \varnothing$; or

(2) x belongs to a worse class than Cl_t while it P-dominates an object y belonging to class *Clt* or better, in other words, $x \notin Cl_t^{\geq}$ but $D_p^-(x) \bigcap Cl_t^{\geq} \neq \varnothing$.

In that case, it is said that x belongs to $Cl_t^{\geq}$ with *some ambiguity*. In contrast, if $x \in Cl_t^{\geq}$ and there is no inconsistency, it is said that x belongs to $Cl_t^{\geq}$ *without any ambiguity*. That is, all objects *P-dominating* x belong to $Cl_t^{\geq}$, namely, $D_p^+(x) \subseteq Cl_t^{\geq}$.

Then, for $P \subseteq C$, the set of all objects belonging to $Cl_t^{\geq}$ without any ambiguity forms the ***P****-lower approximation* of $Cl_t^{\geq}$, denoted by $\underline{P}(Cl_t^{\geq})$, and the set of all objects that have the possibility of belonging to $Cl_t^{\geq}$ constitutes the ***P****-upper approximation* of $Cl_t^{\geq}$, which is denoted by $\overline{P}(Cl_t^{\geq})$. These approximations are defined as follow:

$$\underline{P}(Cl_t^{\geq}) = \left\{ x \in U : D_p^+(x) \subseteq Cl_t^{\geq} \right\}, \tag{6}$$

$$\overline{P}(Cl_t^{\geq}) = \left\{ x \in U : D_p^-(x) \subseteq Cl_t^{\geq} \neq \varnothing \right\} = \bigcup_{x \in Cl_t^{\geq}} D_p^+(x), \qquad t = 1, \ldots, n. \tag{7}$$

Analogously, the ***P****-lower approximation* and ***P****-upper approximation* of $Cl_t^{\leq}$ can be defined as follows:

$$\underline{P}(Cl_t^{\leq}) = \left\{ x \in U : D_p^-(x) \subseteq Cl_t^{\leq} \right\}, \tag{8}$$

$$\overline{P}(Cl_t^{\leq}) = \left\{ x \in U : D_p^+(x) \subseteq Cl_t^{\leq} \neq \varnothing \right\} = \bigcup_{x \in Cl_t^{\leq}} D_p^-(x), \qquad t = 1, \ldots, n. \tag{9}$$

Also the ***P****-upper approximations* of $Cl_t^{\geq}$ and $Cl_t^{\leq}$, by complement of $\underline{P}(Cl_t^{\geq})$ and $\underline{P}(Cl_t^{\leq})$ with respect to *U* can be obtained as follows:

$$\overline{P}(Cl_t^{\geq}) = U - \underline{P}(Cl_{t-1}^{\leq}), \tag{10}$$

$$\overline{P}(Cl_t^{\leq}) = U - \underline{P}(Cl_{t+1}^{\geq}). \tag{11}$$

Therefore, the classification patterns to be discovered in the dominance-based rough sets are functions representing $Cl_t^{\geq}$ and $Cl_t^{\leq}$ by granules $D_p^+(x)$ and $D_p^-(x)$.

3.2.2. Decision Rules

The ultimate result of the DRSA is obtaining some simple ‘‘if . . ., then. . .” decision rules from the information contained in the data table. For a given upward union of classes $Cl_t^{\geq}$, the decision rules, inferred under a hypothesis that actions belonging to $\underline{P}(Cl_t^{\geq})$ are positive and all the others are negative, suggest an assignment to “at least class Cl_t”. Analogously, for a given downward union $Cl_t^{\leq}$, the rules inferred under a hypothesis that actions belonging to $\underline{P}^{l}(Cl_t^{\leq})$ are positive and that all others are negative suggest an assignment to “at most class Cl_s”. On the other hand, the decision rules inferred under a hypothesis that actions belonging to the intersection $\overline{P}(Cl_t^{\leq})\bigcap\overline{P}(Cl_t^{\geq})$ are positive and that all the others are negative suggest an assignment to some class between Cl_s and Cl_t (s<t). Each rule has three parts in the premise. The first one relates to dominance on a subset of criteria, the second to indiscernibility on a subset of qualitative attributes, and the last to similarity on a subset of quantitative attributes. The following three types of decision rules can be considered:

1. $D_{\geq}$-decision rules supported only by objects from ***P****-lower approximations* of the upward unions of classes $Cl_t^{\geq}$, that is $\underline{P}(Cl_t^{\geq})$. They have the following form: If $f(x, q_1) \geq r_{q1}$ and $f(x, q_2) \geq r_{q2}$ and . . . $f(x, q_p) \geq r_{qp}$, then $x \in Cl_t^{\geq}$. In our example (see Figure 1A), this is represented as the cone with solid lines.

2. $D_{\leq}$-decision rules supported only by objects from the ***P****-lower approximation* of the downward unions of classes $Cl_t^{\leq}$, that is $\underline{P}(Cl_t^{\leq})$. They have the following form: if $f(x, q_1) \leq r_{q1}$ and $f(x, q_2) \leq r_{q2}$ and . . . $f(x, q_p) \leq r_{qp}$, then $x \in Cl_t^{\leq}$. In our example (see Figure 1A), this is represented as the cone with dotted lines.

4. EMPIRICAL STUDIES

This section presents the findings from three empirical studies looking at the application of the proposed approach to detect inconsistent choices. In each case, the following methodology was used to identify inconsistent choices that a respondent may have made. First, we estimated a base basic multinomial logit with the entire dataset in Stata (StataCorp, 2011). Second, the Stated Choice data was transformed in the same way as described in the section “Modelling Inconsistent Behaviour”, to derive mutual dominance relations, so that the individual decision cones could be identified. Third, the transformed data were used as inputs to the process whereby the proposed approach produced decision cones for each individual. A code written in Visual Basic (available from the authors), using the proposed approach, was then used to produce decision rules for each individual. By examining the individual decision rules and individual respondent choices, it was possible to identify those choices that were inconsistent with an individual’s other choices, as well as to identify with which other choices the choice was consistent. In fact, it was possible to identify, for each choice, whether it was consistent or inconsistent with all of the other choices. As such, it was also possible to identify the degree to which a given choice was consistent or inconsistent by identifying with how many other choices it was inconsistent. So, for example, supposing a choice set with two alternatives and six choice tasks, it is possible to establish whether a given choice is consistent with all, all but one, all but two, etc. other choice tasks. As a result, a choice task inconsistent with three other choice tasks is considered more inconsistent than a task inconsistent with only one other task. We then removed responses using different thresholds of inconsistency (e.g. inconsistent with more that 50% of all responses) and then re-estimated the basic logit models and compared them with the base model.

4.1. Pedestrian Preferences With Respect to Roundabouts Data

The first empirical analysis makes use of data collected for a study of pedestrian preferences with respect to roundabouts (PPRR) carried out in Canada (Perdomo, et al., 2014). The study was based on an unlabelled, video-based stated preference survey. Each task showed two alternative roundabouts that were characterized by the following attributes: presence of signs (no sign, regular sign and flashing sign); number of lanes (one or two); presence of a pedestrian island (present or absent), presence of pedestrian crossing (no crossing, crossing at roundabout entrance, crossing five meters from entrance); traffic volume (100 and 500 vehicles per hour); and finally, traffic speed (average speed through roundabout of traffic of 22 and 65 km/h). Six choice tasks were presented to each respondent. The online survey was conducted during the first week of July, 2013. The sample available for estimation contains 3005 observations collected from 501 respondents.

4.1.1 Empirical Results

As can be seen in Table 2, the results from 5 different basic MNL models are presented. In each case, a simple linear-in-parameters specification of the MNL model was used. The first model is estimated using all the observations originally collected. The second is the model after having removed observations using the same data cleaning strategy as explained in Perdomo et al. (2014). For the rest of the models, inconsistent responses were removed using different thresholds. Considering model 1, all coefficient signs are intuitively reasonable. Also, they are significant at 10% confidence level except for the case of regular signs and traffic speed. Model 2 was estimated after removing 14% of respondents in the data cleaning process. All coefficients estimated have intuitively reasonable signs and are significant at the 5% confidence level, except the regular sign coefficient that is significant at 10% confidence level. Also, the ρ^2 of the model is 0.43, showing an improvement in the goodness of fit compared to Model 1. Model 3 was estimated after removing responses inconsistent with more than 2/6th (33.3%) of the respondent's other choices. This resulted in the removal of 1% of all responses. Model 4 is a model estimated after removing choices inconsistent with more than 1/6th (17.6%) of the respondent's other choices. That is, 2.2% of responses were removed. While the percentage of the data removed to estimate this model was almost one seventh of that in Perdomo et al. (2014) (i.e. 2.2% vs. 14% in the Perdomo et al. study), the goodness of fit of the new model is almost the same, even though the coefficient of "Regular sign" attribute is still insignificant. Finally, the threshold was set so that all responses inconsistent with any other of the respondent's choices were removed (6.4% of responses). This left only responses with entirely consistent choices. While the percentage of the responses removed to estimate this model is almost half of that in Perdomo et al. (2014), the majority of coefficients of the model using only the consistent choices are significant at higher confidence levels than the coefficients of Model 2. Moreover, the resulting model provides by far the best performance in terms of the ρ^2. Furthermore, a closer inspection of the models estimated shows differences in model coefficients. Model 2 and model 5 result in very similar coefficient estimates for the presence of a pedestrian island and pedestrian crossing at the entrance. However, excluding inconsistent responses results in lower values of pedestrian sensitivity to regular signs, traffic volume and traffic speed; and greater sensitivity to flashing signs, number of lanes and having a pedestrian crossing 5m from entrance.

4.1.2 Discussion

The different estimated models highlight the significant effect that inconsistent behaviour has on model estimates for the PPRR data. Further, removing inconsistent responses leads to universal gains in model fit. As such, the evidence would speak in favour of removing such responses from the data, given the potential effect on model estimates that their inclusion can produce.

TABLE 2 Estimation Result on PPRR Data

	Model 1 Base logit model All observations		Model 2 Perdomo et al. 2014 Removing 14.2%		Model 3 Removing responses with more than 2 inconsistent Responses (1% of data)		Model 4 Removing responses with more than 1 inconsistent Responses (2.2% of data)		Model 5 Removing responses with more than 0 inconsistent Responses (6.4% of data)	
Variables	Coeff.	*t*-stat	Coeff.	*t*-stat	Coeff.	*t*-stat	Coeff.	*t*-stat	Coeff.	*t*-stat
No sign	-	-	-	-	-	-	-	-	-	-
Regular signs	0.140	0.92	0.311	1.80	0.164	1.05	0.187	1.16	0.287	1.66
Flashing signs	0.521	3.41	0.706	4.07	0. 580	3.69	0.661	4.09	0.862	4.93
Number of lanes	-0.318	-5.34	-0.531	-7.65	-0.381	-6.17	-0.436	-6.84	-0.574	-8.19
Presence of Island	0.305	5.11	0.506	7.29	0.385	6.21	0.447	7.00	0.494	7.15
No pedestrian crossing	-	-	-	-	-	-	-	-	-	-
Crossing at entrance	1.843	10.73	2.009	10.18	1.968	11.04	1.985	10.84	2.039	10.28
Crossing 5m from entrance	2.571	14.72	2.845	14.17	2.757	15.16	2.844	15.19	3.080	15.16
Traffic volume	-0.070	-4.68	-0.117	-6.80	-0.088	-5.71	-0.102	-6.43	-0.131	-7.58
Traffic speed	-0.227	-1.65	-0.605	-3.79	-0.375	-2.64	-0.453	-3.10	-0.565	-3.57
Observations	3006		2580		2978		2941		2813	
ρ^2	0.3703		0.4329		0.4004		0.4210		0.4742	

4.2. Shipper Preferences Data

The second data set considered is from a stated choice survey of shippers with respect to carriers in the Quebec City – Windsor Corridor in Canada (Patterson, et al., 2007). The survey was administered online in the summer of 2005. There were 18 choice tasks, each with three unlabelled alternative carriers. The carriers were characterized by five attributes: cost of shipment (low - 10% below; medium and high - 10% above); on-time reliability (85%, 92%, and 98%); damage risk (0.5%, 1%, and 2%); security risk (0.5%, 1%, and 1.5%); and whether or not the shipment would be carried by truck only, or by truck and intermodal train. The sample available for estimation contains 7,074 observations collected from 393 respondents.

4.2.1. Empirical Results

A linear-in-parameter utility function is again used. Also, additional alternative specific constants for the first and second alternatives are included to capture order effects. A detailed summary of the models estimated using the shipper preference data is presented in Table 3. The first model was estimated using all observations, without implementing any data cleaning. All coefficients estimated are significant at the 1% confidence level and have intuitively reasonable signs. The other three models were estimated after removing the most inconsistent responses. As described in the section “Modelling Inconsistent Behaviour”, the methodology described here can be used with experiments that have choice tasks with more than two alternatives, which was the case with the shipper data. To do so, each choice was transformed into two binary choices. This allowed the detection of inconsistencies across derived mutual dominance relations. Consequently, in this case, each response can be inconsistent with up to 72 mutual dominance relations. To be comparable with what was done with the Roundabout data, responses inconsistent with 24/72nd (33.3%), 12/72nd (17.6%), 6/72nd (8.8%) of a respondent’s other mutual dominance relations were removed.

Investigation and comparison of the models reveals that removing more inconsistent respondents improves model performance in term of ρ^2, so that the ρ^2 of Model 4 is far better than that for Model 1. Also, some trends in coefficients value are observed. Both ASCs have become slightly lower when removing inconsistent respondents implying smaller order effects in more consistent respondent data. At the same time, however, there is a marked increase in the other coefficient values when moving across the models, probably due to decreases in the relative weight of the unobserved utility components, and consequently increases in the scale parameter value. This is consistent with the results obtained in other research (Hess, et al., 2010). However, the particularly large increase in the security risk coefficient compared to other coefficients shows that failing to exclude inconsistent respondents can result in under estimation this coefficient.

TABLE 3 Estimation Result on Shippers Data

Variables	Model 1 Base logit model All observations		Model 2 Removing responses with more than 24 inconsistent Responses (0.2% of data)		Model 3 Removing responses with more than 12 inconsistent Responses (1.1% of data)		Model 4 Removing responses with more than 6 inconsistent Responses (4.4% of data)	
	Coeff.	*t*-stat	Coeff.	*t*-stat	Coeff.	*t*-stat	Coeff.	*t*-stat
ASC1	0.475	13.53	0.472	13.39	0.468	13.10	0.466	12.48
ASC2	0.499	14.28	0.495	14.12	0.495	13.94	0.471	12.67
Cost level	-4.311	-24.79	-4.382	-25.07	-4.640	-26.02	-5.335	-28.07
On-time reliability	0.116	40.76	0.118	40.95	0.123	41.70	0.138	43.26
Damage risk	-0.526	-22.23	-0.538	-22.60	-0.573	-23.56	-0.672	-25.85
Security risk	-0.109	-3.21	-0.119	-3.53	-0.163	-4.72	-0.284	-7.58
Intermodal carrier	-0.726	-24.12	-0.739	-24.43	-0.780	-25.28	-0.890	-27.13
Observations	7074		7061		6999		6763	
ρ^2	0.2369		0.2410		0.2557		0.2976	

4.2.2. Discussion

The analysis on the Shipper preferences data has again highlighted the impact that inconsistent behaviour can have on model results. The most apparent change to model results relates to gains in model fit resulting from the removal of inconsistent responses. Finally, as with the PPRR data, the model results show the advantage of removing inconsistent responses from the data, given the potential effect on model estimates that their inclusion can produce.

4.3. Neighborhood Choice Project Data Set

The third analysis makes use of data collected for a neighbourhood location choice study in Montreal, Canada (Mostofi_Darbani, et al., 2014). Each task showed two alternative neighbourhoods that were characterized by the following attributes: Dwelling type (Apartment, Detached houses, Townhouses, and Triplexes); front yard depth (6 feet, 9 feet), space between buildings (no space, 20 feet); average home value (low - 20% below base price; medium and high - 20% base price); travel time to work by car (20, 35, 50 minutes); travel time to work by transit (5% below, 30% above travel time to work by car); and finally, travel time to nearby shops on foot (5, 15, 25 minutes). The surveys were administered at coffee shops in June 2013 and also in February 2014. The sample available for estimation contains 2,430 observations collected from 405 respondents.

4.3.1. Empirical Results

After running the inconsistency detection test only 47 responses (1.93%) were found to contradict mutual dominance relations. Table 4 presents the coefficients of the models estimated on the survey data. The models were estimated 1) using all the observations originally collected, 2) after removing responses that were inconsistent with more than 2/6th (33.3%) of a respondent's other choices, 3) after removing responses that were inconsistent with more than 1/6th (17.6%) a respondent's other choices and 4) after removing all inconsistent responses. Considering model 1, all coefficient signs are intuitively reasonable and significant at 10% confidence level. The significant alternative specific constant implies the existence of an order effect in responses. While the percentage of the data removed to estimate model 2 is very small, the goodness of fit, ρ^2, of the new model is slightly better than the first model. To estimate Model 3 only responses with, at most, one inconsistent choice within each individual's decision were used. All coefficients of this model, except the ASC, are significant at a higher confidence level compared to those of the base model. Model 4 is a model estimated after removing all responses with inconsistent dominance relations. The performance of the model in terms of ρ^2 is much better than the first model. The insignificant ASC coefficient shows that the order effect problem has been resolved. Also, in general, other coefficients are more significant (they are all significant at 1% confidence level) compared to the model estimated using all data. Like the previous case studies, there is a slight increase in the coefficient values when moving across the models, probably due to decreases in the scale parameter. But, a larger increase is observed in the case of the triplex and front yard depth coefficients that could be a result of underestimating these coefficients when including the inconsistent responses in the estimation.

4.3.2. Discussion

As was the case with the previous data sets, the models showed increases in model fit and increasing significance of coefficients after removing responses identified as being inconsistent with respondents' other choices, with respect to the dominance relations. In particular we found better model and more significant coefficients. We also found that coefficient values in general increase, and some coefficients change more than the rest.

TABLE 4 Estimation Result on Virtual Reality Data

Variables		**Model 1 Base logit model All observations**		**Model 2 responses with more than 2 inconsistent responses (0.08%)**		**Model 3 responses with more than 1 inconsistent responses (0.37%)**		**Model 4 responses with more than 0 inconsistent responses (1.93%)**	
		Coeff.	*t*-stat	Coeff.	*t*-stat	Coeff.	*t*-stat	Coeff.	*t*-stat
ASC		0. 092	1.99	0.092	1.99	0.087	1.88	0.78	1.65
Dwelling type	Apartment	-	-	-	-	-	-	-	-
	Detached houses	0. 770	7.76	0.780	7.85	0.790	7.91	0.844	8.28
	Townhouse	0.540	6.41	0.549	6.52	0.585	6.87	0.626	7.22
	Triplex	0.169	1.72	0.177	1.79	0.218	2.18	0.277	2.71
Average home value (thousands CDN)		-1.65e-3	-3.60	-1.70e-3	-3.69	-1.74e-3	-3.76	-1.87e-3	-3.99
Front yard depth (feet)		8.75e-3	2.94	9.07e-3	3.04	9.49e-3	3.16	0.011	3.47
Space between buildings (in feet)		0.025	6.15	0.026	6.18	0.028	6.58	0.029	6.77
Travel time to work by car (minutes)		-0.027	-4.89	-0.027	-4.91	-0.028	-4.98	-0.029	-5.13
Travel time to work by transit (minutes)		-0.013	-3.24	-0.013	-3.28	-0.014	-3.42	-0.014	-3.44
Travel time to nearby shops on foot (minutes)		-0.024	-6.52	-0. 025	-6.59	-0.026	-6.77	-0.028	-7.29
Observations		2430		2428		2421		2383	
ρ^2		0.1683		0.1704		0.1775		0.1897	

5. CONCLUSION

This paper proposed a systematic approach to test the axiom of transitivity in data derived from Discrete Choice Experiments, which is essential in consumer theory and yet little considered in the literature. An approach using dominance rules (Greco, et al., 2001) was proposed to detect inconsistent choices of respondents in the case of more complex experiments than those that have been investigated previously in the literature. This provides the opportunity to examine problematic choices systematically in the context of more complex experiments. The empirical analysis suggests that inconsistent choices are common in SP surveys with multiple tasks and attributes. Moreover, more inconsistent behaviour is detected in more complex experiments – e.g. the shipper dataset compared to the neighbourhood choice project dataset. The analysis also suggests that such choices have a significant impact on the valuation of respondent sensitivity to attributes in estimated models. Another important finding is that excluding inconsistent responses results in significant improvement in model fit. Together, the results suggest that removing inconsistent responses can result in better models.

Further investigation can use this approach to consider how the complexity of experiments influences the share of inconsistent choices, and possibly optimal complexity levels for these surveys. Similarly, the approach could be used to evaluate optimal numbers of tasks in these surveys.

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